AFRL-AFOSR-UK-TR-2018-0039

Finite Set Statistics on Manifolds for Space Object Detection, Tracking, Identification and Characterization

John Kent
THE UNIVERSITY OF LEEDS

08/20/2018
Final Report

DISTRIBUTION A: Distribution approved for public release.

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ IOE
Arlington, Virginia 22203
Air Force Materiel Command

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*
20-08-2018

2. REPORT TYPE
Final

3. DATES COVERED *(From - To)*
01 Dec 2015 to 30 Nov 2017

4. TITLE AND SUBTITLE
Finite Set Statistics on Manifolds for Space Object Detection, Tracking, Identification and Characterization

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-16-1-0099

5c. PROGRAM ELEMENT NUMBER
61102F

6. AUTHOR(S)
John Kent, Islam Hussein

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
THE UNIVERSITY OF LEEDS
LEEDS, WEST YORKSHIRE, LS2 9JT
WOODHOUSE LA, LS2 9JT GB

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
EOARD
Unit 4515
APO AE 09421-4515

10. SPONSOR/MONITOR'S ACRONYM(S)
AFRL/AFOSR IOE

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-AFOSR-UK-TR-2018-0039

12. DISTRIBUTION/AVAILABILITY STATEMENT
A DISTRIBUTION UNLIMITED: PB Public Release

13. SUPPLEMENTARY NOTES

14. ABSTRACT
New statistical methods have been developed to improve the treatment of uncertainty in space situational awareness. First a 'Fisher -Bingham-Kent (FBK)' distribution on the unit sphere has been developed to describe uncertainty in angles-only position for a propagated space object. This distribution has been applied to association problems. Second a new 'adapted structural (AST)' coordinate system has been developed to represent the uncertainty of a state vector. Uncertainty in AST coordinates is approximately Gaussian under a wide set of circumstances and this property facilitates the construction of an unscented Kalman lter for the filtering problem.

15. SUBJECT TERMS
EOARD, Finite Set Statistics, resident space objects, space tracking, FISST theory, space situational awareness, SSA

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	SAR		MILLER, KENT

19b. TELEPHONE NUMBER *(Include area code)*
011-44-1895-616022

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18

Final Report

US Air Force Grant: AFOSR FA9550-16-1-0099
Title: Finite Set Statistics on Manifolds for Space Object Detection, Tracking, Identification and Characterization
PI: John Kent, University of Leeds
Funded collaborator: Islam Hussein, Applied Defense Solutions
Duration: 2 years from 1 December 2015 (plus 6-month extension) until 31 May 2018
Amount granted: $299,995
Date of report: 17 August 2018

Contents

1 Summary

New statistical methods have been developed to improve the treatment of uncertainty in space situational awareness. First a "Fisher-Bingham-Kent (FBK)" distribution on the unit sphere has been developed to describe uncertainty in angles-only position for a propagated space object. This distribution has been applied to association problems. Second a new "adapted structural (AST)" coordinate system has been developed to represent the uncertainty of a state vector. Uncertainty in AST coordinates is approximately Gaussian under a wide set of circumstances and this property facilitates the construction of an unscented Kalman filter for the filtering problem.

2 Introduction

The project fits within the theme of finite set statistics (FISST) for space situational awareness (SSA). More specifically, the objective of the grant was to adapt ideas from directional statistics to improve the methods and algorithms for tracking and identification in SSA. The focus of the work is on improved tracking of space debris. That is the resident space object (RSO) is regarded as non-cooperative and only occasional measurements are taken. Hence the uncertainties in the state can be nontrivial.

The key consideration in our work is a suitable choice of coordinate system. None of the standard coordinate systems is entirely satisfactory and many of them have severe limitations. Such limitations may not be important for certain states with small enough errors and short enough propagation times, but none of the standard coordinate systems works well for all initial states. Our approach is to represent uncertainty in terms of a notional point cloud about a "central" initial state. Then local coordinates are used to parameterize the uncertainty about this initial state. With an appropriate choice, these local coordinate will be approximately Gaussian whatever the choice of central initial state. This approach to representing uncertainty is closely related to the use of first-order Taylor expansions for certain representations of orbital elements.

The second step is to use these local coordinates to represent the propagated state, again in an approximately Gaussian form.

If the coordinate system is ill-chosen, then initial Gaussian perturbations give rise to extremely non-Gaussian distributions at a future time. However, with a good choice of coordinate system, approximate Gaussianity holds for much longer propagation times.

So far the grant has been a great success with our ideas receiving considerable interest at space engineering conferences. The two major contributions are a new "Fisher-Bingham Kent" distribution on the sphere and a new "AST" set of coordinates for the state of an orbiting object in order to facilitate filtering.

3 Methods, Assumptions and Procedures

We have developed new statistical distributions and new coordinate systems to give improved methods in SSA to tackle problems such as association and tracking. For the most part Keplerian dynamics has been assumed, partly for simplicity, and partly so that underlying patterns can be seen more clearly. The methods have been assessed through computer simulation and mathematical expansions. More detail is in the next section.

4 Results and Discussion

(a) One main contribution is a new directional distribution (which we now call the "Fisher-Bingham-Kent (FBK)" distribution) on the unit sphere, which is well-suited to the description of uncertainty in the propagated angular position of an RSO in many circumstances. This distribution is important for the data association problem and for characterizing maneuvers. In addition it forms the building block needed to carry out mixture modelling on the sphere analogous to Gaussian mixture modelling.

(b) The FBK distribution is widely applicable in space applications to describe propagated angular uncertainty. However, there is an important exception under a break-up scenario, where a single object splits (e.g. explodes) into many pieces. If the initial state is well-determined, then for the break-up pieces, the initial position is very well-determined, but the initial velocity has appreciable uncertainty. In this setting the cloud of orbital debris passes very close to the initial location (and its antipode) at multiples of the half period for each object in the cloud. This feature gives rise to "pinching effects" in plots of the angles-only positions at these times. The underlying mathematical reason is now understood and the work is currently being written up.

(c) Another main contribution is a new "adapted structural (AST)" coordinate system to represent the uncertainty in a state vector. It is a "local" coordinate system: if we imagine uncertainty as a point cloud of possible states and choose a "central" point near the middle of the point cloud, then AST coordinates represent states of the point cloud in terms of their differences from the central state. Of course AST coordinates encode the same information as the classic Keplerian elements, but they have a distribution which is approximately Gaussian under a much wider set of circumstances.

(d) The main use of AST coordinates is in the development of a tractable and accurate algorithm for the filtering problem for orbital trajectories. That is, given a sequence observations of an orbiting object (so far we have focused on angles-only observations so that the observer can see the direction in the sky of the object, but not how far away it is), the objective is to predict its orbital path. Unfortunately, the classic Kalman filter in earth-centered inertial (ECI) coordinates is not well-suited to this setting because as one propagates uncertainty further into the future, the uncertainty becomes more spread out along the ellipse of the most likely orbital path. Such a spread is sometimes known as a "banana shape" in Euclidean coordinates, and violates the multivariate Gaussian assumption. Other problems can occur with other conventional coordinate systems such as ECI, Keplerian or equinoctial.

    The state-of-the-art methods for the filtering problem are based on particle filters and Gaussian mixtures. However, such methods are both time-consuming and somewhat cumbersome. The approximate Gaussianity of AST coordinates facilitates the construction of an analytically tractable filter close in character to the classic the Kalman filter. This new AST filter has been implemented and indications so far indicate that it is a substantial improvement over existing filters.

(e) The methods we have developed lend themselves to problems of ambiguity. The simplest case of ambiguity is the association problem where a current measurement is compatible with two or more objects in a catalog. A version of discriminant analysis (or classification) based on the FBK distribution has been developed to measure the compatability of the measurement to each catalog object. Also, a mixture of Gaussians (one component for each object in the catalog) can be incorporated into a UKF, with only a few observations typically needed to resolve the ambiguity.

    Another type of ambiguity occurs when the propagated uncertainty is so large in the in-track direction that it winds at least once around the whole orbital path. A Gaussian mixture model (with one component for each plausible choice of winding number) can be used to represent the uncertainty, and as in the association problem, the ambiguity is typically resolved after a few observations.

(f) For the most part we have worked in Keplerian dynamics, though the broad conclusions carry over largely unchanged if non-Keplerian effects such as non-sphericity of the earth, drag, solar radiation, etc. are incorporated. Some numerical work has been carried out to support this conclusion, but a more systematic study is needed.

(g) The use of the FBK distribution for data association has been extended to maneuver problems, at least in principle. Some initial demonstrations have been carried out, but there is still a need to assess performance in realistic situations.

We have written up our work in 13 conference papers [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13], and four more papers are in preparation for conferences this autumn [14, 15, 16, 17]. Two journal papers consolidating and extending this work are nearly ready to submit. In addition the conference paper [15] may be selected for journal publication.

5 Expenditure and Conferences

The main expenditure is for my time and the support for my PhD student Shambo Bhattacharjee at the University of Leeds, for the time of Islam Hussein and Weston Faber at ADS, and for conference attendance. We have so far presented and discussed our work at the following conferences and other meetings in Statistics and Space Engineering.

1. 19th International Conference on Information Fusion, Heidelberg, Germany, 5-8 July 2016.
2. 27th AAS/AIAA Space Flight Mechanics Meeting, San Antonio, Texas. 5-9 February 2017.
3. 7th European Conference on Space Debris, ESA/ESOC, Darmstadt, Germany, 18-21 April 2017.
4. AAS/AIAA Astrodynamics Specialist Conference, Stevenson, WA, 20-24 August 2017.
5. 68th IAC Adelaide, Australia, 25-29 Sept 2017.
6. 18th AMOS Conference, 19-22 Sept 2017.
7. First IAA Conference on Space Situational Awareness (ICSSA) Orlando, FL, 13-15 November 2017.
8. 28th AIAA/AAS Space Flight Mechanics Meeting, Kissimmee, FL, 8-12 January 2018.
9. Workshop on Statistics for Data with Geometric Structure, Oberwolfach, Germany, 21-27 Jan 2018.
10. Junkins Dynamical Systems Symposium and AFOSR Astrodynamics Planning Workshop, College Park, TX, 20-23 May 2018.
11. 4th Conference of the International Society for Nonparametric Statistics, Salerno, Italy, 11-15 June 2018.
12. 5th European workshop on Space Debris Modeling and Remediation. Paris, France, 25-27 June 2018.
13. 21st International Conference on Information Fusion, Cambridge, UK, 10-13 July 2018.

6 Conclusions

The SSA community has shown considerable interest in this work. But being "interested" is not the same as being "convinced". More work is needed by us to produce evidence that is strong enough to convince this audience that our algorithms reliably provide improved speed and accuracy over existing methods, and hopefully for our algorithms to be adopted by the community. One important goal is to produce more journal papers to publicize and explain our contributions more effectively.

References

[1] J. T. Kent, I. Hussein, and M. K. Jah. Directional distributions in tracking of space debris. In *Proceedings of the 19th International Conference on Information Fusion (FUSION), Heidelberg, Germany*, pages 2081–2086. IEEE, 2016.

[2] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. Orbital error propagation analysis using directional statistics for space objects. In *Proceedings of the 27th AAS/AIAA Space Flight Mechanics Meeting, San Antonio, Texas. February 5-9, 2017*, 2017. Paper AAS 17-390.

[3] J. T. Kent, S Bhattacharjee, I. Hussein, and M. K. Jah. Angles-only data association using directional discriminant analysis. In *Proceedings of the 27th AAS/AIAA Space Flight Mechanics Meeting, San Antonio, Texas. February 5-9, 2017*, 2017. Paper AAS 17-393.

[4] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. Geometric restructurization of the space object tracking problem for improved uncertainty representation. In *Proceedings of the 7th European Conference on Space Debris, ESA/ESOC, Darmstadt,Germany, 18–21 April 2017*, 2017.

[5] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. The performance of a direction-based Bayesian filter in the orbital tracking problem. In *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Stevenson, WA, August 2017*, 2017. Paper AAS-17-745.

[6] W. R. Faber, I. I. Hussein, J. T. Kent, S. Bhattacharjee, and M. K. Jah. Optical data association in a multi-hypothesis framework with maneuvers. In *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference, Stevenson, WA, August 2017*, 2017. Paper AAS-17-639.

[7] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. Bayesian filtering using directional statistics for the orbital tracking problem. In *Proceedings IAC Australia, Sept 2017*, 2017.

[8] W. R. Faber, I. I. Hussein, J. T. Kent, S. Bhattacharjee, and M K Jah. FBK optical data association in a multi-hypothesis framework with maneuvers. In *Proceedings AMOS, Sept 2017*, 2017.

[9] S. Bhattacharjee, J. T. Kent, I. I. Hussein, W. R. Faber, and M K Jah. Application of directional statistics to problems in SSA. In *First IAA Conference on Space Situational Awareness (ICSSA) Orlando, FL, USA, November 2017*, 2017. Paper IAA-ICSSA-17-01-01.

[10] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. Fisher-Bingham-Kent mixture models for angles-only observation processing. In *Proceedings of the 28th AIAA/AAS Space Flight Mechanics Meeting, Kissimmee, Florida, January 8-12, 2018*, 2018.

[11] J. T. Kent, S. Bhattacharjee, I. Hussein, and M. K. Jah. Nonlinear filtering using directional statistics for the orbital tracking problem with perturbation effects. In *Proceedings of the 28th AIAA/AAS Space Flight Mechanics Meeting, Kissimmee, Florida, January 8-12, 2018*, 2018.

[12] W. R. Faber, I. Hussein, J. T. Kent, S. Bhattacharjee, and M. K. Jah. Optical data processing using directional statistics in a multi-hypothesis framework with maneuvers. In *Proceedings of the 28th AIAA/AAS Space Flight Mechanics Meeting, Kissimmee, Florida, January 8-12, 2018*, 2018.

[13] J. T. Kent, S. Bhattacharjee, I. Hussein, W. R. Faber, and M. K. Jah. Filtering when object custody is ambiguous. In *Proceedings of the 21st International Conference on Information Fusion, 10–13 July 2018, Cambridge, UK.*, 2018.

[14] J. T. Kent, S. Bhattacharjee, W. R. Faber, I. I. Hussein, , and M. K. Jah. Filtering under ambiguity for the debris tracking problem. In *69th International Astronautical Congress, Bremen, Germany, 1–5 October, 2018*, 2018. to appear.

[15] J. T. Kent, S. Bhattacharjee, W. R. Faber, I. I. Hussein, and M. K. Jah. Understanding the effect of perturbations on the gaussianity of various coordinates for the space object tracking problem. In *AMOS conference, Hawaii, USA, 11–14 September 2018*, 2018. to appear.

[16] J. T. Kent, S. Bhattacharjee, W. R. Faber, I. I. Hussein, and M. K. Jah. Tackling association and tracking problems using directional statistics to model uncertainty. In *69th International Astronautical Congress, Bremen, Germany, 1–5 October 2018*, 2018. to appear.

[17] J. T. Kent and S. Bhattacharjee. Uncertainty propagation analysis for orbital debris using directional statistics. In *Tenth International Triennial Calcutta Symposium on Probability & Statistics, Kolkata, India, 27–29 December 2018*, 2018. to appear.

7 List of Symbols, Abbreviations and Acronyms

AST: (adapted structural) coordinates to describe the state of an orbiting object.

FBK: (Fisher-Bingham-Kent) distribution on the unit sphere to describe angles only data

FISST: finite set statistics

RSO: resident space object

SSA: space situational awareness