



Passive Snapshot Remote Imaging of Object Velocity

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Final Report

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Final Report

Passive Snapshot Remote Imaging of Object Velocity

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1.0 Introduction

Quantifying a target's velocity is critical for rapid trajectory calculations. Two common active techniques for object tracking in low earth orbit (LEO) includes RADAR and LiDAR, in which velocity is calculated by measuring the returning field's Doppler shift and/or the delivered pulse's time of flight. However, active techniques are often challenged when measuring small or distant objects in medium earth orbits (MEOs), geostationary orbits (GEOs), or geostationary transfer orbits (GTOs) since the detected signal power is reduced both propagating *to* and *from* the target [1,2].

Our central hypothesis is that naturally occurring narrow-band solar absorption features can be used to passively gauge an object's radial velocity using a single frame of data. The goal is to estimate an orbit using only two passive measurements of angular position and Doppler velocity. To this end, we have developed a novel sensing technique that can passively measure an object's radial velocity without the need for gated detectors or active illumination systems.

1.1 Background

All light, emitted by the sun, passes through the solar atmosphere. The hydrogen, helium, and other elements in the sun's atmosphere absorb light at characteristic wavelengths, creating dark lines in the solar spectrum, as illustrated in Fig. 1. These *Fraunhofer lines* occur at specific, well-known frequencies and are typically very narrow in width, on the order of 0.1 angstrom [3]. Their occurrence indicates the presence of specific chemical species, and they can act as reliable wavelength markers. Visible light, reaching Earth from the sun, contains thousands of Fraunhofer lines. The common names of some especially prominent lines, along with their chemical sources, are listed in Table 1.

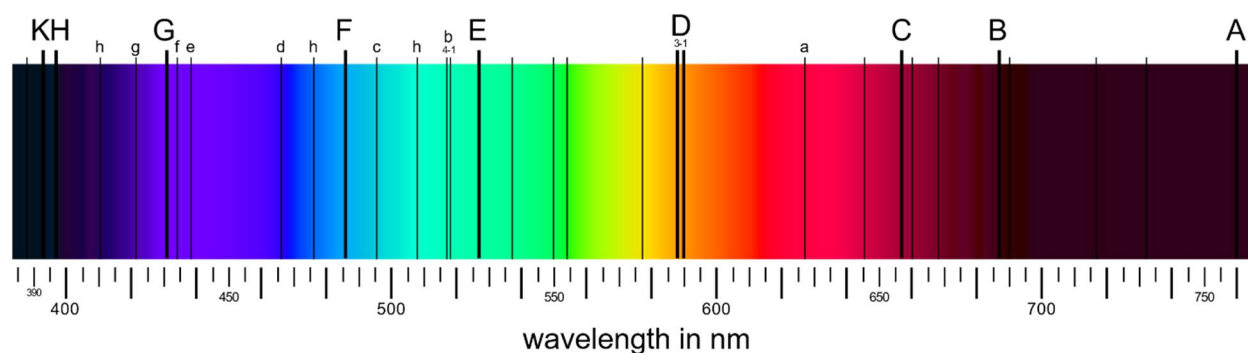


Fig. 1. Fraunhofer absorption lines in the visible spectrum [4].

Table 1. Fraunhofer line designations, sources, and wavelengths [5].

Designation	Element	λ (nm)	Designation	Element	λ (nm)
A	O2	759.370	b4	Mg	516.733
B	O2	686.719	c	Fe	495.761
C	H α	656.281	F	H β	486.134
a	O2	627.661	d	Fe	466.814
D1	Na	589.592	e	Fe	438.355
D2	Na	588.995	f	H γ	434.047
D3 or d	He	587.562	G	Fe	430.790
e	Hg	546.073	G	Ca	430.774
E2	Fe	527.039	h	H δ	410.175
b1	Mg	518.362	H	Ca+	396.847
b2	Mg	517.270	K	Ca+	393.368
b3	Fe	516.891			

Fraunhofer lines are useful not only for their ability to identify unknown elements — the basis for spectroscopic analysis — but also for providing information about the motion of a radiant object. When two objects are in relative motion, waves traveling between them are stretched or compressed. This *Doppler shift* causes a change in the measured wavelength, such that

$$\lambda' = \lambda \left(1 + \frac{v}{c} \right), \quad (1)$$

where λ' is the measured wavelength, λ is the original wavelength, v is the radial velocity between the source and observer, and c is the speed of light. (In general, this expression works for any non-relativistic object that emits or reflects wave energy, provided the constant c is replaced by the propagation speed of the waves.) When the source of light is the sun, the Doppler shift can be determined by comparing an image of shifted Fraunhofer lines to an unshifted reference image. This is a common technique for determining the velocity of stellar objects; unseen planets orbiting other stars can be detected by the small changes in velocity they induce in the host star [6].

When sunlight is reflected by an orbiting object, the object is in motion relative to both the sun and an observer on Earth. Consider a solar Fraunhofer line at some wavelength, λ , as illustrated in Fig. 2(a). When the light is reflected from an orbiting body, the wavelength is shifted twice. The first shift, occurring along the optical path between the sun and the body, results in a wavelength of λ' incident on the body, depicted in Fig. 2(b). The second shift, introduced along the path of reflection between the orbiting body and the observer, results in a measured wavelength of λ'' , depicted in Fig. 2(c). The two wavelength shifts are not necessarily in the same direction, and the final, observed wavelength is highly dependent on the geometry and relative velocities of the sun, body, and observer. Applying Eq. (1),

$$\lambda'' = \lambda \left(1 + \frac{\dot{\rho} + v_s}{c} + \frac{\dot{\rho} v_s}{c^2} \right), \quad (2)$$

where v_s and $\dot{\rho}$ (the *range-rate*) are the target's radial velocities with respect to the sun and the ground station, respectively.

Although the above treatment considers only a single wavelength, a practical implementation would average the Doppler shift over many wavelengths by way of an optical correlator [7,8]. The

accuracy of the final measurement will improve with the number of Fraunhofer lines included in the average.

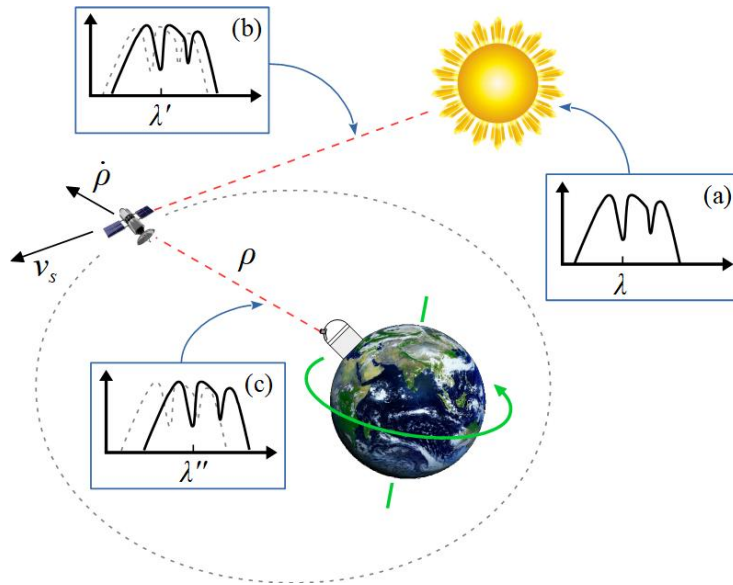


Fig. 2. Components of Doppler shift along the optical path.

1.2 Overview

In the remainder of this report, we present the final summary of the work that was completed during the project. Chapter 2 introduces several optical models. First, a radiometric transfer model is developed. Next, two system design options are explored: a *full-spectral* approach and a *dual-band ratio* approach. Finally, a variety of implementation scenarios are discussed and compared for each system.

Chapter 3 investigates using the optical techniques of *spatial heterodyning* and *signal aliasing*. Optical simulations of both techniques are described, and the results are presented and discussed.

Chapters 4-6 discuss computer simulations of geocentric orbits and calculation of radial velocity given wavelength and line-of-sight angle measurements. Specifically, chapter 4 describes modeling and simulation of equatorial orbits, for the purpose of bounding the required SNR. Next, chapter 5 extends this work to general, three-dimensional orbits, using more accurate orbital models. Finally, chapter 6 introduces the use of *admissible region estimation* as a way to improve the starting values of the orbital solution.

Chapter 7 explores several filter options for use in the dual-band system design. Examined filters include multi-stage Lyot, multi-stage Solc, fiber Bragg gratings, and volume phase holographic gratings. Basic theory of each type of filter is presented, along with simulation results.

Chapter 8 begins with a review of the spatial light modulator, which is a key component in the design of our optical correlator hardware. Next, the design of the correlator itself is explained, followed by the criteria used for selection of an appropriate optical passband for the application.

Chapter 9 discusses assembly and calibration of the hardware prototype. Benchtop measurements of frequency stability and spectral resolution are presented.

Chapter 10 discussed the assembly and calibration of a Fraunhofer line discriminator based on a potassium Voigt filter.

Finally, chapter 11 summarizes our conclusions and presents a brief discussion of potential improvements and future project work.

2.0 Modeling the Optical System

2.1 Radiometric Transfer Model

A basic radiometric transfer model was created to simulate signatures reflected from satellites with simple geometries. This was conducted to estimate the optical signal, for a specific spectral passband (or spectral resolution element), incident onto the optical system's pupil. First, light was simulated using the blackbody exitance equation such that

$$M_{BB}(T) = \int_{\lambda_1}^{\lambda_2} \frac{c_1}{\lambda^5} \frac{d\lambda}{\exp(c_2/\lambda T) - 1} \quad (3)$$

where λ_1 and λ_2 represent the minimum and maximum wavelengths in the spectral passband of interest, respectively, c_1 is a constant that equals $3.74151E8$, c_2 is a constant equaling $1.43879E4$, and T is the temperature of the blackbody emitter in Kelvin. This equation was used to estimate the radiance from the sun assuming $T = 6000$ K around wavelengths in the proximity of the Magnesium b1 Fraunhofer line ($\lambda_0 = 518.362$ nm).

Light from the sun was modeled, using this equation, and propagated according to the earth-solar distance and the solar diameter such that the radiance incident on a satellite orbiting earth can be calculated as

$$L_{Sun} = M_{BB}(T) / \pi E_{Photon}, \quad (4)$$

where E_{Photon} is the energy contained within photons at a wavelength equal to λ_0 in units of (J/photon), yielding a solar radiance in units of (photons/s/m²/sr). Note that it is assumed that passbands are relatively small such that $\lambda_0 \approx \lambda_1 \approx \lambda_2$ in this conversion from (W/m²/sr); otherwise, the photon energy conversion would need to be integrated into Eq. (3) directly.

Propagating this radiance to the satellite is then accomplished by multiplying the solid angle subtended by the sun, such that the incident irradiance onto the satellite can be calculated using

$$E_{inc} = L_{Sun} \Omega_{Sun}, \quad (5)$$

where the solid angle subtended by the sun is

$$\Omega_{Sun} = A_{Sun} / D_{Sun}^2. \quad (6)$$

Here, $A_{Sun} = \pi(695.8E6)^2$ m² and $D_{Sun} = 149.6E9$ m. The satellite is assumed to have a square cross section of side length $S_{Sat} = 1$ m, such that the area of the satellite A_{Sat} is 1 m². The incidence angle

of the sunlight onto the satellite is modeled as θ_i such that the reflected radiance can be modeled as

$$L_{\text{Ref}} = E_{\text{inc}} \rho_{\text{Sat}} \cos(\theta_i) / \pi, \quad (7)$$

where ρ_{sat} is the lambertian reflectance of the satellite and L_{Ref} is in units of (photons/s/m²/sr). Note that unless otherwise noted, θ_i is assumed to be 0 degrees and the reflectance of the satellite is assumed to be 70%. This light is then transferred to the surface of the earth. Assuming that the satellite has a distance from the center of the earth to the satellite of O_{Sat} and that the radius of the earth $R_e = 6371$ km, then the distance between the observer and the satellite can be assumed to be

$$D_{\text{Sat}} = O_{\text{Sat}} - R_e. \quad (8)$$

The solid angle subtended by the satellite from earth can be calculated as

$$\Omega_{\text{Sat}} = A_{\text{Sat}} / D_{\text{Sat}}^2. \quad (9)$$

Thus, the incident irradiance on earth can be calculated as

$$E_{\text{Sat}} = \tau_{\text{atmo}} L_{\text{Ref}} \Omega_{\text{Sat}}, \quad (10)$$

where τ_{atmo} is the transmittance of the atmosphere in the spectral passband of the simulation. For all simulations spanning 500 nm - 600 nm, this is assumed to be 90%.

2.2 Optical System Modeling – Full Spectral Approach

Several designs were generally approached in order to implement the spatial heterodyning technique. Two of which that showed the most promise, in terms of radiometric throughput, sampling efficiency, and ability to be augmented with the spatial heterodyning optics included division of aperture (or pupil-based) sampling and integral field sampling.

2.2.1 SHIFT Pupil-Based Design

The SHIFT spectrometer's pupil-based design is illustrated below in Fig. 3. This consists of the telescope, with a focal length f_T and aperture diameter D_T , which creates an intermediate image of the scene at its focus. A collimation lens L1, with focal length f_1 , collimates the intermediate image into a lenslet array (LA). This creates an array of sub-images (replicas) of the scene at the intermediate focus of each lenslet in the array. These intermediate images are then collimated by lens L2, with focal length f_2 , into an interferometer. In this optical space, the beams are split and encoded with the spectral information before being re-imaged by lens L3, with focal length f_3 , onto a focal plane array (FPA) for final measurement. Note that, for the purposes of this analysis, the lenslet array is assumed perfectly filled; although, there will be underfilling in its center due to the annular pupil of the telescope.

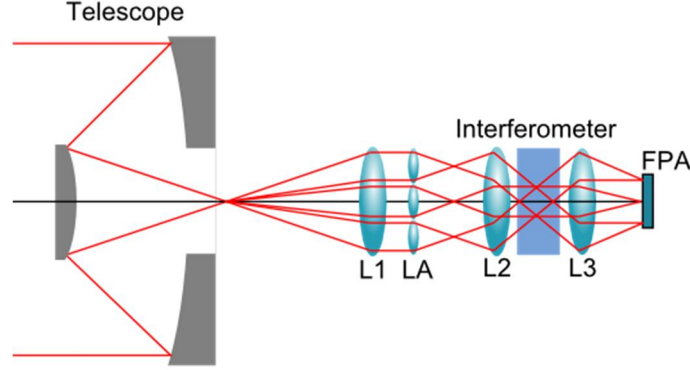


Fig. 3. Pupil-based design for the SHIFT spectrometer in which the lenslet array and interferometer both reside within collimated optical spaces.

In order to model the optical system, we assume an aperture-sampled system as shown in Fig. 4. In this case, each lenslet **LA** can be considered separately as an aperture projection onto the main telescope objective. If we assume that the diameter of the lenslets is chosen to maximize coverage of the objective, then the pupil of each lenslet will have a projected area of

$$A_{lenslet} = \frac{A_{tel}}{N} ff, \quad (11)$$

where A_{tel} is the area of the telescope's aperture, N is the total number of lenslets in the array, and ff is a fill factor to compensate for incomplete coverage due to the lens geometry. By attempting to cover the circular aperture with a square array of lenslets, we can estimate a fill factor of

$$ff = \frac{A_{array}}{A_{tel}} = \frac{2(D/2)(D/2)}{\pi(D/2)^2} = \frac{D^2/2}{\pi(D^2/4)} = \frac{2}{\pi} \approx 0.64, \quad (12)$$

where A_{array} is the area of the lenslet array, and D is the diameter of the telescope's aperture.

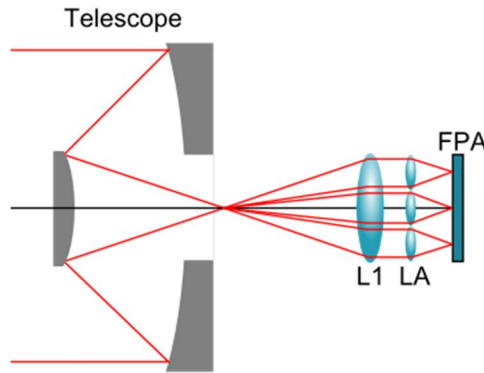


Fig. 4. Simplified aperture-sampled optical design.

To determine the correct formula for finding photon flux, we must first know whether the reflected light from the satellite will overfill or underfill a single pixel at the detector. This can be determined by comparing the solid angle formed by the satellite at the telescope aperture and the solid angle of the telescope optics:

$$\Omega_{sat} = \frac{A_{sat}}{d_{sat}^2} \quad \text{and} \quad \Omega_{optics} = \frac{A_{lenslet}}{f_T^2}, \quad (13)$$

where A_{sat} is the reflected area of the satellite, d_{sat} is the distance to the satellite, and f_T is the focal length of the telescope objective lens. If Ω_{sat} is greater than Ω_{optics} , the telescope is overfilled. However, in the typical case we expect an underfilled condition. Finally, the radiometric flux on the detector can be found by

$$\Phi = L_{ref} \tau_{atmo} \tau_{optics} A_{lenslet} \Omega_{optics} \quad (14)$$

for an overfilled condition, or

$$\Phi = L_{ref} \tau_{atmo} \tau_{optics} A_{lenslet} \Omega_{sat} \quad (15)$$

for an underfilled condition. L_{ref} is the reflected radiance from the satellite, and τ_{atmo} and τ_{optics} are the optical transmittance of the atmosphere and optics, respectively.

There are three main sources of noise in the system: photon or “shot” noise, which is an inherent property of the incoming photons; dark noise, which is caused by thermal generation of carriers in the detector; and read noise, which is caused by inaccuracies in counting electrons as they are clocked out of the detector. The theoretical signal current caused by light on the detector is

$$i_{sig} = (QE)\Phi t, \quad (16)$$

where i_{sig} is the current in electrons, QE is the quantum efficiency of the detector, and t is the integration time (i.e., exposure time) in seconds. It should be noted that the quantum efficiency is highly dependent on the frequency of the incident light.

Because shot noise follows a Poisson statistical distribution, the corresponding variation in measurement is equivalent to the square root of the signal (including thermally generated carriers) so that

$$i_{shot} = \sqrt{i_{sig} + i_{dark} t}, \quad (17)$$

where i_{shot} is the shot noise current, and i_{dark} is the dark current rate (electrons/s) specified by the manufacturer. When read noise is included, the total noise current can be found as

$$i_{noise} = \sqrt{i_{shot}^2 + i_{read}^2}, \quad (18)$$

where i_{read} is the read current, also specified by the manufacturer. Finally, the signal to noise ratio of the system is

$$SNR = i_{sig} / i_{noise}. \quad (19)$$

Simulations of this design were created using the parameters defined in Table 2 below.

Table 2. Parameters for the simulation of the Pupil-based SHIFT design.

Parameter	Value	Parameter	Value
τ_{optics}	0.20	R_T	0.45 m
τ_{atmo}	0.90	O_{Sat}	42.2E6 m
t	40 Minutes	N	10
QE	0.90 e ⁻ /photon	M	10
i_{read}	2.3 e ⁻	S_{Sat}	1 m
i_{dark}	0.0001 e ⁻ /sec/pixel	A_{Sat}	1 m ²
R_e	6.371E6 m	ρ_{Sat}	0.70
D_{Sun}	149.6E9 m	R_{sun}	695.8E6 m
T	6000 K	$\Delta\lambda$	0.01 nm
λ_0	518.362 nm	$A_{detector}$	13.5 micron ²

Results from these simulations yielded an SNR of 4.41 for a spectral resolution of 0.01 nm. Note that this assumes that a polarization interferometer is used to perform the measurement and that only one Fraunhofer line is being used (e.g., the polarization gratings are not being used to alias multiple spectral bandwidths to be coincident). If the PGs were used to alias 3 Fraunhofer lines together, then this SNR would increase to 8.2.

2.2.2 SHIFT Integral Field-Based Design

A view of the integral field based design is presented in Fig. 5. This design consists of the same telescope as before, except that now the interferometer is placed within the converging optical beam of the telescope. Additionally, a lenslet array is now used just in front of the FPA to re-image the telescope's pupil onto the FPA.

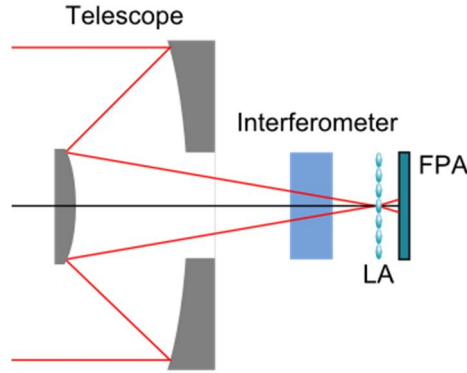


Fig. 5. Optical layout for an integral field spectrometer. In this case, the interferometer is placed in the converging beam of the optical telescope.

In this design, it may be conceivably more light efficient in that all of the light from the telescope is collected by the lenslets in the lenslet array LA. However, this has to be justified against the fact that the light is then subsequently spread out by the LA onto the FPA, thereby reducing the magnitude of the incident irradiance. In this model, the flux incident onto a single lenslet in the lenslet array, from the satellite, can be calculated by

$$\Phi = \tau_{optics} E_{Sat} A_T, \quad (20)$$

where Eq. (20) is identical in form to Eq. (15) except that now the total area of the telescope, A_T , is integrating the light. Light is then transmitted by the lenslet array LA onto the focal plane array, where an image of the pupil is generated that has approximately the same area as the lenslet itself. In this case, the focal length of the lenslets in the array are calculated by

$$f_L = D_{Lenslet} / \theta_{Aperture}, \quad (21)$$

where $D_{Lenslet}$ is the diameter of a single lenslet and $\theta_{Aperture}$ is the angle subtended by the aperture of the telescope from the lenslet. The area of the telescope's pupil, generated by the lenslet on the FPA, is thus

$$A_I = \pi \left(f_L \theta_{Aperture} / 2 \right)^2. \quad (22)$$

The image plane irradiance can then be calculated by

$$E_I = \Phi / A_I. \quad (23)$$

Finally, the incident flux can be calculated by multiplying the incident irradiance by the detector's area such that

$$\Phi_{Detector} = E_I A_{Detector}. \quad (24)$$

It turns out that the flux density given by this design is identical to the previous design. Using identical parameters to the previous SNR simulation yields that this design performs essentially identically, yielding an SNR of ~ 4.94 without any assistance from the multiband polarization gratings. Conversely, if 3 bands were aliased to increase the incident flux, then the SNR would increase to 9.1.

2.2.3 Signal to Noise Ratio Analysis and Line Fitting

Fraunhofer line fitting was also studied in terms of the signal to noise ratios that were calculated in the previous section. In this case, the Magnesium b1 Fraunhofer line was simulated per the spectrum shown in Fig. 6.

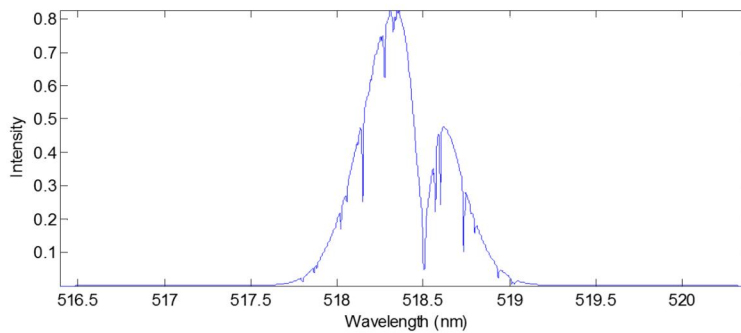


Fig. 6. Fraunhofer spectrum of the magnesium b1 Fraunhofer line.

In this case, several line fitting procedures were conducted, including fitting a Gaussian or Lorentzian line to the Mg b1 line, as well as fitting the solar spectrum, via interpolation, onto the measured (noisy) spectrum. Results of this analysis are provided in the subsequent sections.

2.2.3.1 Lorentzian Line Fitting

Both a Gaussian and a Lorentzian function were studied to fit and/or match the Mg b1 line; however, the Lorentzian line shape matches this particular line most optimally. Measured spectra were generated, using the aforementioned radiometric transfer model and noise model, and the subsequently measured results were spectrally filtered to reduce noise. A view of the measured spectra for three different values of the SNR are presented below in Fig. 7 (a-c) for SNR values of 5, 10, and 20, respectively.

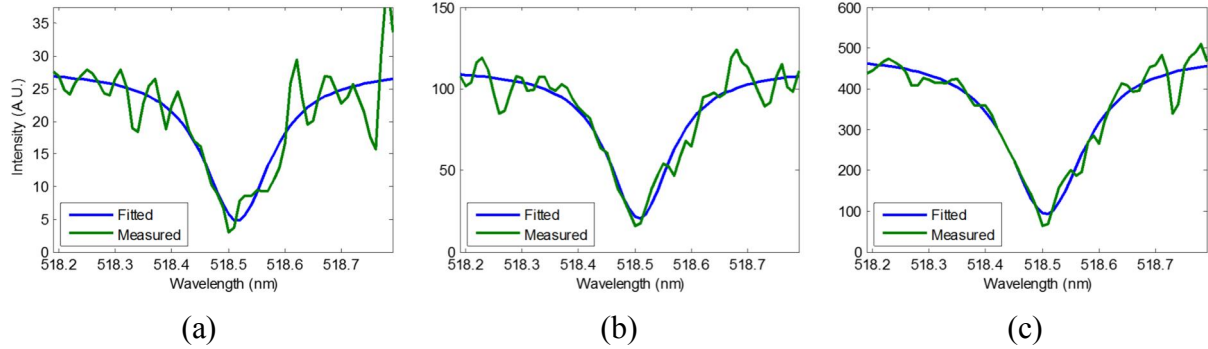


Fig. 7. Simulated fitting procedure to noisy spectra for SNR values of (a) 5; (b) 10; and (c) 20.

This procedure, of measurement and fitting, was run 1000 times and the statistics of the fitting procedure were quantified in terms of the center wavelength of the fit. Note that this center wavelength is what determines the total measured Doppler shift. These statistics can be seen summarized below in Fig. 8 (a-c). In this case, an SNR of 5 yields a standard deviation of 0.0059 nm, and SNR of 10 yields a standard deviation of 0.0028 nm, and an SNR of 20 yields a standard deviation of 0.0014 nm. Thus, each time the SNR doubles, the fitting procedure becomes 2 times more accurate.

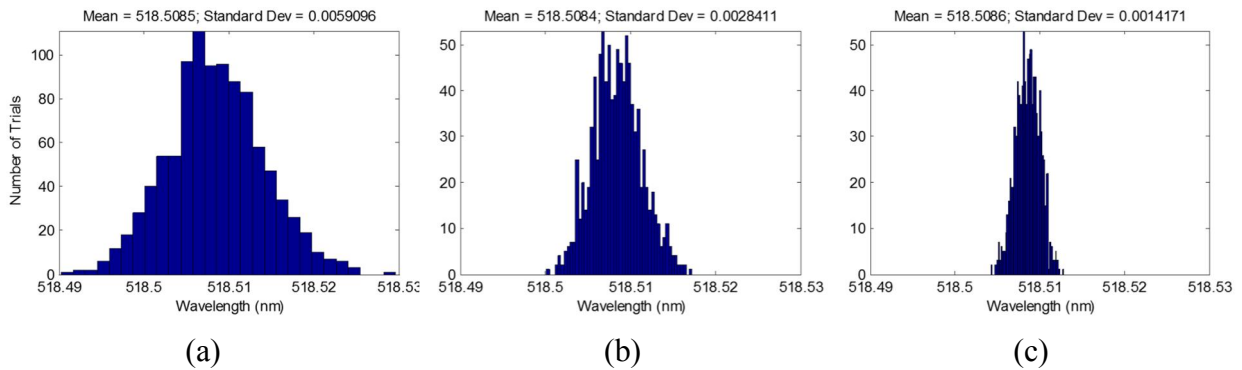


Fig. 8. Statistics of the fitting procedure.

This is illustrated in Fig. 9 below in which the wavelength shift, in nm, is related to the velocity of the measured satellite. Overlaid are the 2σ deviations (e.g., 95% of all trials around the mean), which can readily be related to the standard deviation in velocity measurement. Ultimately, an

SNR of 20 yields an error in measured velocity of ± 400 m/s while an SNR of 5 yields an error exceeding ± 1000 m/s.

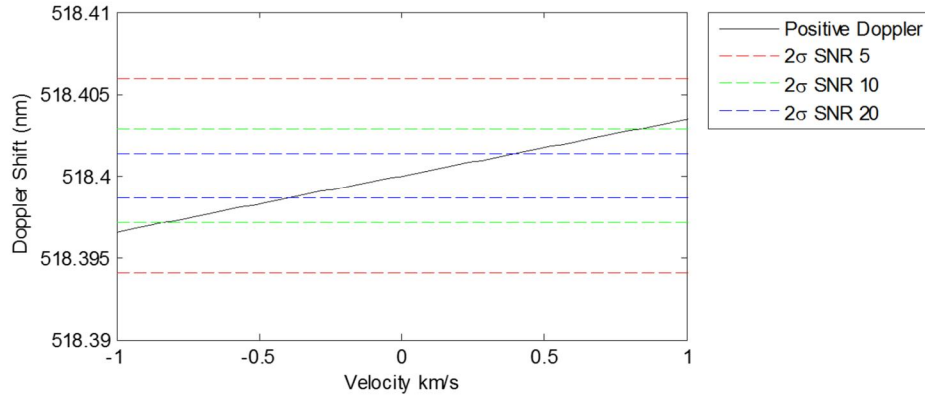


Fig. 9. 2 standard deviations from the mean, plotted versus the Doppler shifted velocity assuming a target is approaching the observer.

2.2.3.2 Full Spectrum Line Fitting

Since the solar spectrum is well known and well characterized, fitting the solar spectrum directly, via interpolation, was also studied to see if errors above could be reduced even further. A view of the measured spectra for three different values of the SNR are presented below in Fig. 7 (a-c) for SNR values of 5, 10, and 20, respectively. Notable is that the fitted curve more closely matches that of the actual measured curve, since the solar spectrum is being fit directly.

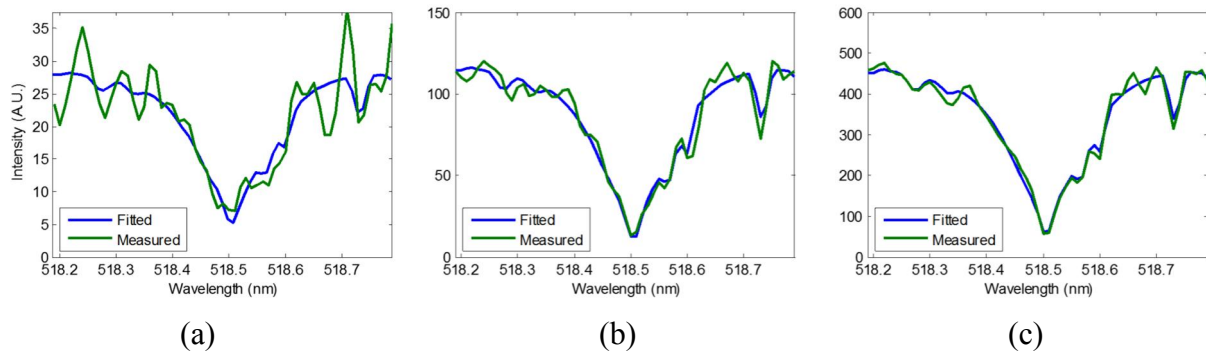


Fig. 10. Simulated fitting procedure to noisy spectra for SNR values of (a) 5; (b) 10; and (c) 20.

Similar to the previous analysis, this interpolation fitting procedure was executed 1000 times and the statistics of the fitting were quantified in terms of the center wavelength. These statistics can be seen summarized below in Fig. 8 (a-c). In this case, an SNR of 5 yields a standard deviation of 0.0039 nm, and SNR of 10 yields a standard deviation of 0.0018 nm, and an SNR of 20 yields a standard deviation of 0.0009 nm. Thus, similar to before, each time the SNR doubles, the fitting procedure becomes 2 times more accurate. Additionally, accuracy is increased in general over the previous single line fitting due to the fact that several lines are being fit simultaneously.

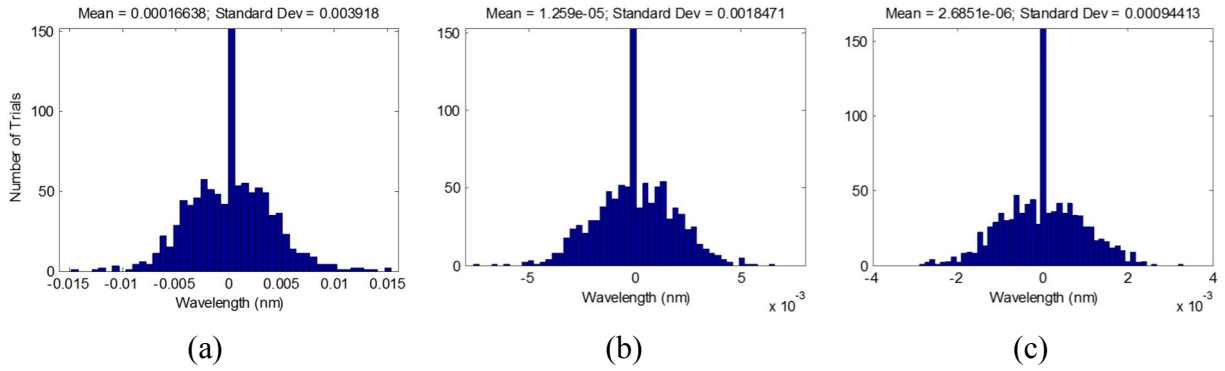


Fig. 11. Statistics of the fitting procedure when using the entire spectrum.

Relating these errors to the error in the detected Doppler shift is shown in Fig. 9. Overlaid are the 2σ deviations (i.e., 95% of all trials around the mean). In this case, an SNR of 20 yields an error in measured velocity of ± 280 m/s while an SNR of 5 yields an error of approximately ± 1100 m/s.

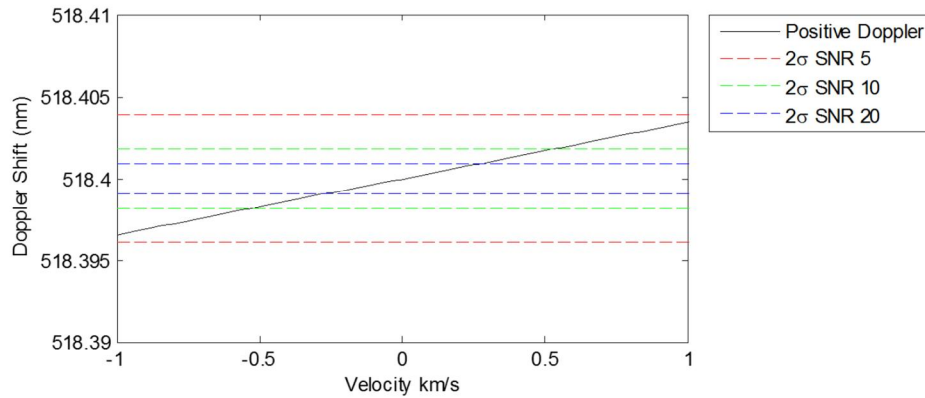


Fig. 12. Two standard deviations from the mean, plotted versus the Doppler shifted velocity assuming a target is approaching the observer.

2.2.4 Zemax Optical Design Simulations

Three designs and implementations for the interferometer have been studied for feasibility. The first is the interferometer design originally proposed in the grant. Second is a Sagnac interferometer that contains a configuration as illustrated previously in Fig. 3. Third is a Sagnac interferometer that contains the configuration of Fig. 5.

2.2.5 Birefringent Interferometer

The design originally proposed is illustrated schematically in Fig. 13. A telescope or objective lens forms an image on a field stop to restrict the field of view, which is then collimated into the system. This collimated light is then incident onto an $N \times M$ lenslet array (LA) to create the sub-images that will be used to measure the spectral image data.

The beams exiting the LA are collimated by a second collimation lens (note, only one of the lenslet's beams is illustrated for clarity). This collimated light is polarized by a linear polarizer (LP) before transmission through the first Wollaston prism (WP_1). WP_1 splits the light into two orthogonally linearly polarized beams, which propagate through a slab waveguide (refractive

index n_g) encapsulated by two high reflectivity dichroic mirrors (M_1 and M_2). Upon exiting the waveguide, the light transmits through the orthogonal prisms WP_2 or WP_3 . This serves two purposes: (1) dispersion induced by WP_1 is compensated and (2) emerging rays propagate parallel to the optical axis. This portion of the interferometer increases the shearing distance S between the two beams, allowing for large OPDs to be achieved. However, up until this point, the light has not been "heterodyned" and transmission of these two beams through a lens would create such a high spatial frequency as to not be measurable by a conventional detector.

In order to heterodyne the beams, the light exiting WP_3 and WP_2 transmits through PG_3 and PG_2 , respectively. This diffracts the light into either the $+1^{st}$ or -1^{st} orders and causes it to propagate back towards the optical axis. An identical waveguide structure is used in which dichroic mirrors, M_3 and M_4 , guides the light into an additional polarization grating (PG_1). This "undiffracts" the two orders, causing them to emerge parallel to the optical axis. Focusing them by a re-imaging lens enables the light to interfere at a much lower (heterodyned) spatial frequency on the detector array, thereby preserving the spectral resolving power of the original beams.

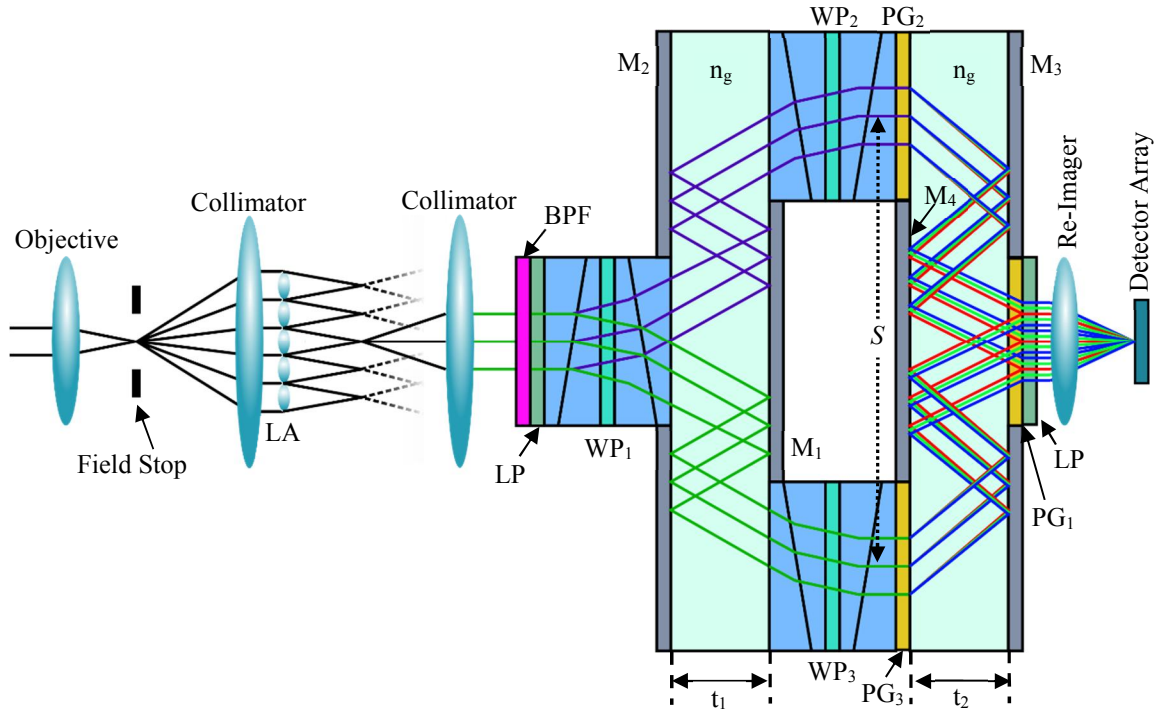


Fig. 13. Fraunhofer imaging spectrometer. The Wollaston prism's shear S is heterodyned to a smaller shear (e.g., a lower frequency) by the polarization gratings.

2.2.5.1 Birefringent Interferometer – Zemax Simulation

A view of the Zemax simulation for the aforementioned design is depicted in Fig. 14. This design produces a shear of approximately 222 mm, which would yield a maximum optical path difference (OPD) of

$$OPD_{\max} = Sx_{\max}/f. \quad (25)$$

where $x_{\max}$ is the maximum x coordinate of the camera, f is the focal length of the re-imaging lens, and S is the interferometer's shear. With $S = 222$ mm, $x_{\max} = 13$ mm, and $f = 100$ mm yields

$OPD_{\max} = 28.8$ mm. This yields a maximum spectral resolution of $\Delta\sigma = 1/OPD_{\max} = 34.65$ m^{-1} or 0.3465 cm^{-1} . At 518 nm, this produces a spectral resolution of

$$\Delta\lambda = \Delta\sigma\lambda^2 = 0.0093 \text{ nm}. \quad (26)$$

Thus, this system does reach the target spectral resolution of 0.01 nm, for a spectral resolving power of approximately 55,000. However, this design suffers from some significant drawbacks; specifically, cost related. In this case, a high birefringence material is needed to obtain a sufficient shear in combination with a large prism wedge angle. This example uses calcite prisms with a wedge angle of 30 degrees and a clear aperture of 50 mm. Attempts have been made in the design to decrease the wedge angle and/or decrease the clear aperture size while maintaining the shear. However, minimal success has been achieved with this. Thus, it is estimated that, due to the large clear aperture, each prism would cost approximately \$3,000 each for a total of \$36,000. Since such a design is impractical on the proposed budget, alternative designs are being investigated. However, these alternative designs will still be able to leverage the multiband polarization grating technique since they can still be designed to act on more than one polarization state.

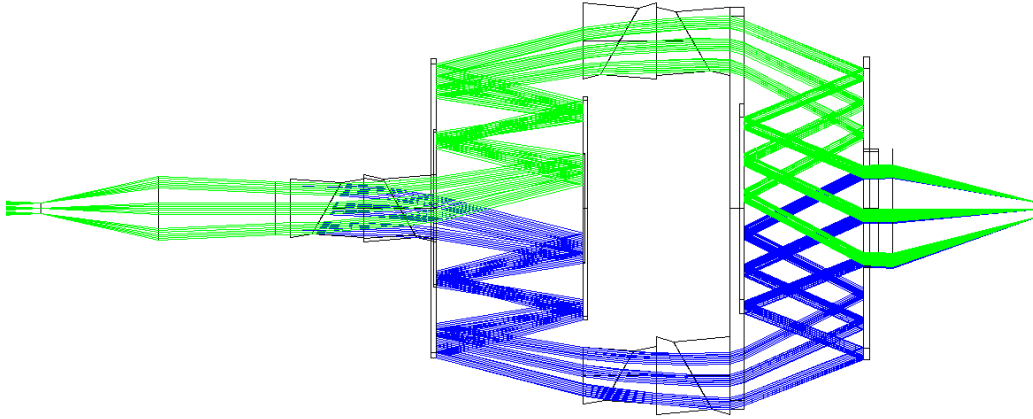


Fig. 14. Zemax simulation of the birefringent interferometer.

2.2.6 Sagnac Interferometer with Pupil-Based Lenslet Array

A schematic of the Sagnac interferometer based concept is illustrated in Fig. 15. In this system, a telescope creates an intermediate image that is then collimated by lens L1. Light is then transmitted through a lenslet array (LA) to create an $N \times M$ array of sub-images at the lenslets' focal point. These sub-images are then collimated by lens L2 into a Sagnac interferometer. This interferometer contains two blazed diffraction gratings, G1 and G2, in its original configuration per Megis for spatial heterodyne interferometry and later by Kudenov for polarization imaging. These gratings produce the heterodyning effect within the interferometer, which allows significant space savings over the separated design of the birefringent interferometer. These two heterodyned beams are recombined by a reimaging lens L3 and are focused onto an FPA.

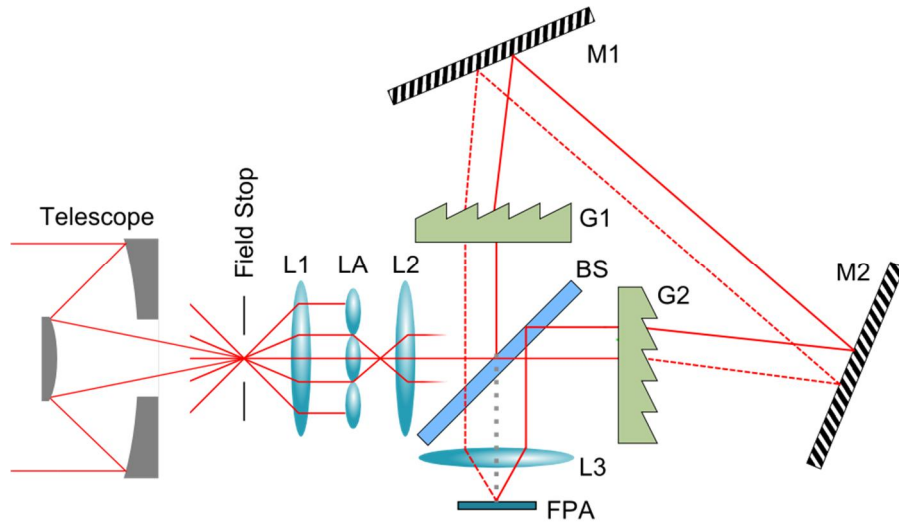


Fig. 15. Schematic of the Sagnac-based interferometer with the LA in a relayed pupil.

A Zemax optical schematic of this system is provided in Fig. 16. This design enables a shear of approximately 200 mm, thus yielding a slightly lower spectral resolution when compared to the previous design. However, the shear can be readily increased by translating mirror M2 further away from the beamsplitter.

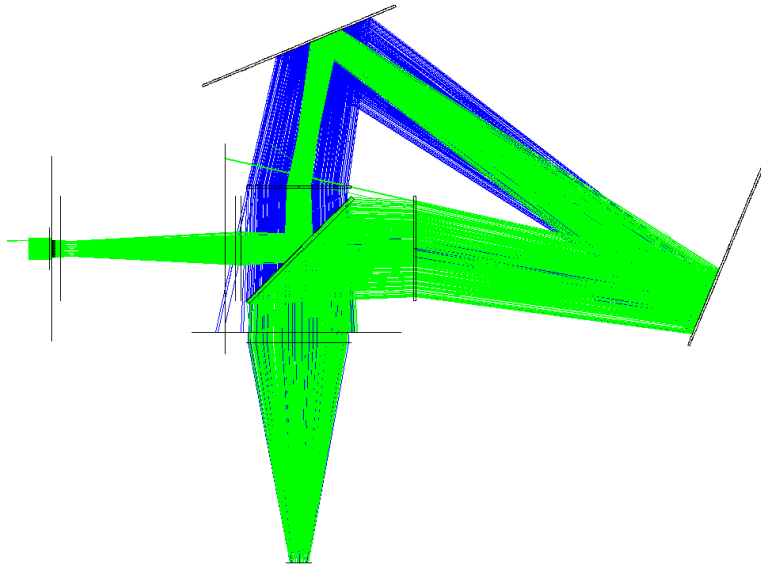


Fig. 16. Zemax simulation of the interferometer above.

2.2.7 Synergistic Work – Sagnac-Based Spatial Heterodyne Spectrometer

As part of a non-related project in our lab, we have also been developing a spatial heterodyne spectrometer for use in agricultural sensing. This work, which was sponsored via North Carolina State University startup funding, has been leveraged in this program to help us understand some of the potential challenges and implementation issues of the aforementioned Sagnac-based approach. Excerpts from this work is summarized here, but was originally published in Ref. [9].

2.2.7.1 Heterodyne Sagnac Interferometer - Theory

The system used in our Fraunhofer line discriminator is based on the multiple-image Fourier transform spectrometer (MIFTS) and a white-light Sagnac interferometer [10–13]. This can realize, for the first time, snapshot imaging FLD and leverage the snapshot advantage for improved radiometric throughput [14]. A conventional two mirror triangular Sagnac interferometer is depicted in Fig. 17(a). Here, two mirrors, M1 and M2, are separated by a distance d_1 and d_2 , respectively, from a plate beamsplitter (BS). Incident light is split by the BS such that the reflected and transmitted beams propagate through the interferometer clockwise and counter-clockwise, respectively. Offsetting mirror M2 by a distance α , measured along the line of sight to the beamsplitter, creates an achromatic shear S_{ach} between the two beams. These two beams are then recombined by a reimaging lens, with focal length f_{Re} , onto a focal plane array (FPA).

For a conventional Sagnac interferometer, the optical path difference (OPD) can be expressed as,

$$OPD = \frac{x S_{ach}}{f_{Re}}, \quad (27)$$

where S_{ach} is the shear. The achromatic shear is given by, [13]

$$S_{ach} = \sqrt{2} \alpha, \quad (28)$$

where α is the displacement of M2 away from the nominal distance d_2 as shown in Fig. 17(a).

Heterodyning the Sagnac Interferometer's interference fringes can be realized with the introduction of two blazed diffraction gratings as in Fig. 17(b) [15]. A well-known method to create white light interference fringes is by using blazed diffraction gratings [15]. In this case the incident spectrally broadband light, after being split by the BS, propagates through the diffraction gratings G1 and G2 and is diffracted to different angles. The clockwise and counterclockwise diffracted beams travel through the interferometer and transmit through gratings G2 and G1, respectively. After recombination and emergence from the BS, they are focused onto the FPA by the reimaging lens.

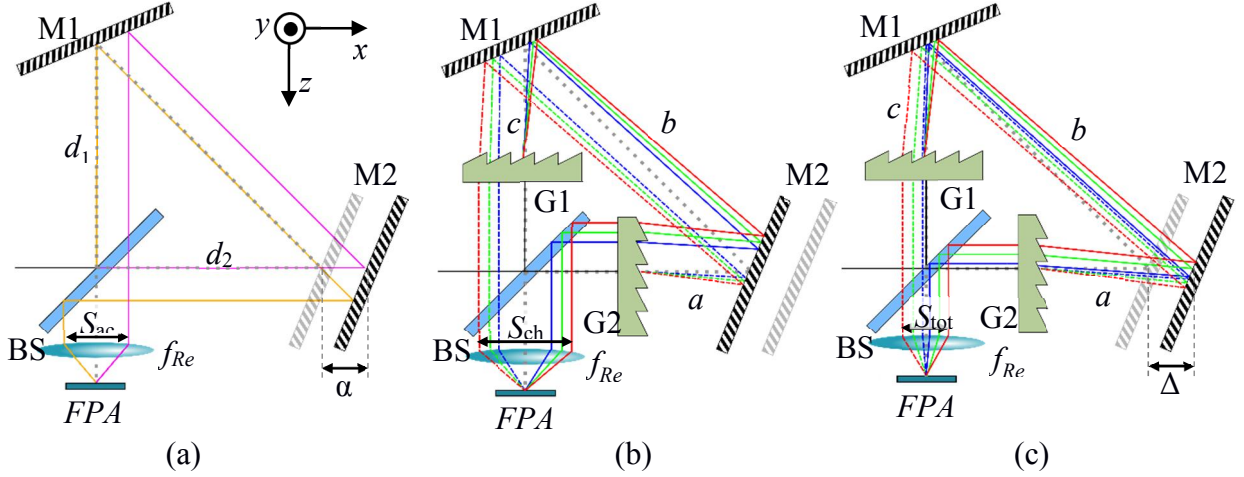


Fig. 17. Schematic of (a) a conventional two-mirror triangular Sagnac interferometer, (b) a dispersion compensated Sagnac interferometer as was used in Ref. [15], and (c) a Sagnac-based SHI based on (b) and Ref. [16].

For dispersion compensated Sagnac interferometer (DCSI), shear S_{ch} is given by

$$S_{ch} = \frac{2m\lambda}{\Lambda}(a + b + c), \quad (29)$$

where m is the order of diffraction, λ is the wavelength, Λ is the period of the gratings (both are identical), and a , b and c are the distances between G1 and M2, M2 and M1 and M1 and G2, respectively [15,16]. Thus, the shear is directly proportional to wavelength. In spatial heterodyne interferometers, this dispersive shear is added to the achromatic shear, which is inherent to the system when the Sagnac interferometer is unbalanced (e.g., when $d_1 \neq d_2$). This unbalanced and heterodyned configuration is depicted in Fig. 17 (c); thus, the total shear (S_{tot}) of the system is

$$S_{tot} = S_{ach} + S_{ch}. \quad (30)$$

From the total shear we can get the OPD according to equation (27) from which the phase difference (Δ) with the total shear (S_{tot}) can be determined as

$$\Delta = \frac{2\pi}{\lambda} \frac{x}{f_{Re}} \left(\sqrt{2}\alpha + \frac{2m\lambda}{\Lambda}(a + b + c) \right) \quad (31)$$

From this phase difference, the intensity can be expressed as,

$$I \propto \cos \left(\frac{2\pi x}{\lambda f_{Re}} \left[\sqrt{2}\alpha - \frac{2\lambda}{\Lambda}(a + b + c) \right] \right). \quad (32)$$

In our system, $S_{tot} = 0$ mm can be achieved by compensating the achromatic shear by chromatic shear. It should be noted that since the chromatic shear is proportional to λ , the heterodyne wavelength can be changed by adjusting the various distances in the system. Therefore, the heterodyne wave number σ_0 is,

$$\sigma_0 = \frac{\sqrt{2}}{\alpha\Lambda}(a + b + c), \quad (33)$$

where $m = -1$ in Eq. (29) and λ_0 is the heterodyne wavelength, expressed as $\lambda_0 = 1/\sigma_0$. Substitution of the heterodyne wavenumber into Eq. (32) yields

$$I(\sigma) \propto \cos \left[\frac{2\pi x}{f_{Re}} \sqrt{2} \alpha (\sigma - \sigma_0) \right]. \quad (34)$$

Thus, the detected interferogram is spatially heterodyned by a constant value σ_0 .

2.2.7.2 Preliminary Calibration Results from Prior Work

The FLD is calibrated using a wavelength tunable light source around 656 nm. Initially, the alignment and calibration was performed with a HeNe laser (633 nm), as it is readily available and has a long coherence length. Fig. 18 shows the experimental calibration setup for our snapshot imaging FLD.

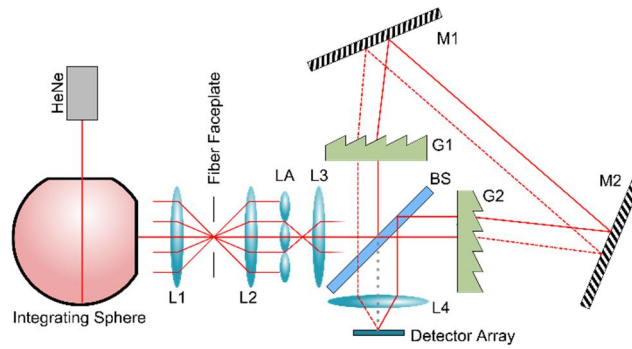


Fig. 18. Experimental setup to calibrate the system using a HeNe laser (633 nm).

This calibration setup consists of the DCSI that images an integrating sphere. As is depicted in Fig. 19, illuminating the sphere with a HeNe laser allows interference fringes to be imaged across the entire field of view of the sensor. It should be mentioned that vignetting is occurring on the last two columns of lenslets in the array; however, this is an issue that is currently being resolved.

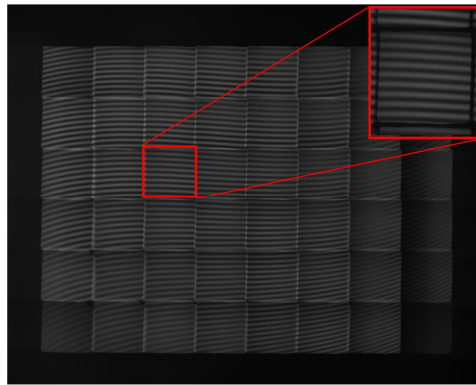


Fig. 19. Image of an integrating sphere illuminated by HeNe laser. The fringes are visible uniformly over all the lenses. Note that the system had been optically aligned once this image had been taken.

Aligning the system is challenging. For instance, when we first illuminated the sensor with the high coherence laser, fringes were not present all over the lenslets as previously shown in Fig. 19. Realistically, only a few of the fringes, near 0 OPD, were visible. To align the system, L1, L2, L4,

the detector and the fiber face plate were removed and a collimated laser beam was shone directly into the system's lenslets and the image was observed on a screen. As depicted in Fig. 20, many of the lenslets' pupils contain more than one fringe. This is ultimately far from the ideal value of less than 1 fringe, which is needed to maintain high visibility interference at the focus of each lenslet. This inevitably results in the reduced visibility of the observed interference fringes.

Initially we had high frequency fringes at the corners and low frequency fringes at the center. The adjustment of the mirrors started with tilting the mirror M1 by clockwise rotation of knob M1_2. This adjustment pushed the high frequency fringes away from the center such that we had one fringe in all the lenses along one diagonal. Along the other diagonal, the fringe frequency increased outward from the center with a highest frequency of three fringes in the corner lenses. It was followed by rotating M2_2 clockwise, which resulted in the high frequency fringes moving upward. This ultimately caused the top 2 rows of the 8×6 lenslets to produce 2 fringes each while the bottom rows had 1 fringe each. Now, counterclockwise rotation of M1_1 reduced the fringe frequency in the upper rows but also increased the fringe frequency of the bottom most row. At this point, the position of the lenslets was adjusted until we had the lowest fringe frequency all over the lenslet array. Once aligned, full-field interference (sans the vignetted regions) were obtained with the HeNe laser.

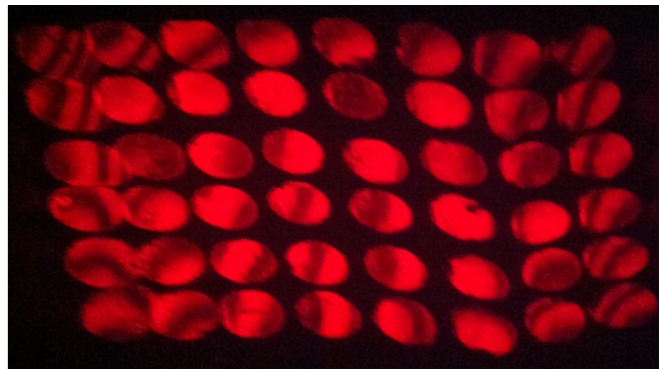


Fig. 20. The image on the screen of the system before alignment.

Additionally, imaging “white light” with the system demonstrates its behavior under low coherence illumination. In this case, “white light” (or spectrally broadband) light was created using a 4 nm bandpass Hydrogen- α filter (Astrodon), which was placed in the collimated space between lens L2 and the lenslet array. The integrating sphere was illuminated with a tungsten halogen lamp and the interference fringes recorded. This image is depicted in Fig. 21.

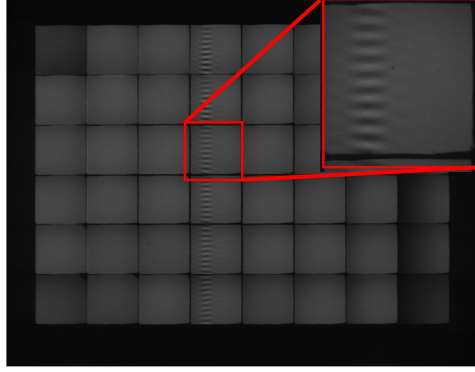


Fig. 21. The image of diffuse white light that was filtered using a $H\alpha$ filter.

To test the system, we used two different light sources to prove that the system is sensitive to wavelength. Fig. 22 shows the images of fringes for 659.6 nm and 669.9 nm. In this case, a tunable laser with a coherence length of approximately 3 mm is being used to illuminate the system, while the maximum OPD in the system is approximately 2.5 mm. As the wavelength changes, the shape and frequency of the fringes change. So we have minimum fringe frequency at 633 nm and thus our system is calibrated such that it is heterodyned at that wavelength and is sensitive to any change of wavelength.

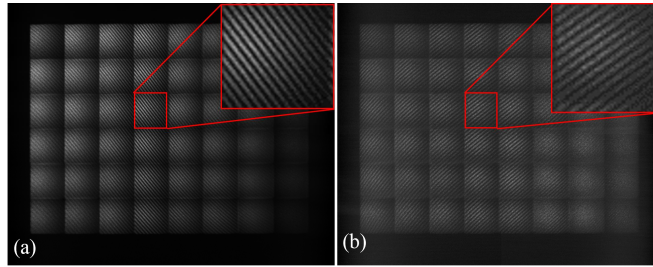


Fig. 22. Images of fringes seen through the system for two different wavelengths (a) 659.6 nm, (b) 669.9 nm.

Finally, a scene with high spatial information was imaged to check if the system can be used to take advantage of the snapshot imaging technique as depicted in Fig. 23.

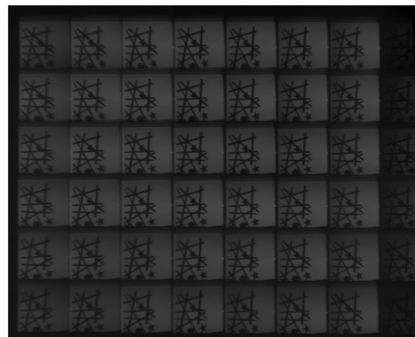


Fig. 23. Image of a scene containing spatial information.

2.2.7.3 Calibration Results Performed on This Program

As of the paper published in [9], we had not yet performed a full calibration. However, as part of the current program, we spent some time with the tunable diode source to investigate whether or

not such a calibration could be performed. This yielded mixed results. Specifically, as the laser diode temperature is changed, so does the thickness of the laser cavity. This shifts the wavelengths at which the various interference orders occur, and complicates the calibration procedure. Additionally, while the system is indeed common path, significantly more engineering may be required in order to ensure the system's stability. As illustrated in the photo per Fig. 24, the system currently exists using off-the-shelf optomechanical components.

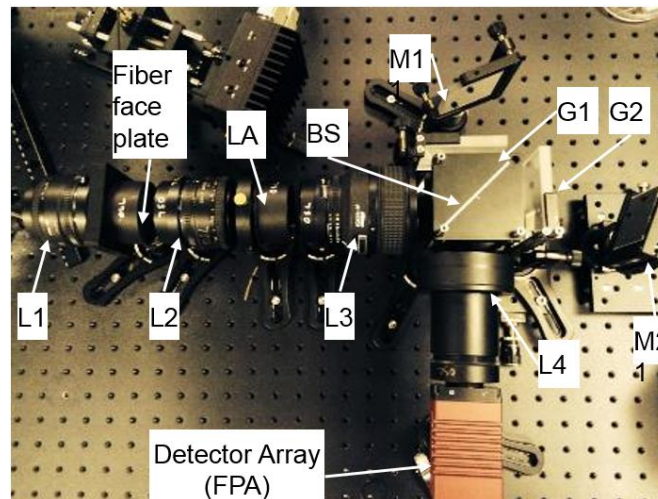


Fig. 24. Photo of the benchtop Fraunhofer line discriminator, based on a Sagnac interferometer.

It is thought that this could be yielding small temperature dependencies during calibration, which are further complicating the calibration procedure. Finally, while any design leveraged for this program would be polarization-based, the current system is not. Thus, it is nearly impossible to obtain a 'pure' flatfield, in which interference fringes are not present. This is a critical dual-beam calibration requirement, in which systematic differences across the various lenslets in the system can be characterized. Ultimately, these complexities may make such an embodiment challenging to fully develop within the confines of this program; however, it is not fully ruled out yet. Thus, alternative systems have also been investigated, as lead by the simulated requirements for the signal to noise ratio and orbital fitting procedures.

2.3 Optical System Modeling – Dual Band (or Dual Beam) Doppler Ratio

As an alternative to fully sampling the spectra of one or two Fraunhofer lines, using one of the two aforementioned techniques, other designs were also investigated. In these subsequent systems, one or several passbands are transmitted through a Lyot filter. These birefringent filters can achieve high spectral resolution. While such a filter would not be snapshot if it were used to fully sample a spectrum, such sampling may not be necessary since only the Doppler shift is of interest. In this case, the Doppler shift would be quantified based on a measurement of in- and out- of band radiance. Similar systems have been configured before, using molecular absorption filters, to measure the Doppler shift of incident (or reflected) laser light. Such an example is in Laser Doppler Velocimetry, in which the motion of molecules creates a Doppler shift in the scattered incident laser light [17]. Transmission of this scattered light, as compared to the incident laser light, yields a ratio that can then be used to measure the velocity of the scattering particles.

2.3.1 Design Variant 1 – Independent Lyot Stages Using Beamsplitters

The first approach offers the most degrees of freedom within the design, and allows for the greatest sensitivity to the detected Doppler shift being measured from the input light. A view of the system is depicted in Fig. 25. It consists of two dichroic beamsplitters (DBS1 and DBS2), which reflect and transmit relatively wide (~ 70 to 100 nm FWHM) spectral passbands into three optical sub-systems. Each sub-system is denoted with its own narrow spectral bandpass filter (BPF), denoted as BPF1, BPF2, and BPF3. These bandpass filters isolate one or two resonant modes of the subsequent Lyot filter, which is shown in this image as a two-stage birefringent filter as created by retarders R1 and R2. It should be noted that each retarder has an additional subscript, since each retarder's thickness can be optimized for its own spectral passband. Beyond this, each stage is identically configured with respect to its exact sequence of polarization elements. First, a linear polarizer (LP1) polarizes the incident light to be parallel to the y axis, and also cleans up the polarization artifacts that may have been introduced in transmission through the DBS or BPF. This light then transmits through the first retarder, with fast axis oriented at 45 degrees with respect to the x axis in stage 3, or the z axis in stages 1 or 2 (for clarity, only stage 3 will be discussed from hereon). Light then transmits through a first quarter wave plate (QWP 1), which converts the linear polarization exiting the retarder into circular polarization. This light then transmits through a second linear polarizer LP2, which has a transmission axis that is nominally parallel to the y axis. A second Lyot stage is also included, in which a second retarder (R2), with twice the thickness as retarder R1, retards the light before a similar conversion into circular polarization through QWP2 and transmission through a linear polarizer LP3. Subsequent Lyot stages can be added by inserting additional retarders with 2^N increments in retarder thickness, and the phase of the interference effect can be manipulated by rotating the polarizers and retarders in a similar fashion in 2^N increments [18].

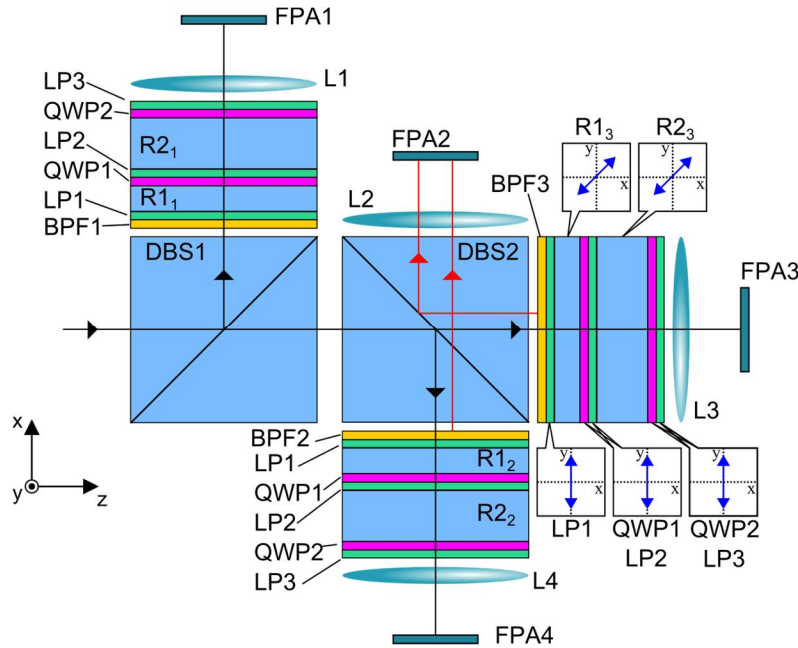


Fig. 25. Optical schematic of a three-stage filter (FPA1, FPA3, FPA4) with a pseudo-broadband intensity reading using FPA2.

Three narrowband intensity measurements are recorded at FPA1, FPA3, and FPA4. Additionally, FPA2 is included to measure light that is initially rejected from BPF2 and/or BPF3. In regions of reflection and transmission where DBS2 has close to a 50/50 transmission to reflection ratio, FPA2 will measure the incident light. This light can be used to capture an ‘unfiltered’ measurement to which all other intensity measurements can be normalized. It should be mentioned that co-registration of the image may be a concern when merging the light from BPF2 and BPF3 onto a single camera; thus, tip and tilt adjustability may need to be included in at least one of these filters.

It should be noted that while the design, depicted previously in Fig. 25, contains three spectral bands that contain a two-stage Lyot filter within each, any number of bands can be combined to form a single system. This can include multiple two- or three-stage filters, filters that act on two passbands simultaneously, or a filter optimized to act exclusively on a single passband. For example, BPF1 may transmit narrowband light around two separate spectral passbands, such that light is integrated by FPA1 from both passbands. Alternatively, BPF2 may transmit only light from one passband, such that the corresponding Lyot filter only has to be optimized for that particular Fraunhofer line.

2.3.1.1 Filter Optimization

Several designs for single, double, and triple filter stages were produced. Some included designs in which two passbands were filtered by one optimized Lyot filter, while others were optimized for three individual spectral passbands. Optimization of a Lyot filter for a particular Fraunhofer line or band was accomplished using the following procedure:

- 1) Simulate correlation function within the band to choose highest relative SNR using *HarmonicAnalysis*
- 2) Simulate the best thickness d to phase the optimal phase theta using *Calcite_Mg_MainFitter*.
- 3) Simulate the SNR performance of the filter using *Calcite_DualbandPlotter*.

The functionality of each program is described in the subsequent sections.

2.3.1.1.1 Function: *HarmonicAnalysis.m*

This function loads the solar spectrum, per Reference [19]. The user specifies the spectral passband across which to correlate a range of Lyot filter parameters. Lyot filter parameters are simulated using the birefringence data from Ref. [20] and by stepping across a range of thicknesses, d , of the first stage. This stage sets the free spectral range (FSR) of the filter. As an example, scanning the thickness of the first stage d from 1 mm to 3 mm in 50 increments yields the following output per Fig. 26. In this case, the top panel of Fig. 26 contains the correlation function. Two quality metrics are then defined from these data. The first is a visibility (or contrast) function, defined as

$$V = (C_{\max} - C_{\min}) / (C_{\max} + C_{\min}), \quad (35)$$

where $C_{\max}$ and $C_{\min}$ represent the local max and min of the correlation function. Meanwhile, the Noise Contrast is defined relative to the total integrated signal, and yields a *relative* notion for the background limited performance of a particular system. This is defined as

$$NC = (C_{\max} - C_{\min}) / \sqrt{C_{\max} + C_{\min}}. \quad (36)$$

This metric is included since high correlations, or high visibilities, can be achieved for Lyot filters that contain narrow spectral transmission peaks. In such cases, the signal may contain a large background radiance, yielding a lower SNR than would be achieved with a band that has a lower visibility, but similarly lower background radiance.

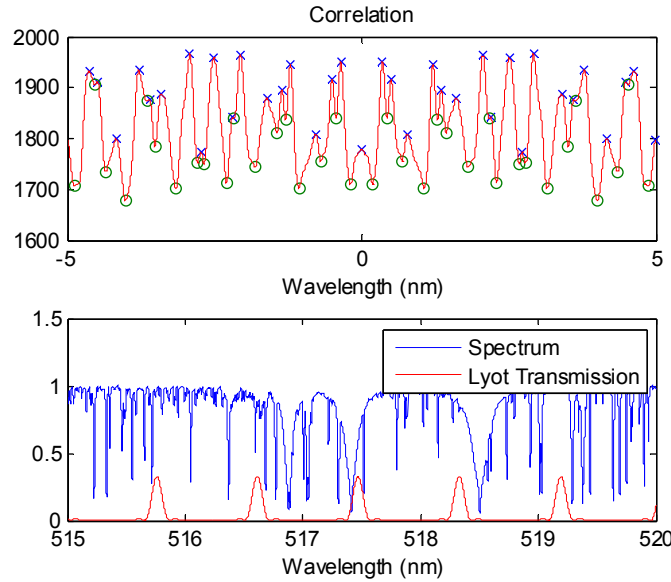


Fig. 26. (Top) Correlation function vs wavelength and (bottom) spectral transmission spectrum of the sun plotted alongside the Lyot transmission of the filter under test.

Plotting these metrics (V and NC) versus filter thickness yields a metric plot depicted in Fig. 27.

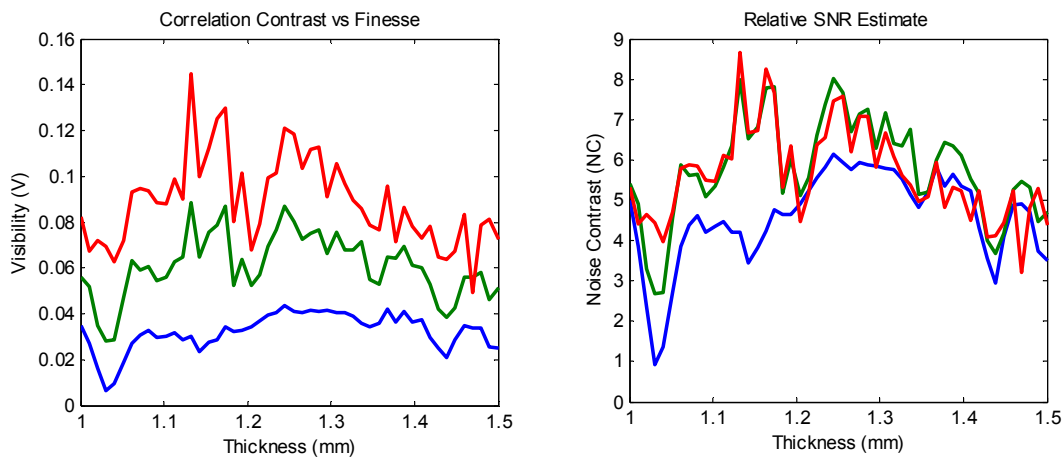


Fig. 27. (Left) Visibility (V) and (Right) Noise Contrast (NC) versus the thickness of the first stage. Blue, green, and red lines indicate 1, 2, and 3 stage filters.

Meanwhile, the critical parameters for this function are provided in Table 3.

Table 3. User defined inputs for function *HarmonicAnalysis*.

Name	In/Out	Units	Description
d	In	Meters	Vector containing thickness of first stage to scan over.
Sigmaview	In	nm^{-1}	Wavenumber range, in inverse nanometers, across which to perform the correlation analysis.
Tx	In	Unitless	X-transmission (or peak transmission) for the linear polarizer used in the simulation.
Ty	In	Unitless	Y-transmission (or peak transmission) for the linear polarizer used in the simulation.
Vismax	Out	Unitless	Visibility (or contrast) of the correlation for each local maxima and minima in the correlation function.
NoiseContrast	Out	Unitless	Calculated as the maximum minus the minimum
Bestcoeffs	Out	Degrees	Contains the optimal value of thetao, which relates to the rotation angle of each stage within the Lyot filter to set the correct filter phase.

2.3.1.1.2 Function: Calcite_Mg_MainFitter.m

The next step in the design optimization process is determining the appropriate phase of the Lyot filter. From the correlation plots, the phase is calculated via Fourier transformation to increase speed. Otherwise, the phase must be adjusted by rotating the Lyot filter stages, and corresponding quarter wave plates and linear polarizers, by appropriate amounts. In order to determine the phase, a least squares fitting procedure is used within the *Calcite_Mg_MainFitter* function. This function calls a sub-function, *Calcite_Mg_MainFitFunction*, which takes the phase offset coefficient as a variable and sends it into Matlab's built-in *fminsearch* subroutine. Within this function, parameters for the bandpass filter model are passed, including primarily the refractive index of the high and low layers. This ensures a realistic model for the bandpass filter by using the matrix thin film calculation.

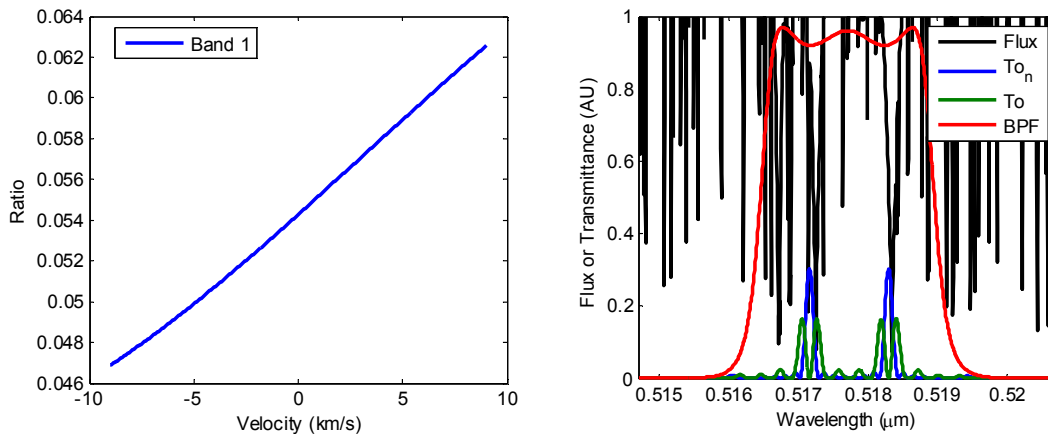


Fig. 28. (Left) Ratio versus velocity during optimization and (Right) the spectral passband of the Lyot filter, shown for analyzing linear polarizer orientations of 90 degrees (blue, To_n) and 0 degrees (green, To). Additionally, the flux spectrum and bandpass filter spectra are illustrated.

A summary of the variables, passed between the different functions for this stage of optimization, are provided below in Table 4.

Table 4. User defined inputs for function *Calcite_Mg_MainFitter*.

Name	In/Out	Function	Units	Description
do	In	Calcite_Mg_MainFitter.m	Meters	Thickness of the Lyot filter's first stage.
coeff	In	Calcite_Mg_MainFitter.m	Degrees	Starting estimate for the phase shift. Phase shift is twice this value; e.g., inserting 0 to 180 will phase-shift the passbands from 0 to 360.
Lambda_bp1o	In	Calcite_Mg_MainFitFunction.m	Microns	Center wavelength at which to simulate the filter.
StageType	In	Calcite_Mg_MainFitFunction.m	Unitless	0 if absorption filter, 1 if Lyot filter. Usually 1.
Nstages	In	Calcite_Mg_MainFitFunction.m	Unitless	Number of stages to simulate in the filter. Higher numbers yield narrower lines, but also less transmission.
Polfactor	In	Calcite_Mg_MainFitFunction.m	Unitless	Factor by which to multiply the signal to account for the polarization converter.
NH	In	Calcite_Mg_MainFitFunction.m	Unitless	Refractive index of high index layer in the bandpass filter. Increasing this narrows the filter, decreasing widens.
NL	In	Calcite_Mg_MainFitFunction	Unitless	Refractive index of the low index layer in the bandpass filter. This is typically left at 1.33.
RTelescope	In	Calcite_Mg_MainOptimizer	Meters	Radius of the telescope to simulate. This is fed into the radiometric model.
Tauint	In	Calcite_Mg_MainOptimizer	Seconds	Integration time to simulate for the camera. This also goes into the radiometric model function.
Dlambda	In	Calcite_Mg_MainOptimizer	Microns	Bandwidth over which to sample for the simulation. Be sure Dlambda > the BPF width, set by NH above.
Velocity	In	Calcite_Mg_MainOptimizer	Meters / Second	Velocity range over which to simulate. Some absorption filters are best simulated starting at 0.
T	In	Calcite_Mg_MainOptimizer	Celcius	Temperature of the Calcite birefringent filter in C. This can be used to study temperature dependence. The coefficient of linear expansion is also applied.
Tx	In	Caclite_Filter_Simulation_Lyot_Function	Unitless	Tx transmission, set within the function that builds the radiometric model.
Ty	In	Caclite_Filter_Simulation_Lyot_Function	Unitless	Ty transmission, similar to Tx above.

Table 5. User defined inputs for function *Calcite_DualBandPlotter*.

Name	In/Out	Units	Description
dV	In	Meters	Thickness of the Lyot filter's stages. Vector has dimensions 1xNb, where Nb is the number of bands.
thetaV	In	Degrees	Phase offset for each of the Lyot filter's stages. Dimensions 1xNb.
StageType	In	Unitless	0 or 1. 0 for absorption, 1 for Lyot. Generally set to 1. Dimensions 1xNb.
NH	In	Unitless	High refractive index for each of the stage's bandpass filters. Each filter can be defined uniquely by this vector. Dimensions 1xNb.
NL	In	Unitless	Same as NH above, but for the low refractive index. Dimensions 1xNb.
Nstages	In	Unitless	Number of Lyot filter stages to simulate within each band. Dimensions 1xNb.
Plotval	In	Unitless	1 or 2. 2 sets optimization (previous step) while 1 is best for plotting (this step)
Lambda_bp1o	In	Microns	Sets the center wavelength for each filter stage. Dimensions 1xNb.

2.3.2 Noise Simulations Performed on Optimal Lyot Filter

These designs are summarized here assuming the following telescope parameters per Table 6.

Table 6. Parameters for simulation of Lyot filter. Modeled after the Sony ICX694 sensor in the Apogee Ascent 694.

Parameter	Value	Parameter	Value
τ_{optics}	0.66 (w/ Pol. Converter)	R_T	0.45 m
τ_{atmo}	0.90	O_{Sat}	42.2E6 m
Integration Time	5 Minutes	N	1
Quantum Efficiency	0.71 electrons/photon	M	1
Read Noise	5 Electrons	S_{Sat}	1 m
Dark Noise	0.005 electrons/sec/pixel	A_{Sat}	1 m ²
R_e	6.371E6 m	ρ_{Sat}	0.70
D_{Sun}	149.6E9 m	R_{sun}	695.8E6 m
T	6000 K	$\Delta\lambda$	Lyot Filter Setting
λ_0	517.7, 656.3, and 589.25 nm	$A_{detector}$	13.5 microns square

Following the aforementioned optimization procedure yielded three Lyot filters, optimized for the Mg I (~518 nm), H-Alpha (~656 nm), and Na I (~589 nm) Fraunhofer lines. Parameters for these filters are provided below in Table 7.

Table 7. Parameters for the optimal Lyot filters for each of the three bands.

Band	Thickness	Orientation	Lamba (nm)	NH*	NL	NStages
------	-----------	-------------	------------	-----	----	---------

1 (Mg)	1.206 mm	-11.50 Deg.	517.70	2.6	1.33	3
2 (H-a)	1.206 mm	40.21 Deg.	656.30	2.7	1.33	3
3 (Na)	2.933 mm	-32.90 Deg.	589.25	2.9	1.33	3

*Note: While these refractive indices are not physically realizable, the necessary full-width at half maximum bandwidth of the passband, defined by these values, is realizable with more optimal thin film filter designs. Such designs will be requested from filter vendors.

It should be noted that, before constructing any such filters, the true temperature dependence of the calcite crystal will be tested first on samples from our chosen vendor. However, some tunability will be designed into the filter stages through the use of temperature tuning. Results from this simulation are depicted below in Fig. 29 in which optimal Lyot filters, consisting of three stages each, are illustrated. Here, Fig. 29 (a-c) represent the Lyot filter and Bandpass filter transmission spectra for the Mg I, H-alpha, and Na I Fraunhofer lines. Meanwhile, Fig. 29 (d) shows the calculated ratio for each of these bands.

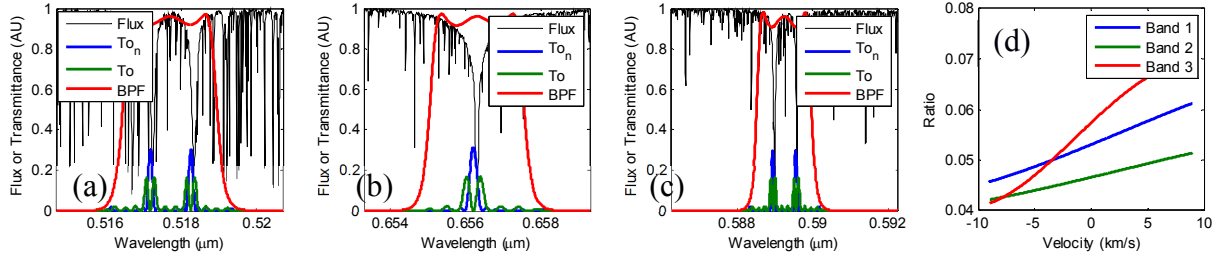


Fig. 29. Lyot filter transmission functions, To_n and To , along with the bandpass filter transmission function (BPF) and the incident normalized solar flux for the (a) Magnesium I; (b) Hydrogen-alpha; and (c) Sodium Fraunhofer lines. (d) represents the intensity ratio, taken relative to the BPF passband, versus velocity for the three bands (a-c) representing bands (1-3), respectively.

Independent velocity measurements are calculated using these three bands, and then averaged to create a single value for the simulated measured velocity. Simulating shot noise on the detected signal current yields the simulated measured velocity versus true velocity in Fig. 30. Noise on this signal yields a standard deviation of 404 m/s with a single 5 minute measurement, 128 m/s for a 50 minute integration time, and 40 m/s given a 500 minute integration time.

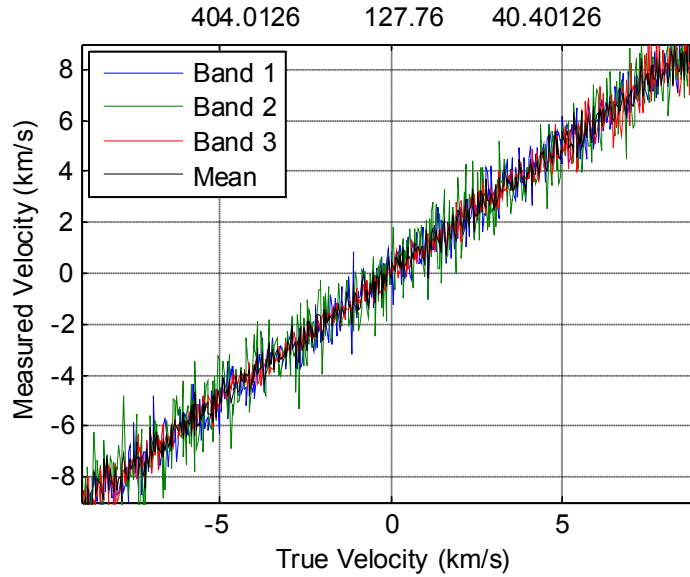


Fig. 30. Simulated measured velocity, with noise, versus true velocity in km/s. velocity is assumed to contain the maximum Doppler frequency shift (e.g., 2 times the true velocity). Simulation is performed assuming a 0.9 meter aperture.

Alternatively, use of a larger telescope (3.5 meter diameter) yields the velocity plot depicted in Fig. 31. For this larger aperture, the standard deviation is calculated to be 97 m/s for a single 5 minute exposure, 31 m/s for 50 minutes, and 9.7 m/s for 500 minutes.

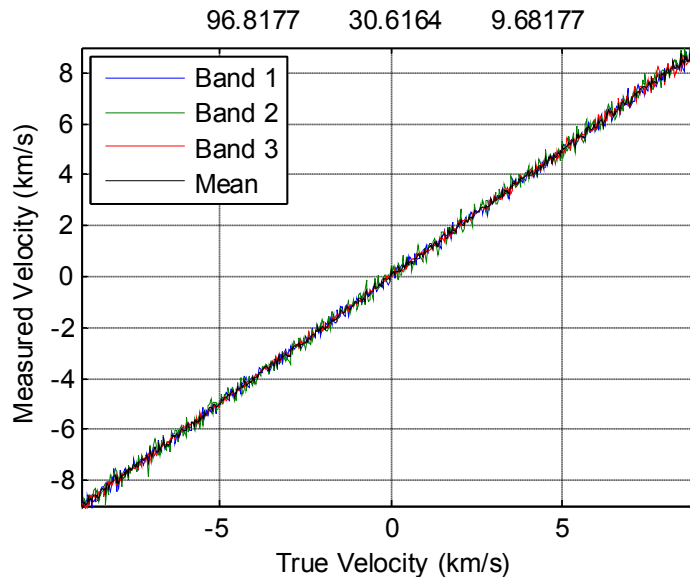


Fig. 31. Simulated measured velocity, with noise, versus true velocity in km/s for a 3.5 meter diameter aperture.

2.3.3 Lyot Filter Field of View Dependence

Zemax was used to assess the field of view limitations of the Lyot filter under a variety of different operating conditions. A simulation of the Mg I or Hydrogen-alpha Lyot filter was conducted for a telescope operating at F/6 over a ± 0.3 degree field of view. First, a non-field widened Lyot filter

was simulated, in which the filter was placed between a negative and positive barlow lens. A view of the optical system is depicted in Fig. 32. This yielded the results per Fig. 33 for a 3 stage calcite filter, where the first stage had a thickness of 1.2 mm, the second 2.4 mm, and the third 4.8 mm.

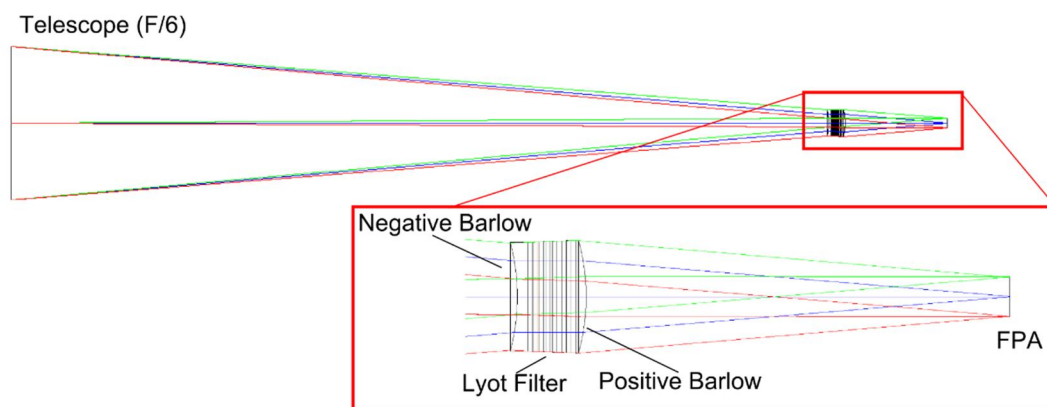


Fig. 32. Optical system for the double-Barlow Lyot filter configuration.

A simulation of the filter performance was conducted to study the filter's field of view dependencies. This required that a polarization ray trace be performed from Matlab, using the MZDDE direct data exchange toolbox. Linear polarizers were simulated in the optical design, and the intensity of the polarization ray trace read directly into Matlab. Rays were traced across the entire pupil of the telescope's F/6 entrance aperture for 8 fields of view, spanning -0.3 to 0.3 degrees. Polarization plots of the pupil, for the transmitted intensity, are depicted in Fig. 33. In this case, a distinct hyperbolic pattern is visible, where significant nonuniformity is present as function of field position.

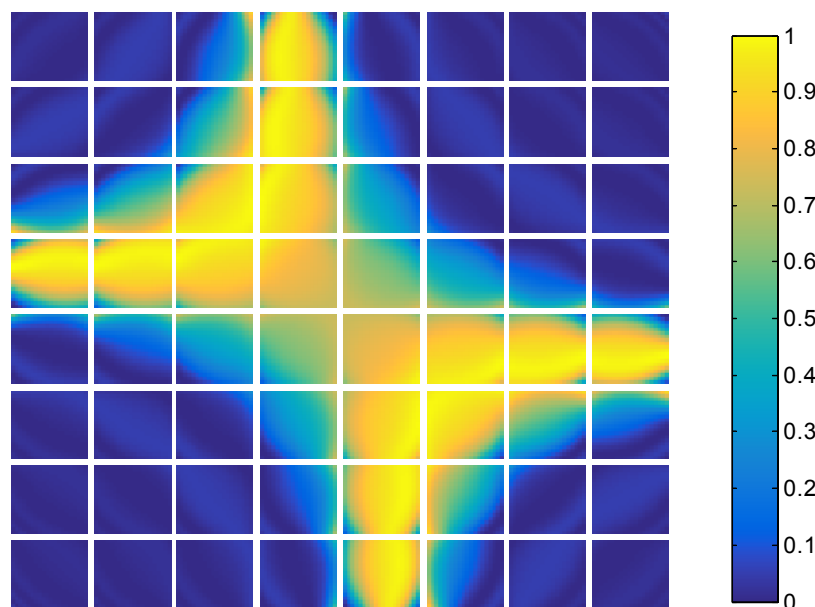


Fig. 33. Simulated field of view dependence in the Lyot filter. Shown are pupil maps, as would be viewed for light transmitted through crossed linear polarizers. Meanwhile, each sub-image within the array denotes a different field angle; *e.g.*, the upper-left sub-image shows $h_x, h_y = (-0.3, -0.3)$ degrees, while the lower-right sub-image shows the pupil map for $h_x, h_y = (0.3, 0.3)$ degrees.

In order to better compensate the field of view, field widened retarders were incorporated into the design by splitting each birefringent plate into two sections. A half wave plate was then inserted between the two plates at an angle 45 degrees with respect to the calcite crystal's fast axes. The configuration included two 0.6 mm thick retarders in stage 1, two 1.2 mm thick retarders in stage 2, and two 2.4 mm thick retarders in stage 3. Simulations of this field widened retarder yields the results per Fig. 34. In this case, the performance off-axis is significantly enhanced; however, it still yields much to be desired in terms of uniformity within the pupil and across the field of view.

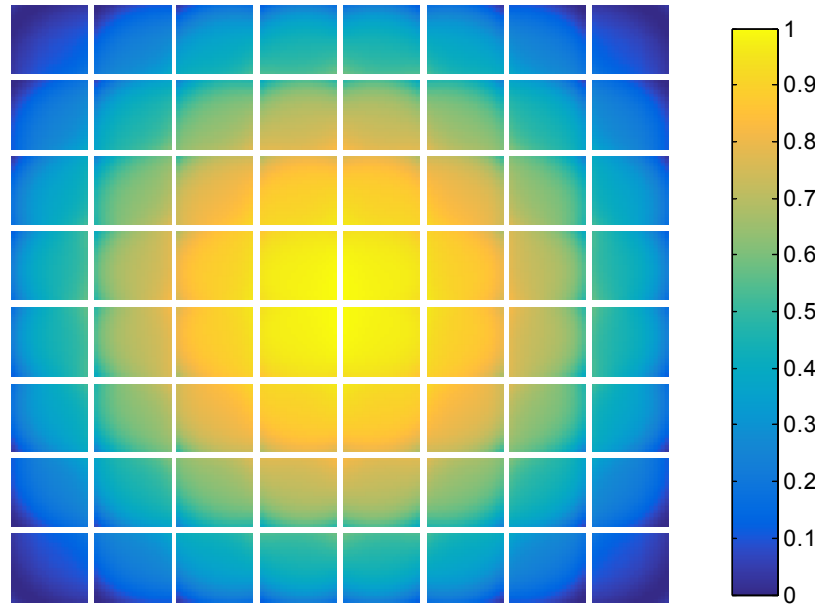


Fig. 34. Simulated results of the field widened Lyot filter. In this case, only barlow compensation was used to compensate the focal ratio of the telescope.

The nonuniformity is also depicted in plots of the spectral transmission for the same fields of view. Results of this are depicted in Fig. 35, indicating transmission spectra for a linear polarizer oriented at 90 degrees and 0 degrees.

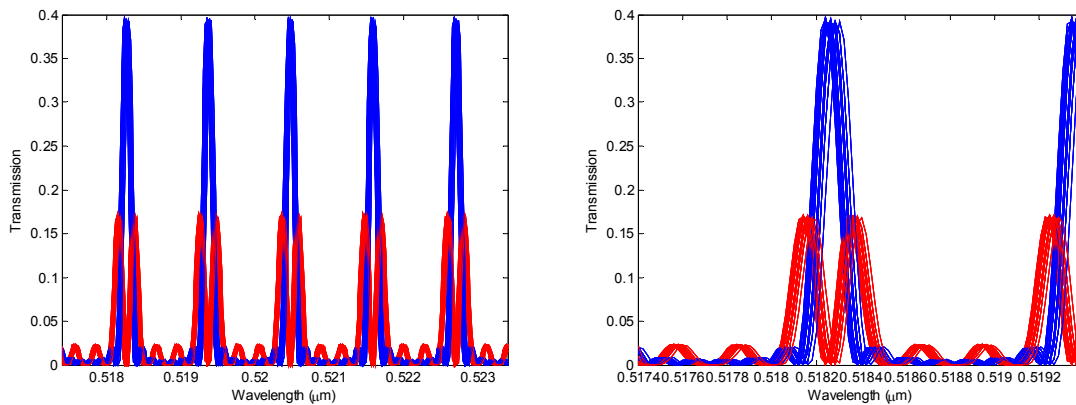


Fig. 35. Simulated measured velocity, with noise, versus true velocity in km/s for a 3.5 meter diameter aperture.

In order to further remedy issues with field of view and pupil uniformity, additional optics are required. While the negative and positive barlows correct for angular issues across the pupil, they do not compensate for angle introduced by field. To compensate for this, the rays exiting the telescope must be telecentric [21]. To achieve this, a positive lens was inserted into the intermediate image of the telescope's focal plane and a negative barlow lens was added to increase the telescope's focal ratio from F/6 to F/16. A view of the first order optical design for this concept is illustrated in Fig. 36.

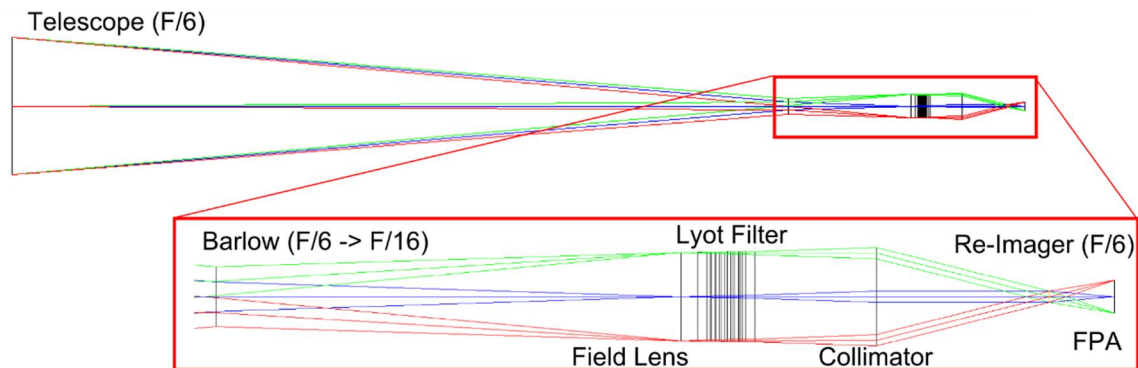


Fig. 36. First order optical design, showing a 3 stage Lyot filter with a 1.2 mm thick first stage.

Performance of this design is significantly improved, in that there is no variation in the performance of the Lyot filter versus field of view. However, the performance across the pupil maps varies by approximately 5% peak-to-peak. This yields a reduction in the contrast of interference fringes from a visibility of 1.0 to a visibility of xx.

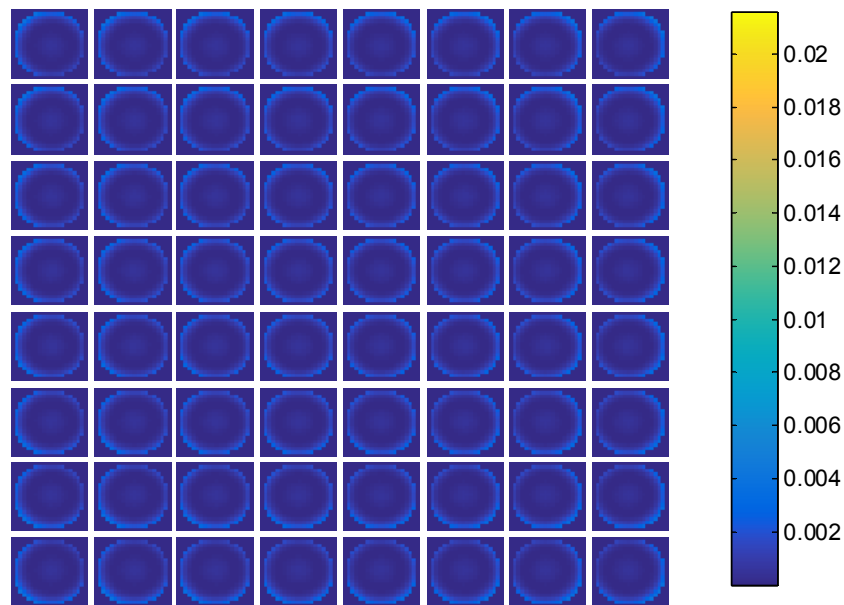


Fig. 37. Simulated results of the field widened Lyot filter with both focal ratio compensation and telecentric correction. A circular mask has been applied to these results to trim off regions of the pupil that would not exist on a rectangular normalized coordinate system.

To demonstrate the effect of the remaining field of view dependence, a simulation was performed for the $h_x = h_y = 0$ degree field of view. Two transmission functions were calculated at several wavelengths by (1) selecting only one position in the pupil, $P_x = P_y = 0$; or (2) averaging across all pupil positions. The results for these two cases are illustrated in Fig. 38, along with the scaled absolute transmission error. Ultimately, integrating across the pupil serves to shift the passband by a small amount, equal to 0.0028 nm. Ultimately, this small shift can be accommodated for by calibration.

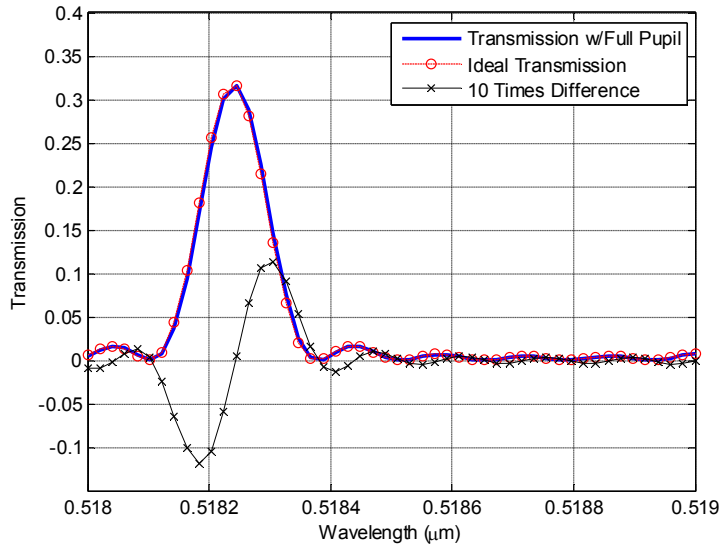


Fig. 38. Lyot filter transmission when averaging across the entire pupil (Transmission w/Full Pupil, solid blue line) or when calculated using only the $P_x = P_y = 0$ position (Ideal Transmission, red circles). The absolute error, scaled by a factor of 10, is also shown (solid black x line).

2.3.4 Design Variant 2 – Molecular Absorption Filters

One disadvantage with Lyot filters, or interference filters in general, relates to their angular dependence. As demonstrated in the previous section, the filter's field dependence can be reduced to negligible levels. Varying angle through the filter will create a change in the optical path difference as a function of both field of view and the telescope's focal ratio. While some of the Lyot filter stages, presented previously in Table 7, would be relatively thin and are unlikely to experience significant effects, one of them – the sodium filter – has a third stage thickness of approximately 11.6 mm. In such a case, it may be more optimal to have absorption filters, made of other gaseous materials, that can be used to estimate the value of the Doppler shift.

To investigate this, a sodium Faraday rotation filter passband was simulated to probe the sodium Fraunhofer line [20]. The typical bandwidth of this type of filter is on the order of 16 GHz full width (defined as 10% of the maximum), while full width at half maximum values are typically 2 GHz. From Ref. [20], the typical transmission spectrum for a filter, optimized at the D2 line, is illustrated below per Fig. 39.

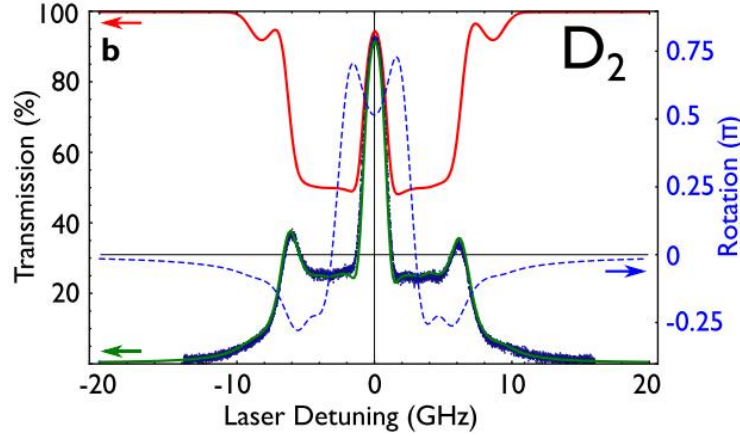


Fig. 39. Typical D2 sodium line transmission spectrum for a sodium filter at 200 mT magnetic field strength and a temperature of 153 degrees C. Note that the calculated transmission of the filter excludes the first linear polarizer's transmission losses. Figure repeated from Ref. [20].

Simulating this transmission spectrum, as applied to observations made of a satellite at the D2 Fraunhofer solar line, are illustrated below in Fig. 40 (c), next to the Mg I and H-alpha Lyot filter transmission spectra per Fig. 40 (a-b). Meanwhile, the detected intensity ratio vs velocity for each of these filters is provided in Fig. 40 (d). Notable is that the slope of the ratio for the absorption filter is nearly identical in magnitude to the previous Lyot filter designs; however, the offset in the ratio is significantly lower. This means that there is less background signal, which should yield a smaller background noise contribution than the previous filters.

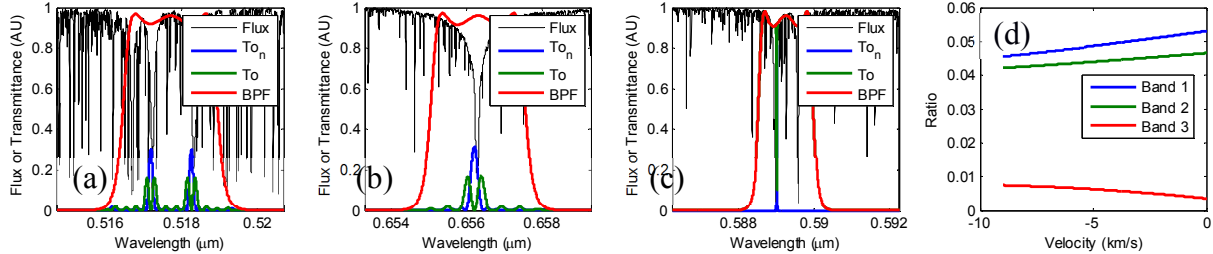


Fig. 40. Sodium Faraday filter transmission spectrum performance, as calculated using a 0.9 meter telescope aperture with the same aforementioned simulation parameters.

To investigate this effect, the simulated Doppler velocity measurement error for the single band Na filter is illustrated below in Fig. 41. In this case, the velocity was simulated from -9 km/s to 0 km/s, illustrating a disadvantage of the absorption-based approach. When the gas in the cell is based on the same constituent as the observed Fraunhofer line, the filter's transmission is always centered on the line when the Doppler shift $D = 0$ km/s. For a symmetric Fraunhofer line, this yields minimum sensitivity at 0 km/s, with increasing sensitivity for greater absolute (positive or negative) Doppler shifts. Additionally, the *sign* on the Doppler cannot be quantified with a single measurement. In the case of the Sodium D2 line, the minimum sensitivity occurs for positive Doppler values close to 4 km/s, due to the presence of a nearby iron absorption feature (Fe I). Thus, a second filter system, based on either a different (but nearby) absorption feature, or based on one of the Lyot designs depicted previously, would need to be included in the system. This yields sufficient degrees of freedom to enable the passband to be offset slightly from the center, such that a unique ratio can be obtained for a finite range of Doppler shifted frequencies.

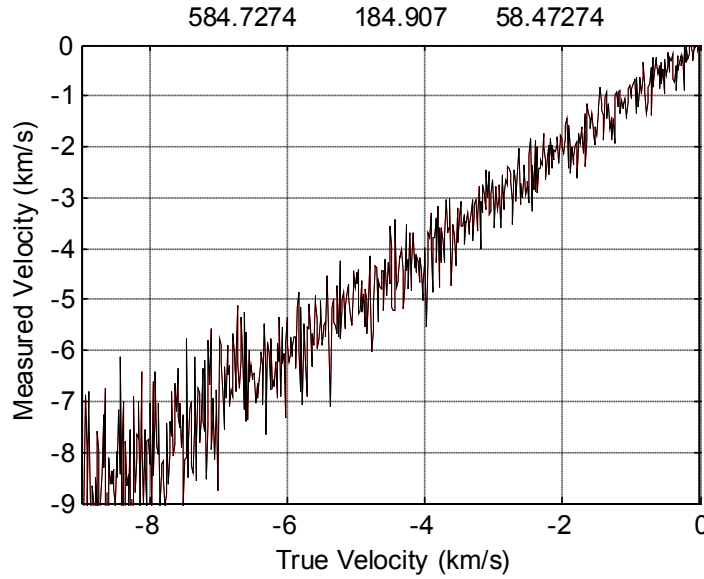


Fig. 41. Simulated measurement error vs true velocity for the single Na filter band. A standard deviation of 585 m/s, 185 m/s, and 59 m/s is obtained for 5, 50, and 500 minute exposures.

Unfortunately, sodium itself may not be a viable Fraunhofer line to observe, due to the presence of sodium emission (skyglow) in the terrestrial atmosphere [22]. Further simulations will be conducted in our future work to assess this risk. Additionally, since the Lyot filter can be tuned to be off-resonance with the dominant peak emission of any constituent, it will also be re-assessed in the presence of Na skyglow.

Finally, other absorption filter designs will be assessed, including designs that could be used to ‘stack’ appropriate molecular or atomic absorption lines from other constituents. In this way, it may be feasible to create a gas filter that, while strictly absorption-based, optimally transmits the chosen Fraunhofer band.

3.0 Spatial Heterodyning and Signal Aliasing

3.1 Spatial Heterodyning

For the purpose of improving the signal-to-noise ratio (SNR) and sensitivity of the Doppler instrument, we have explored the use of spatial heterodyning of the optical signal. In a conventional two-beam Michelson interferometer, the relationship between the optical path difference and the fringe order is

$$OPD = m\lambda, \quad (37)$$

where OPD is the optical path difference between the beams, m is the diffraction order, and λ is the wavelength. For fixed OPD , the zero-order diffraction fringe represents infinite wavelength; in other words, the Fourier transform of the spatial frequency signal will be centered around

$$\sigma = \frac{1}{\lambda} = \frac{m}{OPD} = 0, \quad (38)$$

where σ is the wavenumber, typically measured in inverse centimeters (cm^{-1}).

A heterodyning interferometer replaces the two plane mirrors of the Michelson with reflective diffraction gratings. In this case the wavelength associated with the Littrow angle of the gratings is retro-reflected back to the beamsplitter and becomes the 0 cm^{-1} wavenumber. Other wavelengths are diffracted to either side of the central wavenumber and interfere to create higher order fringes. In this way, the Fourier transform can be centered around any desired wavelength, simply by careful selection of the gratings' blaze angle.

To take the heterodyning concept one step further, multiple-layer gratings can be used, where each layer retro-reflects a specific wavelength. This results in the overlay of two or more narrow band spectra, one on top of the other. If the bands are chosen wisely, such that the Fraunhofer lines in one band overlap, or nearly overlap, the lines in the other bands, the SNR will benefit from the combination of intensities in each band. This overlapping may also improve the sensitivity of Doppler velocity measurements, as discussed in the next section.

The observed change in wavelength is directly proportional to the wavelength under observation, or in other words, longer wavelengths λ_0 will experience a larger offset $\Delta\lambda$. If several Fraunhofer lines are overlapped, the longer wavelength will be shifted farther than the shorter wavelength for the same velocity. Thus, the amount of overlap between the two lines, which can be directly estimated from an intensity measurement, directly depends on the target velocity.

3.1.1 Heterodyne Simulation

As previously mentioned, there are thousands of Fraunhofer absorption lines within the visible solar spectrum. The criteria for getting the greatest utility, in terms of sensitivity improvement, from overlapping two or more narrow-band segments are not immediately obvious. To help predict which overlapping combinations are most beneficial, an empirical approach was tried. A simple MATLAB simulation was created to Doppler shift three narrow-band samples of measured solar intensity data, then overlap the resulting signals. The steps in the simulation were as follows:

- Randomly select three wavelengths between 358 nm and 684 nm. Although the data file used [23] includes wavelengths up to 741 nm, some of the data points above 684 nm seem to be corrupted (show intensities of -1) and so were eliminated from consideration.
- Extract 11 000 sample points around each central wavelength. The samples are interpolated onto a wavenumber axis such that the distance between each sample is constant.
- For each band, calculate the Doppler shifted spectra for a range of velocities between 0 and 10 000 m/s. This covers the entire range of satellite velocities from all reasonable orbits and angles of observation.
- For each Doppler velocity, average the intensities of all three bands.
- Calculate the differential of average intensity vs. velocity (dI/dv_r) and find the wavenumber at which it is a maximum. Record this value, the three center wavelengths, and the mean and standard deviation for dI/dv_r at this wavenumber.
- Repeat N times starting from the top.

Larger values of dI/dv_r imply greater sensitivity to target velocity.

3.1.2 Results

The simulation results can be loaded from disk and sorted according to their statistics, and interesting sets can be plotted for visual inspection. Over 5000 trials were performed by the simulation. The results for just one of the randomly chosen frequency triplets are plotted in Fig. 42. The interpretation of these results is not immediately obvious, and so deserves some elaboration.

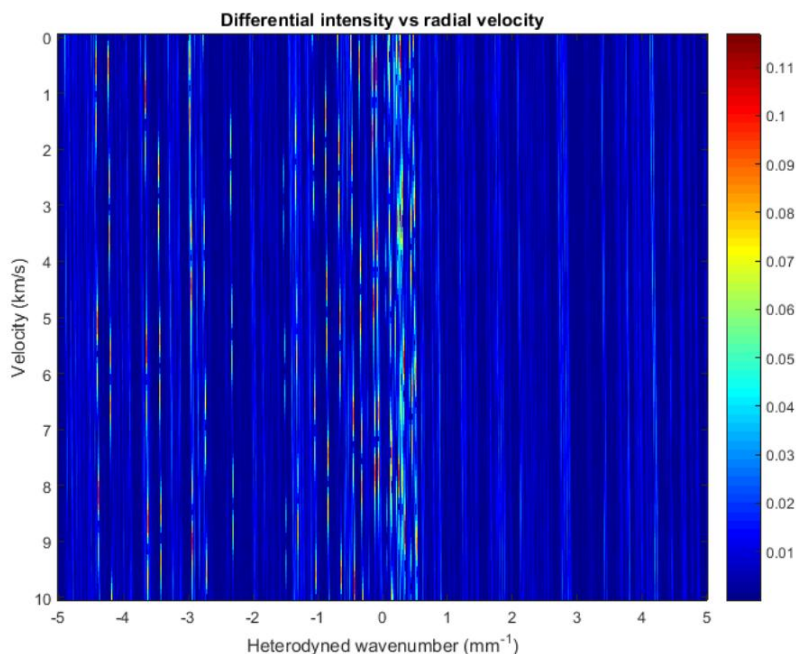


Fig. 42. Differential intensity vs. Doppler velocity for heterodyned frequency bands with centers at 424 nm, 530 nm, and 628 nm.

The x-axis in Fig. 42 represents the wavenumber range of each of the three selected frequency bands after heterodyning to 0 cm^{-1} . The y-axis represents a hypothetical satellite velocity. The color represents the differential intensity of the “stacked” light energy as the velocity is swept from 0 to 10 km/s in steps of 100 m/s. If one observes a “hot” vertical stripe in Fig. 42, it indicates that the change in measured intensity as the velocity changes will be relatively large. This translates to a high sensitivity with respect to Doppler measurement. Maintaining a high sensitivity across the entire range of velocity requires observation at multiple wavenumbers, since as the heterodyned wavelengths shift relative to one another, some lines will overlap at first, then “cross” and separate. From Fig. 42 it is clear that even within a wavenumber range of 10 mm^{-1} , there are enough overlapping lines in the three bands to create “hot stripes” covering every part of the velocity range of interest. These results confirm the hypothesis that the benefit of spatial heterodyning is dependent on the judicious selection of the overlapping Fraunhofer lines.

3.2 Signal Aliasing

Although the Fraunhofer line simulations provided a clear indication of the potential benefit of heterodyning, additional simulations were needed to estimate the amount of sensitivity gain. The effects of noisy measurement data and the overall signal-to-noise ratio also needed to be taken into

account. The main goal of additional simulation was to identify unique features in the solar spectrum that might provide an improvement in sensitivity when measuring Doppler shifts, which are used to estimate the velocity of a target satellite. The simulator code was designed to account for the following situations:

- Arbitrary selection of center frequency and bandwidth
- Spatial heterodyning of two or more spectral bands
- Variable aliasing of spectral information
- Different SNR metrics for the optical system

The velocity fitting simulation, which incorporates the radiometric and noise models developed in section 2.0, is organized as illustrated in Fig. 43.

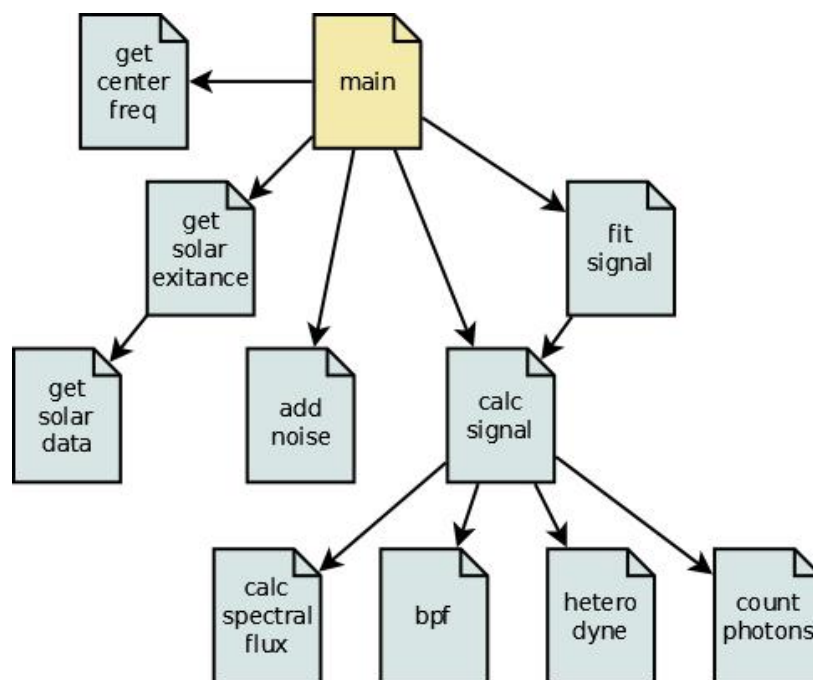


Fig. 43. Structural organization of the Doppler velocity fitting simulation.

Each simulation run is defined by the parameters listed in Table 8. A description of each parameter and reasons for the values chosen are described below. At the start of each simulation run, an output directory is created based on the current date and time. The value of each simulation parameter is saved to a summary file, “summary.mat.”

Table 8. Simulation parameters for Doppler velocity fitting.

Parameter	Description	Range	Units
<i>lambdao</i>	Center frequency.	360 to 738	nm
<i>width</i>	Bandwidth.	130	cm ⁻¹
<i>shift</i>	Heterodyne offset.	0	cm ⁻¹
<i>bins</i>	Number of frequency bins.	100	-
<i>Vdop</i>	Doppler velocity.	0 to 8000	m/s
<i>tauint</i>	Integration time.	240	s
SNR	Signal/noise ratio.	{5, 10, 20}	-

The available solar data [23] includes frequencies from 357.2 to 740.9 nanometers, so center frequencies *lambdao* were drawn randomly from this range. The initial set of simulation runs only considered one band at a time; *i.e.*, multiple overlapping bands were not simulated. Later simulations considered two and three overlapping bands.

When randomly selecting simulation parameters such as center frequencies, it should be noted that MATLAB (version R2015a) reproduces the same sequence of pseudo-random numbers every time it starts. This makes it possible to reproduce results, but care must be taken so that “randomized” parameters are not duplicated over multiple simulation sessions. This can be ensured by placing the MATLAB command *rng('shuffle')* near the top of the simulation script. This action seeds the random number generator with a value from the PC clock, effectively randomizing the start of the sequence.

The default *width* of each spectral band was chosen to be 130 cm⁻¹ because this value represents a bandwidth of approximately 4 nm at the center of the frequency range. This is a conservative estimate of the expected input bandwidth after filtering. Smaller bandwidth values were also simulated; these showed some improvement in sensitivity, but we are unlikely to obtain spectral bandwidths below 40 cm⁻¹ due to limitations of the filters.

Heterodyning is controlled by the *shift* parameter in the main simulation script; the value of this parameter selects the spatial frequency, in inverse centimeters, that will be mapped to the center frequency of each band. For the initial set of simulation runs, the center frequency of each band was spatially heterodyned down to 0 cm⁻¹. This causes a complete overlap of each band with an aliased (spatially reversed) version of itself.

The simulation parameter *bins* represents the number of separate frequency bins captured by the multi-spectral imager. Increasing the number of bins would improve the sensitivity of the simulation, but this value was chosen as 100 to match the current optical design. With a 4 nm bandwidth, this corresponds to an approximate sensitivity of 0.04 nm.

The integration time period, *tauint*, was set equal to 240 seconds (40 minutes). This was chosen as a compromise between signal quality and required observation time. The overall signal to noise ratio of the system is simulated by the SNR parameter. Each simulation run is repeated for SNR values of 5, 10, and 20.

The Doppler velocity of the simulated signal is determined by the V_{dop} parameter. Because orbital velocity is inversely proportional to the orbital height, maximum speed occurs at low earth orbit. For a circular orbit, the satellite velocity with respect to earth is found as

$$v = \sqrt{GM / r}. \quad (39)$$

Since orbits below 150 km are unstable due to atmospheric drag [24], the maximum satellite velocity in a circular low earth orbit can be estimated as 7750 m/s. For this reason, each simulation run is repeated for values of V_{dop} from 0 to 8000 m/s at intervals of 500 m/s.

Finally, for each simulation run defined by these parameters, 1000 separate trials were performed with different values of randomly generated noise added to the signal. After 1000 trials, the estimated Doppler values, based on the fitted spectral bands, are saved to an output file, “snr-[xx]-vdop-[xxxx].mat,” where [xx] and [xxxx] are replaced with the SNR and Doppler velocity values for the simulation run, respectively. The mean and standard deviation of all 1000 trials are calculated and added to the “summary.mat” file for convenience.

3.2.1 Aliasing Simulation

Implementation details of the simulation are discussed in the next sections. Each section refers to a MATLAB script or function represented in the overview diagram of Fig. 43.

3.2.1.1 Script: main

This is the main script which initializes all parameters and controls the simulation. The most important simulation parameters are listed in Table 9. The *shift* parameter deserves special mention. It is the offset value used for heterodyning the observed frequency bands. Each band is heterodyned around its center wavenumber, with an offset of *shift* inverse centimeters. If this parameter is zero, the center of each frequency band will be mapped to zero spatial frequency, effectively overlapping the entire spectrum with an aliased version of itself. If the absolute value of *shift* is greater than zero but less than the half-width of the frequency band, then there will be partial aliasing. Otherwise, there will be no overlap of positive and negative frequencies, and thus, no aliasing.

Table 9. List of simulation parameters for script *main.m*.

Name	Units	Description
tauint	s	Integration time.
Nsamples	-	Number of sample points per frequency band.
width	1/cm	Width of each frequency band in wavenumbers (inverse cm).
shift	1/cm	Heterodyne wavenumber offset.
Vdop	m/s	Doppler velocity of the simulated measurement.
lambdao	nm	Vector of center frequencies of every frequency band.
SNR	-	The desired signal to noise ratio for the simulation.

After initializing the parameters, the main loop is entered. First, the noiseless signals generated by each frequency band on the detector are generated. Next, the signal amplitudes are adjusted to represent the desired SNR being simulated. Finally, simulated noise is added to the signals. The

goal is to fit the known solar spectrum to the simulated measurements, and thus determine the Doppler velocity of the reflecting object. The fitting procedure uses three independent variables for fitting: the estimated Doppler velocity, a signal magnitude, and a magnitude offset. The Matlab function *fminsearch* iteratively calls *fit_function* until the value it returns has been (locally) minimized. At each iteration, the initial estimates of the orbital parameters are adjusted to optimize the fit. Once the known spectral data has been fitted to a noisy signal, the estimated velocity is recorded and the loop is repeated. After 1000 iterations, the results are analyzed.

3.2.1.2 Function: *getCenterFreq*

This function calculates random center frequencies for simulation. It insures that the selected frequencies are non-overlapping and do not include the range of missing data points in our solar spectrum data file. The function's input and output parameters are described in Table 10.

Table 10. Input/output parameters for function *getCenterFreq*.

Name	In/Out	Units	Description
lambda	Out	nm	A vector of randomly selected center frequencies.
Nfreqs	In	-	The number of random frequencies to generate.
width	In	cm ⁻¹	The full width of the pass band in inverse centimeters.

3.2.1.3 Function: *getSolarExitance*

This function calculates the spectral exitance of a given frequency band. It applies the intensity data from disk file to a blackbody model of the sun. The function's input and output parameters are described in Table 11.

Table 11. Input/output parameters for function *getSolarExitance*.

Name	In/Out	Units	Description
lambdao	In	nm	Center frequency in units of nanometers.
Nsamples	In	-	Number of sample points to return.
width	In	cm ⁻¹	Full width of spectral band in units of wave number.
sigma	Out	cm ⁻¹	Vector of equally-spaced sample points.
Msun	Out	W/m ² /μm	Spectral exitance within the selected pass-band.

3.2.1.4 Function: *getSolarData*

This function reads the solar intensity data from disk file and returns the spectral data sampled over the requested bandwidth. The data is taken from a file provided in conjunction with [23]. The function's input and output parameters are described in Table 12.

Table 12. Input/output parameters for function *getSolarData*.

Name	In/Out	Units	Description
lambda	In	nm	Center frequency in units of nanometers.
Nsamples	In	-	Number of sample points to return.
width	In	cm ⁻¹	Full width of spectral band in units of wave number.
sigmaout	Out	cm ⁻¹	Linear vector of wave number sample points.
specout	Out	-	Vector of intensities at the specified wave numbers.

3.2.1.5 Function: *fit_signal*

This function is called repeatedly by MATLAB's built-in *fminsearch* function to minimize the error between the estimated Doppler shift and the measured spectral data. The goal is to determine the Doppler velocity that best fits the measured signal.

Each time it executes, the *fit_signal* function calls *calc_signal* with the current estimate of the Doppler velocity. The resulting Doppler-shifted spectral signal is interpolated onto that of the measured signal, and a sum-of-squares error value is calculated. The error value is returned to *fminsearch*, and the process is repeated with a new velocity estimate until the error is minimized.

The function's input and output parameters are described in Table 13.

Table 13. Input/output parameters for function *fit_signal*.

Name	In/Out	Units	Description
coeffs	In	unitless	Three element vector containing the parameters under optimization.
coeffs(1)	In	m/s	The current guess for the Doppler velocity.
coeffs(2)	In	unitless	The current guess regarding relative signal intensity.
coeffs(3)	In	unitless	An additional free parameter representing an additive magnitude offset.
signal	In	unitless	The measured, noisy signal to which the estimated signal is fit.
sigma	In	cm ⁻¹	A vector containing the wavenumbers of each measured spectral band. These bands are heterodyned to overlap in the detector.
solarspec	In	W/m ²	The solar exitance (reference data) for each bandpass region contained in <i>sigma</i> .
shift	In	cm ⁻¹	The heterodyne wave number offset.
fval	Out	unitless	Scalar value used by <i>fminsearch</i> to determine convergence of the optimization algorithm.

3.2.1.6 Function: *calc_signal*

This is where most of the work takes place. The *calc_signal* function accepts the solar exitance data, Doppler velocity, and heterodyne offset as inputs. The next steps are contained in a loop and repeated for each frequency band:

- The exitance wavelengths are shifted according to the specified Doppler velocity.
- Shifted wavelengths are passed to the radiometric model to estimate the spectral power on the detector.
- The *heterodyne* subroutine is called to shift the spectrum down to the desired spatial frequencies, overlapping with negative frequencies if aliasing occurs.
- Subroutine *count_photons* is called to construct a signal representing the output of the detector device (without noise).
- The resulting signal is added to a running total, and the function loops to repeat for the next frequency band, if any.

The function's input and output parameters are described in Table 14.

Table 14. Input/output parameters for function *calc_signal*.

Name	In/Out	Units	Description
sigma	In	cm ⁻¹	Vector of wavenumbers corresponding to spectral samples.
Msun	In	W/m ² /μm	The solar exitance within the selected pass-band.
Vdop	In	m/s	The Doppler velocity of the satellite.
shift	In	cm ⁻¹	The heterodyne wave number.
SNR	In	-	The simulated signal-to-noise ratio.
tau	In	s	The integration time.
bins	In	-	The number of frequency bins.
signal	Out	-	The estimated photon flux incident on the detector.

3.2.1.7 Function: *calc_spectral_flux*

This function uses the radiometric model to calculate spectral power (flux) incident on the detector. The target area is taken to be the area of a single lenslet in the array as projected onto the telescope's aperture. The calculated flux is therefore the flux over the area of one sub-image on the detector. The following assumptions are made when calculating the flux:

1. Lenslets are sized such that the pupil area of all lenslets projected onto the telescope aperture will exactly cover the aperture area, less some fill factor assumed to be $\pi/4$, which is the area of a circular (telescope aperture) divided by the area of a square (lenslet area).
2. Aperture sampling is performed, with each lenslet illuminating a single pixel on the detector.

The function's input and output parameters are described in Table 15.

Table 15. Input/output parameters for function *calc_spectral_flux*.

Name	In/Out	Units	Description
Msun	In	W/m ² /μm	The spectral exitance of the sun in the pass band of interest.
phi	Out	W/μm	The resulting spectral power on the detector.

3.2.1.8 Function: *heterodyne*

This function performs heterodyning of spectral data around the selected wave number. The results are returned in the form of an “upper” and “lower” spectral portion. This is so that the correct frequency values are used for calculating the photon flux. The function's input and output parameters are described in Table 16.

Table 16. Input/output parameters for function *heterodyne*.

Name	In/Out	Units	Description
sigma	In	cm ⁻¹	Vector of wavenumbers corresponding to spectral samples.
spectrum	In	W/m ² /μm	The corresponding vector of spectral intensities.
sigmao	In	cm ⁻¹	The reference wavenumber for heterodyning.
shift	In	cm ⁻¹	The heterodyne wave number.
upper	Out	W/m ² /μm	The result vector containing the positive frequencies.
lower	Out	W/m ² /μm	The result vector containing the negative frequencies.

3.2.1.9 Function: *count_photons*

This function estimates the number of photons incident on each pixel of the detector. It first reduces the incoming spectral power by the quantum efficiency of the detector. Next, it interpolates the resulting intensity data onto a new spatial axis representing the individual pixels of the detector. The spectral power is then integrated over the frequencies covered by each pixel. The resulting radiant flux is multiplied by the integration time, and this energy is used to determine the photon flux on each pixel. More specifically,

$$\begin{aligned}
 \Phi &= \int_{\lambda_1}^{\lambda_2} \Phi_{\nu} d\lambda, \\
 E &= hc / \lambda, \text{ and} \\
 N_{\text{photons}} &= \frac{\Phi t}{E}.
 \end{aligned} \tag{40}$$

where Φ and Φ_{ν} are the radiant and spectral power, respectively. The limits on the integral, λ_1 and λ_2 , are the upper and lower wavelengths incident on a particular pixel; this is directly related to the spatial frequency of the dispersed spectrum and the width of each pixel. E is the energy of a single

photon of frequency λ , where λ is the center frequency, i.e., $\lambda = (\lambda_1 + \lambda_2) / 2$. Finally, $N_{photons}$ represents the total number of photons collected on that pixel during the integration time, t .

The function's input and output parameters are described in Table 17.

Table 17. Input/output parameters for function *count_photons*.

Name	In/Out	Units	Description
sigma	In	cm ⁻¹	Vector of wavenumbers corresponding to spectral samples.
spectrum	In	W/m ² /μm	The corresponding vector of spectral intensities.
sigmao	In	cm ⁻¹	The reference wavenumber for heterodyning.
shift	In	cm ⁻¹	The heterodyne wave number.
upper	Out	W/m ² /μm	The result vector containing the positive frequencies.
lower	Out	W/m ² /μm	The result vector containing the negative frequencies.

3.2.1.10 Function: *add_noise*

This function adds noise to the ideal signal by implementation of the noise model discussed in sections 2.2.3 and 2.3.2. The function's input and output parameters are described in Table 18.

Table 18. Input/output parameters for function *add_noise*.

Name	In/Out	Units	Description
phi	In	photons/OPD	A vector containing the (noiseless) photon count at the detector.
tau	In	s	The signal integration time.
Itotal	Out	e ⁻ /s/bin	Noisy output of the detector in electrons per second for each bin.

3.2.2 Results

3.2.2.1 One Band

A high-level overview of all single-band simulation results is shown in Figs. 44 - 46. Each line in these figures represents a single spectral band. The horizontal axis represents the Doppler velocity of the simulated signal. The vertical axis represents the standard deviation of the fitted velocity over 1000 trials. There are two important features apparent in these plots. First, there is a large variation of the standard deviation over the 40 simulated frequency bands. For the system in Fig. 44 with an SNR of 20, values range from around 120 to over 2500 m/s. This indicates that some bands provide much better sensitivity than others for determination of Doppler velocity. Second, standard deviation values seem to scale almost linearly with SNR, approximately doubling with every 3 dB decrease in the SNR. This can be seen by comparing the system SNR values of 10 and 5, shown in Figs. 45 and 46, respectively.

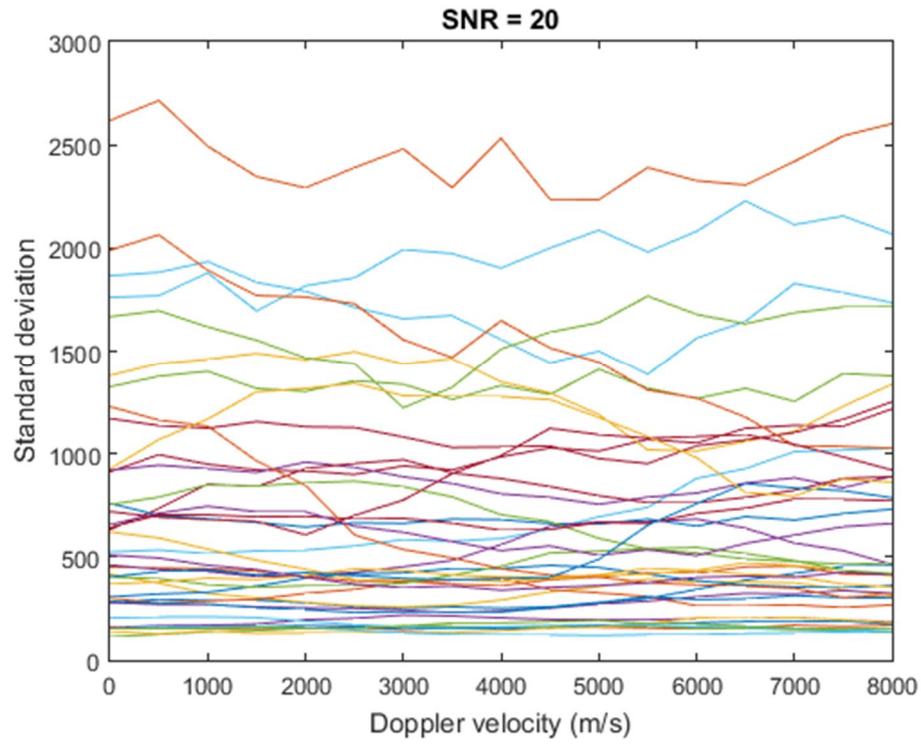


Fig. 44. Overview of all single-band simulation results for a system SNR of 20.

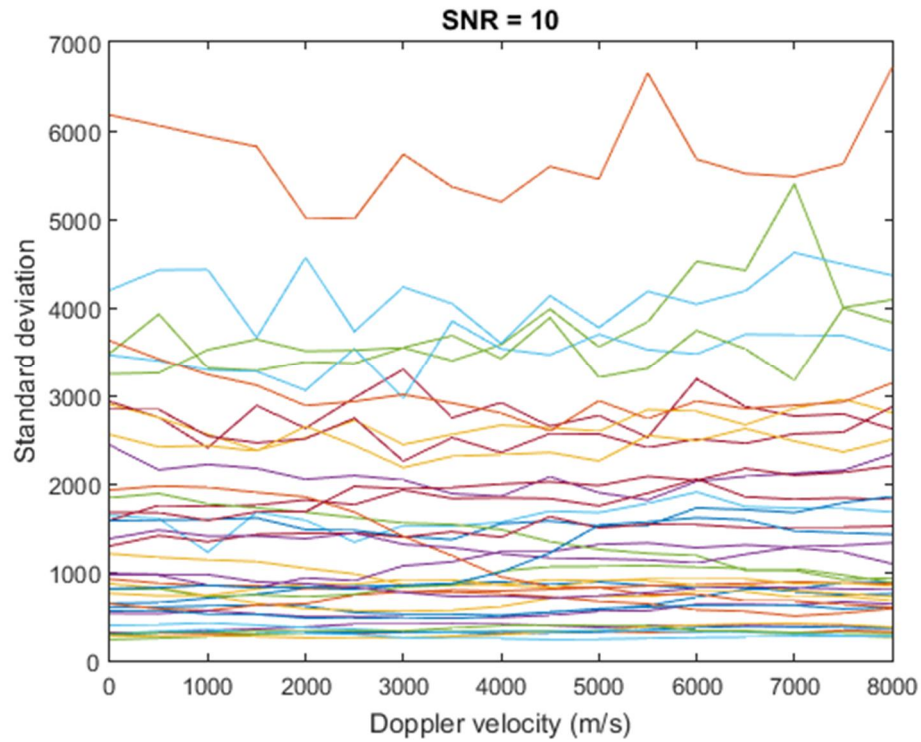


Fig. 45. Overview of all single-band simulation results for a system SNR of 10.

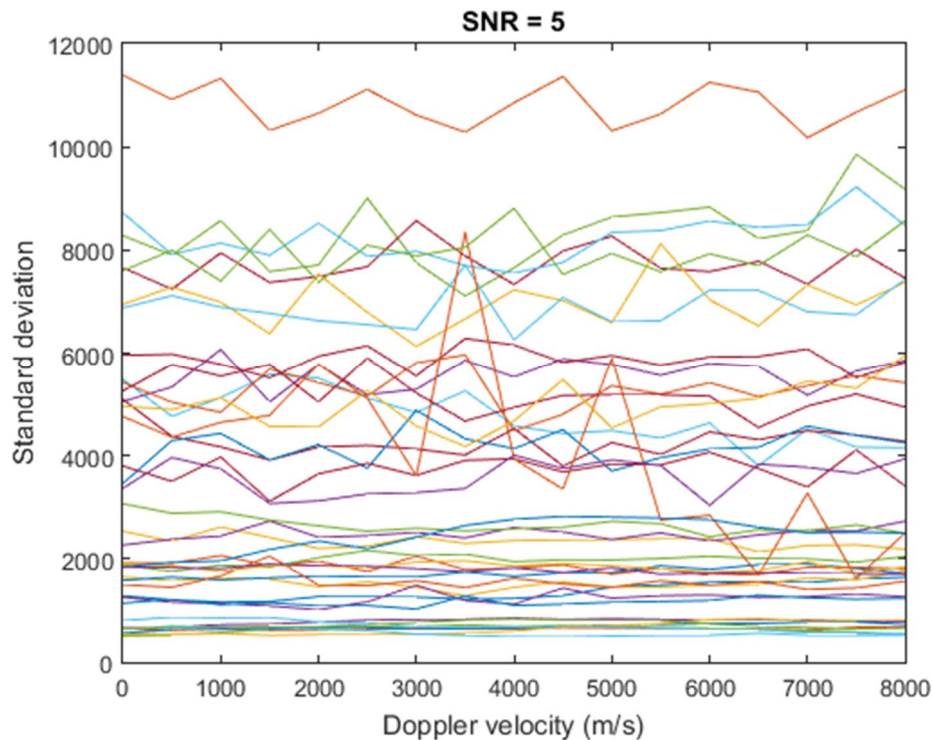


Fig. 46. Overview of all single-band simulation results for a system SNR of 5.

To better see details in the data, the next three figures zoom in to show only the seven best frequency bands. System SNR values of 20, 10, and 5 are represented in Figs. 47, 48, and 49, respectively. At this scale, several additional features become apparent. First, the overall shape of the response curve does not change with changes in system SNR. This lends credence to the idea that the differences in sensitivity are due to features in the spectral bands themselves, and not a result of the random addition of noise. Second, there is no clear trend in standard deviation vs velocity. This reinforces the idea that standard deviation is mainly a function of the selected frequency band.

It can be seen from these figures that, for Doppler velocities greater than 3000 m/s, observation of the frequency band around 391.0 nm might be a good choice. For velocities less than 3000 m/s, the band with center at 397.6 nm might be better. It is possible that these two bands could be heterodyned to overlap, creating a composite signal with good sensitivity in both velocity ranges.

Finally, Figs. 50 - 53 show histograms for the frequency band centered around 391.0 nm. Each plot represents 1000 individual results for a particular combination of simulation parameters plus random noise. Simulated Doppler velocities of 2000, 4000, 6000, and 8000 m/s are plotted for an SNR value of 20. Other parameters were chosen as described in Table 2.1. As expected, the shape of these plots resembles a normal distribution of the fitted velocities.

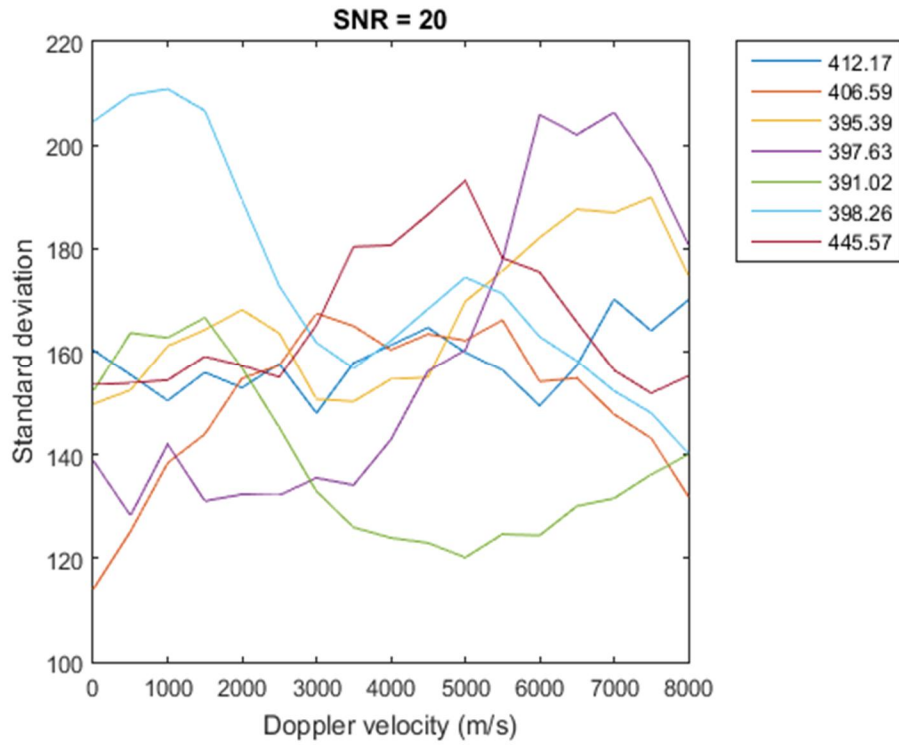


Fig. 47. Standard deviation vs Doppler velocity for the seven best single frequency bands. The legend indicates center wavelength values in nanometers. System SNR value 20.

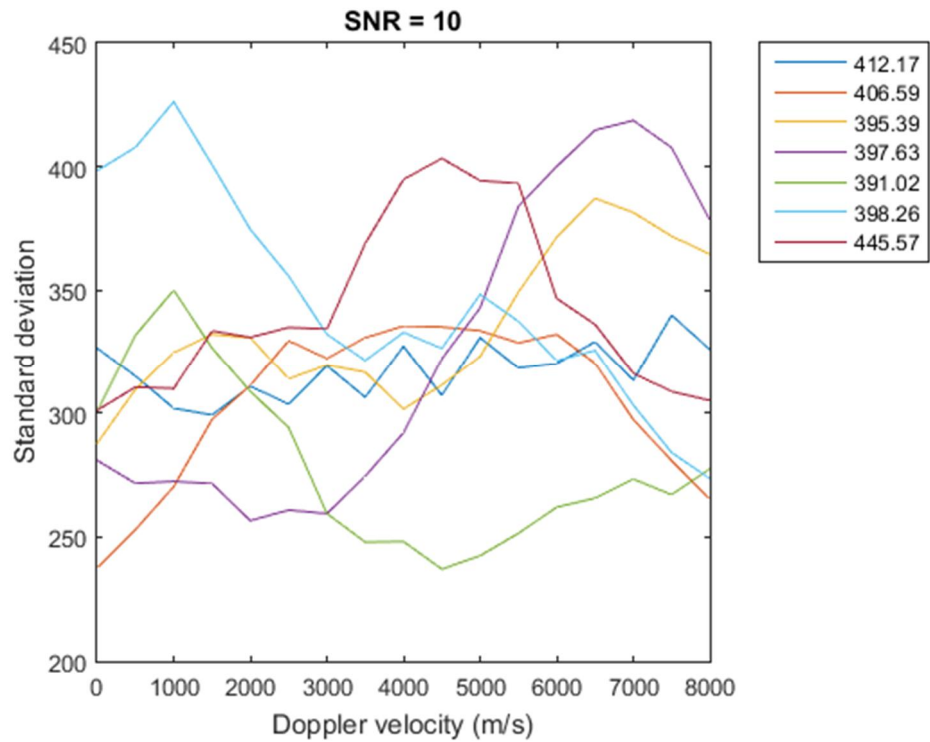


Fig. 48. Standard deviation vs Doppler velocity for the seven best single frequency bands. The legend indicates center wavelength values in nanometers. System SNR value 10.

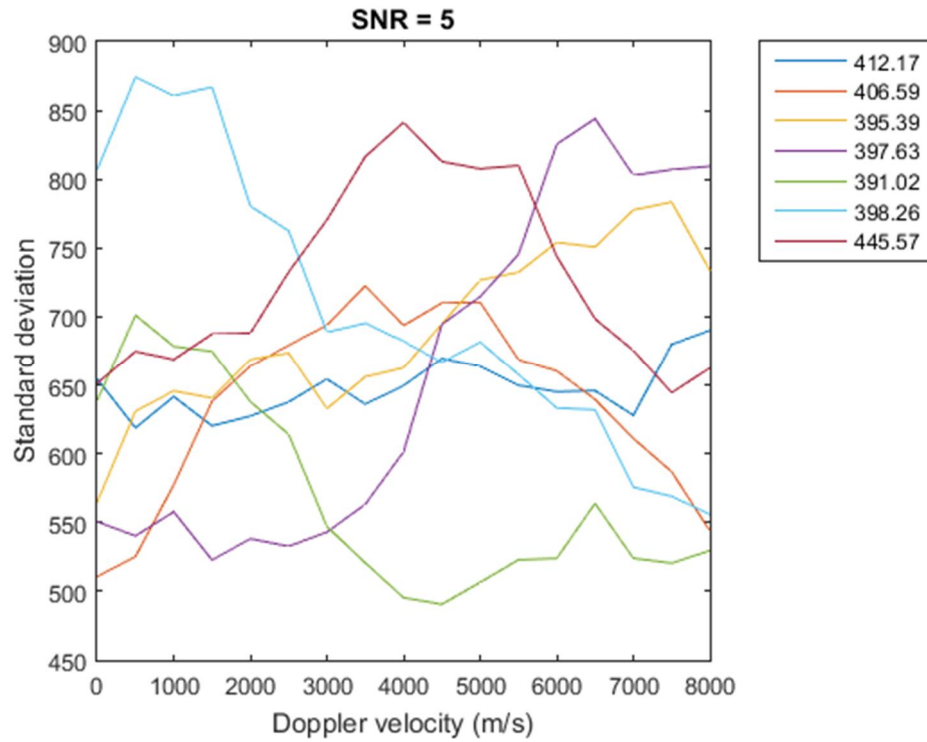


Fig. 49. Standard deviation vs Doppler velocity for the seven best single frequency bands. The legend indicates center wavelength values in nanometers. System SNR value 5.

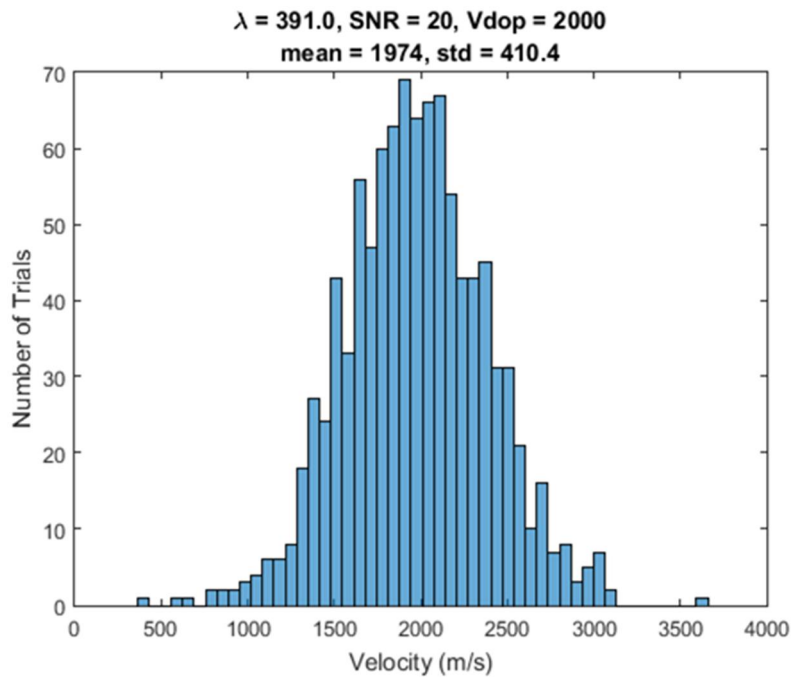


Fig. 50. Histogram for Doppler velocity of 2000 m/s.

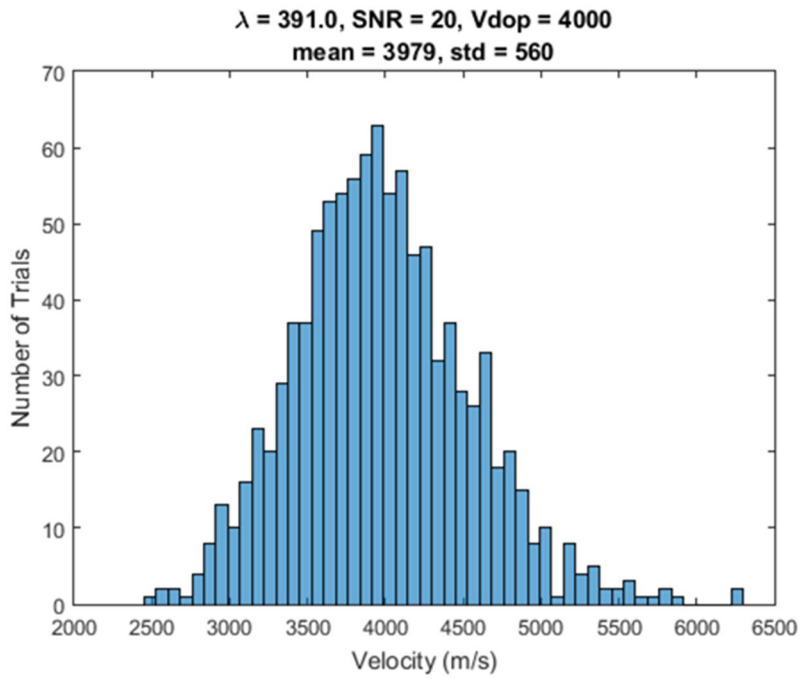


Fig. 51. Histogram for Doppler velocity of 4000 m/s.

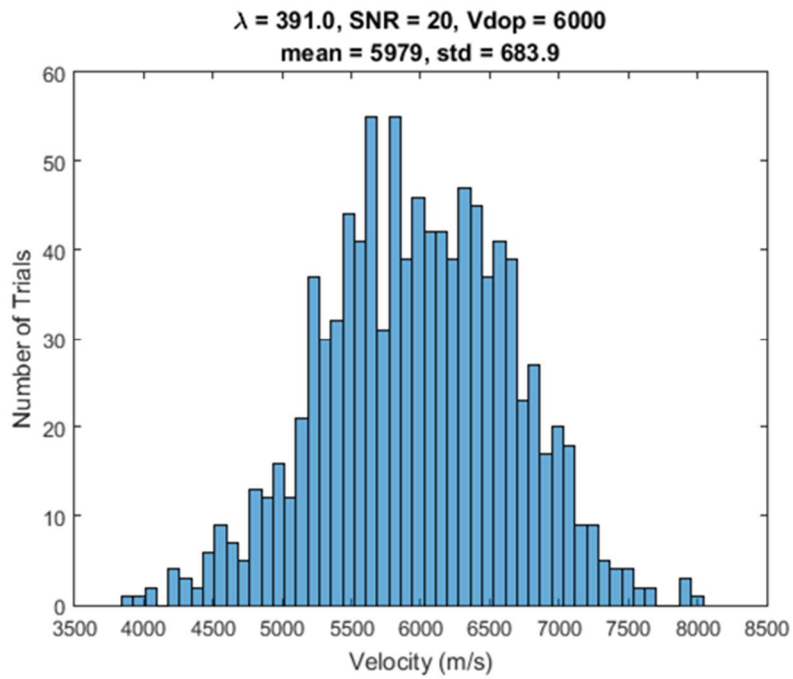


Fig. 52. Histogram for Doppler velocity of 6000 m/s.

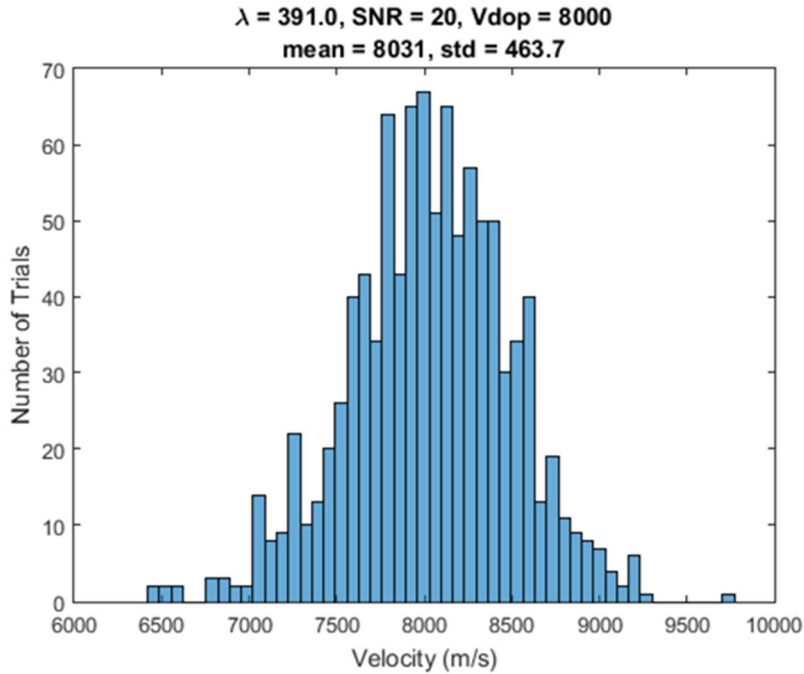


Fig. 53. Histogram for Doppler velocity of 8000 m/s.

3.2.2.2 Two bands

The combined results of 36 randomly selected pairs of overlapping bands are summarized in Fig. 54. For the case of two and three overlapping bands, we show only the results for a system SNR of 20. The overall trends for SNR values of 10 and 5 are the same as seen in the single-band case. Many of the results charted in Fig. 54 exhibit a concave shape with a minimum standard deviation near 4000 m/s. This interesting and unexpected result might warrant further investigation; however, it is likely caused by the spectral bandwidth of the various lines under test, relative to the observed Doppler shift. The seven best results of the two-band trials are reproduced in Fig. 55. In this case, the lowest standard deviation is found from the overlap of the two center wavelengths at 514.97 nm and 387.83 nm. This pairing has an average standard deviation of 365 m/s, which is slightly worse than the best single band result.

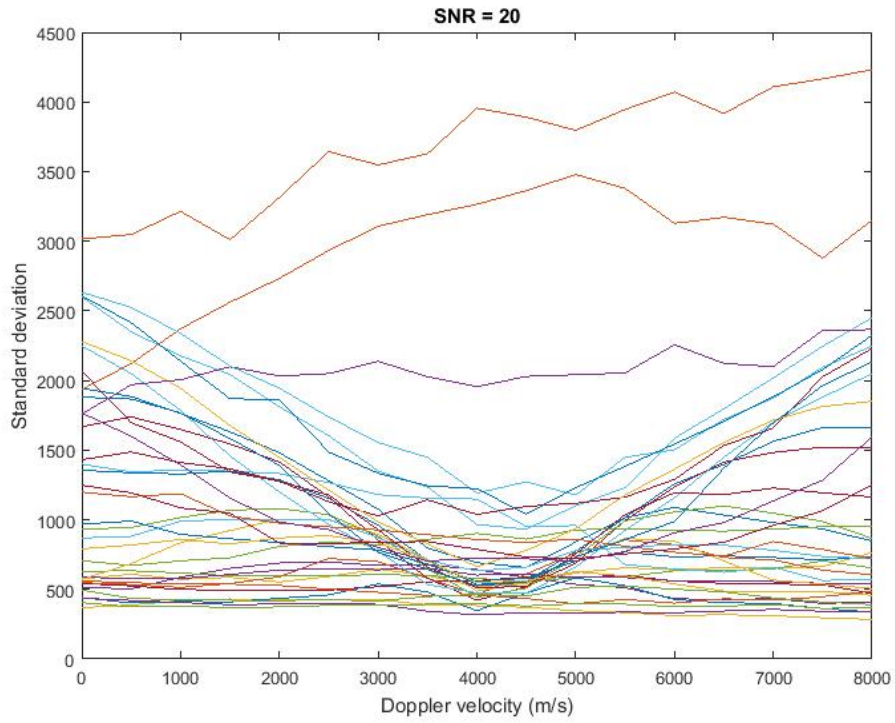


Fig. 54. Overview of all single-band simulation results for a system SNR of 20.

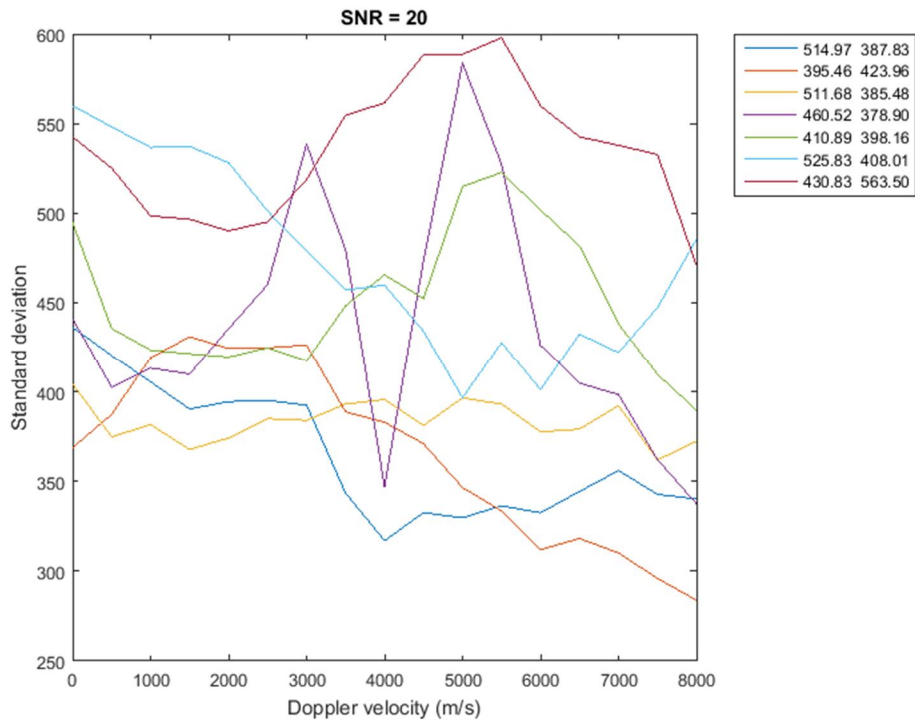


Fig. 55. Standard deviation vs Doppler velocity for the seven best trials of two overlapping bands. The legend indicates overlapping center wavelength values in nanometers. System SNR value is 20.

3.2.2.3 Three bands

Three overlapping bands were the largest number to be simulated. The results for 100 randomly generated triplets, assuming a signal-to-noise ratio of 20, are shown in Fig. 56. Each line in the graph represents the average of 1000 trials with random Poisson noise added to the signal. In general, the three-band trials do not seem to show any improvement over the two-band trials. Some of the trials in Fig. 56 exhibit the same unexplained dip in the standard deviation, this time at a slightly higher velocity of 4500 m/s. Roughly 20 of the trials also show a new feature: the standard deviation appears linear with respect to velocity. In about six of the trials, this relationship is very consistent across the entire range, with a slope of about 0.5 standard deviations per meter per second. This is likely caused by the relative positions of the lines with respect to each other, in which the sensitivity of the detected correlation will vary as a function of line placement. For example, placing two lines directly on top of each other for a Doppler shift of 0 m/s would place a 0 slope in the correlation function at 0 m/s, with higher, more rapidly varying slopes on either side. A rapid slope yields a large signal difference, which is easier to detect in the presence of noise.

Zooming in for a closer look, the best seven trials are shown in Fig. 57. At this scale the two-band trials in Fig. 55 seem to have a slight advantage. However, the best seven trials from both experiments lie in roughly the same range of 250 m/s - 500 m/s of standard deviation. At this point we could draw a general conclusion from all of the Doppler velocity simulations to date: Although stacking two bands does improve the accuracy of the simulated measurement, the addition of a third band does not afford any further advantage. The cause of this is likely due to reduced coherence between the interference of the multiple bands.

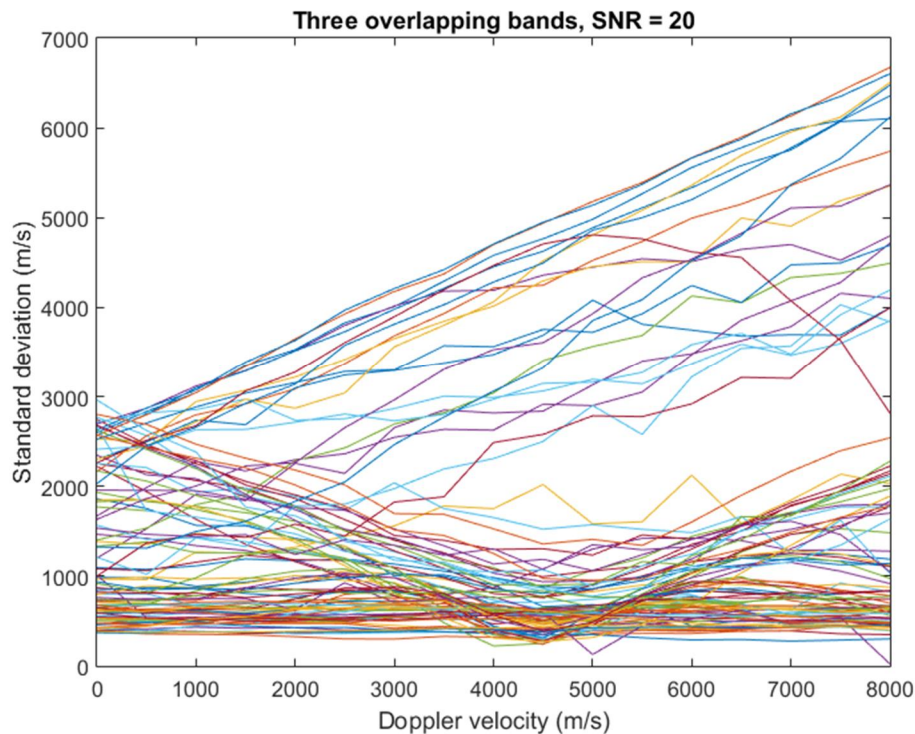


Fig. 56. Overview of all three-band simulation results for a system SNR of 20.

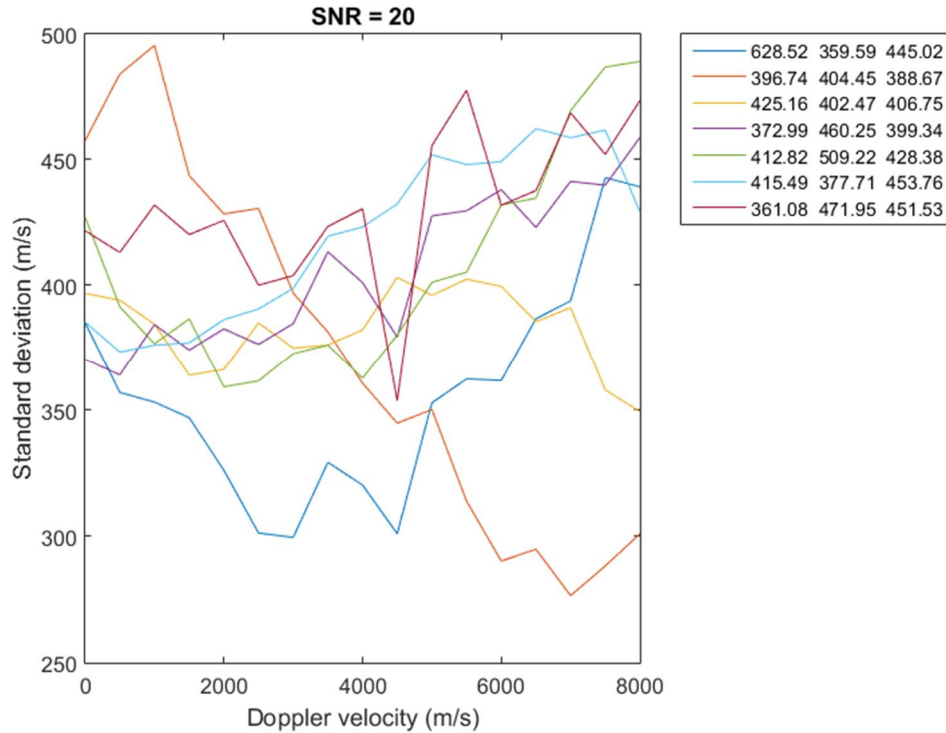


Fig. 57. Standard deviation vs Doppler velocity for the seven best trials of three overlapping bands. The legend indicates overlapping center wavelength values in nanometers. System SNR value is 20.

3.2.3 Discussion

In order to more directly compare the effects of spatial heterodyning, a particular set of frequencies was chosen for simulation using two, three, and no overlapping bands. The following simulations were performed:

- A single band centered around 361.06 nm.
- A single band centered around 375.26 nm.
- A single band centered around 391.23 nm.
- A heterodyned (“stacked”) combination of 361.06 and 391.23 nm bands.
- A heterodyned (“stacked”) combination of all three bands.

These particular bands were chosen because they were among the best results, in terms of standard deviation, of all the previous single band simulations. The results are summarized in Fig. 58, which includes the five simulations listed above. The highest three lines represent the single-band simulations at center wavelengths of 361.06, 375.26, and 391.23 nanometers. The fourth simulation (purple) is a two-band overlap of center wavelengths 361.06 and 391.23 nanometers. Finally, the fifth simulation (green) is a three-band overlap of all three wavelengths.

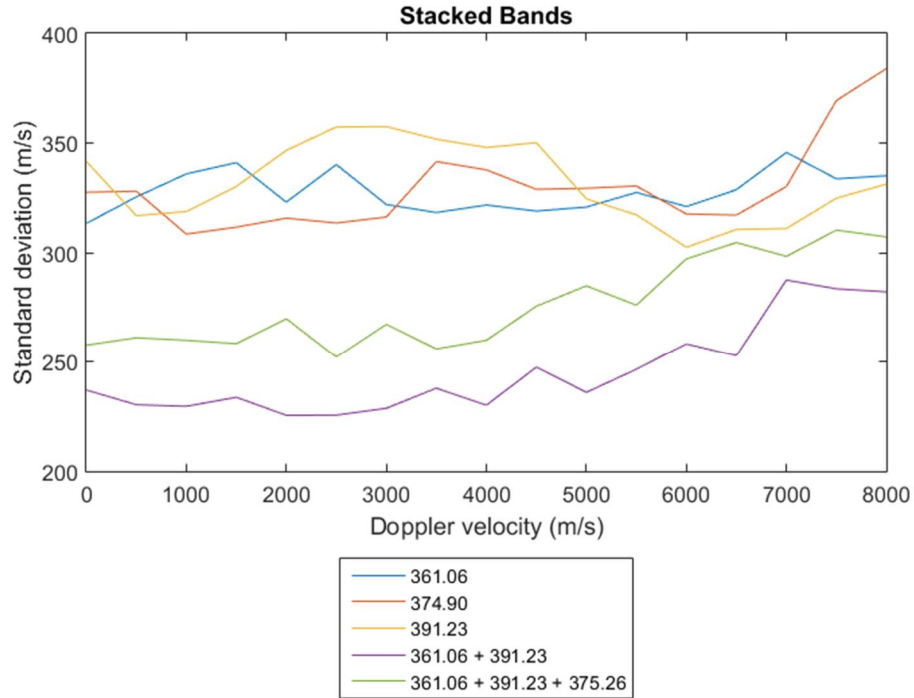


Fig. 58. Dispersive simulation of three frequency bands taken separately and together. The legend indicates wavelength in nanometers.

The simulations using multiple overlapping bands (purple and green on the chart) show a definite improvement in sensitivity compared to the three single-band simulations. The best result here is an average standard deviation of 276 m/s, which is significantly better than the best single-band results of 328 m/s. These results seem to validate the idea that Doppler measurement sensitivity can be improved by overlapping multiple spectral bands. Interestingly, the two-band overlap shows a better sensitivity (in terms of average standard deviation) than the three-band overlap. This suggests that overlapping bands need to be carefully selected for maximum benefit.

4.0 Equatorial Orbital Simulation

The goal of this task was to quantify the required Doppler measurement accuracy and the overall benefit of this technique. An orbital simulation model was developed, including a numerical fitting procedure for estimation of orbital parameters using two or more passive measurements.

One of the goals of this project is to show that the inclusion of velocity measurements can reduce the number of satellite observations required to make an initial orbit determination. However, traditional orbital models do not include velocity as an input parameter. Therefore, in order to evaluate our hypothesis, we constructed an orbital fitting model that includes velocity as a parameter. This model, implemented in MATLAB, enables simulation of orbital fitting using measurement of angles only (AO) or both angles and velocity (AV).

The main script, *suite.m*, initializes all parameters and controls the simulation. When run, it defines a global Cartesian axis to represent the orbital plane of a target satellite. It also defines the semi-major and semi-minor axes of the satellite's elliptical orbit. The relationship between these separate axes is illustrated in Fig. 59. The green circle represents Earth, which lies on the origin of the global x - y plane and is also one focus of the orbital ellipse. The blue ellipse represents the orbit

to be determined; its geometric center lies at point C. The vector $\mathbf{r}$ represents the distance and direction from Earth center to the instantaneous position of the satellite at P. Finally, the angle θ indicates the angle subtended between the satellite and the horizontal x -axis. Zero radians is a position on the x -axis to the right of the origin. Increasing the angle moves θ counter-clockwise towards the positive y -axis. Thus, π radians (180 degrees) is a location on the negative x -axis on the opposite side of the Earth. Together, $\mathbf{r}$ and θ specify the particular location of the satellite at any point on the orbital ellipse.

When the orbital ellipse is rotated with respect to the global coordinates, as it is in Fig. 60, this creates an angle θ_{rot} between the two coordinate systems. The angle φ is used to differentiate angles relative to the elliptical axis from angles θ relative to the global axis. Finally, the angles α_n represent measurements taken at the observation point R relative to the global x -axis. Typically, orbital frames are specified with the Earth to the right of the ellipse's center. This is called a *perifocal* reference frame, and places perigee (the satellite's closest approach) on the positive X -axis at 0 radians. However, our simulation specifies Earth to the left of center, so that perigee occurs at π radians.

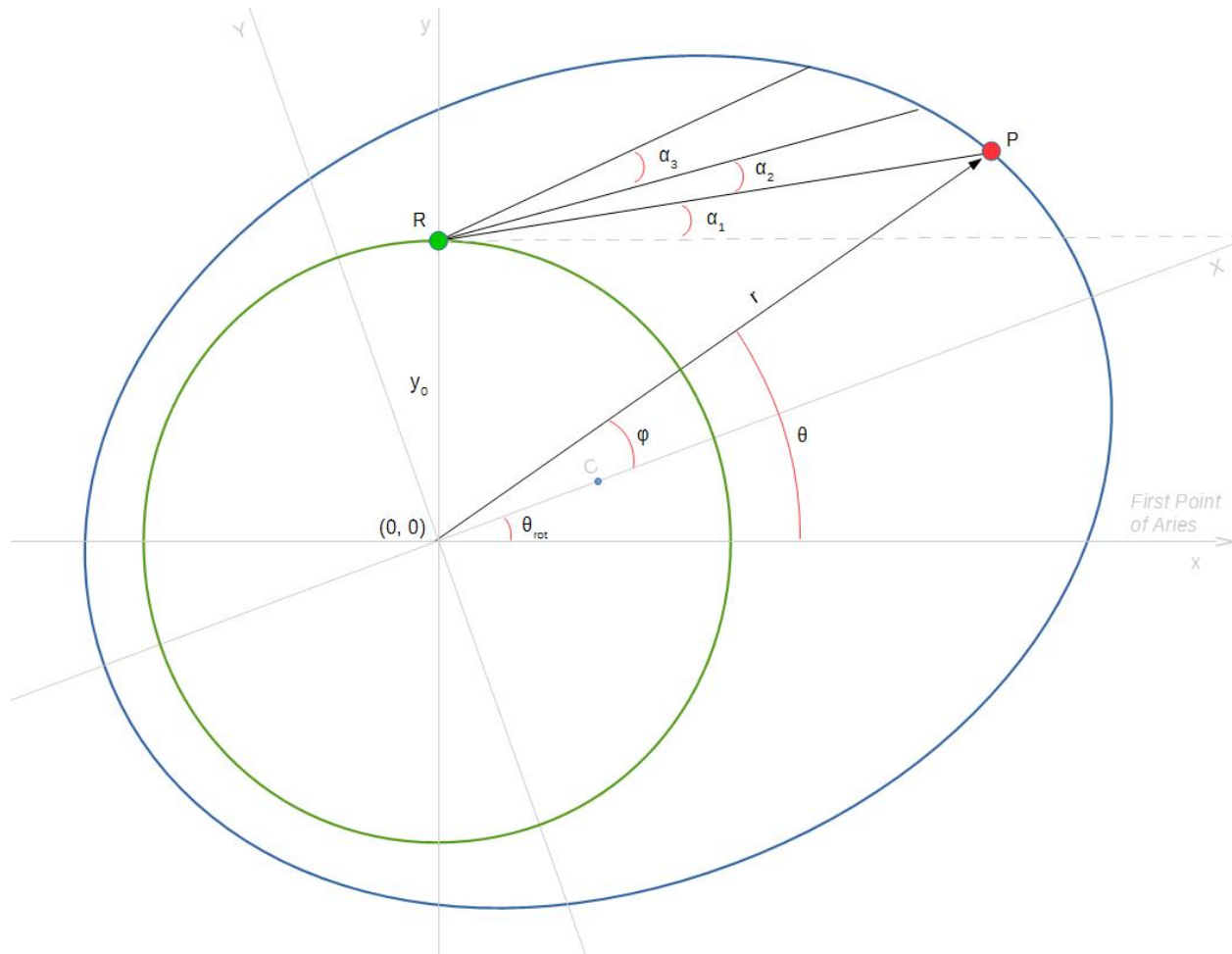


Fig. 59. Global reference axis (x, y) and elliptical axis (X, Y) used in simulations. The green circle represents Earth, with the observer's position at point R. The blue ellipse represents the orbit to be determined, with the satellite's position at point P. (Not to scale.)

The initialization portion of the script also defines the location of the Sun relative to the orbital plane. The relative position of the Sun is critical when simulating the Doppler shift of light reflected from the satellite. The parameters defining the Sun's location are illustrated in Fig. 60. In this sketch, the Earth-Sun distance is indicated by the vector $\mathbf{r}_{\text{sun}}$, while θ_{sun} represents the angle subtended between the Sun and the horizontal x -axis. Furthermore, the vector $\mathbf{d}_{\text{sun}}$ is the projection of $\mathbf{r}_{\text{sun}}$ onto the orbital plane. The angle formed by these two vectors, which is the angle between the Sun and the orbital plane, is represented by θ_z . This angle can be used to find z_{sun} , which is the Sun's height above the plane.

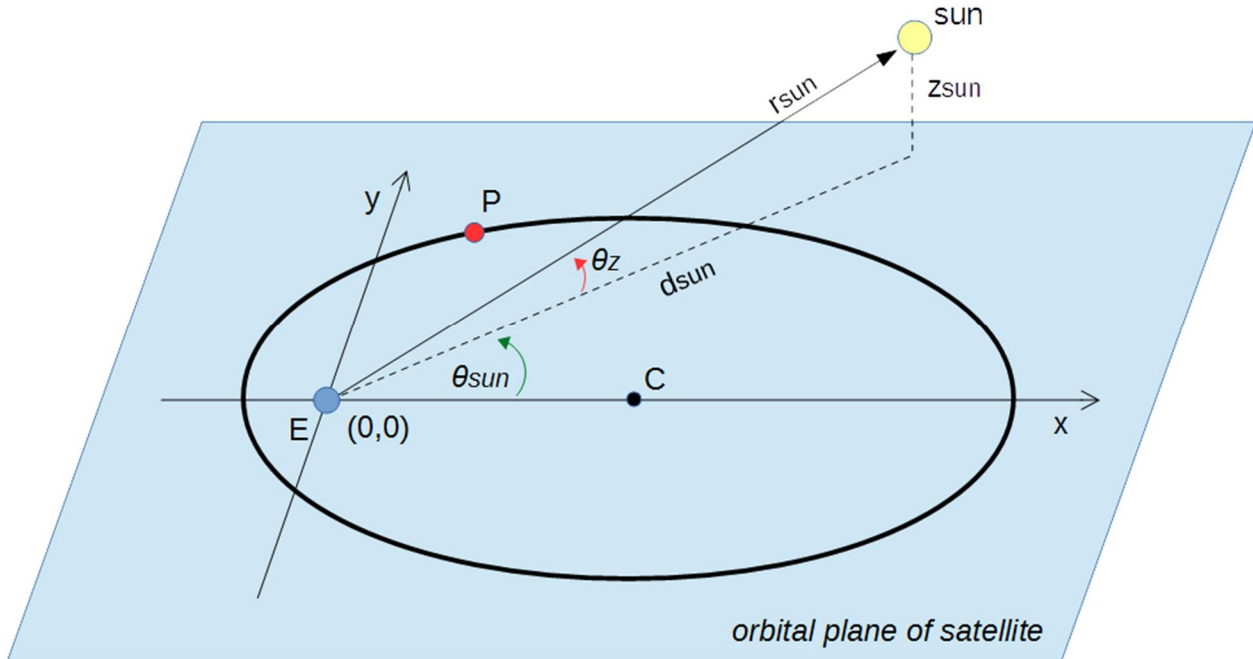


Fig. 60. Position of the Sun relative to the orbital plane of the satellite. The Earth-Sun distance is indicated by the vector $\mathbf{r}_{\text{sun}}$ which is projected onto the plane as $\mathbf{d}_{\text{sun}}$. The angle θ_z formed by these two vectors can be used to determine z_{sun} which is the Sun's height above the orbital plane.

After the orbital frame and parameters have been initialized, the main script calls the *calc_orbit* function, providing it with the length, width, and rotation of the simulated orbit, plus the location of the Sun. The function returns the orbital time, Doppler velocity, and observation angles for every sample point on the orbital path.

Now that a simulated orbit has been established, the script picks a few sample points to use as simulated measurement data. The goal is to fit an estimated orbit to these measurements and compare it to the original model orbit. The fitting procedure uses values of the observation angle and Doppler velocity at various times during the orbital period. Because there are no absolute time or angle references for the “unknown” orbit, all times and angles are specified relative to the first observation. The built-in MATLAB function *fminsearch* is used to fit an orbit to this sample data by iteratively calling the *fit_orbit* function until the value it returns has been (locally) minimized. At each iteration, the initial estimates of the orbital parameters are adjusted to optimize the fit.

4.1 Implementation Details

Implementation details of the main simulation script were described in the previous section. For completeness, Table 19 contains a data dictionary that catalogs all of the important variables used in the main script. Supporting functions called by the main script are described in detail in the following subsections.

Table 19. Data dictionary for the main simulation script, *suite.m*.

Name	Units	Description
a	m	Semi-major axis of elliptical orbit model.
a0	m	Initial estimate of semi-major axis.
alpha	rad	Observation angle relative to theta.
angle_err	arcsec	Standard deviation error of angle measurement.
b	m	Semi-minor axis of elliptical orbit model.
b0	m	Initial estimate of semi-minor axis.
coeff	-	Vector holding the three initial values $a0$, $b0$, $rot0$.
doppler_err	m/s	Standard deviation error of Doppler measurement.
DopplerV	m/s	Calculated Doppler velocity at each point in the orbit.
dt	s	Time between successive observations.
EA	rad	Calculated value of eccentric anomaly at each point on the orbit.
GSO	m	Vector: semi-major and minor axes of a geostationary orbit.
GTO	m	Vector: semi-major and minor axes of a geosynch. transfer orbit.
LEO	m	Vector: semi-major and minor axes of a typical low earth orbit.
Nobs	-	Number of observations, i.e., angle or velocity measurements.
noisyAngles	rad	Vector: simulated angle measurements, including noise.
noisyDoppler	m/s	Vector: simulated Doppler measurements, including noise.
obs	m	Vector: (x,y) location of observation point on Earth's surface.
obsAngles	rad	Ideal (noiseless) angles measured at each observation point.
obsDoppler	m/s	Ideal (noiseless) Doppler measured at each observation point.
obsTimes	s	Time of each observation, relative to the first measurement.
output	-	Matrix: stores fitted a , b , and rot values for each simulated trial.
rot	rad	Rotation angle of the orbital ellipse.
rot0	rad	Initial estimate of the rotation angle.
t0	s	Time of first measurement relative to apogee.
theta	rad	Angular axis for the simulation.
theta_sun	rad	Angle of the sun with respect to x-axis.
theta_z	rad	Angle of the sun above orbital plane.
time0	s	Interpolated orbital time, relative to perigee, corresponding to an observed angle of $\alpha = 0$ radians.
timesum	s	Cumulative time at each point in the orbit, relative to perigee.

trials	-	Number of random trials for each set of orbital parameters.
useVelocity	bool	Flag: fit orbit using angles only (false) or angles and Doppler (true).
x	m	Horizontal axis for the simulation.

4.1.1 Function: `calc_orbit`

This function calculates an elliptical orbit based on the given input parameters. First, it calculates the radius (or height) of the orbit at every angle indicated by the input vector *theta*. Next, the so-called *vis-viva* equation is used to find the orbital velocity corresponding to each radial value:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}, \quad (41)$$

where v is the instantaneous orbital velocity, r is the orbital radius, and a is the semi-major axis. The parameter μ is the standard gravitational parameter for Earth. Although μ can be determined by multiplying the mass of Earth by Newton's gravitational constant, it can also be measured independently and is known with greater accuracy than either of the other two values. Ideally in a two-body problem, the masses of both bodies should be used to determine μ , but in the case of an artificial satellite orbiting the Earth, the satellite's mass is negligible.

After the orbital velocity has been calculated, the function finds the eccentric anomaly for each point in the orbit. This is used to calculate components of the satellite's velocity in the direction of Earth, Sun, and tangential to the orbit. All of these velocities are expressed as multiples of unit vectors in the x and y directions relative to the original orbital frame. The magnitude of the Doppler velocity measured by an observer on Earth then becomes the sum of the satellite's velocities relative to Earth and Sun.

Next, for each point on the orbit, the function calculates the angle that would be measured by the observer on the Earth. This angle is based on the observer's position and the global x and y coordinates of the satellite at each point in its orbit.

Finally, the function calls the *elliptic12* function to determine the arc length at each point in the orbit. From the arc length and the tangential velocity already calculated, orbital time at each point in the orbit is found. In our model, an orbital time of zero corresponds to the satellite at apogee.

To summarize, the complete list of input and output parameters for the function `calc_orbit` are described in Table 20.

Table 20. Data dictionary for input/output parameters of `calc_orbit.m`.

Name	In/Out	Units	Description
a	In	m	The length of the semi-major axis of the orbital ellipse.
b	In	m	The length of the semi-minor axis of the orbital ellipse.
rot	In	rad	Rotation of the orbit with respect to the global axis.
obs	In	m	A two-element vector representing the (x, y) location of the observer on the surface of Earth.

theta	In	rad	The range of angles over which simulated orbits are generated. It is measured from the horizontal x-axis with Earth at the origin. Counter-clockwise rotations are positive.
theta_sun	In	rad	The angular direction of the sun with respect to the horizontal x-axis.
theta_z	In	rad	The angle of the sun above the orbital plane.
EA	Out	rad	The eccentric anomaly, which can be used to define the position of a satellite along its orbital path.
timesum	Out	s	The elapsed orbital time, in seconds, at each angle of eccentric anomaly. An orbital time of 0 corresponds to apogee.
alpha	Out	rad	A vector of angles between the global x-axis and the satellite as measured by the observer on the surface of Earth.
vdop	Out	m/s	the calculated Doppler velocity of the satellite as seen by the observer on the surface of Earth.
coords	Out	m	A vector of (x,y) pairs corresponding to points on the orbital ellipse. This is used for calculation of relative error and for plotting output diagrams.

4.1.2 Function: *fit_orbit*

The *fit_orbit* function is not called directly from the main script. Instead, it is passed as a function handle to MATLAB's built-in *fminsearch* function. The purpose of *fit_orbit* is to compare the simulated measurement data with the current orbit estimation, and then return a value representing the goodness of the fit (smaller is better). It is called iteratively from within *fminsearch* until the output value is minimized. The *fit_orbit* function starts by calling *calc_orbit* with the current coefficients to calculate the most recent estimate of the orbit. After each iteration, the estimates of the orbital parameters *a*, *b*, and *rot* are adjusted to optimize the fit. The input variables for the *fit_orbit* function are catalogued in Table 21.

Table 21. Input parameters for the *fit_orbit* function.

Name	Units	Description
coeff	m m rad	This vector holds the estimated orbital coefficients, which are adjusted by the optimization process in an attempt to minimize the output of the function. The coefficients are mapped as follows: coeff (1) : Estimated length of the semi-major axis. coeff (2) : Estimated length of the semi-minor axis. coeff (3) : Estimated rotation of the orbital ellipse.
obs	m	A two-element vector representing the (x, y) location of the observer, relative to our global coordinate system with origin at the center of the Earth.
time_meas	s	A vector with the differential time of each observation relative to the first.
alpha_meas	rad	A vector with the measured (noisy) angle for each observation.

vdop_meas	m/s	A vector with the satellite's measured (noisy) Doppler velocity for each observation.
theta	rad	The range of angles over which simulated orbits are generated. It is measured from the horizontal x-axis with Earth at the origin. Counter-clockwise rotations are positive.
theta_sun	rad	The angle of the sun with respect to the x-axis.
theta_z	rad	The angle of the sun above the orbital plane.

This function returns a scalar value used by *fminsearch* to determine convergence of the optimization algorithm. The value returned is the sum-of-squares difference between the fitted and actual angles and, optionally, the fitted and actual Doppler velocity at each observation point.

If the Doppler velocity (*vdop_meas*) is provided as an input to this function, it will be used as an additional parameter to fit the orbit. This is what we are trying to introduce with our new system. However, if the input argument is left empty (empty square brackets [] in MATLAB syntax) then Doppler velocity will **not** be used to fit the orbit. This is the more conventional way of doing things. Thus, it takes more observations to fit the orbit accurately.

4.1.3 Function: plot_orbit

The *plot_orbit* function calculates the relative error between the optimal fitted orbit and the actual simulated orbit. It also generates (x, y) data for plotting the orbital ellipse. Input parameters for this function are listed in Table 22.

Table 22. Input parameters for the *plot_orbit* function.

Name	Units	Description
a	m	The length of the semi-major axis of the orbital ellipse.
b	m	The length of the semi-minor axis of the orbital ellipse.
rot	rad	Rotation of the orbit with respect to the global axis.
obs	m	A two-element vector representing the (x, y) location of the observer on the surface of Earth.
theta	rad	The range of angles over which simulated orbits are generated. It is measured from the horizontal x-axis with Earth at the origin. Counter-clockwise rotations are positive.
theta_sun	rad	The angular direction of the sun with respect to the horizontal x-axis.
theta_z	rad	The angle of the sun above the orbital plane.
output	-	The accumulated results including <i>a</i> , <i>b</i> , and <i>rot</i> for each random trial. These values are used to determine the average relative error and to generate density plots of the estimated orbits.

4.1.4 Function: `elliptic12`

The `elliptic12` function integrates an elliptical curve over a specified range of angles. There are three input parameters: u , which specifies the angles at which to sample the curve; m , which specifies the eccentricity of the ellipse; and an optional tolerance, tol . Arc lengths at each angle specified by u are found through calculation of incomplete elliptic integrals of the second kind. Although MATLAB has a built-in function, `ellipke`, that performs the same calculation, this highly optimized function is an order of magnitude faster.

The function returns three vectors corresponding to the evaluation of an ellipse with eccentricity m at angles specified by u . The first two vectors represent the incomplete elliptic integrals of the first and second kind; the third vector represents Jacobi's zeta function. Our simulation uses only the incomplete elliptical integral of the second kind to determine arc length, and from that, the elapsed orbital time. Table 23 summarizes the parameters for the `elliptic12` function.

Table 23. Input/output parameters for the `elliptic12` function.

Name	In/Out	Units	Description
E	Out	-	Incomplete elliptic integration over u of the second kind.
F	Out	-	Incomplete elliptic integration over u of the first kind.
m	In	-	The eccentricity of the ellipse under integration.
tol	In	-	Optional tolerance value to specify algorithmic completion.
u	In	rad	A vector of phase angles over which to integrate the solution.
Z	Out	-	Jacobi's zeta function over u .

4.2 Experimental Design

Our orbital fitting simulation takes into account a large number of variable parameters. These parameters are associated with both the simulated orbit and the details of how the angular and velocity measurements are modeled. The simulation software was designed to vary all of these parameters over a given range, recording the results for each combination of parameters. All of the key parameters and their corresponding ranges are visually represented in Fig. 61. A detailed explanation of each parameter follows.

- The **orbit** parameter represents the size and shape of the simulated orbit in terms of the semi-major (a) and semi-minor (b) elliptical axes. “GSO” is a geostationary orbit with axes $a = b = 42,164$ km. “GTO” is a geosynchronous transfer orbit with $a = 24,582$ km and $b = 17,161$ km. “LEO” is a typical low Earth orbit with $a = 7780$ km and $b = 7741$ km.
- The angle **theta_z** represents the angle of the Sun above the orbital plane, in degrees. It is shown as θ_z in Fig. 60. This angle, along with the phase angle of the Sun – which is fixed at 270 degrees in these simulations – affects the Doppler shift viewed by an observer.
- The **rotation** is the rotation, in degrees, of the orbit within the orbital plane. The reference for the angle is taken from the global axis depicted in Fig. 59, where this parameter is shown as θ_{rot} .

- The number of measurements used as input to the orbital fitting algorithm is denoted by the **observations** parameter, which ranges from 2 to 3.
- The parameter **dt** represents the elapsed time between successive measurements, which is modeled as 30, 45, or 60 minutes.
- Parameter **t0** indicates the starting time, in seconds, of the first observation. This is measured starting from the point at which the satellite crosses an observation angle (α in Fig. 59) of 0 degrees.
- **Dop_error** represents the measurement error of the Doppler velocity, in units of m/s of standard deviation. The values modeled are informed by the previous Doppler velocity simulations; we chose to use values of 50, 100, and 200 m/s.
- The **angle_error** represents the error inherent in the angle measurements. In relative terms, the angular errors are insignificant when compared to the errors in Doppler velocity, so we only deemed it necessary to model a single error value of 5 arcsec of standard deviation.
- Finally, the **method** parameter indicates whether the fitting algorithm should use angles only (AO) or include velocity measurements (AV) when evaluating estimated orbits.

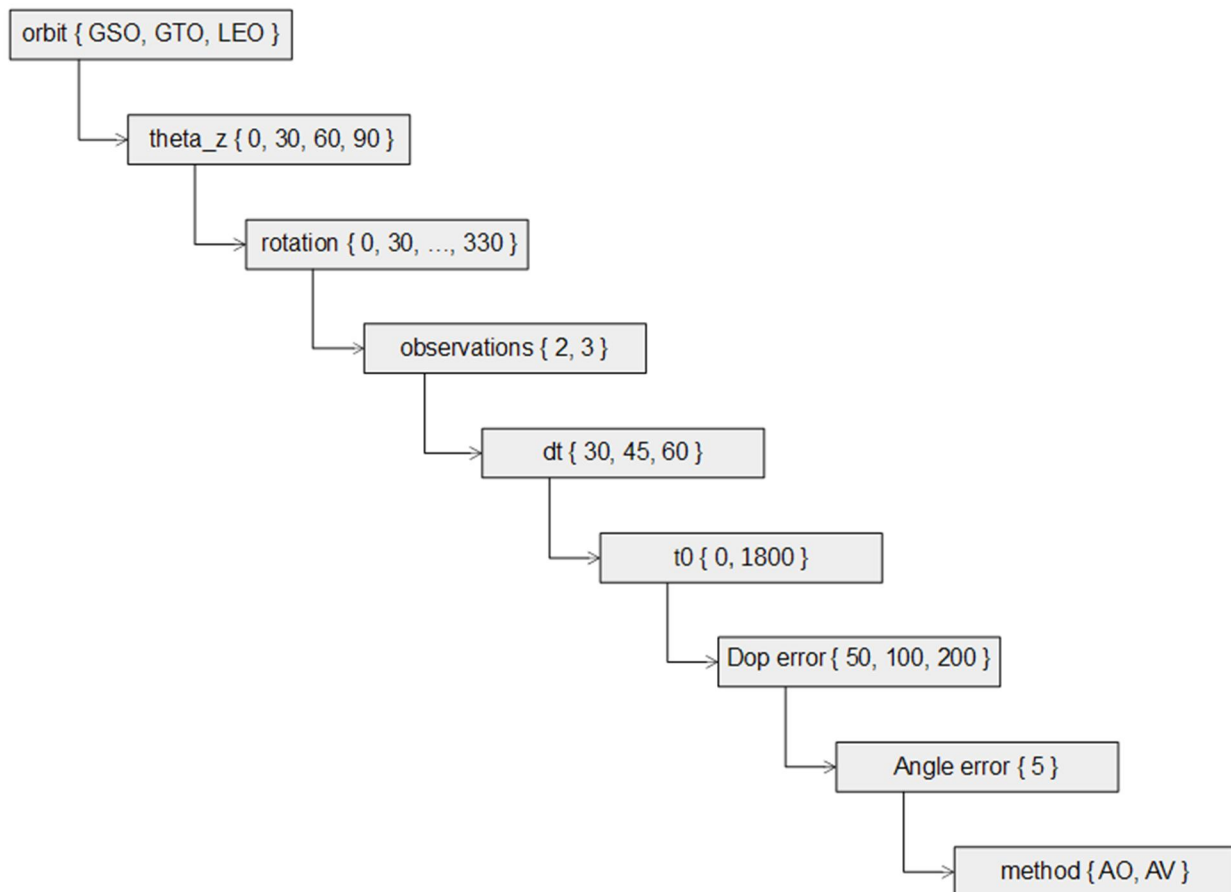


Fig. 61. Parameters used in the design of simulation experiments. See the text for a complete description of each parameter.

For each combination of parameters generated by this experimental design, the simulation is run 1000 times using different values – chosen from a normal distribution – for the angle and Doppler measurement errors. The fitting algorithm is provided with three initial guess for a , b , and rotation angle, as described in section 4.1.2. The initial guesses used for a and b are equal to 1.10 and 0.90 times their actual values, respectively. The initial guess used for the rotation angle is the actual angle plus 10 degrees. Although these initial guesses are unrealistically close to the actual, unknown values, the point of the simulation is to compare the accuracy of the fitting with and without the inclusion of the Doppler measurements.

4.3 Results

We have found that visualization of the estimated orbits and their associated parameters is an effective way to understand the results. The following charts include both density plots and histograms to describe the statistical variation of results over the full range of 1000 random trials. Currently, these results are still being investigated and methods of enabling our algorithm to converge to the global minimum, given a wider range of initial starting estimates, is being investigated. Additionally, all the current results were made by simulated observations approximately 30 minutes apart. Greater improvement in the accuracy of many orbital fits is observed when an observation is made at least one (or more) orbital periods away from any previous observations (or set of observations). Thus, these results demonstrate the performance that can be expected for a truly short window for observation time on targets with various orbital geometries. The plots with dark blue background show several features on a single chart. The central green circle represents the Earth, marked with a '+' at its center and an 'x' at the location of the observer. The red ellipse surrounding Earth represents the actual simulated orbit. The fuzzy, multi-colored ellipse is a density plot of 1000 estimated orbits. Its brightness indicates the density of estimated orbital points in a given location; brighter areas are more densely populated.

The histograms provide a less intuitive, but more precise, numerical distribution of the same data. Each chart shows simulation results for a , b , and rotation angle versus the empirical probability of finding a given range of values within 1000 simulations. For reference purposes, the red dashed line indicates the actual value of the parameter for each simulation.

4.3.1 Two Observations

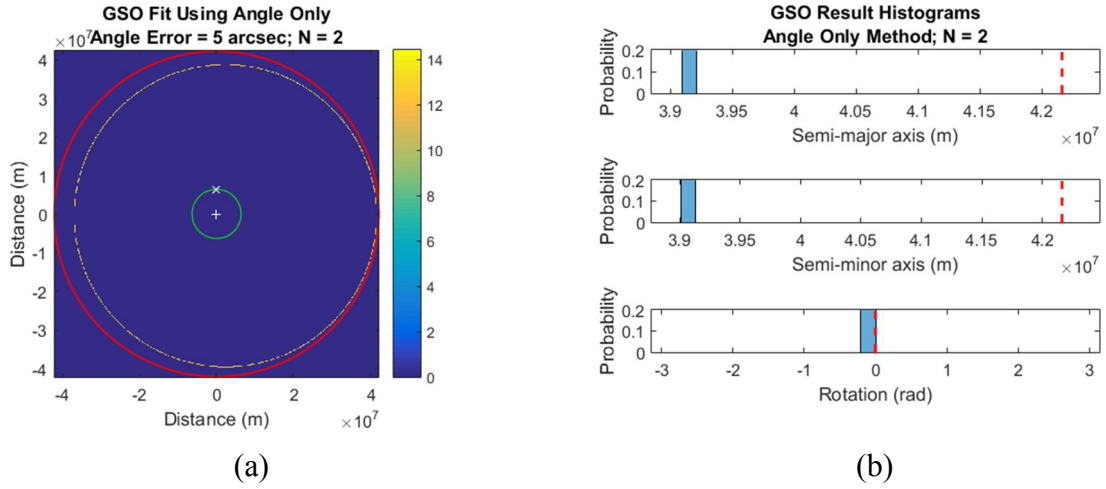


Fig. 62. (a) Density plot of 1000 estimated orbits using the angle-only method. Two observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 7.3%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 39150 km, 39080 km, and -0.1887 rad; standard deviation values are 1 km, 1 km, and 0.0000 rad, respectively.

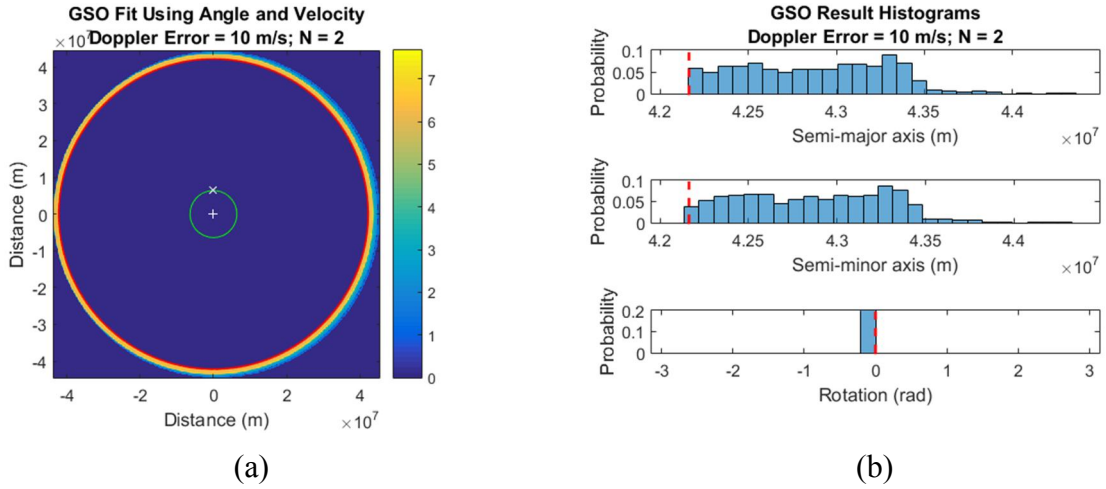


Fig. 63. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Two observations were used for the fitting. The average error of the fit is 1.7%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 42871 km, 42870 km, and -0.1314 rad; standard deviation values are 409 km, 408 km, and 0.0055 rad, respectively.

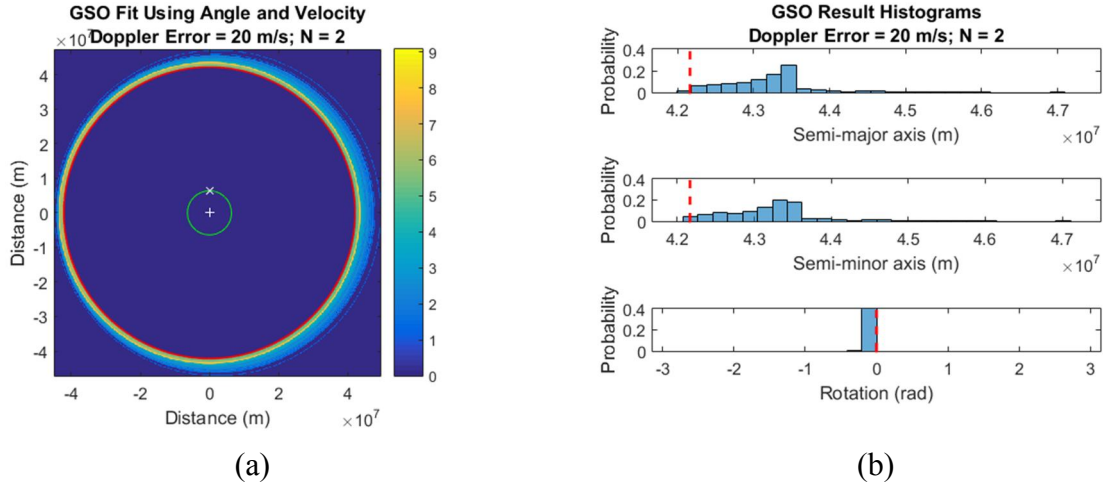


Fig. 64. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Two observations were used for the fitting. The average error of the fit is 2.5%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 43218 km, 43216 km, and -0.1271 rad; standard deviation values are 624 km, 620 km, and 0.0104 rad, respectively.

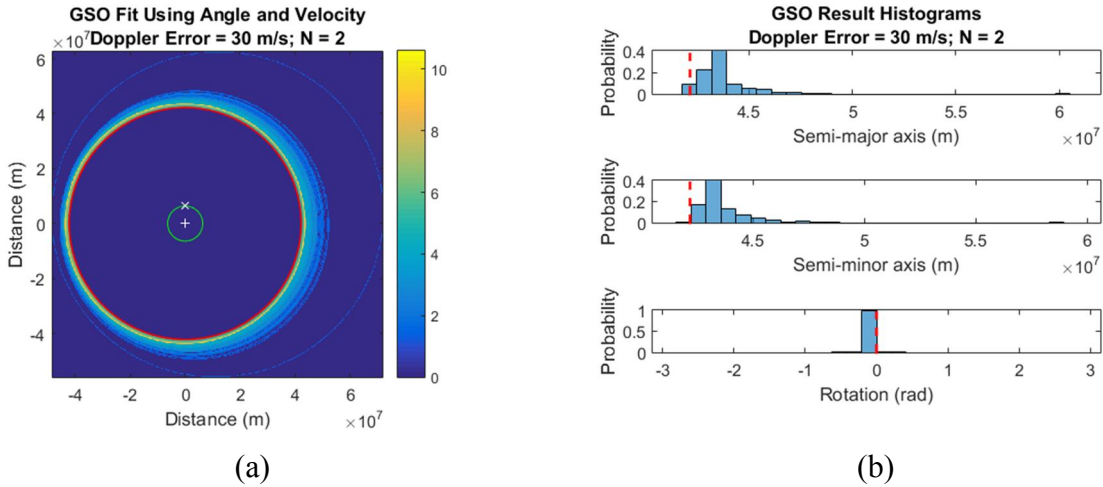


Fig. 65. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Two observations were used for the fitting. The average error of the fit is 3.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 43593 km, 43586 km, and -0.1241 rad; standard deviation values are 1091 km, 1063 km, and 0.0255 rad, respectively.

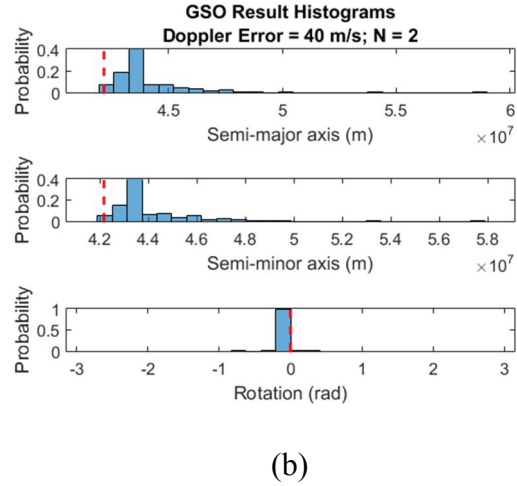
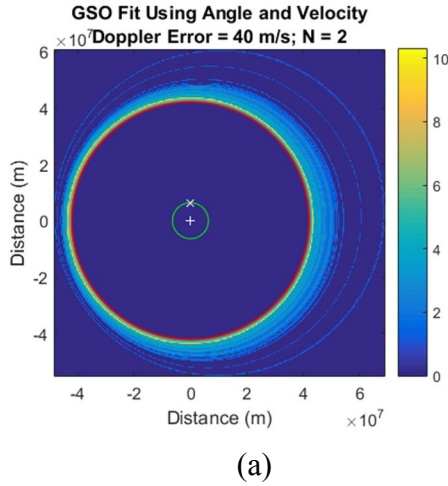


Fig. 66. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Two observations were used for the fitting. The average error of the fit is 4.1%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 43914 km, 43902 km, and -0.1213 rad; standard deviation values are 1456 km, 1418 km, and 0.0396 rad, respectively.

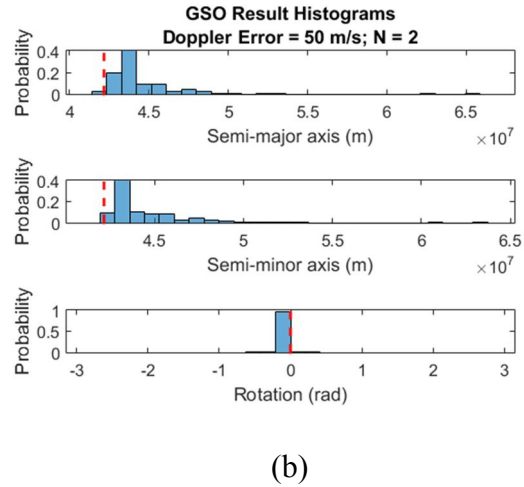
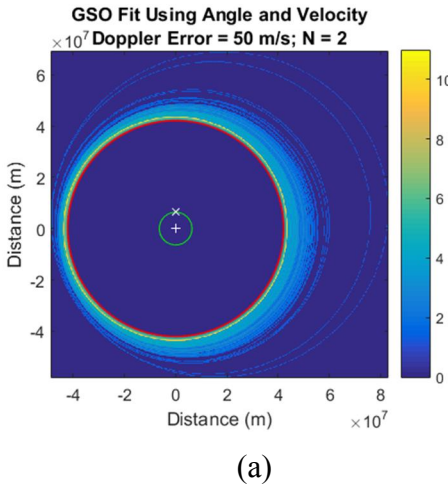
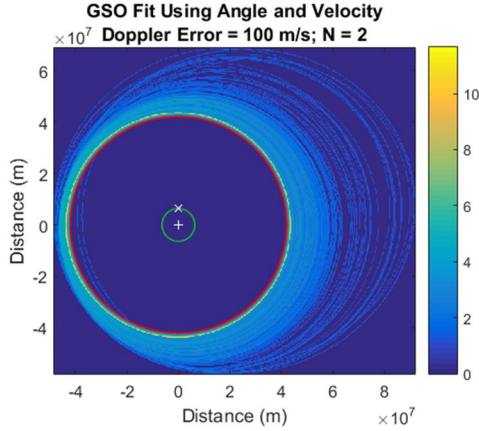
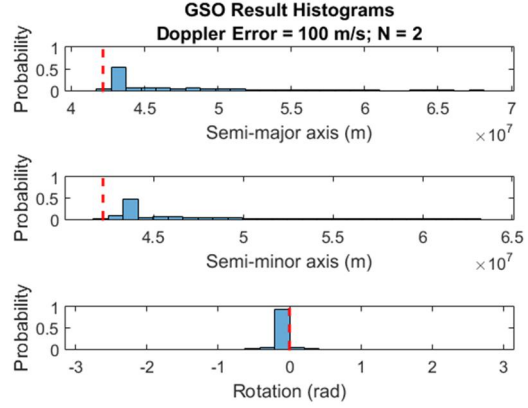


Fig. 67. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Two observations were used for the fitting. The average error of the fit is 4.8%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 44217 km, 44198 km, and -0.1211 rad; standard deviation values are 1831 km, 1758 km, and 0.0413 rad, respectively.

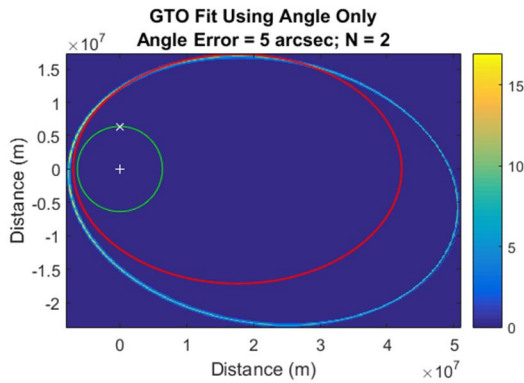


(a)

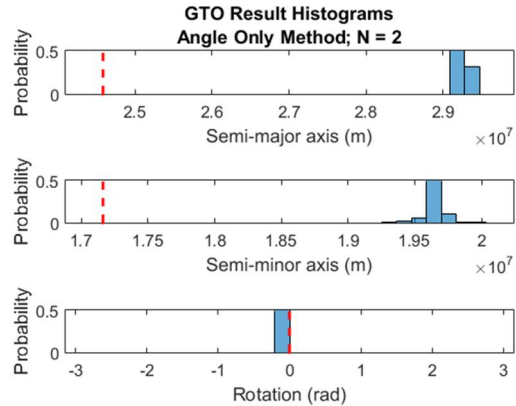


(b)

Fig. 68. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Two observations were used for the fitting. The average error of the fit is 8.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 45658 km, 45543 km, and -0.1166 rad; standard deviation values are 3805 km, 3448 km, and 0.0645 rad, respectively.

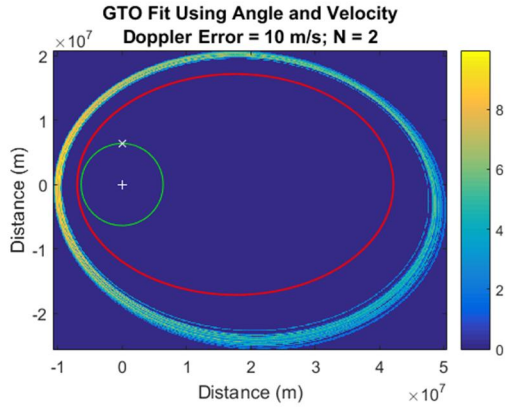


(a)

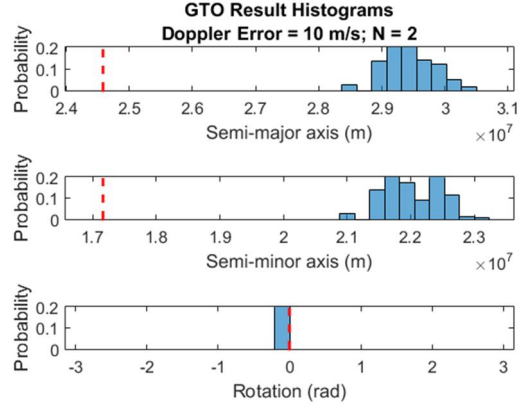


(b)

Fig. 69. (a) Density plot of 1000 estimated orbits using the angle-only method. Two observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 11.8%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29276 km, 19640 km, and -0.1581 rad; standard deviation values are 23 km, 57 km, and 0.0004 rad, respectively.

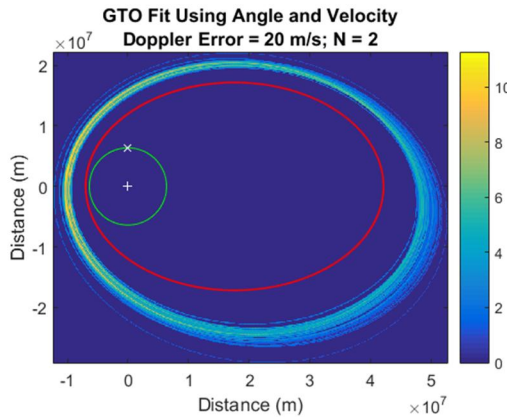


(a)

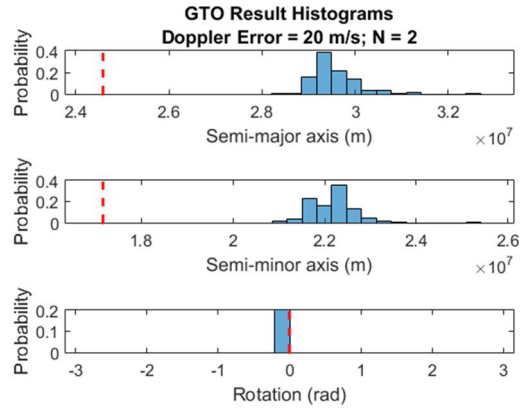


(b)

Fig. 70. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Two observations were used for the fitting. The average error of the fit is 34.1%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29437 km, 22044 km, and -0.1011 rad; standard deviation values are 361 km, 402 km, and 0.0081 rad, respectively.

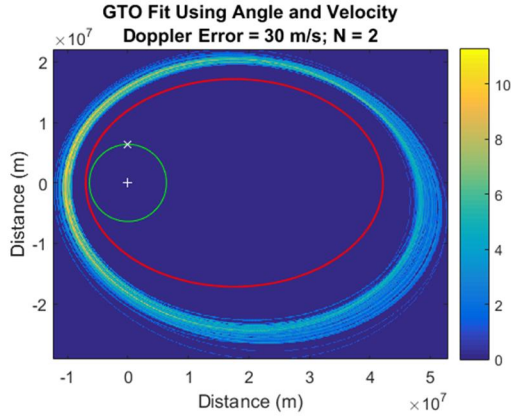


(a)

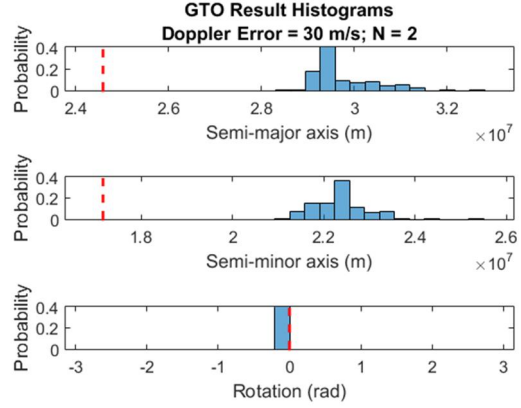


(b)

Fig. 71. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Two observations were used for the fitting. The average error of the fit is 35.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29539 km, 22170 km, and -0.1035 rad; standard deviation values are 447 km, 442 km, and 0.0091 rad, respectively.

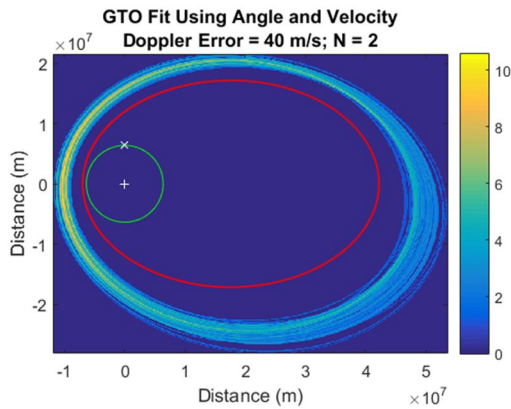


(a)

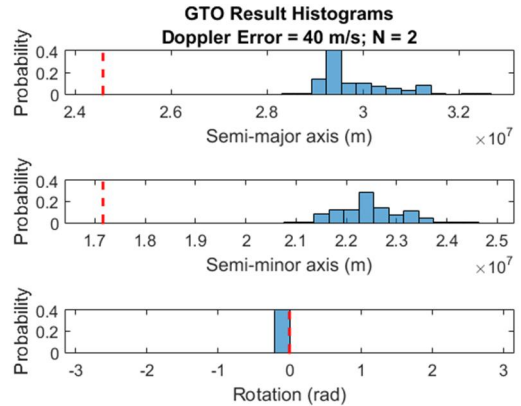


(b)

Fig. 72. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Two observations were used for the fitting. The average error of the fit is 36.2%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29707 km, 22337 km, and -0.1070 rad; standard deviation values are 630 km, 538 km, and 0.0118 rad, respectively.

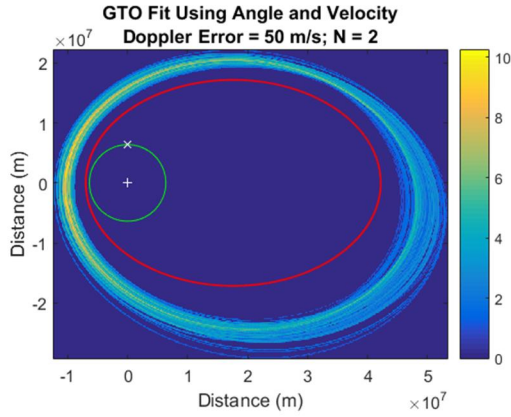


(a)

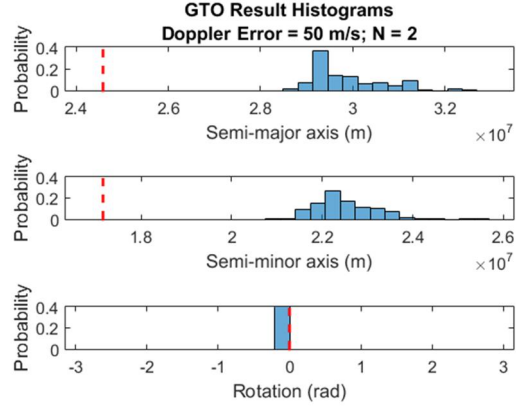


(b)

Fig. 73. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Two observations were used for the fitting. The average error of the fit is 36.9%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29789 km, 22433 km, and -0.1089 rad; standard deviation values are 702 km, 598 km, and 0.0128 rad, respectively.

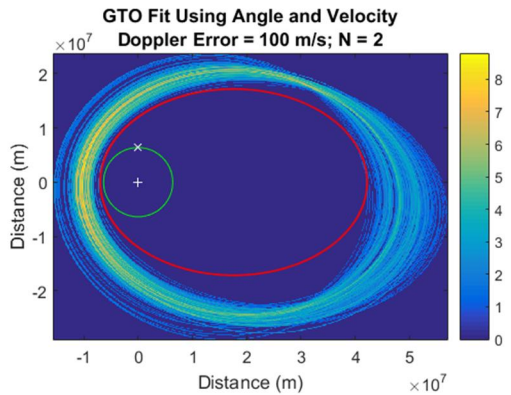


(a)

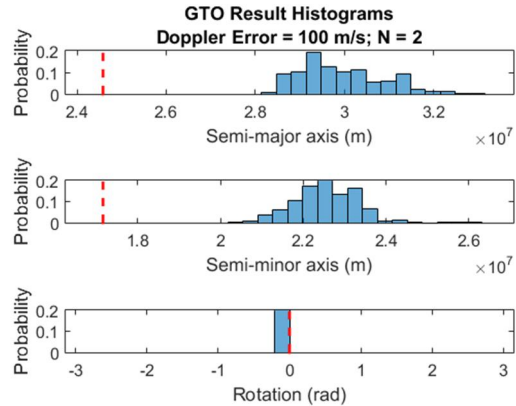


(b)

Fig. 74. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Two observations were used for the fitting. The average error of the fit is 37.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29838 km, 22492 km, and -0.1101 rad; standard deviation values are 730 km, 627 km, and 0.0131 rad, respectively.



(a)



(b)

Fig. 75. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Two observations were used for the fitting. The average error of the fit is 37.9%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29936 km, 22573 km, and -0.1118 rad; standard deviation values are 935 km, 770 km, and 0.0137 rad, respectively.

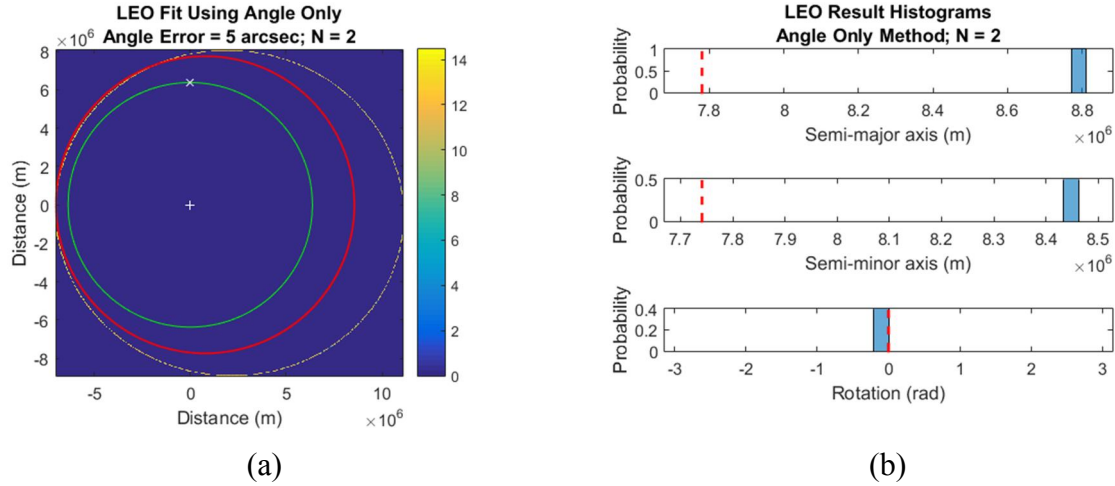


Fig. 76. (a) Density plot of 1000 estimated orbits using the angle-only method. Two observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 12.6%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 8781 km, 8455 km, and -0.1803 rad; standard deviation values are 0 km, 0 km, and 0.0000 rad, respectively.

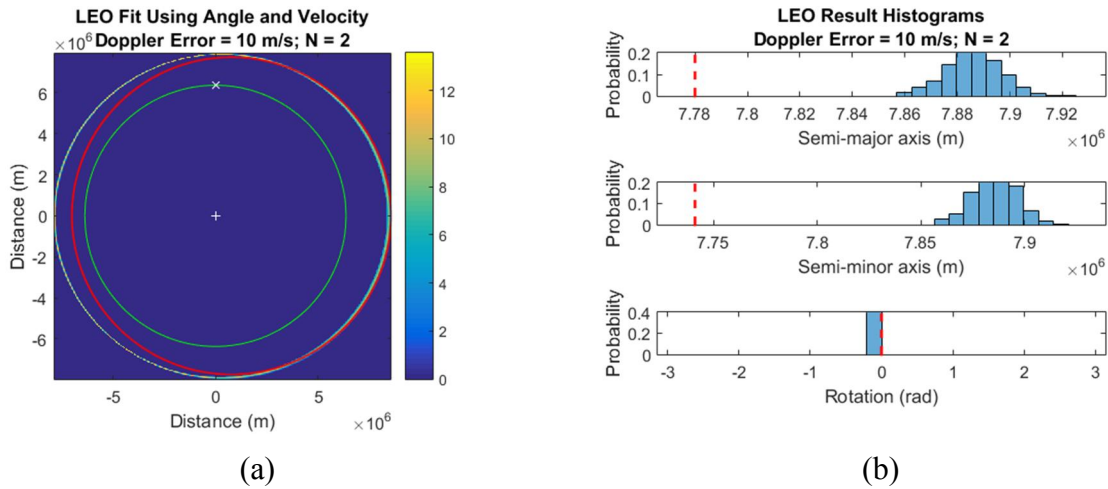


Fig. 77. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Two observations were used for the fitting. The average error of the fit is 6.2%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7886 km, 7886 km, and -0.2033 rad; standard deviation values are 11 km, 10 km, and 0.0024 rad, respectively.

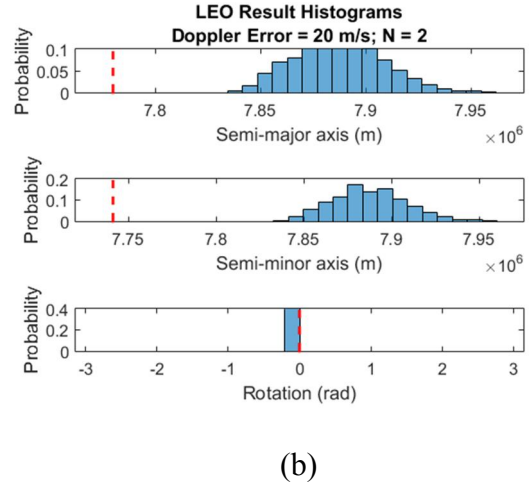
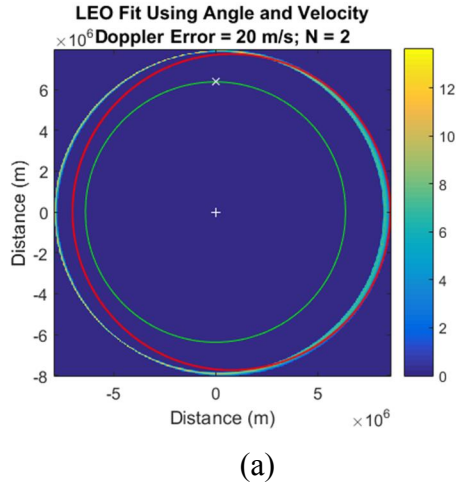


Fig. 78. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Two observations were used for the fitting. The average error of the fit is 6.1%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7887 km, 7887 km, and -0.2027 rad; standard deviation values are 21 km, 21 km, and 0.0019 rad, respectively.

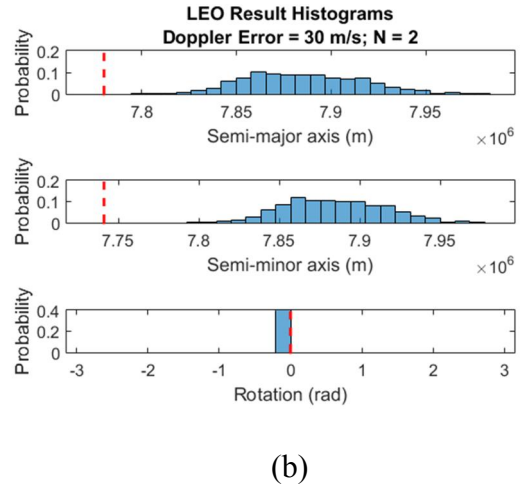
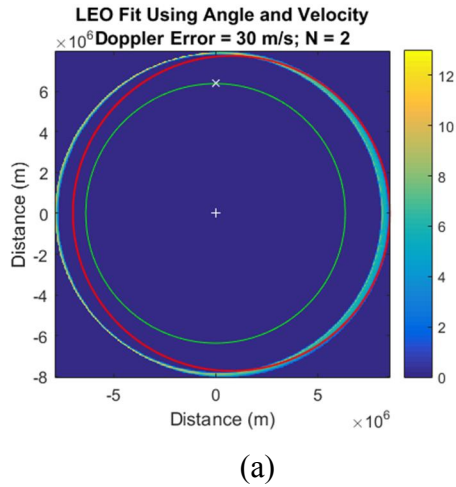


Fig. 79. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Two observations were used for the fitting. The average error of the fit is 6.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7885 km, 7885 km, and -0.2026 rad; standard deviation values are 31 km, 30 km, and 0.0017 rad, respectively.

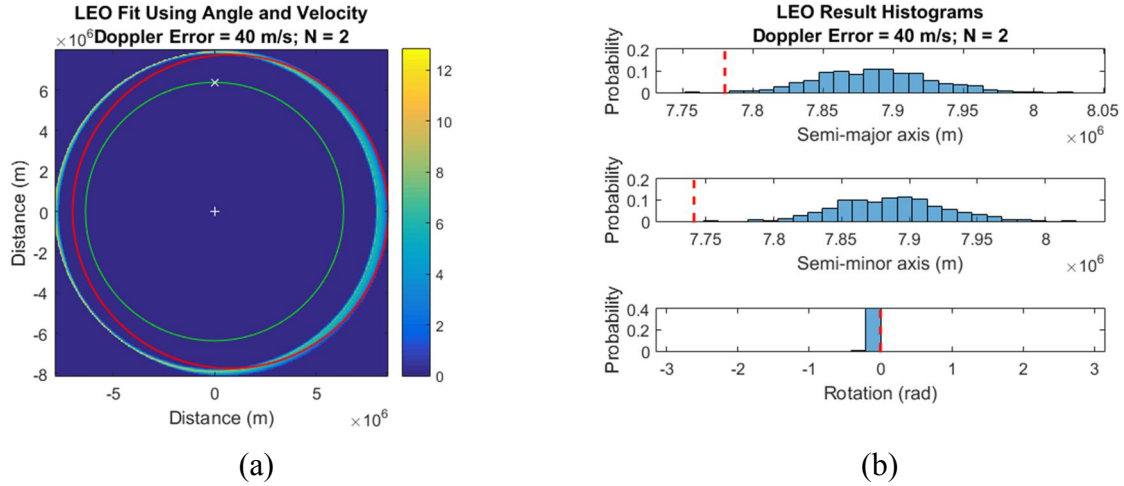


Fig. 80. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Two observations were used for the fitting. The average error of the fit is 5.9%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7887 km, 7887 km, and -0.2023 rad; standard deviation values are 39 km, 38 km, and 0.0022 rad, respectively.

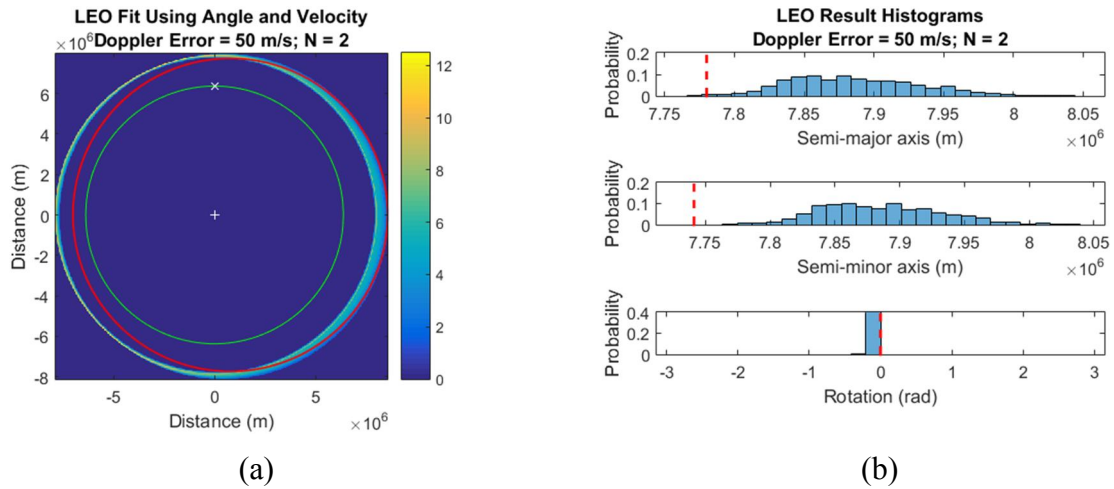


Fig. 81. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Two observations were used for the fitting. The average error of the fit is 5.8%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7886 km, 7885 km, and -0.2022 rad; standard deviation values are 47 km, 46 km, and 0.0026 rad, respectively.

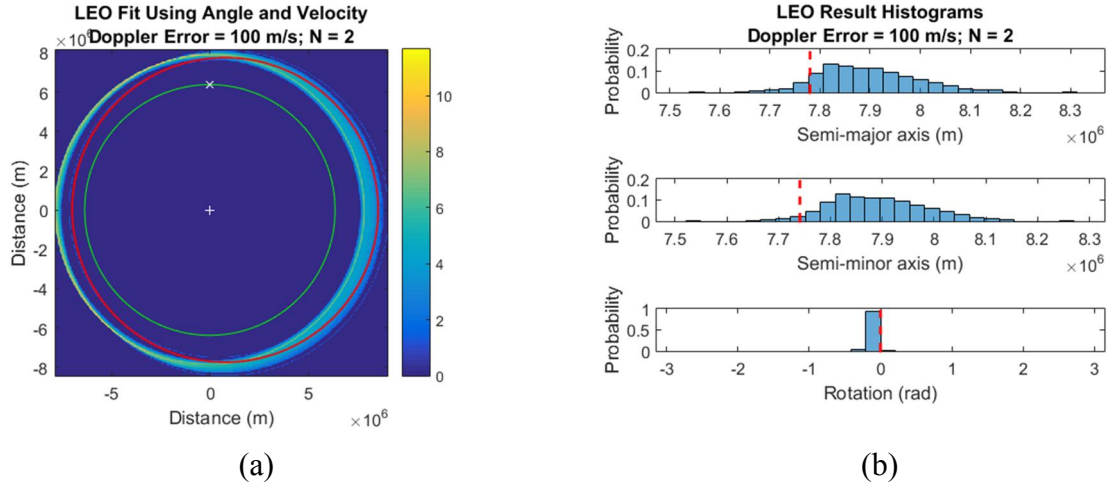


Fig. 82. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Two observations were used for the fitting. The average error of the fit is 5.1%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7896 km, 7893 km, and -0.2012 rad; standard deviation values are 97 km, 94 km, and 0.0118 rad, respectively.

4.3.2 Three Observations

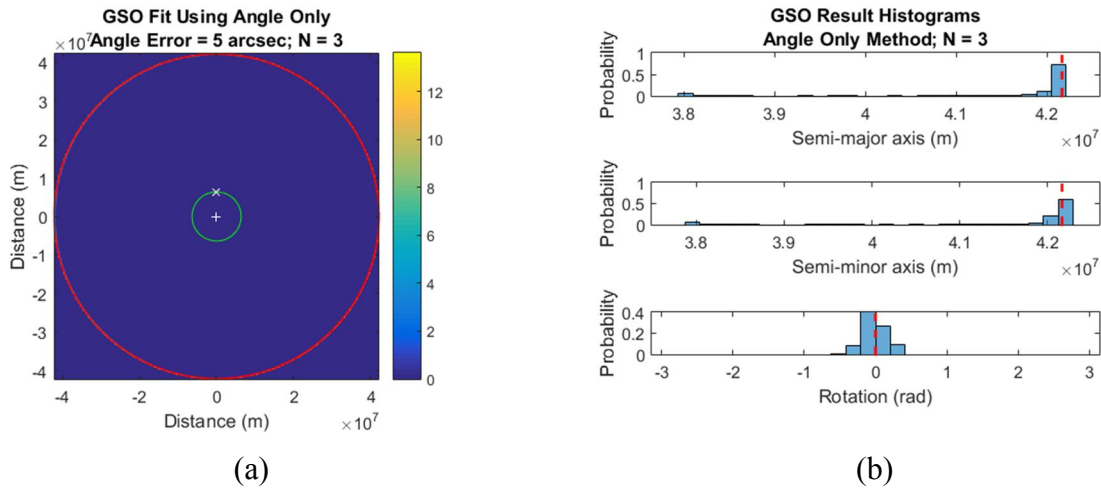


Fig. 83. (a) Density plot of 1000 estimated orbits using the angle-only method. Three observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 1.8%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 41674 km, 41661 km, and -0.0466 rad; standard deviation values are 1166 km, 1203 km, and 0.1718 rad, respectively.

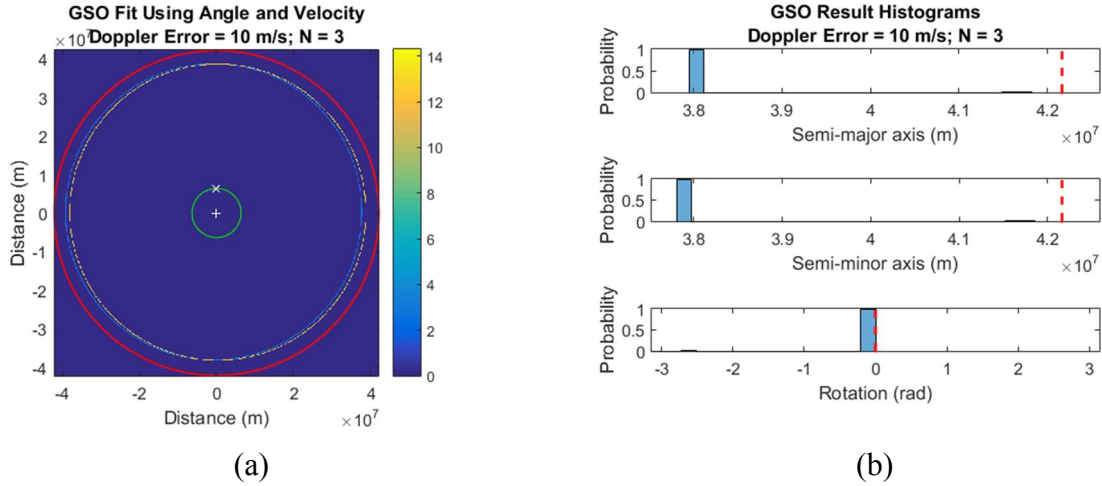


Fig. 84. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Three observations were used for the fitting. The average error of the fit is 9.9%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 37974 km, 37973 km, and -0.2497 rad; standard deviation values are 309 km, 309 km, and 0.4386 rad, respectively.

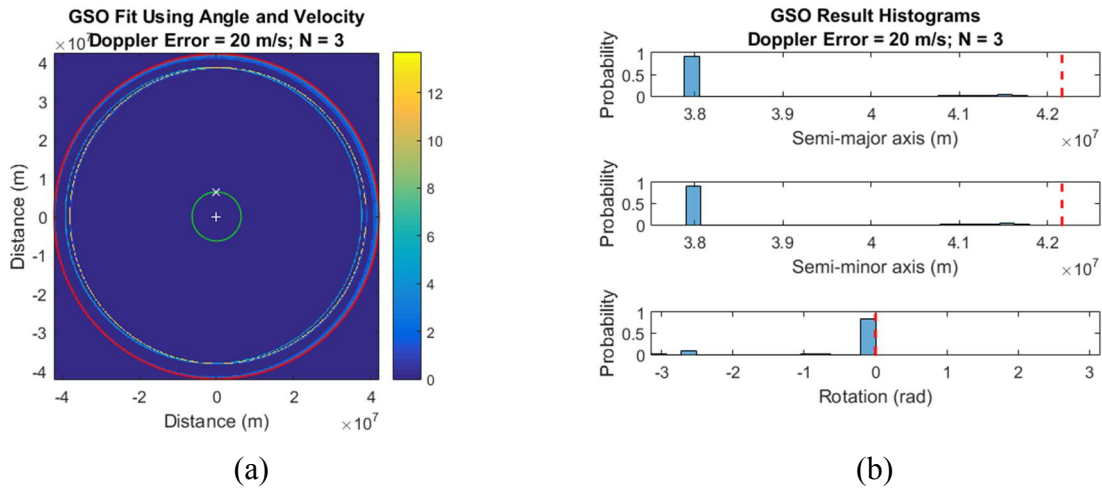


Fig. 85. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Three observations were used for the fitting. The average error of the fit is 9.3%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 38252 km, 38250 km, and -0.6126 rad; standard deviation values are 985 km, 984 km, and 0.9869 rad, respectively.

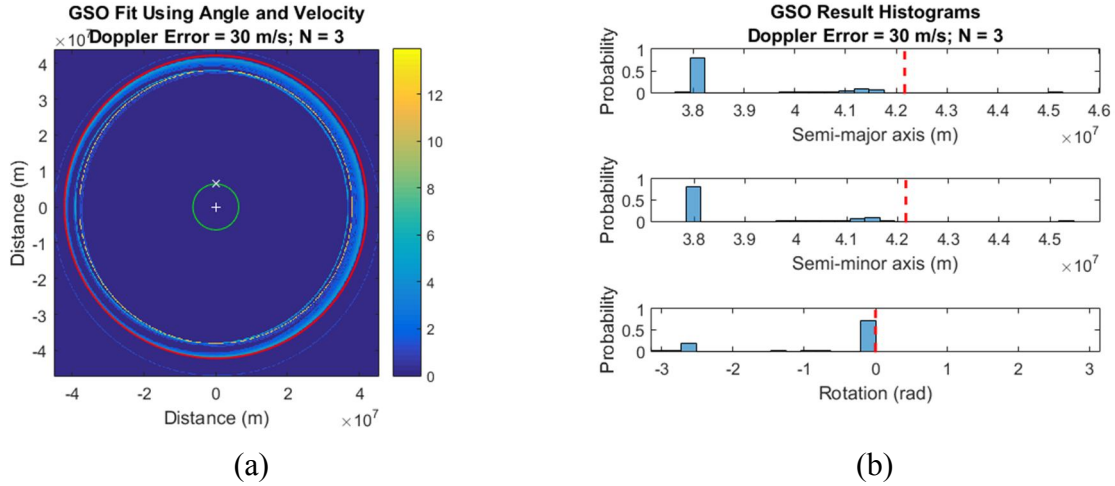


Fig. 86. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Three observations were used for the fitting. The average error of the fit is 8.5%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 38579 km, 38576 km, and -0.9066 rad; standard deviation values are 1321 km, 1319 km, and 1.1703 rad, respectively.

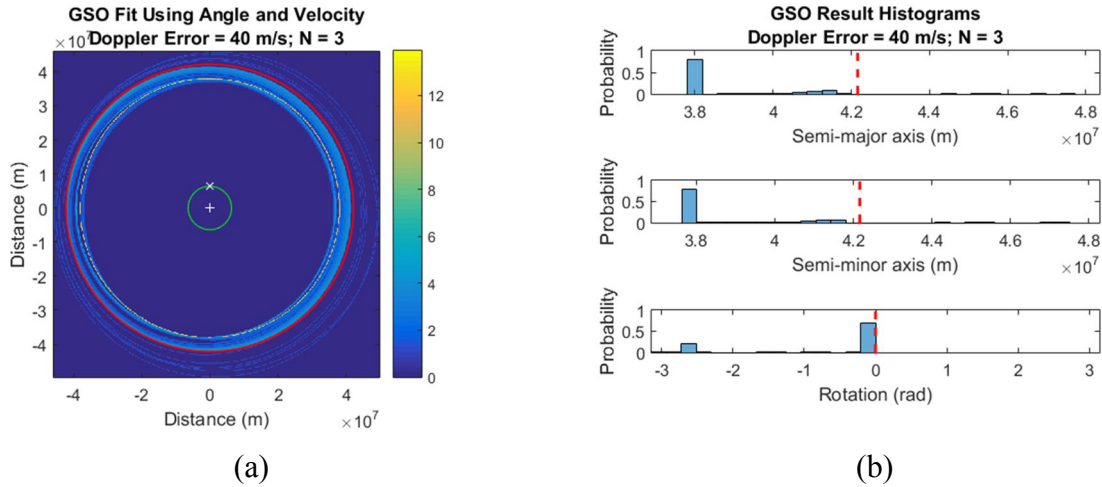


Fig. 87. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Three observations were used for the fitting. The average error of the fit is 8.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 38629 km, 38625 km, and -0.8942 rad; standard deviation values are 1384 km, 1380 km, and 1.1374 rad, respectively.

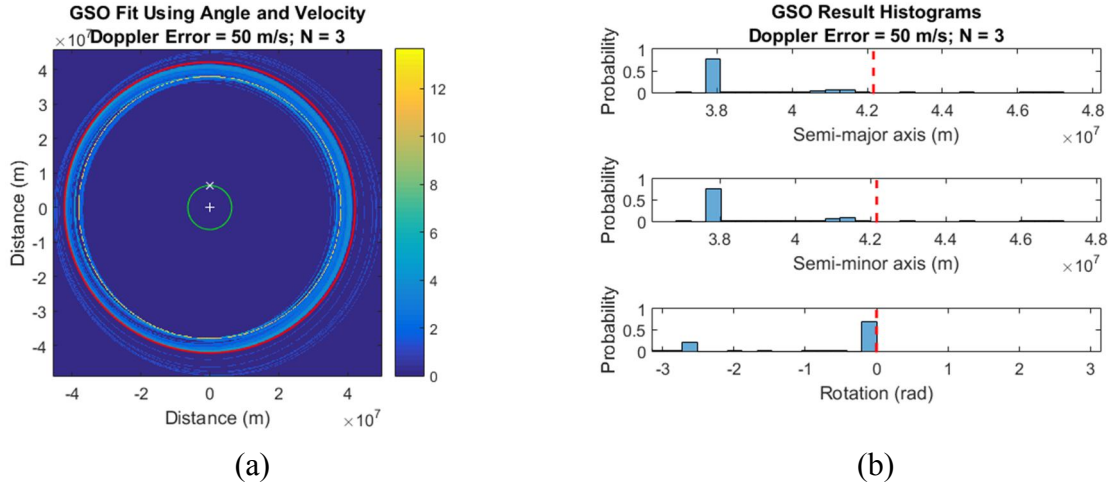


Fig. 88. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Three observations were used for the fitting. The average error of the fit is 8.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 38627 km, 38622 km, and -0.9464 rad; standard deviation values are 1370 km, 1366 km, and 1.1637 rad, respectively.

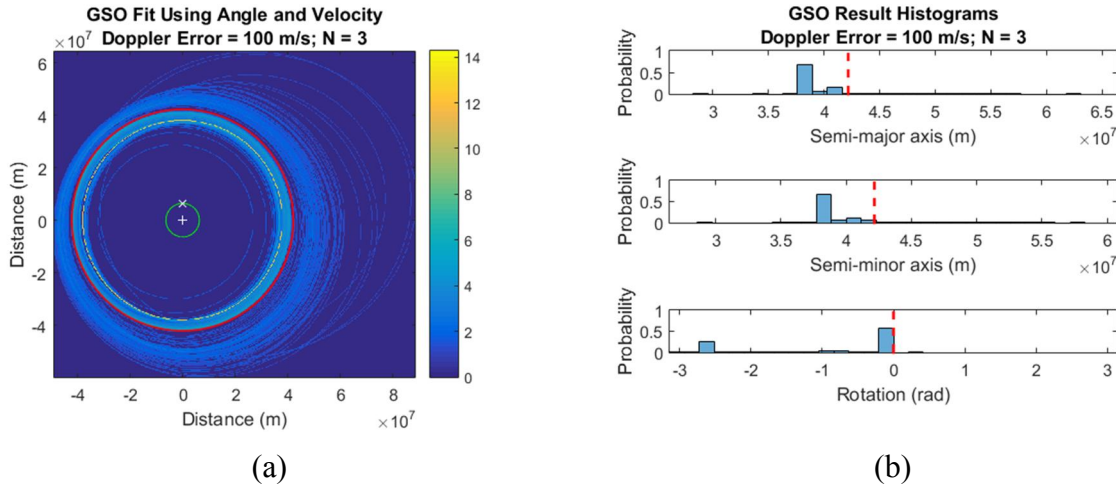
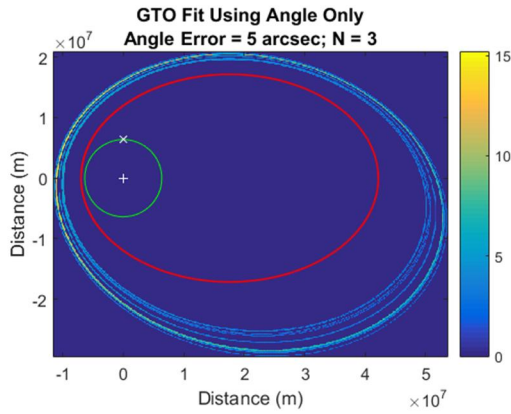
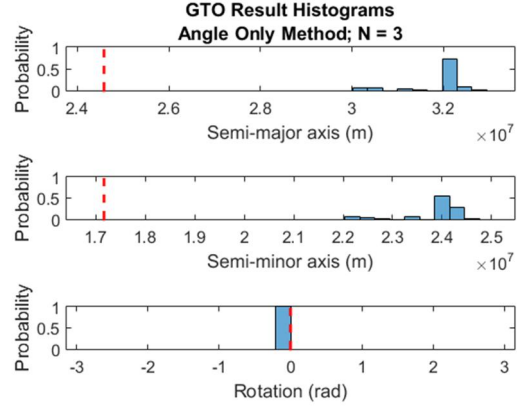


Fig. 89. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Three observations were used for the fitting. The average error of the fit is 6.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 39528 km, 39484 km, and -1.0345 rad; standard deviation values are 3393 km, 3230 km, and 1.1381 rad, respectively.

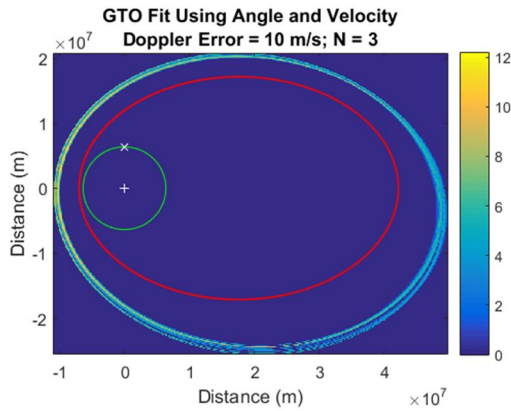


(a)

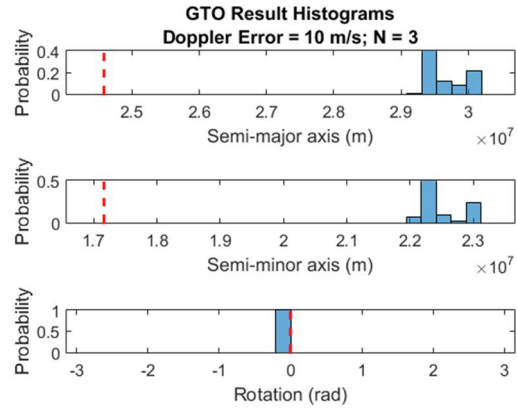


(b)

Fig. 90. (a) Density plot of 1000 estimated orbits using the angle-only method. Three observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 45.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 31923 km, 23906 km, and -0.1836 rad; standard deviation values are 633 km, 585 km, and 0.0151 rad, respectively.

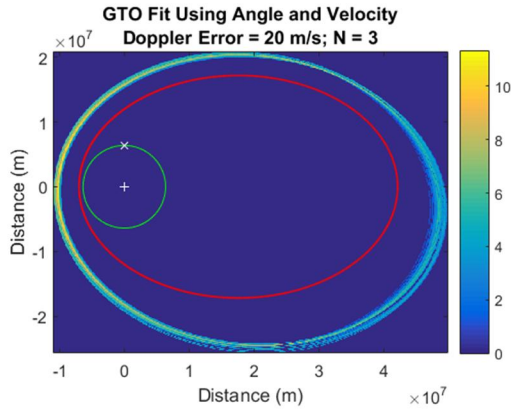


(a)

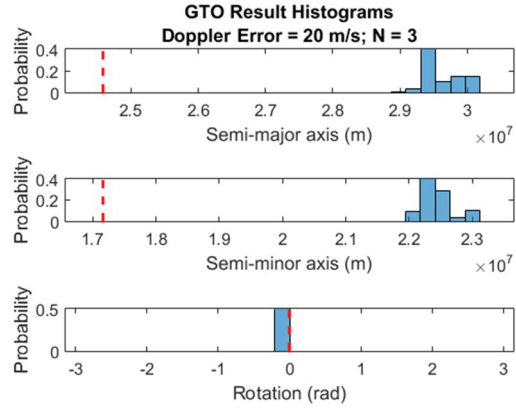


(b)

Fig. 91. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Three observations were used for the fitting. The average error of the fit is 37.5%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29647 km, 22451 km, and -0.1118 rad; standard deviation values are 264 km, 322 km, and 0.0067 rad, respectively.

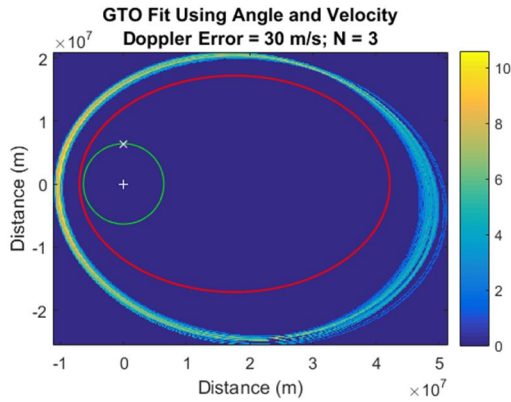


(a)

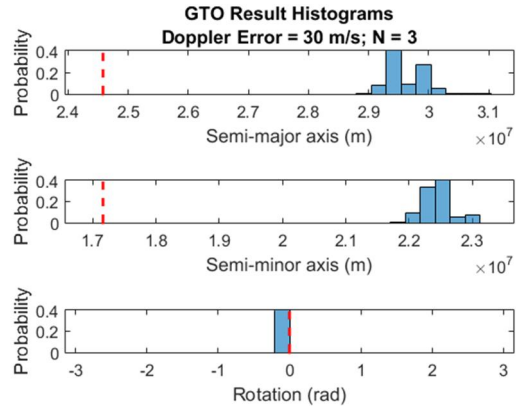


(b)

Fig. 92. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Three observations were used for the fitting. The average error of the fit is 37.2%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29610 km, 22405 km, and -0.1108 rad; standard deviation values are 251 km, 245 km, and 0.0051 rad, respectively.

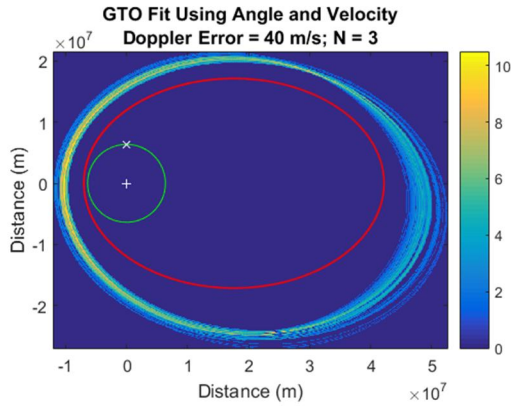


(a)

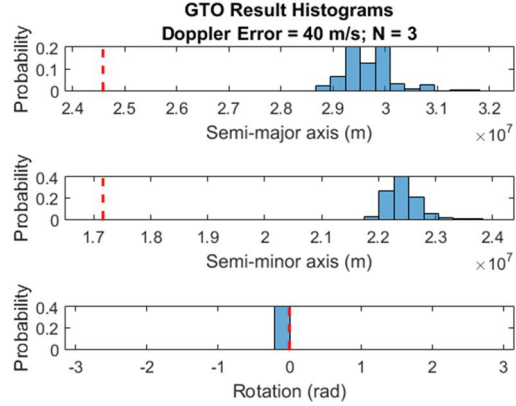


(b)

Fig. 93. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Three observations were used for the fitting. The average error of the fit is 37.4%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29618 km, 22433 km, and -0.1112 rad; standard deviation values are 296 km, 238 km, and 0.0049 rad, respectively.

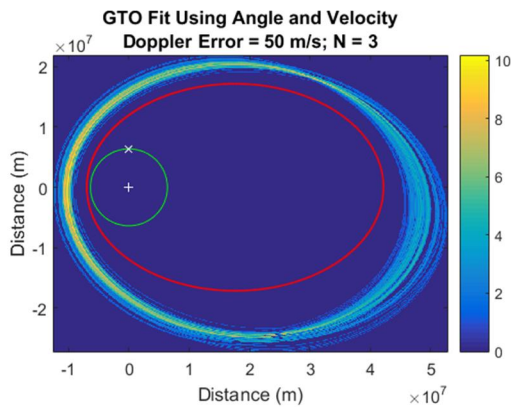


(a)

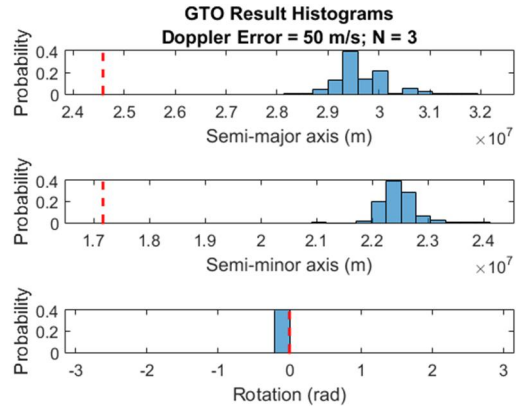


(b)

Fig. 94. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Three observations were used for the fitting. The average error of the fit is 37.3%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29634 km, 22427 km, and -0.1112 rad; standard deviation values are 392 km, 270 km, and 0.0057 rad, respectively.

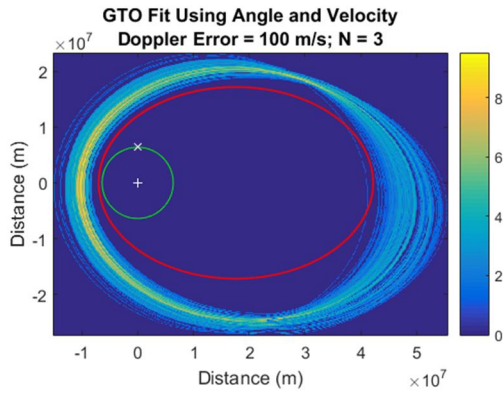


(a)

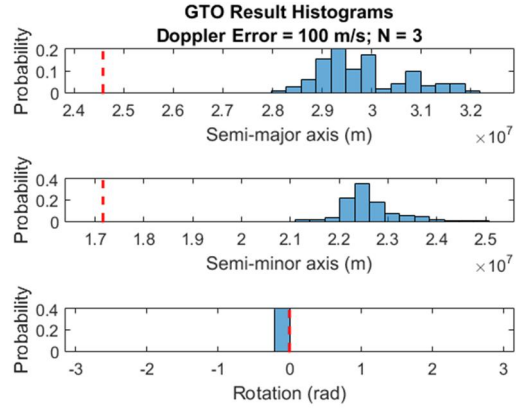


(b)

Fig. 95. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Three observations were used for the fitting. The average error of the fit is 37.6%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29658 km, 22462 km, and -0.1118 rad; standard deviation values are 490 km, 303 km, and 0.0067 rad, respectively.

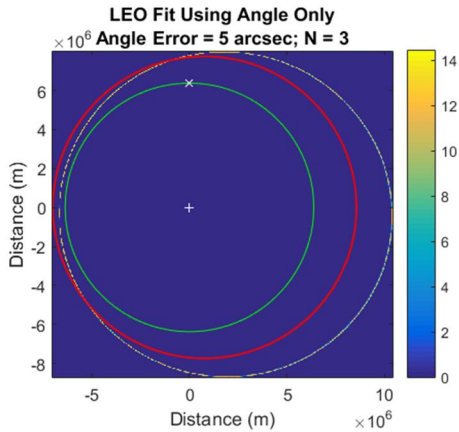


(a)

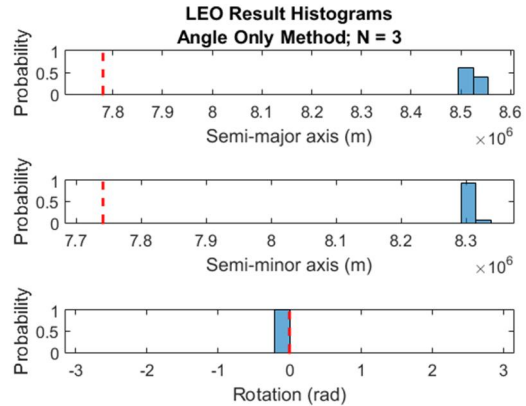


(b)

Fig. 96. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Three observations were used for the fitting. The average error of the fit is 38.7%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 29813 km, 22619 km, and -0.1150 rad; standard deviation values are 857 km, 509 km, and 0.0101 rad, respectively.



(a)



(b)

Fig. 97. (a) Density plot of 1000 estimated orbits using the angle-only method. Three observations with a measurement error of 5 arcsec (standard deviation) were used for the fitting. The average error of the fit is 9.2%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 8524 km, 8311 km, and -0.1896 rad; standard deviation values are 3 km, 2 km, and 0.0005 rad, respectively.

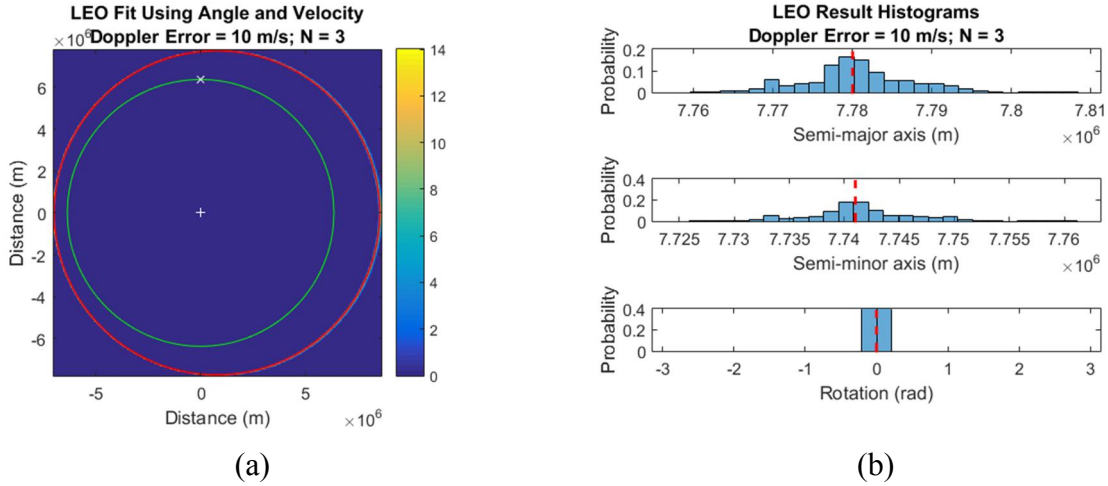


Fig. 98. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 10 m/s. Three observations were used for the fitting. The average error of the fit is 0.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7780 km, 7741 km, and -0.0000 rad; standard deviation values are 7 km, 5 km, and 0.0013 rad, respectively.

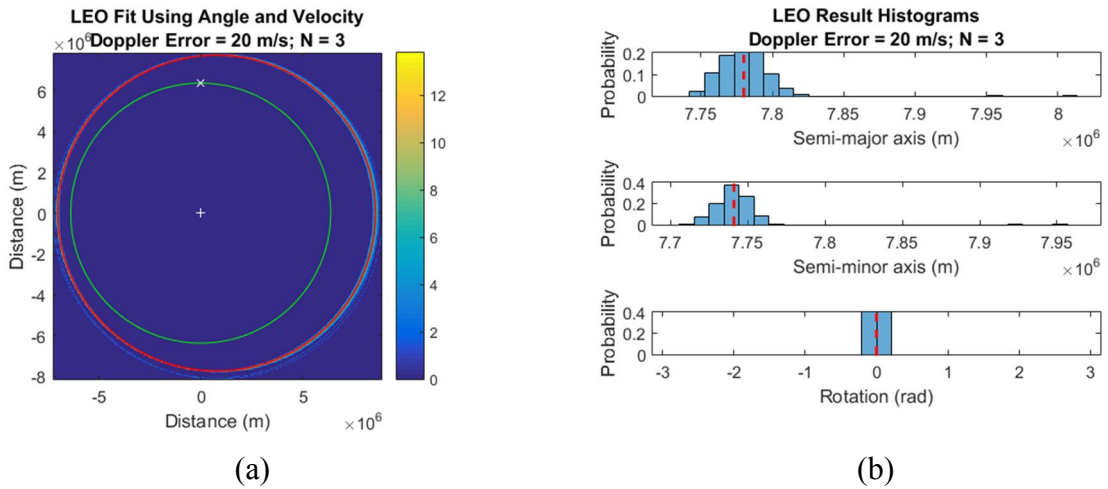


Fig. 99. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 20 m/s. Three observations were used for the fitting. The average error of the fit is 0.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7780 km, 7741 km, and -0.0002 rad; standard deviation values are 18 km, 15 km, and 0.0111 rad, respectively.

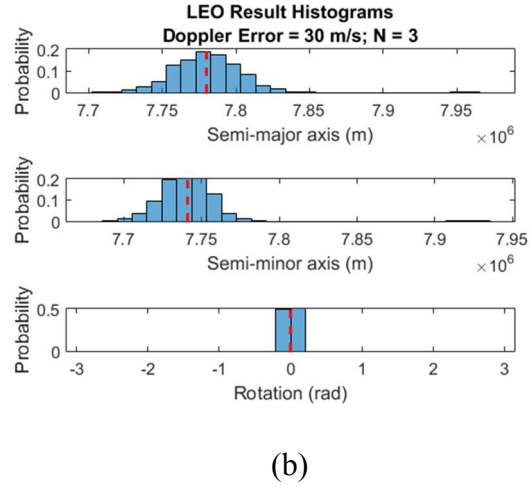
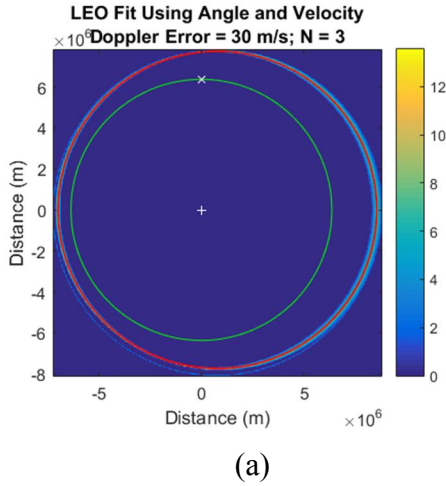


Fig. 100. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 30 m/s. Three observations were used for the fitting. The average error of the fit is 0.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7781 km, 7742 km, and -0.0002 rad; standard deviation values are 24 km, 20 km, and 0.0150 rad, respectively.

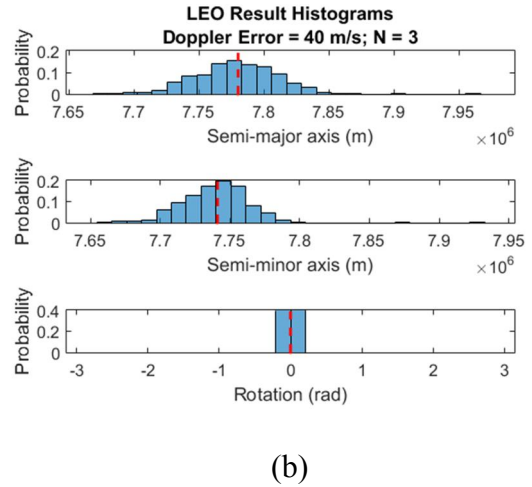
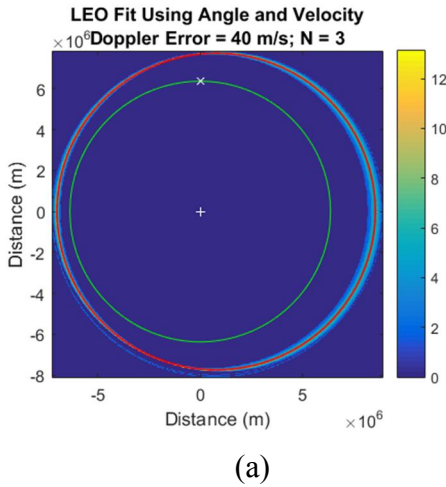


Fig. 101. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 40 m/s. Three observations were used for the fitting. The average error of the fit is 0.0%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7779 km, 7740 km, and 0.0013 rad; standard deviation values are 32 km, 24 km, and 0.0145 rad, respectively.

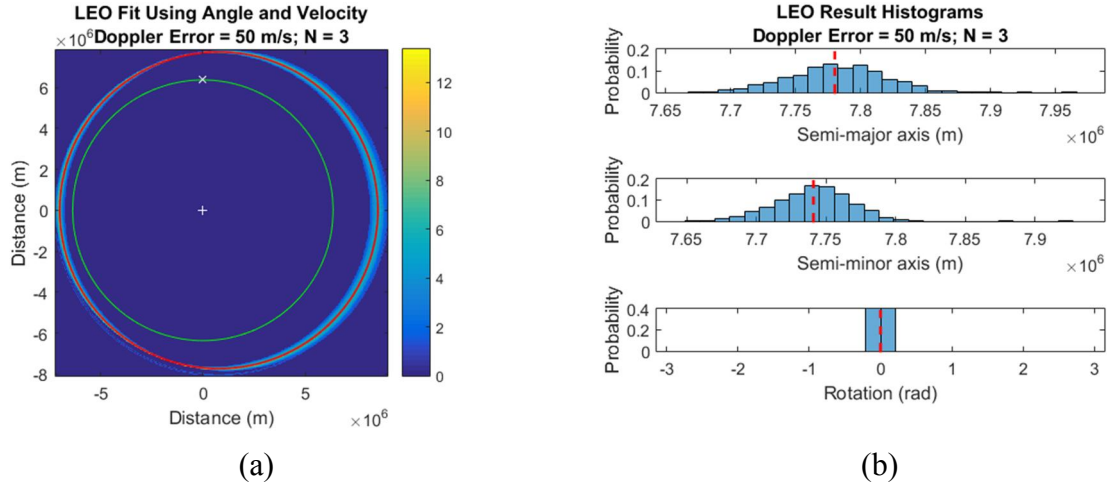


Fig. 102. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 50 m/s. Three observations were used for the fitting. The average error of the fit is 0.1%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7781 km, 7741 km, and 0.0020 rad; standard deviation values are 37 km, 28 km, and 0.0126 rad, respectively.

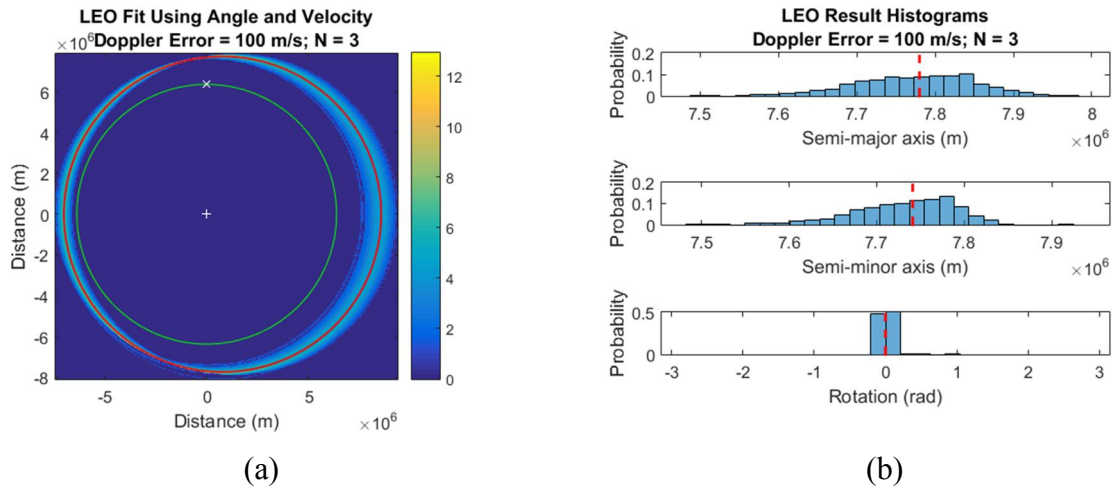


Fig. 103. (a) Density plot of 1000 estimated orbits using both angle and velocity measurements. Standard deviation of the velocity error is 100 m/s. Three observations were used for the fitting. The average error of the fit is 0.2%. (b) Histograms of the estimated orbital parameters. Dashed red lines indicate the actual values. Mean values for semi-major axis, semi-minor axis, and orbital rotation are 7773 km, 7732 km, and 0.0164 rad; standard deviation values are 76 km, 57 km, and 0.0582 rad, respectively.

5.0 General Orbital Simulation

The goal of this task was to quantify the required Doppler measurement accuracy and the overall benefit of this technique. An orbital simulation model was developed, including a numerical fitting procedure for estimation of orbital parameters using two or more passive measurements.

5.1 Orbital Parameters

Doppler shift is dependent on the radial velocity between an object and the source of illumination. In other words, the change in wavelength is proportional to the component of velocity in a direct line between the two points, regardless of the absolute speed of the object. Therefore, the angle formed between the velocity vector and the direction to the source is critical to correctly evaluating the Doppler shift.

In the case of orbiting bodies, a second angle must be taken into account. That is the angle between the target's velocity vector and the direction to the observer on the surface of Earth. In order to simulate all of these angles, the model must represent the target's orbit in three dimensions. A full description of state of an orbital body requires six independent parameters. Three spatial parameters (x , y , and z) represent position, and their time derivatives (dx , dy , and dz) represent velocity. Using this Cartesian representation of an orbit, each of the six parameters takes on a unique value at each point in the orbit.

However, there are other ways to represent a three dimensional orbit mathematically. One of the most commonly used is the Keplerian system shown in Fig. 104. In contrast to the continually changing Cartesian parameters, five of the six Keplerian parameters are constant for any given orbit. Only the value of *true anomaly* changes to indicate a particular position on the orbital ellipse.

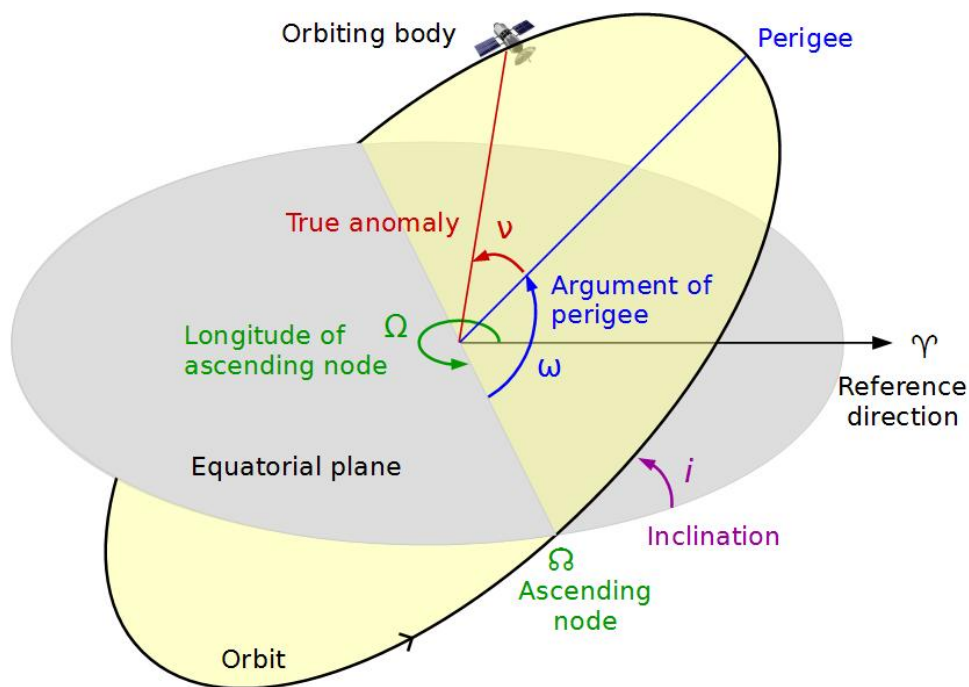


Fig. 104. Keplerian orbital parameters.

The six Keplerian parameters are described below, and summarized in Table 24 along with the corresponding variable names used in the simulation code.

- The *semi-major axis* (a) describes the spatial length of the orbit. It is measured along the long axis of the orbital ellipse, from the center to the edge.
- The *semi-minor axis* (b) describes the width of the orbit. It is measured along the short axis of the ellipse, from the center to the edge. Although our simulation uses the semi-minor axis, it is more usual to specify the *eccentricity* (e) which relates a and b by

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}. \quad (42)$$

- The *ascending node* (δ) is the point in space where the orbiting body crosses the equator in an upward (ascending) direction. The *longitude of the ascending node* (Ω) is the angle formed by the ascending node and a reference direction ($\mathcal{V}$). The usual reference direction is defined by a line from Earth to Sun on the day of the spring equinox.
- The *inclination* (i) is the angle between the equatorial plane and the orbital plane. Inclination ranges from 0° to 180° . An equatorial orbit, where the orbital plane is coincident with the equator, has an inclination of 0° . Due to orbital symmetry, an inclination of 180° is also an equatorial orbit, but with the orbiting body travelling in the opposite direction. By convention, a 0° or *prograde* orbit travels west to east, while a 180° *retrograde* orbit travels east to west. An inclination of 90° represents a polar orbit.
- *Perigee* is the point on an orbit that is closest to the Earth. The *argument of perigee* (ω) is the angle between the ascending node and perigee.
- Finally, the *true anomaly* (v) is the angle between perigee and the current position of the orbiting body.

Table 24. Keplerian parameters and the variable names used in the simulation.

Symbol	Variable	Name
a	a	Semi-major axis
b	b	Semi-minor axis
Ω	Omega	Longitude of the ascending node
i	inc	Inclination
ω	w	Argument of perigee
v	f	True anomaly

It is important to distinguish between the *description* of an orbit, provided by the Keplerian parameters, and an orbital *measurement*. While a measurement provides the immediate position of an object, it has no power to predict future location. Two angles and a range (distance) to the object, relative to the Earth's equatorial plane, give a measurement. The two angles are called *right ascension* (α) and *declination* (δ). Declination is the height of an object (in degrees) above the equator. Right ascension is the angle formed by the reference direction and a vector projection onto the equatorial plane. This is demonstrated in Fig. 105, which regards the position of the Sun in terms of right ascension and declination.

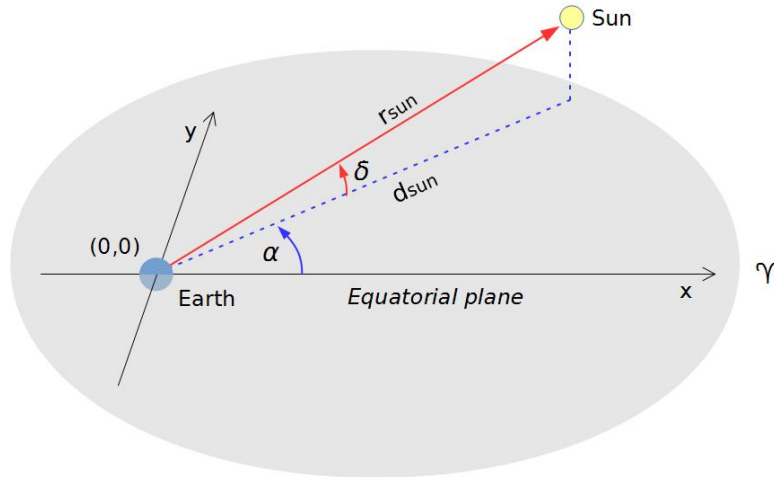


Fig. 105. Locating the Sun with respect to the simulation's reference frame. Angles α and δ are the right ascension and declination, respectively. The symbol for Aries (♈) indicates the direction of the Sun at vernal equinox, which is the reference direction for right ascension. Earth's equator is the reference for angles of declination.

5.2 Benefit of Doppler Velocity

In the classical two-body problem, an orbiting body is described by a state vector having three positional components and three velocity components in the x , y , and z directions. This reflects the fact that six degrees of freedom are required to describe an object moving in three dimensions. When the only available measurements are the angles of right ascension and declination, three separate observations are required to unambiguously determine an orbital solution.

However, the light reflected from a moving object holds additional information in the form of its spectral signature. In particular, reflected sunlight experiences an optical Doppler shift directly proportional to object velocity, as expressed in Eq. (1). It is important to note that the v in that expression is not the true velocity of the object; it is the radial, or Doppler, velocity in the direction of the light wave propagation. While knowledge of this Doppler velocity does not directly provide us with an object's true velocity, it is an additional piece of information that can be used for initial orbit determination (IOD).

5.3 Simulation Software

Since traditional orbital models do not include velocity as an input parameter, we constructed a numerical orbital fitting model that does. This model, implemented in MATLAB, enables simulation of orbital fitting using measurement of angles only (AO) or both angles and velocity (AV). The simulation software performs two main functions: orbit generation and orbit prediction. The goal is to estimate the complete orbit based on simulated observation of only a few points in the orbit.

The original version of the simulation suffered from two limitations. First, only equatorial orbits were considered. In contrast, the updated software is able to model any physically realizable geocentric orbit. Second, the original version did not include the Doppler contribution of the Earth's rotation, which was determined to be significant. However, the current version takes into account the observer's motion as a function of latitude.

A simplified block diagram of the simulation procedure is shown in Fig. 106. Keplerian parameters for the orbit to be simulated, plus reference coordinates for the Sun and the observing station, are provided as inputs to the simulation. Based on this data, orbital positions, velocities, and Doppler velocities are calculated. From the generated orbit data, two, three, or four sample points are chosen to represent observational data. These data values are corrupted by the addition of Gaussian noise in order to simulate real measurements.

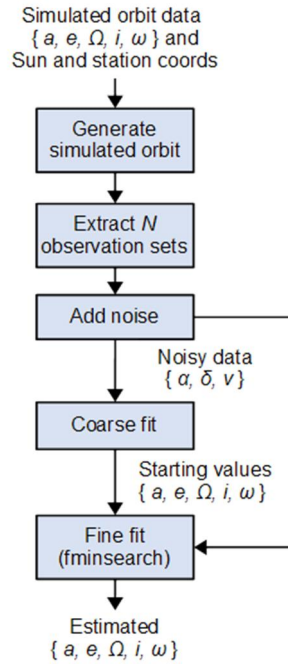


Fig. 106. Block diagram of the orbit generation and prediction algorithm.

Orbital prediction is performed by minimizing a cost function representing the goodness of fit between the measurement data and a set of estimated orbital parameters. Minimization is performed through a two-part procedure, also represented in Fig. 106. The first part of the procedure, called the “coarse fit,” searches through a database of pre-calculated orbits to find one that best fits the simulated measurements. The second part of the procedure is a “fine fit” iterative search. The fine fit incorporates the Nelder-Mead simplex algorithm implemented by the MATLAB *fminsearch* function. Because Nelder-Mead can be sensitive to the starting values [25], the coarse fit is used to provide it with good initial guesses. This reduces the likelihood of convergence on local minima in the solution space. After successful optimization of the orbital parameters, the results represent the best estimate of an orbit fitting the measured sample values.

The cost function that drives both the coarse fit and fine fit procedures is based on differences between the estimated orbits and the simulated measurements of right ascension, declination, and Doppler velocity (α , δ , and ν in Fig. 106, respectively). The angular error is calculated by first constructing unit vectors pointing to the estimated and actual target positions. Then the angular difference a_{err} between them is simply the magnitude of their cross product

$$a_{err} = \|\hat{u}' \times \hat{u}\|, \quad (43)$$

where

$$\hat{u}' = [\cos(\delta')\cos(\alpha'), \cos(\delta')\sin(\alpha'), \sin(\delta')]^T, \quad (44)$$

and

$$\hat{u} = [\cos(\delta)\cos(\alpha), \cos(\delta)\sin(\alpha), \sin(\delta)]^T, \quad (45)$$

such that $\hat{u}'$ and $\hat{u}$, α' and α , and δ' and δ are the estimated and actual unit vectors, right ascension angles, and declination angles, respectively. Doppler velocity error v_{err} is calculated as

$$v_{err} = \frac{|v - v'|}{v}, \quad (46)$$

where v' and v are the estimated and measured Doppler velocity values, respectively. The final cost function representing the overall estimation error is

$$fval = \sum_{i=1}^N (a_{err,i} + v_{err,i})^2, \quad (47)$$

where N is the total number of observations.

The relations between various functions used by the simulation are illustrated in Fig. 107. Each function exists in a separate file. Directed arrows between functions indicate code dependencies. The script *main* initializes all parameters and starts the simulation.

The remainder of this section describes each function in detail. Conscious effort has been made to explicitly specify the measurement unit associated with each data variable. However, unless otherwise specified, units of time are seconds, distances are meters, and angles are radians.

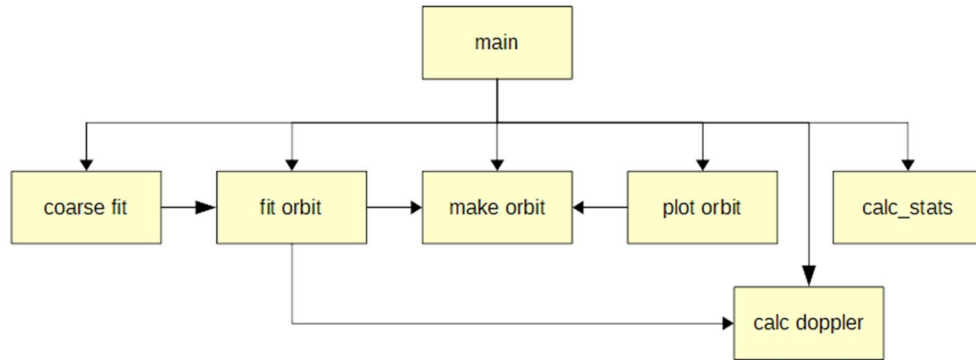


Fig. 107. Architectural overview of the simulation software.

5.3.1 Main

This is the main script, which initializes all parameters and controls the simulation. While Table 25 lists the most important variables, the following description of the orbital generation and prediction algorithm is represented by the block diagram of Fig. 106.

The initialization portion of the script defines the simulated orbit in terms of its orbital parameters. It also defines the locations of the Sun (see Fig. 105) and the Earth-bound observer in terms of their right ascension and declination. Once these parameters are determined, the script calls the

make_orbit function, which returns the time, position, and velocity at each sample point in the orbit. Sample points are referenced by the true anomaly in the range of 0 to 2π radians.

Now that a model orbit has been established, the script picks a few sample points to use as simulated measurement data. The goal is to calculate an estimated orbit from these measurements and compare it to the original simulated orbit. The fitting procedure uses values of right ascension (α), declination (δ), and Doppler velocity at various points in the orbit. Because the observer has no *a priori* knowledge of the orbit, measurement times are specified relative to the first measurement. Finally, the sample values are corrupted with noise in order to simulate the margin of error in a real measurement.

Because the orbital estimation is very sensitive to initial guesses, the process is split into two parts. The first part, which we call *coarse fitting*, attempts to choose the best match to the measurements from a coarse sampling of circular orbits. The results of the coarse fitting are used as the initial guesses for the *fine fitting* procedure. This method allows the initial conditions to be chosen from the full range of possible orbits. Otherwise, the fine fitting procedure has a tendency to converge on a local minimum, rather than finding the best solution among all possible orbits.

The fine fitting procedure uses the MATLAB function *fminsearch* to iteratively refine the initial guesses. The sample measurement data is compared to the estimated orbit, and an error metric is calculated. At each iteration, the estimates of the orbital parameters are adjusted to reduce the error and optimize the fit. Once the orbit has been fitted, data plots are generated to compare the original (simulated) and final (estimated) orbits.

Table 25: Data dictionary for *main* simulation script.

Name	Units	Description
a, b	m	Semi-major and semi-minor axes of the simulated orbit.
Omega	rad	Longitude of ascending node of the simulated orbit.
w	rad	Argument of perigee of the simulated orbit.
inc	rad	Inclination of the simulated orbit.
f	rad	Range of true anomaly ($0, 2\pi$) over which orbit is simulated.
angle_err	rad	Standard deviation for sampled angle measurements.
doppler_err	m/s	Standard deviation for sampled Doppler measurements.
RA, DEC	rad	Right ascension and declination at each point in the simulated orbit.
time	s	Time-of-orbit relative to perigee.
Vdop	m/s	Observed Doppler velocity (not frequency shift) at each point in the orbit.
P	m	Matrix 3-by-1000 holds x, y, z position at each point in the orbit.
V	m/s	Matrix 3-by-1000 holds x, y, z velocity at each point in the orbit.
t0	s	Time-of-orbit of the first sample measurement. This is solely used to determine the RA and DEC measurement values; it is unknown to the observer and is not used in the orbit estimation.
dt	s	Time between two consecutive measurements.
Nobs	-	Number of sample measurement sets to use for orbital fitting.

obsRA, obsDEC	rad	Vectors of length Nobs hold the sampled right ascension and declination measurements before noise is added.
obsTime	s	Vector of length Nobs holds relative time of measurements. First value corresponds to first measurement and is always zero.
obsDOP	m/s	Vector of length Nobs holds sampled Doppler measurements before noise is added.
sun, sta	struct	Structures with fields “ra” and “dec” hold the initial right ascension and declination values, in radians, for the Sun and observer.
useDoppler	boolean	If true, use Doppler velocity measurements when fitting the orbit.

5.3.2 Make Orbit

The *make_orbit* function generates an elliptical orbit based on the given input parameters, which are the six Keplerian parameters described in Table 24. The full set of input and output variables is described in Table 26.

Table 26. Data dictionary for the *make_orbit* function.

Name	Units	In/Out	Description
a	m	In	Length of the semi-major axis (major radius) of the orbit.
b	m	In	Length of the semi-minor axis (minor radius) of the orbit.
Omega	rad	In	Longitude of the ascending node.
inc	rad	In	Inclination of the orbital plane.
w	rad	In	Argument of perigee.
f	rad	In	Vector of true anomaly values. This specifies the range of angles over which the orbits are generated (typically $0 - 2\pi$).
t	s	Out	Vector of time-of-orbit values. This is the elapsed time since perigee at each specified value of true anomaly.
RA	rad	Out	Right ascension of the object's angular position at each angle of true anomaly.
DEC	rad	Out	Declination of the objects's angular position at each angle of true anomaly.
P	m	Out	Matrix of Cartesian coordinates of the satellite at each each angle of true anomaly. Rows 1, 2, and 3 are x, y, and z coordinates, respectively.
V	m/s	Out	Matrix of object velocity at each angle of true anomaly. Rows 1, 2, and 3 are x, y, and z velocity components, respectively.

The first step of this function is to calculate the height of the orbit as a distance from the center of the Earth. The height, which is calculated for every angle of true anomaly specified by the input vector f , is found as

$$r = \frac{a(1 - e^2)}{1 + e \cos(f)}, \quad (48)$$

where a is the semi-major axis and e is the ellipticity found by Eqn. (42). To find the orbital velocity at each sample point, the *vis-viva* equation is used:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}, \quad (49)$$

where μ is the standard gravitational parameter for Earth, which is known empirically as $3.986 \times 10^{14} \text{ m}^3/\text{s}^2$. Next, the Cartesian x , y , and z positions for the orbit are calculated from the values of orbital height and true anomaly:

$$x = r \cos(f), \quad (50)$$

$$y = r \sin(f), \text{ and} \quad (51)$$

$$z = 0. \quad (52)$$

At this point, the z coordinates are zero because we have assumed an equatorial orbit; we have not yet rotated the two-dimensional orbital plane into three-dimensional space. This is accomplished by an Euler rotation through the orbital angles Ω , i , and ω . The rotation matrix is calculated as

$$R = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 \\ c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 \\ s_2 s_3 & c_3 s_2 & c_2 \end{bmatrix}, \quad (53)$$

where c and s indicate sine and cosine functions; and subscripts 1, 2, and 3 indicate the angles Ω , i , and ω , respectively. When used as a pre-multiplication operator, the resulting R matrix performs intrinsic rotations around the spatial axes:

$$P = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad (54)$$

where P is the position matrix containing the rotated x , y , z coordinates at each sample point in the orbit. Next, the function calculates the right ascension and declination angles at each orbital position. These are found from the arctangent ratios of the positional coordinates:

$$\alpha = \tan^{-1} \left(\frac{y'}{x'} \right) \text{ and} \quad (55)$$

$$\delta = \tan^{-1} \left(\frac{z'}{\sqrt{(x')^2 + (y')^2}} \right), \quad (56)$$

where α and δ are right ascension and declination, respectively. Some additional logic in the code ensures that the α vector is monotonic. Otherwise, later attempts to interpolate time-of-orbit values between sample points may fail.

Finally, the time-of-orbit is calculated from the *eccentric anomaly* for each orbital position. The eccentric anomaly, which is the orbital angle measured from the center of the ellipse, is found as

$$E = \tan^{-1} \left[\frac{\sqrt{1-e^2} \sin(f)}{e + \cos(f)} \right], \quad (57)$$

where E is the eccentric anomaly, e is the eccentricity, and f is the true anomaly. Time-of-orbit is then calculated by the following formula

$$t(E) = [E - e \sin(E)] \sqrt{a^3 / \mu}, \quad (58)$$

where a is the semi-major axis and μ is the standard gravitational parameter for Earth, which is $3.986 \times 10^{14} \text{ m}^3/\text{s}^2$.

5.3.3 Calc Doppler

The *calc_doppler* function calculates the Doppler velocity at each sample point in the orbit. This calculation considers four things:

- (1) The orbital velocity at each sample point,
- (2) the location of the Sun,
- (3) the location of the observer, and
- (4) the motion of the observer due to Earth's rotation.

We can determine the Sun's location by

$$d_{sun} = r_{sun} \cos \delta \quad (59)$$

$$x_{sun} = d_{sun} \cos \alpha \quad (60)$$

$$y_{sun} = d_{sun} \sin \alpha \quad (61)$$

$$z_{sun} = r_{sun} \sin \delta \quad (62)$$

where r_{sun} is the Earth-Sun distance, α is the Sun's right ascension, δ is the Sun's declination, and d_{sun} is the projection of r_{sun} onto the equatorial plane. The Cartesian coordinates x_{sun} , y_{sun} , and z_{sun} represent the Sun's location with respect to Earth, where the x -axis points in the direction of the spring equinox. This was illustrated previously in Fig. 105, above.

The location of the observer is determined in the same way, with the substitution of Earth radius r_{obs} in place of r_{sun} . We account for motion of the observer by changing its right ascension angle as the Earth spins on its axis:

$$d_{obs} = r_{obs} \cos \delta \quad (63)$$

$$\alpha' = \alpha + \left(\frac{2\pi}{86164} \right) t \quad (64)$$

$$x_{obs} = d_{obs} \cos \alpha' \quad (65)$$

$$y_{obs} = d_{obs} \sin \alpha', \quad \text{and} \quad (66)$$

$$z_{obs} = r_{obs} \sin \delta \quad (67)$$

where α and δ refer to the observer's initial right ascension and declination, t is the elapsed time in seconds since the initial position, and α' is the right ascension of the rotating observer. Note that

the declination does not change as the Earth spins. The value 86 164 refers to the number of seconds in a sidereal day, as opposed to the longer, solar day of 86 400 seconds. We can now find the observer's velocity as

$$v_{obs} = \frac{2\pi d_{obs}}{86164} \begin{bmatrix} \cos(\alpha') \\ \sin(\alpha') \\ 0 \end{bmatrix}, \quad (68)$$

where the quantity $(2\pi d_{obs})$ is the Earth's circumference at the observer's latitude, representing the distance traveled by the observer in one sidereal day. The velocity in the z -direction is zero, corresponding to the fact that the declination does not change.

Next, we calculate the radial (Doppler) velocity of the satellite with respect to the Sun by taking the scalar product of the orbital velocity with a unit vector pointing to the Sun:

$$\hat{u} = \frac{1}{r_{sun}} \begin{bmatrix} x_{sat} - x_{sun} \\ y_{sat} - y_{sun} \\ z_{sat} - z_{sun} \end{bmatrix}, \quad v_{sun} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \cdot \hat{u}, \quad (69)$$

where x_{sat} , y_{sat} , and z_{sat} are the Cartesian coordinates of the satellite; v_x , v_y , and v_z are the satellite's velocity components; u is the unit vector pointing to the Sun; and v_{sun} is the Doppler velocity with respect to the Sun.

We calculate the radial velocity with respect to the observer in a similar way:

$$r_{sat} = \sqrt{(x_{sat} - x_{obs})^2 + (y_{sat} - y_{obs})^2 + (z_{sat} - z_{obs})^2}, \quad (70)$$

$$\hat{u} = \frac{1}{r_{sat}} \begin{bmatrix} x_{sat} - x_{obs} \\ y_{sat} - y_{obs} \\ z_{sat} - z_{obs} \end{bmatrix}, \text{ and} \quad (71)$$

$$v_{rel} = (v_{sat} - v_{obs}) \cdot \hat{u}, \quad (72)$$

where r_{sat} is the distance between satellite and observer, and v_{rel} is the relative velocity between satellite and observer along a line drawn between them. Finally, the total Doppler velocity is found by summing the velocities along the two legs of the optical path:

$$v_{dop} = v_{sun} + v_{rel}. \quad (73)$$

A list of the input and output parameters for the *calc_doppler* function is shown in Table 27.

Table 27. Data dictionary for the *calc_doppler* function.

Name	Units	In/Out	Description
sun	struct	In	Structure with fields "ra" and "dec" which hold the right ascension and declination values, in radians, for the location of the Sun.
station	struct	In	Structure with fields "ra" and "dec" which hold the right ascension and declination values, in radians, for the initial location of the observer.

position	m	In	Matrix with three rows and n columns holding the x, y, z , coordinates of the satellite at each sample point in the orbit.
velocity	m/s	In	Matrix with three rows and n columns holding the x, y, z , components of satellite velocity at each sample point in the orbit.
t	s	In	Vector holding the time-of-orbit at each sample point in the orbit.
Vdoppler	m/s	Out	The accumulated Doppler velocity along the optical path from Sun, to satellite, to observer on the surface of Earth.

5.3.4 Coarse Fit

The *coarse_fit* function runs a coarse orbital matching procedure against a library of pre-defined orbits. The goal is to find a set of orbital parameters for use as initial guesses for the *fit_orbit* fine fitting routine. This method allows the initial conditions to be chosen from the full range of possible orbits. Otherwise, the fine fitting procedure has a tendency to converge on a local minimum, rather than finding the best solution among all possible orbits. Table 28 lists input and output parameters for the function.

Table 28. Data dictionary for the *coarse_fit* function.

Name	Units	In/Out	Description
obsTime	s	In	Vector containing the elapsed time at each separate observation.
obsRA	rad	In	Vector containing the measurement of right ascension at each observation.
obsDEC	rad	In	Vector containing the measurement of declination at each observation.
obsDOP	m/s	In	Vector containing the measurement of Doppler velocity at each observation.
sun	struct	In	Structure with fields “ra” and “dec” which hold the right ascension and declination values, in radians, for the location of the Sun.
sta	struct	In	Structure with fields “ra” and “dec” which hold the right ascension and declination values, in radians, for the initial location of the observer.
bestfit	1 x 5	Out	A five-element vector containing (1) the semi-major axis, (2) the semi-minor axis, (3) the longitude of right ascension, (4) the inclination, and (5) the argument of perigee. These will be used as initial guesses for the main orbital fitting procedure.

The *coarse_fit* function itself comprises two separate loops. The first loop iterates over the Keplerian parameters a , Ω , and i . The b parameter is set equal to a , and the w parameter is set equal to zero. Each set of five parameters is then passed to the *fit_orbit* function, which returns a scalar value representing the goodness-of-fit between this circular test orbit and the measured values of right ascension, declination, and (optionally) Doppler velocity. Parameters a , Ω , and i corresponding to the closest fit are saved and passed on to the second loop.

The second loop iterates over the remaining Keplerian parameters b and w , with the other three parameters held constant at the values discovered during the first loop. The values corresponding to the closest fit are combined with the other three to become the final coarse fit solution.

5.3.5 Fit Orbit

The *fit_orbit* function is called iteratively from within the MATLAB *fminsearch* function until the output value is minimized. It compares measured data values to an estimated orbit and returns a value representing the goodness of fit. At each iteration, the estimates of the orbital parameters are adjusted to optimize the fit. Parameters for the *fit_orbit* function are listed in Table 29.

Table 29: Data dictionary for the *fit_orbit* function.

Name	Units	In/Out	Description
X(1)	m	In	Semi-major axis (a) of the estimated orbit.
X(2)	m	In	Semi-minor axis (b) of the estimated orbit.
X(3)	rad	In	Longitude of the ascending node (Omega) of the estimated orbit.
X(4)	rad	In	Inclination (inc) of the estimated orbit.
X(5)	rad	In	Argument of perigee (w) of the estimated orbit.
obs	struct	In	Structure that holds the measurement data for comparison to the estimated orbit. Each field is an <i>n</i> element vector, where <i>n</i> is the number of separate observations of the satellite. The fields are as follows: obs.ra is right ascension, in radians obs.dec is declination, in radians obs.vdop is Doppler velocity, in meters per second obs.time is elapsed time since the first observation, in seconds
f	rad	In	Vector of true anomaly values representing the sample points of the orbit.
sun	struct	In	Structure with fields “ra” and “dec” which hold the right ascension and declination values, in radians, for the location of the Sun. Used in calculation of Doppler velocities for the estimated orbit.
station	struct	In	Structure with fields “ra” and “dec” which hold the right ascension and declination values, in radians, for the initial location of the observer. Used in calculation of Doppler velocities for the estimated orbit.
t	s	In	This optional structure is the time-of-orbit corresponding to the parameters in X. It is provided by the coarse-fit routine, which reads time-of-orbit from a disk file, in order to speed up the process. When not provided by the caller (the usual case) it is calculated by <i>make_orbit</i> .
debug	bool	In	Optional flag used for debugging the function.
fval	-	Out	Goodness of fit error value as described in the text.

If Doppler velocity is provided as an input to this function, it will be used as an additional parameter to fit the orbit. This is what we are trying to introduce with our new system. However, if the input argument is left empty (that is, [] in MATLAB syntax) then Doppler velocity will not be used to fit the orbit. This is the more conventional way of doing things, and should require more measurements to fit the orbit accurately.

The cost function that drives both the coarse fit and fine fit procedures is based on differences between the estimated orbits and the simulated measurements of right ascension, declination, and (optionally) Doppler velocity. The angular error is calculated by first constructing unit vectors

pointing to the estimated and actual target positions. Then the angular difference a_{err} between them is simply the magnitude of their cross product

$$a_{err} = \|\hat{u}' \times \hat{u}\|, \quad (74)$$

where

$$\hat{u}' = [\cos(\delta')\cos(\alpha'), \cos(\delta')\sin(\alpha'), \sin(\delta')]^T, \quad (75)$$

and

$$\hat{u} = [\cos(\delta)\cos(\alpha), \cos(\delta)\sin(\alpha), \sin(\delta)]^T, \quad (76)$$

such that $\hat{u}'$ and $\hat{u}$, α' and α , and δ' and δ are the estimated and actual unit vectors, right ascension angles, and declination angles, respectively. Doppler velocity error v_{err} is calculated as

$$v_{err} = \frac{|v - v'|}{v}, \quad (77)$$

where v' and v are the estimated and measured Doppler velocity values, respectively. The final cost function representing the overall estimation error is

$$fval = \sum_{i=1}^N (a_{err,i} + v_{err,i})^2, \quad (78)$$

where N is the total number of observations.

5.3.6 Calc Stats

The main script is designed to run a Monte Carlo simulation of multiple trials, using randomly chosen noise values for both the angular and Doppler measurements. At the end of the simulation, the *calc_stats* function is called to calculate the overall position error and standard deviation. For each result generated by the Monte Carlo trials, the position error is calculated as

$$err(\%) = \frac{1}{P} \sum_{i=1}^P \frac{\|\bar{r}_i' - \bar{r}_i\|}{\|\bar{r}_i\|} \times 100, \quad (79)$$

where $\bar{r}_i$ and $\bar{r}_i'$ are the actual and estimated position vectors, respectively, at each of P points spaced evenly around the orbital ellipse. The final position error is calculated as the mean error over all trials. The input/output parameters for *calc_stats* are listed in Table 30.

Table 30. Data dictionary for the *calc_stats* function.

Name	Units	In/Out	Description
a	m	In	Length of the semi-major axis (major radius) of the true orbit.
b	m	In	Length of the semi-minor axis (minor radius) of the true orbit.
Omega	rad	In	Longitude of the ascending node of the true orbit.
inc	rad	In	Inclination of the orbital plane of the true orbit.
w	rad	In	Argument of perigee of the true orbit.

f	rad	In	Vector of true anomaly values. This specifies the range of angles over which the orbits are generated (typically $0 - 2\pi$).
out	$5 \times N$	In	Matrix containing the results of N Monte Carlo trials. Each row of the matrix contains [a, b, Omega, inc, w].
rval	2×2	Out	The average error between the estimated and true orbital positions. The matrix is constructed as [error (km), error (%); std (km), std (%)].

5.4 Simulation Results

The sample orbits listed in Table 31 were selected from published ephemeris data [26] and include a geosynchronous orbit (GSO), Hohmann transfer orbit (GTO), and a global positioning system (GPS) orbit.

Table 31. Orbital parameters used for test simulations.

Orbit	a (km)	e	Ω ($^\circ$)	i ($^\circ$)	ω ($^\circ$)
GSO	42165	0.006887	336	14	185
GTO	40575	0.7599	14	30	180
GPS	26560	0.004216	243	56	116

For each type of orbit, we simulated twelve different cases:

- A. One case each of two, three, and four observations using angular measurements only (AO).
- B. One case each of two, three, and four observations using angular measurements and Doppler velocity (AV) with a simulated Doppler precision of 5 m/s.
- C. Same as B, but with a simulated Doppler precision of 10 m/s.
- D. Same as B and C, but with a simulated Doppler precision of 20 m/s.

Table 32 summarizes the critical constraints placed upon each simulation. Each observation consists of one right ascension measurement, one declination measurement, and for the AV cases, one Doppler velocity measurement. To attain the required signal-to-noise ratio, we assume that the light used to calculate the Doppler shift is integrated for 300 seconds, as informed by forthcoming radiometric and optical simulations. While not reflected in the simulation, this integration time will cause a decrease in the accuracy of the Doppler measurement, since the satellite's velocity is changing over the course of the observation.

Measurement error was simulated by the addition of Gaussian noise with the standard deviations listed in Table 32. The time between observations was 3000 seconds for all orbits. Finally, the initial location of the observer was chosen such that the target object would be in the visible line of sight for each observation, with the Sun in position to provide reflected light for Doppler measurements.

Table 32. Key simulation parameters and their ranges.

Simulation Parameter	Values
----------------------	--------

Type of orbit	GSO, GTO, GPS
Estimation method	AO, AV
Number of observations	N = 2, 3, 4
Time between observations (s)	dt = 3000
Doppler measurement error (m/s)	$\sigma = 5, 10, 20$
Angular measurement error (arcsec)	$\sigma = 10$

For each of the twelve simulation cases, 1000 Monte Carlo simulations were run, using randomly chosen noise values for both the angular and Doppler measurements. For each trial, the position error was calculated as

$$\text{err (\%)} = \frac{1}{P} \sum_{i=1}^P \frac{\|\bar{r}_i' - \bar{r}_i\|}{\|\bar{r}_i\|} \times 100, \quad (80)$$

where $\bar{r}_i$ and $\bar{r}_i'$ are the actual and estimated position vectors, respectively, at each of P points spaced evenly around the orbital ellipses. The position error and other values shown in the following Tables 33-35 represent the mean values of all 1000 trials.

Table 33 summarizes the results for the GSO simulations. Using two observations, without the benefit of Doppler measurements, the AO position error is 17%. When Doppler measurements with a precision of 5 m/s are incorporated, the AV error is reduced to less than 2%. With a Doppler error of 20 m/s, the position error is 2.30%.

To improve upon this, the number of observations is increased to three, in which case the AO position error decreases to 1.08%. Although the AV method also shows improvement, it performs slightly worse than AO, with position errors of 1.65%, 2.11%, and 2.97% for Doppler precisions of 5, 10, and 20 m/s, respectively. Extending the analysis to four observations shows the same pattern, with an AO position error of 0.68% versus AV errors of 1.92%, 2.68%, and 3.65%.

Table 34 shows results for the GTO simulations. In all cases, the AV method performs significantly better than AO when only two or three observations are available. However, the AO position error decreases with each additional observation. At four observations, the AO method performs better than AV at 20 m/s precision; the position errors are 1.05% and 1.22%, respectively.

Finally, Table 35 lists the results for the GPS orbital simulations. These follow the same pattern as the GSO results. For two observations, the AO method results in a position error of 69.88%. The AV method, in comparison, yields results of 1.16%, 1.34%, and 1.59% for Doppler measurements of 5, 10, and 20 m/s precision, respectively. However, when more observations are used, the AO method shows marginally better performance.

Table 33. GSO simulation results.

Method	N	σ (m/s)	a (km)	e	Ω (°)	i (°)	ω (°)	Position error (%)	Std Dev (%)
AO	2	—	43164	0.263978	336.0	14.0	83.7	17.112	1.319
AV	2	5	42497	0.004870	334.3	15.4	233.0	1.965	0.346

AV	2	10	42494	0.006988	334.2	15.4	232.5	2.021	0.570
AV	2	20	42491	0.009368	335.0	15.5	230.7	2.303	1.044
AO	3	—	42618	0.002651	336.0	14.0	304.2	1.079	0.057
AV	3	5	42491	0.008062	335.6	14.9	212.6	1.653	1.024
AV	3	10	42494	0.012007	335.4	15.2	212.8	2.111	1.774
AV	3	20	42498	0.021832	334.4	15.4	213.1	2.970	3.730
AO	4	—	42427	0.003543	336.0	14.0	206.0	0.680	0.235
AV	4	5	42462	0.017448	334.1	14.7	262.9	1.923	2.167
AV	4	10	42618	0.033507	332.7	14.8	264.1	2.682	4.720
AV	4	20	42660	0.057785	332.3	15.1	259.8	3.651	7.564

Table 34. GTO simulation results.

Method	N	σ (m/s)	a (km)	e	Ω (°)	i (°)	ω (°)	Position error (%)	Std Dev (%)
AO	2	—	38815	0.467732	14.0	30.0	197.5	69.876	4.023
AV	2	5	40156	0.759251	13.9	30.1	179.5	2.000	2.002
AV	2	10	40251	0.759468	14.0	30.0	179.5	2.091	1.871
AV	2	20	40308	0.759607	14.1	30.0	179.5	2.725	1.840
AO	3	—	42280	0.762377	14.0	30.0	181.4	3.461	0.636
AV	3	5	40575	0.759868	14.0	30.0	180.0	0.352	0.264
AV	3	10	40588	0.759931	14.0	30.0	180.0	0.699	0.519
AV	3	20	40602	0.759981	14.1	30.0	180.0	1.354	0.959
AO	4	—	40655	0.759985	14.0	30.0	180.1	1.050	0.839
AV	4	5	40591	0.759888	14.1	30.0	180.0	0.492	0.827
AV	4	10	40579	0.759872	14.0	30.0	180.0	0.708	0.745
AV	4	20	40593	0.759867	14.0	30.0	180.0	1.218	0.951

Table 35. GPS simulation results.

Method	N	σ (m/s)	a (km)	e	Ω (°)	i (°)	ω (°)	Position error (%)	Std Dev (%)
AO	2	—	42535	0.769867	242.9	55.7	30.7	62.969	0.094
AV	2	5	26736	0.002439	243.4	55.7	122.1	1.163	0.642
AV	2	10	26720	0.004442	243.4	55.6	122.8	1.337	0.780
AV	2	20	26690	0.008848	243.4	55.6	127.5	1.592	0.888
AO	3	—	26728	0.001079	242.9	55.7	261.7	0.632	0.104
AV	3	5	26741	0.000915	243.2	55.6	123.8	0.966	0.479
AV	3	10	26741	0.001111	243.2	55.6	123.8	0.984	0.507
AV	3	20	26737	0.001569	243.2	55.6	123.3	0.948	0.482
AO	4	—	26742	0.002062	242.9	55.7	305.4	0.687	0.191
AV	4	5	26715	0.000770	243.0	55.8	125.8	0.762	0.389
AV	4	10	26720	0.000815	243.1	55.7	127.4	0.844	0.478
AV	4	20	26721	0.001860	243.1	55.7	129.9	0.907	0.541

In order to visualize these results, Figs. 108, 109, and 110 show density plots of the GSO, GTO, and GPS solutions for two observations, using both AO and AV methods. The central green circle represents the Earth, marked with a line at the equator and a cross at the north pole. The pink ellipse surrounding Earth traces the actual simulated orbit. The remaining ellipses represent a

density plot of the results of all 1000 Monte Carlo trials. Brightness indicates the number of overlapping orbital points in a given location; brighter areas are more densely populated.

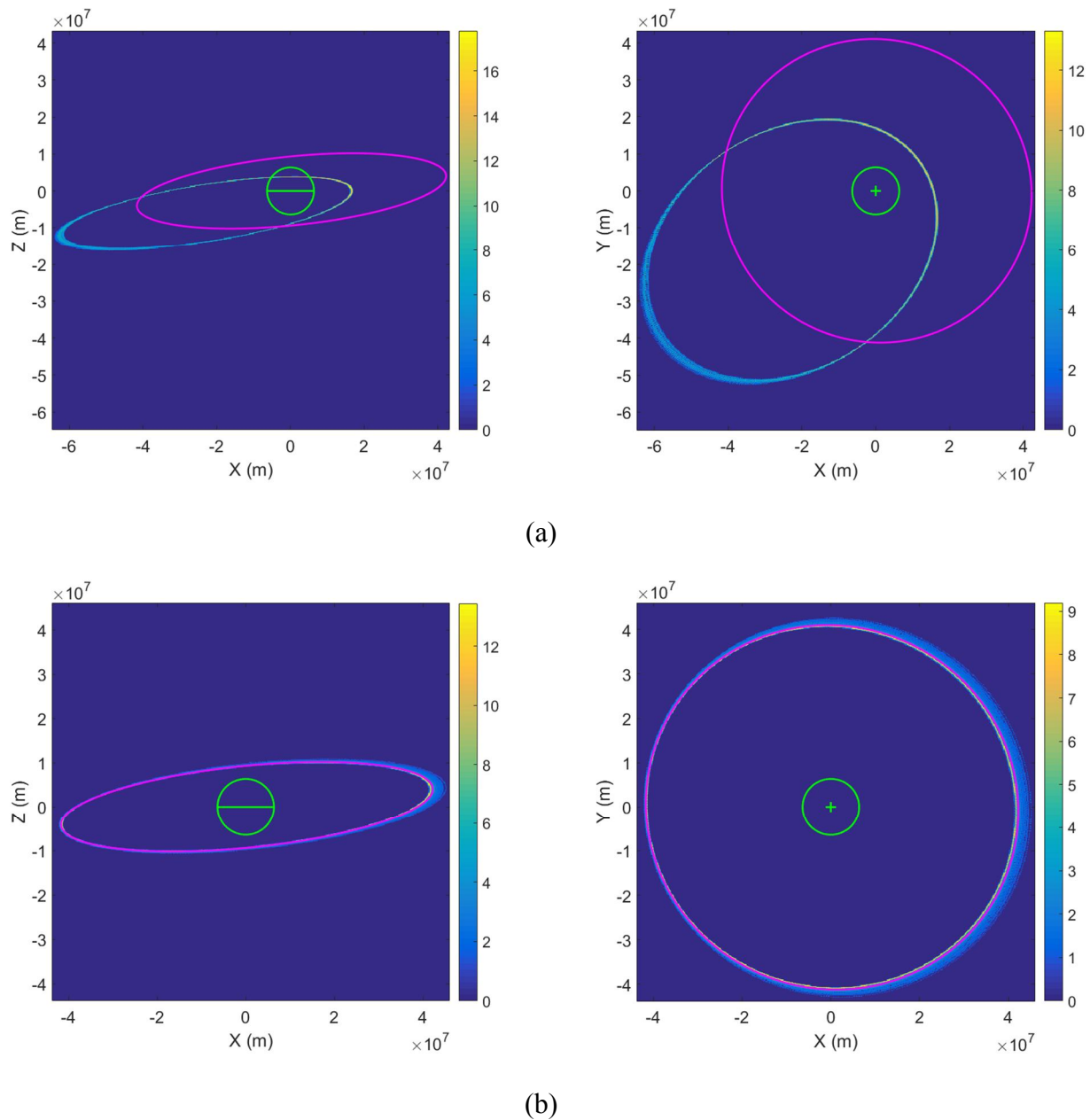
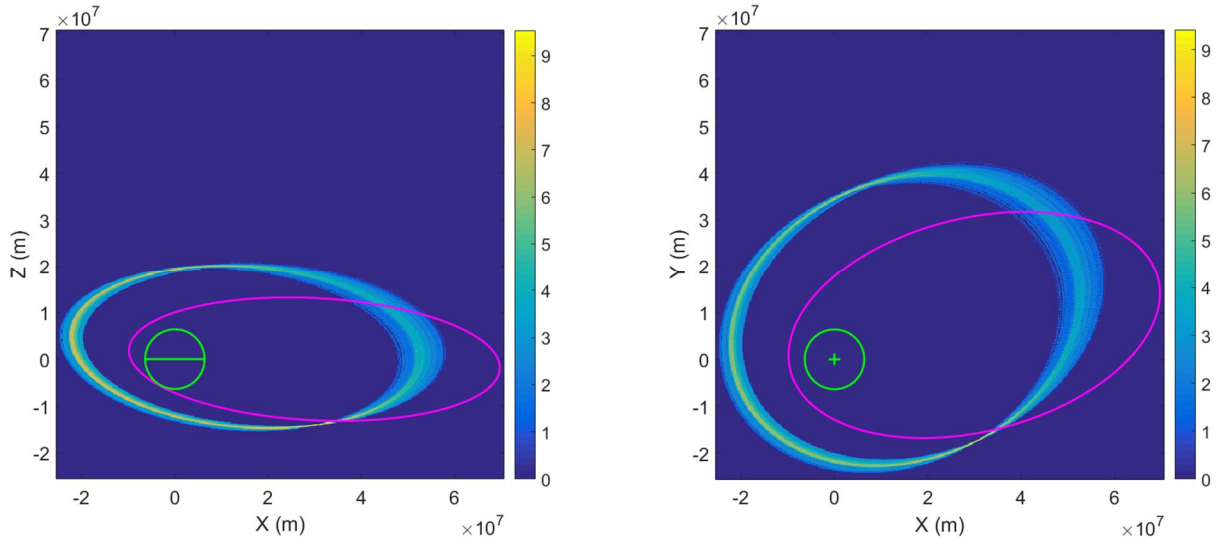
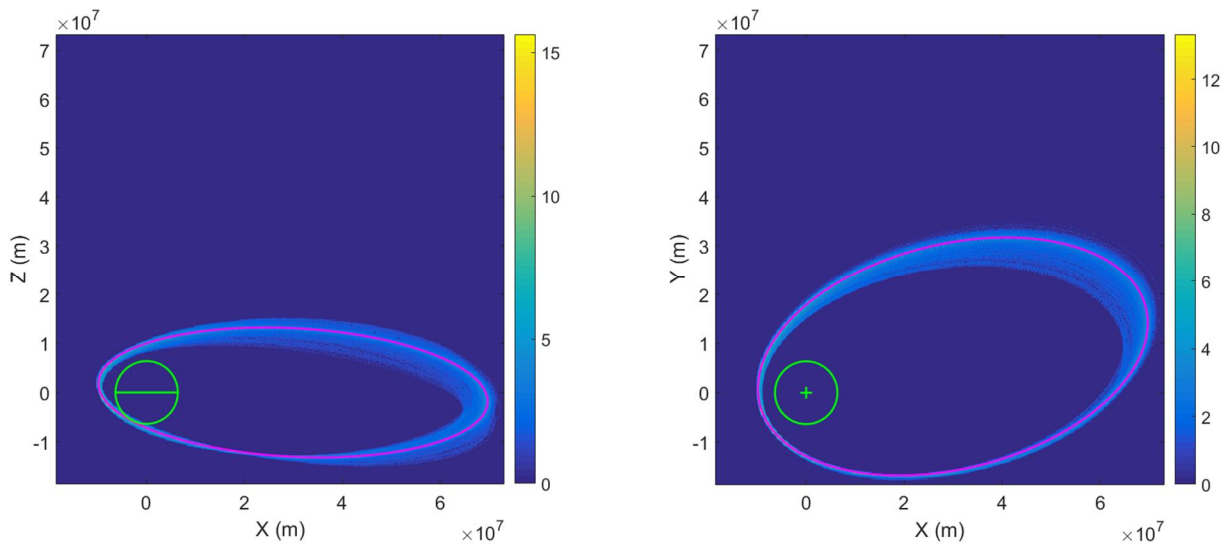


Fig. 108. Density plots of two GSO simulations using (a) the AO method and (b) the AV method with a Doppler precision of 5 m/s. In each case, two independent observations are used to perform the orbital estimation. The pink ellipse is the actual orbit; the other ellipses represent the individual results of the 1000 Monte Carlo simulations. The X-Y plane is the equatorial plane.



(a)



(b)

Fig. 109. Density plots of two GTO simulations using (a) the AO method and (b) the AV method with a Doppler precision of 5 m/s. In each case, two independent observations are used to perform the orbital estimation. The pink ellipse is the actual orbit; the other ellipses represent the individual results of the 1000 Monte Carlo simulations. The X-Y plane is the equatorial plane.

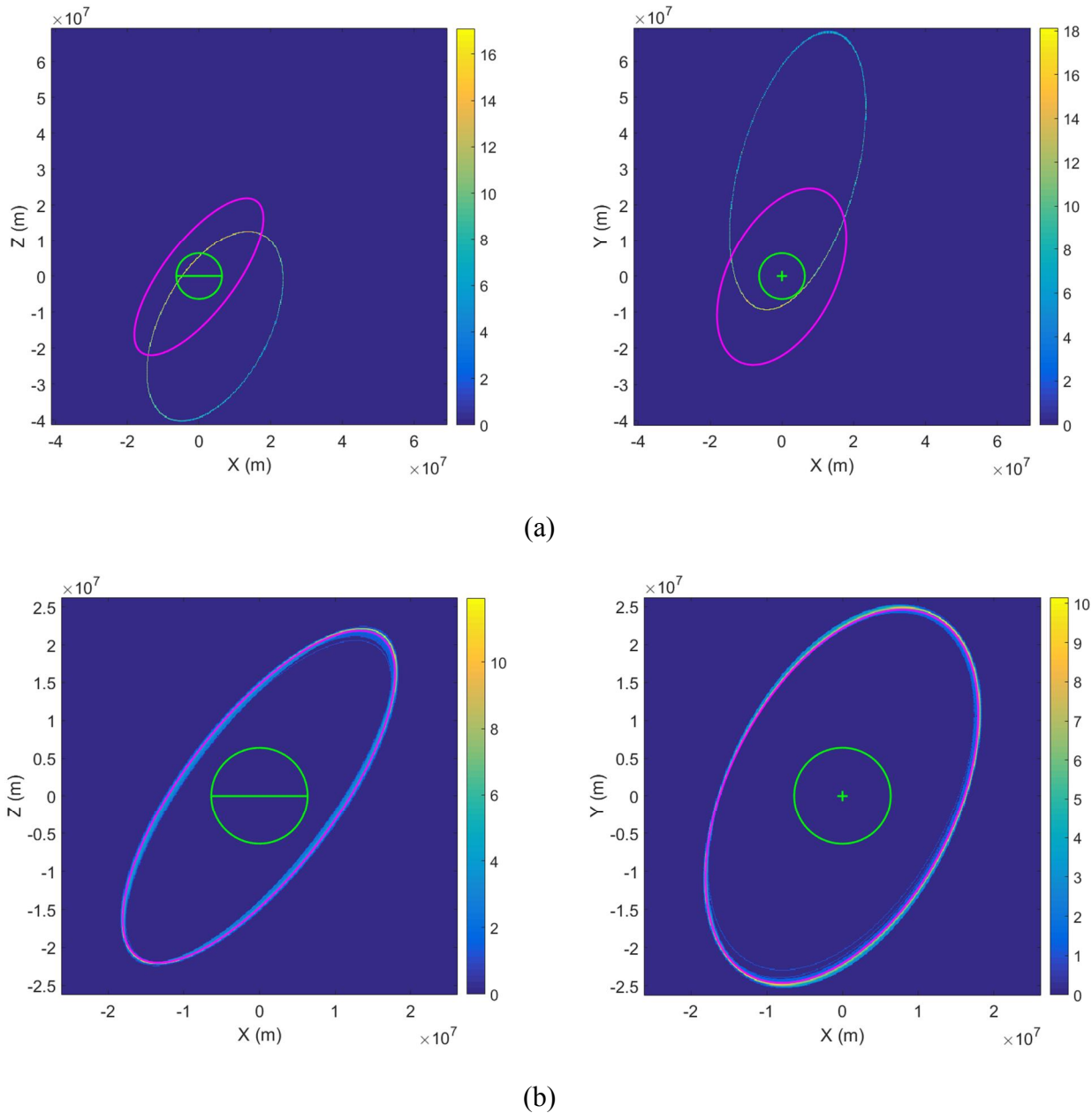


Fig. 110. Density plots of two GPS simulations using (a) the AO method and (b) the AV method with a Doppler precision of 5 m/s. In each case, two independent observations are used to perform the orbital estimation. The pink ellipse is the actual orbit; other ellipses represent the individual results of the 1000 Monte Carlo simulations. The X-Y plane is the equatorial plane.

Although these results indicate the usefulness of incorporating Doppler velocity as part of orbit determination, there are a number of limitations inherent in these simulations. First, the orbital model described here is a first-order approximation of the orbital mechanics. It does not account for second-order perturbations such as atmospheric drag, oblateness of the Earth, or gravitational forces from the Sun, Moon, or other bodies. In this respect, the model is useful for initial orbit determination, but not for long-term tracking of orbital objects. Second, the simulation does not

check that the target object is actually visible from the observer's location at the orbital times specified. Third, time is measured starting at perigee. There is no tracking of solar time, so even if a particular time of orbit places the object in the observer's sky, the location of the sun could make observation impracticable. Finally, location of the observing station is specified by right ascension and declination, relative to the vernal equinox. This simplifies the simulation considerably, but there is no correlation with geographic longitude, since the model is not aware of solar time.

The results of our simulations demonstrate that for mid- and high-altitude orbits, incorporation of Doppler velocity measurements with a precision of 20 m/s or less can generate IOD solutions with smaller position errors than angular measurements alone. For two observations of GSO, GTO, and GPS orbits, Doppler velocity measurements yielded IOD solutions with less than 3% error, while conventional angles-only measurements yielded errors from 17% (GSO) to 70% (GTO). Meanwhile, for three observations, Doppler velocity yielded improved IOD solutions for GTO orbits, with a worst-case error of less than 1.35% for a precision of 20 m/s. In comparison, using only angular measurements, the error was 3.46%. Finally, the AO method shows marginally better results than AV for GSO and GPS orbits when using three observations, and all tested orbits when using four observations. These results are within 1.24% of the AV results with 5 m/s precision.

6.0 Admissible Region Estimation

The algorithms employed in the orbital simulations described in chapters 4-5 are based on the Nelder-Mead simplex method. In this fashion, we are able to use our model, which simulates both the satellite's orbit and the Doppler shift given an arbitrary sun-earth-satellite geometry, for use in our fitting procedure. However, this technique is highly susceptible to error, in that it tends to fit to local minima too easily. Additionally, it does not have a good sense of the total merit function's shape. Thus, we shifted focus to a more robust technique involving two approaches: (1) creating a pre-optimization step, which will estimate, using a global parameter sweep, local minima that should be investigated further; and (2) adaptation of existing IOD techniques, such as observed in Ref. [27] and [28], to include Earth-Sun-satellite geometry and the expected Doppler shift associated with any given observation.

6.1 Mathematical Model

Since the publication of Newton's laws of motion, various solutions have been proposed for solving an orbit from three line-of-sight (LOS) angle measurements. Two classical examples are the methods of Laplace and Gauss. Modifications of these two techniques for improved accuracy and stability are still in use today [29]. Modern methods leverage the availability of computing resources to implement iterative solutions such as Double-r [30] and Gooding's method [28]. More recently, new techniques, based on the use of Lagrangian coefficients, have been developed that circumvent problems contributing to inaccuracies and instability in earlier methods [31].

Development of the present method relies on a modification of Gooding's algorithm, which we briefly outline for context. Gooding begins with three pairs of angles comprising right ascension and declination, represented by α and δ , respectively. These angles define three unit vectors,

$$\mathbf{L}_i = \begin{bmatrix} \cos(\alpha_i)\cos(\delta_i) \\ \sin(\alpha_i)\cos(\delta_i) \\ \sin(\delta_i) \end{bmatrix}; \quad i = 1, 2, 3, \quad (81)$$

which represent measured LOS directions to the target at observation times t_1 , t_2 , and t_3 . We now define three additional quantities, illustrated in Fig. 111. The range value, ρ , is the unknown distance between the observer and the target. The vector $\mathbf{R} = \langle R_x, R_y, R_z \rangle$ represents the Earth-centered inertial (ECI) position of the observing station. Finally, we define

$$\mathbf{r}_i = \mathbf{R}_i + \rho_i \mathbf{L}_i, \quad (82)$$

which is the target's position vector at each observation time.

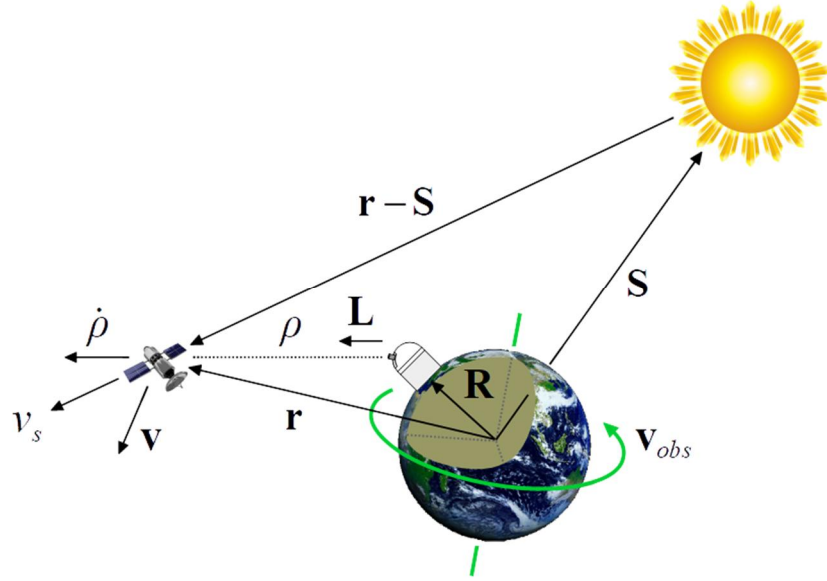


Fig. 111. Geometric relationships between Earth, Sun, and target.

A complete orbital solution can be described by a state vector $\langle \mathbf{r}, \mathbf{v} \rangle$ comprising both the position and velocity at any given time. Gooding's algorithm estimates the state vector using the following procedure:

1. Measure $\mathbf{R}_i$, $\mathbf{L}_i$, and t_i for $i = 1, 2, 3$.
2. Assume values for the unknown ranges ρ_1 and ρ_3 .
3. Calculate the estimated position vectors $\mathbf{r}_1$ and $\mathbf{r}_3$ using (82).
4. Solve Lambert's problem to obtain velocity estimates $\mathbf{v}_1$ and $\mathbf{v}_3$.
5. Solve Kepler's problem to generate a prediction of $\langle \mathbf{r}_2, \mathbf{v}_2 \rangle$.
6. Compute the error in the predicted LOS vector at time t_2 .
7. Minimize the error with Newton's method and generate new estimates of ρ_1 and ρ_3 .
8. Iterate (go to 3) until the error is below a desired tolerance.

The predicted LOS vector in step 6 is calculated as $\mathbf{C} = \mathbf{r}_2 - \mathbf{R}_2$. Two orthogonal error functions determine the difference between the predicted vector $\mathbf{C}$ and the angles measured in step 1. Error functions f and g are defined as

$$f(x, y) = \frac{\mathbf{P} \cdot \mathbf{C}}{\|\mathbf{P}\|}; \quad \mathbf{P} = (\mathbf{L}_2 \times \mathbf{C}) \times \mathbf{L}_2, \text{ and} \quad (83)$$

$$g(x, y) = \frac{\mathbf{N} \cdot \mathbf{C}}{\|\mathbf{N}\|}; \quad \mathbf{N} = \mathbf{L}_2 \times \mathbf{P}, \quad (84)$$

where the independent variables x and y substitute for ranges ρ_1 and ρ_3 , respectively. The error functions are driven to zero by the two-variable Newton's method,

$$\begin{bmatrix} x \\ y \end{bmatrix}_{n+1} = \begin{bmatrix} x \\ y \end{bmatrix}_n - \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}^{-1} \begin{bmatrix} f(x, y) \\ g(x, y) \end{bmatrix}, \quad (85)$$

where the subscript $(n + 1)$ indicates updated range estimates to be used in the next iteration. The partial derivatives f_x , f_y , g_x , and g_y are determined by repeating steps 3-5 using incremental variations of the x and y range values.

Our modified algorithm uses an alternate set of error functions. The new error functions are based on the difference between measured and predicted Doppler shift values. Given an estimated state vector $\langle \mathbf{r}, \mathbf{v} \rangle$ as illustrated in Fig. 111, we calculate the following two velocities:

$$v_s = (\mathbf{v} - \mathbf{v}_{sun}) \cdot \frac{\mathbf{r} - \mathbf{S}}{\|\mathbf{r} - \mathbf{S}\|}, \quad \text{and} \quad (86)$$

$$\dot{\rho} = (\mathbf{v} - \mathbf{v}_{obs}) \cdot \mathbf{L}, \quad (87)$$

where v_s and $\dot{\rho}$ are the target's radial velocities with respect to the sun and the observer, respectively. The vectors $\mathbf{v}_{sun}$ and $\mathbf{v}_{obs}$ are the ECI velocities of Sun and observer, respectively. $\mathbf{S}$ is the sun's position vector. The expected Doppler shift, $\Delta\lambda = \lambda'' - \lambda$, can now be found by substitution of Eqs. (86) and (87) into Eq. (2):

$$\Delta\lambda = \frac{\dot{\rho} + v_s}{c} + \frac{\dot{\rho}v_s}{c^2}. \quad (88)$$

Because the Doppler shift measurements are completely independent of the LOS angle measurements, our method is able to find an orbital solution using only two sets of observations. In other respects, the procedure is similar to the one used by Gooding:

1. Measure $\mathbf{R}_i$, $\mathbf{L}_i$, t_i , and $\Delta\lambda_i$ for $i = 1, 2$.
2. Assume values for the unknown ranges ρ_1 and ρ_2 .
3. Calculate the estimated position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ using Eq. (82).
4. Solve Lambert's problem to obtain velocity estimates $\mathbf{v}_1$ and $\mathbf{v}_2$.
5. Generate independent predictions of $\Delta\lambda_1$ and $\Delta\lambda_2$ using Eq. (88).
6. Use Newton's method to generate new estimates of ρ_1 and ρ_2 .
7. Iterate (go to 3) until the error is below a desired tolerance.

In this case, the error functions f and g are simply the difference between the measured and predicted wavelength shifts at times t_1 and t_2 , respectively. Optimization of the estimated range values, using Newton's method, proceeds as described above. However, the method is easily extended from $N = 2$ telescopic observations to $N = 3$ or more, with an attendant improvement in accuracy. Using three variables, Newton's method becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{n+1} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_n - \begin{bmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ h_x & h_y & h_z \end{bmatrix}^{-1} \begin{bmatrix} f(x, y, z) \\ g(x, y, z) \\ h(x, y, z) \end{bmatrix}, \quad (89)$$

where the additional variable z represents the estimated range value (ρ_3) at time t_3 , and $h(x, y, z)$ is its associated error function.

6.2 Admissible Region Estimation

The mathematical model described in the previous section provides an opportunity to narrow the search-space of the solution by application of “admissible region estimation” as described by Tommei, *et al.* [32]. The admissible region is defined as that portion of the range, range-rate plane within which the two-body energy of the Earth-satellite system is less than zero, implying the potential for a valid orbit. The *range* is the line-of-sight distance between the orbiting object and the observer. The *range-rate* is the time rate of change of the range, or, in other words, the radial velocity toward or away from the observer. In the latest versions of our orbital simulations, these represent the unknown quantities which are solved by nonlinear optimization. In addition to the measurement of right ascension, declination, and their time derivatives, the measurement of optical wavelength shift due to the Doppler effect provides the input values for the solution.

Fig. 112 shows a plot of the admissible region for a hypothetical object in Earth orbit. The curve labeled ϵ_E represents zero geocentric energy. Values of range and range-rate that map to the outside of this curve are not confined to Earth orbit. The lines labeled $\rho_{\min}$ and $\rho_{\max}$ are additional, arbitrary constraints imposed upon the range under consideration. The intersection of the constraint regions, shown by the shaded area, indicates the range of possible orbits to be searched.

Implementation of admissible region estimation has reduced the computation time of our simulations by a factor of ten and improved the accuracy by roughly 3%. The values currently in use for $\rho_{\min}$ and $\rho_{\max}$ are 20e6 m and 60e6 m, respectively.

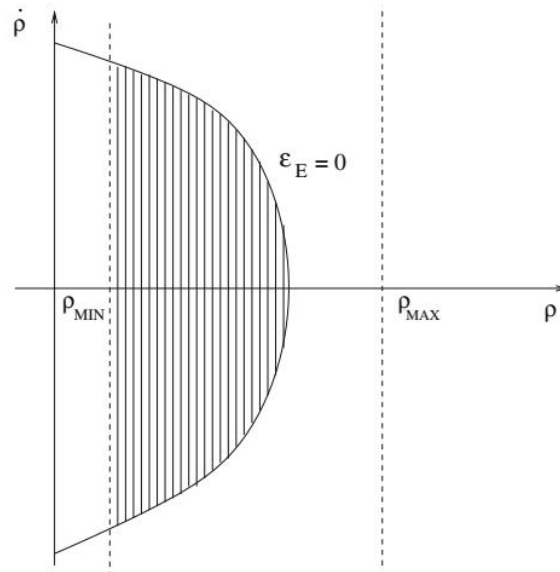


Fig. 112. Admissible region for an orbital object [32].

6.3 Implementation

Gooding's method and the present algorithm both use Newton's method to minimize the error in successive approximations of orbital range values. Because this procedure is sensitive to the initial estimate, better results can be obtained if the starting values are chosen wisely. Therefore, our simulation performed the orbital prediction in two steps: (1) a coarse fit step, which attempted to eliminate invalid starting values; followed by (2) a fine fit step, which optimized the estimates and rendered the final output.

6.3.1 Coarse fit

Operation of the coarse fit procedure is illustrated in Fig. 113. The first step generates an *admissible region* over an area of range and range-rate values [33–35]. Briefly, orbital energy considerations are applied to the angle measurements, and theoretical limits on the target's range and range-rate are determined. This process results in a bounded region, where each pair of values represents a closed geocentric orbit.

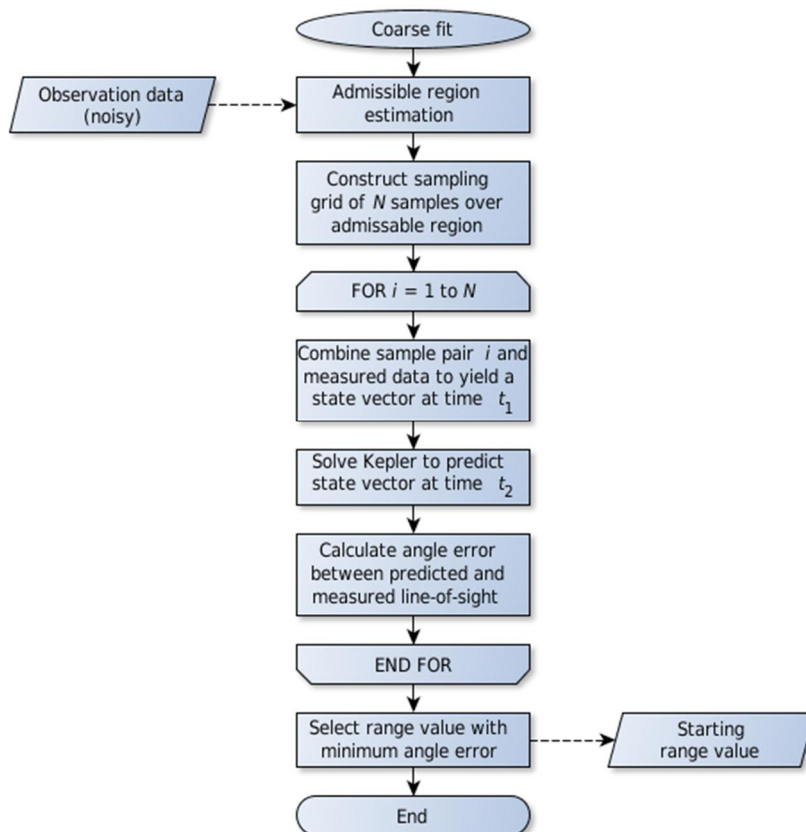


Fig. 113. Block diagram of the coarse fit procedure.

For best results, admissible region estimation requires accurate measurement of the rate of change of the observed angle lines, which we do not presume to have. However, since the goal is elimination of invalid starting values, we could approximate the angular rate of change for right ascension and declination as

$$\dot{\alpha} \approx \frac{\alpha_2 - \alpha_1}{t_2 - t_1}, \text{ and} \quad (90)$$

$$\dot{\delta} \approx \frac{\delta_2 - \delta_1}{t_2 - t_1}, \quad (91)$$

respectively. If three or more observations were available, this approximation could be improved by using Lagrange interpolation or similar methods. The accuracy also improves as $t_2 - t_1$, the separation time between observations, decreases.

The next step of the coarse fit procedure is to overlay a sampling grid on the admissible region. The grid intervals were chosen as 1000 km for the range and 1000 m/s for the range-rate. This process generates a small set of 400-900 sample pairs, depending on the orbit. Smaller sampling intervals have not been shown to improve the starting values obtained.

In the final step of the coarse fit procedure, each pair of range and range-rate samples is combined with the measured direction angles to generate an estimated state vector for time t_1 . A second state vector is predicted for time t_2 by solving Kepler's two-body problem. The predicted position is then compared to the measured angles at time t_2 , and the error is recorded. After iteration over the sampling grid, the range value associated with the smallest error is selected as the starting value for the fine fit procedure.

6.3.2 Fine fit

The fine fit procedure is illustrated in Fig. 114. The output of the coarse fit is used as the initial guess for range estimates x and y at times t_1 and t_2 , respectively. The process then proceeds according to the steps outlined in section 6.1. The final result was an orbital prediction, comprising state vectors $\langle \mathbf{r}_1, \mathbf{v}_1 \rangle$ and $\langle \mathbf{r}_2, \mathbf{v}_2 \rangle$ at observation times t_1 and t_2 , respectively.

6.4 *Other Simulation Improvements*

In addition to the changes described above, many other improvements have been made to the simulation software. These include:

- Right ascension and declination values based on topocentric measurements,
- Recalculation of relative solar position for each observation,
- Elimination of circular orbit assumption for coarse fit procedure,
- Improvement in the accuracy of angle error calculations,
- Specification of observer location by latitude, longitude, and altitude,
- Specification of observation times by Julian date,
- Improvement of solar position accuracy to 36 arcsec,
- Addition of random Gaussian noise to coarse fit input data, and
- Use of modulus of right ascension to avoid interpolation errors.

In addition to these changes to the physical model, several small performance enhancements have been made, such that the simulations run much faster on the computer. This extra speed allows execution of more trials in a shorter time, resulting in better statistical data.

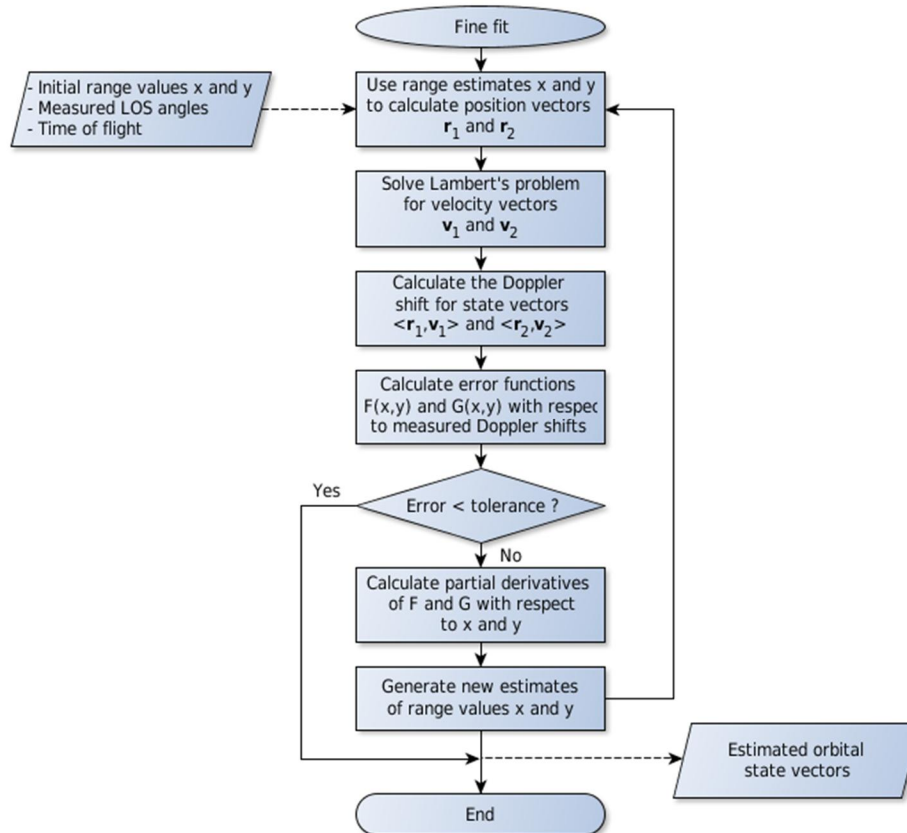


Fig. 114. Block diagram of the fine fit procedure.

6.5 Results

To analyze the performance of the new Doppler shift method, IOD simulation results were compared to those of Gooding's method. A variety of orbital configurations were used to test different aspects of each algorithm:

1. Three geostationary equatorial orbits were selected to test performance with respect to the angle of observation. These orbits were identified as GEO-1, GEO-2, and GEO-3 and were stationed at longitudes of 0° , 45°E , and 45°W , respectively.
2. Three mid-altitude, circular orbits with inclinations of 56° were selected to test performance with respect to orbital height. These orbits were identified as MEO-1, MEO-2, and MEO-3 and were given semi-major axes of 12 000 km, 22 000 km, and 33 000 km, respectively.
3. One geostationary transfer orbit (GTO) was chosen to test performance with respect to eccentricity. The transfer orbit was given a semi-major axis of 24 500 km, an eccentricity of 0.72, an argument of perigee at 0° , and no inclination.
4. One Molniya orbit (MOL) was chosen to test performance in cases of high inclination. This orbit was given a semi-major axis of 26 660 km, an eccentricity of 0.72, an ascending node at 192° , a 270° argument of perigee, and an inclination of 63.5° .

These scenarios were similar to those used elsewhere [36] for comparison of different IOD methods. Each simulated observation consisted of one right ascension angle, one declination angle, and one Doppler shift value.

Gooding's method, relying only on angle measurements, requires a minimum of three observations. The current method requires only two observations, but it was tested with both two and three observations for a more equitable comparison. The time between observations was varied among 4, 8, 16 and 32 minutes to test the effect of angular separation on algorithm performance. However, shorter separations yielded consistently worse predictions for both methods, because noise has a proportionately greater effect on smaller separation angles. In the interest of efficiency, only the results for 16 minute separation times are reported in this section, but similar results were obtained for the other separation times.

Since the angle of reflection affects the Doppler measurement accuracy, simulations were repeated for multiple target positions, covering a minimum of one complete orbit. Results were recorded at 20 minute intervals over a simulated 24-hour period. Although some of these simulations included infeasible daytime observations, the resulting test suite comprehensively characterized the performance of the algorithm over a range of scenarios.

For each simulation described above, 1000 Monte Carlo trials were run using noise values randomly chosen from a normal distribution. Angle measurements were perturbed by 1 arcsec of standard deviation, and Doppler shift measurements were perturbed by amounts equivalent to 2, 5, and 10 m/s of standard deviation.

Since the goal was to predict the target's position at a future time, errors were averaged over the entire orbit. Both the true and estimated orbits were sampled at 100 points, starting at perigee and evenly distributed in time over one complete revolution. The estimation's error was then calculated by

$$\text{err}(\%) = \frac{1}{1000} \sum_{n=1}^{1000} \left(\frac{1}{100} \sum_{m=1}^{100} \frac{\|\mathbf{r}_m - \hat{\mathbf{r}}_{m,n}\|}{\|\mathbf{r}_m\|} \right) \cdot 100, \quad (92)$$

where $\mathbf{r}_m$ is the true position vector at the m^{th} sample point, and $\hat{\mathbf{r}}_{m,n}$ is the estimated position vector at the m^{th} sample point of the n^{th} Monte Carlo trial.

6.5.1 Geostationary orbits

Our implementations of the Gooding and Doppler algorithms were verified by checking the results under noiseless conditions. This also generated a baseline reference reflecting the maximum possible accuracy inherent in each method. Fig. 115 illustrates the zero noise performance for test case GEO-1. In this configuration, the target is stationary above the equator, 16.6° east of the ground station. For this hypothetical noiseless situation, Gooding outperformed Doppler by a factor of 1000. However, in absolute terms, the Doppler method's worst case result is accurate to within 21 cm. The other GEO test cases produce similar results under noiseless conditions and are not shown here to avoid redundancy.

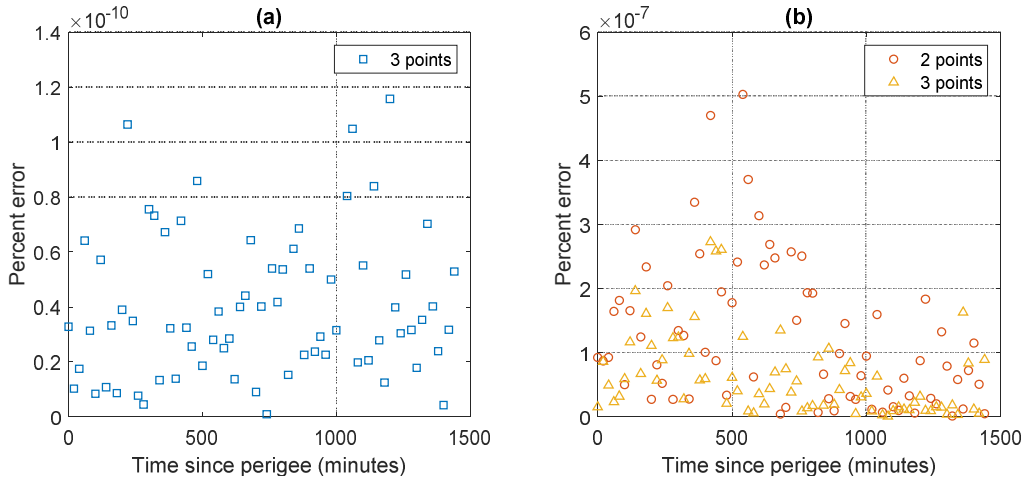


Fig. 115. Zero noise performance of test case GEO-1 for (a) Gooding, (b) Doppler.

Fig. 116 depicts the results after noise was added to the simulated measurements. Gooding's method exhibited an average error of 6.9% over the entire orbit. The average error for the Doppler method, using three observations with a noise level of 2 m/s, was 4.4%. With two observations, the error increased to 8.9%, but Gooding's method, using angles only, was not viable. These plots demonstrate that the accuracy of the Doppler method varies greatly depending on the orbital position of the target. The large error peak occurring 280 minutes after perigee coincides with the maximum solar phase angle between Earth and Sun, with the target at the vertex. The situation is similar to the geometric dilution of precision experienced by global positioning satellites [37].

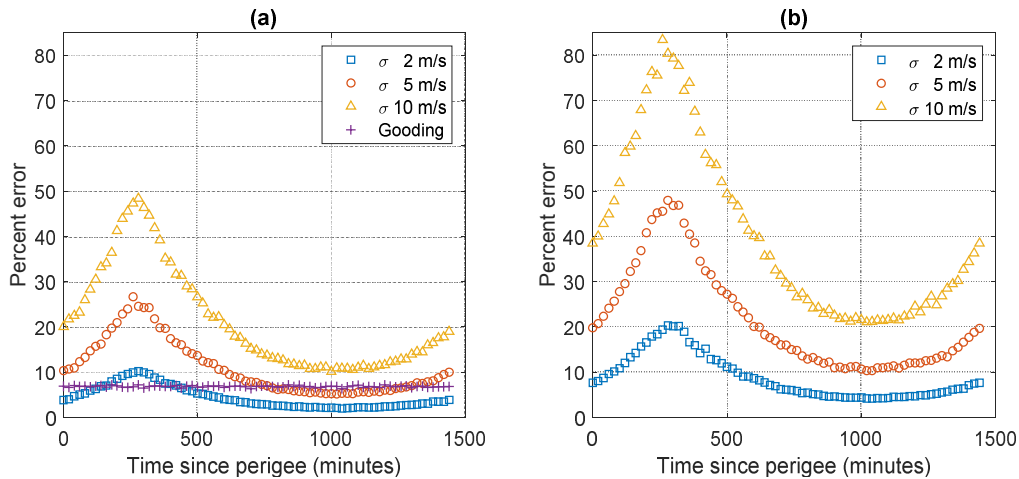


Fig. 116. Results for the GEO-1 test case using (a) three observations, (b) two observations.

Test cases GEO-2 and GEO-3 produced nearly identical results, with the Doppler method's error peaks straddling the maximum phase angle in each case. These results suggest an overall dependence on the solar phase angle, rather than the angle of observation.

6.5.2 Mid-altitude orbits

Fig. 117 illustrates the zero noise performance for test case MEO-1, a 12 000 km circular orbit. Although Gooding's method again outperformed Doppler by a factor of 1000, the maximum Doppler error was less than 4 cm in absolute terms. The noiseless performance for the other two mid-altitude cases was similar.

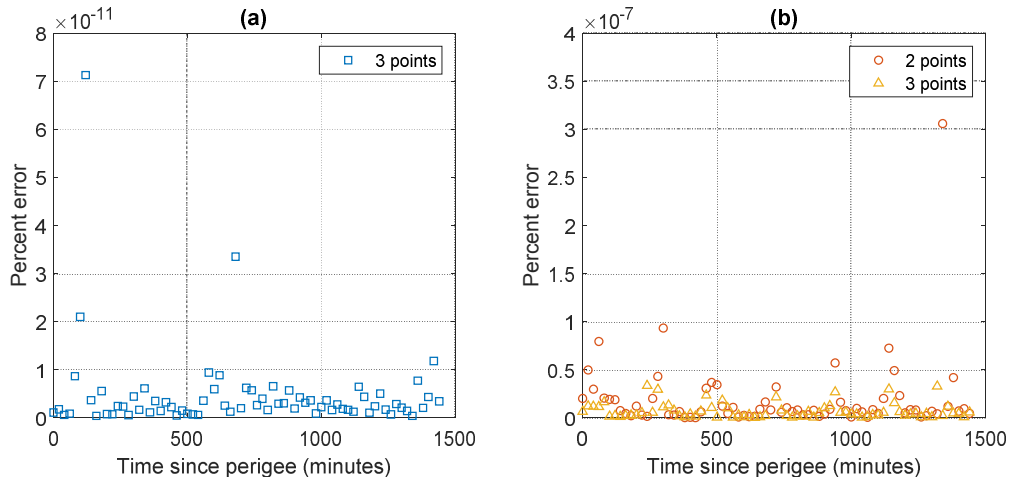


Fig. 117. Zero noise performance of test case MEO-1 for (a) Gooding, (b) Doppler.

Fig. 118 depicts the results after noise was added to the measurements. The average error of Gooding's method was 0.12% compared to 0.46% for the Doppler method, assuming a measurement error of 2 m/s. In the worst case, the average error for the Doppler method, using two observations with a noise level of 10 m/s, increased to 4.4%.

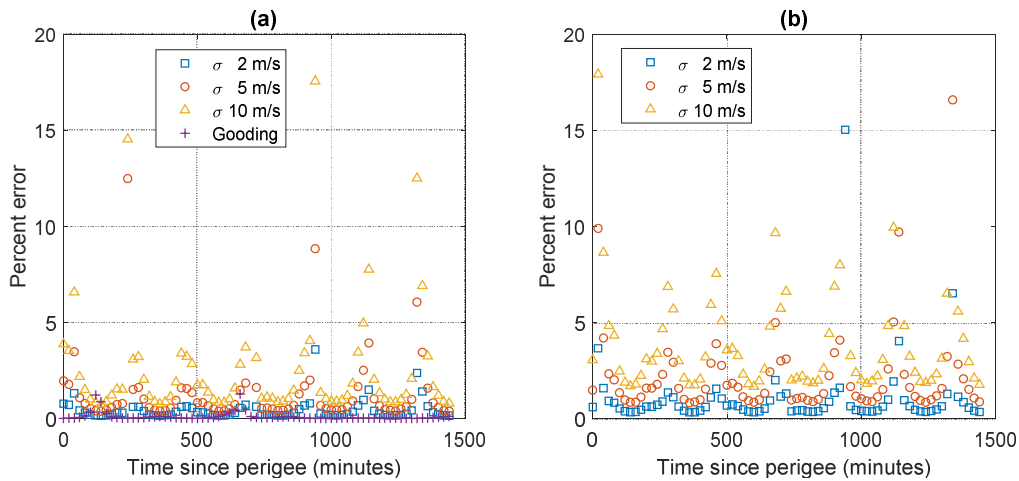


Fig. 118. Results for the MEO-1 test case using (a) three observations, (b) two observations.

Results for the other two mid-altitude orbits, with heights of 22 000 km and 32 000 km, are illustrated in Fig. 119 and Fig. 120, respectively. In all cases, the accuracy of the predicted orbit decreased with orbital height. This decrease is because the measurement error has a relatively larger impact when the angular separation between subsequent observations is small, which is consistent with the case for larger orbits. As with the GEO test orbits, the Doppler method produced error peaks around orbital times when the phase angle was largest.

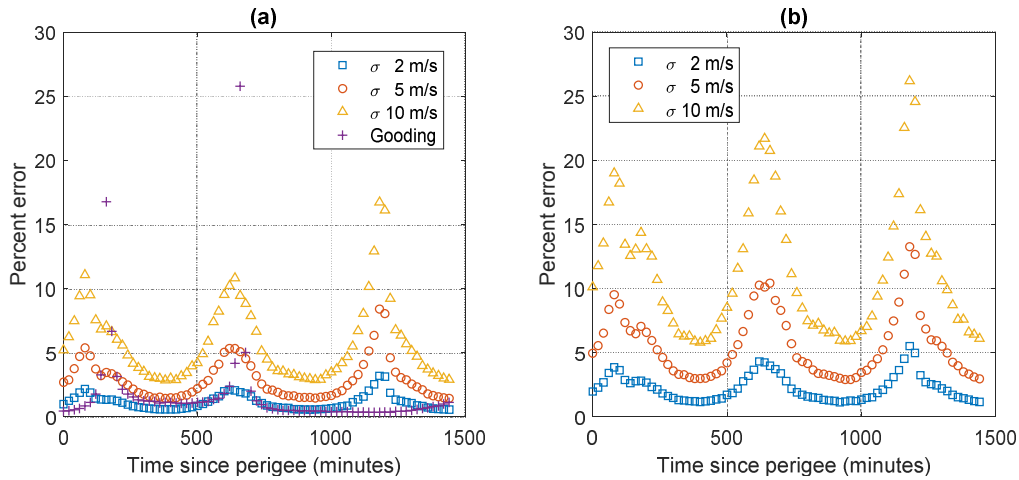


Fig. 119. Results for the MEO-2 test case using (a) three observations, (b) two observations.

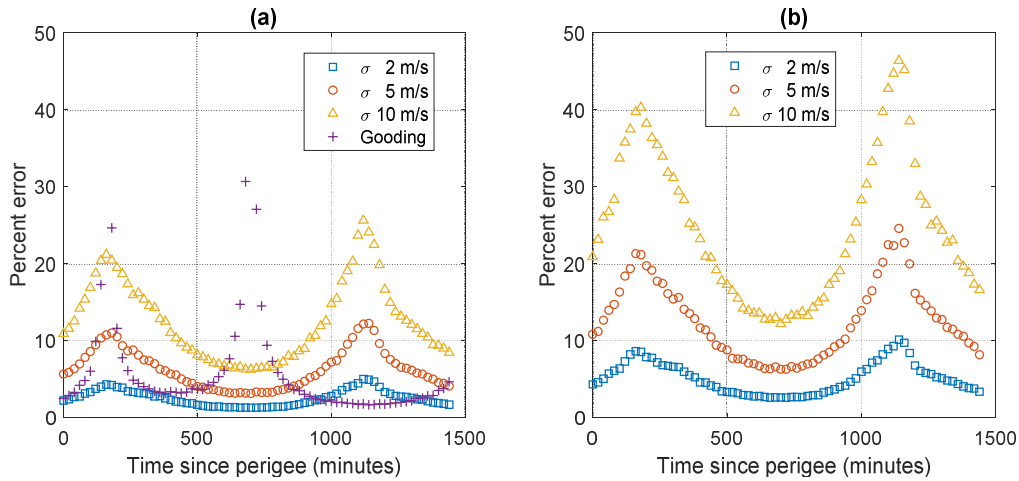


Fig. 120. Results for the MEO-3 test case using (a) three observations, (b) two observations.

6.5.3 Eccentric and inclined orbits

Noiseless performance for the GTO test case is illustrated in Fig. 121. Once again, Gooding's method outperformed Doppler by three orders of magnitude, although the absolute error for the worst case Doppler was less than 2 cm.

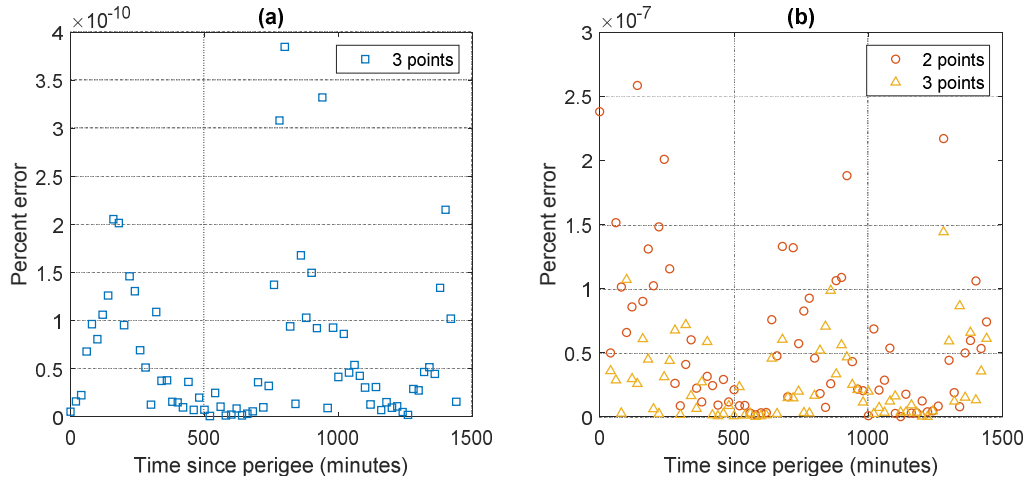


Fig. 121. Zero noise performance of test case GTO for (a) Gooding, (b) Doppler.

Fig. 122 depicts the results after addition of noise. As in the previous test cases, the errors in the Doppler method peaked near the maximum phase angle. Conversely, the errors produced by Gooding's method were dominated by the angular separation of the three observations.

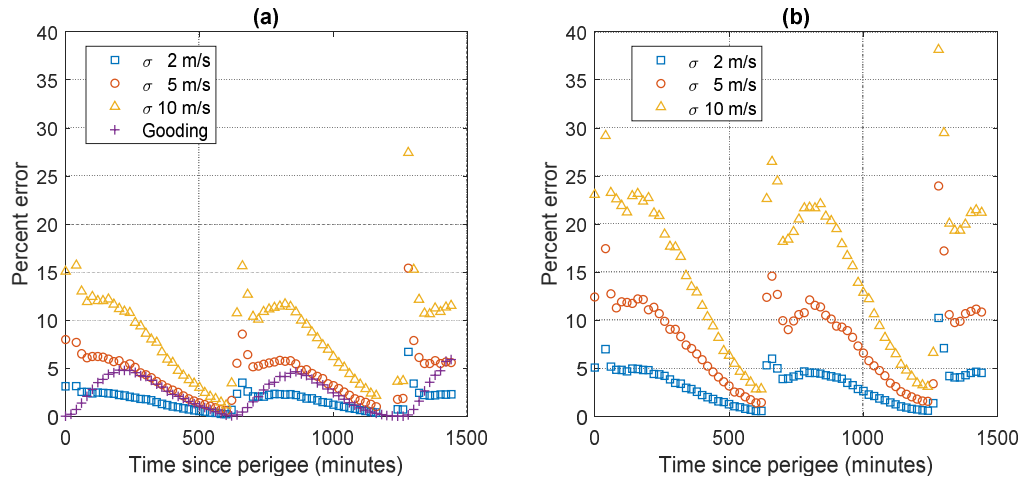


Fig. 122. Results for the GTO test case using (a) three observations, (b) two observations.

Results for the other highly eccentric orbit, the MOL test case, are depicted in Fig. 123 for the zero noise condition. Again, Gooding's method was more accurate for the noiseless case, but the absolute worst case error for the Doppler method is less than 16 cm. Finally, results for the MOL test case, with noise added, are depicted in Fig. 124.

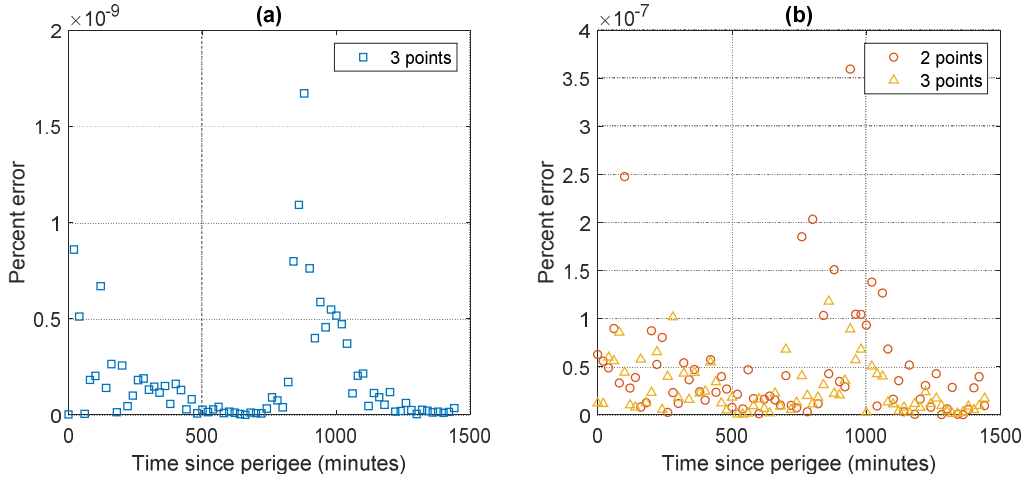


Fig. 123. Zero noise performance of test case MOL for (a) Gooding, (b) Doppler.

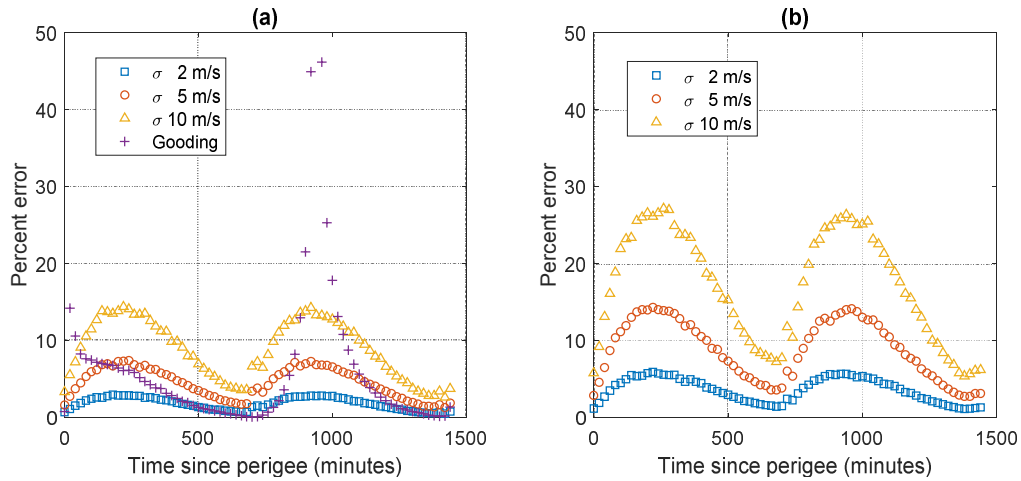


Fig. 124. Results for the MOL test case using (a) three observations, (b) two observations.

6.5.4 Coplanar observer

Angles-only methods are known to perform poorly, or not at all, when the observing station is located on, or very near, the plane of the orbit. With a coplanar observer, all of the angle measurements are constrained to a single plane and are therefore not truly independent, resulting in insufficient information for solving the orbit. However, measurement of wavelength shift is not affected by coplanar geometry. Fig. 125 depicts Doppler method results from the GEO-1 test case, with the observing station located on the equator. These results were nearly identical to the non-coplanar case illustrated in Fig. 116. No results were recorded for Gooding's method, since the algorithm would not converge to a solution under coplanar conditions.

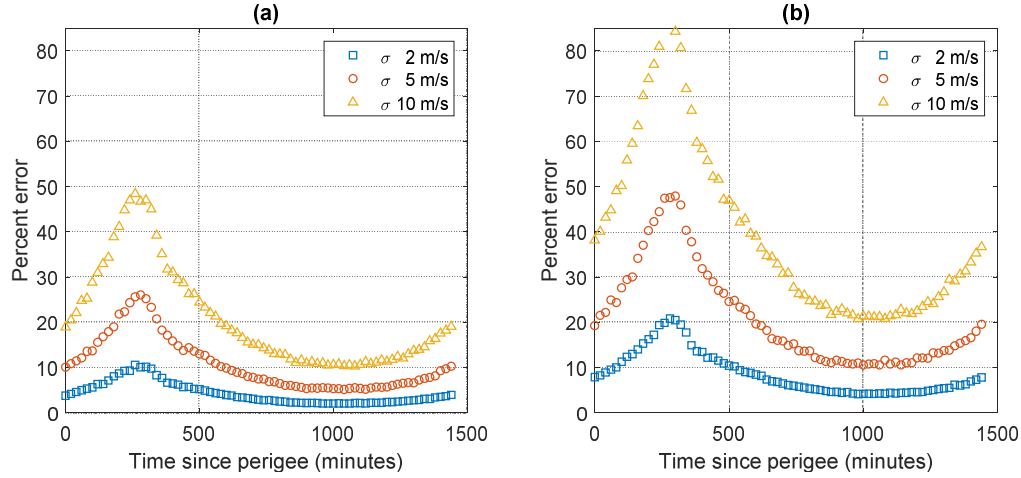


Fig. 125. Results for the GEO-1 test case with a coplanar observer.

6.6 Discussion

The results illustrated in the previous sections exhibit large variations in accuracy across the simulated 24-hour orbital period. However, a pattern emerged when the results were compared to the solar phase angle. Fig. 126 illustrates the pattern for the MEO-1 test case. The prediction errors were higher when the phase angle was large and lower when the phase angle was small. This pattern was typical for all test cases and is explained by the fact that the measured wavelength shift is a function of the reflection angle.

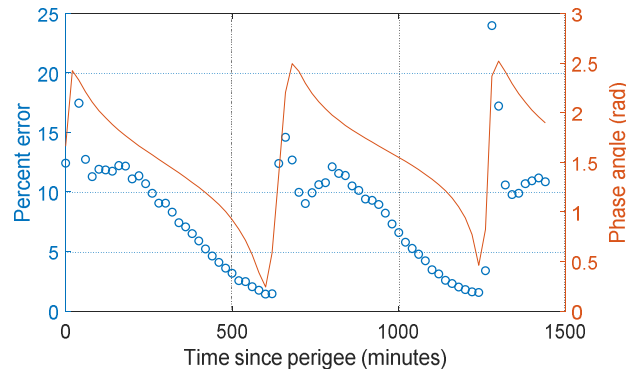


Fig. 126. Orbital prediction error (circles) vs. phase angle (solid line).

The pattern illustrated in Fig. 126 suggests a way to determine when the Doppler method might have an advantage over conventional angles-only methods. When results corresponding to large phase angles were ignored, the average error of the remaining results decreased significantly. Although the exact phase angle (φ) cannot be known at the time of observation, the elongation angle (θ) between Sun and target can be measured directly. For targets in geosynchronous orbit, the approximation

$$\varphi = 180^\circ - \theta \quad (93)$$

is always within 2° of the true phase angle. Based on this approximation of the phase angle, we can estimate the relative accuracy of the Doppler shift measurement.

Figs. 127-132 illustrate the effectiveness of using the phase angle to qualify individual Doppler results. Each chart summarizes the error distribution for a particular test case. Box plot symbols indicate the minimum, maximum, median, and interquartile range over one orbital revolution.

Fig. 127 combines the results for test cases GEO-1, GEO-2, and GEO-3. On the left-hand side, Fig. 127(a) depicts the full set of error statistics including all results, regardless of phase angle. On the right-hand side, Fig. 127(b) depicts only the results generated when the phase angle was less than 90° . From left to right in each chart, the first box plot represents Gooding's angles-only method. The next three boxes represent the three-point ($N = 3$) Doppler method with wavelength measurement errors of 2, 5, and 10 m/s, respectively. The final three boxes represent the two-point ($N = 2$) Doppler method.

Compared to Fig. 127(a), the subset of results in Fig. 127(b) show significant improvement. After measurements during times of large phase angle were excluded, outlying Doppler results with large errors were removed from the statistics, and the error variance was reduced. Conversely, the angles-only results remained unchanged.

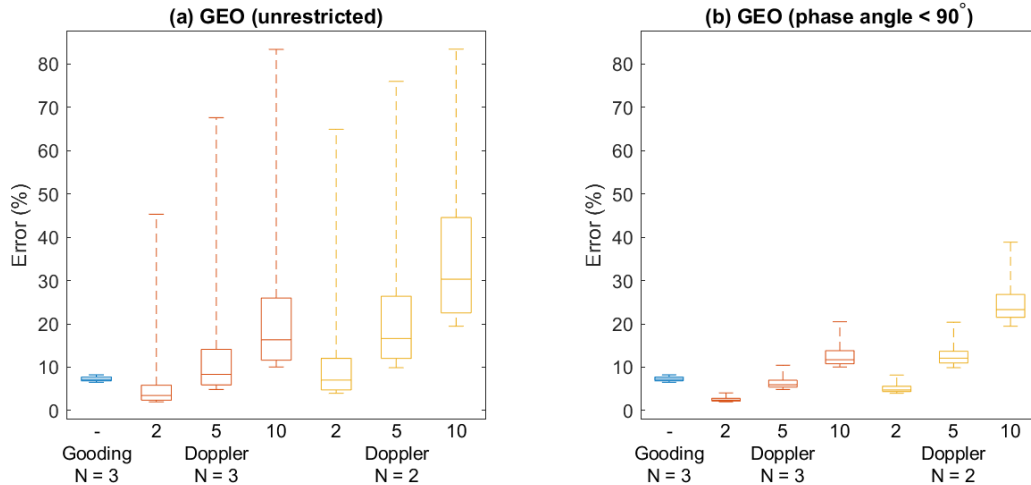


Fig. 127. Summary statistics for combined test cases GEO-1, GEO-2, and GEO-3. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma = 2, 5$, and 10 m/s; and two-point Doppler with $\sigma = 2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

Figs. 128-130 summarize the results for test cases MEO-1, MEO-2, and MEO-3. As with the GEO test cases, application of the phase angle condition has filtered out large, outlying Doppler errors and reduced the variance of the results. However, the effect on the angles-only results were mixed. For the MEO-1 case in Fig. 128, the results were unchanged. Fig. 129 depicts elimination of a large error outlier for the MEO-2 case, but little or no change in the variance. Finally, the error variance of the angles-only method increased after the phase angle condition was applied to the MEO-3 results in Fig. 130.

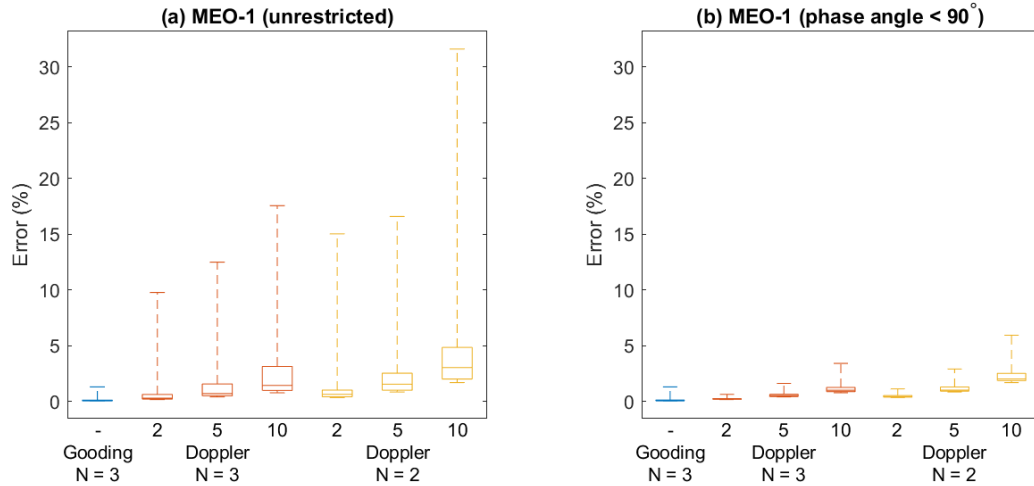


Fig. 128. Summary statistics for test case MEO-1. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma = 2, 5$, and 10 m/s; and two-point Doppler with $\sigma = 2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

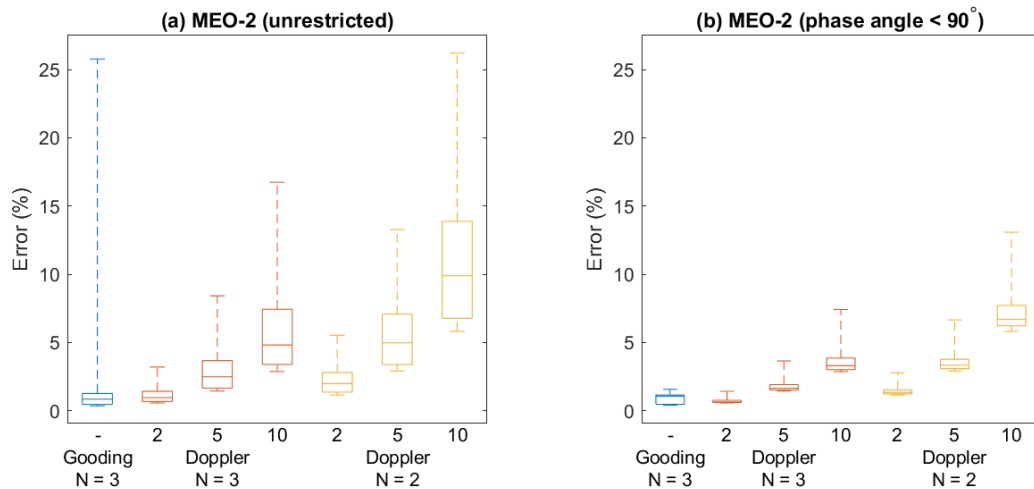


Fig. 129. Summary statistics for test case MEO-2. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma = 2, 5$, and 10 m/s; and two-point Doppler with $\sigma = 2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

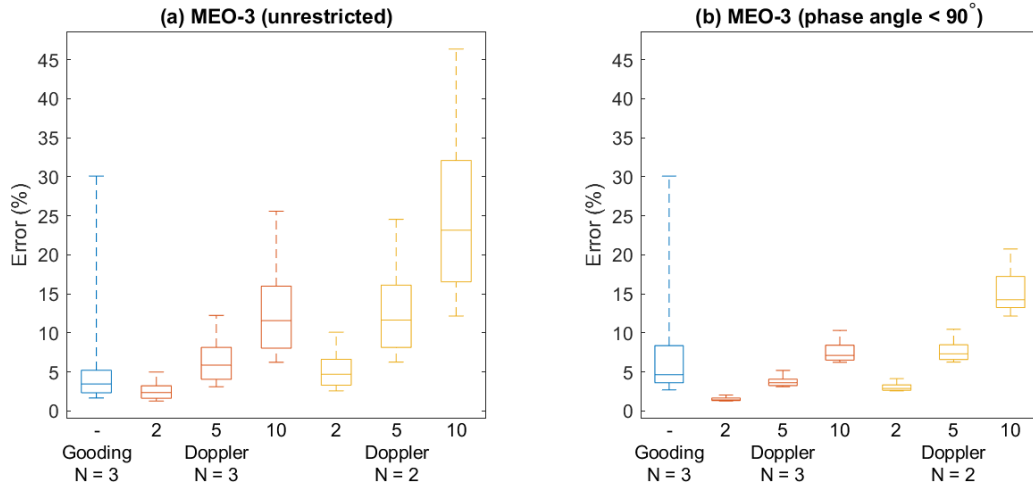


Fig. 130. Summary statistics for test case MEO-3. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma = 2, 5$, and 10 m/s; and two-point Doppler with $\sigma = 2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

Next, Fig. 131 depicts statistics for the GTO test case. The Doppler and angles-only statistics both exhibited a reduction in median error and variance after the phase angle condition was applied. Because the GTO orbit is highly eccentric, the angles-only statistics benefit from removal of observations near apogee.

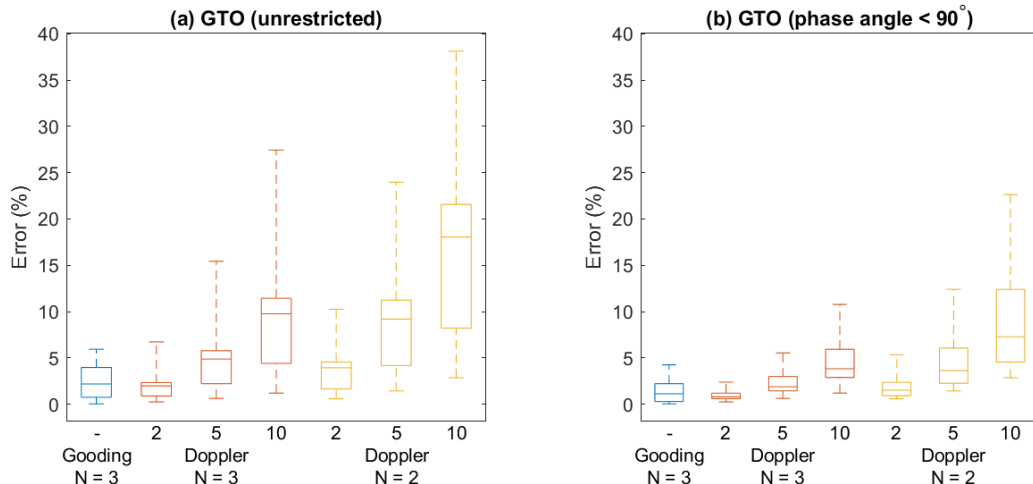


Fig. 131. Summary statistics for test case GTO. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma = 2, 5$, and 10 m/s; and two-point Doppler with $\sigma = 2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

Finally, Fig. 132 depicts statistics for the MOL test case. In this scenario, a large orbital inclination maintained the phase angle within a relatively small range near 90° , so eliminating results above that value had little effect on the statistics. Other than a slight increase in the median error, the

conditional results in Fig. 132(b) demonstrate no major differences when compared to the full set of statistics in Fig. 132(a).

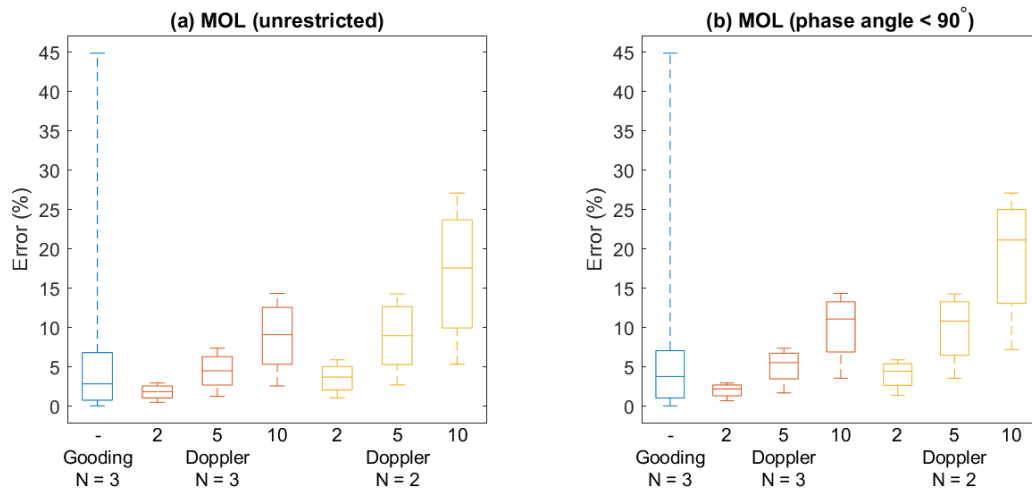


Fig. 132. Summary statistics for test case MOL. Box plots from left to right represent Gooding's method; three-point Doppler method with measurement errors of $\sigma=2, 5$, and 10 m/s; and two-point Doppler with $\sigma=2, 5$, and 10 m/s. (a) Error statistics for all phase angles. (b) Errors conditioned on phase angle $\varphi < 90^\circ$.

7.0 Filter Tradespace

A key aspect of the instrument design is correlation of the received signal with the known, unshifted, Fraunhofer absorption lines. The ratio of in-band (within the bandwidth of an absorption line) to out-of-band signal power provides a measurement of the amount of Doppler shift. Several different optical filters were examined to determine the optimal method for the correlation process.

7.1 Multistage Lyot Filter

A Lyot filter is a refinement of the simple birefringent filter. Any uniaxial birefringent crystal has ordinary and extraordinary axes with indices of refraction n_o and n_e , respectively. If a linearly polarized plane wave enters the crystal parallel to the ordinary axis, with a polarization angle of 45° relative to the extraordinary axis, the vertical and horizontal components of polarization will diverge. Each component will experience a different refractive index traveling through the crystal, such that their relative retardation, in terms of wave number, is [38]

$$n_1 = (d_1 / \lambda) \mu, \quad (94)$$

where d_1 is the thickness of the crystal along the ordinary axis, λ is the wavelength of the light, and μ is the birefringence, specified as $n_e - n_o$. If the light subsequently passes through a linear polarizer, the transmission will be

$$T_1 = \cos^2(\pi n_1). \quad (95)$$

The spectral output of the polarizer is sinusoidal, with peaks located at wavelengths that are integral multiples of n_1 . Thus, we have a simple, periodic filter. However, if a second crystal and polarizer are considered, with the width of the second crystal $d_2 = 2d_1$, the transmission is

$$T_2 = \cos^2(\pi n_2) = \cos^2(\pi 2n_1). \quad (96)$$

As additional filter stages are added, with each subsequent crystal being twice the width of the previous, the transmission through the entire assembly becomes

$$T_L = \left(\frac{t_L}{2} \right) \cos^2(\pi n_1) \cos^2(\pi 2n_1) \cdots \cos^2(\pi 2^{N_L-1} n_1), \quad (97)$$

where N_L is the total number of Lyot filter stages, and the factor $(t_L/2)$ accounts for transmission loss due to absorption and reflection. As N_L increases, the passband will narrow. The resulting comb filter is illustrated in Fig. 133 for three values of N_L . The center wavelength is determined by choosing values for d_1 and μ in (94) such that n_1 is an integer value at the desired wavelength. The first stage thickness d_1 sets the free spectral range, and should be chosen small enough for an ordinary filter to remove the unwanted bands.

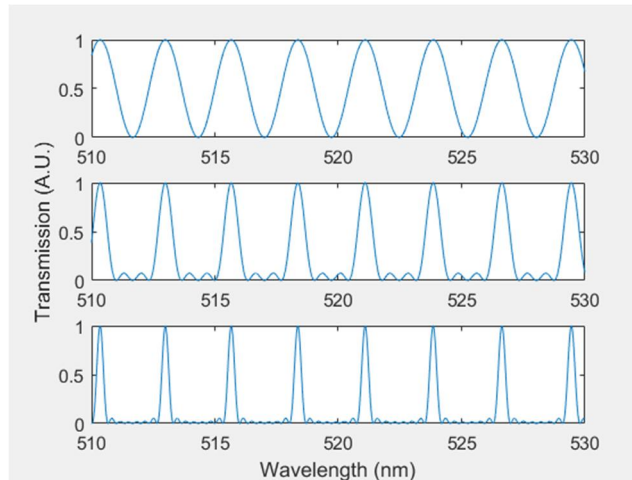


Fig. 133. Illustration of Lyot filter characteristics for one, two, and three stages (top, center, and bottom, respectively). This filter was designed for the Mg I Fraunhofer line at 518.36 nm using quartz (birefringence 0.009) with a first stage thickness of 11 mm.

7.1.1 Lyot Filter Design

Fig. 134 shows a Lyot filter design as part of an optical correlator. There are actually three separate Lyot filters, allowing concurrent filtering of Mg I, H-alpha, and Na Fraunhofer lines. Each Lyot filter resides in a separate optical arm of the device. Collimated light from the telescope enters from the left and is directed by a pair of beamsplitters DBS1 and DBS2 into four separate arms.

In the first arm, a thin-film interference filter formed by BPF1 and LP1 isolate a single Lyot passband around the wavelength of interest. Following that is a two-stage Lyot filter. Crystals R1 and R2 are separated from polarizers LP2 and LP3 by quarter-wave plates QWP1 and QWP2, respectively. This provides a margin of field widening without changing the filter characteristics. Finally, the filter output is focused by lens L1 onto a detector at FPA1. Arms 3 and 4 of the device are identical, save for optimization of the crystal thicknesses for different center wavelengths.

Arm 2 collects a portion of the out-of-band light reflected by bandpass filters BPF2 and BPF3. The power measured by the detector at FPA2 provides an estimate of the total out-of-band power received by the device. The ratio of in-band to out-of-band power is used to determine the amount of Doppler shift.

7.1.2 Lyot Filter Simulation

Fig. 135 shows the results of simulating the filter in MATLAB. The measured Doppler velocity, as calculated by the ratio of in-band vs. out-of-band power received in the filter, is plotted against the true velocity. For a single five-minute exposure on a 0.9 m diameter telescope, the simulation indicates a Doppler sensitivity of ± 404 m/s. Sensitivity increases to approximately ± 100 m/s for a 3.5 m telescope aperture.

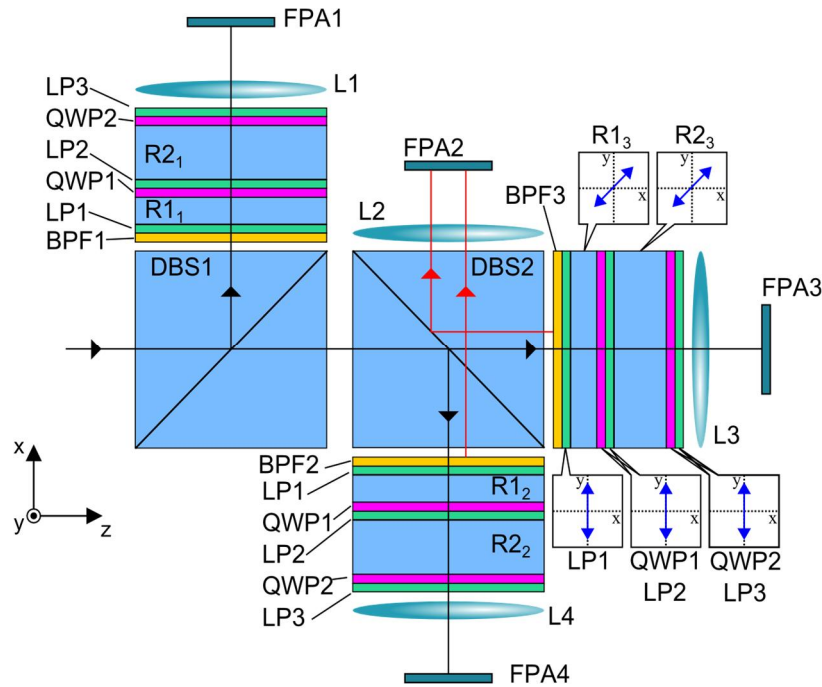


Fig. 134. Design of a multistage Lyot filter with centers at three separate Fraunhofer wavelengths. This filter is designed to measure the received signal power found within the Fraunhofer absorption bandwidths of Mg I, H-alpha, and Na.

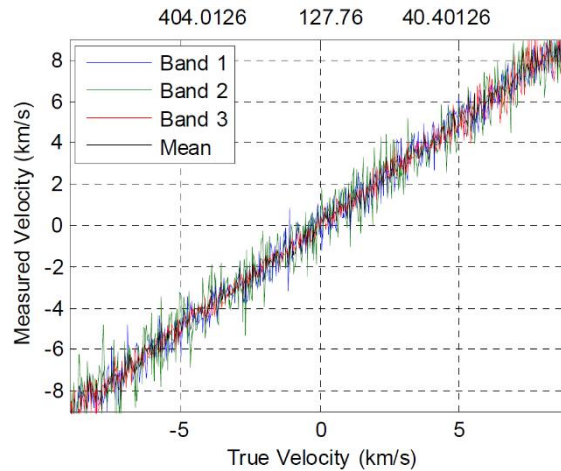


Fig. 135. Results of Lyot filter simulation, showing the actual vs. calculated Doppler velocity for three separate wavelengths, plus the mean value of all three. Numbers above the plot are sensitivity values in m/s for simulated telescope apertures of 0.9 m, 2.85 m, and 9 m, respectively.

7.2 Multistage Solc Filter

Solc filters offer several degrees of freedom over Lyot filters, at the cost of greater sensitivity to tolerancing error. Like the Lyot, the Solc is a modified form of the simple birefringent filter. The

Solc “fan filter” [39] is constructed using a stack of birefringent waveplates, all of equal width. Each element is rotated by an angle of

$$\omega_j = \frac{\pi}{2N_s} \left(j - \frac{1}{2} \right), \quad (98)$$

where N_s is the number of Solc filter elements, and j represents each element’s order in the stack. Parallel linear polarizers cap each end of the stack. A derivation [39] using Jones matrix calculus results in a transmission through the stack of

$$T_s = \frac{t_s}{2} \left[\frac{\sin(N\chi)}{\sin(\chi)} \cos(\chi) \tan\left(\frac{\pi}{2N}\right) \right]^2, \quad (99)$$

where t_s accounts for transmission loss due to absorption and reflection, and χ is defined as

$$\cos(\chi) = \cos(\pi n_1) \cos(\pi / 2N), \quad (100)$$

with n_1 coming from (94). A comparison of this result with the previous result for the Lyot filter produces the plot in Fig. 136. In this example, the filters are constructed from quartz. All of the Solc waveplate elements, and the smallest Lyot element, have a width of 11 mm. In general, Lyot and Solc filters will have the same peak transmission wavelengths, bandwidth, and free spectral range whenever the width of the Solc elements equal the width of the first Lyot element and the number of Solc elements $N_s = 2^{N_L}$. The Solc filter transmits more light than the Lyot filter, but shows slightly more leakage in the sidebands.

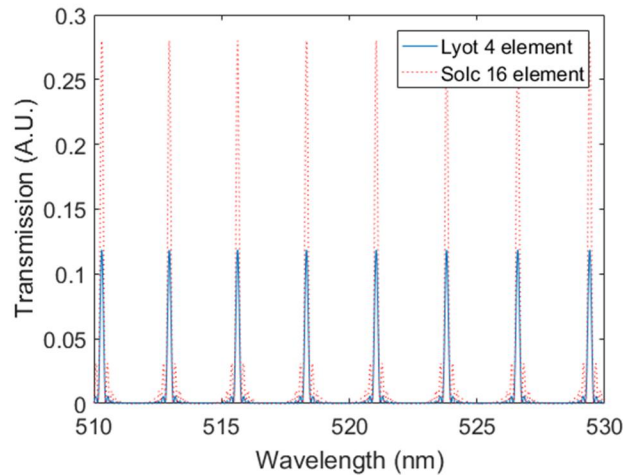


Fig. 136. Comparison of 4-element Lyot and 16-element Solc filters. The smallest quartz waveplate width is 11 mm. The loss parameters t_L and t_s are assumed to be dominated by the linear polarizers.

A multistage Solc filter design is illustrated in Fig. 137. In this design, multiple filters are stacked together with a half-wave plate (HWP) inserted between each section. Each segment implements a separate comb filter with center wavelengths at

$$\lambda_n = d\mu / n, \quad (101)$$

where d is the thickness of a single waveplate, n is an integer, and μ is the birefringence of the material. In this configuration, subsequent stages are additive within the polarization state introduced by linear polarizer LP1. The combined output of a hypothetical filter with three stages is shown in Fig. 138.

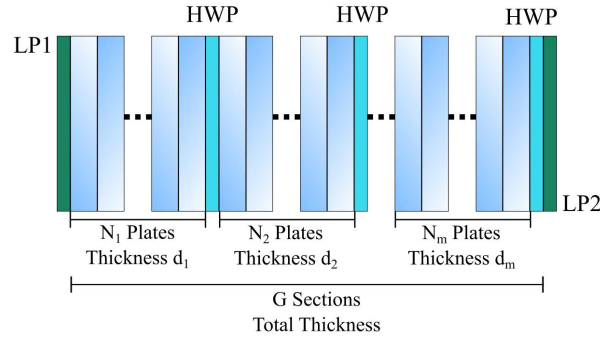


Fig. 137. Multistage Solc filter design. Each stage implements a comb filter.

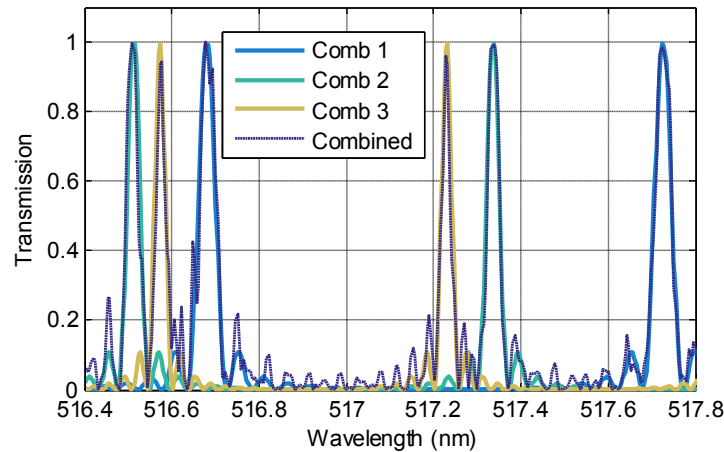


Fig. 138. Example characteristic of a multistage Solc filter using three stages.

7.2.1 Solc Filter Design

The Solc filter can be further optimized based on the signal-to-noise ratio in the correlation function between the filter and the incident solar spectrum. Fig. 139 shows a filter design implementing four discrete comb functions. The filter parameters were determined by Monte Carlo simulation, maximizing the filter correlation in the range of 501.9-516.5 nm. Instead of capping the filter stack with a linear polarizer, this design uses a Wollaston prism (WP) to separate the output into in-band (i.e., within the Fraunhofer bands) and out-of-band components. This technique leverages all of the received light to improve the sensitivity of the instrument.

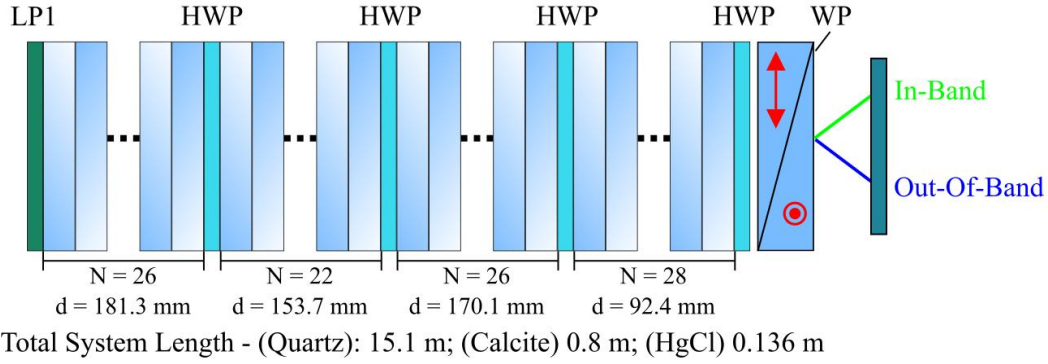


Fig. 139. Optimized four-stage Solc filter design. Total system length is given based on three potential birefringent materials.

7.2.2 Solc Filter Simulation

The performance of the filter in Fig. 139 was estimated by a radiometric simulation across a wavelength range of 501.9-516.5 nm. The following values were assumed:

- Target is a 1 m² object in geosynchronous orbit.
- Target is a Lambertian reflector with 70% reflectance.
- Telescope apertures simulated as 0.9 m and 3.5 m diameters.
- Detector is a CCD with QE 0.71, dark current 0.0014 e/s, and read noise 5 e.
- Integration time is 5 min.

Results are summarized in Fig. 140, which shows (a) the in-band and out-of-band flux components of the measured signal, (b) the ratio of in-band to out-of-band power plotted against Doppler velocity, and (c) the measured vs. true Doppler velocity. The sensitivity, calculated as the standard deviation of all trials, was ± 172 m/s for the 0.9 m telescope and ± 55 m/s for the 3.5 m telescope.

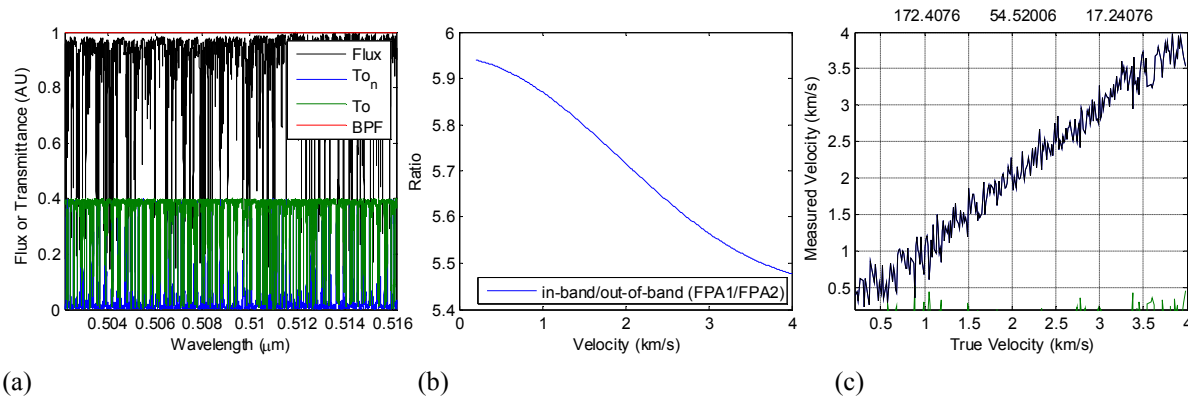


Fig. 140. Results of the radiometric simulation, showing (a) in-band, out-of-band, and total flux; (b) ratio of in-band to out-of-band power vs. Doppler velocity; and (c) the measured vs. true Doppler velocity.

7.3 Fiber Bragg Gratings

The optical fiber has been one of the standard technology in communication systems. The advantages of the optical fiber include low loss transmission, immunity to EM interference, light weight, and electrical isolation. Besides the application of transmitting data, the optical fiber is used for filtering applications in the telecommunication domain. The currently used optical filter in Wavelength Division Multiplexing is one of the key example of the importance of optical filter. Most of the optical filter are designed in the infrared region due to the low attenuation window of about 0.2 dB /km at 1550 nm and 0.5 dB /km at 1300 nm respectively. As compared to the infrared region, the attenuation in the visible range is much higher considering the absorption and scattering losses in the optical fiber. Due to this, not much research has been performed in the designing of visible range optical filter.

However, the visible range provides some of the spectrum parameters which can be of practical relevance. One of them is the Fraunhofer solar spectrum wavelengths in the visible range of 400 - 700 nm. Some of the Fraunhofer wavelengths fall in the visible range and are especially significant in the range of 500 nm - 600 nm. This report highlights the designing of the visible range optical filter for the application of Passive Remote estimation of object velocity using Fiber Bragg Gratings. At the Fraunhofer wavelengths, the received solar intensity is very less as compared to the solar intensity at non-Fraunhofer wavelengths. By designing a Bragg filter for reflecting the solar power at each of the Fraunhofer wavelengths, we can estimate the velocity of the object based on the difference in the solar power intensity at the Fraunhofer and non-Fraunhofer wavelengths. The object's relative velocity is determined based on the Doppler shift of the object velocity and the Bragg filter will capture the net difference in the reflected output power.

In 1978, Hill et al. first demonstrated the formation of gratings in an optical fiber [40,41]. They observed that the argon-ion laser radiation which was incident into the fiber was completely reflected back after several minutes of launching into the fiber. Later, Lam and Garside [42] showed that the increase in the grating strength was directly proportional to the square of the writing power. Then, in 1989, Meltz et al. [43] showed the formation of gratings by the side illumination using two intersecting beams of Ultraviolet light (UV). This marked the beginning of the direct optical inscription of high-quality gratings into the core of optical fibers using various techniques such as interferometric, phase mask, and point-by-point exposure to UV laser light.

7.3.1 Theory and Modeling

A Fiber Bragg grating (FBG) is formed when periodic perturbation of the refractive index occurs along the fiber length and is formed by exposure of the core to an optical interference pattern.

The principle of the fiber Bragg grating is that light is made to reflect along an optical fiber due to a refractive index modulation being printed on the fiber core. If this modulation describes a grating, light can be made to reflect back with high efficiency in a narrow wavelength interval. If the grating period is given by Λ the reflection efficiency is maximized at the Bragg wavelength

$$\lambda_B = 2 n_{eff} \Lambda, \quad (102)$$

where n_{eff} is the effective refractive index of the core.

This growth in back reflected light was explained in terms of a new nonlinear effect called "photosensitivity". When ultraviolet light radiates an optical fiber, the refractive index of the fiber is changed permanently; which enables an index grating to be written in the fiber. Coherent light

propagating in the fiber interferes with a small amount of light reflected back from the end of the fiber and sets up a standing wave pattern which writes an index grating in the fiber core.

The figure 1 below shows the reflection of a certain narrow wavelength called Bragg wavelength through a grating.

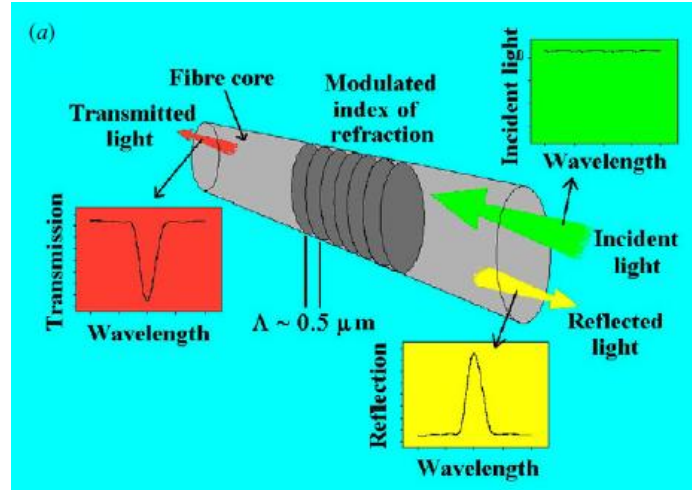


Fig. 141. The reflection and transmission of light through the fiber Bragg grating [44].

These gratings consist of a periodic refractive index pattern in the fiber core. The gratings operation can be understood as the coherent interaction of the Fresnel reflections caused by every single refractive index step. The basic parameter that controls the behavior of a given fiber filter is the magnitude of the induced refractive- index change in the fiber core. This index modulation is typically achieved by the photosensitivity of fiber core under illumination by UV laser light or by exposure with high-intensity IR femtosecond laser pulses.

The retro-reflective effect of FBG's is based on the fulfilment of the phase-matching condition at a certain wavelength between the incident and reflected core-guided modes. In the simplest case (first-order grating) this Bragg condition reduces to $\lambda_B = 2n_{eff}\Lambda$, where λ_B is the Bragg wavelength, and Λ is the period of the grating.

The propagation of light along a waveguide is described in terms of a set of guided electromagnetic waves called the modes of waveguide. In optical fibers, the core-cladding boundary conditions lead to coupling between the electric and magnetic field components and each mode has its specific propagation constants. If the periodic perturbation is introduced alongside the fiber the mode will exchange its power. This phenomenon is known as mode coupling. Hence, this coupling occurs between modes traveling in opposite directions in Fiber Bragg Gratings. For reflection of a certain Bragg wavelength, there is only one mode traveling in the fiber core and this mode travels in forward direction and this same mode gets reflected back, known as counter-propagating mode in the opposite direction. Due to this interaction of counter-propagating modes, the power transfer occurs such that maximum power gets reflected back at a certain specific wavelength, while rest of it gets transmitted in the forward direction. Due to this coupling nature, fiber grating are simply an optical diffraction grating, and thus its effect upon a light wave incident on the grating at an angle can be described by the grating equation as [45]

$$n_{co} \sin(\theta_2) = n_{co} \sin(\theta_1) + m(\lambda / \Lambda) \quad (103)$$

Where θ_2 the angle of the diffracted wave and the integer m determines the diffraction order.

Bragg grating of a mode with a bounce angle of θ_1 reflects into the same mode traveling in the opposite direction with a bounce angle of $\theta_2 = -\theta_1$

The mode propagation constant β is given by:

$$\beta = (2\pi/\lambda) n_{eff} \text{ where } n_{eff} = n_{co} \sin(\theta) \quad (104)$$

For guided modes, $\beta_2 = \beta_1 + m(2\pi/\Lambda)$, for the first order diffraction which dominates in the fiber Bragg grating, $m = -1$; Negative β values describe modes that propagate in the -z direction and recognizing $\beta_2 < 0$; we find that the resonant wavelength for reflection of a mode of index n_{eff1} into a mode of index n_{eff2} is

$$\lambda = (n_{eff1} + n_{eff2})\Lambda \quad (105)$$

and if the two counter-directional modes are identical, then we obtain the Bragg condition.

In the fiber, a small amount of incident light is reflected at each periodic refractive index change. The entire reflected light waves are then combined into one large reflection at the Bragg wavelength. Light propagated along the core of an optical fiber is scattered by each optical grating plane and if the Bragg condition is not satisfied, then the reflected light from each of the subsequent planes becomes progressively out of phase and eventually cancels out. Where the Bragg condition is satisfied, the contributions of the reflected light from each of the grating plane adds up constructively in the backward direction and forms a back reflected peak with a center wavelength defined by the grating parameters such as grating length (L), grating period (Λ) and refractive index modulation given by δn_{eff} . The index modulation $\delta n_{eff}(z)$ to the effective refractive index n_{eff} is given by the below equation [46]

$$\delta n_{eff}(z) = \underline{\delta n_{eff}(z)} [1 + v \cos((2\pi z/\Lambda) + \varphi)], \quad (106)$$

where $\underline{\delta n_{eff}(z)}$ is the “dc” index change spatially averaged over a grating period, v is the fringe visibility of the index change and φ describes the grating chirp. As we are considering uniform grating where the grating sections are uniformly spaced and hence the phase term or grating chirp reduces to zero..

For a fiber with a step-index profile, where the core-cladding index show a step-index variation; and a uniform induced index change across the core $\delta n_{co}(z)$, we get the equation

$$\delta n_{eff} = \phi \delta n_{co}, \quad (107)$$

where ϕ is the core power confinement factor for the mode of interest. Typically, the fundamental propagation mode for a single mode fiber, for which the normalized frequency $V < 2.4$, gives a confinement factor of 0.8 and an effective index parameter $b = 0.5$ [47],

$$V = (2\pi/\lambda) a (n_{co}^2 - n_{cl}^2), \quad (108)$$

where a is the core radius, n_{co} and n_{cl} are the core and cladding indices respectively. The effective refractive index is related to the parameter b as [47]

$$b = (n_{eff}^2 - n_{cl}^2) / (n_{co}^2 - n_{cl}^2). \quad (109)$$

The coupling interaction between modes in the Fiber Bragg Gratings can be explained using Coupled Mode Theory (CMT) which is a method to analyze the light propagation in perturbed or weakly coupled waveguides between two identical modes propagating in opposite direction. First, the modes of the unperturbed or uncoupled structures are defined and solved first. Then, a linear combination of these modes is used as a trial solution to Maxwell's equations for complicated perturbed or coupled structures. After that, the derived coupled mode equations can be solved analytically or by numerical methods.

In the Fiber Bragg Gratings, the reflection occurs from a mode of amplitude $A(z)$ traveling in the forward direction into an identical counter propagating mode of amplitude $B(z)$ in the backward direction. Hence, we get the coupled mode equations which describe their complex amplitudes as:

$$dA^+ / dz = j\sigma A^+(z) + j\kappa B^+(z) \text{ and} \quad (110)$$

$$dB^+ / dz = -j\sigma B^+(z) - j\kappa A^+(z), \quad (111)$$

where

$$A^+(z) = A(z) \exp(j \delta_d z - \varphi/2) \text{ and} \quad (112)$$

$$B^+(z) = B(z) \exp(-j \delta_d z + \varphi/2). \quad (113)$$

σ is the general “dc” self-coupling coefficient defined as

$$\sigma = \delta_d + \zeta - 0.5 d\varphi / dz . \quad (114)$$

δ_d is known as the detuning parameter and is defined as

$$\delta_d = \beta - \pi / \Lambda = 2\pi n_{eff} (1 / \lambda - 1 / \lambda_B) . \quad (115)$$

The term ζ is known as dc coupling coefficient and is defined as

$$\zeta = 2\pi \frac{\delta n_{eff}}{\lambda} , \quad (116)$$

and

$$\kappa = (\pi / \lambda) v \frac{\delta n_{eff}}{\lambda} \quad (117)$$

is the “ac” coupling coefficient. For a uniform Bragg grating along the z direction, δn_{eff} is constant along z and the differential of grating chirp $d\varphi / dz = 0$; hence, κ , σ and ζ are constants. So, this simplifies the coupled mode equations into coupled first order ordinary differential equations with constant coefficients. Hence, we get a closed-form solution to these equations given the appropriate boundary conditions.

7.3.2 Transfer matrix equations

There are several methods to solve the coupled mode equations numerically inside the gratings in order to calculate the reflection and transmission spectra of Fiber Bragg Gratings. The most convenient method to analyze the coupling phenomenon in Bragg Gratings is through the direct numerical integration of the coupled mode equations using Transfer Matrix Method. In this method, the modeling of the grating structures is based on identifying 2 by 2 matrices for each uniform section of the grating and then multiplying all of them together to obtain a single 2 by 2 matrix that describes the whole grating. The grating is divided into a sufficient number M of

sections so that each section can be approximately treated as uniform with A_k^+ and B_k^+ being the field amplitudes after traversing the section k [46]. Let each of the section length be

$$dz = L / M \quad (118)$$

By starting with the boundary conditions,

$$A_0^+ = A^+(L/2) = 1, B_k^+ = B^+(L/2) = 0 \quad (119)$$

and calculating the final matrix component

$$A_M^+ = A^+(-L/2) \text{ and } B_M^+ = B^+(-L/2), \quad (120)$$

the propagation through each of the uniform sections k is described by a matrix T_k defined as:

$$T_k = \begin{pmatrix} \cosh(\gamma dz) - j(\sigma/\gamma) \sinh(\gamma dz) & -j(\kappa/\gamma) \sinh(\gamma dz) \\ j(\kappa/\gamma) \sinh(\gamma dz) & \cosh(\gamma dz) + j(\sigma/\gamma) \sinh(\gamma dz) \end{pmatrix}. \quad (121)$$

σ and κ are the local coupling coefficients for the kth section.

The parameter defining the coupling coefficients is

$$\gamma = \sqrt{\kappa^2 - \sigma^2}. \quad (122)$$

The total grating structure is expressed as $T = T_M \cdot T_{M-1} \dots T_K \dots T_1$, so that

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}. \quad (123)$$

The reflection and transmission coefficients are given by $R = T_{21}/T_{11}$ and $T = 1/T_{11}$, respectively.

Here, the number of sections M cannot be made arbitrarily large since the coupled mode theory approximations are no longer valid when a uniform grating section is only a few grating periods long. Hence, the above approximation is valid for $dz \gg \Lambda$ and for which the condition is

$$M \ll (2n_{eff}L) / \lambda_B. \quad (124)$$

The number of sections determines the accuracy of the piecewise-uniform calculation. For most cases, $M > 50$ sections gives sufficient accuracy. The number of sections considered for our simulations are $M = 100$.

Using the coupled mode theory, the required refractive index modulation in the core of the fiber considering the fiber grating parameters are as follows [48]

$$\Delta n_{AC} = \frac{\arctan h(\sqrt{R}) \lambda_B (\lambda_B^2 + \pi^2 d^2 N A^2)}{\pi^3 d^2 L N A^2}. \quad (125)$$

For a grating with uniform index modulation and uniform period, the reflectivity is given by:

$$R(L, \lambda) = \kappa^2 \sinh^2(\gamma L) / (\Delta \beta^2 \sinh^2(\gamma L) + \kappa^2 \cosh^2(\gamma L)), \quad (126)$$

where R is the grating reflectivity as a function of both grating length and input wavelength.

$\Delta \beta$ is the detuning parameter and is given by

$$\Delta \beta = \beta - (\pi/\Lambda). \quad (127)$$

Simplifying, the reflectivity function as a function of grating length and wavelength becomes

$$R(L, \lambda) = \tanh^2(\gamma L). \quad (128)$$

The maximum reflectivity of a uniform FBG is determined by the refractive index modulation

$$R = \tanh^2(kL) = \tanh^2\left(\frac{\pi \delta n_{eff} L}{\lambda_B}\right), \quad (129)$$

where k is the coupling coefficient, L is the grating length, δn_{eff} being the induced refractive index modulation and λ_B is the Bragg wavelength. The grating strength R (or transmission loss value $T = 1 - R$) monotonically increases (decreases) with the rise of the refractive index modulation. The grating strength or δn_{eff} a function of how long the fiber has been exposed to UV illumination. The bandwidth of the Bragg reflection depends on two things: the number N of the grating planes, and the strength of the index modulation δn_{eff} .

A certain proportion of the incident light is reflected at each grating plane; if the Bragg condition is not satisfied, then the wavelets reflected at each subsequent plane become progressively more and more out of phase (“dephased”) ; if however, the grating strength is sufficient, a substantial proportion of the incident power can be reflected before dephasing becomes important. If it is too weak, then very little power is reflected before dephasing sets in. The balance between dephasing length, the physical length of the grating, and the length (at exact phase-matching) needed for substantial reflection determines the bandwidth of the Bragg reflection. A general expression for the approximate full-width-half-maximum (FWHM) bandwidth of a grating is [49]

$$\Delta\lambda / \lambda_B = v\sqrt{(\delta n_{eff} / 2 n_{eff})^2 + (1/N)^2}, \quad (130)$$

where $\Delta\lambda$ is the FWHM bandwidth of Bragg reflection, $v = 1$ for strong gratings (with near 100% reflection) and 0.5 for weak gratings. Multiple reflection to and fro between opposite ends of the grating region cause side-lobes to appear off-resonance; particularly noticeable for strong gratings.

The grating period and the length, together with the strength of the modulation of the refractive index, determine whether the grating has a high or low reflectivity over a wide or narrow range of wavelengths. For normal uniform fiber gratings, by properly choosing the period, length, index modulation amplitude, one can flexibly design and optimize grating reflection or transmission spectra for Bragg filter.

Thus, there are mainly four parameters that characterize a Fiber Bragg Grating:

- The grating strength, which is limited by how much UV-induced refractive index change is achievable in a given fiber. Higher induced index changes increase the bandwidth. Hence, the bandwidth is directly proportional to the induced index change.
- The grating length.
- The refractive index profile which denotes the variation in the effective refractive index of the fiber core.
- The grating period.

7.3.3 Simulation

We present a new approach for passive remote detection of object velocity based on the difference in solar spectrum intensity at the Fraunhofer wavelengths of the solar spectrum. The approach mentioned below is based on the Bragg filter design by Bland-Hawthorn et al. for the atmospheric OH suppression in the paper [50].

The basic idea is that light from the sky is focussed onto the entrance aperture of a single mode, step index photosensitive (Ge or B doped) optical fiber. A complex fiber Bragg grating printed onto the fiber then rejects specific wavelengths corresponding to the narrow Fraunhofer wavelength lines. The reflected light travels back towards the source and leaks out from the entrance aperture. This device acts as a pre-filter ahead of an imager or spectrograph. This reflected light at Fraunhofer wavelength is of prime importance as it gives the solar spectrum intensity which is used for further processing for the passive estimation of relative velocity of the satellite.

In this initial demonstration, the Bragg filter simulations were performed for the 30 Fraunhofer lines in the wavelength interval 500 nm - 600 nm. Our source wavelengths come from the solar flux data mentioned in the paper [51]. The set of 30 Fraunhofer wavelengths in the visible range of 500 nm - 600 nm have been selected from the list of Fraunhofer wavelengths given in the paper.

Based on the theory of Fiber Bragg Gratings mentioned above, the Bragg filter simulations for visible range were performed. For this, we used MATLAB as a modeling software and based on the program written in MATLAB, the gratings were simulated for reflecting at specific wavelengths. The purpose of simulations was first, to achieve the set of design parameters for the fabrication of visible range Bragg gratings. Second, the filter was used to filter out the specific Fraunhofer lines and based on the filtering efficiency obtained for different lines, we could get the output of solar power intensity in photon/sec reflected at these lines. Hence, using this data of output solar intensity and considering the set of relative velocity of satellite from 1000 m/s to 10000 m/s, the filter was simulated for measuring the output solar intensity at Bragg wavelength. Due to velocity of satellite, there occurs a doppler shift in the Fraunhofer line. This shift in center wavelength and new output solar intensity is measured by the filter. Then, both the output solar intensities are compared and we get the net output power which can be used for further calculations of tracking the object velocity.

At the Fraunhofer lines, when the object velocity is zero, the output solar intensity obtained must be low due to the dark Fraunhofer lines. But when the object velocity is considered, the shift in the lines results in a large output solar intensity at the original Bragg wavelength. This increase in output power was successfully measured by the designed Bragg filter.

Initially, the solar spectrum data for plotting of Fraunhofer lines were used in the program. Then for designing the Bragg filter, the coupled mode equations were solved using transfer matrix equations. For this, the theoretical expected filter reflectivity was assumed to be 90%.

The following Table 36 illustrates the initial set of parameters used for the simulations of Bragg filter reflectivity and net output solar intensity.

Table 36. Set of input parameters for simulation

Parameter	Value
Grating Length (L)	1 cm
Expected Reflectivity (Ref)	90 %
Fringe visibility (v)	1
Number of Transfer Matrix sections (M)	100
Core diameter of fiber (d)	4 um
Numerical Aperture (NA)	0.2
Gap length between two Gratings	10 cm
Attenuation loss dB / km	30 dB / km

Using the above input parameters, the gratings were simulated for peak reflection at Bragg wavelength. The obtained results of design parameters are mentioned in the Results section. The simulations were based on the set of equations in the theory and with multiple iterations using Transfer matrix equations, the final set of design parameters were obtained. Then, the losses in the gratings were considered. There are many types of losses inside the optical fiber. Considering the very small length of gratings, we considered only two main losses – Absorption loss and Rayleigh scattering loss. The attenuation loss depends on multiple factors including the type of fiber, range of input wavelength etc. For the visible range, the attenuation was assumed to be 30 dB / km. The Rayleigh scattering loss in optical fiber is given by the equation:

$$\alpha_{scat} = (8\pi^3 n^8 p^2 k T_f \beta) / 3\lambda^4, \quad (131)$$

where n is the refractive index of fiber core = 1.46, p is the photo elastic coefficient of glass = 0.286, β is the isothermal compressibility = 7×10^{-11} , T_f is the fictive temperature = 1950K, λ is the input wavelength, k is the Boltzmann constant = 1.38×10^{-34}

Hence, we get the losses inside the fiber gratings.

Then, the shift in the Fraunhofer lines is given by

$$\text{shift} = \lambda (1 + (v / c)), \quad (132)$$

where v is the object's velocity and c is the speed of light.

Based on the shift, the filter simulations produced the output solar intensity to be used for estimating the range of relative velocity of object.

7.3.4 Results

The results obtained after simulating the grating are as mentioned below. The Table 2 highlights the obtained set of design parameters for the list of 30 wavelengths. For any grating to be designed, the grating length, pitch, index modulation and the number of sections are required. The Table illustrates the corresponding values of each parameter. It is observed that as the Bragg wavelength to be reflected increases, the grating pitch increases, the number of grating sections decreases and required maximum index modulation increases, keeping the grating length constant. Also, mentioned are the Reflectivity, side lobe strength and FWHM. From Table 37, the filter reflectivity varies according to the Bragg wavelength, and so does the side lobe strength with the reflectivity. But the FWHM remains fairly constant to about 0.01 nm.

The grating reflectivity depends on the grating length and as the grating length increases, the reflectivity and side lobe strength also increases. So we have taken the grating length to be 1 cm.

Table 37. Results of Fiber Bragg Grating design parameters. Length = 1 cm, Effective Refractive index = 1.47, $v=1$, Expected Reflectivity = 0.9.

Bragg Wavelength λ_B (nm)	Grating pitch Λ (um)	No of grating sections N	Max ref index modulation δn_{eff}	Max Ref of filter (%)	Grating strength (1/cm)	Side lobe strength (%)	FWHM (nm)
505.160	0.1718	58200	2.9240×10^{-5}	89.98%	1.8184	13.27%	0.0100
509.227	0.1732	57740	2.9475×10^{-5}	65.87%	1.8184	5.68%	0.0102
513.556	0.1747	57250	2.9726×10^{-5}	89.89%	1.8184	13.37%	0.0104
514.080	0.1749	57190	2.9756×10^{-5}	82.29%	1.8184	9.76%	0.0104
516.358	0.1756	56940	2.9888×10^{-5}	76.32%	1.8184	7.94%	0.0105
518.590	0.1764	56700	3.0017×10^{-5}	86.43%	1.8184	11.31%	0.0106
520.451	0.1770	56490	3.0125×10^{-5}	82.34%	1.8184	9.70%	0.0106
520.605	0.1771	56470	3.0134×10^{-5}	1.42%	1.8184	0.0006%	0.0107
523.464	0.1780	56160	3.0299×10^{-5}	59.36%	1.8184	4.69%	0.0108
530.421	0.1804	55430	3.0702×10^{-5}	16.57%	1.8184	0.87%	0.0111
530.845	0.1806	55380	3.0726×10^{-5}	59.66%	1.8184	4.74%	0.0111
532.558	0.1811	55200	3.0825×10^{-5}	86.86%	1.8184	11.66%	0.0111
534.274	0.1817	55030	3.0925×10^{-5}	18.52%	1.8184	0.99%	0.0112
534.905	0.1819	54970	3.0961×10^{-5}	66.54%	1.8184	5.74%	0.0112
536.857	0.1826	54760	3.1074×10^{-5}	65.78%	1.8184	5.69%	0.0113
537.154	0.1827	54730	3.1091×10^{-5}	56.42%	1.8184	4.28%	0.0113
538.342	0.1831	54610	3.1160×10^{-5}	70.62%	1.8184	6.61%	0.0114
539.472	0.1835	54500	3.1226×10^{-5}	89.85%	1.8184	13.39%	0.0114
540.212	0.1837	54420	3.1268×10^{-5}	0.85%	1.8184	0.0004%	0.0115
541.412	0.1841	54300	3.1338×10^{-5}	89.00%	1.8184	12.87%	0.0115
543.592	0.1849	54080	3.1464×10^{-5}	66.68%	1.8184	5.83%	0.0116
545.658	0.1856	53880	3.1584×10^{-5}	34.84%	1.8184	2.13%	0.0117
549.966	0.1871	53460	3.1833×10^{-5}	83.02%	1.8184	9.88%	0.0119
552.924	0.1881	53170	3.2004×10^{-5}	4.93%	1.8184	0.24%	0.0120
557.712	0.1897	52720	3.2281×10^{-5}	52.94%	1.8184	3.84%	0.0122
558.306	0.1899	52660	3.2316×10^{-5}	79.91%	1.8184	8.90%	0.0122
558.886	0.1901	52600	3.2349×10^{-5}	87.47%	1.8184	12.00%	0.0123
560.505	0.1906	52450	3.2443×10^{-5}	88.93%	1.8184	12.83%	0.0123
568.959	0.1935	51670	3.2932×10^{-5}	0.6%	1.8184	0.0002%	0.0127
589.612	0.2005	49860	3.4128×10^{-5}	64.94%	1.8184	5.55%	0.0137

The plots of simulation figures are mentioned below. Fig. 142 shows the Fraunhofer lines in the wavelength range of 500 nm – 600 nm. The plot describes Input wavelength in um vs Input solar flux in photons/s.

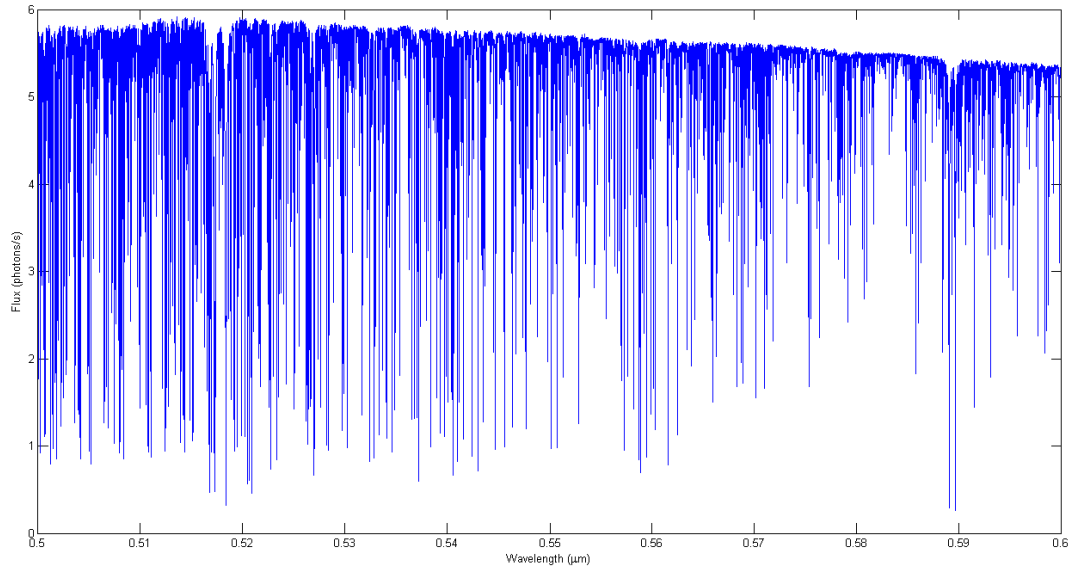


Fig. 142. Fraunhofer lines ranging from 500 - 600 nm

Fig. 143 shows the reflected Bragg wavelengths with the corresponding reflectivity as mentioned in Table 37 above.

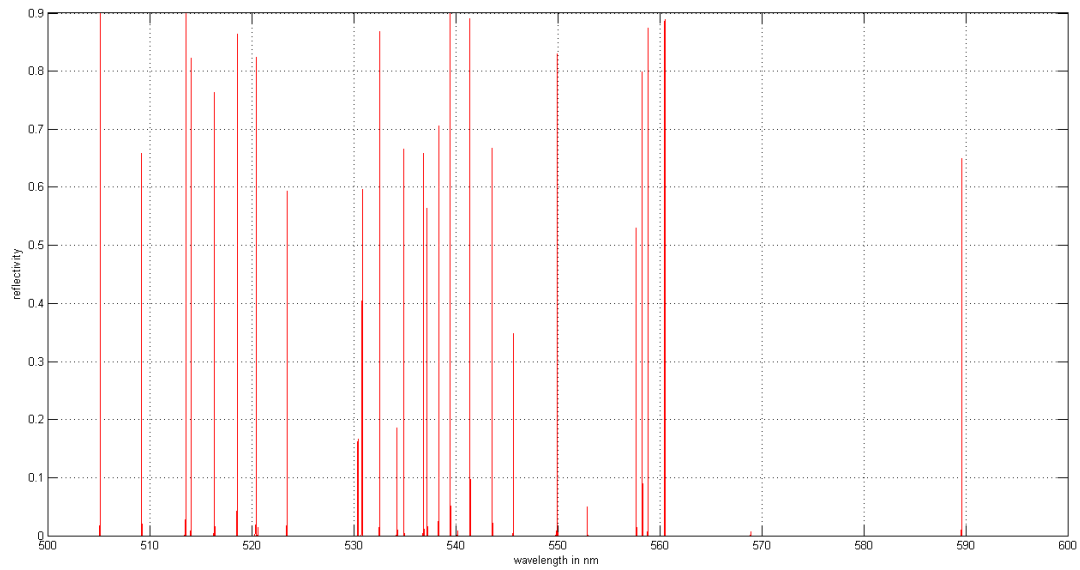


Fig. 143. Bragg Reflected wavelengths

Fig. 144 shows the variation of maximum refractive index modulation with the input wavelength. The plot shows a linear response with the increase in input wavelength. Hence, we get the value of required index modulation for designing a grating for reflecting specific wavelengths.

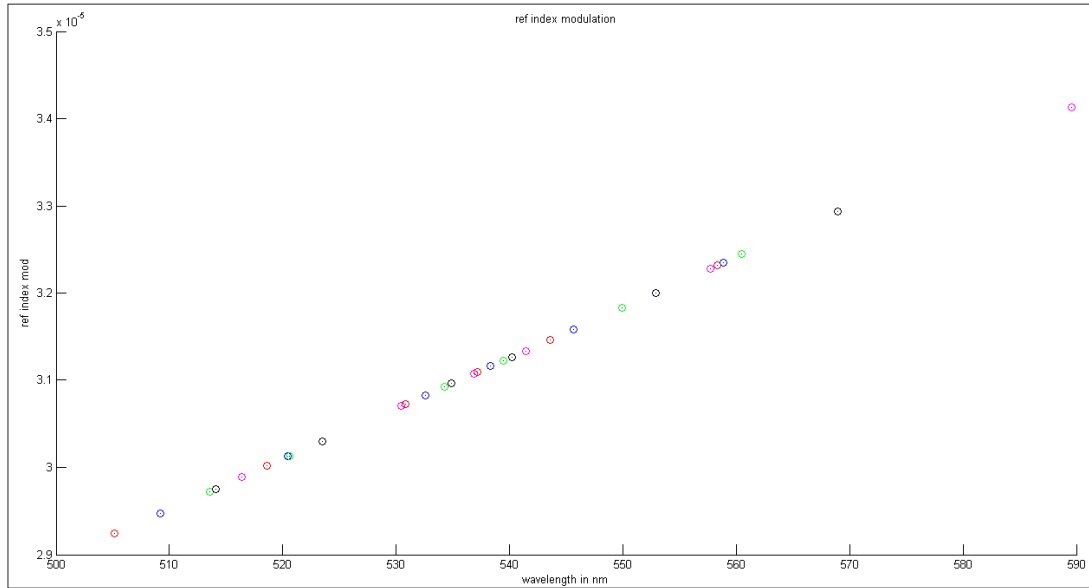


Fig. 144. Refractive index modulation as function of wavelength

Table 38. Results of Bragg filter output solar spectrum (photons/s) for velocity of (a) 1000 m/s (minimum) and (b) 10000 m/s (maximum).

Bragg wavelength (nm)	Input photons/s @ Bragg wavelength	Output photons/s for $v = 1$ km/s (min)	Output photons/s for $v = 10$ km/s (max)	Reflectivity magnitude	Transmission in dB
505.160	0.0732	0.0070	0.3963	0.8998	-19.9819
509.227	0.4552	0.0025	0.0250	0.6587	-9.3384
513.556	0.4871	0.0078	0.0497	0.8989	-19.9021
514.080	0.4393	0.0186	0.0866	0.8229	-15.0366
516.358	0.4902	0.0046	0.0258	0.7632	-12.5141
518.590	0.1722	0.0452	0.2991	0.8643	-17.3479
520.451	0.0522	0.0032	0.3407	0.8234	-15.0582
520.605	0.0470	5.2×10^{-5}	0.0044	0.0142	-0.1238
523.464	0.1408	0.0128	0.2380	0.5936	-7.8203
530.421	0.4633	0.0018	0.0161	0.1657	-1.5739
530.845	0.4132	0.0108	0.0847	0.5966	-7.8861
532.558	0.2981	0.0190	0.2201	0.8686	-17.6263
534.274	0.3669	0.0075	0.0359	0.1852	-1.7792
534.950	0.1215	0.0131	0.2731	0.6654	-9.5100
536.857	0.5221	0.0024	0.0248	0.6578	-9.3154
537.154	0.0586	0.0028	0.2027	0.5642	-7.2133
538.342	0.0870	0.0063	0.2771	0.7062	-10.6402
539.472	0.2892	0.0023	0.2459	0.8985	-19.8741
540.212	0.4808	8.0×10^{-5}	7.0×10^{-4}	0.0085	-0.0743
541.412	0.3898	0.0156	0.1504	0.8900	-19.1686

543.592	0.2383	0.0359	0.2175	0.6668	-9.5455
545.658	0.5247	0.0021	0.0135	0.3484	-3.7199
549.966	0.5512	0.0031	0.0164	0.8302	-15.4003
552.924	0.4651	5.1×10^{-4}	0.0043	0.0493	-0.4392
557.712	0.5009	0.0066	0.0374	0.5294	-6.5472
558.306	0.5498	0.0014	0.0153	0.7991	-13.9402
558.886	0.0703	0.0115	0.3917	0.8747	-18.0443
560.505	0.5584	0.0031	0.0151	0.8893	-19.1163
568.959	0.4997	8.0×10^{-5}	4.1×10^{-4}	0.0060	-0.0525
589.612	0.0232	0.0025	0.1839	0.6494	-9.1041

The obtained data from Table 38 shows the input solar flux in photons/s, and the corresponding output solar flux values for minimum velocity of 1000 m/s to maximum velocity of 10000 m/s.

As the object velocity increases, the shift also increases which results in more light to enter the fiber. So we get more light and more solar flux intensity at 10000 m/s. Thus, the output solar flux depends on the shift in the Fraunhofer lines as well as the filter reflectivity, which is different for different lines.

Fig. 145 below shows the transmission spectrum in dB. The values extend as low as -20 dB.

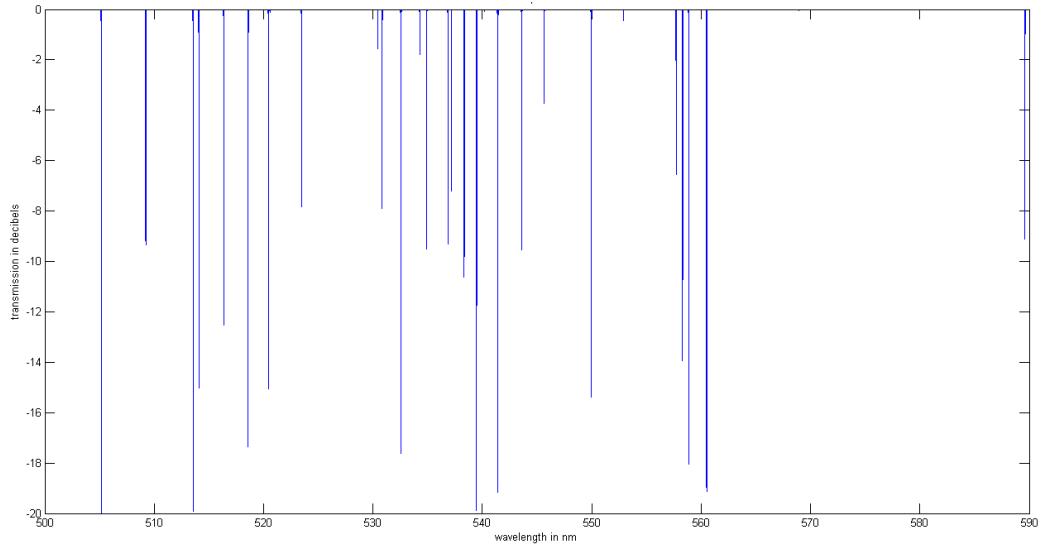


Fig. 145. Transmission in dB.

Fig. 146 shows the plot of output solar flux at $v = 1000$ m/s with respect to input wavelength.

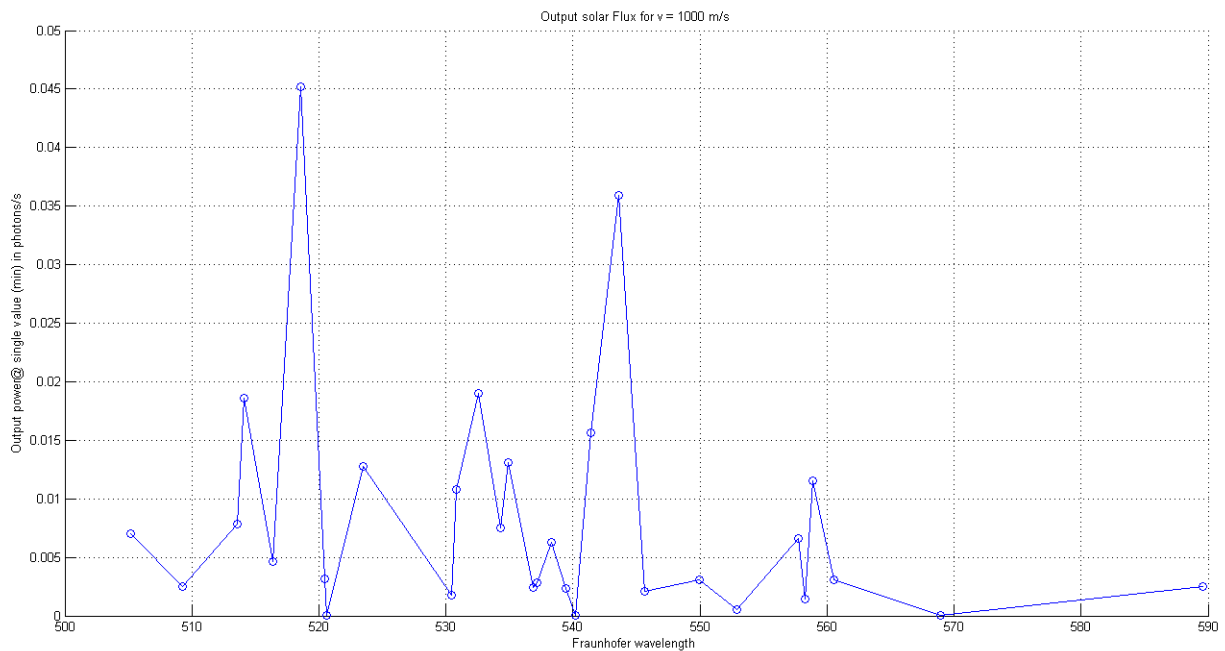


Fig. 146. Input wavelength vs Output solar flux.

Fig. 147 shows the plot of input wavelength vs output solar flux at $v = 10000$ m/s.

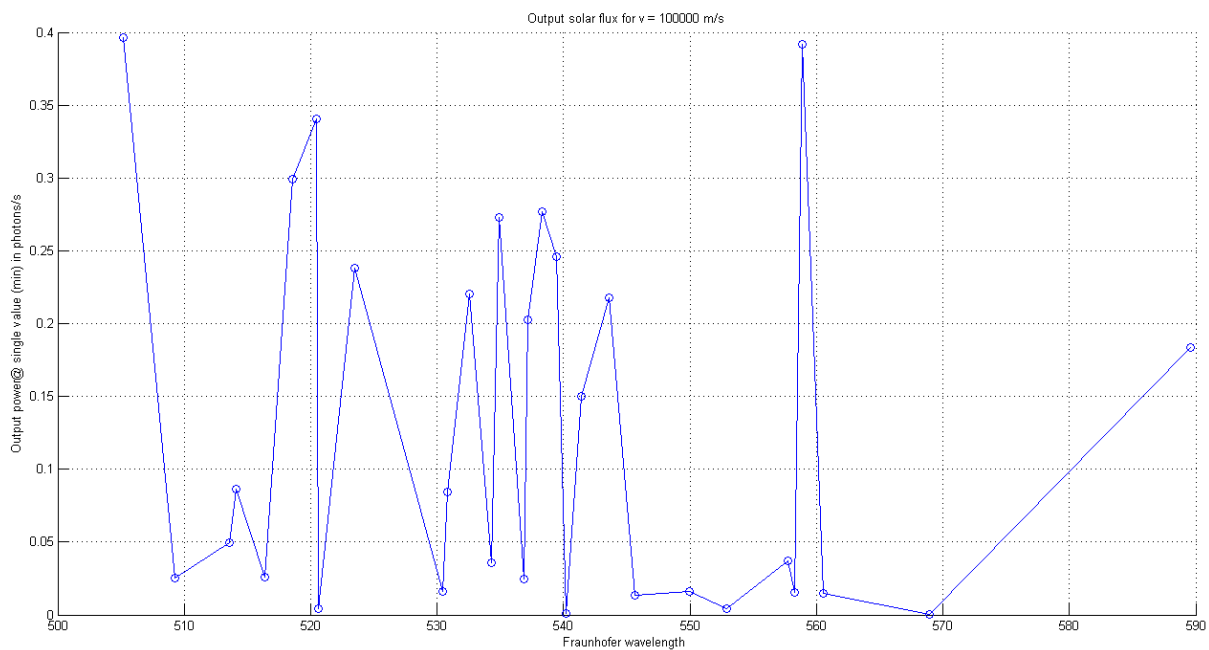


Fig. 147. Input wavelength vs output solar flux

Fig. 148 below shows the non-linear response of input velocity of object vs net output solar flux encompassing all the Bragg wavelengths from 500 – 600 nm.

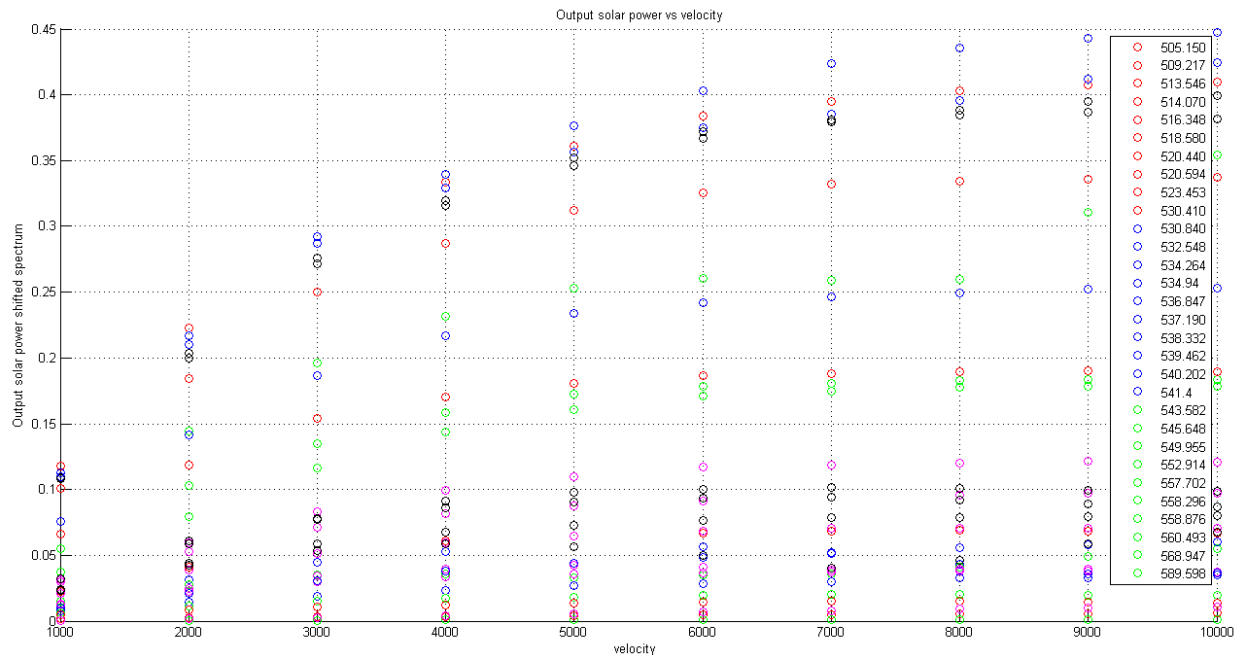


Fig. 148. Input velocity vs output solar flux

7.3.5 Summary

The visible range Bragg filter simulations were performed and it showed that it is possible to design the Bragg filter under the set of design parameters. The Reflectivity as large as 90% were obtained, though it is possible to obtain even higher reflectivity, which would lead to increase in grating length and also increase in side lobe strength which is not desirable.

A key insight we obtain from the simulation results is the similarity between the FWHM and the shift in the Fraunhofer lines. The increment in the FWHM value with the Bragg wavelengths is very small, it actually remains constant near to 0.01 nm. Also, the shift in the Fraunhofer lines due to input velocity is about 0.017 nm. This indicates that the Bragg filter gives the peak reflectivity in the FWHM wavelength range. And for obtaining maximum output flux, the shift in Fraunhofer lines has to be equal to or greater than FWHM. As the simulation results provides us the required values, thus the filter is designed properly to work at peak reflectivity within the FWHM range and hence filter out the solar flux due to shift in object velocity.

Thus, the successful study and simulation of visible Bragg gratings was performed and the results obtained have given the required object velocity as expected. This was confirmed after comparing with other optical filter called a Solc filter. The next step would be the practical implementation of the filter and to conduct more research in visible grating designs.

The filter design parameters were compared with the paper [42] and based on the comparisons, we get that for designing of 1 cm long grating, the writing power of laser would be about 35 – 140 mW. While the paper [43] gives the data of about 4 - 20 mW of writing power required using holographic based writing technique. Besides, the fabrication conditions were specified for actual designing of visible range filter. As reported by Becker, *et al.* [52], the interferometric set-up technique can be used for inscription of visible gratings. As only simulations are specified in this report, one of the future work would be the fabrication of visible range Bragg Gratings for the characterization and verification of the designing process. Various fabrication techniques are

specified in the literature and it will be relevant to make a thorough study of the most efficient fabrication technique for visible gratings.

7.4 Volume Phase Holographic Grating

To further increase the strength of the correlation filter, a design using a volume phase hologram (VPH) for high efficiency is underway. Volume phase gratings are constructed from a layer of dichromated gelatin (DCG) or other photosensitive material sandwiched between glass plates, as shown in Fig. 149. Exposure to a UV interference pattern creates a sinusoidal variation of refractive index within the DCG.

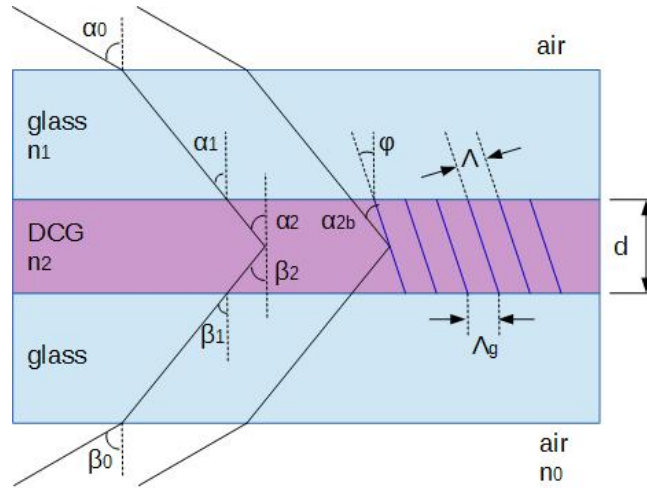


Fig. 149. Key parameters for a volume phase hologram.

Diffraction in a VPH follows the same equation as for surface gratings, such that

$$\sin(\alpha_0) - \sin(\beta_0) = \frac{m\lambda}{\Lambda_g}, \quad (133)$$

where α_0 is the incident beam angle, β_0 is the diffracted beam angle, m is the diffraction order, λ is the wavelength, and Λ_g is the spacing between modulation planes (fringes) at the surface of the DCG. However, reflection at the fringe planes insures that diffraction occurs only for beams that satisfy the Bragg condition. When the fringes are orthogonal to the surface, the Bragg condition is

$$\sin(\theta_B) = \frac{m\lambda_B}{2\Lambda}, \quad (134)$$

where θ_B is the incident angle corresponding to the Bragg angle, m is the Bragg order, λ_B is the Bragg wavelength, and Λ is the perpendicular distance between fringe planes within the DCG. Diffraction will occur for a small bandwidth of wavelengths and angles near the Bragg condition. The full-width, half-maximum (FWHM) spectral bandwidth can be approximated as [53]

$$\Delta\lambda_{FWHM} \approx \lambda\Lambda / d \tan(\alpha_{2B}), \quad (135)$$

where α_{2B} is the incident angle between the incoming beam and the fringe plane. The FWHM angular bandwidth is approximately [53]

$$\Delta\theta_{FWHM} \approx \Lambda / d, \quad (136)$$

where Λ is the perpendicular distance between fringe planes within the DCG. Rearranging (133) and differentiating the diffracted angle with respect to wavelength yields an expression for angular dispersion as a function of wavelength:

$$\frac{d\beta_0}{d\lambda} = -\frac{m}{\Lambda_g} \left[1 - \sin^2(\alpha_0) - \frac{m^2 \lambda^2}{\Lambda_g^2} + \frac{2m\lambda}{\Lambda_g} \sin(\alpha_0) \right]^{-1/2}. \quad (137)$$

Our lab has obtained a visible wavelength VPG from Wasatch Photonics with a fringe spacing of 1800 mm^{-1} and a Bragg wavelength of 532 nm. Substituting the first-order parameters of this grating into equations (134)-(137), we obtain the results shown in Table 39.

Table 39. First order parameters at the Bragg condition for the VPH grating from Wasatch Photonics.

Parameter	Symbol	Value	Units
Grating period	Λ_g	1/1800	mm
Fringe plane angle	φ	0	deg
Fringe plane spacing	Λ	1/1800	mm
Bragg wavelength	λ_B	532	nm
Bragg angle	θ_B	28.61	deg
Thickness	d	3	μm
Spectral bandwidth	$\Delta\lambda_{FWHM}$	180	nm
Angular bandwidth	$\Delta\theta_{FWHM}$	10.6	deg
Dispersion	$d\beta_0/d\lambda$	0.117	deg/nm

To determine the “off Bragg” behavior of the VPH grating requires use of coupled wave theory. Kogelnik’s 1969 paper [54] uses a coupled wave analysis to derive five parameters for use in the characterization of VPH gratings. The first parameter Γ quantifies the “dephasing” of the incident wave from the Bragg condition. It is calculated as

$$\Gamma = \frac{2\pi \Delta\alpha \cos(\pi/2 + \alpha - \varphi)}{\Lambda} - \frac{\pi \Delta\lambda}{n_2 \Lambda^2}, \quad (138)$$

where $\Delta\alpha$ is the difference (in radians) between the Bragg angle and angle of incidence, $\Delta\lambda$ is the difference (in nanometers) between the Bragg wavelength and incident wavelength, φ is the slant angle of the fringe planes with respect to grating normal, Λ is the perpendicular spacing between fringe planes, and n_2 is the bulk refractive index of the DCG layer.

Next, Kogelnik defines two “obliquity factors,”

$$c_R = \cos \alpha, \text{ and} \quad (139)$$

$$c_S = \cos \alpha - \frac{\lambda}{n_2 \Lambda} \cos \varphi, \quad (140)$$

where c_R and c_S represent coefficients of the coupled wave equations for the reference (incident) beam and signal (diffracted) beam, respectively.

Kogelnik then derives a general expression for the diffracted wave in a lossless medium,

$$S = \frac{-j\sqrt{c_R/c_S} e^{-j\xi} \sin(\sqrt{\nu^2 + \xi^2})}{\sqrt{1 + \xi^2/\nu^2}}. \quad (141)$$

The expression in (141) introduces the final two Kogelnik parameters, ν and ξ , defined as

$$\nu = \frac{\pi \Delta n d}{\lambda \sqrt{c_R c_S}}, \text{ and} \quad (142)$$

$$\xi = \frac{\Gamma d}{2c_S}, \quad (143)$$

where Δn is the modulation amplitude of the refractive index. Now, the first order diffracted beam efficiency for both S-polarized and P-polarized waves can be expressed as

$$\eta_S = \frac{\sin^2(\sqrt{\nu^2 + \xi^2})}{1 + \xi^2/\nu^2}, \text{ and} \quad (144)$$

$$\eta_P = \frac{\sin^2(\sqrt{\nu^2 \cos^2[\pi + 2\alpha - 2\varphi] + \xi^2})}{1 + \xi^2/(\nu^2 \cos^2[\pi + 2\alpha - 2\varphi])}. \quad (145)$$

Using expressions (138) through (145), the following plots were generated. Fig. 150 shows the first order diffraction efficiency (as defined in [54]) for S-polarized, P-polarized, and unpolarized waves. Similarly, Fig. 151 shows diffraction efficiency as a function of incidence angle. Finally, Fig. 152 plots diffraction efficiencies at the Bragg condition as a function of both grating strength (amplitude of the index modulation) and thickness of the DCG material.

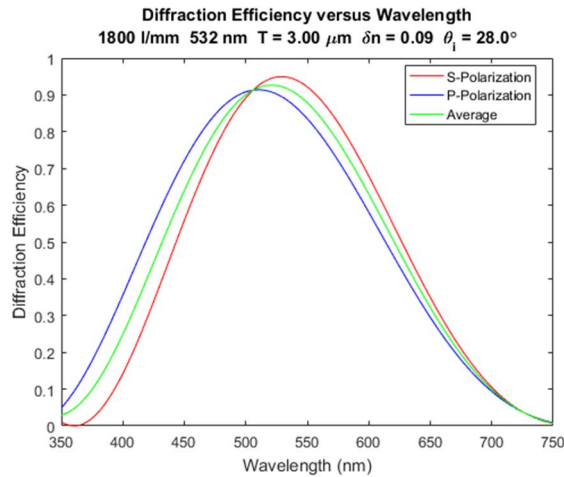


Fig. 150. First order diffraction efficiency versus wavelength. These results include an additional 3% loss in both S and P waves due to absorption.

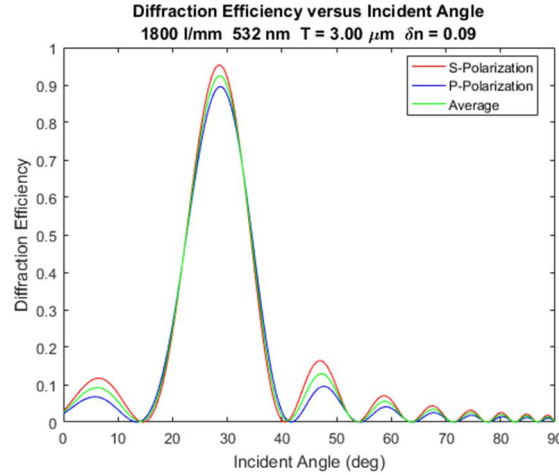


Fig. 151. First order diffraction efficiency versus incident angle. These results include an additional 3% loss in both S and P waves due to absorption.

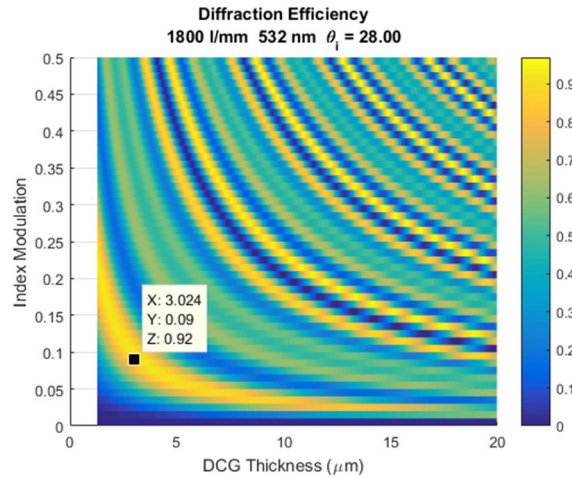


Fig. 152. First order diffraction efficiency versus index modulation (vertical axis) and DCG thickness (horizontal axis). Color maps to the average efficiency of unpolarized light. These results include an additional 3% loss due to absorption. The marker indicates the design parameters for the VPH grating described in the previous section.

7.4.1 VPH Correlator Design

The design of a correlator with VPH grating is shown in Fig. 153. This design uses a liquid-crystal based spatial light modulator (SLM) to separate the in-band and out-of-band signal power into orthogonal polarization states. From the left, light from the telescope is collimated and travels through a bandpass filter. The VPH diffracts the light into its spectral components. The diffracted light is focused by a relay lens onto the SLM, which is programmed with a spatial pattern representing Fraunhofer lines in the unshifted solar spectrum. The portions of the diffracted beam falling within the absorption line areas experience a relative polarization shift of 90° with respect to the out-of-band light. The SLM reflects all of the light back through the relay lens and onto the VPH, which reintegrates it into a single beam. A polarization beamsplitter enables separation of the two polarization components, which can then be measured separately by the detector.

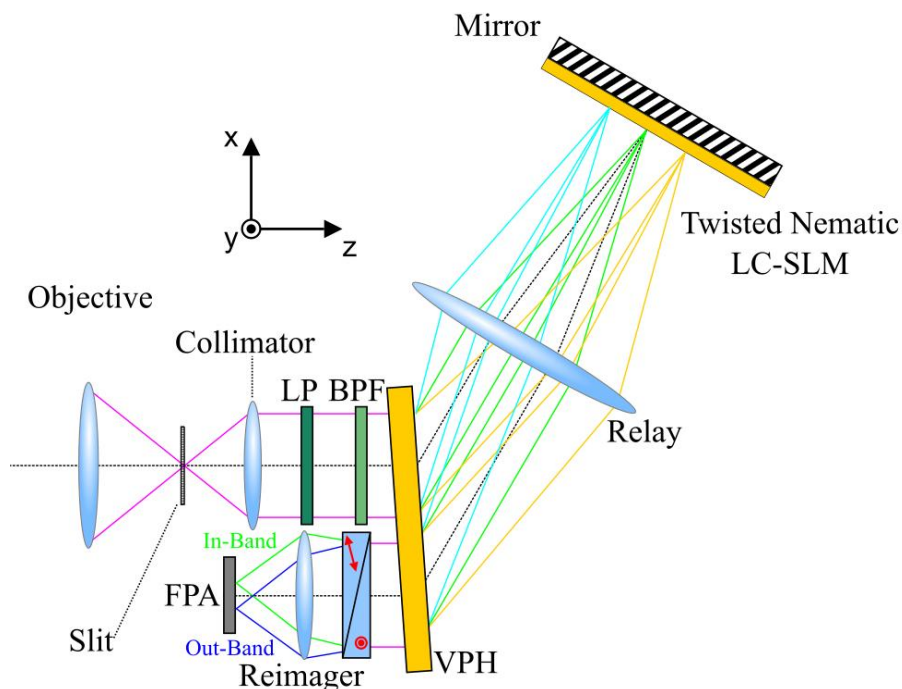


Fig. 153. Illustration of the VPH correlator.

7.4.2 VPH Correlator Simulation

The current VPH correlator design has been simulated assuming a 150 mm diameter aperture, 1800 mm focal length Cassegrain telescope. Other components of the design are assumed to be as specified in Table 40. The optical layout is shown in Fig. 154.

Table 40. Component specifications used in radiometric simulation of the VPH correlator.

Component	Specifications
Collimator lens	$f = 200$ mm
Relay lens	$f = 200$ mm
VPH grating	Wasatch Photonics WP-1800/532-50.8 $G = 1800$ lines/mm, $\lambda_B = 532$ nm
SLM	Santec SLM-100 Twisted nematic, reflective

The solar radiance for the wavelength range of 504-526 nm is simulated based on the data provided by Wallace, et al. [23]. Simulations were run assuming 5 minute integration time on a 0.15 m, 0.9 m, and 3.5 m telescope. The results are shown in Table 41.

Table 41. Summary of simulation results.

Telescope Aperture (m)	Doppler Sensitivity (m/s)
0.15	± 200
0.90	± 40
3.50	± 10

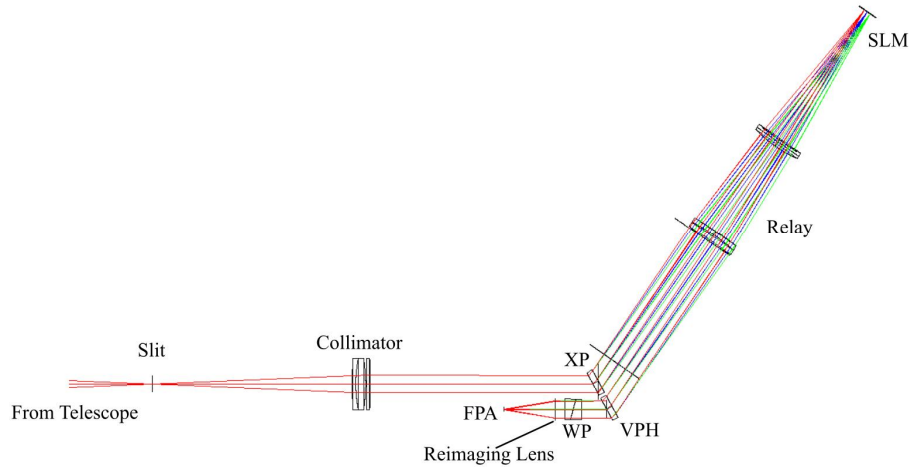


Fig. 154. Zemax layout of the VPH correlator.

Additional simulation output is provided Fig. 155, which shows (a) the in-band and out-of-band flux components of the measured signal, (b) the ratio of in-band to out-of-band power plotted against Doppler velocity, and (c) the measured vs. true Doppler velocity.

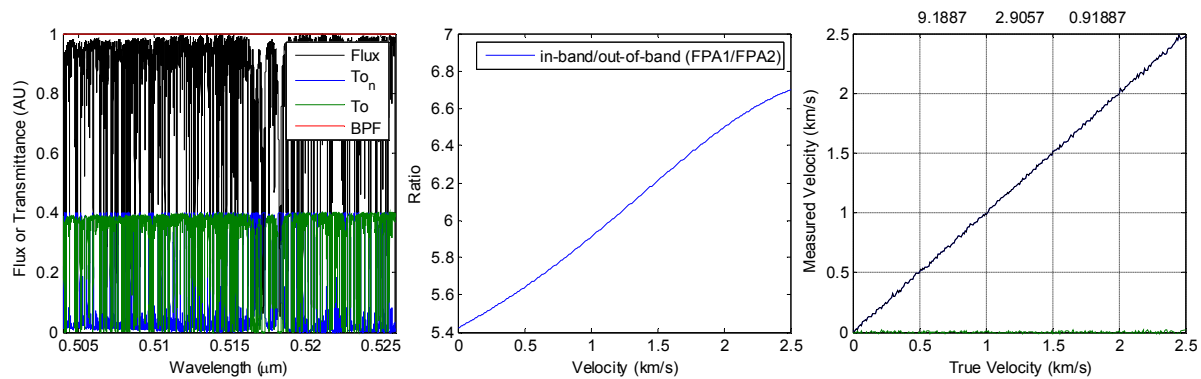


Fig. 155. Results of the radiometric simulation, showing (a) in-band, out-of-band, and total flux; (b) ratio of in-band to out-of-band power vs. Doppler velocity; and (c) the measured vs. true Doppler velocity.

8.0 Direct Correlation Spectrometer

A key aspect of the instrument's design is the direct optical correlation of the received signal with the known, unshifted, Fraunhofer absorption lines. The ratio of in-band (within the bandwidth of an absorption line) to out-of-band signal power provides a measurement of the amount of Doppler shift. The investigation of several different optical filters for use in the correlator – as described in the previous section – a filter based on a volume phase holographic grating was chosen for the final design. Advantages of this design includes

- Improved SNR with high spectral resolution (~ 0.025 nm).
- Enables in-band and out-of-band light to be directed into orthogonal polarization states.
- Simultaneous detection of 155 deep Fraunhofer lines spanning 520-540 nm.

One major component of the correlator is a spatial light modulator (SLM) used to map a shifted spectral image to a reference spectrum. Because of its importance to the design, a brief review of SLM technology is provided in the next section.

8.1 *Spatial Light Modulators*

A spatial light modulator is a device that can modify the intensity, phase, and/or polarization of incident light over a two-dimensional optical wavefront. The active area of a typical device is divided into horizontal and vertical pixels, each having an individual birefringence controlled by a programmed operation $T(x, y)$. This results in modulation of the wavefront properties across the surface of the SLM [55].

Depending on the construction and intended application, SLMs can be either transmissive or reflective. The physical method of modulating the pixel birefringence can be mechanical, chemical, or optical [55]. We consider only optical-mode devices, in which pixels are electrically programmed, because they are the most common type and are employed in the present work. In this type of device, each pixel is composed of a liquid crystal (LC) cell, illustrated in Fig. 156.

The liquid crystal molecules are bipolar, so that their orientation can be controlled with an applied electric field. The molecules are also birefringent, with the slow optical axis oriented along the polar axis of the molecule. With no electric field applied, the molecules at either end of the cell self-align with crossed layers of a highly polar polyimide film. This self-alignment creates a twisted stack of LC molecules, as shown in Fig. 156(a). Light entering the cell experiences a rotation of its polarization state as it travels along the stack. When an external field is applied between a pair of transparent electrodes, the molecules in the cell line up end-to-end, as shown in Fig. 156(b). In this configuration, the optical axis of the cell is parallel to the direction of wave propagation, and the wavefront polarization is unaffected [56].

Light passing through each pixel of the SLM experiences a different index of refraction depending on the voltage applied to each cell. Programming the cells with a specific voltage pattern creates a phase modulation across the pixel array. When polarizing filters are used in the path of the input and output wavefronts, the polarity or intensity of the output can be controlled in the same way. This makes SLMs useful for a variety of optical tasks, including beam shaping, pulse shaping, and aberration correction [57].

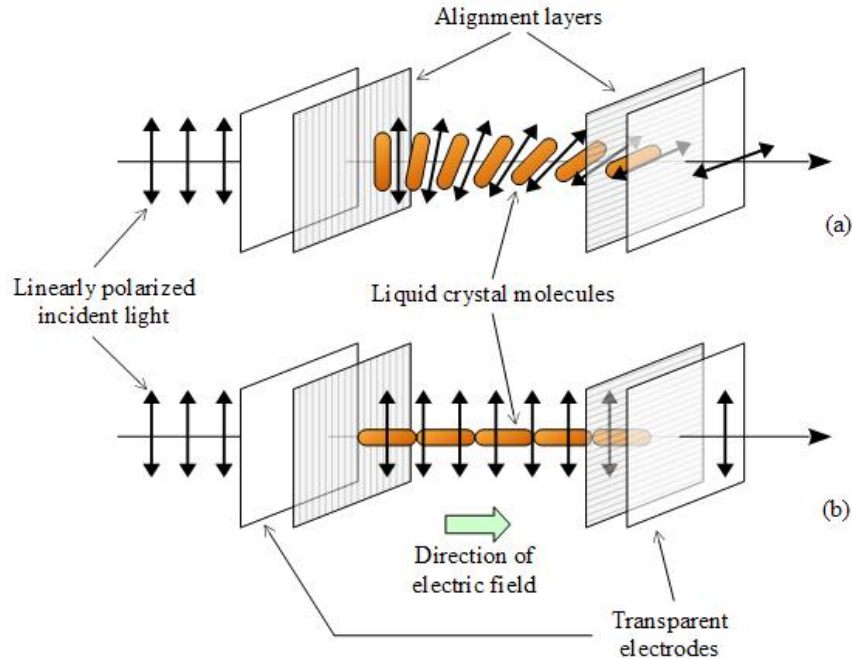


Fig. 156. Transmissive liquid crystal (LC) cell. In the absence of an electric field, the LC molecules at either end orient themselves with the polyimide alignment layers. (a) The orientation of alignment layers is orthogonal, causing the polar LC molecules to form a twisted helix structure, rotating the polarization of the incoming light. (b) When a voltage is applied to the electrodes, the molecules align with the electric field, and the wavefront is unrotated.

Although Fig. 156 illustrates a transmissive cell, the preceding description also applies to reflective cells, which are constructed with a dielectric mirror at one end. The incoming wavefront is reflected by the mirror, experiencing additional phase shift.

8.2 Design of the Optical Correlator

When programmed with a reference pattern of interest, an SLM can be used as a photomask for an optical correlator. If the SLM is placed at an image plane, the portions of the image coincident with the pattern on the SLM will experience a phase shift relative to the rest of the image. When the incoming light is linearly polarized, it will experience a rotation of polarization state as described above. Our correlator design makes use of this polarization difference to measure the wavelength change caused by Doppler shift.

The design of the correlator is illustrated in Fig. 157. First, light from a telescope is focused onto a slit. This intermediate image is collimated, polarized, and dispersed by a volume-phase holographic (VPH) grating. The VPH is a high-efficiency diffraction element with high dispersion and an effective grating frequency of 1800 lines/mm. The diffracted light is relayed through a lens, and an image of the (Doppler shifted) spectrum is focused onto a liquid crystal spatial light modulator.

The SLM can be programmed with any desired spectral image as a correlation reference. Fig. 158(a) illustrates a sample pattern for wavelengths in the range 533.8-535.2 nm. The black line illustrates an unshifted reference spectrum [23]. The red line represents a pattern on the SLM. Pixels adjacent to absorption lines are programmed to induce a polar rotation of 90 degrees. Fig.

158(b) shows a close-up of the mask pattern, which more clearly illustrates the concept behind the correlator. If the spectral image slides to the right due to Doppler shift, the “in-band” light energy (spatially overlapping the SLM pattern) is subject to polar rotation. Conversely, if the image shifts to the left, the “out-of-band” light (the rest of the spectral continuum) is unaffected.

Next, the light reflected by the SLM is recombined by the same VPH dispersion grating. A birefringent Wollaston prism (WP) splits the light into two orthogonal polarization states. These two beams form separate images on the camera, representing the in-band and out-of-band energy. The ratio of the optical powers in each image is related to the Doppler shift observed at the telescope. This power ratio is plotted in Fig. 159 as a function of simulated wavelength shifts in the range -0.005 to 0.005 nm. Fitting a polynomial to this curve allows the observed wavelength shift to be directly determined from the measured power ratio.

The main advantage of this method is that the light associated with many lines can be integrated for a SNR improvement of more than an order of magnitude over 20 nm of spectral bandwidth.

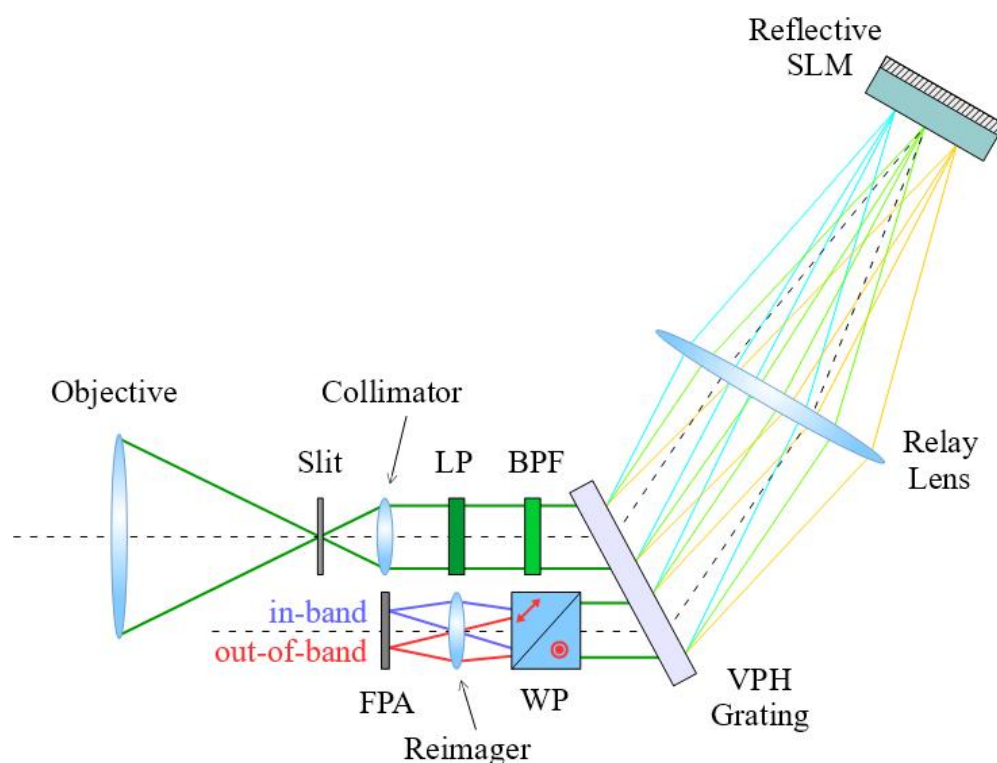


Fig. 157. High level concept design of the optical correlator. Light is dispersed by a hologram (VPH) onto a spatial light modulator (SLM). The SLM can be programmed to match the absorption lines in a dispersed solar spectrum.

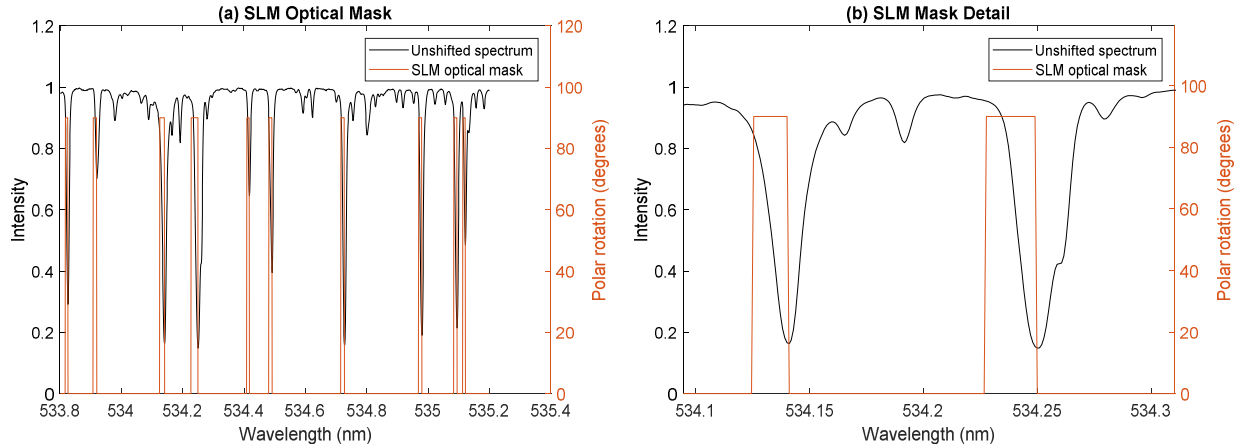


Fig. 158. (a) Conceptual model of the photomask pattern used on the SLM. (b) A detail of the pattern, showing the relationship between wavelength of the input and phase modulation introduced by the SLM.

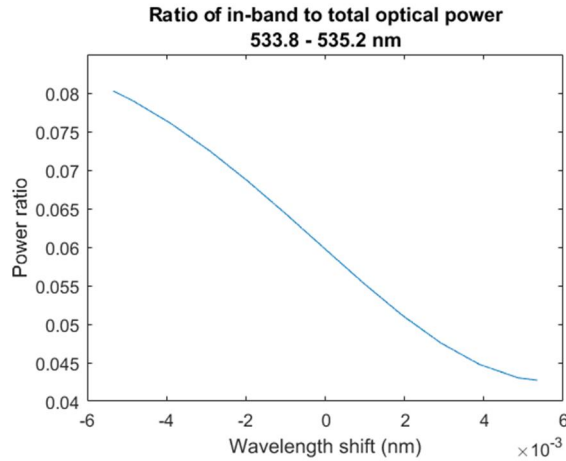


Fig. 159. Ratio of in-band optical power to total power as a function of simulated wavelength shifts.

8.2.1 Final Design Schematic

The final design incorporated additional optics for folding the optical path on the benchtop. A view of this design is illustrated in Fig. 160 in which light is first incident onto a telescope. This focuses light onto a entrance slit, which is collimated by a collimator lens. Light is then dispersed by a VPH into a set of fold mirrors (M1 and M2), before being incident onto a 400 mm focal length, 75 mm diameter relay lens. This focuses the dispersed light onto the reflective SLM, where it first passes through an achromatic quarter wave plate (AQWP) to rotate the SLM's eigenvectors. Light is then reflected back towards the relay, where it is collimated and folded by mirrors M2 and M1, respectively. An image of the VPH is then re-formed on the VPH, allowing the dispersion to be compensated. Finally, the orthogonal polarizations of each beam are split into an in-band (IB) and out-of-band (OB) component, before being reflected by a third fold mirror (M3). A final linear polarizer (L2) is used to reduce the intensity of the OB light relative to the weaker IB light, and a disperser with a spatial period of approximately 8 microns is used to create a small low spectral resolution spectrum on the FPA. This spectrum can be used to increase stability against low spectral resolution changes in the target spectra, and may provide the potential for single-frame correlation measurements in the future. Finally, it should be mentioned that mirror M2 is attached to a piezo transducer to enable the spectral passband to be swept across the SLM, similar to how

conventional direct correlation spectrometer measurements have been made with non-polarization sensitive masks [58].

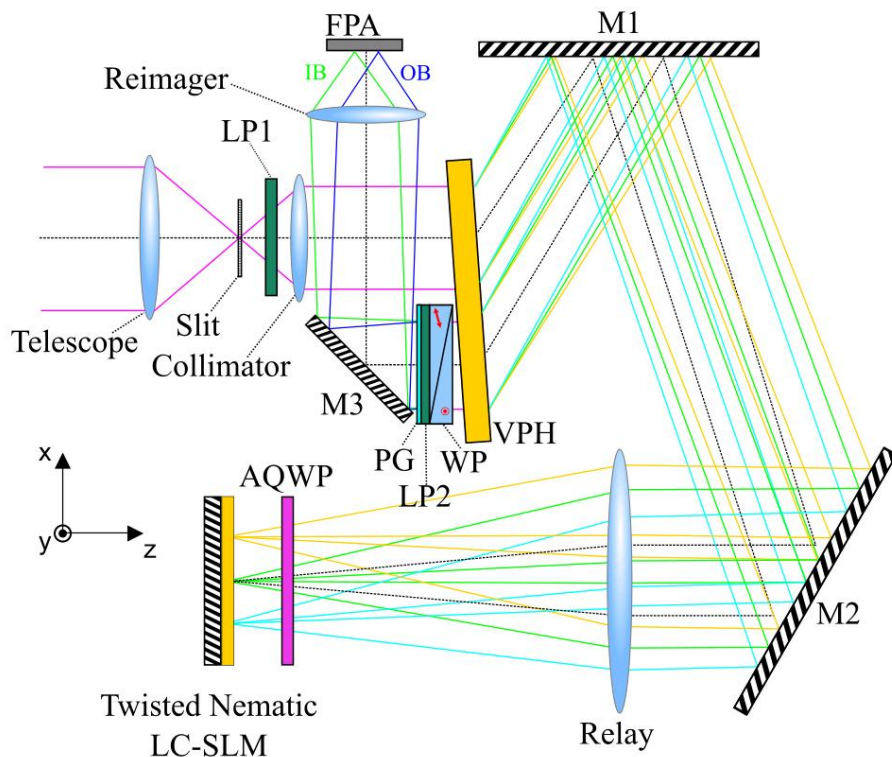


Fig. 160. System schematic for the final implementation.

8.3 Selection of Passband

A key aspect of the instrument design is correlation of the received signal with the known, unshifted, Fraunhofer absorption lines. The ratio of in-band (within the bandwidth of an absorption line) to out-of-band signal power provides a measurement of the amount of Doppler shift. The density of solar Fraunhofer lines and their profiles vary with the portion of the spectrum being observed. Because the SNR of our instrument depends partly upon the number of lines within its passband, as well as the profiles and spacing of those lines, it is important to select a spectral region that provides the best performance. In our previous simulations, Mg I, H-alpha, and Na I absorption lines were chosen. However, upon further study, it is unlikely that Na can be used due to atmospheric skyglow. This is unfortunate, since this band performs significantly better than any of the other Fraunhofer lines studied to date (this is likely caused by the Doppler bandwidth of these lines being well matched to the 0-10 km/s range of our investigation, in addition to the fact that both lines can be observed simultaneously with one filter). Therefore, the spectral region near a potassium (K I) line at ~532 nm has been selected as a possible alternative.

To test the new spectral selection, radiometric simulations were run using 1 nm passbands in the range of 517 nm - 545 nm. The simulations were based on the following assumptions:

- Target object in geosynchronous orbit
- Area of target 1 m²
- Lambertian reflectivity of 90%

- Telescope aperture 8-inch diameter
- Exposure time 10 minutes

The results of all simulations are illustrated in Fig. 161, which shows the error in the target's measured radial velocity as a function of the instrument's center frequency. For radial velocities of -500 to 2500 m/s, a 1 nm spectral passband centered at 519.5 nm seems best. This is most likely due to the relatively substantial number of narrow iron lines within that region. In contrast, the strong magnesium lines near 517 nm yield much larger errors, probably because of their broad profiles.

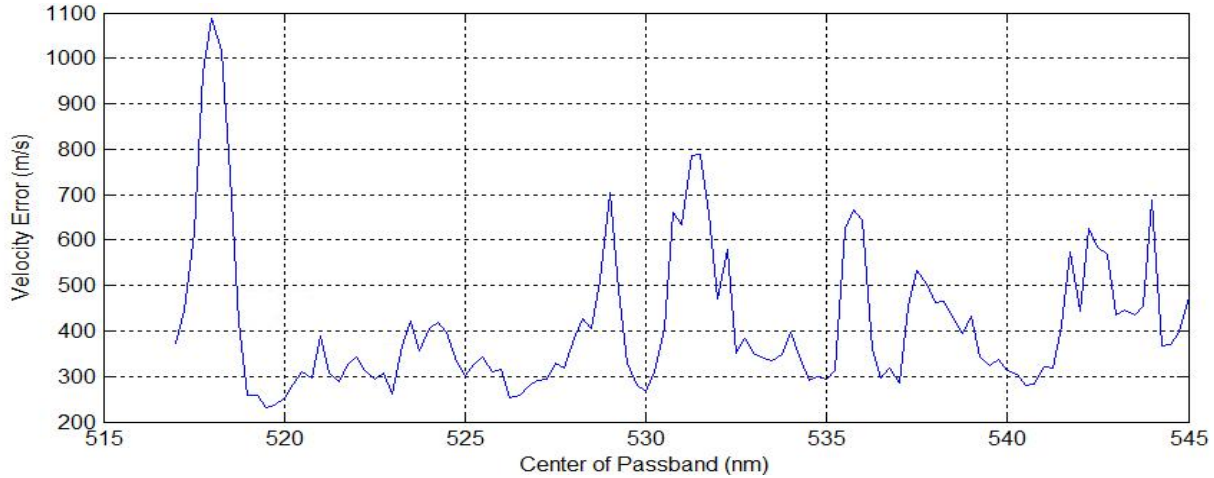


Fig. 161. Aggregated results of passband selection simulations.

Fig. 162 shows the best performing passband in detail. The black line represents the solar flux [23] within the range of 519-520 nm. The red line represents an ideal band-pass filter used to extract the 1 nm-wide spectral band. The green and blue lines represent the instrument's total flux transmittance and its stopband, respectively. The narrow, closely-spaced iron lines provide good reference markers for measurement of wavelength shift.

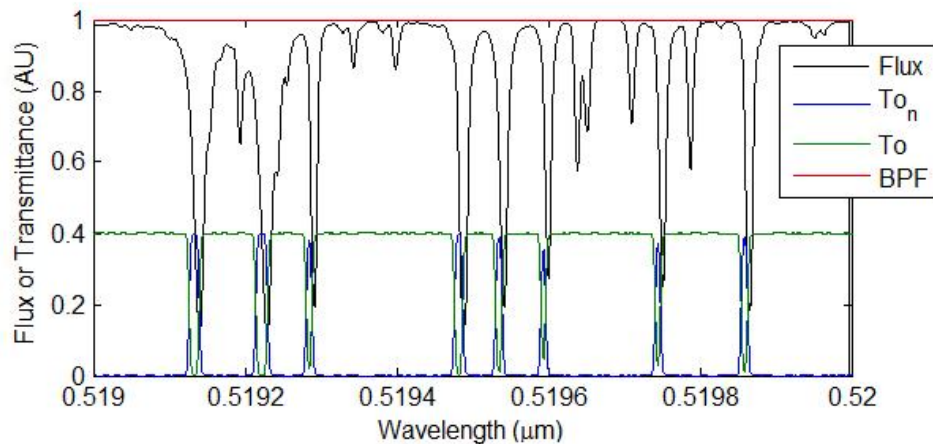


Fig. 162. Characteristics of the best performing passband. The black line represents the solar flux. The green and blue lines represent the instrument's radiometric transmittance and stopband, respectively.

The estimated accuracy of the velocity measurement is shown in Fig. 163. The black line charts the measured vs. actual velocity, while the green line represents the error value. The overall trend of the measured results coincides linearly with the actual velocities.

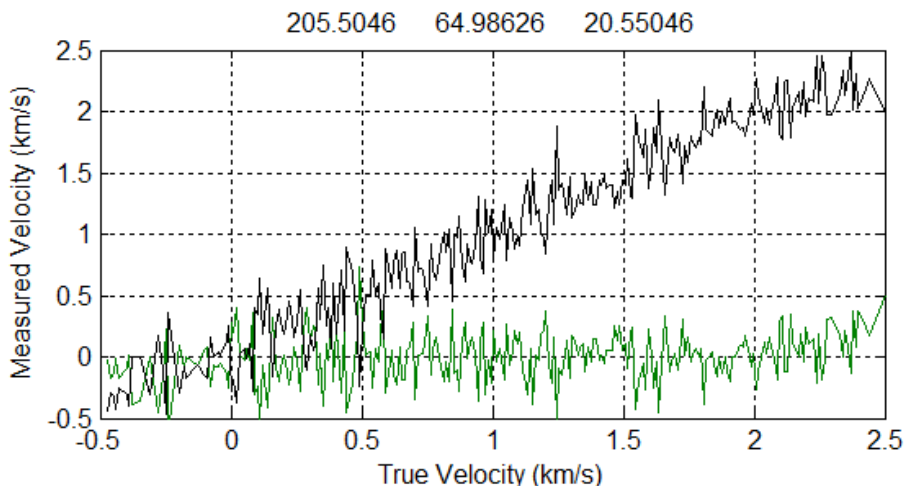


Fig. 163. Measured velocity (black) vs. true velocity and the associated error (green).

Finally, Fig. 164 shows the ratio of in-band to out-of-band optical power with respect to the observed radial velocity of the target. A curve like this one will be used to map data from the image sensor to measured Doppler shift of the reflected sunlight.

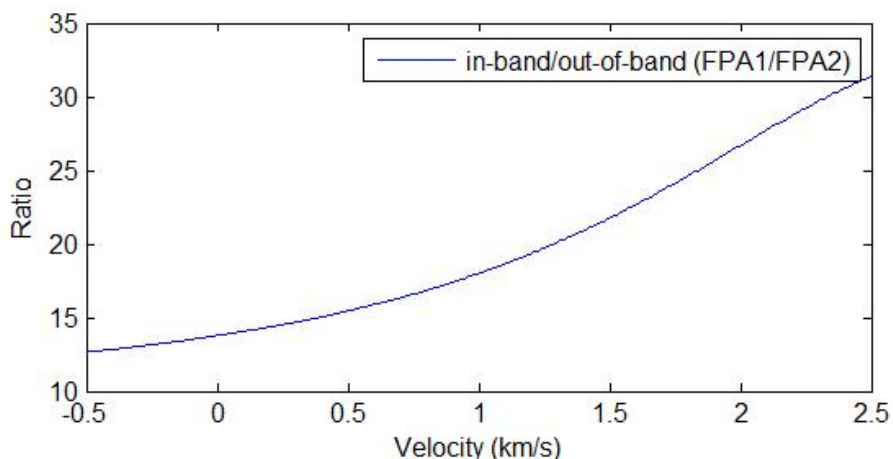


Fig. 164. In-band/out-of-band power ratio vs. observed radial velocity of the target.

9.0 Assembly and Calibration

The bill of materials for the prototype instrument is listed in Table 42. Additional details about the imaging sensor and spatial light modulator are shown in Table 43 and Table 44, respectively.

Table 42. Bill of materials.

Item	Description	Qty.	Price	Total	Source
SLM-100	Spatial Light Modulator	1	12500	12500	Santec
WP-1800/532-25.4	Volume Phase Hologram	1	800	800	Wasatch Photonics
Apogee A694	CCD Camera	1	3281	3281	Allied Scientific Pro
KPC064AR.14	Newport Lens (Main Barlow)	1	81	81	Newport
SLB-50.8B-200NM	Optosigma Lens (Alternative Barlow)	1	134	134	Optosigma
SLB-50.8-1000PM	Optosigma Lens (Collimator / Reimgr)	3	86	257	Optosigma
49665	Aspherized Achromat (Objective)	1	115	115	Edmund Optics
49287	Edmund Optics Achromat (Collimator)	1	140	140	Edmund Optics
48249	Edmund Optics PCX (Reimgr)	1	68	68	Edmund Optics
49289	Edmund Optics Achromat (Reimgr)	1	140	140	Edmund Optics
5GCT0	Foam Sheet, 220 Poly, 1 x 24 x 18"	6	19	116	Grainger
AC254-050-A	50 mm Achromat Lens (FPA Obj.)	1	73	73	Thorlabs
LPVISE100-A	Polarizing Filter	2	89	178	Thorlabs
MBC2412	Water-Cooled Breadboard	1	814	814	Thorlabs
VA100	Adjustable Mechanical Slit	1	248	248	Thorlabs
RS3	1" Pillar Post	6	24	141	Thorlabs
RSH3	1" Post Holder with Flexure Lock	6	34	204	Thorlabs
XT95P11	Drop-On Rail Carriage for 95 mm Rails	5	80	400	Thorlabs
XT95SP-1000	95 mm One-Sided Construction Rail	1	201	201	Thorlabs
RSHT3	1" Post Holder with Flexure Lock	1	34	34	Thorlabs
LMR2	Lens Mount with Retaining Ring	5	25	123	Thorlabs
PF175	Clamping Fork	6	17	105	Thorlabs
SM2L10	SM2 Lens Tube	1	29	29	Thorlabs
PF20-03-P01	Protected Silver Mirror	1	101	101	Thorlabs
POLARIS-K2F3	Polaris Low Distortion Mirror Mount	1	290	290	Thorlabs
PLS-P150	1" Post for Polaris Mirror Mounts	1	30	30	Thorlabs
PFR10-P01	Protected Silver Mirror	1	80	80	Thorlabs
FMP1	Fixed 1" Optical Mount	1	15	15	Thorlabs
LMR1	Lens Mount with Retaining Ring	1	15	15	Thorlabs
PH3E	1/2" Pedestal Post Holder	7	24	168	Thorlabs
CF125C	Clamping Fork	7	11	74	Thorlabs
TR3	1/2" Optical Post	7	5	38	Thorlabs
XE25L09	25 mm Construction Rail	4	15	59	Thorlabs
MB612	Aluminum Breadboard	1	130	130	Thorlabs
XE25-CUST-22	25 mm Construction Rails, 22" Long	4	85	340	Thorlabs
RM1G	Counterbored Construction Cubes	4	17	67	Thorlabs
MB4	Solid Aluminum Breadboard	2	41	82	Thorlabs
CAM2	Right-Angle Bracket	2	97	194	Thorlabs
AP90	Right-Angle Mounting Plate	2	78	156	Thorlabs
TB4	Black Hardboard, 24 x 24"	2	63	125	Thorlabs
5GCT0	Foam Sheet, 220 Poly, 1 x 24 x 18"	6	19	116	Grainger
P826A	Hollow Cathode Lamp (Fe)	1	264	264	Photron
P209-USB	HCL Power Supply	1	2487	2487	Photron

Table 43. Andor Apogee Ascent A694 specifications.

Sensor	Sony ICX694
Pixels	2750 × 2200
Sensor size	12.5 × 10 mm
Pixel pitch	4.54 × 4.54 μm
Well depth	21,000 e^- / pixel
Read noise	5.9 e^-
Binning	1 × 1 to 8 × 200
Quantum efficiency	77% @580 nm
Temperature stability	± 0.1 $^{\circ}\text{C}$
Dark current	0.0041 e^- / pixel / s
Digital resolution	16 bits
Exposure time	100 ms to 183 min
Computer interface	USB 2.0

Table 44. Santec spatial light modulator specifications.

Type	Reflective LCOS
Operation mode	Amplitude and phase modulation
Bandwidth	500-700 nm
AR coating	BBAR, R < 0.5% average, R < 1.0% peak
Active area	10 x 10 mm
Resolution	1440 x 1050 pixels
Pixel fill factor	90%
Panel reflectivity	> 80%
Gray levels	1024
Phase depth	> 2π rad
Phase fluctuation	< 0.002π rad
Operating temperature range	15-35 $^{\circ}\text{C}$
Temperature stability	< 0.001π rad/ $^{\circ}\text{C}$
Repeatability	r < 0.002π rad

The assembled prototype instrument is shown in Fig. 165. As mentioned previously, the prototype differs slightly from the design of Fig. 157, as per Fig. 160, in that two mirrors are used to fold the optical space. This change was made to allow all the components to fit on an optical breadboard small enough to fit behind an 8-inch Schmidt-Cassegrain telescope while still being able to re-image the VPH back onto itself. This re-imaging is important, since it prevents vignetting of the dispersed light. The telescope is then coupled onto the main system via a dove-tailed mounting bracket, which was bolted to the underside of the breadboard's optical rail.

To guard against thermal drift caused by temperature fluctuations in the air, the system is built upon a water-heated breadboard and enclosed in an insulating foam box. The interior of the box is maintained at 30 $^{\circ}\text{C}$ by a separate thermal control unit. The entire system – including water circulator, computer controller, telescope mount, and power supply for the calibration lamp – fits on a small custom lab cart. The final assembled system is depicted in Fig. 166 just before observations of Venus were made. Meanwhile, an image of the calibration section of the telescope are provided in Fig. 167. Here, a fold mirror can be inserted temporarily to re-direct light from an iodine gas cell, set to a temperature of 100 $\pm$ 0.1 $^{\circ}\text{C}$, to enable cross-reference to the 0 correlation point. Additionally, an iron lamp can also be re-directed into the entrance slit for this purpose. Generally, the iron lamp was used during our experiments, since iron lines are abundant in the

solar spectrum. This enabled the same SLM pattern to remain on the SLM between our observations and calibration, thereby mitigating against phase stability issues. Finally, key specifications of the assembled correlator prototype are listed in Table 45.

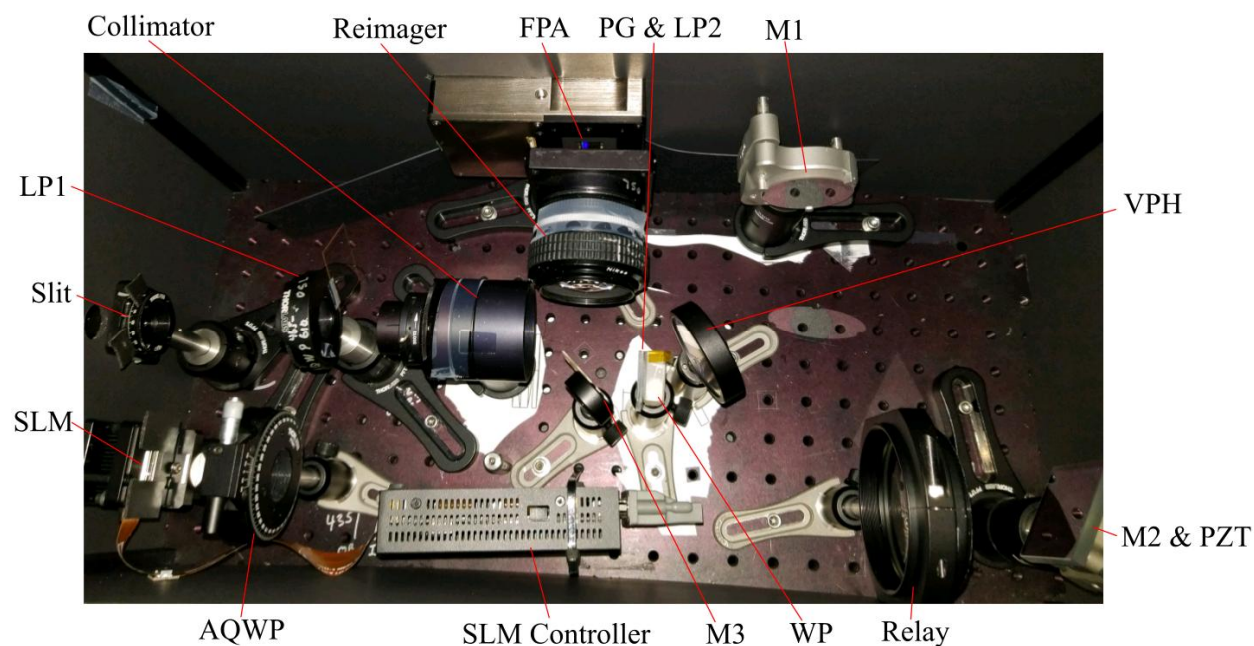


Fig. 165. Laboratory prototype of the optical correlator.

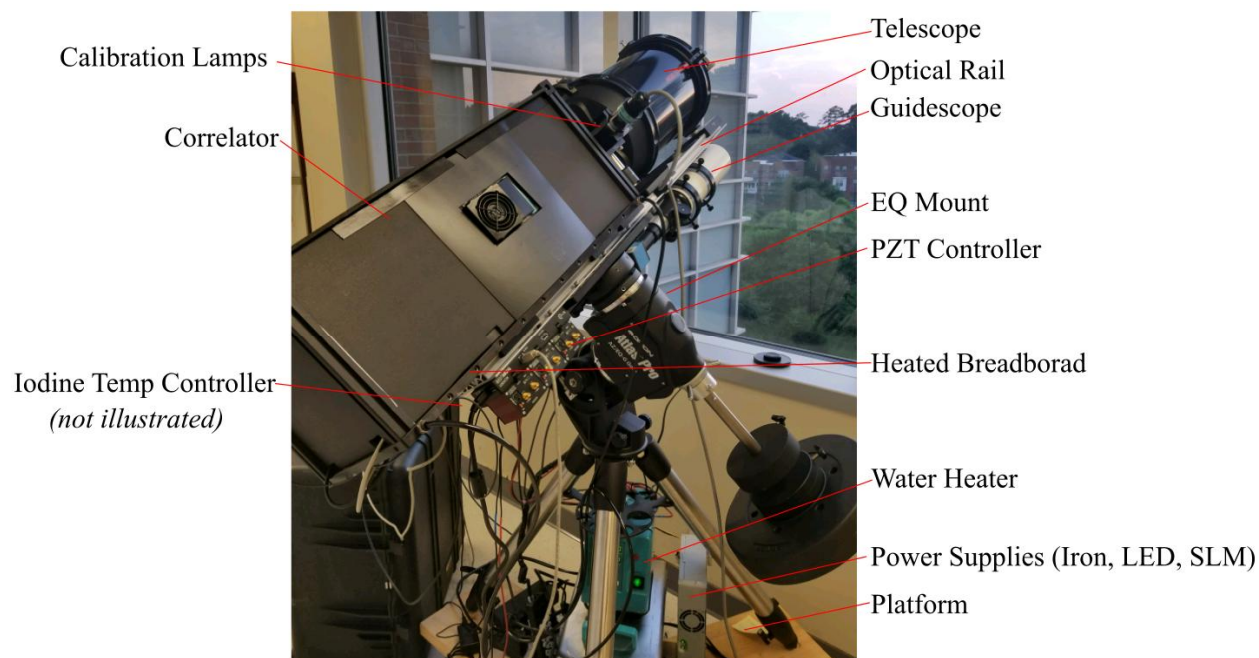


Fig. 166. Laboratory prototype on the EQ mount and platform.

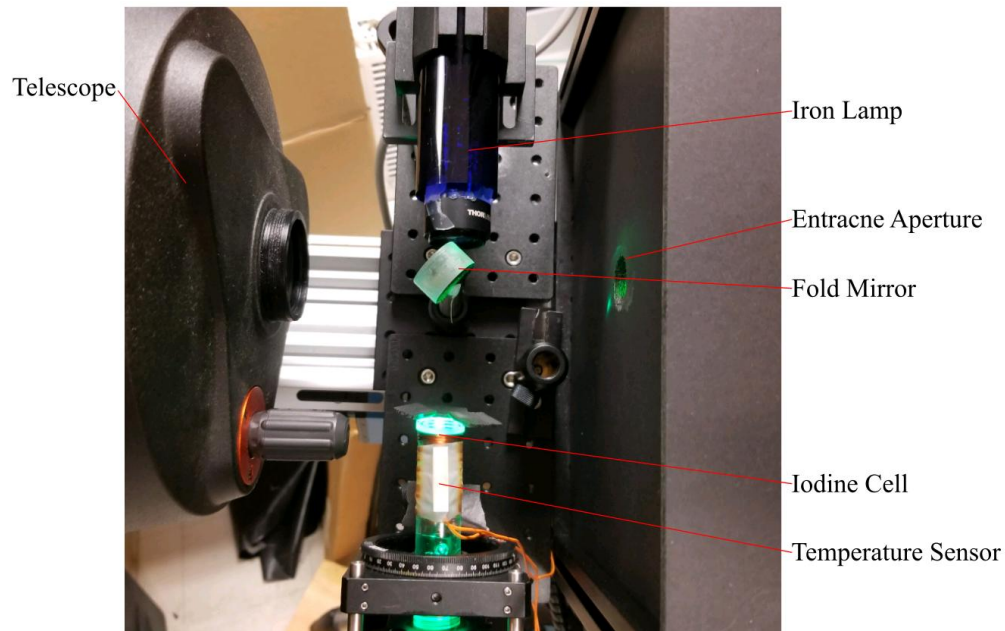


Fig. 167. Calibration section of the telescope.

Table 45. Key specifications of the prototype correlator.

Resolution	$R \sim 0.003 \text{ nm}$
Bandwidth	20 nm
SLM	Santec reflective LCOS, 1440 x 1050 pixels
Imaging sensor	Apogee A694, 2750 x 2200 pixels
Dispersive element	Wasatch Photonics VPH, 1800 lines/mm
Temperature stabilization	Thorlabs water-cooled breadboard

9.1 Frequency Stability

Measurements of a known source (halide lamp) were taken over a period of 16 hours to provide an estimate of the instrument's wavelength stability. For reference, the approximate spectral response of the lamp is shown in Fig. 168. For these tests, the camera was placed at the focal plane of the SLM so that the dispersed spectrum could be imaged directly.

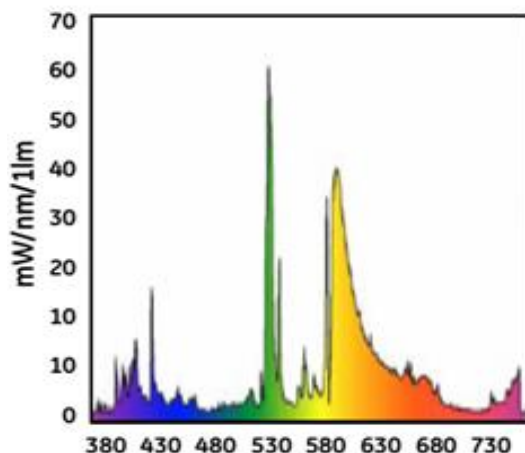


Fig. 168. Power spectral distribution for halide lamp.

The thermal control unit (TCU) for the instrument was set to circulate water through the breadboard at a constant temperature of 30 °C. Each of the resulting 1000 spectral images was averaged along the y-pixel axis to generate a set of intensity vs. x-pixel relationships. An isolated intensity peak near $x = 1340$ pixels was chosen as a reference line. For each image, a Gaussian curve was fitted to the reference line, and the parameters of height (peak amplitude), width (standard deviation), and center pixel (mean) were extracted. The center pixel values for every image are plotted with respect to time in Fig. 169.

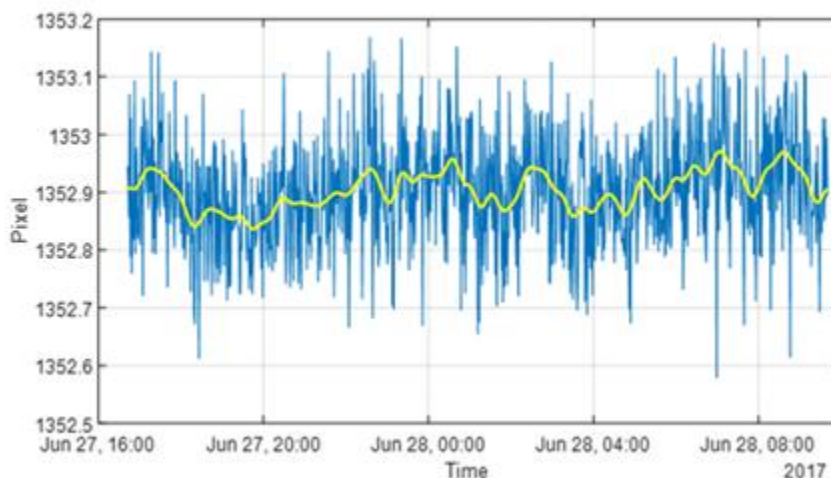


Fig. 169. Center pixel values (Gaussian fit) as a function of time.

The extremes of the graph show a maximum swing of less than 0.7 pixels, while the smoothed data (yellow line) is well within a range of 0.2 pixels, corresponding to a wavelength stability of $\pm 4.23 \times 10^{-5}$ nm. For wavelengths around 532 nm, this implies a Doppler velocity error of

$$v = c \frac{\Delta\lambda}{\lambda} = 3 \times 10^8 \frac{4.23 \times 10^{-5}}{532} = \pm 24 \text{ m/s.} \quad (146)$$

Extensive testing has indicated that the setting of the camera's thermoelectric cooling system (TEC) can significantly affect the wavelength stability. For best results, the TEC was set to a temperature of 18 °C, or 2 °C below ambient. A chart of the ambient temperature and the instrument temperature (measured near the center of the insulated enclosure) during the stability test is shown in Fig. 170.

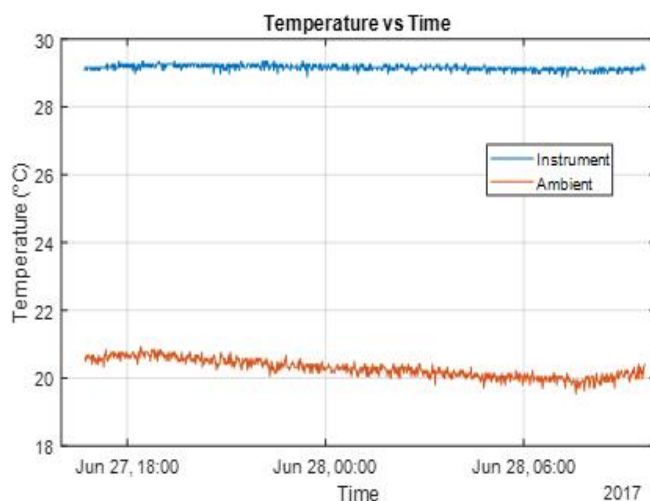


Fig. 170. Instrument and ambient temperatures during stability test.

9.2 Calibration

Alignment consisted of the following steps:

- 1) First, the slit was positioned at the focal point of the collimator by configuring a camera, with an 85 mm focal length objective lens, for infinity focus. This camera was placed at the output of the VPH to observe the 0 order transmitted beam. The position of the slit was then adjusted for best focus in the camera system.
- 2) Next, the mirrors M1 and M2 needed to be adjusted such that the diffracted beam from the VPH was successfully directed to one edge of the relay lens's clear aperture. A 532 nm single longitudinal mode diode laser and a 520 nm multiple mode diode laser (OSRAM PL520) were simultaneously coupled into the slit via a 50/50 beamsplitter. The total spectral resolution of the system is approximately 20 nm, and these two diodes can be easily viewed as being near either end of the operational passband. The beams are followed through the system, using a white card, until both beams arrive at the relay lens without

vignetting. These are then followed to the SLM, where the SLM's position is approximately verified by visualizing focus on the card.

- 3) Next, the SLM should be rotated in the xz plane of Fig. 160 until the reflected laser beams are re-directed back towards the relay lens. The beams should exit the lens on the other side of its clear aperture, without vignetting. These then propagate into mirrors M2 and M1 and should recombine at the VPH. The SLM should be tilted to ensure that these beams arrive on the other side of the VPH's clear aperture.
- 4) The SLM's focus should be more precisely positioned. This can be done by adjusting the distance between the SLM and the relay lens. The relay lens is in a slip mount to enable fine adjustments to this distance. For this adjustment, only the 532 nm laser is used, since it has a single longitudinal mode. The SLM is loaded with a periodic pattern in which each pixel (or every 3rd pixel) produces a retardance difference of half-wave. In the pupil of the relay lens, this will create two sheared beams. Adjusting the relay lens's position will cause the fringes between these beams to tip and tilt. The goal is to position the relay such that these fringes are oriented vertically (parallel to the y axis).
- 5) Finally, the SLM's focus should be precisely positioned. This is done by scanning the SLM such that a single column of pixels is turned on over the course of a time-sequence, diverting in-band light to one order of the camera's output, and out-of-band light to the other order. The ratio of IB/OB will provide a spectrum of the laser diode(s) after the scan is completed. For this procedure, both the 532 and 520 nm diode lasers can be used simultaneously. This will also provide an approximate wavelength calibration, if desired. Finally, the multi longitudinal modes of the 520 nm diode creates a frequency comb at the SLM, making this diode an ideal choice for this calibration step. Ideally, the output should produce a spectrum in which the longitudinal modes of this diode span approximately 2-3 pixels FWHM when in best focus.

Once aligned, calibration consisted of the following steps:

- 1) Linearity Coefficient Calculation. The linearity of the camera must be corrected. This was achieved by focusing a 532 nm green LED onto the system's entrance slit. A linear polarizer was mounted onto the LED and the camera's exposure time was adjusted to be near saturation when this linear polarizer was parallel to the system's polarizer LP1. The polarizer was then rotated from 0 to 180 degrees in 10 degree increments. An intensity profile was then produced and plotted against the ideal transmission of the crossed polarizers via. Malus's law. This produced a 2nd order polynomial that was used to linearize the intensity of all subsequent measurements.
- 2) Iron Lamp Calibration. The wavelength axis of the SLM was then calibrated to the emission spectrum of the iron lamp. Reference spectra [59] of an iron hollow cathode lamp were converted into Matlab and compared to a single pixel scan from the SLM. Similar to the laser diode spectra acquired previously, acquiring the iron lamp's spectrum involved scanning a half-wave column of pixels across the SLM. The ratio of IB to OB light was then used as the spectrum, and manually fitted to the reference spectrum using a second order polynomial. This polynomial established the wavelength axis of all subsequent spectra that were loaded onto the SLM for direct correlation measurements.

- 3) Piezo Transducer Calibration. To accurately measure radial velocity, the PZT's voltage is adjusted to tip mirror M2 by a repeatable amount. This laterally translates the spectrum on the SLM along the x -axis, enabling an effective Doppler shift to be induced on the incoming light. This calibration was achieved by heating an iodine reference cell to 100 C. A 532 nm green LED was collimated through the cell and re-directed into the system's slit. The SLM was then loaded with the iodine's reference spectra per an available atlas [60] such that

$$I(\lambda) = I_R(\lambda + \Delta\lambda), \quad (147)$$

where I_R is the reference spectrum, $\Delta\lambda$ is a spectral shift in the reference spectra that can be incorporated via. interpolation, and I is the spectrum on the PZT. According to the manufacturer's specifications, the PZT should have an angular sensitivity of 5 microradians per volt and can accept inputs spanning 0 to 150 V. When imaging through the 400 mm focal length lens, this should produce a spectral displacement of $\Gamma = 2.597\text{E}-3$ nm/V. This number was verified by scanning the PZT from 0 to 150 V, in 1 V increments, for two spectra I_1 and I_2 with $\Delta\lambda_1 = -0.1$ nm and $\Delta\lambda_2 = +0.1$ nm, respectively. From these DCS measurements, the voltage at which the minimum value occurred was taken and the new calibration coefficient calculated as

$$\Gamma = \left| \frac{\Delta\lambda_1 - \Delta\lambda_2}{V_1 - V_2} \right|, \quad (148)$$

where V_1 and V_2 are the voltages at the minimum value for shifts of $\Delta\lambda_1$ and $\Delta\lambda_2$, respectively. This yielded a new value for $\Gamma = 3.571\text{E}-3$ nm/V. Doppler shift can now be related to PZT voltage by

$$d\lambda = \Gamma V, \quad (149)$$

where V is the voltage applied to the PZT. Since this spans 0 to 150 V, the total range of induced effective Doppler shift can span +/- 0.268 nm, which corresponds to radial velocities spanning approximately +/- 81.4 km/s.

- 4) Low Spectral Resolution Calibration. As depicted previously in Fig. 160, a polarization grating (PG) is used to disperse the light of the slit before it is imaged onto the FPA. This produces a low spectral resolution (~2 nm) spectrum, which can be used to spectrally resolve the in-band to out-of-band ratio. The axis of this low-resolution spectrum is calibrated by converting 100 pixel wide columns of the SLM to half-wave plates, enabling each small spectrum to be calibrated when referenced back to the SLM's spectral calibration. While this small dispersion produces a reduced signal to noise ratio, it enables low spectral resolution features of the source to be normalized between the in-band and out-of-band measurement. While this normalization is not necessary when the PZT is used, it may be important in the future for single-shot radial velocity measurements, or for single-shot Fraunhofer line discriminator measurements. Further system modeling and simulations are needed to identify error and uncertainty sources for such measurements.
- 5) Direct Correlation Measurements. Once the calibrations are completed, DCS measurements can be made with the system. For solar spectra, this is achieved by taking a sequence of measurements of the iron lamp. This sequence is collected by varying PZT voltages from 35 to 115 V, in 2 V increments, to identify the 0 radial velocity voltage. A

subsequent measurement is then taken of the object under test by performing the same sequence of measurements. A Gaussian function of the form

$$I = 1 - \exp\left[-\frac{(V - V_0)^2}{2w^2}\right], \quad (150)$$

is then fitted, using Matlab's *fminsearch*, to identify the central minimas' voltage V_0 of both the iron lamp and the target spectra. This offset voltage of both DCS spectra are then converted into a radial velocity, using Eq. (149) and Eq. (1), and the difference between them is taken to extract the target's absolute radial velocity.

From these results, we calculated a FWHM spectral resolution of 0.04 nm. Ideally, a value of 0.01 nm was desired; however, the limited aperture of the VPH and temperature controlled breadboard restricted the minimum value we could achieve with the materials available. Regardless, we are still able to obtain good DCS results with this system, as is highlighted in the subsequent section.

9.3 Results

Testing was conducted by observing light from Venus. Observations were made on 6/29/2018 at 9:30 PM EST. DCS spectra, obtained from both the iron lamp and of venus, are illustrated below in Fig. 171. The camera integration time, for each PZT measurement position, was set to 20 seconds. Since we were limited in mobility of the measurement setup, observations were made through a window under turbulent atmospheric seeing ranged from poor to average, partially related to the fact that observations were made through the building window (and thus, conduction from the surrounding building interfered with the seeing). Despite this, the polarization method, which captures both in-band and out-of-band light, demonstrated itself to be relatively insensitive to these variations. Additionally, guiding was done manually through the attached guidescope, since precise polar alignment could not be achieved indoors.

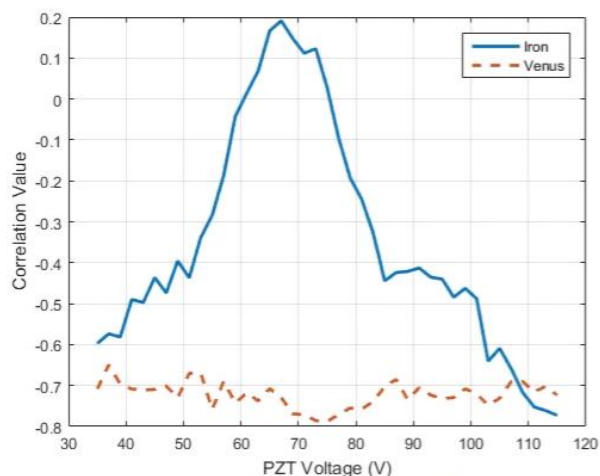


Fig. 171. DCS spectra obtained from the instrument of the iron calibration source (solid blue line) and Venus (dashed red line).

Notable from Fig. 171 is the inverse relationship between the iron lamp and Venus, which is a result of emission versus absorption lines. The data were first filtered using a Hamming window before being normalized to span values of 0 to 1. Additionally, the iron lamp DCS spectrum was

inverted to match the shape of the Venus DCS spectrum. After filtering and normalization, the spectra appeared as per Fig. 172.

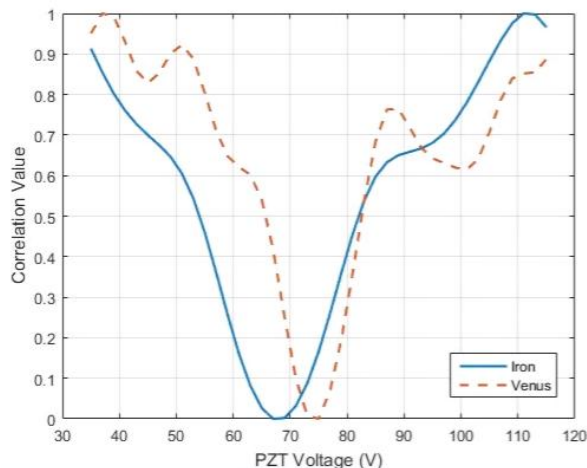


Fig. 172. Filtered, normalized, and in the case of the iron lamp, inverted DCS spectra obtained from the instrument. Iron lamp (solid blue line) and Venus (dashed line) correlation spectra.

In this case, the minima is clearly evident in the Venus spectrum, and a Gaussian function as defined previously in Eq. (150) is fit to a cropped version of these DCS spectra. A view of the fitted Gaussian function, as compared to the cropped filtered data, is provided in Fig. 173 (a) and (b) for Venus and the iron lamp, respectively. Residual mean square error for the fit to the Venus and iron data is $1.39\text{E-}3$ and $1.36\text{E-}4$, respectively.

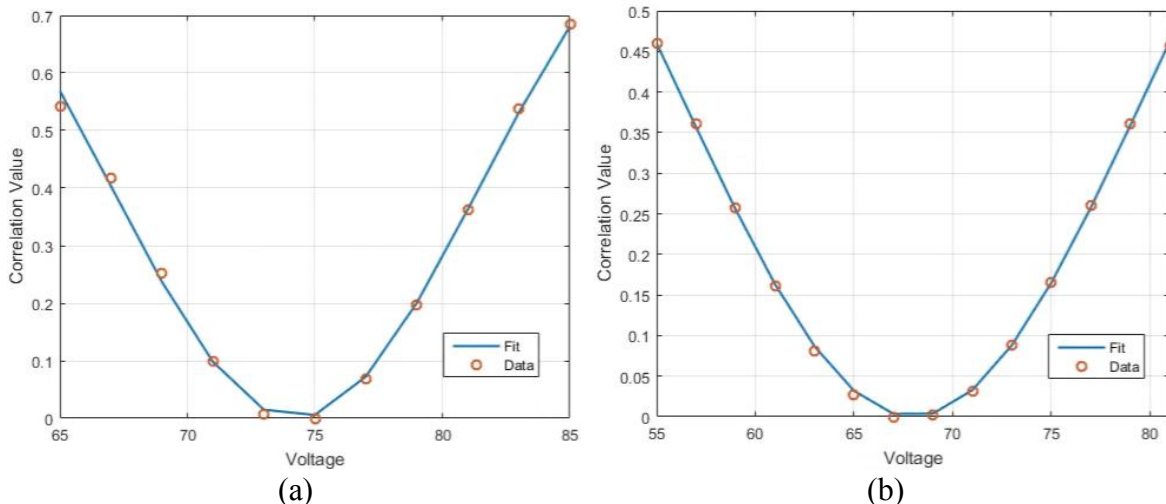


Fig. 173. Fitted and filtered measured data versus voltage for (a) Venus and (b) the iron lamp.

From the fitted Gaussian functions, the offset PZT voltages were 67.978 V and 74.237 V for the iron and Venus data. This corresponds to a velocity difference of 12.74 km/s. An ephemeris of the relative velocity between Venus and Earth, given an observation position in Raleigh, NC, and accounting for earth's rotation, is illustrated below for the year 2018 in Fig. 174. Indicated on the plot is the calculated theoretical "true" value of 12.79 km/s versus our measured result of 12.74 km/s.

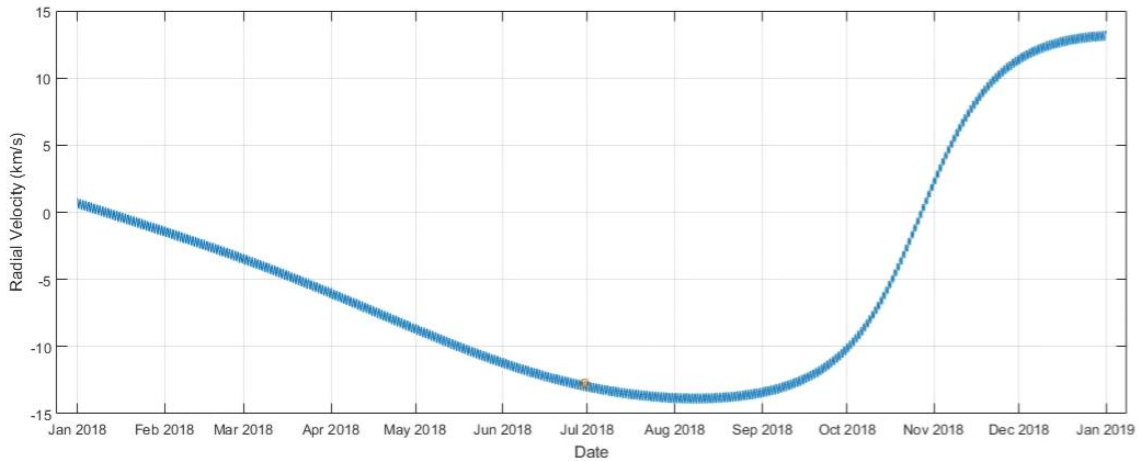


Fig. 174. Fitted and filtered measured data versus voltage for (a) Venus and (b) the iron lamp. The actual value (o) of -12.78918 km/s is indicated next to our measured value (x) of -12.74 km/s.

A magnified view of the region, illustrating the actual value and measured value, are depicted in Fig. 175. Error between our observation and the theoretical value is 0.38%.

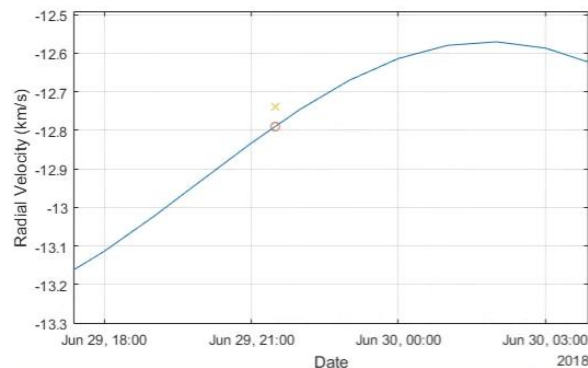


Fig. 175. Magnified view of the ephemeris data comparing the actual value on the curve (o) to the measured value (x) at the measurement time (21:30 hours).

Note that the rotation rate and cloud velocity of Venus was not included in our ephemeris and radial velocity calculation. However, given that the rotation rate of Venus is low (~ 6.5 km/h), error is primarily due to unknown cloud velocity effects (0.1 km/s estimated) and reduced signal caused by the low atmospheric seeing or other systematic errors related to temperature effects and SLM phase stability. We hope that future measurements can be conducted outdoors when feasible.

10.0 Potassium (K) Voigt Filter

This section details the results of a narrow band atomic line filter that can be used for detecting potassium signatures. The polarization filter was derived from the dual-beam concept (in-band and out-of-band) that was previously leveraged for the optical correlator. It ultimately came from our tradespace analysis. The filter, which is based on a magneto-optical Voigt configuration [61,62], enables narrow-band emission lines to be imaged through the filter. Advantages include:

- 1) Spectrally self-referenced. The atomic absorption of potassium dictates its transmission.
- 2) Dual spectral-band measurements. Both in-band and out-of-band light can be captured simultaneously, enabling rejection of solar glints and glare.

- 3) Wide field of view. Since the spectral transmission is dictated by the gas's absorption and the incident linear polarization, the FOV is constrained by the polarizer, which is large compared to narrow-band interference filters, etalons, and birefringent filters.
- 4) Increased solar rejection. The filter operates within a solar Fraunhofer line, which further increases contrast for targets and decreases in-band light transmission.

A summary of the optical design and preliminary results are provided below.

10.1 Fraunhofer Line Discrimination (FLD)

FLD is a technique that is used to detect weak signals in the presence of intense solar background light. Often it is used within the context of measuring fluorescence. In order to reduce power requirements and system complexity associated with active fluorescence measurements, a measurement of passive fluorescence can be achieved by using excitation from the incident sunlight. However, high spectral resolution is important for the process since passive measurements of plant fluorescence are made by quantifying the “filling” of solar Fraunhofer lines [63].

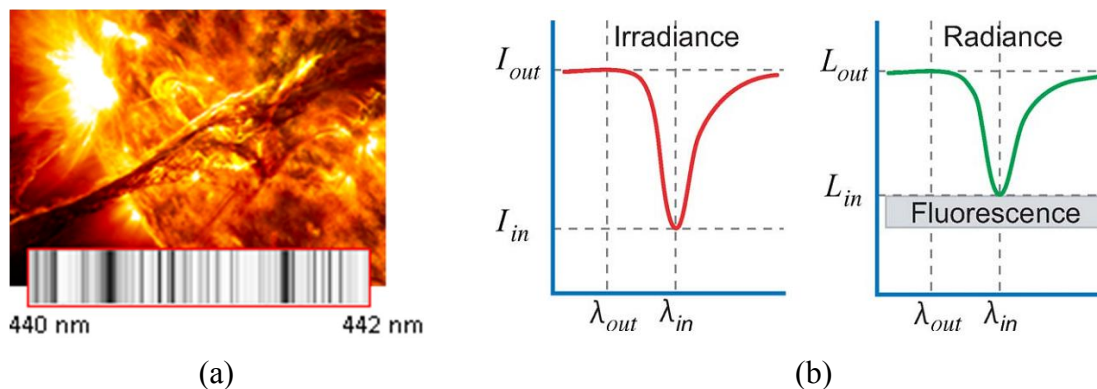


Fig. 176. (a) Solar Fraunhofer lines are dark absorption features which require a high spectral resolution to measure. (b) If a measurement can be made within a line, band filling within that line represents terrestrial fluorescence signatures [64].

These extremely narrow lines (< 0.01 nm full-width at half maximum spectral bandwidth) are created by intense atomic absorption features within the solar atmosphere. Many such lines are present, as illustrated in the 2 nm wide solar spectrum per Fig. 176 (a). Measurements made within one of these lines, at a high spectral resolution per Fig. 176 (b), thus represent a small window in which incident solar irradiance is minimal. Thus, any light detected within one of these bands represents terrestrial light caused by fluorescence in FLD. However, it can also provide a signal to noise ratio advantage when detecting atomic emission within these regions, as is the case when detecting plumes with solar atomic species present within them. Potassium is a species that has absorption appearing within the incident solar spectrum, as indicated in the spectral curve of Fig. 177. Also provided in Fig. 177 is the transmission spectrum of the potassium D1 line at 769.89646 nm for the parameters of our experimental configuration [62]. Note that while Fig. 177 shows the results for a Faraday filter, Voigt filter profiles can also be simulated using similar numerical tools [65].

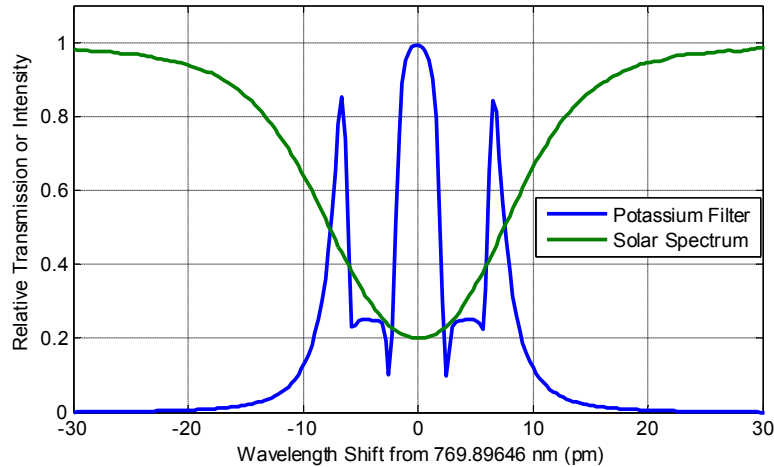


Fig. 177. Solar spectrum (green solid line) and filter transmission spectrum (blue solid line), illustrating the narrow bandwidth of both the potassium filter and the incident potassium Fraunhofer line [66].

10.2 Optical Design

A view of the optical design is illustrated below in Fig. 178. It consists of a bandpass interference filter (BPF) with a center transmission wavelength of 770 nm and full-width at half maximum (FWHM) spectral bandwidth of 10 nm. This enables both potassium lines, centered at 766.4899 nm and 769.8964 nm (air wavelength), to transmit into the system [67]. Light is first focused by an objective lens into a field stop, which is then collimated into a magneto-optical filter (MOF) of the Voigt configuration. A linear polarizer, oriented at 45 degrees, first polarizes the light into the potassium cell. Two permanent neodymium magnets create a magnetic field, oriented parallel to the x -axis (0 degrees), with a strength of 1150 Gauss. The 71.8 mm long cell has three temperature controllers that maintain the reservoir, cell, and window temperatures at T_R , T_C , and T_W , respectively.

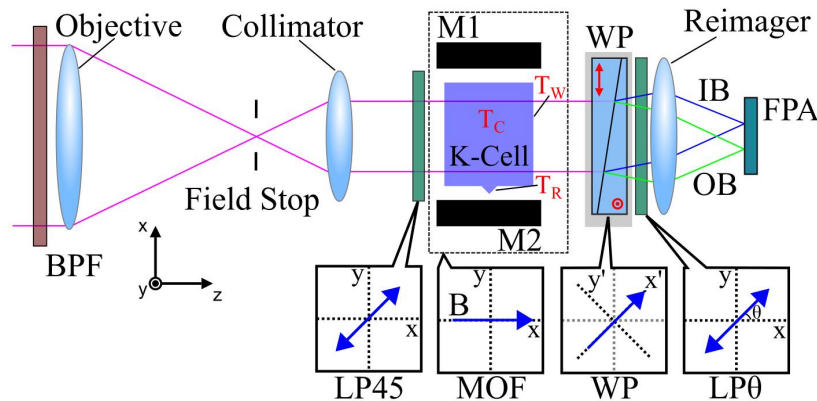


Fig. 178. Magneto optical filter (MOF) configuration for dual-band imaging. In-band (IB) and out-of-band (OB) light creates an image above and below the optical axis, respectively.

Light that has absorption within potassium's absorption wavelengths experiences a half a wave of retardance through the filter. Light that undergoes this rotation (in-band light) will then refract in the Wollaston prism (WP) above the optical axis, while light that does not undergo rotation (out-of-band light) will refract below the optical axis. After transmitting through a re-imaging lens, this forms two images of the scene on the focal plane array (FPA). In-band (IB) light corresponds to

the potassium absorption lines, while out-of-band (OB) light corresponds to the transmission spectrum of the BPF sans the energy contained within the IB component. Since the OB component is significantly brighter than the IB component, a second linear polarizer can be oriented at an arbitrary angle θ . This can be used to attenuate the OB component, such that both can be imaged simultaneously on the FPA.

Finally, a view of the benchtop proof of concept is indicated in Fig. 179 (a) and (b) for the top and side views, respectively. The current concept is limited in size by the use of commercial off-the-shelf heater control units, as well as optomechanics. Presently the sensor weighs 20 kg and measures (excluding the breadboard but including the fore-optics) approximately $40 \times 20 \times 30 \text{ cm}^3$; however, this can be reduced to less than 2 kg and $35 \times 10 \times 10 \text{ cm}^3$ with optimization.

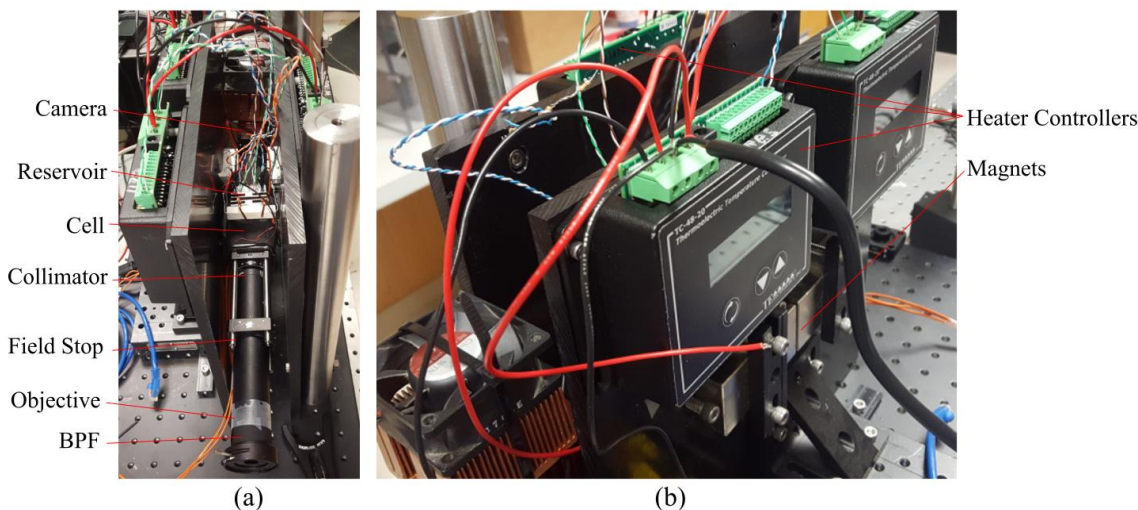


Fig. 179. Photos of the experimental proof of concept on the benchtop. (a) Top view illustrating the optical path and (b) side view illustrating the location of the magnets. Note that magnets are present on both sides of the cell to create a linear uniform magnetic field across its width and length.

10.3 Preliminary Results

Preliminary results have been acquired on similar test targets to our previous work [68]. This included a true solar glint, created by a mirror, and a potassium-based model rocket motor. A photo of the test setup is illustrated below in Fig. 180 (a) in which a mirror was used to produce a sun glint, while the engine was mounted in place for observation. After a radiometric calibration, linear unmixing was performed on the raw dual-band imagery. This resulted in the two spectral components per Fig. 180 (b), which indicates the solar spectral contribution, and Fig. 180 (c), which indicates the potassium spectral contribution. Notable is the high level of isolation for both components. These results are achieved with no spectral calibration requirements – only radiometric considerations are needed to account for the finite extinction ratios of the linear polarizers and Wollaston Prism, which creates some cross-talk of OB light into the IB image.

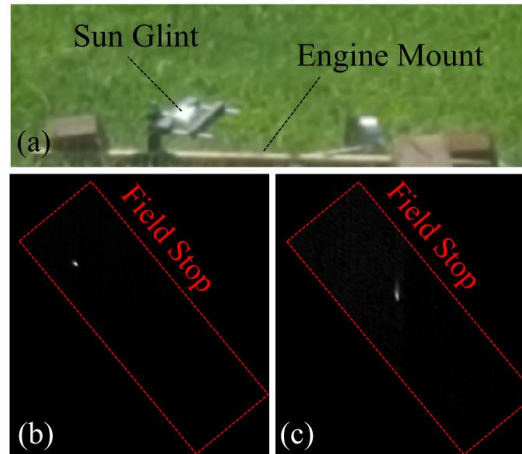


Fig. 180. (a) Photo of the experimental configuration for the sun glint and engine mount. Results after linear unmixing for (b) solar glint and (c) potassium emission. These results were acquired with a 200 ms exposure. Note that the field stop is indicated and has been rotated to fit the orientation of the photo in (a).

10.4 Future Improvements

Future work would focus on increasing the radiometric throughput of the sensor and augmenting the sensor to a more advanced FPA or intensified CCD. This would allow for higher framerates and shorter measurement times across a larger range of standoff distances. We estimate that with these changes, the radiometric throughput could be increased by a factor of 4 with the optical design, a factor of 2 with the FPA's quantum efficiency, and a factor of 2 over the existing FPA's dark and readout noise. This would reduce the minimum integration time from 200 ms to 12.5 ms, increasing the framerate from 5 frames per second (FPS) to 80 FPS.

11.0 Conclusions

We have presented the design of an optical correlator based on discrimination of orthogonally polarized wavefronts. Using an optical mask aligned with multiple Fraunhofer absorption lines helps to improve the SNR compared to measurement of a single line.

The prototype unit has not yet been field tested. However, simulations based upon bench performance indicate that for geostationary orbits, an acceptable IOD estimate can be obtained with only two telescopic observations, instead of three as usually required by angles-only methods. The potential benefit to space situational awareness is obvious, but instruments based on this correlator could have applications in many other areas of remote sensing, when sub-angstrom resolution and high throughput is required.

Future steps will involve creation of an optical mask based on direct solar measurements, rather than bench-top reference lamps. A field test with a small telescope will follow, possibly using the Moon as an easy test target.

Finally, the following publications and conference proceedings, either completed or pending, have been pursued since the previous report. The first two are included in the Appendix.

1. B. A. Pantalone, M. W. Kudenov, "Fraunhofer line optical correlator for improvement of initial orbit determination," *Proc. SPIE in Polarization Science and Remote Sensing VIII*, (Completed, 2017).

2. B. A. Pantalone, M. W. Kudenov, "Initial Orbit Determination Using Doppler Shift of Fraunhofer Lines," *Celestial Mechanics and Dynamical Astronomy*, (In peer-review, Resubmitted 5/22/2018).
3. M. W. Kudenov, B. A. Pantalone, "Potassium Voigt Filter for Dual-Band Atomic Line Imaging," *Appl. Opt.* (Peer review, Pending 2018).
4. C. G. Scarboro, B. A. Pantalone, M. W. Kudenov, "High Throughput Fraunhofer Line Discrimination using Optical Correlation" *Appl. Opt.* (Peer-review, Pending 2018).

11.1 Future Work

Future work will focus on a modification of the VPH/SLM correlator to enable two-dimensional imaging, as illustrated in Fig. 181. The addition of a 2D to 1D fiber array will permit coupling of the 2D optical field into the 1D slit. A custom-designed spatial correlator comprising of a geometric phase hologram (GPH) can be fabricated in-house by Dr. Michael Escuti. Similar to the SLM, the GPH will enable capture of both in-band and out-of-band light. Integration time can be reduced to less than 5 minutes due to the increase in the spectral passband afforded by the GPH. Furthermore, multiband GPH technology may enable new methods for expanding the dynamic range of measured Doppler velocity, and coupling the GPH to an Echelle spectrograph may enable, for the first time, polarization-compensated DCS measurements. Prior methods of DCS sensors required additional mechanical compensators to correct for atmospheric seeing conditions, which a polarization system is inherently immune to [58,69]. These advantages will be studied in future work.

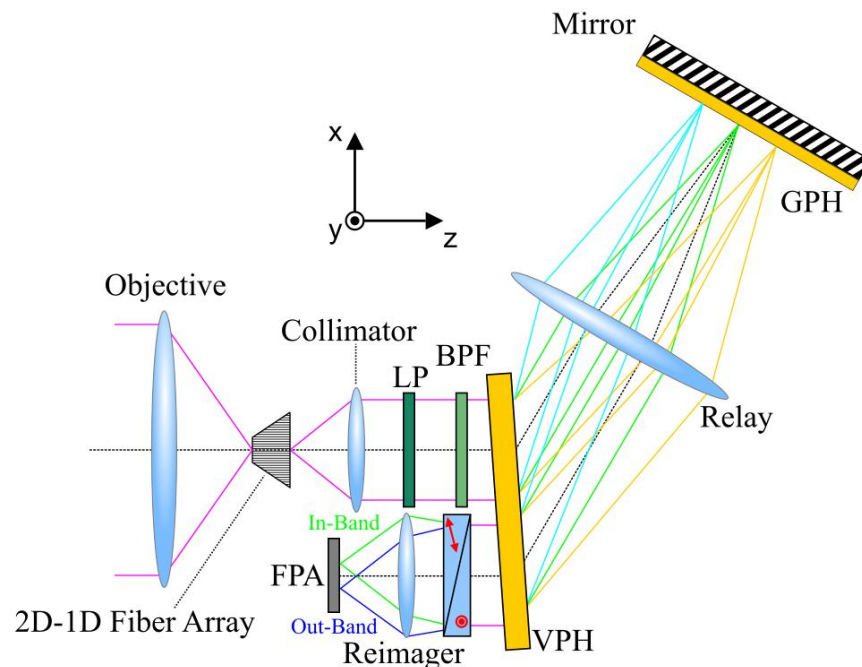


Fig. 181. VPH correlator with imaging functionality.

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