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**Modeling, Observability and Change Detection in Space Situational Awareness**

**Daniel Scheeres**  
**REGENTS OF THE UNIVERSITY OF COLORADO THE**

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**Final Report**

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# Modeling Observability and Change Detection in Space Situational Awareness

## Final Report

Grant# FA9550-14-1-0348

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## Overview

The objective of this study was to perform basic research into the observable consequences of natural forces and commanded actions (such as changes in orbit or attitude) for resident space objects (RSO) in Earth orbit. By studying and developing improved models of these activities, the goal was to be able to identify their observable consequences and devise optimal ways in which they can be detected or constrained based on existing astrometric and radiometric observation techniques. These research goals were chosen as they would advance the application of the scientific method in Space Situational Awareness (SSA) and support the development of unambiguous observation strategies to properly characterize space objects.

The research objectives consisted of three fundamental research questions. In the following report, progress in each of these themes will be presented.

**Modeling and Dynamics** “How can we reliably and rigorously estimate, model and predict how space objects react to their environment?”

**Observations and Observability** “What is the accessible information content in remote space object observations that can be utilized for SSA?”

**Maneuver Detection and Constraints** “How can changes be reliably detected in a space object’s state and what can be said about what actions that object took?”

Results from this research program can be applied to yield improved understanding of what effects are important in the geospace environment, how such effects can be measured and constrained, how commanded actions can be identified and used as predictive models, and how future observation capabilities and cadences can be optimized to enable these detections. Ultimately, this research will substantially aid in the development and maintenance of the increased space object catalog, which will result from ongoing improvements to SSA. The development of more accurate models for the catalog will also enable longer time spans between tracks and more reliable correlations between tracks, with a commensurate decreased taxing of the SSA sensing network. Research methods used in this study relied on fundamental astrodynamics, optimal control theory and estimation theory.

This was a multi-investigator and multi-institution proposal. At the University of Colorado the research was led by co-PI Prof. Daniel J. Scheeres. At Texas A&M University the research is led by co-PI Prof. Kyle T. Alfriend. At Purdue University research was led by Assistant Professor Dr. Carolin Frueh. The current research performed continued and extended research that was earlier carried out by the same team under AFOSR grant FA9550-11-1-0188 with The University of Colorado.

## Results

Progress was made in all areas, as documented by PhD theses, peer-reviewed journal papers, and conference papers, all listed at the end of this report and described in brief below.

## Modeling and Dynamics

### **Accurately Mapping Uncertainty Distributions: Scheeres**

The thesis and research of Dr. Park was focused on leveraging higher-order analytical averaging theories to enable rapid and accurate mapping of orbit uncertainty over long time spans. This research posed the question of what level of model fidelity is needed to accurately capture the statistical moments of a distribution. To answer this question he implemented a high-order Deprit-Lie series expansion of perturbing forces and used these to map the secular dynamics of an orbit uncertainty distributions. This required development of complex analytical functions, a mean to osculating transformation, and mapping functions to evaluate the higher-order averaged dynamics. Comparisons of these results with precise mappings using Monte Carlo methods showed that the secular terms accurately capture the statistical moments as mapped in time. This is an important realization as it provides a pathway to the rapid (near instantaneous) calculation of orbit uncertainty mappings over long periods of time with high accuracy. These advancements are documented in [13,14].

In a similar vein, the research of Dr. McMahon on the use of enhanced, yet simple, models of solar radiation pressure showed how model fidelity can be expanded to improve orbit determination [12]. That work applied research developed under a previous AFOSR grant to test how simple extensions of an unknown object's SRP model can improve orbit estimation and prediction. It was found that a minimal extension to the "cannonball" model enables a great increase in filter accuracy.

## Characterizing Non-Gaussianity of a Distribution: Alfriend

The orbit covariance of a space object is, or can be used, in important space situational awareness (SSA) processes including a) conjunction assessment and probability of collision [A1,A2], b) uncorrelated track (UCT) association [A3], c) maneuver detection [A4], and d) sensor tasking [A5]. The algorithms for these SSA processes all assume that the orbit uncertainty remains Gaussian. It is assumed, and the evidence supports it, that the uncertainty is Gaussian at epoch. However, the equations of motion are nonlinear and, hence, the uncertainty will become non-Gaussian at some time. When the uncertainty becomes non-Gaussian depends on the coordinate system and the "magnitude" of the initial uncertainty. Research has shown that the uncertainty remains Gaussian much longer in non-singular orbital element space than in Cartesian coordinates. In catalog maintenance of LEO it has been shown that the uncertainty typically remains Gaussian in orbital element space up to 10 days, whereas in Cartesian coordinates only about 2-3 days. Current methods for determining when the uncertainty becomes non-Gaussian require the propagation of the full 6-D covariance. A simpler tool for determining when the uncertainty becomes non-Gaussian would be very useful to users such as those performing conjunction assessment.

The primary nonlinearity that causes the uncertainty to become non-Gaussian is the error in the semi-major axis. This error causes secular growth in the in-track direction, that is an error in the mean and true anomaly. Secular growth in the out of plane direction is primarily caused by an inclination error, but this secular growth is proportional to the equatorial bulge term,  $J_2=0.00108$ , which means it should be much smaller than the in-track secular growth. The research here addressed the following key questions:

1. What metric should be used to determine when the uncertainty is non-Gaussian?
2. Since the secular in-track error growth is due primarily to the semi-major axis error can the 2D semi-major axis-mean anomaly space be used to determine when the uncertainty becomes Gaussian?
3. Can an analytic result be obtained to estimate when the uncertainty becomes non-Gaussian? If so, this would be a very useful tool.

Based on research by Horwood [A6] the Cramer-von Mises metric was selected for determining when the uncertainty becomes non-Gaussian. Figure 1-Left compares the CVM and Anderson-Darling metric for determining the time when the uncertainty becomes non-Gaussian with 99.9% confidence for nominal initial conditions as a function of the semi-major axis standard deviation. These results were obtained using the numerical form for the metric, Eq. (6), with 10,000 samples. The results show that both metrics give the same answer. Figure 1-Right compares the time when the uncertainty becomes non-Gaussian for the full 6D case to the 2D semi-major axis-mean anomaly space and shows that using the 2D space is a valid approach for determining when the uncertainty will become non-Gaussian. Research on developing an analytic expression analytic for when the uncertainty becomes non-Gaussian is continuing.

## References

- A1. Foster, J., "A Parametric Analysis of Orbital Debris Collision Probability and Maneuver Rate for Space Vehicles," NASAJSC-25898, European Space Agency, Aug. 1992.
- A2. Akella, M.R. and Alfriend, K.T., "The Probability of Collision Between Space Objects?," AIAA Journal of Guidance, Control and Dynamics, Vol. 23, No. 5, pp. 769-772, September 2000.
- A3. Hill, K., Sabol, C. and Alfriend, K.T., "Comparison of Covariance-Based Track Association Approaches With Simulated Radar Data," AAS J. of the Astronautical Sciences, Vol. 59, Nos. 1 & 2, Jan-June 2012, pp. 287-306.
- A4. Holzinger, M.J., Scheeres, D.J. and Alfriend, K.T., "Object Correlation, Maneuver

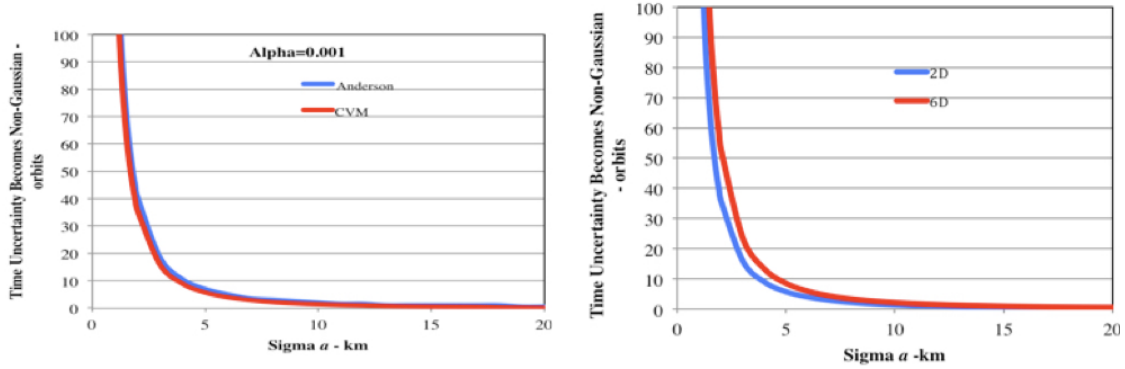


Figure 1: Left: Metric Comparison. Right: 2D and 6D Orbital Element Comparison

Detection, and Characterization Using Control Distance Metrics with Uncertain Systems,? AIAA J. of Guidance, Control and Dynamics, Vol. 35, No. 4, July-August 2012, pp. 1312-1325.

A5. Hill, K, Sydney, P., Hamada, K. Cortez, R., Luu, K.K., Schumacher, P.W. and Jah, M., ?Covariance Based Scheduling of a Network of Optical Sensors, ? Paper No. AAS 10-325, 2010 Space Flight Mechanics Conference, Kyle T. Alfriend Astrodynamics Symposium, Monterey, CA, May 2010.

A6. Horwood, J.T, Aristoff, J.M., Singh, N., Poore, A.B. and Hejduk, M.D. ?Beyond Covariance Realism: A New Metric for Uncertainty Realism?, SPIE Proceedings: Signal and Data Processing of Small Targets, 2014, Vol. 9092.

## Observations and Observability: Frueh

Observability is a well established concept in control theory. In this research, the concept of observability has been made available for Space Situational Awareness. In order to do so, two crucial advancements had to make. For one the formulation had to be extended to include measurement noise. And secondly, extending the classical state formulation to include what is called an extended state, including not only position and velocity but further orbit relevant or characterization information, such as for example area to mass ratio.

Using Cholesky whitening procedure allows for a redefinition of the output or measurement equation as:

$$\tilde{\mathbf{y}} = \hat{\mathbf{H}} \mathbf{x} + r, \quad (1)$$

where  $r$  is the measurement noise with zero-mean and covariance equal to identity because of the whitening process. Moreover, the measurement matrix and output are re-defined as:

$\hat{\mathbf{H}} = \mathbf{L}^T \mathbf{H}$  and  $\tilde{\mathbf{y}} = \mathbf{L}^T \mathbf{y}$ . Use of pre-whitening and reformulation of the measurement matrix enables measurement noise to be incorporated into the observability analysis. Using the Cholesky pre-whitening, the observability [5]: matrix with measurement noise is given by:

$$\mathcal{O} = \sum_{k=1}^m \Phi(t_k, t_0)^T \hat{\mathbf{H}}(t_k)^T \mathbf{R}(t_k)^{-1} \hat{\mathbf{H}}(t_k) \Phi(t_k, t_0). \quad (2)$$

It has been shown that the resulting matrix has the same form as the information matrix used within the linear, unbiased, minimum variance estimate and can be directly transformed into the normal equations matrix derived in a least squares concept, instead from the observability perspective, showing a direct connection of the observability to estimation. For the measurement noise representation, a new derivation for the measurement noise of optical sensors has been derived using Fisher information gain of the centroiding in the pixel grid [15,16].

Observability analysis was performed for the range of variance values given above and the times to become observable are shown in Figure 2. The times to become observable versus  $\log(\text{variance ratios})$  are fit well with a quartic polynomial as seen by the coefficient of determination ( $R^2$ ) value. Using five different Geosynchronous Orbits (GEO), observability has

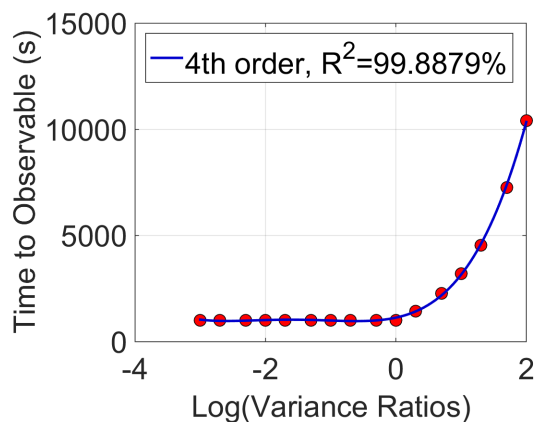


Figure 2: Exemplary geostationary object continuous observability analysis with different variance ratios and including a quartic fit [25].

been investigated for the regular state, and extended state vector by area-to-mass ratio and for the known, unobservable case of independent determination of area-to-mass ratio and reflection coefficient, see Figure 3 and Table 1. The last column of Table 1 gives the times for the seventh singular value of the AMR &  $C$  extended state case to cross the tolerance line. It clearly shows the the observability analysis potent to sort out the cases where knowledge is clearly absent, and also is a direct tool for estimation; more measurements are needed in order to observe the extended state. It hence directly lends its way to use in consider filter and sensor tasking.

Table 1: Times to become observable for the GEO objects with extended states and without measurement noise, times given in hours (Friedman, Frueh 2016,2017).

Orbit #	$[\mathbf{r}; \mathbf{v}]$	AMR	(AMR· $C$ )	AMR & $C$ (7th S)
1	0.47	0.51	0.51	0.54
2	4.20	4.29	4.29	4.37
3	2.97	3.08	3.08	3.18
4	4.87	4.96	4.96	5.04
5	1.39	1.48	1.48	1.57

On problem that remains for observability analysis over longer time spans is the

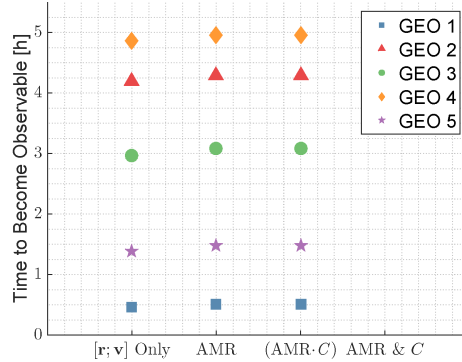


Figure 3: Five GEO objects featuring different inclinations without measurement noise, discrete analysis [24,25].

non-linearity of the orbit problem. This is particularly relevant for sensor tasking over an extended time horizon. Furthermore, the use of observability theory for further characterization, such as light curve inversion is subject to further research.

### Maneuver Detection and Constraints: Scheeres

A series of papers and PhD theses were developed that focused on the detection, estimation and incorporation of maneuver and other change events in a space object’s orbit. Two different approaches were taken, one based on representation of the event and the other on developing a new approach to estimation in the presence of uncertain maneuvers.

The thesis and papers by Dr. Ko modeled unknown events by a 6-dimensional representation that effectively interpolates between two states. Background theoretical work was able to show that the state transition matrix for the nominal trajectory from such a simplified interpolation accurately matches the actual state transition matrix, meaning that it can be used in estimation and other filtering approaches. This representation method was then applied to develop new algorithms that allow for the detection of an event relative to an assumed background level of noise and orbit determination uncertainty. It was also shown to be effective in allowing a maneuvering vehicle to be tracked through such a change in its state, even when the maneuver occurs when the satellite is not tracked (under some basic assumptions). The method was also demonstrated on tracking a maneuvering vehicle. This approach enables a simple and efficient manner in which a maneuvering vehicle can be tracked, its past trajectory reconstructed, and its future ballistic trajectory predicted. Papers documenting this research are found in [8, 9, 10, 11].

The thesis and papers by Dr. Lubey developed a new approach to orbit determination filtering, incorporating thrusting profiles into the nominal dynamics and solving them as an optimal control problem. The theory developed a linearized filtering approach that was shown to have improved properties as compared to existing approaches that use process noise to artificially boost filter sensitivity. The optimal control filter derived by Lubey is able to produce a least-squares, optimal acceleration profile that represents any maneuvers or unmodeled forces acting on a tracked space vehicle. It is also able to detect maneuver events relative to a background threshold noise level. Conference papers that document these advances are [28, 29, 30, 31, 32].

## Personnel Supported

- D.J. Scheeres: direct salary support.
- K.T. Alfriend: direct salary support.
- C. Frueh: direct salary support.
- J.W. McMahon: Assistant Research Professor, direct salary support
- Daniel Lubey: Graduate Student Researcher, PhD Student  
Advisor: D.J. Scheeres  
Thesis: Maneuver Detection and Reconstruction in Data Sparse Systems with an Optimal Control Based Estimator  
Defense date: November 2015.
- Antonella Albuja Graduate Student Researcher, PhD Student  
Advisor: D.J. Scheeres  
Thesis: Rotational Dynamics of Inactive Satellites as a Result of the YORP Effect  
Defense date: November 2015.
- Hyun Chul “Ddard” Ko Graduate Student Researcher, PhD Student  
Advisor: D.J. Scheeres  
Thesis: Representation of Unknown and Unmodeled Space Events for Satellites: Characteristics and Applications  
Defense date: November 2015.
- In-Kwan Park Graduate Student Researcher, PhD Student  
Advisor: D.J. Scheeres  
Thesis: Dynamical Realism and Uncertainty Propagation  
Defense date: December 2015. Subsequent Post-Doc at Texas A&M with K.T. Alfriend.
- Nicola Baresi Graduate Student Researcher, PhD Student  
Advisor: D.J. Scheeres  
Thesis: Spacecraft Formation Flight on Quasi-periodic Invariant Tori  
Defense date: May 2017.
- PhD Candidates/MS students supported or contributing to research:  
Alex Friedman, Advisor C. Frueh  
Francois Sanson, Advisor C. Frueh  
Conor Benson, Advisor D.J. Scheeres  
Marielle Pellegrino, Advisor D.J. Scheeres  
Chandrakanth Venigalla, Advisor D.J. Scheeres

## Transitions

1. Our approach to modeling non-gravitational forces is being further developed by Space-NAV, a local Boulder-based SSA company.

## Achievements

1. Scheeres was elected to the National Academy of Engineering, Class of 2017.
2. Scheeres named a Distinguished Professor of The University of Colorado by the Board of Regents, 2014.

## Publications

The following publications were supported, at least in part, by the AFOSR grant.

### Peer-Reviewed Journals

1. A. Albuja, D.J. Scheeres, R.L. Cognion, W. Ryan and E.V. Ryan. 2018. "The YORP Effect on the GOES 8 and GOES 10 Satellites: A Case Study," *Advances in Space Research* 61: 122-144.
2. A. Albuja, D.J. Scheeres and J.W. McMahon. 2015. "Evolution of Angular Velocity for Defunct Satellites as a Result of YORP: An Initial Study," *Advances in Space Research* 56: 237-251.
3. N. Baresi and D.J. Scheeres. 2017. "Bounded relative motion under zonal harmonics perturbations," *Celestial Mechanics and Dynamical Astronomy* 127(4): 527-548.
4. N. Baresi and D.J. Scheeres. "Design of Bounded Relative Trajectories in the Earth Zonal Problem," *Journal of Guidance, Control, and Dynamics*, in press 5/2017
5. A. Friedman, C. Frueh, Determining Characteristics of Artificial Near-Earth Objects, Using Observability Analysis, *Acta Astronautica* Volume 144, March 2018, Pages 405-421, DOI: 10.1016/j.actaastro.2017.12.028
6. K. Fujimoto and D.J. Scheeres. 2015. "Tractable Analytical Expressions for Non-Linearly Propagated Uncertainties," *Journal of Guidance, Control and Dynamics* 38(6): 1146-1151. doi: 10.2514/1.G000795
7. M.J. Holzinger, D.J. Scheeres and J. Hauser. 2015. "Reachability Using Arbitrary Performance Indices," *IEEE-Transactions on Automatic Control* 60(4): 1099-1103.
8. H.C. Ko and D.J. Scheeres. 2016. "Tracking Maneuvering Satellite Using Thrust-Fourier-Coefficient Event Representation," *Journal of Guidance, Control and Dynamics* 39(11): 2551-2559.
9. H.C. Ko and D.J. Scheeres. 2016. "Maneuver Detection with Event Representation using Thrust-Fourier-Coefficients," *Journal of Guidance, Control and Dynamics* 39(5): 1080-1091.
10. H.C. Ko and D.J. Scheeres. 2016. "Orbit Determination Across Unknown Maneuvers Using The Essential Thrust-Fourier-Coefficients," *Acta Astronautica* 118: 90-95.
11. H.C. Ko and D.J. Scheeres. 2015. "Event Representation-Based Orbit Determination Across Unknown Space Events," *Journal of Guidance, Control and Dynamics* 38(12): 2351-2365.

12. J.W. McMahon and D.J. Scheeres. 2015. "Improving Space Object Catalog Maintenance Through Advances in Solar Radiation Pressure Modeling," *Journal of Guidance, Control and Dynamics* 38(8), 1366-1381.
13. I.-K. Park, K. Fujimoto and D.J. Scheeres. 2015. "Effect of Dynamical Accuracy for Uncertainty Propagation of Perturbed Keplerian Motion," *Journal of Guidance, Control and Dynamics* 38(12): 2287-2300. doi: 10.2514/1.G000956
14. I.-K. Park and D.J. Scheeres. 2018. "Hybrid Method for Uncertainty Propagation of Orbital Motion," *Journal of Guidance, Control and Dynamics* 41(1): 240-254.
15. F. Sanson, C. Frueh, Space Object Position Uncertainty Quantification in Non-Resolved Images. In *Advances in Space Research*, submitted April 2018
16. F. Sanson, C. Frueh, Noise Estimation and Probability of Detection in nonresolved Images: Application to Space Object Observation. In *Journal of Astronautical Sciences*, submitted Sep. 2017.
17. J. Worthy, M.J. Holzinger and D.J. Scheeres. "An Optimization Approach for Observation Association with Systemic Uncertainty Applied to Electro-Optical Systems," *Advances in Space Research*, in press 2/2018.

### Conference Papers and Presentations

18. A.A. Albuja and D.J. Scheeres. "Representation of Short Period Variations in an Inactive Satellite's Rotational State Due to the YORP Effect," paper presented at the International Symposium on Space Technology and Science, July 2015.
19. A. Albuja and D.J. Scheeres. "Short Period Variations in Angular Velocity and Obliquity of Inactive Satellites Due to the YORP Effect," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-264
20. A. Albuja, R. Cognion, W. Ryan, E. Ryan and D.J. Scheeres. "Rotational Dynamics Of The Goes 8 And Goes 10 Satellites Due To The Yorp Effect," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Napa Valley, California, February 2016. Paper AAS 16-416
21. Alfriend, K.T. and Park, I., "When Does the Uncertainty Become Non-Gaussian?," 2016 AMOS Technology Conference, Wailea, HI, Sept. 2016.
22. Benson C, Scheeres DJ, Ryan WH, Ryan EV, Moskovitz N. "Rotation State Evolution of Retired Geosynchronous Satellites." *Advanced Maui Optical and Space Surveillance (AMOS) Technologies Conference*, 2017
23. Benson C, Scheeres DJ, Moskovitz N. "Light curves of retired geosynchronous satellites." *Proceedings of the 7th European Conference on Space Debris*, 2017
24. A. Friedman, C. Frueh, Determining Debris Characteristics from Observability Analysis of Artificial Near-Earth Objects, *European Conference on Space Debris*, Darmstadt, Germany, April 18-21, 2017.
25. A. Friedman, C. Frueh, Observability Analysis Applied to Artificial Near-Earth Objects with Realistic Noise, *Space Flight Mechanics Meeting*, San Antonio, TX, Feb 5-9, 2017.

26. H.C. Ko and D.J. Scheeres. "Orbit Determination and Maneuver Detection Using Event Representation with Thrust-Fourier-Coefficients," paper presented at the 2015 AMOS Conference, Wailea, Maui, September 2015.
27. H.C. Ko and D.J. Scheeres. "Maneuver Detection with Event Representation using Thrust-Fourier-Coefficients," paper presented at the AAS/AIAA Astrodynamics Meeting, Vail, Colorado, August 2015. Paper AAS 15-631
28. D. Lubey, A. Doostan and D.J. Scheeres. "Estimating Object-Dependent Natural Orbital Dynamics with Optimal Control Policies: A Validation Study," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-252
29. D. Lubey and D.J. Scheeres. "Robust Tracking and Dynamics Estimation with the Automated Optimal Control Based Estimator," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-251
30. D. Lubey and D.J. Scheeres. "Automated State and Dynamics Estimation in Dynamically Mismodeled Systems with Information From Optimal Control Policies," paper presented at the 18th International Conference on Information Fusion, Washington, D.C., July 2015. Paper AAS 15-252
31. D.P. Lubey and D.J. Scheeres. "Towards Real-Time Maneuver Detection: Automatic State and Dynamics Estimation with the Adaptive Optimal Control Based Estimator," paper presented at the 2015 AMOS Conference, Wailea, Maui, September 2015.
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# AFOSR Deliverables Submission Survey

Response ID:9976 Data

1.

**Report Type**

Final Report

**Primary Contact Email**

Contact email if there is a problem with the report.

scheeres@colorado.edu

**Primary Contact Phone Number**

Contact phone number if there is a problem with the report

7205441260

**Organization / Institution name**

University of Colorado Boulder

**Grant/Contract Title**

The full title of the funded effort.

Modeling Observability and Change Detection in Space Situational Awareness

**Grant/Contract Number**

AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-14-1-0348

**Principal Investigator Name**

The full name of the principal investigator on the grant or contract.

Daniel J. Scheeres

**Program Officer**

The AFOSR Program Officer currently assigned to the award

Stacie Williams

**Reporting Period Start Date**

09/01/2014

**Reporting Period End Date**

03/31/2018

**Abstract**

The objective of this study was to perform basic research into the observable consequences of natural forces and commanded actions (such as changes in orbit or attitude) for resident space objects (RSO) in Earth orbit. By studying and developing improved models of these activities, the goal was to be able to identify their observable consequences and devise optimal ways in which they can be detected or constrained based on existing astrometric and radiometric observation techniques. These research goals were chosen as they would advance the application of the scientific method in Space Situational Awareness (SSA) and support the development of unambiguous observation strategies to properly characterize space objects.

**Distribution Statement**

This is block 12 on the SF298 form.

Distribution A - Approved for Public Release

**Explanation for Distribution Statement**

If this is not approved for public release, please provide a short explanation. E.g., contains proprietary information.

DISTRIBUTION A: Distribution approved for public release.

## SF298 Form

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[AFOSR\\_SSA\\_final.pdf](#)

**Upload a Report Document, if any. The maximum file size for the Report Document is 50MB.**

### Archival Publications (published) during reporting period:

Journal Papers:

1. A. Albuja, D.J. Scheeres, R.L. Cognion, W. Ryan and E.V. Ryan. 2018. "The YORP Effect on the GOES 8 and GOES 10 Satellites: A Case Study," *Advances in Space Research* 61: 122-144.
2. A. Albuja, D.J. Scheeres and J.W. McMahon. 2015. "Evolution of Angular Velocity for Defunct Satellites as a Result of YORP: An Initial Study," *Advances in Space Research* 56: 237-251.
3. N. Baresi and D.J. Scheeres. 2017. "Bounded relative motion under zonal harmonics perturbations," *Celestial Mechanics and Dynamical Astronomy* 127(4): 527-548.
4. N. Baresi and D.J. Scheeres. "Design of Bounded Relative Trajectories in the Earth Zonal Problem," *Journal of Guidance, Control, and Dynamics*, in press 5/2017
5. A. Friedman, C. Frueh, Determining Characteristics of Artificial Near-Earth Objects, Using Observability Analysis, *Acta Astronautica* Volume 144, March 2018, Pages 405-421, DOI: 10.1016/j.actaastro.2017.12.028
6. K. Fujimoto and D.J. Scheeres. 2015. "Tractable Analytical Expressions for Non-Linearly Propagated Uncertainties," *Journal of Guidance, Control and Dynamics* 38(6): 1146-1151. doi: 10.2514/1.G000795
7. M.J. Holzinger, D.J. Scheeres and J. Hauser. 2015. "Reachability Using Arbitrary Performance Indices," *IEEE-Transactions on Automatic Control* 60(4): 1099-1103.
8. H.C. Ko and D.J. Scheeres. 2016. "Tracking Maneuvering Satellite Using Thrust-Fourier-Coefficient Event Representation," *Journal of Guidance, Control and Dynamics* 39(11): 2551-2559.
9. H.C. Ko and D.J. Scheeres. 2016. "Maneuver Detection with Event Representation using Thrust-Fourier-Coefficients," *Journal of Guidance, Control and Dynamics* 39(5): 1080-1091.
10. H.C. Ko and D.J. Scheeres. 2016. "Orbit Determination Across Unknown Maneuvers Using The Essential Thrust-Fourier-Coefficients," *Acta Astronautica* 118: 90-95.
11. H.C. Ko and D.J. Scheeres. 2015. "Event Representation-Based Orbit Determination Across Unknown Space Events," *Journal of Guidance, Control and Dynamics* 38(12): 2351-2365.
12. J.W. McMahon and D.J. Scheeres. 2015. "Improving Space Object Catalog Maintenance Through Advances in Solar Radiation Pressure Modeling," *Journal of Guidance, Control and Dynamics* 38(8), 1366-1381.
13. I.-K. Park, K. Fujimoto and D.J. Scheeres. 2015. "Effect of Dynamical Accuracy for Uncertainty Propagation of Perturbed Keplerian Motion," *Journal of Guidance, Control and Dynamics* 38(12): 2287-2300. doi: 10.2514/1.G000956
14. I.-K. Park and D.J. Scheeres. 2018. "Hybrid Method for Uncertainty Propagation of Orbital Motion," *Journal of Guidance, Control and Dynamics* 41(1): 240-254.
15. F. Sanson, C. Frueh, Space Object Position Uncertainty Quantification in Non-Resolved Images. In *Advances in Space Research*, submitted April 2018
16. F. Sanson, C. Frueh, Noise Estimation and Probability of Detection in nonresolved Images: Application to Space Object Observation. In *Journal of Astronautical Sciences*, submitted Sep. 2017.
17. J. Worthy, M.J. Holzinger and D.J. Scheeres. "An Optimization Approach for Observation Association with Systemic Uncertainty Applied to Electro-Optical Systems," *Advances in Space Research*, in press 2/2018.

Conference papers:

18. A.A. Albuja and D.J. Scheeres. "Representation of Short Period Variations in an Inactive Satellite's
- DISTRIBUTION A: Distribution approved for public release.

- Rotational State Due to the YORP Effect," paper presented at the International Symposium on Space Technology and Science, July 2015.
19. A. Albuja and D.J. Scheeres. "Short Period Variations in Angular Velocity and Obliquity of Inactive Satellites Due to the YORP Effect," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-264
  20. A. Albuja, R. Cognion, W. Ryan, E. Ryan and D.J. Scheeres. "Rotational Dynamics Of The Goes 8 And Goes 10 Satellites Due To The Yorp Effect," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Napa Valley, California, February 2016. Paper AAS 16-416
  21. Alfriend, K.T. and Park, I., "When Does the Uncertainty Become Non-Gaussian?," 2016 AMOS Technology Conference, Wailea, HI, Sept. 2016.
  22. Benson C, Scheeres DJ, Ryan WH, Ryan EV, Moskovitz N. "Rotation State Evolution of Retired Geosynchronous Satellites." Advanced Maui Optical and Space Surveillance (AMOS) Technologies Conference, 2017
  23. Benson C, Scheeres DJ, Moskovitz N. "Light curves of retired geosynchronous satellites." Proceedings of the 7th European Conference on Space Debris, 2017
  24. A. Friedman, C. Frueh, Determining Debris Characteristics from Observability Analysis of Artificial Near-Earth Objects, European Conference on Space Debris, Darmstadt, Germany, April 18-21, 2017.
  25. A. Friedman, C. Frueh, Observability Analysis Applied to Artificial Near-Earth Objects with Realistic Noise, Space Flight Mechanics Meeting, San Antonio, TX, Feb 5-9, 2017.
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  27. H.C. Ko and D.J. Scheeres. "Maneuver Detection with Event Representation using Thrust-Fourier-Coefficients," paper presented at the AAS/AIAA Astrodynamics Meeting, Vail, Colorado, August 2015. Paper AAS 15-631
  28. D. Lubey, A. Doostan and D.J. Scheeres. "Estimating Object-Dependent Natural Orbital Dynamics with Optimal Control Policies: A Validation Study," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-252
  29. D. Lubey and D.J. Scheeres. "Robust Tracking and Dynamics Estimation with the Automated Optimal Control Based Estimator," paper presented at the AAS/AIAA Spaceflight Mechanics Meeting, Williamsburg, Virginia, January 2015. Paper AAS 15-251
  30. D. Lubey and D.J. Scheeres. "Automated State and Dynamics Estimation in Dynamically Mismodeled Systems with Information From Optimal Control Policies," paper presented at the 18th International Conference on Information Fusion, Washington, D.C., July 2015. Paper AAS 15-252
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**New discoveries, inventions, or patent disclosures:**

**Do you have any discoveries, inventions, or patent disclosures to report for this period?**

No

**Please describe and include any notable dates**

**Do you plan to pursue a claim for personal or organizational intellectual property?**

**Changes in research objectives (if any):**

None

**Change in AFOSR Program Officer, if any:**

Original officer was Julie Moses, current officer is Stacie Williams.

**Extensions granted or milestones slipped, if any:**

One NCE was provided, from September 2017 to March 2017

**AFOSR LRIR Number**

**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, \$K)**

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

**Report Document**

**Report Document - Text Analysis**

**Report Document - Text Analysis**

**Appendix Documents**

**2. Thank You**

**E-mail user**

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