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We have carefully studied the problem of elastodynamics cloaking in both linear and nonlinear regimes. We present a mathematically precise definition of cloaking an object from elastic waves and prove a proposition. This result will be the mathematical foundation of our future work on cloaking object from stress waves.

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Final Report: Nonlinear Elastodynamics Cloaking

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We have carefully studied the problem of elastodynamics cloaking in both linear and nonlinear regimes. We present a mathematically precise definition of cloaking an object from elastic waves and prove a proposition. This result will be the mathematical foundation of our future work on cloaking object from stress waves.

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Names of Post Doctorates

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Scientific Progress

Since 2006 it has been claimed in the literature that the governing equations of linear elasticity (Navier's equations) are not form-invariant under coordinate transformations. One immediate consequence of this claim has been that transformation cloaking is not possible for linear elasticity. In this work, we show that this claim is false. Staring from nonlinear elasticity, we first show how the governing equations are invariant under both arbitrary time-dependent spatial and time-independent referential coordinate transformations. We next show that the same invariance property holds when one linearizes the governing equations of nonlinear elasticity with respect to any finitely-deformed configuration. In particular, the governing equations of classical linear elasticity are invariant under both spatial and referential coordinate transformations. We then formulate the problem of cloaking of elastic waves in both nonlinear and linear elastodynamics. In particular, it is noted that a cloaking transformation is neither a spatial nor a referential change of frame (coordinates); a cloaking transformation maps the boundary-value problem of an isotropic and homogeneous body with a desired mechanical response (virtual problem) to that of an anisotropic and inhomogeneous body that is to be designed (physical problem). The virtual body has a desired mechanical response while the physical body is designed to mimic the same response outside the cloak using the cloaking transformation. Finally, we present some examples of elastic cloaks.

Technology Transfer

Final Report for Nonlinear Elastodynamics Cloaking

ARO GRANT NUMBER W911NF-16-1-0064

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1 Summary

During this 9-month project, we studied the tools that we would need in formulating the elastodynamics cloaking problem. By cloaking of an object, e.g., an inclusion, to stress waves, we mean designing a cover (cloak) that deflects the elastic waves as if the object did not exist. To achieve this, it is first essential to investigate how the presence of an inclusion affects the stress and deformation fields in its vicinity, and in the matrix it is contained. As a particular example, we studied the stress field of a solid torus made of an incompressible isotropic solid containing a toroidal inclusion that is concentric with the solid torus and has a finite uniform distribution of pure dilatational eigenstrains [Golgoon and Yavari, 2017b]. Using a perturbation analysis, we calculated the residual stresses to the first order in the thinness ratio (defined as the ratio of the radius of the generating circle and the overall radius of the solid torus). Specifically, we showed that the stress field inside the inclusion is not uniform. This result is in contrast with the corresponding results for infinitely-long and finite circular cylindrical bars and spherical balls containing cylindrical and spherical inclusions, respectively.

As the elastic properties of the elastic cloak is anisotropic and inhomogeneous, in general, it is necessary to study the inclusion problem in nonlinear anisotropic solids as well. To this end, we investigated nonlinear inclusions in anisotropic solids. Particularly, we considered finite eigenstrains in orthotropic cylindrical bars and transversely isotropic spherical balls in the case of both incompressible and compressible solids [Golgoon and Yavari, 2017a]. We showed that the stress field in a spherical inclusion with uniform pure dilatational eigenstrain contained in a spherical ball made of an incompressible transversely isotropic solid with the radial material preferred direction is uniform and hydrostatic. Similarly, if the radial and circumferential eigenstrains are equal and the axial stretch is equal to a value given by the axial eigenstrain, the stress inside a cylindrical inclusion in an incompressible orthotropic cylindrical bar is uniform hydrostatic. Also, we proved that for a compressible isotropic spherical ball and a cylindrical bar containing a spherical and a cylindrical inclusion, respectively, with uniform eigenstrains if the radial and circumferential eigenstrains are equal the stress in the inclusion is uniform (and hydrostatic in the case of spherical inclusion). For compressible transversely isotropic and orthotropic solids, we found that the stress field in the inclusion with uniform eigenstrain, in general, is not uniform. Nonetheless, in some special cases, the material can be designed such that a uniform stress field is maintained in the inclusion. To investigate such special cases, as particular examples, we considered compressible Mooney-Rivlin and Blatz-Ko reinforced models and found analytical expressions for the stress field in the inclusion.

There have been many works on elastodynamics cloaking in the past ten years. None of these works has properly formulated this problem. The following abstract from a paper [Yavari and Golgoon, 2017] that is in preparation summarizes our progress in this exciting field: "Since 2006 it has been claimed in the literature that the governing equations of linear elasticity (Navier's equations) are not form-invariant under coordinate transformations. One immediate consequence of this claim has been that transformation cloaking is not possible for linear elasticity. In this paper, we show that this claim is false. Staring from nonlinear elasticity, we first show how the governing equations are invariant under both arbitrary time-dependent spatial and time-independent referential coordinate transformations. We next show that the same invariance property holds when one linearizes the governing equations of nonlinear elasticity with respect to any finitely-deformed configuration. In particular, the governing equations of classical linear elasticity are invariant under both spatial and referential coordinate transformations. We then formulate the problem of cloaking of elastic waves in both nonlinear and linear elastodynamics. In particular, it is noted that a cloaking transformation is neither a spatial nor a referential change of frame (coordinates); a cloaking transformation maps the boundary-value problem of an

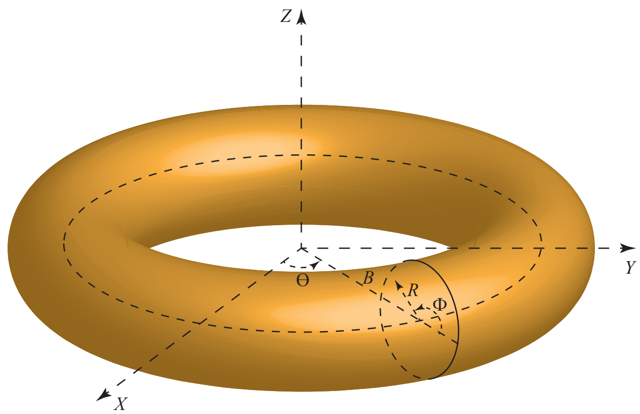

Figure 1: *A solid torus and its toroidal coordinates in the undeformed configuration.*

isotropic and homogeneous body with a desired mechanical response (virtual problem) to that of an anisotropic and inhomogeneous body that is to be designed (physical problem). The virtual body has a desired mechanical response while the physical body is designed to mimic the same response outside the cloak using the cloaking transformation. Finally, we present some examples of elastic cloaks."

2 Overview of the Important Results

In this section, we briefly review some important results obtained in [Golgoon and Yavari, 2017a,b]. Section 2.1 deals with the nonlinear inclusion problem in the case of a solid torus as one of the simplest examples of non-simply connected bodies.[1] Nonlinear inclusion problem in anisotropic solids is discussed in section 2.2.

2.1 A nonlinear inclusion in a solid torus as a non-simply connected body

Let us consider a solid torus generated by rotating a circle with radius R_o about a line in its plane, where the distance from the origin to the center of the circle is B. Let us denote the material and spatial toroidal coordinates with (R, Θ, Φ) and (r, θ, ϕ), respectively, as shown in Figure 1. In the toroidal coordinates (R, Θ, Φ), the material metric for the torus with an axially-symmetric (Θ-independent) eigenstrain distribution reads

$$\mathbf{G} = e^{\Omega(R,\Phi)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & (B + R\cos\Phi)^2 & 0 \\ 0 & 0 & R^2 \end{pmatrix}, \tag{2.1}$$

where $\Omega(R, \Phi)$ is a function representing the inhomogeneous dilatational eigenstrain distribution in the torus. We then consider a class of axially-symmetric deformations of the form

$$r = r(R, \Phi), \quad \theta = \Theta, \quad \phi = \phi(R, \Phi). \tag{2.2}$$

Next, we restrict our attention to the radially-symmetric dilatational eigenstrain distributions such that $\Omega = \Omega(R)$. Furthermore, we assume that $\frac{r}{b}$ and $\frac{R}{B}$ are small enough such that the higher powers of $\frac{r}{b}$ can be ignored, and find the solutions to the first order in the thinness ratio $\varepsilon = \frac{R_o}{B} \ll 1$. The zero-order problem, i.e., when $\varepsilon \to 0$, is equivalent to the case in which the torus becomes a cylinder with the cylindrically-symmetric distribution of purely dilatational eigenstrains, where, in the cylindrical coordinates, $r = r(R)$, $\phi = \Phi$, $z = \frac{b}{B}Z$, and $p = p(R)$ (Lagrange multiplier or pressure corresponding to the incompressibility constraint). After solving the perturbation problem, we simplify the governing equations (see [Golgoon and Yavari, 2017b] for details) in the neo-Hookean case and consider the following distribution of eigenstrains in the torus

$$\Omega(R) = \begin{cases} \Omega_o, & 0 \le R < R_i \\ 0, & R_i < R \le R_o \end{cases}. \tag{2.3}$$

[1]Isotropic inclusion problem in cylindrical and spherical geometries have been addressed in [Yavari and Goriely, 2013].

In this case, we find closed form solutions for the stress and deformations fields in the torus.

In a nutshell, we used a perturbation analysis, and to the first order in the thinness ratio, we obtained the stress and displacement fields. We observed that the stress field in the toroidal inclusion is nonuniform, as opposed to cylindrical and spherical inclusions in infinitely-long and finite circular cylindrical bars and spherical balls, respectively, where the stress field inside the inclusion is uniform. For a neo-Hookean solid torus containing an inclusion with a uniform pure dilatational eigenstrain distribution, we presented some numerical results. Particularly, we found that all the first-order stress components in the inclusion depend linearly on the referential radial coordinate. Furthermore, the maximum shear stress in the torus is first increasing, then decreasing as the relative size of the inclusion increases from zero. We observed that the shear stress has some concentration regions across the inclusion-matrix interface in the case of a torus with a negative pure dilatational eigenstrain distribution. It is interesting to note that the torus exhibits different responses for positive and negative eigenstrain values. We observed that for the positive eigenstrians, b/B monotonically increases as the eigenstrain Ω_o increases such that the increase is more rapid for inclusions with larger relative sizes. Nevertheless, for negative eigesntrains, the ratio b/B reaches a minimum such that its value decreases as the relative size of the inclusion becomes larger. Also, we found that the deformed shape of the outer boundaries of the matrix and the inclusion are eccentric circles with their radii being equal to their corresponding zero-order radii in the first-order approximation in the thinness ratio. We finally proved that the stress field in a toroidal inclusion with nonzero uniform infinitesimal pure dilatational eigenstrains in an isotropic incompressible linear elastic solid torus with any size is always nonuniform.

2.2 Nonlinear anisotropic inclusions

We studied several examples of inclusions in transversely isotropic spherical balls and orthotropic cylindrical bars. Starting with spherically and cylindrically symmetric distributions of finite dilatational eigenstrains in a spherical ball and a solid cylinder, respectively, we studied the anisotropic inclusion problem considering uniform distribution of finite anisotropic eigenstrains in the inclusion region. For instance, the material metric for the ball with dilatational eigenstrains in the spherical coordinates (R, Θ, Φ) is given as

$$\mathbf{G} = \begin{pmatrix} e^{2\omega_R(R)} & 0 & 0 \\ 0 & R^2 e^{2\omega_\Theta(R)} & 0 \\ 0 & 0 & e^{2\omega_\Theta(R)} R^2 \sin^2\Theta \end{pmatrix}, \tag{2.4}$$

where ω_R and ω_Θ describe the radial and circumferential eigenstrains, respectively. Similarly, in the cylindrical coordinates (R, Θ, Z), we construct the following material metric for the bar with eigenstrains

$$\mathbf{G} = \begin{pmatrix} e^{2\omega_R(R)} & 0 & 0 \\ 0 & R^2 e^{2\omega_\Theta(R)} & 0 \\ 0 & 0 & e^{2\omega_Z(R)} \end{pmatrix}, \tag{2.5}$$

where ω_R, ω_Θ, and ω_Z are functions representing the radial, circumferential, and axial eigenstrains, respectively. Next, we consider the following distribution of eigenstrains corresponding to a cylindrical inclusion with radius R_i along the axis of the cylindrical bar with radius R_o.

$$\omega_R(R) = \begin{cases} \omega_1\,, & 0 \le R \le R_i \\ 0\,, & R_i \le R \le R_o \end{cases}, \quad \omega_\Theta(R) = \begin{cases} \omega_2\,, & 0 \le R \le R_i \\ 0\,, & R_i \le R \le R_o \end{cases}, \quad \omega_Z(R) = \begin{cases} \omega_3\,, & 0 \le R \le R_i \\ 0\,, & R_i \le R \le R_o \end{cases}. \tag{2.6}$$

We investigated conditions under which the stress field inside the inclusion is uniform. Also, we identified those cases exhibiting stress singularities, depending on the values of the radial and circumferential eigenstrains, as well as the axial eigenstrain in the case of cylindrical bars. We proved the following propositions in [Golgoon and Yavari, 2017a]:

Proposition 2.1. *Consider a nonlinear incompressible transversely isotropic spherical ball with the material preferred direction being radial at any material point. Assume that the ball is subject to a uniform pressure on its boundary. Suppose that a spherical inclusion with uniform radial and circumferential eigenstrains is contained at the center of the ball. The stress field inside the inclusion exhibits a logarithmic singularity at the origin unless the radial and circumferential eigenstrains are equal or the energy function is special. Furthermore, the stress inside the inclusion is uniform and hydrostatic if the eigenstrains are pure dilatational.*

Proposition 2.2. *Let a spherical ball be made of a compressible isotropic solid subject to a uniform pressure on its boundary sphere. Suppose that the ball contains a spherical inclusion at its center with uniform radial and circumferential eigenstrains. The stress field inside the inclusion is uniform and hydrostatic if the eigenstrains are pure dilatational.*

Proposition 2.3. *Consider a finite incompressible orthotropic elastic solid cylinder, where the material orthotropic axes lie in the radial, circumferential, and longitudinal directions of the cylinder. Assume that the bar is subject to a uniform pressure on its boundary cylinder and contains an inclusion along its axis with uniform radial, circumferential, and longitudinal eigenstrains. The Cauchy stress exhibits a logarithmic singularity at the centerline of the cylinder unless the radial and circumferential eigenstrains are equal and the axial stretch* α *is equal to* e^{ω_3} *or the energy function is special. If the radial and circumferential eigenstrains are equal and* $\alpha = e^{\omega_3}$*, then the stress inside the inclusion is uniform and hydrostatic.*

Proposition 2.4. *Let a cylindrical bar be made of a compressible isotropic solid subject to a uniform pressure on its boundary cylinder. Suppose that a cylindrical inclusion with uniform radial, circumferential, and axial eigenstrains is contained along the axis of the bar. The stress field inside the inclusion is uniform if the radial and circumferential eigenstrains are equal.*

In summary, we have shown that the stress field in a spherical inclusion with uniform eigenstrains placed in an incompressible transversely isotropic spherical ball with the radial material preferred direction is uniform and hydrostatic if the radial and circumferential eigenstrains are equal. Similarly, this result holds for cylindrical inclusions contained in incompressible orthotropic cylindrical bars with orthotropic axes being radial, circumferential, and axial, provided that the axial stretch is equal to a value given by the longitudinal eigenstrain. Except in some special cases that the energy function is constrained depending on the eigenstrains, in the case of incompressible solids, a logarithmic singularity in the stress field emerges due to a mismatch between radial and circumferential eigenstrains at the center of a ball or the centerline of a bar.

In addition, we generalized the results given in [Yavari and Goriely, 2013] pertaining to some special classes of compressible isotropic materials to any compressible isotropic material. Specifically, we proved that for compressible isotropic spherical balls and cylindrical bars with spherical and cylindrical inclusions, respectively, with uniform eigenstrains, if the radial and circumferential eigenstrains are equal, the stress field inside the inclusion is uniform (and hydrostatic in the spherical case).

Moreover, we observed that for compressible transversely isotropic and orthotropic solids, the stress field in the inclusion with uniform dilatational eigenstrains is not uniform, in general. We showed, nevertheless, that there are some special cases, for which for a given applied pressure on the outer boundary, the ratio R_o/R_i is determined in order for a uniform stress field to be maintained in the inclusion. Analogously, in such special cases, fixing R_o/R_i uniquely determines the pressure that has to be applied on the outer boundary to maintain a uniform stress field in the inclusion. Additionally, material parameters need to satisfy certain conditions depending on the eigenstrains (and the axial stretch in the case of cylindrical inclusions).

2.3 Nonlinear and linear elastodynamics cloaking

Let us consider a body $\mathcal{B}$ with a hole $\mathcal{H}$ (see Fig.2). An object is placed inside $\mathcal{H}$ and needs to be hidden from elastic waves. The hole is reinforced by a cloaking region $\mathcal{C}$, which we can assume is an annulus (or spherical annulus). Elastic properties and mass density of $\mathcal{C}$ are, in general, inhomogeneous and anisotropic. For the sake of simplicity and without loss of generality, let us assume that in $\mathcal{B} \setminus \mathcal{C}$ the body is homogeneous and isotropic. This means that the mass density ρ_0 is a constant and the body has an energy function $W = W(I_1, I_2, I_3)$, where I_i are the principal invariants of the left or right Cauchy-Green strain. Motion of $\mathcal{B}$ is represented by a map $\varphi_t : \mathcal{B} \to \mathcal{S}$ in Fig.2. A spatial diffeomorphism leaves the governing equations form invariant. However, spatial diffeomorphisms (spatial coordinate transformations) would not be useful in designing a cloak because a change of spatial coordinates is nothing but looking at the same problem using a different observer. The cloaking transformation is assumed to be a time-independent map $\Xi : \mathcal{B} \to \tilde{\mathcal{B}}$ such that the annulus $\mathcal{C}$ is transformed to a disk with a very small hole (or a spherical ball with a very small hole in 3D). The mapping Ξ is assumed to be the identity in $\mathcal{B} \setminus \mathcal{C}$. It is also assumed that the virtual body (the virtual structure) has the uniform and isotropic mechanical properties of $\mathcal{B} \setminus \mathcal{C}$.

We have been able to prove the following proposition for the construction of an elastic cloak in a nonlinear elastic solid.

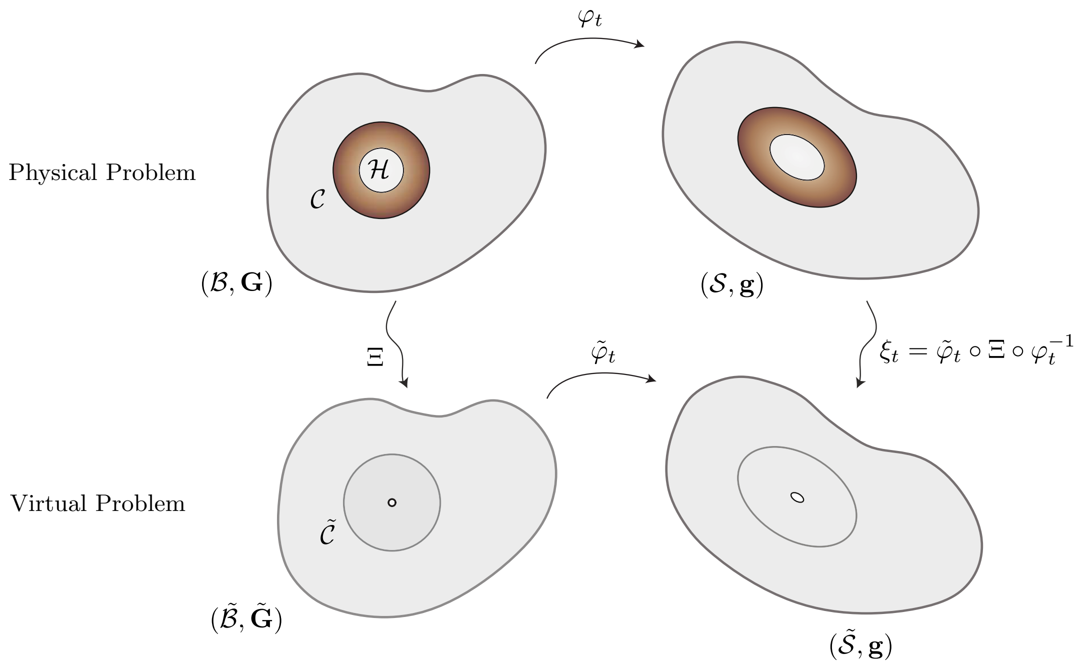

Figure 2: *The mapping Ξ is a cloaking transformation. It transforms a body with a hole $\mathcal{H}$ to another body that is homogeneous and isotropic with an infinitesimal hole. The cloaking transformation is defined to be the identity map outside the cloaking region $\mathcal{C}$. Note that Ξ is not a referential change of coordinates and ξ is not a spatial change of coordinates.*

Proposition 2.5. *Consider a diffeomorphism $\Xi : \mathcal{B} \to \tilde{\mathcal{B}}$ that shrinks a hole in the physical body $\mathcal{B}$ to an infinitesimal hole in the virtual body $\tilde{\mathcal{B}}$. The hole is surrounded by a cloak $\mathcal{C}$ in the physical body $\mathcal{B}$. Assume that $\Xi|_{\mathcal{B}\setminus\mathcal{C}} = id$ and that on the outer boundary of the cloak $T\Xi = id$. Assume that the (finite) displacement vectors in the physical and virtual bodies, mass densities, body forces, and the first Piola-Kirchhoff stresses are related as $\tilde{\mathbf{U}} = \mathbf{U} \circ \Xi^{-1}$, $\tilde{\rho}_0 = J_\Xi^{-1} \rho_0$, $\tilde{\mathbf{B}} = \mathbf{B} \circ \Xi^{-1}$, and $\tilde{\mathbf{P}} = J_\Xi^{-1} \mathbf{P} \mathbf{F}_\Xi^\star$ (or, equivalently, $\mathbf{S} = J_\Xi \mathbf{F}^{-1} \tilde{\mathbf{F}} \, \tilde{\mathbf{S}} \, \mathbf{F}_\Xi^{-\star}$). The virtual body is assumed to be homogeneous and isotropic. The constitutive equations of the physical body inside the cloak are anisotropic and inhomogeneous, in general, and are given by*

$$\mathbf{S}_\mathcal{C} = 2 J_\Xi \mathbf{F}^{-1} \left[\tilde{\mathbf{I}} + (\mathbf{F} - \mathbf{I}) \mathbf{F}_\Xi^{-1} \right] \left\{ \tilde{W}_{\tilde{I}_1} \tilde{\mathbf{G}}^\sharp + \tilde{W}_{\tilde{I}_2} \left(\tilde{I}_2 \tilde{\mathbf{C}}^{-1} - \tilde{I}_3 \tilde{\mathbf{C}}^{-2} \right) + \tilde{W}_{\tilde{I}_3} \tilde{I}_3 \tilde{\mathbf{C}}^{-1} \right\} \mathbf{F}_\Xi^{-\star} . \tag{2.7}$$

Outside the cloak the physical body is homogeneous and isotropic and has a constitutive equation identical to that of the virtual body. We assume that the two problems have identical boundary conditions on the outer boundaries of $\mathcal{B}$ and $\tilde{\mathcal{B}}$, and the physical hole and the virtual hole are both traction free. Under the above assumptions the two boundary-value problems are equivalent. In other words, the governing equations of the physical problem are satisfied if and only if those of the virtual problem are satisfied. In addition, the two bodies have identical current configurations outside the cloak $\mathcal{C}$.

3 Personal Supported During Duration of Grant

Ashkan Golgoon   Graduate Student, Georgia Institute of Technology, Atlanta

4 Journal Publications Acknowledging the ARO Grant

1) A. Yavari and A. Golgoon, Nonlinear and linear elastodynamics transformation cloaking, in preparation.

1) A. Golgoon and A. Yavari, Nonlinear elastic inclusions in anisotropic solids, *Journal of Elasticity*, DOI: 10.1007/s10659-017-9639-0.

2) A. Golgoon and A. Yavari, On the stress field of a nonlinear elastic solid torus with a toroidal inclusion. *Journal of Elasticity* **128**(1), 2017, pp. 115-145.

3) A. Yavari and A. Goriely, The anelastic Ericksen's problem: Universal eigenstrains and deformations in compressible isotropic elastic solids. *Proceedings of the Royal Society A* **472**, 2016, 20160690.

4) S. Sadik and A. Yavari, Small-on-large geometric anelasticity. *Proceedings of the Royal Society A* **472**, 2016, 20160659.

5) F. Sozio and A. Yavari, Nonlinear mechanics of surface growth for cylindrical and spherical elastic bodies, *Journal of the Mechanics and Physics of Solids* **98**, 2017, pp. 12-48.

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A. Golgoon and A. Yavari. On the stress field of a nonlinear elastic solid torus with a toroidal inclusion. *Journal of Elasticity*, 128:115–145, 2017b.

A. Yavari and A. Golgoon. Nonlinear and linear elastodynamics transformation cloaking. 2017.

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