AIMdyn, Inc.
Engineering Consulting & Software Development

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Project Title: Analysis and Prediction of Sea Ice Evolution using Koopman Mode Decomposition Techniques
Subject: Monthly Progress Report
Period of Performance: July 31, 2018 – August 30, 2018

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AIMdyn, Inc. respectfully submits Progress Report 5 for contract N00014-18-P-2004. Please direct any Technical questions on this report to the undersigned.
V/r

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Business Status Report

(1) Resource Status:
Resourced to plan.

(2) Contributions by AIMdyn Inc:
See technical report.

(3) Resource status VS Original schedule:
On plan to original scope of work. Effort is fixed priced.

PROGRAM FINANCIAL STATUS

Work Breakdown	Cumulative to Date	At Completion
CLIN#0001-CLIN#0005	$93,435	$112,125

Structure or Task Element	Planned Expend	Actual Expend	% Budget Compl	At Compl	Latest Revised Estimate	Remark Remark
CLIN#0005	$18,687	$18,687	83.3%	100 %	N/A	N/A

Subtotal: $18,687
Management Reserve: N/A
Or Unallocated Resources: N/A
TOTAL: $18,687

Technical Status Report

Abstract

Prediction of sea ice concentration in particular geographic regions was performed using Koopman Mode Decomposition. The predicted sea ice concentrations were in good agreement with actual data for many regions.

ACCOMPLISHMENTS

Summary

Koopman Mode Decomposition (KMD) was used to predict sea ice concentration in particular seas and other geographic regions in the northern and southern hemispheres. Previous prediction work in this project was applied to the entire region covered by the available data in each hemisphere. By focusing on particular geographic regions, the successes and shortcomings of the KMD prediction approach are most visible. It is found that the predictions are most accurate for regions with large seasonal variations in sea ice concentration and show good general agreement in most cases.

Introduction

KMD reconstruction techniques were applied to the prediction of future sea ice concentrations in particular geographic regions. Because the dynamics of the seasonal variation in sea ice concentration differ greatly between high latitude regions, which have nearly complete sea ice coverage in the winter and retain some sea ice through the summer, and lower latitude regions, which do not necessarily reach complete sea ice coverage in the winter nor retain any sea ice through the summer, it is of interest to examine the KMD prediction results in particular geographic regions. To that end, the maps of the NSIDC Sea Ice Index for the northern and southern hemispheres were divided into approximate seas and other geographic regions (see Figures 1 and 2) and the KMD prediction results for each region were examined separately.

The focus in this month was visualizing the "goodness" of the KMD prediction results by comparison with actual sea ice concentration data for the time period the prediction was performed over.

Methods

As described in the previous report, reconstruction of the N_p sea ice concentration pixel values $\mathbf{C}$ at discrete time step k is performed using the Koopman eigenvalues λ_j and the Koopman modes $\mathbf{v}_j$ obtained from applying KMD to the concentration values over N time steps (months, in this case):

$$\mathbf{C}_k = \sum_{j=1}^{N} \lambda_j^{k-1} \mathbf{v}_j$$

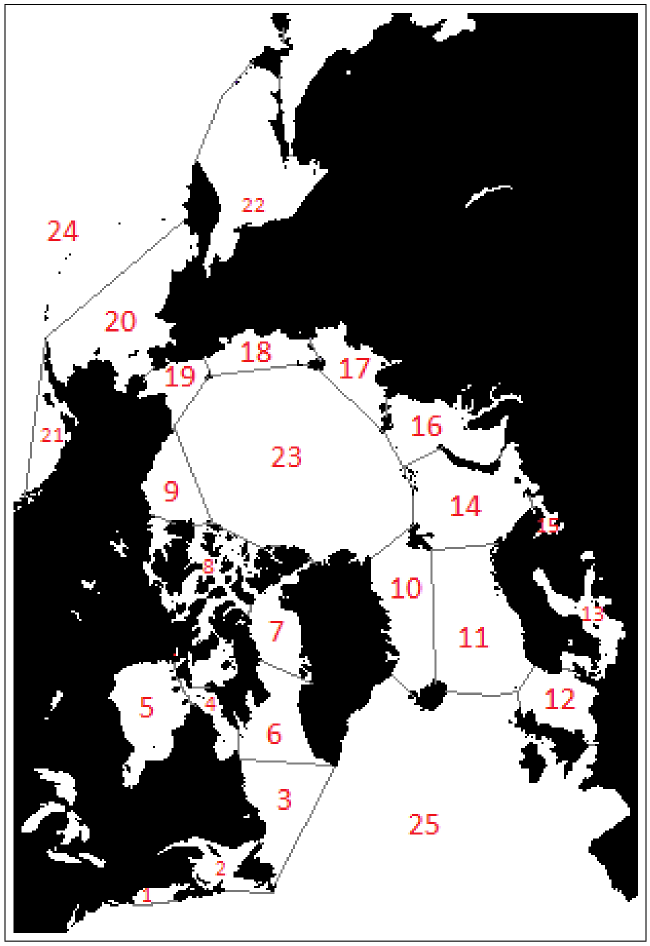

Figure 1: Northern hemisphere geographic regions considered in reconstruction results shown here. 1: Bay of Fundy, 2: Gulf of St Lawrence, 3: Labrador Sea, 4: Hudson Strait, 5: Hudson Bay, 6: Davis Strait, 7: Baffin Bay, 8: Northwestern Passages, 9: Beaufort Sea, 10: Greenland Sea, 11: Norwegian Sea, 12: North Sea, 13: Baltic Sea, 14: Barents Sea, 15: White Sea, 16: Kara Sea, 17: Laptev Sea, 18: East Siberian Sea, 19: Chukchi Sea, 20: Bering Sea, 21: Gulf of Alaska, 22: Sea of Okhotsk, 23: Arctic Ocean, 24: North Pacific Ocean, 25: North Atlantic Ocean.

Here, there are N Koopman eigenvalues and Koopman modes, where each Koopman eigenvalue is a single complex number and each Koopman mode has dimensions 1 by N_p.

For $1 \leq k \leq N$, $\mathbf{C_k}$ is termed a reconstruction of the kth time step in the original data $\mathbf{C}$, as the Koopman eigenvalues and modes came from a decomposition of the observations over this time range and should simply reproduce the data used as input to the KMD. For $k > N$, $\mathbf{C_k}$ is a prediction of the future behavior of the sea ice concentration for the (future) kth time step, based on the system dynamics deduced from decomposition of earlier observations.

KMD reconstruction was performed on all data from the given hemisphere, then the pixels corresponding to each region were identified and compared with actual data (when available). Note that an alternate approach, not shown here, is to apply KMD to only the data from a particular region and then perform reconstruction for that region using

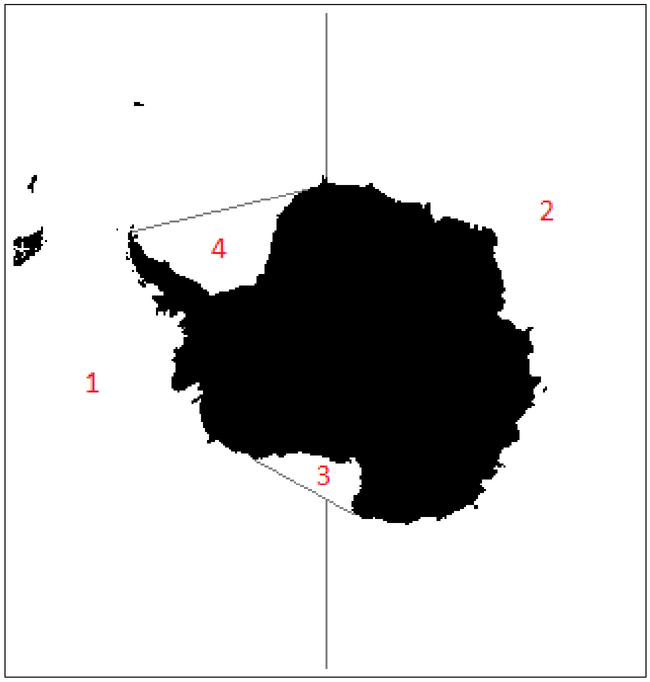

Figure 2: Southern hemisphere geographic regions considered in reconstruction results shown here. 1: West Antarctica, 2: East Antarctica, 3: Ross Sea, 4: Weddell Sea.

those KMD results. It was found that the predictions produced using only the data from a particular region were less accurate than predictions from applying KMD to the entire hemisphere of data, presumably due to the lower dimensionality of the former case.

Two methods were applied to visualize the “goodness” of the KMD reconstruction results. The first was based on a direct pixel-to-pixel comparison of the actual data and prediction results. In this case, the calculation consisted of, for each month and for each region, taking the mean of the absolute value of the difference between the actual data and prediction results for all pixel values for which either the actual data or prediction results contained a sea ice concentration value greater than zero. The reason for restricting consideration to nonzero concentration pixels was to avoid giving a misleading impression of the goodness of the prediction results, particularly when there is a large fraction of ice-free pixels in lower latitude regions and in the summer. The second visualization method was to plot the mean sea ice concentration value of the actual data and prediction results in each region. This approach does not judge the exact geographic precision of the prediction results but provides a broader sense of whether the predictions match the actual increase or decrease of sea ice in a particular region.

Results and Discussion

Figure 3 shows the actual mean sea ice concentrations for the 39 year period of data for the northern hemisphere regions considered here. The breakdown of the sea ice concentration into different regions shows the geographic variation in dynamics. Some regions, such as the Hudson Strait, Hudson Bay, and Chukchi Sea, consistently vary between nearly 100% sea ice concentration in the winter and 0% coverage in the summer, while most other regions show greater year-to-year variation in maximum or minimum sea ice concentration. The regions in this latter case are of greatest interest for prediction techniques, as it is desirable to, for example, correctly predict the future behavior of the summer sea ice coverage in the Arctic Ocean (bottom row, center in Figure3 and subsequent northern hemisphere figures) .

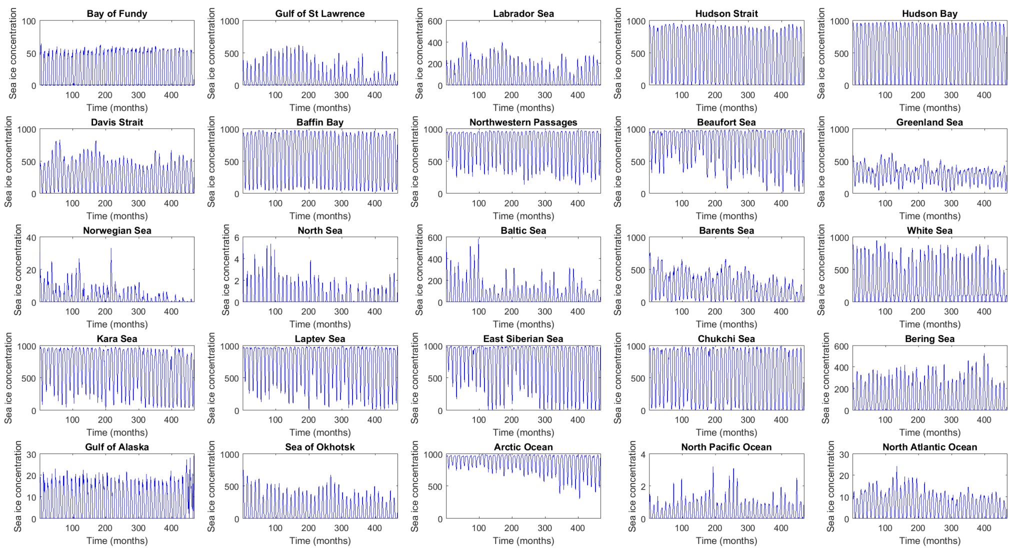

Figure 3: Time series of monthly average sea ice concentration (1000=100%) for northern hemisphere regions.

Figure 4 shows the actual mean sea ice concentrations for the 39 year period of data for the southern hemisphere regions. The character of the southern hemisphere differs from the northern hemisphere, as expected due to the large scale geographical differences. The ocean regions off of West Antarctica and East Antarctica show consistent seasonal variation, and the Ross Sea behaves somewhat like an Arctic sea with its large consistent seasonal variations, but the Weddell Sea shows a large degree of year-to-year variability in mean sea ice concentration.

Figure 5 shows the results for the first method of judging the goodness of the prediction (the mean of the absolute difference between the actual and predicted results), for the northern hemisphere for predictions for 2014-2017 using the previous 30 years of data. This plot shows, in effect, the expected error of the prediction for a given pixel in each region. Seasonal variation is apparent, but interestingly the error does not always increase year to year. The error is also generally largest in the seas around the Arctic Ocean (the Kara, Laptev, East Siberian, Chukchi, etc.) where the seasonal variation in sea ice concentration is most pronounced.

Figure 6 shows a different view of the goodness of the northern hemisphere prediction, showing the mean of the actual data (blue lines) and predictions (red lines) for each region. This shows the general trends of sea ice concentration in each region, rather than being a pixel-to-pixel comparison of the actual and predicted results. Here we see that the various

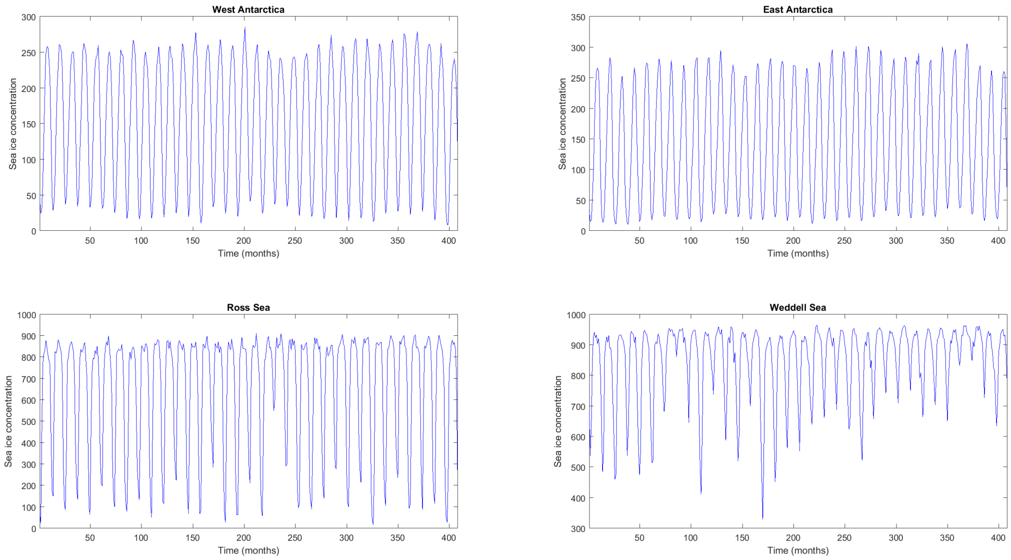

Figure 4: Time series of monthly average sea ice concentration (1000=100%) for southern hemisphere regions.

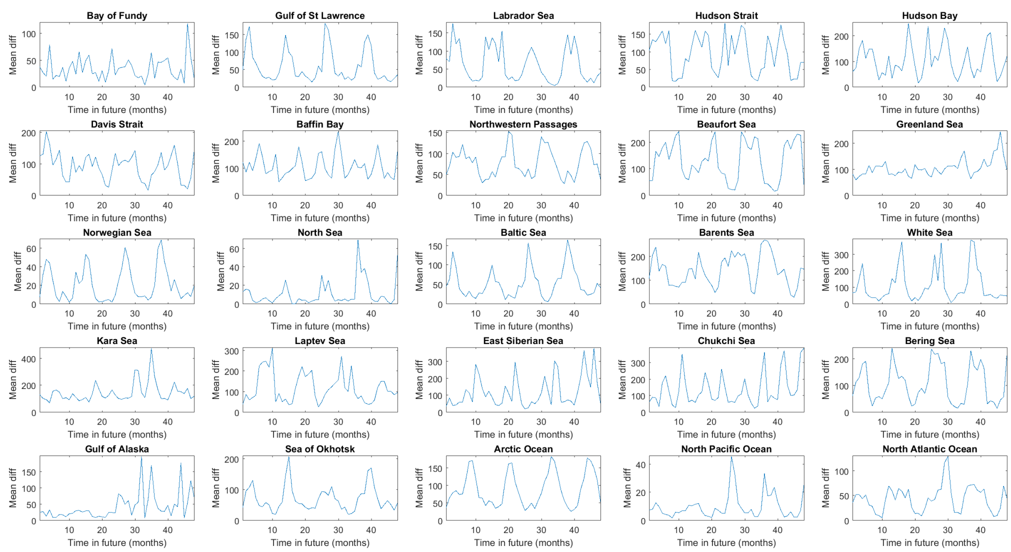

Figure 5: Goodness of northern hemisphere prediction results, based on summing the absolute value of the difference between the actual data and prediction results for all pixel values for which either the actual data or prediction results contained a sea ice concentration value greater than zero. Prediction is based on KMD of 30 year data range (1984 to 2013) and prediction of four future years. Vertical axis units are sea ice concentration (1000=100%).

Arctic seas mentioned above and other regions with large seasonal variations show good agreement between the actual and predicted results. This implies that while the prediction may not always be geographically precise in predicting the distribution of sea ice concentration within a particular region, it is generally successful at predicting the average sea ice concentration within the region. Close examination of regions of interest, particularly the Arctic Ocean, shows that the prediction does not always closely match the summer minimum in sea ice concentration, but in this case the prediction does match the trend of the actual result (i.e., the summer minimum decreases year to year for the first three years, then it increases in the fourth year).

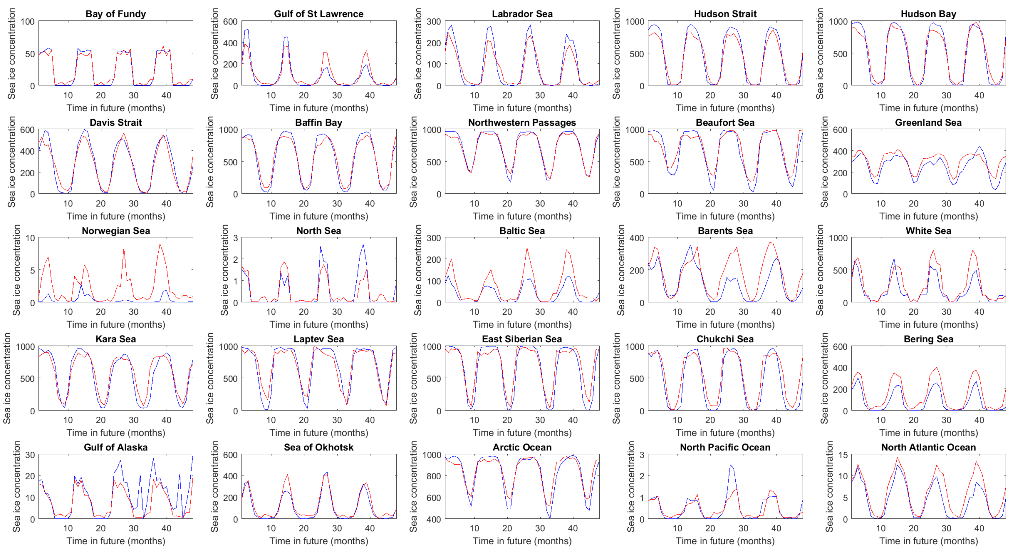

Figure 6: Comparison of actual (blue) and predicted (red) northern hemisphere mean sea ice concentration. Prediction is based on KMD of 30 year data range (1984 to 2013) and prediction of four future years. Vertical axis units are sea ice concentration (1000=100%).

Figures 7 and 8 show the same goodness measures for the southern hemisphere. Similar comments apply here as in the northern hemisphere, where the predictions for some regions are generally close to the actual data and some show greater deviations.

Conclusions

The examination of predictions of future sea ice concentration in particular geographic regions using Koopman Mode Decomposition was found to work well for some regions, as Arctic seas, and produce useful predictions for the Arctic Ocean. Future work is planned to consist of organization of results for publication.

Personnel Supported

Dr. Maria Fonoberova, Dr. Igor Mezic, Dr. James Hogg

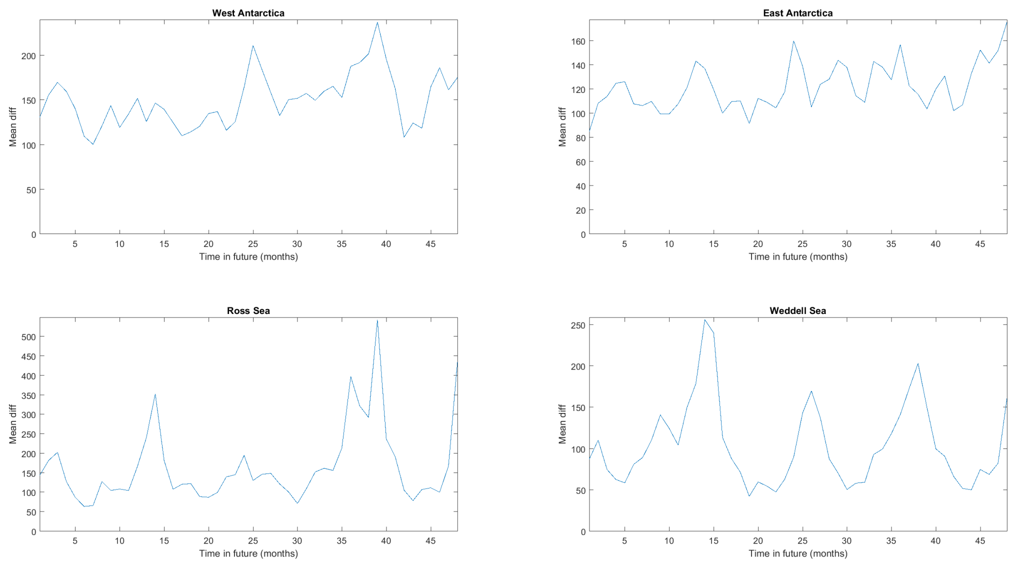

Figure 7: Goodness of southern hemisphere prediction results, based on summing the absolute value of the difference between the actual data and prediction results for all pixel values for which either the actual data or prediction results contained a sea ice concentration value greater than zero. Prediction is based on KMD of 30 year data range (1984 to 2013) and prediction of four future years. Vertical axis units are sea ice concentration (1000=100%).

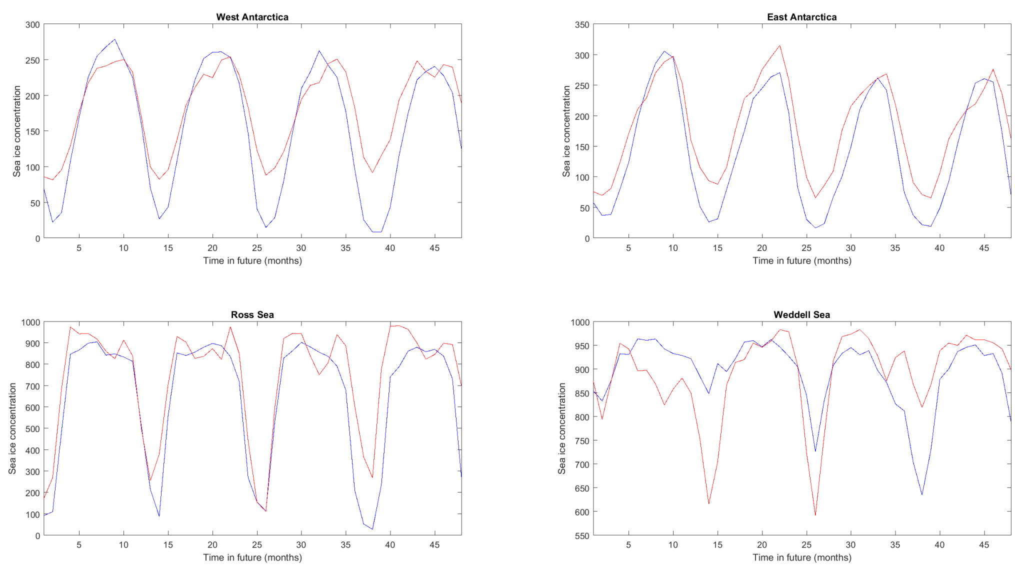

Figure 8: Comparison of actual (blue) and predicted (red) southern hemisphere mean sea ice concentration. Prediction is based on KMD of 30 year data range (1984 to 2013) and prediction of four future years. Vertical axis units are sea ice concentration (1000=100%).