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**Further Developments in the Global Resolution of Convex Programs with Complementarity Constraints**

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Final Report**

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<b>14. ABSTRACT</b> This projects aim to study the class of convex programs with complementarity constraints for the design of algorithms for their global and local resolution and to investigate applications of such problems. The major accomplishments during the reporting period are several fold. Specifically, we have (1) completed a comprehensive investigation of the difference-of-convex approach for solving a linear program with linear complementarity constraints; this is the first step in utilizing this approach for nonlinear programs with same constraints; (2) developed a logical Benders based algorithm for the global resolution of convex quadratic programs with complementarity constraints, possibly with additional binary variables.					
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Final Report: AFOSR FA9550-15-1-0126

Report period: March 15, 2015 to July 15, 2018

Principal Investigator: **Jong-Shi Pang**, University of Southern California

For the duration of this grant, we have

- (1) completed a comprehensive investigation of the difference-of-convex approach for solving a linear program with linear complementarity constraints; this is the first step in utilizing this approach for nonlinear programs with same constraints;
- (2) developed a logical Benders based algorithm for the global resolution of convex quadratic programs with complementarity constraints, possibly with additional binary variables; connection of the method with branch-and-bound procedures is made and improved sparsification schemes are developed;
- (3) investigated sparse optimization problems based on its formulation using complementarity constraints and carried out numerical experimentation with solving these formulations using existing nonlinear programming solvers that are publicly available;
- (4) studied certain bi-parametric linear complementarity constrained linear programs that arose from the parameter selection in support vector machine regression; and
- (5) developed models and designed solution methods for biofuel supply chain design under land use constraints formulated as bilevel programs with integer locational variables.

In addition, we have presented our results in international conferences including the triennial International Symposium on Mathematical Programming and the Institute for Operations Research and Management Science annual meetings. We have worked with graduate students on the research of this award and written joint papers with them. We have published seven papers and supervised two doctoral thesis on work supported by this grant.