REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY) 27-04-2017	2. REPORT TYPE Final Report	3. DATES COVERED (From - To) 1-May-2016 - 30-Apr-2017

4. TITLE AND SUBTITLE
Final Report: Workshop proposal for consideration by the Complex Dynamics and Systems Program, ARO -- The Future of Vibration Energy Transfer in Solids and Structures: Needs and Opportunities

5a. CONTRACT NUMBER
W911NF-16-1-0293

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
611102

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S)
ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
68282-EG-CF.1

12. DISTRIBUTION AVAILIBILITY STATEMENT
Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT

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15. SUBJECT TERMS
Vibration, dynamics, acoustics, solids, structures, nonlinearity, instability

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Nicholas Boechler
UU	UU	UU	UU		19b. TELEPHONE NUMBER 206-221-6515

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Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

See attached final workshop report (conference proceedings).

Technology Transfer

Final Report: The Future of Vibration Energy Transfer in Solids and Structures: Needs and Opportunities Workshop

18-19 October 2016 at the University of Washington, Seattle, WA, USA

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Executive Summary

This report describes the outcomes of an early-career workshop, held at the University of Washington from October 18-19, 2016, where the participants sought to identify essential research needs, emergent concepts, and collaborative opportunities from the wide field of vibration energy transfer in solids and structures. Particular emphasis was placed on systems involving nonlinearity and instability, and forward-looking and out-of-the-box ideas were strongly encouraged. The focus of this two-day workshop was on identifying and formulating critical current fundamental research challenges, major open questions, and future opportunities, and identifying symbiotic, cross-disciplinary concepts that promise transformative advancements in knowledge and performance potential. Topics included snapping and multistable systems, amplitude-dependent dynamics, experimental challenges and opportunities (material characterization and fabrication), design and modeling challenges, the interplay of nonlinearity with new phenomena in linear systems such as topological insulators, and the dynamics of related biological systems. A list of several critical, future, open questions identified as part of this workshop are detailed at the conclusion of the report.

1 Introduction and Motivation

Vibrational energy transfer in solids and structures has been an area of significant research interest for several decades, with recent focus areas that include novel material systems such as acoustic metamaterials and phononic crystals, and proposed applications extending from energy harvesting to signal processing. While much of this research has been associated with linear dynamics, recent developments have suggested there are untapped opportunities arising in connection with nonlinearity and instability. Such nonlinear and multistable systems include materials that involve multifunctional, active or living components, or that mimic atomic features at mesoscopic structural scales. While the latter has become popular for linear mechanical phenomena such as acoustic dispersion relations, its extension to nonlinear concepts is still in an early stage.

For many of the aforementioned emergent concepts, classical theories, existing numerical tools and experimental capabilities are insufficient, calling for new, exploratory research. It is through this early-career workshop, held at the University of Washington, Seattle, Washington, from October 18-19, 2016, that we sought to identify essential research needs and emergent concepts – as well as collaborative opportunities – from the wide field of vibration energy transfer in solids and structures. The focus of this two-day workshop was on identifying and formulating critical current fundamental research challenges, major open questions, and future opportunities – not only for new research directions but also by the symbiotic, cross-disciplinary combination of mechanical, physical, chemical and biological concepts that promise transformative advancements in knowledge and performance potential. The workshop discussed how such research directions may be exploited to advance our understanding of the rich dynamics of complex (meta) material systems, and their potential future benefits for U.S. Army missions.

The two-day workshop, which brought together 18 early-career U.S. faculty members as well as ARO representatives Samuel C. Stanton and David M. Stepp, featured short presentations by all participants. In addition, extended break-out sessions enabled discussions and brain-storming to foster exciting concept visions and identify the grand challenges that lay ahead for the scientific communities engaged in research on topics of vibration energy transfer in solids and structures.

Topics ranged from snapping or multistable systems and amplitude-dependent dynamics to experimental challenges and opportunities (both concerning the experimental characterization of high-speed dynamic phenomena and metamaterial fabrication by additive manufacturing and self-assembly), new and exciting phenomena in linear systems (such as topological insulators and advanced wave guides or atomic-scale, phononic energy transfer) and in nonlinear systems (from wood stacks and helical metamaterials to 3D-printed mechanical logic gates), all the way to related biological systems (such as the time-dependent evolution of large ant populations). Since particular emphasis was placed on systems involving nonlinearity and instability, a recurring feature of most concepts was the deliberate 'leaving behind' of classical, linear wave mechanics. Exceptionally innovative, forward-looking and out-of-the-box ideas were strongly encouraged and featured in presentations and discussions.

The outcomes of this workshop are summarized in the following sections, which align with major themes that emerged during the workshop, namely: nonlinearity and instabilities, advancing fundamental understanding, transformative new ideas, and grand challenges.

2 Roles of Nonlinearities and Instabilities

In recent years, we have witnessed scientists and engineers exploit nonlinearities and instabilities to successfully enhance the performance of materials and structural systems, often achieving extraordinary properties that were not possible in conventional linear and/or stable systems. This is in sharp contrast to traditional approaches in the design of materials/structures, in which nonlinearities and instabilities were considered to be roadblocks that should be avoided. Here we briefly summarize opportunities enabled by the use of nonlinearities and instabilities.

2.1 Storage and release of energy

One of the most straightforward uses of instability is to store input elastic energy in a manner equivalent to energy absorption. The superb capabilities of instable and/or multi-stable lattices for energy absorption have been reported in references [1] [2] [3]. In recent years, the unstable nature of such engineered material systems has been exploited not only for the absorption of energy, but for the designed release of energy, revealing highly interesting dynamic phenomena. One example is the propagation of solitons in a one-dimensional lattice made of bistable, post-buckled beam components [4]. More recently, the propagation of such nonlinear waves was enabled in soft material systems [5]. This is a notable achievement, since soft materials are conventionally considered not to be suitable for forming and propagating stress waves due to their high dissipation. Rapid actuation in soft materials was also achieved exploiting energy release in snap-buckling [6]. Another noticeable point of interest in this research direction is that multi-stable structures can reveal opportunities in nonlinearity (and nonconvexity) by generating nonlinear waves (e.g., solitons) triggered by the large deformation of the constituent materials/structures due to their intrinsic instability. Thus, instability and nonlinearity are closely linked with each other, and it will be an interesting subject of study to investigate the interplay between these two. Also, there is no doubt that nonlinear nature of materials/structures offer an enhanced degree of freedom in manipulating the dynamic responses of structures. In the next section, we briefly discuss the potential of un- and multistable/nonlinear systems for tuning their effective material properties.

2.2 Potential for tuning system properties

Previous studies have successfully shown that nonlinearities can enable new toolsets for tuning system and effective material properties. From the viewpoint of wave dynamics, one of the most advantageous features enabled by the nonlinearity is amplitude-dependence. This can imply a simple tunablility capability, e.g., changing frequency band structures of systems that depend on the amplitude of external excitations or propagating waves [7] [8]. But such amplitude-dependent properties can also affect speed, waveform, and even energy transmission efficiency of nonlinear waves [9], and perhaps even more broadly, spatiotemporal and spectral distribution of acoustic energy [10]. For instance, researchers have shown that amplitude-dependent properties enabled by nonlinearities can alter the landscape of wave localization in lattice structures [11], which can be exploited for energy harvesting applications (e.g., breathers characterized by spatially localized, temporally oscillating structures [12].

Another potential of nonlinearity from the dynamics standpoint is a feature of breaking reciprocity. By leveraging this feature, researchers have shown one-way transport of energy in the setting of nonlinear granular crystals [13]. The lessons learned can be possibly extended to different length scales of phononics, e.g., opening a new way to controlling heat transfer in micro- and nano-scales [14]. Along these lines, recent investigations have begun to investigate the dynamics of micro- to nanoscale analogs of such geometrically nonlinear structures [15].

In a similar vein, it has been shown that it is possible to transmit or block a selected range of frequencies in nonlinear, buckled structures by exploiting their tunable frequency bands, not only by leveraging the aforementioned amplitude-dependent property, but also by using geometrical nonlinearity of wave-guiding structures [16] [17]. These structures may be useful for numerous engineering applications, such as adaptive noise shielding materials for aircraft structures that undergo harsh vibrations with varying frequencies even during a single flight operation.

Last but not least, the key advantage of nonlinear systems is the richness of nonlinear wave structures that can be formed in such systems. Harmonic generations is a phenomenon that can be exploited for controlling the responses of nonlinear systems. However, there are a number of other possibilities, such as mode hopping and mixing [18]. In addition, the propagation of nonlinear waves has been reported in the context of topological metamaterials undergoing buckling [19]. In the nonlinear regime, energy cascades in temporal and spatial domains are also enabled, which is phenomenologically similar to turbulence generated in fluidic systems [20]. Such features can be highly useful in mitigating impact in structures not by relying on material damping or plasticity, but by redistributing energy in space/time and spectral domains.

2.3 Challenges associated with instability/nonlinearity

There are a number of challenges associated with implementing unstable/nonlinear features in engineered structures. They include the large deformations typically needed to achieve multi-stability, low system durability for long life-cycles (again stemming from the large deformations), inverse design of systems exhibiting the desired stability and nonlinearity characteristics, and finally the fabrication of such structures. Regarding fabrication, there could be several avenues to achieving efficient construction of such systems. One of them is the use of additive manufacturing (3D printing) technology [21]. Control of bistability [6] and buckling modes [22] have been demonstrated using 3D printing. This technique can also incorporate multi-functionality (e.g., piezoelectric actuating systems [23] [24] into 3D printed multi-stable nonlinear systems. Other platforms can be also explored, e.g., liquid crystalline elastomers for excellent energy dissipation properties, and composite materials with soft matrix materials enabling large deformations, self-healing and adapting materials such as mechanoresponsive polymers [25], or self-assembled geometrically nonlinear microstructured materials [11]. In the long run, biological systems (e.g., fire ants [26]) may also provide a unique avenue for designing materials with tailored nonlinear characteristics and unprecedented properties.

3 Advancing Fundamental Understanding: Challenges and Strategies for Overcoming Them

While the workshop underscored apparent potentials enabled by leveraging nonlinear phenomena, several challenges to effectively capitalize on such features by obtaining deep insight on the problems at hand were simultaneously identified. Additional challenges were also exemplified through presentations and discussions, including the appropriate use of continuum and lattice-type model frameworks (and approaches in between). Workshop participants also assessed ways by which such challenges may be overcome towards establishing new fundamental understanding of novel vibration and wave energy transport properties. The following sections summarize the key outcomes of the workshop pertaining to identified current challenges in fundamental understanding, and potential strategies to achieve these goals.

3.1 Establishing improved analytical tools and optimization methods

The recent evidence that nonlinearity and instabilities may cultivate enriched opportunities for tuning and tailoring vibration energy transport in solids and structures has motivated researchers to build beyond classic, linear theories of such dynamic phenomena to provide insight via analyses that account for the nonlinear characteristics. The use of instabilities, a subset of nonlinearity, poses particularly unique challenges since far-from-equilibrium behaviors are induced that are inherently not permitted in traditional nonlinear analysis based on perturbation methods [27]. Similar challenges may be posed for systems containing non-differentiable or discontinuous nonlinearities. Examples of this include bifurcations encountered near the loss of contact in granular crystals, used for instance in non-reciprocal acoustic devices [13]. For systems modeled effectively by low order representations, so as to be reduced to single or few degree-of-freedom (DOF) governing equations, new strategies are being established and experimentally validated for their efficacy to predict strongly nonlinear behavior ascribed to far-from-equilibrium dynamics, such as the snap-through of post-buckled structural elements [28]. While greatly simplifying the dynamic response, low order representations may provide useful guidance in the design and implementation of structural constituents, since such analyses are often orders of magnitude faster to compute than corresponding simulations. Such modeling practices have recently been extended to the study of periodic structures having constituents that may be far-from-equilibrium. For instance, analyses have predicted the capability for soliton propagation in lattice-type architecture of bistable constituents [29] thus cultivating unique non-reciprocal wave propagation behavior [4].

These efforts reveal that more advanced theoretical analysis may illuminate a domain of nonlinear response that previously was not amenable to clear understanding and who's access has been restricted to numerical simulation. Yet, the use of reduced order analysis must inevitably accommodate the fact that enticing possibilities are realized when considering large, multi DOF systems [30] [31]. Such strategies are of particular relevance in the research communities involved in nonlinear system identification [32], and may serve important roles to obtain fundamental insight on such high-dimensional systems [33] [34].

As the emergent potential to harness strong nonlinearities becomes evident, a need correspondingly emerges to establish approaches by which the deployment of such far-from-equilibrium behaviors may be optimized. This includes both in optimizing the type of the nonlinearity to a given performance metric, as well as the structural design needed to obtain such a response. Within the context of the latter, traditional topology optimization tools have been established for linear vibration and wave propagation behaviors [35]. A need exists to transition these concepts to the investigation of nonlinear response. Yet, the complexity of lattice compositions and desire to scrutinize the roles of geometry, since such features are relevant to experimental realizations [36], puts limits on the extension of traditional optimization tools to nonlinear characteristics of solids and structures. Fortunately, efforts are uncovering how to fashion topology optimization approaches to empower prescribed nonlinear properties [37], including features characteristic of instability mechanisms, and to provide guidance for the cultivation of novel wave-guiding behaviors including non-reciprocal propagation [38]. Nevertheless, much progress lay ahead for the rigorous establishment of optimization approaches that accommodate all realizations of far-from-equilibrium dynamics and non-unique response.

3.2 Limits of modeling methods between continuous and discrete systems

Mass-spring models, assumption of infinite domains, and continuum approximations are historical core foundations in the analytical evaluation and understanding of intriguing wave propagation behavior and energy transport [39]. Yet, in many settings, analysis by infinite domains of mass-spring constituents may not accurately replicate experimentally observed dynamic response [40]. As a result, balances need to be found between continuum and discrete analyses that delivers the requisite insight within computational cost practicality. For instance, shock wave development in strongly nonlinear granular media may be ineffectively predicted by continuum approximations, albeit delivering a more straightforward theoretical derivation, when compared to mass-spring models that more accurately represent the inevitably experimental platform [41]. Experimental validations are also needed to serve as essential assessment means for the efficacy of the outcomes.

3.3 Incorporation and accounting for defects and impurities in periodic systems

Understanding of vibration and wave energy transfer properties is often achieved analytically via the assumptions of periodicity and symmetries. Indeed, certain phenomena are inhibited if the periodicity is perturbed by defects within the repeated architecture. As a result, there are numerous motivations to explore the influences of asymmetries and impurities in the solids and structures under consideration. For instance, experimental evidence has revealed opportunities enabled by specifically leveraging such non-ideal compositions. Namely, unique propagation of mechanical waves can be induced by leveraging asymmetries in experimental platforms [5], while reflection-less transmission of waves through granular chains may still be achieved in the presence of multiple impurities [43]. Researchers have also characterized how the unique wave propagation behavior in lattices of bistable constituents [29] is influenced by the presence of defects [44], helping to classify the roles of clustered or isolated defects within the system. The existence and dynamic triggering of metastable states in a mechanical platform have also been analyzed with respect to the roles of asymmetry in inhibiting such phenomena or simplifying the picture of dynamic response [13] [45]. Emergent capabilities that are prohibited or disabled by defects demand greater study, particularly within the context of nonlinear dynamic vibration phenomena.

4 Emerging Themes with Transformative Potential

The workshop identified several novel, emerging themes with transformative potential, which may be grouped into four categories and will be summarized in the following.

4.1 Beyond classical metamaterials: from topological insulation to programmable solids

Mechanical analogs of topological insulators have attracted attention over the past few years and resulted in various designs of mechanical metamaterials that come with the typical features of topological insulation: breaking of symmetry and reciprocity, wave motion through edge states, and immunity to defects. Indeed, what has become popular in the quantum world is still in its infancy in the mechanical realm, where specific quantum mechanical Hamiltonians are mimicked by structural mass-spring systems. Examples include structural lattices whose nodes are equipped with gyroscopes [46], circular fluid motion [47], cross-links between duplicate lattices [48], or arrays of pendula [49], to name a few. Although these and more examples exist, additional fundamental understanding is needed. Promising approaches make use of well-understood system components such as vibrating plates and membranes, which may result in programmable wave guides [38] and can also include weak nonlinearity to break time-reversal symmetry. Breaking symmetry can also be achieved through a breaking of periodicity by a careful control of parametric gradients in the unit cell topology, geometry, and scale of graded structural metamaterials [13].

A recurring scheme of workshop presentations, the combination of regular and stochastic arrangements in a metamaterial can provide new ways to control wave propagation, including the independent control of wave motion at several different wavelength and frequency regimes through hierarchical or multiscale architecture (besides wave benefits, stochasticity can also produce beneficial combinations of strength and toughness of mechanical metamaterials). While classical phononic and photonic metamaterials have focused on wave dispersion in periodic lattices, the presence of defects, imperfections and random variations poses new challenges but also offers new, untapped opportunities. Another related, auspicious topic of recent interest is the exploration of wave motion at the smallest scales, viz. terahertz vibrations in atomic lattices [50]. The ultimate prospect is wave control spanning all time and length scales, from acoustics to the thermal regime (and all scales in between).

Nonlinearity is not only of interest within the context of instability and multi-welled energy landscapes but it can also enhance directivity in nonlinear lattices. For instance, by stretching the wave response of the system and thereby smearing it out over a broad frequency interval, one may strategically distribute the system response over all available modes and generate higher harmonics not found in classical, linear systems [51]. Tensegrity structures are one example system whose soft connections with tensioncompression asymmetry yield a controllable nonlinear response of interest both for wave direction and impact mitigation [52].

4.2 Active, reconfigurable and living (meta)materials

New concepts of active, reconfigurable or self-healing metamaterials are expected to offer new avenues for the generation of smart system responses. Several workshop contributions highlighted the potential of learning from biological systems. Large ant aggregations are an excellent example that behaves, in an effective-medium sense, as a meta-"material" halfway between fluid and solid [26]. It behaves like a viscoelastic or viscoplastic active granular medium that displays, e.g., nonlinear density waves and features of ideal gases (so it can be described by classical theories of active suspensions and particles in the presence of Brownian motion). This concept goes considerably beyond prior and most current approaches for "smart" impact mitigation.

Equally important to such innovative concepts is the development of new theories and models to describe their effective behavior. This is where reduced order models [53] gain importance, e.g. to learn from mechanical computation in nature. Classical theories have relied heavily on the continuum limit and work with a uniform unit cell topology. However, tendon networks (as one example in nature) are non-isotropic and rampant and may adapt to external loads, which requires memory and learning, whose fundamental understanding is largely lacking. Further examples of interesting biological dynamical systems recently studied include running cockroaches, or horse and dog motion on rough and flat terrains. An integral requirement, namely the robustness of the outcome (against imperfections or perturbations, flaws and system damage) is crucial, and once more, calls for techniques that can cope with random variations. A natural application lies in self-healing media (made of living or non-living elements).

4.3 New trends in additive manufacturing

Additive manufacturing techniques – from polymerization-based lithography to laser sintering – have gained ubiquitous importance and popularity across disciplines. While recent advances have enabled the manufacturing of finer and finer feature sizes, there is still a lack of techniques that can tackle the special requirements of smart metamaterials; also missing are advanced theoretical and computational tools to optimize "printable" metamaterial architectures for complex, nonlinear system responses.

On the experimental side, present obstacles include the scalability of additive manufacturing techniques, oftentimes confined to small volumes and printable volumes usually scaling with the minimum feature resolution (e.g., ultralight lattices showing high-resolution nano-scale features [54] but can print at most millimeter-sized objects, while microlattices can print meter-sized patches but the resolution lies on the millimeter-scale). This scalability obstacle is further amplified, when considering the need in many cases for large areas and volumes of metamaterial structure. Recent studies have explored the use of self-assembly for creating scalable acoustic metamaterial structures [55]. The separation of scales between macroscopic architecture and microscopic features may also be exploited in multiscale metamaterials if random or systematic defects at the small scales can be accounted for by an effective-medium theory at the macroscale. This could take advantage of fabrication tolerances and relax design constraints by providing robustness of the system response (this allows achieving macroscopic characteristics with reduced challenges in fabrication). Another topical challenge is the additive manufacturing of active and transformable structures by using stimuli-responsive base materials. The manufacturing/printing process typically destroys or, at least, interferes with the active control mechanism of the base material. Additive manufacturing using, e.g., hydrogels or shape memory polymers is therefore an open challenge with enormous potential. Similarly, the printing of active materials such as ferroelectric ceramics has only recently been tackled but offers tremendous potential for 3D-printed electric actuators that may provide self-adaptive functionality.

Besides new fabrication techniques, theoretical and computational tools are urgently needed to optimize the system response via tailored architecture. For example, 3D networks of structural building blocks with built-in snap-through instabilities were shown to provide superior energy absorption and impact mitigation capabilities, going beyond the well-studied quasistatic system response. The complex dynamics of the system calls for topology optimization methodologies that can deal with multiple materials, high dimensionality, structural hierarchy, geometric and material nonlinearity, strain rate- and path-dependence, coping with defects and imperfections, and optimizing strongly nonlinear and dynamic system behavior. Alternatives to topology optimization should be exploited by borrowing concepts from, e.g., combinatorics, the materials genome approach, big data tools, or complex, pre-defined libraries of unit cells.

4.4 Translational physics: sharing scientific principles across scales and disciplines

A hot topic of this workshop (also reflected in the grand challenges) is what was we coined translational physics, viz. the emergence of physical principles, governing equations or phenomena across length and/or time scales, and across various disciplines. As a recent example, nonlinear metamaterials that exploit snapping instability to produce nonlinear transition waves in periodic structures [5] were shown to follow the same Allen-Cahn governing equations [56] commonly used to describe the time-dependent evolution of domain walls at the atomic level of ferroelectrics, ferromagnetics and during phase separation. Here, two very different systems (structural metamaterials vs. atomic crystal lattices) at two distinct scales (at the atomic and mesoscopic levels vs. at the structural macroscale) and from two very different disciplines (solid state physics vs. engineering mechanics) obey seemingly similar system behavior. This can be (but has not been) exploited (i) to understand atomic-scale phenomena by instructive, structural-level experiments and (ii) to create structures and devices that reproduce phenomena commonly found in materials (e.g., for mechanical logic, soft robotics, impact mitigation, morphing surfaces, and more).

A similar correspondence was shown between governing equations describing cosmology vs. meta-materials, black hole evaporation vs. nonlinear waves in fireants.

The workshop participants discussed both advantages and risks associated with “translational physics” ideas. On the one hand, there is danger in associating the physics of very distinct systems with one another because principles that apply to one may not apply to the other. The common continuum limit used to predict the system response in a mathematically tractable fashion may apply well to some systems but may lead to wrong conclusions for others if discreteness effects overweigh or if active, smart system components (such as in ant aggregations) may disrupt the idea of a local continuum description using the principle of local action. Similarly, there is incomplete understanding about the continuum hypothesis in concepts where symmetry is broken and consistency may be lost. This calls for new theoretical and computational research that can exploit the effects of discreteness and, again, random or systematic system variations. In addition, size effects are important to understand in this context as the usual size independence may break down when applying macroscale concepts at microscales, and vice-versa (this especially applies to nonlinear phenomena).

Overall, the idea of translational physics across disciplines requires interdisciplinary research, linking physics, chemistry, biology, materials science and engineering to assess the challenges and opportunities across length and time scales and the feasibility of exporting concepts to other scales and scientific disciplines.

5 Identifying and Addressing Grand Challenges

One of the goals of the workshop, which was highlighted in the breakout sessions on the second day, was to identify grand challenges in the area of vibration energy transfer in solids and structures, particularly with relevance to nonlinearity and instability. Several of the identified grand challenges are summarized in the following.

5.1 Understanding the capacity for spectral and spatiotemporal energy redistribution

Linear systems have shown the capacity for spectral filtering, waveguiding, and spatiotemporal energy dispersion through dispersive mechanisms. Nonlinear systems, including systems exhibiting multistability, exhibit several potential advantages, including the ability redistribute vibrational energy between different spectral content, and additional mechanisms, such as self-localization and delocalization, for spatiotemporal energy redistribution. However, understanding of the capacity of nonlinear systems for such energy control remains in its early stages. What are the fundamental limitations on vibrational energy redistribution in nonlinear systems (potentially in the context of thermodynamic limits)? What are the limits in the context of controlled energy storage capacity and release rates? What new analytical tools would enable the directed conversion of energy from a given to desired spatiotemporal state? How does this enable unprecedented performances, in the context of metrics such as coefficient of restitution? Such energy redistribution has further implications in the context of vibrational energy scales. For instance, can ultra-dissipation be enabled by directed energy conversion between disparate scales (acoustic to high frequency phonon / thermal energy)? What is the role of nonlinearity in broadband functionality? And what is the robustness of nonlinearity-enabled performance features?

5.2 Understanding the role and potential for non-traditional structures/architectures, including asymmetries, defects, disorder, and hierarchy

The majority of studies aiming to design materials and structures for tailored vibration energy transfer properties have been centered upon periodic geometries. While many studies have explored the effects of asymmetries, defects, disorder, and hierarchy, understanding of the potential capabilities and performance enhancements that arise at the intersection between such features and nonlinearity and instability remains in its infancy. Indeed, many examples suggest enhanced performance in systems containing disorder, be it the preference for biological growth to favor systems with imperfections or the failure resistance of hierarchical structures. A grand challenge, is the need for new tools to understand multiscale order and disorder in the context of nonlinearity and instability. This challenge relates directly to the next challenge, namely, understanding the interplay of discrete/continuum limits.

5.3 Understanding the interplay between discrete and continuum limits, in the context of complex nonlinear systems

As eluded to in the previous two challenges, the concept of scale becomes particularly challenging in the context of nonlinear systems. What is the best method of description for nonlinear systems, where energy is redistributed drastically among scales? When do defects and disorder yield continuum or local behavior, and what is the capacity for controlling this interplay? How does nonlinearity enable defect sensitivity or amplify insensitivity? Are there new metrics that can be extracted that emphasize non-scale dependent properties (e.g. energetic properties vs. typically scale-dependent kinetic properties)? Furthermore, what is the relevance and potential of universality in the context of nonlinear and disordered systems, and what are the limitations on exploiting analogies in the context of disparate scales? Are there advantages to be gained in the context of amplifying nonlocality through nonlinearity?

5.4 What is the role of nonlinearity in topological contexts?

Topological insulators are one of the most recent, exciting developments in the context of vibration energy transfer. While many topologically designed structures leverage linear functionalities, the concept has also been explored, although to limited extent, in the context of highly nonlinear and multistable systems. What is the performance potential for nonlinear systems in the context of topology, and what new insights does topological analysis confer?

5.5 What is the potential at the intersection of active, or otherwise individually complex, constituents with nonlinear and multistable systems?

Within a linear vibrational energy transfer control context, active constituents have enabled new capabilities, ranging from extreme tunability to non-reciprocity. Furthermore, systems composed of interacting active elements, such as agglomerations of fire-ants have shown unique collective mechanical and nonlinear dynamic responses. What unique phenomena are enabled by active, or individually complex, elements in nonlinear systems that are not present in linear systems? Furthermore, how can active elements enable effective material nonlinearities and instability? Such active elements have particular potential relevance in the context of enabling new smart and active structures with extreme energy redirection, adaptive, environmentally-responsive, and self-correcting functionalities.

5.6 How can we design and manufacture any desired nonlinearity?

While the potential for systems exhibiting nonlinearity and instability is visible, the realization of such systems lags drastically behind. This is further amplified in the context of systems across different scales (macro- to sub-nanoscales, few Hz to THz thermal vibrations), and raises several related questions, and grand challenges. Understanding how transferable nonlinear concepts are to different system classes remains an open challenge, and has implications for the portability of other physics (e.g. from optics or quantum mechanics) to vibrational systems. One major challenge in this area is the ability to intentionally design structures that present a given nonlinear property (including active and multistable properties). Topology optimization strategies have been shown as an effective material design approach however due to the inherent complexity of problem, such topological optimization strategies lag behind. Other approaches such as combinatorics, materials genome, big data, unit cells libraries have been suggested. This gap is further amplified in the context of disordered systems. In addition to the design challenge, there lies a major challenge in the way of scalability. First, the realization of macroscale physics and sub-micron to near-atomic scales remains challenging. For instance, the ability to continuously tailor strong nonlinearities at the near-molecular level remains an open challenge. A second major challenge in the way of manufacturing scalability lies in the need to produce large quantities of microstructured material (e.g. nanoscale control over potentially kilometer scales). While the flexibility of additive manufacturing has been suggested as a solution for tailoring nonlinear microstructures at the macroscales, self-assembly manufacturing has emerged as a possible solution for systems with sub-micron features (however, the implementation of strong nonlinearities and instabilities in a self-assembled context remains open).

5.7 Nonlinearity in high-energy settings

Many ARO relevant applications are anticipated to involve high-energy settings. While strong nonlinearities are well matched with such contexts, most of the existing studies have involved low amplitude vibrations. Understanding the potential for nonlinear vibrational energy transfer strategies in extreme settings including high energy/strain/strain rate and destructive regimes remains an open challenge.

5.8 Nonlinearity beyond nearest neighbor potentials

As the investigations into vibrational energy control via nonlinearity are relatively recent, many studies have been focused on the effects of systems wherein the nonlinearity is manifested as nearest neighbor stiffness terms of the constitutive response. The investigation of nonlinear effects with increasing degrees of complexity, and the realization of materials supporting such effects, remains an open challenge. For instance, how can strongly nonlinear viscosities be continuously tailored, and what are the implications in the context of previously mentioned areas such as disorder, topology, and active elements? What are the possibilities in terms of coupling to other physical fields (e.g. electromagnetism)? Can this have implications for long range, magnetic-like interactions? What are the implications in terms of previously discussed features (again, disorder, topology, and active elements)? In addition to long-range interactions, what implications does this have for multibody dynamics, and the resulting effective response (e.g. 3-body nonlinear or buckling interactions).

6 Conclusions

This report summarizes the essential research needs and emergent concepts identified by 18 early-career U.S. faculty members who participated in the "Future of Vibration Energy Transfer in Solids and Structures: Needs and Opportunities Workshop," held on 18-19 October, 2016 at the University of Washington, Seattle, WA, USA. The discussions identified several future avenues of thematically aligned research attention and discovery potential deserving of continued and focused study:

- Exploration of nonlinearities and instabilities for achieving new functionalities of materials and structures in engineering applications, e.g., energy storage/release, and system property tuning.
- Incorporation of emerging themes, such as defects, impurities, disorder, multi-functionality, and topology, into the mechanical settings of solids and structures
- Necessity of advanced manufacturing technologies, interdisciplinary combination of mechanical, physical, chemical, and biological concepts, and appropriate modeling/computational/optimization tools.

In addition to these broadly defined needs for continued investigation, several grand challenges were identified that pose transformative potential if successfully developed. In conclusion, based on the workshop activities and outcomes, the field of energy transfer in solids and structures faces a new era with the emergence of numerous high-risk and high-reward concepts that may enable researchers to move beyond the bounds and notions of classical mechanics by leveraging previously-untapped dynamical concepts, emerging fabrication capabilities, and novel experimental frameworks.

Acknowledgements

The organizers are grateful for the support of the U.S. Army Research Office that enabled this workshop.

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