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# Array Realizations of Complex-Source Beams

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**14. ABSTRACT**
The inverse source problem for the complex-source beam is formulated in terms of an array radiating in free space or radiating in front of an impenetrable conformal or impedance surface. The array elements are non-resonant radiators. The physical dimension of the array is typically smaller than the diameter of the branch-cut disk of the complex point source. The array excitation coefficients are determined by solving a matrix equation that enforces the condition that the array fields match the fields of the complex point source in the far zone. For conformal and impedance surfaces the additional condition of zero internal fields is imposed. The SVD provides a powerful tool for obtaining the best-behaved solution.

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# 1 Introduction

Complex-source beams are exact solutions to the wave equation that are radiated by complex point sources. These beams, which are used in both acoustic and electromagnetic wave theories, are often referred to as Gaussian beams in the frequency domain and as pulsed-beam wavelets or complex-source pulsed beams in the time domain. The most basic introduction to complex point sources goes as follows. The far-field pattern of a scalar time-harmonic point source located at $\mathbf{r}'$ has the form $e^{-ik\,\mathbf{r}'\cdot\hat{\mathbf{r}}}$, where $\hat{\mathbf{r}}$ is the direction of observation and $k$ the wavenumber ($e^{-i\omega t}$ time dependence is suppressed with $k = \omega/c$ where $c$ is the wave speed and $\omega$ the frequency). Except for a phase variation, this pattern is isotropic when $\mathbf{r}'$ is real.We can introduce directivity if $\mathbf{r}'$ is allowed to be complex. Indeed, $\mathbf{r}' = ia\hat{\mathbf{x}}$ with $a > 0$ produces the directive pattern $e^{ka\,\hat{\mathbf{x}}\cdot\hat{\mathbf{r}}}$, which has no sidelobes and peaks in the direction $\hat{\mathbf{r}} = \hat{\mathbf{x}}$ (the unit vector in the $x-$direction is denoted by $\hat{\mathbf{x}}$). The minimum is attained in the opposite direction $\hat{\mathbf{r}} = -\hat{\mathbf{x}}$. Also, it can be shown that the near field on any sphere outside the reactive zone of the complex point source behaves almost as its far-field pattern. In particular, the near field exhibits the same directivity as the far-field pattern. Hence, the complex point source radiates fields that are unlike electromagnetic and acoustic fields encountered in nature.

Directive near fields are of great interest in a number of areas. For example, in wireless communications (and in machine to machine communications), an ever greater number of devices must operate simultaneously in close proximity to each other. Therefore, there is a growing interest in tailoring the near fields of devices to (for example) suppress unwanted interactions in certain near-field regions.

Complex-source beams are also useful for providing solutions to scattering and propagation problems in complex configurations. They are suitable as basis functions in exact expansions of arbitrary acoustic and electromagnetic fields with finite source regions in both the time and frequency domains. Recently, complex sources were employed to obtain new exact plane-wave expansions that led to directional translation operators for the fast multipole method.

Another potential advantage of employing complex point-source fields is found in simulations. An antenna or transducer that radiates the directional field of a complex point source is fully characterized by a point dipole or scalar point source in complex space. Hence, the field in both near and far zones is given by the free-space Greens function with a complex source point and the output of such an antenna or transducer is simply the incoming field at a complex point in space. (General antennas and transducers need to be characterized by their plane-wave or spherical-wave transmission and reception functions.)

The goal of the project is to solve the inverse source problem for the complex-source beam in terms of an array. Specifically, we determine excitation coefficients of typical non-resonant array elements so that the total array field equals the field of a complex point source. The following sections describe the findings of the project with the published papers listed at the end of the report. References [1]–[6] are refereed journal papers, and references [7]–[9] are refereed conference papers.

# 2 Free-space and conformal arrays

We formulated the inverse source problem for the complex-source beam as an array optimization problem with the array elements distributed evenly over a surface. Both array elements in free space [1], [9] and array elements over an impenetrable surface [3] were considered. By employing a recently-developed theorem on the spatial bandwidth of complex-source beams, we show that the physical dimension of the array is typically smaller than the diameter of the branch-cut disk of the complex point source. This was proven through the use of spherical-wave expressions for both the complex point-source field and the field generated by typical non-resonant sources of finite extent. Specifically, the spherical-wave coefficients of the array field must match the spherical-wave coefficients of the field of the complex point source.

These detailed studies reveal that the complex point source is indeed fundamentally different from the typical sources encountered in nature. Their spherical-wave coefficients behave very differently, so the matching of spherical-wave expansions require that the expansion coefficients for the array elements almost cancel over a broad range of indices. Also, through the use of the Wilcox expansion we discovered that it is the null-free far-field pattern of the complex point source that causes the near and far fields of the complex point source to be nearly identical [3]. (Typical non-resonant sources produce far-field patterns with nulls and consequently their near and far fields are very different.)

In [1] we employed simple non-resonant array elements with sidelobes to reproducing (to within a specified error limit) the far-field pattern of complex-source beams in both frequency and time domains. The numerical examples demonstrated the validity of the theory using array elements with a typical sidelobe structure. Hence, the results of the paper [1] can be reproduced with ordinary electro-acoustic transducers as array elements. Also, the least-squares approach used to determine the array excitation coefficients applies to any surface of sufficient physical extent. The time domain arrays were determined through the use of the Fourier transform by matching the array and complex point-source fields one frequency at a time. This study was extended to Hertzian dipoles with complex source points in [9].

Achieving a large dynamic range with an array of small elements in free space is challenging because (i) an entire closed surface needs to be populated, and (ii) the effective dynamic range of each array element is by its nature small. Hence, if no other structure is present, the large dynamic range of the complex-source beam must be accomplished through extremely precise interference among array elements (fully populating a closed surface) over a wide angular region. Inaccuracies in array-element patterns and array excitation coefficients make it challenging in practice to create such large dynamic ranges with an array. Fortunately, the array problem becomes better posed if we consider a conformal array with array elements mounted on a large structure (for example a fuselage) [3]. Because of interactions with the large structure, each mounted array element is now a large antenna or transducer with a considerable dynamic range that improves the numerical conditioning of the array-realization problem for the complex-source beam. The number of array elements required to achieve a certain accuracy of the radiated field can be dramatically reduced when a conformal array structure is employed with the array elements mounted on an impenetrable object. Only the part of the impenetrable object that faces in the beam direction need to be populated with array elements. This was demonstrated in [3] using both standard and weighted least-squares techniques.

## 3 Small array in front of impedance surface

In [4] we explored the possible physical realization of a complex point-source in two dimensions using an impenetrable impedance surface that is illuminated by an array of real point sources or a plane wave. Such a device can be implemented using a metasurface that is characterized by a transversely inhomogeneous impedance profile. The problem of determining a physically realizable impedance profile was cast in the form of a constrained inverse source problem that can be solved using a least-squares approach. Naturally, the impedance profile contains free parameters that allow the number of array elements to be significantly reduced without sacrificing accuracy.

It was shown that no impedance profile residing on a circular cylindrical surface and illuminated by near-zone array elements can create a total field that equals the field of a complex point source. The root of this problem lies in the fact that the radius of the cylinder is smaller than the distance from the origin to the array elements. However, the configuration where an incident plane wave illuminates the circular cylinder can reproduce a scattered field that equals the field of the complex point source to machine precision.

An impedance cylinder in the shape of a Cassini oval illuminated by an array of real point source can reproduce the near and far fields of a complex point source to an arbitrarily high degree of accuracy. The impedance profile was determined so that the total field (incident plus scattered) match the complex point source field in the far zone and equals zero on an inner surface located inside the surface on which the impedance profile resides. One of these solutions has 99% efficiency but exhibits both gain and loss locally.

## 4 Arrays determined from singular-value decomposition

The singular-value decomposition (SVD) can be used in inverse source problems to obtain a range of solutions with different properties. For example, with the SVD one can obtain the smoothest (most easily implementable) solution that ensures a required accuracy of the radiated far-field pattern. One can also obtain the least-squares solution as a special case. In [6] we presented a general SVD formulation for inverse source problems that applies directly in the dime domain. This formulation uses a sinc-function expansion that explicitly imposes a temporal bandlimit on the solutions. In other words, the frequency spectrum of the source is guaranteed to be zero outside a specified bandlimit.

This SVD formulation was applied to obtain an array solution for complex point sources with time dependence given by analytic $\delta-$pulses. To determine the proper array size, we derived a time-domain spherical-harmonics truncation formula for this type of complex point source. This truncation formula depends on the required relative accuracy, the beam parameter, and the parameters of the analytic $\delta-$pulse.

The source consists of an array of point sources evenly distributed over a smooth array surface. To make the inverse source problem numerically tractable, we imposed the condition that the interior field of the array be zero on a surface that lies inside the array surface. The array surface and the inner surface are bodies of revolution with cross sections in the shape of superellipses. The field of the array is matched to the complex point-source field (with analytic $\delta-$pulse time dependence) in the far zone and set to zero on the inner surface.

We thus obtained a matrix that relates the unknown space-time source values to the far-field pattern and near field on the inner surface. We investigated properties of solutions obtained with different truncations of the SVD. In particular, two solutions were examined: the first solution (i) included all singular values (this is the least-squares solution), and the second solution (ii) used only about half of the singular values. Remarkably, the power required in solution (i) is orders of magnitude greater than the power required in solution (ii). Yet, the solution (ii) still produces a relatively accurate far-field pattern. The higher power required in solution (i) is due to the reactive (non-radiating) fields that are stored in the near zone of the array. In solution (ii), only the less rapidly varying singular functions are included and therefore the array excitation coefficients exhibit smoother behavior.

We further used the time-domain SVD formulation to examine the accuracy of the far-field pattern that can be achieved with a given number of array elements distributed evenly over the same superelliptic body of revolution. The inner surface on which the condition of zero field is imposed also remains the same. For this configuration, all singular values were included so that the the least-squares solution is obtained. Interestingly, in a certain parameter range, the pattern accuracy on a log scale depends linearly on the number of array elements.

# 5 Application of complex point sources in near-field scanning

We also employed complex point sources to further develop near-field scanning techniques. In [7] the Gaussian translation operator was employed to speed up the fast irregular antenna field transformation algorithm. Since the Gaussian translation operator is directive with exponential decay in the sidelobe region for large translation distances, the necessary number of plane wave samples within the translation process can be reduced significantly. Reducing the number of translated plane waves has a large impact on the overall performance of the algorithm since the number of plane wave samples considered in the aggregation/disaggregation and interpolation/anterpolation processes can also be reduced. Moreover, the number of required levels within the hierarchical multilevel representation can be considerably reduced in many cases.

In [2], [8] a fast inverse equivalent source technique was presented for the fully probe-corrected transformation of measured radiation or scattering fields utilizing directive sources. The directivity of the expansion sources is either obtained by combining electric and magnetic surface current densities in a directive Huygens radiator or by shifting the source locations into complex space. The latter approach is known to generate Gaussian-beam-like directive radiators. These two techniques can be used separately or in combination to obtain three different solutions that approximately satisfy the null-field condition (also known as the Love condition) outside the solution domain, i.e., inside the volume of the original device under test. The directive sources lead to a better conditioning of the inverse problem and especially the Huygens radiator concept leads to good conditioning for a small number of unknowns. Thus, considerable reduction in computation times and better solution accuracies are achieved.

In [5] we discovered the root cause of the ill-conditioned nature of the linear system of equations often achieved with the fast irregular antenna field transformation algorithm. This occurs when certain sets of model parameters lie outside the spatial bandwidth of the operator that computes the probe output. One remedy is to restrict the sets of model parameters allowed and perform up-sampling if needed to achieve the desired accuracy.

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