



NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

INTERESTS, INTENSITY, AND RED LINES MODEL (I²R)

PRELIMINARY REPORT

by

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January 2017

Approved for public release; distribution is unlimited

Prepared for: Major Jason Hansberger, J-5, Joint Staff, Pentagon, Washington DC

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The report entitled “*Interests, Intensity, and Red Lines Model (I2R) Preliminary Report*” was prepared for Major Jason Hansberger, from the J5 of the Joint Staff and was funded by both the J5 and J8.

Further distribution of all or part of this report is authorized.

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ABSTRACT

This report is a summary of the I2R work for the time period 22 August 2016-September 30, 2016. I2R is a modeling effort for planning purposes by the Joint Staff. Guidance provided by the J5 was used to build a model that both graphically and through other modeling efforts would identify possible red line issues with targets that may trigger military action. Through many discussions with Major Hansberger, we decided on a logistical regression surface where the red line level is a 0-1 binary variable indicating whether military escalation occurred. We initially decided on one and two independent variables. When three independent variables are required the model uses only the equations without any graphical interface. The binary value one represents a red line issue where the action causes a military response. The model builds the equation by finding the optimal coefficients. It also provides a 3D image, although not as useful as other software since it is prepared in MS-EXCEL. The model also has a planning evaluator where the user inputs targeting information and the output is the probability of an action based upon the target. A slider bar is used to alter the probability levels for success and failure for the model. Although real data will most likely be classified, the examples used in this report and model are all unclassified.

This technical report includes a methodology for red line analysis of a target or set of targets using an EXCEL model along with the description of the model to perform red line analysis. The model utilizes input data to create a logistical surface in 3 dimensions and also plots the model for limited visualization analysis. The model allows the user to input targeting data for x_1 and x_2 and obtain an output which represents a probability measure for the effects of targeting in terms of red line analysis. A value of one indicates a 100% chance of a response and a value of 0 represent an almost 0% chance of a military response. A key input is the probability levels or increments in which we call the results a zero or one. A slider bar has been placed in the model for adjusting the probability of a red line within the template for quick turn-around analysis. Additionally

sensitivity analysis and specificity analysis has been added to bring more clarity to the red line probability value to be used in the analysis.

EXECUTIVE SUMMARY

The I2R Model was created for the Joint Staff. Its purpose is to assist in targeting planning. By analyzing previous data that either lead to escalation or no escalation, we are able to build a baseline model using binary logistic regression. This model might and most likely could change, based upon the adversary. Therefore, multiple baseline models might exist. Next, a targeting list is created and run through the predictive model to determine if escalation might occur. Sensitivity and specificity models are used to obtain probability values to be used similar to the medical field analysis. The results of the model determine whether selected targets are above or below the red line. Two methods are available for the red line (1) a sliding probabilistic scale is available for the user as well as (2) the sensitivity and specificity probability values. We envision targets can be all of the following options at a minimum: military, political, diplomatic, and economic. This analysis and modeling package is meant to assist the decision maker.

I. INTRODUCTION

A. BACKGROUND FROM J5

Interests, Intensity, and Red Lines Model (I2R)

The I2R model is designed to answer an intuitive question, “What are their red lines?” by analyzing two competitors who seek to pursue and defend their own interests but avoid escalations with each other. In the model, responses are based how the each side perceives the other’s action affects its interests, which is a function of three variables: geographic significance, organizational significance, and the intensity of the original action. The significance variables define a player’s interest and form the two-dimensional base of the model. Simply put, we assume that actions taken close to or within a homeland or against a national leader would be interpreted as more threatening to a player’s interests than actions taken outside a player’s area of interest or against an unrelated individual or institution. The third variable, intensity, measures actions on a scale from mild criticisms to nuclear weapons. Taken together, the variables generate a three dimensional red surface that assesses where, against whom, and what kind of actions will generate a response, thereby allowing strategy and planning teams to better approximate the opposing side’s response.

Although the I2R model was inspired by state to state international relations, analysts can use it to evaluate relations between any two players including people or organizations like businesses, political parties, or terrorist groups. We will therefore present the model under a generic contest between two competitors, A and B, but will commonly refer to international relations. For naming convention, A will always initiate action against B, which must then decide whether or not to respond. To explain the model, we will (1) provide definitions and explain each of the variables separately; (2) address the model’s assumptions; (3) describe the model as a whole; and (4) use the model to analyze behavior.

Definitions

Red Line: A stated or unstated set of conditions that trigger a response.

Nation: A group of people who claim a sovereign right to self-rule.

Interests: The places, people, and institutions that allow for the continuation of the nation and the state.

Institution: A set of values or ideals that a group of people share and abide by. An institution does not necessarily occupy a physical space.

Existential Nexus: The most important institution and geographic space to a group’s continued existence.

International Relations: The interactions between sovereign states that take place in an anarchic space. There is no hierarchical authority to govern interactions or adjudicate disputes.

Action Variables

1. Geographic Significance. A measurement of how far away from B’s existential nexus the action takes place. In the simplest sense, it can be expressed as linear miles, but significance could also be measured in time to travel since features like mountainous terrain or oceans can make short distances relatively long. We also consider strategic

features like straits and man-made features like borders when analyzing geographic significance.

2. Organizational Significance. A measurement of how closely related the target of the action is to B's existential nexus. Organizationally significant targets embody B or its dearest values, while distant targets become increasingly less relevant to B as an entity. Close targets include heads of state, headquarter buildings, or ways of life. Increasingly distant targets move through allies, annoyances, and eventually to entities completely unassociated with B.

3. Intensity. A measurement of how intense A's action is. For a general presentation, we assume a universal scale of intensity, no matter the actors. However, we allow for two multipliers. The first addresses major organizational differences between A and B. The multiplier is based on how B perceives A's status as either a revolutionary, revisionist, or status quo actor. We assume that B perceives a revolutionary's actions as more intense than it does a status quo's. Allowing for B's perceptions enables the multiplier to capture trends towards cooperation or antagonism as well. For example, if A and B found ways to cooperate despite their differences, B would eventually upgrade A's status from revolutionary to revisionist, which would lower the perceived intensity of A's actions. On the other hand, continuous or increased antagonism would move A's status and B's perception of A's intensity in the opposite direction. The second multiplier accounts for action frequency. We assume that if A takes actions against B in close succession, B would group those actions together and view the trend as increasingly intense.

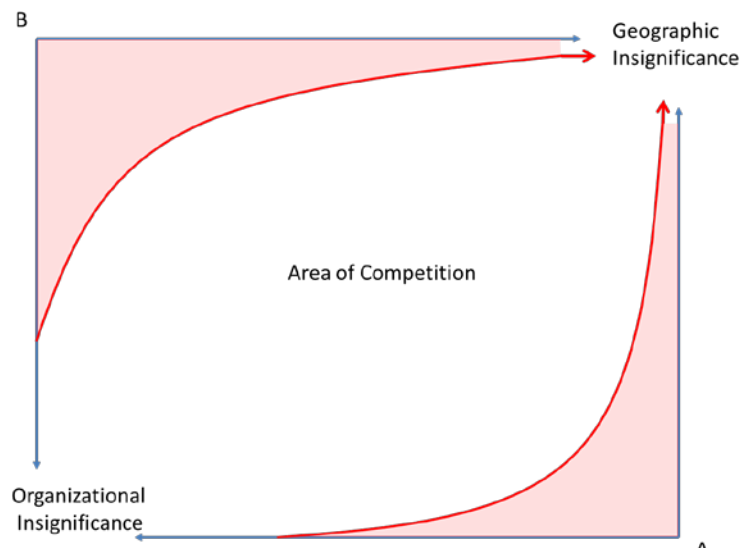


Figure 1. Interests worth protecting are determined by Geographic and Organizational Significance.

Assumptions

Assumption 1:

Both A and B have an existential nexus that must be protected. The model would not apply to a perfectly networked group with no geographic or hierarchical structure since the network lacks a nexus to protect (i.e. hacking groups like Anonymous would be difficult to model).

Assumption 2: Both A and B seek to avoid escalation. Players on both sides tolerate some level of an opponent's efforts that run counter to their own interests in what we call the "competitive sphere." Doing so allows the opponents to conserve resources for use outside of the bilateral relationship or for a threat closer to their nexus of interests. A player like the Islamic State, which seeks escalation per se, does not have clear red lines, but analyzing its responses may help point to important centers of gravity.

Assumption 3: Provocative actions can be normalized over time.

Building the Model

The Distance Dimension – The model is built on the assumption that interests worth protecting are determined by the geographic and organizational significance with respect to the existential nexus. We therefore use the significance variables as the base of the model. Figure 1 shows the bilateral contest between A and B and how the players' red lines relate to increasing insignificance. (For this section, intensity of action is constant). In the white area of competition, players can take actions that do not threaten the other's interests to the point of triggering a response from the other side. Conversely, actions taken closer to an existential nexus fall in the red area and thus have crossed a red line. At this level of intensity, the red line's intersection with the organizational distance axis shows that, eventually, organizationally distant actions no longer elicit a response, even if that action were to take place in the opponent's homeland. Conversely, as shown by the asymptotic approach to the geographic significance axis, actions taken against extremely important entities will elicit a response no matter where the actions take place. For ease of presentation, we only include one red line in Figure 1, but as a model between two specific actors becomes more complete, multiple red lines emerge as actions taken increasing close to B's nexus elicit corresponding increases in response intensity. In international relations terms, each state constructs informational, diplomatic, economic, and military responses to a competitor's actions against it.

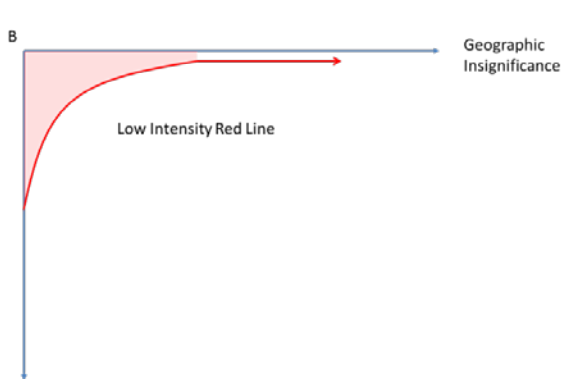


Figure 2. Low Intensity Response Function

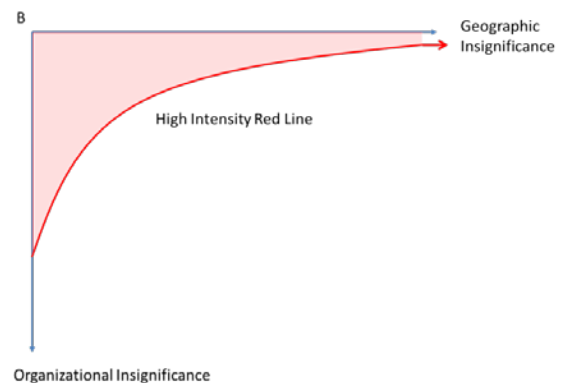


Figure 3. High Intensity Response Function

Adding the Intensity Variable – Intuitively, two actions at the same geographic and organizational distances but with differing levels of intensity have the potential to induce different responses. For example, an American citizen being detained or jailed by a foreign government is different than an American citizen being executed. The visual representation of that response function is presented in Figures 2 and 3, which show how higher intensity actions bring B's red lines further away from its existential nexus. Figure 4 represents B's side of the relationship.

The three dimensional chart combines all three of the model's variables into a single chart and results in a red surface that depicts (1) how an increase in intensity of action correspondently draws B's red lines away from the nexus and (2) how an action with the same level of intensity can eventually cross a red plane as it takes place increasing close to B's nexus. Not depicted is A's side of the relationship or the theoretical sphere of contention that would lie above the area of contention depicted in Figure 1.

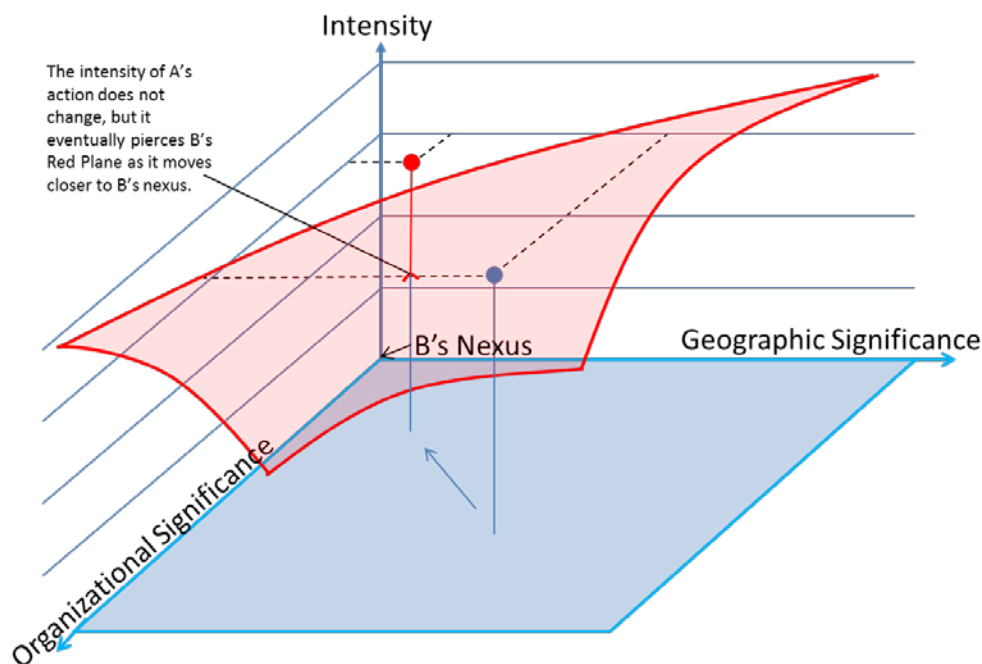


Figure 4. The I2R Model

Employing the Model

In the most basic sense, we believe that there is value in using the model as a concept to help guide strategic decision making simply by understanding how both a competitor and oneself construct response functions. Once understood, the model not only provides more clarity in a competitive bilateral relationship, it can guide and unify efforts across an organization by providing a common frame of reference for multiple relationships. Specifically, if A is able to gather enough intelligence or information about B, A can employ the model to (1) design operations that do not inadvertently trip B's red lines; (2) conduct preparation activities that lower B's perception of significance or intensity; (3) more accurately understand their own red lines; (4) understand how B may interpret actions taken outside of the A-B model; and (5) finally, construct the sphere of contention to a level of detail that would allow A to manipulate the sphere in its favor. Overtime, if

A is able to successfully operate close to, but not cross B's red plane we assume that B would eventually be normalized to the behavior and would slowly perceive A's actions as less intense. If that is true, A could move the sphere of contention closer to B and thereby open up their options and limit B's. Mastering the sphere would not only help A issue and defend realistic red lines; A could also advance its own interests while simultaneously limiting B's ability or willingness to respond.

Often it is difficult to match exactly what the sponsor desires in a research effort with the results of that research. In this article, we discuss our methodology for targeting analysis for minimal response that was built for elements of the Department of Defense (DOD). We begin by discussing the DOD research objectives. Then we discuss our approach, results, proof of principle model, and conclusions.

What is a red line? In some ways, it used to be called the line in the sand, a line that if crossed brought an action or reaction. Several recent articles have addressed red lines in Syria and Iran dealing with interests of the United States. For example, the use of chemical weapons by Syria was a red line of U.S. involvement that eventually became a missile strike or Iran's misuse of oil in their region.^{1, 2, 3, 4}

2. Red Line Modeling Approaches

Initially many possible modeling approaches were considered. These included discreet dynamical system models, system dynamics models, and game theory models. Although portions of these type models might be used in future additions, we decided to use a logistics regression modeling approach and provide ability to use MADM to obtain values of targets if many targets may be similar in nomenclature. Our approach is a multi-layered approach. Since many targets tend to have the same nomenclature, we decided to use our AHP-TOPSIS hybrid modeling approach to obtain ranked values for targets based upon their weighted CARVER criteria using decision maker's inputs. After these values are obtained they become an independent variable in the binary logistics regression model where the dependent variable y is a binary value indicating escalation or no-escalation. Other independent variables of interest for each target are used as well as the ability for interaction terms. These variables are at the discretion of the model user's but can include distance, importance, or other critical factors. We will describe our approach in more detail. We note that we have built an Excel workbook with multiple worksheets to assist the users in building and obtaining the model as well as obtaining a predictive model application for future missions as a function of the independent variables used.

2.1 Red Line Model Methodology

In our approach we use and describe two parts. Part I concerns the targeting data and converting the target name into a cardinal data values through TOPSIS. Part 1 can be ignored if good, descriptive, cardinal data exists already for target descriptions. Part 2 concerns the binary logistics regression model and the predictive red line analysis model. Since this is to be used by the Department of Defense no specialized packages were used other than MS-EXCEL and free EXCEL add-ins such as RealStats ©.

2.1. 1 Targeting data^{5,6,7,8,9,}

Step 1. Using experts and the CARVER Matrix explained in FM 34-36 as a targeting guide for analysis, choose the target list and fill in the values as directed by the matrix CARVER⁸.

NOTE: CARVER can be substituted for by any targeting scheme methodology that has criteria.

Step 2. Using experts perform a pairwise comparison among the criteria list under CARVER⁸:

CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, AND RECOGNIZABILITY FACTORS.

Step 2a. Prioritize these variables as to their importance in the mission.

Step 2b. Fill in the pairwise comparison worksheet, comparing each pair A to B , and indicate intensity values on Saaty's 1-9 scale^{11,12,13}.

Step 2c. Insure the consistency ratio, CR , value is less than 0.1. If not, then some of the values in step 2b were inconsistent^{12,13,14}. This means that it violates $A > B$, $B > C$, so we conclude $A > C$ or violates the intensity levels. This is fixed by trial and error.

Step 3. Reorder the current target CARVER matrix so that the columns match the prioritized order used in Step 2. This could be ACVERR, for example.

Step 4. The TOPSIS computes the TOPSIS values and completes the ranking of the targets and these values will be used as the target variable x_i in the red line model^{13,14,15,16,17}.

For those not familiar with the TOPSIS methodology, we present the methodology steps.

Step 4-1 Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criterion given as x_{ij} , giving us a matrix $(X_{ij})_{m \times n}$.

$$D = \begin{matrix} & \begin{matrix} x_1 & x_2 & x_3 & \cdot & \cdot & \cdot & x_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \cdot \\ \cdot \\ \cdot \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdot & \cdot & \cdot & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdot & \cdot & \cdot & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdot & \cdot & \cdot & x_{3n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{m1} & x_{m2} & x_{m3} & \cdot & \cdot & \cdot & x_{mn} \end{bmatrix} \end{matrix}$$

Step 4-2 The matrix shown as D above then is normalized to form the matrix $R = (R_{ij})_{m \times n}$ as shown using the normalization method

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum x_{ij}^2}} \text{ for } i=1,2,\dots,m; j=1,2,\dots,n$$

Step 4-3 Calculate the weighted normalized decision matrix. First we need the weights. Weights can come from either the decision maker or by computation.

Step 4-3 a. Use either the decision maker's weights for the attributes $x_1, x_2, \dots, x_n$ or compute the weights through the use of Saaty's (1980) AHP decision maker weights method to

obtain the weights as the eigenvector to the attributes versus attribute pairwise comparison matrix.

$$\sum_{j=1}^n w_j = 1$$

The sum of the weights over all attributes must equal 1 regardless of the method used.

Step 4-3b. Multiply the weights to each of the column entries in the matrix from *Step 2* to obtain the matrix, T .

$$T = (t_{ij})_{m \times n} = (w_j r_{ij})_{m \times n}, i = 1, 2, \dots, m$$

Step 4-4 Determine the worst alternative (A_w) and the best alternative (A_b) : Examine each attribute's column and select the largest and smallest values appropriately. If the values imply larger is better (profit), then the best alternatives are the largest values, and if the values imply smaller is better (such as cost), then the best alternative is the smallest value.

$$A_w = \{ \langle \max(t_{ij} | i = 1, 2, \dots, m | j \in J_-), \langle \min(t_{ij} | i = 1, 2, \dots, m) | j \in J_+ \rangle \rangle \}$$

$$\equiv \{ t_{wj} | j = 1, 2, \dots, n \},$$

$$A_{wb} = \{ \langle \min(t_{ij} | i = 1, 2, \dots, m | j \in J_-), \langle \max(t_{ij} | i = 1, 2, \dots, m) | j \in J_+ \rangle \rangle \}$$

$$\equiv \{ t_{bj} | j = 1, 2, \dots, n \},$$

where,

$J_+ = \{ j = 1, 2, \dots, n | j \text{ associated with the criteria having a positive impact, and}$

$J_- = \{ j = 1, 2, \dots, n | j \text{ associated with the criteria having a negative impact.}$

We suggest that if possible make all entry values in terms of positive impacts.

Step 4-5 Calculate the L2-distance between the target alternative i and the worst condition A_w

$$d_{iw} = \sqrt{\sum_{j=1}^n (t_{ij} - t_{wj})^2}, i=1, 2, \dots, m$$

and then calculate the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\sum_{j=1}^n (t_{ij} - t_{bj})^2}, i=1, 2, \dots, m$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.

Step 4-6 Calculate the similarity to the worst condition:

$$s_{iw} = \frac{d_{ib}}{(d_{iw} + d_{ib})}, 0 \leq s_{iw} \leq 1, i = 1, 2, \dots, m$$

$s_{iw}=1$ if and only if the alternative solution has the worst condition; and

$S_{iw}=0$ if and only if the alternative solution has the best condition.

Step 4-7 Rank the alternatives according to their value from S_{iw} ($i=1,2,\dots,m$) .

2.1.2 Red Line Analysis

Step 1. Obtain the data for the independent variables and binary dependent variables. Choose either the appropriate logistics regression model template for as many variables as desired or use the add-in for binary logistics regression with RealStats©.

Step 2. The variable x_1 can be automatically populated into the logistics model to be used unless otherwise directed by the user. We note that two columns are initially available in the worksheet for x_1 but only one of these columns that include either the TOPSIS values or user inputs will be used.

Step 3. Enter in the remaining columns of data in the appropriate worksheet.

Step 4. Enter the binary y_i variable where 1 implies unacceptable military escalation and 0 implies no unacceptable military escalation.

Step 5. As directed in the Excel template, the Excel *Solver* needs to be engaged. In the order as directed inside the template highlight the MLFs to be maximized and then appropriate coefficients and for each in order call the solver and click solve^{16,17,18,19,20,21,22}. The template worksheet is updated automatically. The user must examine the output for significance. The user might use RealStats→ Reg→ Binary Logistics Regression by opening that dialog box and filling in the required cells.

Step 6. If using only the two variable worksheet template you may view the 3-dimensional red line plot otherwise with 3 or more independent variable there is no plot available. However, if hold one variable constant, say x_3 , then for each constant value of x_3 , we can obtain a surface to view.

Step 7. You can now enter your predictive analysis portion where for each possible real target you enter the data for x_1, x_2, x_3 .

Step 7a. We note that you may need to run TOPSIS again for the values of x_1 in the prediction model with CARVER criteria to obtain values of x_1 for your target list.

Step 7 b. Using the slider bar select the probability level for the Red Line, $r, 0 \leq r \leq 1$.

Step 7c. The model compares the predicted probability with your slider value. Any values greater than the red line value is indicated as **above the red line**.

Step 8. Interpret all results.

Step 9. Perform sensitivity and specificity analysis.

Step 10. Use as a predictive model

2.2 Discussion of Binary Logistic Regression

Binary logistics regression has often been used in research when the dependent variable is binary $\{0,1\}$ and ordinary least squares (OSL) is inadequate. {list some here}. This is another case where OSL is adequate and a binary regression model is required.

In data analysis, logistic regression (sometimes called the logistic model or logit model) is a type of regression analysis used for predicting the outcome of a binary dependent variable (a variable which can take only two possible outcomes, e.g. "yes" vs. "no" , "success" vs. "failure" , or “escalation” vs “no escalation”) based on one or more predictor variables. Logistic regression attempts to model the probability of a "yes/success" outcome using a linear function of the predictors. Specifically, the log-odds of success (the logit of the probability) are fit to the predictors using linear regression.

Logistic regression is one type of discrete choice model, which in general predict categorical dependent variables. For our needs we use binary.

Like other forms of regression analysis, logistic regression makes use of one or more predictor variables that may be either continuous or categorical. Also, like other linear regression models, the expected value (average value) of the response variable is fit to the predictors where the expected value of a Bernoulli distribution is simply the probability of success. Unlike ordinary linear regression, however, logistic regression is used for predicting binary outcomes (Bernoulli trials) rather than continuous outcomes, and models a transformation of the expected value as a linear function of the predictors, rather than the expected value itself.

For example, logistic regression might be used to predict whether a patient has a given disease (e.g. diabetes), based on observed characteristics of the patient (age, gender, body mass index, results of various blood tests, etc.). Another example might be to predict whether a voter will vote Democratic or Republican, based on age, income, gender, race, state of residence, votes in previous elections, etc. Logistic regression is used extensively in numerous disciplines: the medical and social sciences fields, natural language processing, marketing applications such as prediction of a customer's propensity to purchase a product or cease a subscription, etc. Yes, even in the military and Department of Defense a logistic regression has utility as we will see.

The model for just one predictor is

$$y = \frac{e^{b_0 + b_1 x_1}}{1 + e^{b_0 + b_1 x_1}}$$

where the error term are independent and identically distributed (*iid*) as normal random variables with constant variance.

For more than one predictor we use the model form:

$$y = \frac{e^{b_0 + \sum_{i=1}^n b_i x_i}}{1 + e^{b_0 + \sum_{i=1}^n b_i x_i}}$$

Logistic Regression calculates the probability of the event occurring, such as the escalation caused by an event or the purchase of a product. In general, the object being predicted in a regression equation is represented by the dependent variable or output variable and is usually labeled as the Y variable in the regression equation. In the case of Logistic Regression, this “Y” variable is binary. In other words, the output or dependent variable can only take the values of 1 or 0. The predicted event either occurs or it doesn’t occur – your target’s destruction either causes escalation or does not cause escalation.

To meet the request of the sponsor, when available we produce either the “S” curve with one predictor or the S surface when we have two predictors. We provide an example of the surface in Figure 5.

In order to evaluate how good or how adequate our model is we require all the relevant statistics including the following:

Estimates of the coefficients, their standard errors, t* statistics, P-values, and some analysis of fit between the full model and a not-full model that includes -2 ln likelihood and chi-squared tests, and if two or fewer independent variables a plot of the regression model as either an S curve or a S surface.

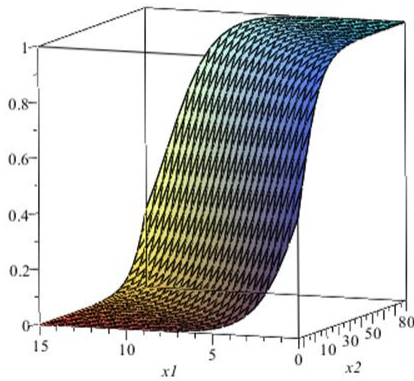


Figure 5. Plot of the Logistic Function in this case a 3D CDF surface.

Before we accept this model, we require a minimum of a few diagnostics. We want to (1) examine the significance of each estimated coefficient $\{b_0, b_1, b_2, \dots, b_n\}$ and (2) compare the full model to an intercept only model and one term model to measure the chi-square differences.

We use pattern recognition and a simplification step to better see what is happening here and let

$$\pi = \exp(b_0 + b_1x_1 + b_2x_2 + \dots + b_nx_n).$$

Let’s define P as follows, $P = -\sum_{i=1}^n \left(\frac{\pi}{1+\pi} \right) - \left(\frac{\pi^2}{(1+\pi)^2} \right)$. Then we can more easily write the Hessian matrix for n terms and its inverse as follows:

$$\begin{bmatrix}
-(P) & -(\sum x_1(i)P) & -(\sum x_2(i)P) & . & \dots & -(\sum x_n(i)P) \\
-(\sum x_1(i)P) & -(\sum x_1^2(i)P) & -(\sum x_1(i)x_2(i)P) & . & \dots & -(\sum x_1(i)x_n(i)P) \\
-(\sum x_2(i)P) & -(\sum x_1(i)x_2(i)P) & -(\sum x_2^2(i)P) & . & \dots & -(\sum x_2(i)x_n(i)P) \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
-(\sum x_n(i)P) & -(\sum x_1(i)x_n(i)P) & -(\sum x_2(i)x_n(i)P) & & & -(\sum x_n^2(i)P)
\end{bmatrix}$$

$$= \begin{bmatrix}
-(P) & -(\sum x_1(i)P) & -(\sum x_2(i)P) & . & \dots & -(\sum x_n(i)P) \\
-(\sum x_1(i)P) & -(\sum x_1^2(i)P) & -(\sum x_1(i)x_2(i)P) & . & \dots & -(\sum x_1(i)x_n(i)P) \\
-(\sum x_2(i)P) & -(\sum x_1(i)x_2(i)P) & -(\sum x_2^2(i)P) & . & \dots & -(\sum x_2(i)x_n(i)P) \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
-(\sum x_n(i)P) & -(\sum x_1(i)x_n(i)P) & -(\sum x_2(i)x_n(i)P) & & & -(\sum x_n^2(i)P)
\end{bmatrix}^{-1}$$

The Hessian is the Variance-Covariance matrix and we take the square root of the entries on the main diagonal as our estimates of the standard error (se) for $\{b_0, b_1, \dots, b_n\}$.
{fox, Log reg, COED)

In our example, we compute the Hessian, H, and its inverse, H^{-1} . We compute H using the sums of the columns in the matrix H. To obtain H^{-1} , we use =MINVERSE command in Excel.

This discussion is essential because EXCEL does not have a binary logistics regression package, so we wrote one using the Solver prior to finding and comparing to the Add-In RealStats©.

Odds-Ratio

Odds-Ratios are important as well. Interpretation of B parameters in the logistic model.

$$\pi^* = B_0 + B_1x_1 + \dots + B_nx_n$$

where

$$\pi^*$$

$$\pi^*$$

B_1 =Change in log-odds π^* for every 1 unit increase in x_1 holding all other x 's fixed.

$e^{B_1}-1$ = Percentage change in odds ratio $\pi/(1-\pi)$ for every 1 unit increase in x_1 holding all other x 's fixed.

For example, if $B_1=-0.7553$, $e^{B_1}=0.47$, $e^{B_1}-1=-0.53$. For each unit of x_1 , we estimate the odds of a fixed contract to decrease by 53% holding x_2 fixed.

For example, if $B_2=0.1122$, $e^{B_2}=1.12$, $e^{B_2}-1=0.12$. For each unit of x_2 , we estimate the odds of a fixed contract to increase by 12% holding x_1 fixed. Odds ratios can help the analyst determine the impact of additional units of the independent variables.

3. Simple Illustrative Example

Lets' start by discussing the decision criteria portion of the model using the CARVER decision variables as defined in FM 34-36⁷: CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, and RECOGNIZABILITY FACTORS.

3.1.1 Decision Criteria

Step 1. Prioritize decision criteria in CARVER. The decision maker or his/her staff must initially prioritize these factors.

CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, AND RECOGNIZABILITY FACTORS

Step 2. Using Saaty's 9 point scale whose meaning are shown in Table 1, compare the criteria in a pairwise fashion. When a criterion is compared to itself, it gets a 1. Only fill in the upper portion of the triangle with the integer values from 1-9.

Table 1. Saaty's 9-Point Scale

Intensity of Importance in Pairwise Comparisons	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements i and j if i is 3 compared to j , then j is 1/3 compared to i .
Rationale	Force consistency; measure

	values available
--	------------------

For example:

	C	A	R	V	E	R
C	1	3	4	5	6	8
A		1	3	4	5	6
R			1	4	5	6
V				1	3	4
E					1	3
R						1

Enter letters for criteria in priority order in the first column and first row. In the upper portion fill in the values using the intensity scale in Table 1.

	C	A	R	V	E	R
C	1					
A		1				
R			1			
V				1		
E					1	
R						1

Doing this makes the use of the decision criteria worksheet in the template easier. We provide an example by including our pairwise numbers.

	C	A	R	V	E	R
C	1	3	4	5	6	8
A		1	3	4	5	6
R			1	4	5	6
V				1	3	4
E					1	3
R						1

Given that we have 31 historical data set or even a survey inspired targets, $t_1, t_2, \dots, t_{31}$. We gather information on target type, called x_1 . Often targets are not clearly distinct in title or name only so we need a way to obtain a numerical value which is why we suggest using TOPSIS to generate these values. Variables x_2 and x_3 might be any measurable component of the target such as distance, ratio of importance to each side or any other measurable variables, significance, estimated casualties, etc. For each target with its associated attributes in x_2 and x_3 , determine if escalation is possible (1) or no escalation (0). This might be highly subjective unless historical information or data is assessable. Currently the J5 believes the data can be obtained through the use of surveys with experts.

Before we enter the data, we obtain a priority of the CARVER elements. In our example, our military experts in the Department of Defense Analysis, prioritized these as CERRAV. We enter this list, in this order, into our model. Let's assume that in our example that the matrix looks like this:

	C	E	R	R	A	V
C	1	2	3	5	5	6
E		1	2	4	5	5
R			1	3	4	4
R				1	2	3
A					1	1
V						1

We input this information into the worksheet page titled we developed for our model that allows the pairwise comparison and values to be input by the user. We developed templates to aid the user to enter the criteria in priority order. Then the user employs Saaty's 9 point pairwise comparison. The computation yields two critical results shown in figure 6: (1) the consistency ratio and (2) the criterion weights. We must check the *CR* value on the worksheet to insure that it is less than 0.10. Having a *CR* value less than 0.10 insures our transitivity is acceptable.

Step 4. Final Criterion Weights (automatically transferred to TOPSIS Tool) and Consistency Ratio:		
	Criterion Name	Weight
1	C	0.4168
2	E	0.2416
3	R	0.1499
4	R	0.0770
5	A	0.0607
6	V	0.0540
7	NA	0.0000
8	NA	0.0000

CR= 0.012 Consistent <0.10

Figure 6. Screenshot of criteria weights and consistency ratio.

In this example, we find the *CR* value is 0.012 which is less than 0.10 so we are consistent and we can use the weights for the criteria. Since we prioritized the CARVER criteria our weights are presented in numerical order displayed in Figure 6. Next we take our targets' CARVER data for our 31 targets and reorganize the columns to match our prioritized criteria order from above CERRAV.

C	A	R	V	E	R				C	E	R	R	A	V
5	1	5	4	6	2				5	6	5	2	1	4
4	9	4	4	7	3				4	7	4	3	9	4
6	5	2	6	8	2				6	8	2	2	5	6
5	9	2	1	6	3				5	6	2	3	9	1
5	3	4	2	4	1				5	4	4	1	3	2
1	2	8	5	9	3				1	9	8	3	2	5
8	9	1	8	2	1				8	2	1	1	9	8
4	2	8	7	9	6				4	9	8	6	2	7
6	7	4	7	5	1				6	5	4	1	7	7
7	3	3	6	6	4				7	6	3	4	3	6
6	5	3	6	1	8				6	1	3	8	5	6
1	1	7	1	7	1				1	7	7	1	1	1
1	5	2	7	5	7				1	5	2	7	5	7
3	9	1	2	4	4				3	4	1	4	9	2
7	4	4	1	9	2				7	9	4	2	4	1
3	7	8	2	4	3				3	4	8	3	7	2

We input these into our TOPSIS template. The outputs are the TOPSIS values that rank the targets from 1 to 31 in order of strategic or tactical importance based upon the criteria used.

Alternatives	C	E	R	R	A	V	NA	NA	Value	Rank	
1	5	6	5	2	1	4	0	0	0.307	29	T1
2	4	7	4	3	9	4	0	0	0.635	10	T2
3	6	8	2	2	5	6	0	0	0.522	16	T2
4	5	6	2	3	9	1	0	0	0.510	17	T3
5	5	4	4	1	3	2	0	0	0.281	31	T4
6	1	9	8	3	2	5	0	0	0.482	22	T5
7	8	2	1	1	9	8	0	0	0.638	9	T7
8	4	9	8	6	2	7	0	0	0.493	20	T8
9	6	5	4	1	7	7	0	0	0.634	11	T9
10	7	6	3	4	3	6	0	0	0.409	25	T10
11	6	1	3	8	5	6	0	0	0.501	19	T11
12	1	7	7	1	1	1	0	0	0.355	27	T12
13	1	5	2	7	5	7	0	0	0.657	6	T13

Figure 7. Screenshot for TOPSIS values for the targeting part of the model.

The ranks are listed under the column titled **Rank**. The model uses the value column in its building of the logistics regression model. We also input the attributes of each target, the other independent variables and the binary y value under the appropriate heading in the logistics worksheet. This is displayed in Figure 7. Currently the values of x_1 are from the TOPSIS worksheet.

Y	x1	x2	x3	x4
0	0.307359	19.2	24.000	0.000
1	0.635242	24.1	22.000	0.000
0	0.522035	-7.1	19.000	0.000
1	0.510246	3.9	15.000	0.000
0	0.280965	4.5	15.000	0.000
1	0.482208	10.6	25.000	0.000
0	0.638221	-3	19.000	0.000
1	0.49337	16.2	19.000	0.000
1	0.634412	72.8	14.000	0.000
0	0.408865	28.7	16.000	0.000
0	0.501083	11.5	28.000	0.000
0	0.354648	56.3	30.000	0.000
1	0.656819	-0.5	25.000	0.000
1	0.562575	-1.3	21.000	0.000

The user tells the worksheet the number of data points and the number of variables used as well as whether or not to use the TOPSIS results as x_1 . Next, we need to run a binary logistics regression where y is binary $\{0,1\}$ and the other x variables beyond x_1 are related to the targets listed in x_1 .

3.2 Discussion to obtain predictive **Red Line** model.

In this portion of the model we only have information on x_1, x_2 for our mission specific targets. As a matter of fact, we suggest using the *TOPSIS_PRED* and *DecisionWeight_PRED* to generate the target list values. These worksheets are available. You do not need the same weights as before in the original data sets analysis although it might be helpful.

For sake of illustration, we change a value from a 5 to a 6.

	C	E	R	R	A	V
C	1	2	3	5	6	6
E		1	2	4	5	5
R			1	3	4	4
R				1	2	3
A					1	1
V						1

The CR is 0.007 and the weights are as shown below.

Step 4. Final Criterion Weights (automatically transferred to TOPSIS Tool) and Consistency Ratio:

	Criterion Name	Weight
1	C	0.4221
2	E	0.2416
3	R	0.1497
4	R	0.0771
5	A	0.0553
6	V	0.0543
7	NA	0.0000
8	NA	0.0000

CR= 0.007 Consistent <0.10

Next, we compute the TOPSIS ranks after we input our CARVER data in the order of CERRVA into the model and using these weights.

Step 5. Enter the criterion values for each alternative (accepts either numerical or categorical data) and view results:

Alternatives	C	E	R	R	A	V	NA	NA	TOPSIS Value	Final Rank	
1	9	3	3	8	7	6	0	0	0.597	3	Bulk Elec Power
2	7	3	7	4	3	8	0	0	0.545	4	Bulk Petroleum
3	3	5	3	7	7	3	0	0	0.624	1	Water Supply
4	3	4	7	2	2	2	0	0	0.352	7	Communication Sys
5	1	1	3	1	2	2	0	0	0.396	6	Air Transport
6	1	1	3	1	1	1	0	0	0.350	8	Ports and waterways
7	2	4	4	1	4	3	0	0	0.506	5	Rail Transport
8	1	5	2	8	2	8	0	0	0.606	2	Road Networks

The TOPSIS values then automatically feed into our *Predictive Model* sheet. We enter the corresponding x_2 data with our calculated x_1 values for our targets. We also may use the slider bar to set the probability levels of acceptance where model values exceeding the probability are interpreted as above the red line.

We already have obtained our model from the baseline data and binary logistics regression model. In figure 8, we show a screen shot of the predictive model and red line analysis.

A	B	C	D	E	F	G	H
			b0	b1	b2		
			-10.9744102	21.67322996	0.028868569		
	Redline Probability			57			
			P(X>n)	0.57	User Inputs probability between [0,1]		
	User Inputs				Scroll Bar		
	Do you have x1 data or want it inputed			by Topisis: 1 yes or 2 no	1		
			x1	x2		Model Calculation	Decision
	Bulk Elec Power		0.59705	16		0.918902206	Above redline
	Bulk Petroleum		0.54505	12		0.765834853	Above redline
	Water Supply		0.62413	5		0.936843421	Above redline
	Communication Sys		0.35165	3		0.036744168	Below redline
	Air Transport		0.3958	11		0.111199879	Below redline
	Ports and waterways		0.3496	4		0.03619513	Below redline
	Rail Transport		0.50559	5.5		0.535477788	Below redline
	Road Networks		0.60598	5		0.909156732	Above redline

Figure 8. Screenshot of predictive model analysis

In the example the probability of being a 1 versus a 0 (above the red line) is set at 0.57. Additionally, if we set the probability at 0.75, we find no change in our predictions. Looking at the values if we set our probability at 0.95 then no target would be above the *red line*. Later we discuss sensitivity analysis and specificity and we suggest this method or process for obtaining the values for this probability.

4. Cases Studies and Analysis

One of two conditions can exist for analysis. First, we might have “real” data for targets that can be used or second, we might have to use our MADM methods with TOPSIS to give us values to use for the targets in both the baseline and predictive models. We illustrate both cases as examples.

4.1 Case 1. One predictor model

Assume we have the following data to use for x_1 and y . The data for the 22 targets has x_1 which comes from intelligence sources and represents the psychological factor [0,100] for the targets according to the enemy. The y -variables represent whether or not escalation occurred (1) or no escalation (0).

target	psych Influence	y
1	66	1
2	70	0
3	69	1
4	68	1
5	67	1
6	72	1
7	73	1
8	70	1
9	57	0
10	63	0
11	70	0
12	67	1
13	53	0
14	57	1
15	75	1
16	70	1
17	81	1
18	76	1
19	79	1
20	75	0
21	76	1
22	58	0

From our binary logistics regression template we obtain found the model,

$$y = \frac{e^{-12.3334+0.200365x_1}}{1+e^{-12.3334+0.200365x_1}}.$$

We examine the statistics for the coefficients and find that the psychological influence is significant ($p=0.041$) while the constant is not significant ($p=0.059$). We decide to keep them both since the model is significant ($p=0.014$) and Hosmer($p=0.585 > 0.05$.)The model itself, is significant with a p -value of 0.005481. We plot the model since there is only one predictor. We obtain a plot of the model in Figure 9.

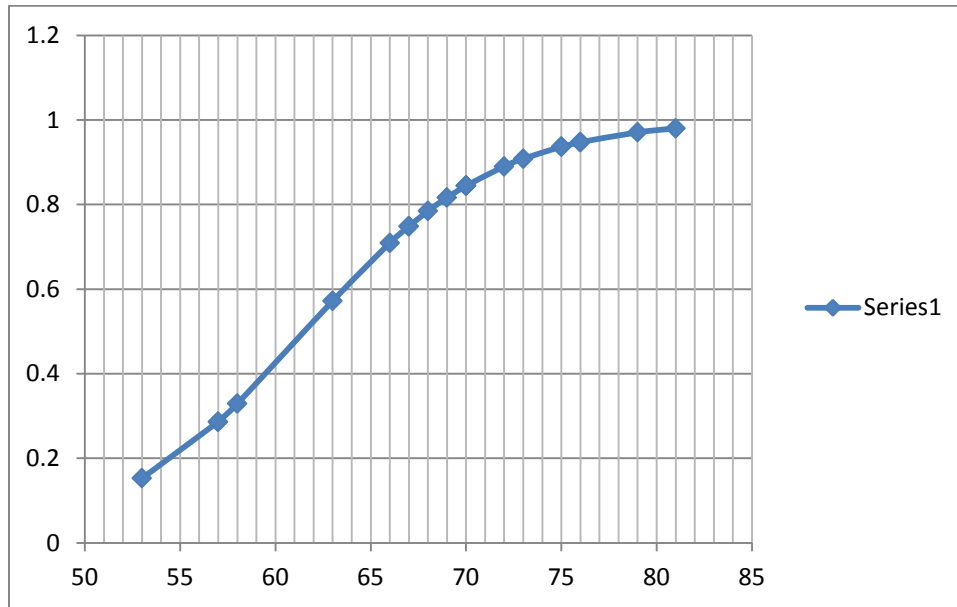


Figure 9. Plot of our binary logistics model

Next, we assume we have target data for which we need to predict y . We find the y values for our x inputs are as follows:

Prediction

x	y
50	0.08988
63	0.5719
72	0.89021
93	0.99817

Next we need to determine the minimum value of probability that will be considered a one (1) in our analysis. For this calculation, we will do a sensitivity and 1-specificity analysis. We provide a plot, figure 10, and an interpolated approximation.

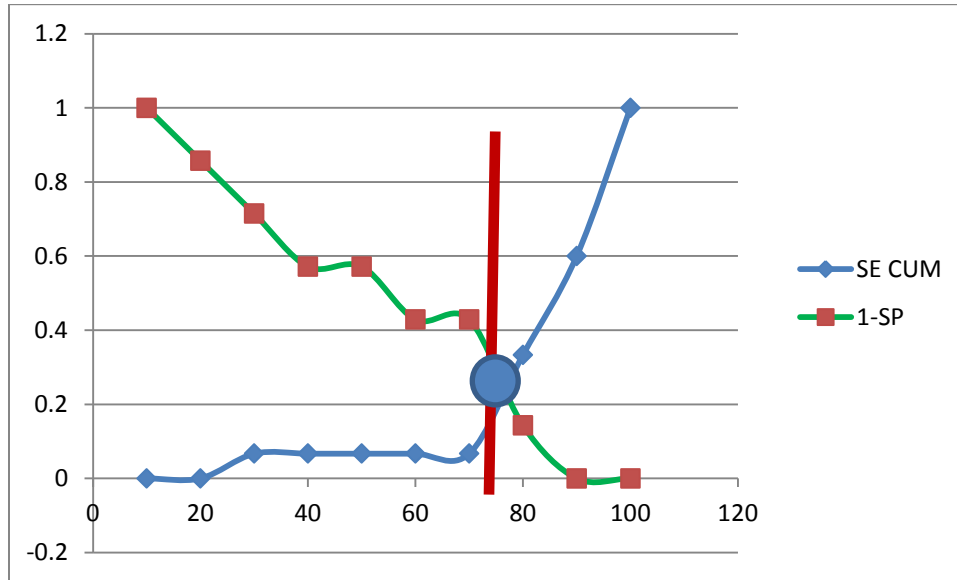


Figure 10. SE vs 1-SP plot for example 1.

We interpret this value to be approximately 78%. Therefore any prediction value greater than or equal to 78% will be excluded from our target set for the mission because of their great potential to cause escalation.

x	y	<i>Red line</i>
50	0.08988	<i>below red line</i>
63	0.5719	<i>below the red line</i>
72	0.89021	<i>above red line</i>
93	0.99817	<i>above ther red line</i>

Thus, the target with $x_1=72$ and 93 should be excluded because they are predicted to cause a red line escalation. Our mission should be able to safely use targets with psychological factors of 50 and 63.

4.2 Case 2. Two predictor model with surface plot we use data for x_1 from using TOPSIS and CARVER doctrine.

We assume we have the following data.

Data:

y	$x1$	$x2$
0	0.38184	19.2
1	0.50436	24.1
0	0.45	-7.1
1	0.44669	3.9
0	0.27432	4.5
1	0.57984	10.6
0	0.19315	-3
1	0.69207	16.2
1	0.31722	72.8
0	0.43595	28.7
0	0.48941	11.5
0	0.45653	56.3
1	0.66977	-0.5
1	0.46114	-1.3
0	0.46339	12.9
0	0.41424	34.1
1	0.49713	6.6
1	0.46971	-2.5
1	0.6789	24.2
0	0.41134	2.3
1	0.26999	36.9
0	0.39846	-11.7
1	0.66981	2.1
1	0.71992	10.4
0	0.47004	9.1
0	0.52333	2
0	0.54026	12.6
1	0.64992	18
0	0.82686	1.5
1	0.72419	27.3

We use the Real Stats: binary regression, to obtain the model

$$y = \frac{e^{(-3.8328882+6.73867424x_1+0.028533151x_2)}}{1+e^{(-3.8328882+6.73867424x_1+0.028533151x_2)}}$$

We find the coefficients b_0 and b_1 are significant ($p_{b0}=0.039$, $p_{b1}=0.047$) and the p -value for b_2 is not significant ($p=0.209$). Since the model in its complete form is significant ($p=0.040$) we will keep all terms and variables. The surface plot is shown in Figure 11 and is created inside the template. At a later time, we will attempt to color code the plot so that the upper portions are red.

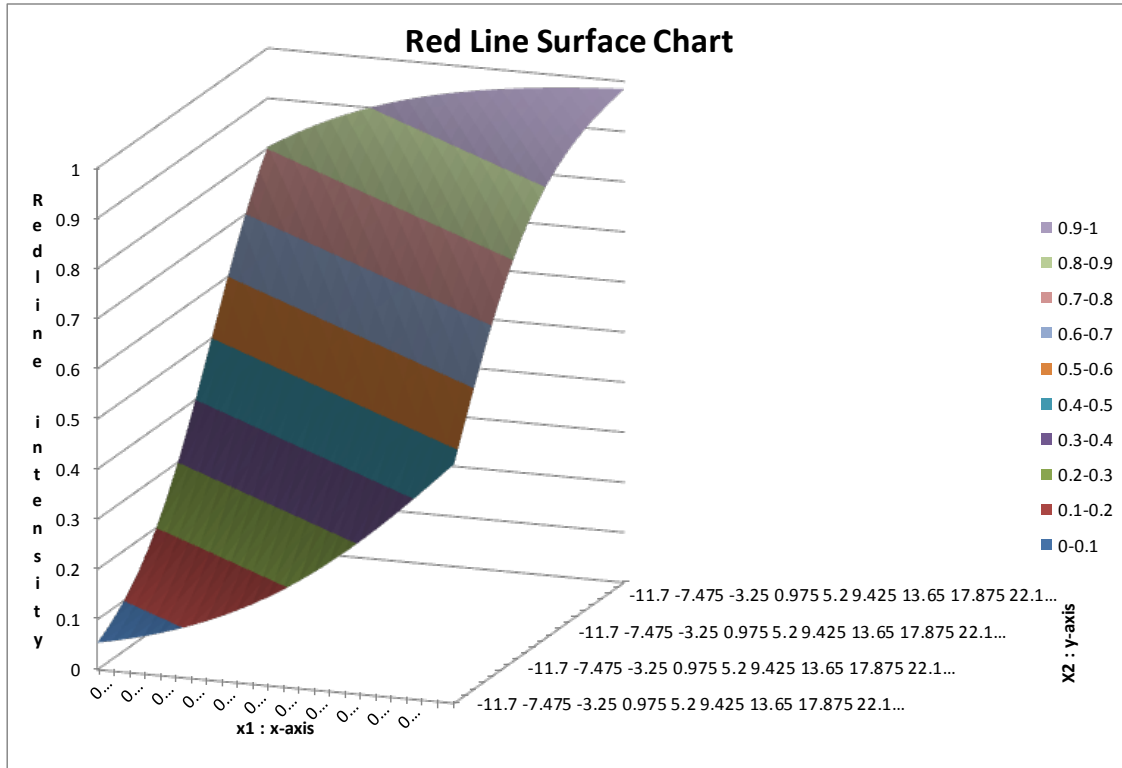


Figure 11. Surface plot for example 2.

Now, that we have a baseline model to use we can address the real issue of possible targets. We have accessed the x_1 and x_2 variables values in the same way we did the baseline variables. Our goal is to predict the y - the probability of escalation for these targets. Then based upon the value of probability between $[0,1]$ that we select to call escalation, we can eliminate targets due to their potential to cause escalation. Our targets and their variables are:

	x_1	x_2
Bulk Elec Power	0.59705	16
Bulk Petroleum	0.54505	12
Water Supply	0.62413	5
Communication Sys	0.35165	3
Air Transport	0.3958	11
Ports and waterways	0.3496	4
Rail Transport	0.50559	5.5
Road Networks	0.60598	5

We use equation 3 to obtain the predictions shown as:

	x1	x2	Model Calculation
Bulk Elec Power	0.59705	16	0.656329133
Bulk Petroleum	0.54505	12	0.545474654
Water Supply	0.62413	5	0.626118869
Communication Sys	0.35165	3	0.201392852
Air Transport	0.3958	11	0.299046636
Ports and waterways	0.3496	4	0.203767361
Rail Transport	0.50559	5.5	0.433165971
Road Networks	0.60598	5	0.597064658

Possible interpretations,

- If we assume that $p(\text{escalation})$ be greater than or equal to 0.50, then we eliminate Bulk Elect Power, Bulk Petroleum, Water Supply, and Road Networks from our target list.
- We can do additional sensitivity and specificity analysis in order to find a better value of the probability of escalation. We find the better value to be greater than 0.60, see Figure 12.

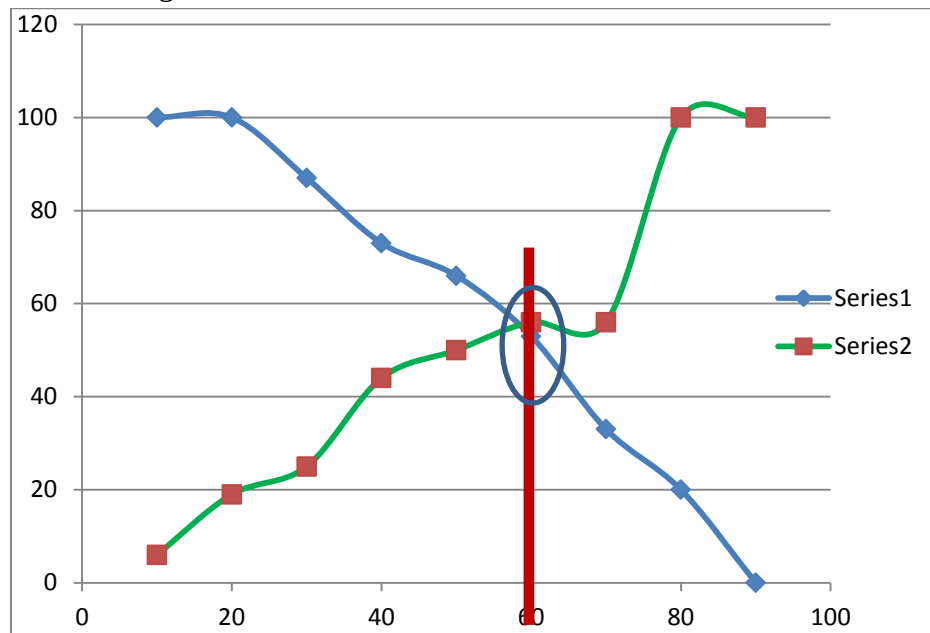


Figure 12. *Se vs 1-Sp* plot

- We can obtain a value from the mission commander.

Case 2 Results

In this illustrative example, we use the value of 0.60, obtained by method 2, for our analysis.

	x1	x2		Model Calculation	Decision
Bulk Elec Power	0.59705	16		0.656329133	Above red line
Bulk Petroleum	0.54505	12		0.545474654	Below red line
Water Supply	0.62413	5		0.626118869	Above red line
Communication Sys	0.35165	3		0.201392852	Below red line
Air Transport	0.3958	11		0.299046636	Below red line
Ports and waterways	0.3496	4		0.203767361	Below red line
Rail Transport	0.50559	5.5		0.433165971	Below red line
Road Networks	0.60598	5		0.597064658	Below red line

Only the targets, Bulk Elec Power and Water Supply, are to be eliminated from the target list because of their potential to cause military escalation.

4.3 Case 3. Using Three or More Variable Binary Logistics' Model

If we choose to use the three variable Logistics' model, all processes are the same except:
(1) No surface can be directly generated for viewing since we can only directly view in 2 or 3 dimensions. NOTE: if needed we can hold one of the variable x_1 , x_2 , or x_3 as a constant value and obtain a surface for the remaining variables. This is the same as taking a slice through the surface.

(2) We added an additional independent variable to the logistics process and thus require an additional b coefficient to the b_i coefficients.

We only provide the screenshots of the model here.

Entering the data: x_i is the target type and can be entered directly into the worksheet labeled **x1** or accepted from the TOPSIS output.

Y	x1	x2	x3
0	0.381845	19.2	24.000
1	0.504357	24.1	22.000
0	0.449995	-7.1	19.000
1	0.44669	3.9	15.000
0	0.274323	4.5	15.000
1	0.579838	10.6	25.000
0	0.193153	-3	19.000
1	0.692072	16.2	19.000
1	0.317215	72.8	14.000
0	0.435951	28.7	16.000
0	0.489406	11.5	28.000
0	0.456532	56.3	30.000
1	0.669774	-0.5	25.000
1	0.461139	-1.3	21.000

The coefficients are found by engaging the Solver twice as directed within the worksheet, or by using the RealStats© binary regression:

The model for this 3 variable case is

$$y = \frac{\exp(-3.35005 + 6.8414x_1 + 0.03015x_2 - 0.025659x_3)}{1 + \exp(-3.35005 + 6.8414x_1 + 0.03015x_2 - 0.025659x_3)}$$

In our spreadsheet model we find a few issues with RealStats© such as the location of the output changing that makes automatics predicting more difficult. So, we continue to use the original Excel model using the Solver directly.

Again, TOPSIS or direct input is used to create the prediction input data for x_1 and we assume we have good data for x_2 , and x_3 that correspond with each x_1 . The scroll bar is initially set at 0.57 for this run of the model.

User Inputs		by TOPSIS: 1 yes or 2 no			Scroll Bar	
Do you have x1 data or want it inputed		x1	x2	x3	1	
					Model Calculation	Decision
	Bulk Elec Power	0.597051617	16	25	0.640052664	Above redline
	Bulk Petroleum	0.545046921	12	23	0.537557241	Below redline
	Water Supply	0.624132466	5	32	0.562068045	Below redline
	Communication Sys	0.351653397	3	31	0.161214113	Below redline
	Air Transport	0.395802233	11	27	0.268289655	Below redline
	Ports and waterways	0.34960048	4	21	0.201568154	Below redline
	Rail Transport	0.505590679	5.5	24	0.415547043	Below redline
	Road Networks	0.605975628	5	22	0.594339165	Above redline

Again, we can adjust the scroll bar. If set at 0.70 then in this example all targets are below the *Red Line*. So what probability value is better to use? We address a method for determining that next using the same process as the previous example.

4.3.1 Sensitivity and Specificity

Sensitivity and specificity can be used to assess discriminatory performance (*DP*) of a binary logistics model. According to Kleinbaum and Klein²², they say that a good *DP* is obtained if the covariates in the model help to predict which inputs will develop the $Y=1$ or $Y=0$. A receiver operating characteristic curve (ROC) can be used to plot sensitivity (*Se*) versus 1-specificity (*Sp*). The intersection of the two curves provides a cut-off value for the probability.

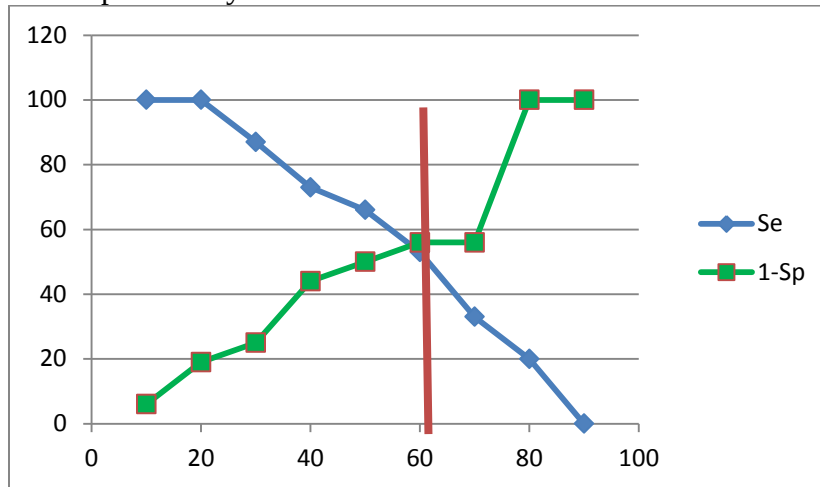


Figure 13. Plot of the curves for *Se* and *1-Sp* identifying the intersection of the two curves.

We see from Figure 13 that the point of intersection appears to be a probability of 0.60. This value will discriminate nicely. Thus, for our possible targets only Bulk Electric power at model prediction value of 0.6400 exceeds this probability criteria of $p=0.60$ for the red line. Our model suggests excluding the Bulk Electric Power from the target selection set since it mathematically appears to cause a crossing of a red line for military escalation.

5. Conclusion

Although the model has a lot of subjectivity woven into the methodology especially since the user employs TOPSIS, the results are promising for use by the Department of Defense. More realistic and, most likely, classified data from intelligence analysts need to be incorporated into the model to more accurately obtain a clearer vision of the possible targets and mission situation and their relationship to a red line. Sensitivity analysis and modeling are important elements in any good analysis^{23,24,25}.

APPENDIX A (SOURCE: FROM FM 34-36)

CARVER Targeting Analysis

TARGET ANALYSIS PROCESS

Adapted from Source:

<http://fas.org/irp/doddir/army/fm34-36/appd.htm>

This explains CARVER, which is an SOF term. CARVER is used by ARSOF SIOs and operational personnel throughout the ARSOF targeting and mission planning process to assess mission, validity, and requirements. It is also used in technical appreciation and target analysis. This appendix provides a step-by- step example of how to use CARVER.

CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, AND RECOGNIZABILITY FACTORS

The CARVER selection factors assist in selecting the best targets or components to attack. As the factors are considered, they are given a numerical value. This value represents the desirability of attacking the target. The values are then placed in a decision matrix. After CARVER values for each target or component are assigned, the sum of the values indicate the highest value target or component to be attacked within the limits of the statement of requirements and commander's intent.

CRITICALITY

Criticality means target value. This is the primary consideration in targeting. A target is critical when its destruction or damage has a significant impact on military, political, or economic operations.

Targets within a system must be considered in relation to other elements of the target system. The value of a target will change as the situation develops, requiring the use of the time-sensitive methods which respond to changing situations. For example, when one has few locomotives, railroad bridges may be less critical as targets; however, safeguarding bridges may be critical to maneuvering conventional forces which require use of such bridges. Criticality depends on several factors:

- Time: How rapidly will the impact of the target attack affect operations?
- Quality: What percentage of output, production, or service will be curtailed by target damage?
- Surrogates: What will be the effect on the output, production, and service?
- Relativity: How many targets are there? What are their positions? How is their relative value determined? What will be affected in the system or complex "stream"?

Table D-1 shows how criticality values are assigned on CARVER matrixes.

Table D-1. Assigning criticality values.

<u>CRITERIA</u>	<u>SCALE</u>
Immediate halt in output, production, or service; target cannot function without it	9-10
Halt within 1 day, or 66% curtailment in output, production, or service	7-8
Halt within 1 week, or 33% curtailment in output, production, or service	5-6
Halt within 10 days, or 10% curtailment in output, production, or service	3-4
No significant effect on output, production, or service	1-2

ACCESSIBILITY

A target is accessible when an operational element can reach the target with sufficient personnel and equipment to accomplish its mission. A target can be accessible even if it requires the assistance of knowledgeable insiders. This assessment entails identifying and studying critical paths that the operational element must take to achieve its objectives, and measuring those things that aid or impede access. SOF must not only be able to reach the target but must also remain there for an extended period. The four basic steps identifying accessibility are-

- Infiltration from the staging base to the target area.
- Movement from the point of entry to the target or objective.
- Movement to the target's critical element.
- Exfiltration.

Factors considered when evaluating accessibility include but are not limited to-

- Active and passive early warning systems.
- Swimmer detection devices.
- Air defense capabilities within the target area.
- Road and rail transportation systems.
- Type of terrain and its use.
- Concealment and cover.

- Population density.
- Other natural or synthetic obstacles and barriers.
- Current and climatic weather conditions.

The analysis along each critical path to the target should measure the time it would take for the action element to bypass, neutralize, or penetrate barriers and obstacles along the way. Accessibility is measured in terms of relative ease or difficulty of movement for the operational element and the likelihood of detection. The use of standoff weapons should always be considered in such evaluations. [Table D-2](#) shows how accessibility values are assigned on CARVER matrixes.

Table D-2. Assigning accessibility values.

<u>CRITERIA</u>	<u>SCALE</u>
Easily accessible, standoff weapons can be employed	9-10
Inside a perimeter fence but outdoors	7-8
Inside a building but on ground floor	5-6
Inside a building but on second floor or in basement; climbing or lowering required	3-4
Not accessible or inaccessible without extreme difficulty	1-2

RECUPERABILITY

A target's recuperability is measured in time; that is, how long will it take to replace, repair, or bypass the destruction of or damage to the target? Recuperability varies with the sources and type of targeted components and the availability of spare parts availability. Factors which should be considered when assessing recuperability include, but are not limited to, the availability of-

- On-hand equipment such as railroad cranes, dry docks, and cannibalization.
- Restoration and substitution through redundancies.
- On-hand spares.
- Equivalent OB equipment sets that backup critical equipment or components, and the effects of economic embargoes and labor unrest.

[Table D-3](#) shows how recuperability values are assigned on CARVER matrixes.

Table D-3. Assigning recuperability values.

<u>CRITERIA</u>	<u>SCALE</u>
Replacement, repair, or substitution requires 1 month or more	9-10
Replacement, repair, or substitution requires 1 week to 1 month	7-8

Replacement, repair, or substitution requires 72 hours to 1 week	5-6
Replacement, repair, or substitution requires 24 to 72 hours	3-4
Same day replacement, repair, or substitution	1-2

VULNERABILITY

A target is vulnerable if the operational element has the means and expertise to successfully attack the target. When determining the vulnerability of a target, the scale of the critical component needs to be compared with the capability of the attacking element to destroy or damage it. In general, the attacking element may tend to-

- Choose special components.
- Do permanent damage.
- Prevent or inhibit cannibalization.
- Maximize effects through the use of onsite materials.
- Cause the target to self-destruct.

Specifically, vulnerability depends on-

- The nature and construction of the target.
- The amount of damage required.
- The assets available; for example, personnel, expertise, motivation, weapons, explosives, and equipment.

[Table D-4](#) shows how vulnerability values are assigned on CARVER matrixes.

Table D-4. Assigning vulnerability values.

<u>CRITERIA</u>	<u>SCALE</u>
Vulnerable to long-range laser target designation, small arms fire, or charges of 5 pounds or less	9-10
Vulnerable to light anti-armor weapons fire or charges of 5 to 10 pounds	7-8
Vulnerable to medium anti-armor weapons fire, bulk charges of 10 to 30 pounds, or very careful placement of smaller charges	5-6
Vulnerable to heavy anti-armor fire, bulk charges of 30 to 50 pounds, or requires special weapons	3-4
Invulnerable to all but the most extreme targeting measures	1-2
EFFECT	

The effect of a target attack is a measure of possible military, political, economic, psychological, and sociological impacts at the target and beyond. This is closely related to the measure of target criticality. The type and magnitude of given effects desired will help planners select targets and target components for attack. Effect in this context addresses all significant effects, whether desired or not, that may result once the selected target component is attacked. Traditionally, this element has addressed the effect on the local population, but now there are broader considerations. Effect is frequently neutral at the tactical (ODA) level.

For example, the primary effect of the destruction of two adjacent long-range radar sites in an early warning system may be to open a hole in the system that is of sufficient size and duration to permit the attacker to launch a successful air or missile nuclear strike against the defender. Effects can also include-

- The triggering of countermeasures.
- Support or negation of PSYOP themes.
- Unemployment.
- Reprisals against the civilian populace.
- Collateral damage to other targets.

Possible effects can be speculative and should be labelled as such. Effects of the same attack may be quite different at the tactical, operational, and strategic levels. For example, the destruction of a substation may not affect local power supply but cuts off all power to an adjacent region. [Table D-5](#) shows how effect values are assigned on CARVER matrixes.

Table D-5. Assigning effect values.

<u>CRITERIA</u>	<u>SCALE</u>
Overwhelmingly positive effects; no significant negative effects	9-10
Moderately positive effects; few significant negative effects	7-8
No significant effects; neutral	5-6
Moderately negative effects; few significant positive effects	3-4
Overwhelmingly negative effects; no significant positive effects	1-2

RECOGNIZABILITY

A target's recognizability is the degree to which it can be recognized by an operational element and/or intelligence collection and reconnaissance assets under varying conditions. Weather has an obvious and significant impact on visibility. Rain, snow, and ground fog may obscure observation. Road segments with sparse vegetation and adjacent high ground provide excellent conditions for good observation. Distance, light, and season must also be considered.

Other factors which influence recognizability include the size and complexity of the target, the existence of distinctive target signatures, the presence of masking or camouflage, and the technical sophistication and training of the attackers. [Table D-6](#) shows how recognizability values are assigned on CARVER matrixes.

Table D-6. Assigning recognizability values.

<u>CRITERIA</u>	<u>SCALE</u>
The target is clearly recognizable under all conditions and from a distance; it requires little or no training for recognition	9-10
The target is easily recognizable at small-arms range and requires a small amount of training for recognition	7-8
The target is difficult to recognize at night or in bad weather, or might be confused with other targets or target components; it requires some training for recognition	5-6
The target is difficult to recognize at night or in bad weather, even within small-arms range; it is easily confused with other targets or components, it requires extensive training for recognition	3-4
The target cannot be recognized under any conditions, except by experts	1 -2

CARVER MATRIX

These CARVER factors and their assigned values are used to construct a CARVER matrix. This is a tool for rating the desirability of potential targets and wisely allocating attack resources.

To construct the matrix, list the potential targets in the left column. For strategic level analysis, list the enemy's systems or subsystems (electric power supply, rail system). For tactical level analysis, list the complexes or components of the subsystems or complexes selected for attack by your higher headquarters. ([Figure D-1](#) shows a sample matrix for a bulk electric power supply facility.)

BULK ELECTRIC POWER SUPPLY							
POTENTIAL TARGETS	C	A	R	V	E	R	TOTAL
FUEL TANKS	8	9	3	8	5	6	41
FUEL PUMPS	8	6	2	10	5	3	34
BOILERS	6	2	10	4	5	4	31
TURBINES	8	6	10	7	5	9	45
GENERATORS	4	6	10	7	5	9	41
CONDENSERS	8	8	5	2	5	4	34
FEED PUMPS	3	8	5	8	5	6	33
CIR. WATER PUMPS	3	8	5	8	5	4	33
GENERATOR STEP UP TRANSFORMER	10	10	10	9	5	9	53

Figure D-1. Completed CARVER matrix.

As each potential target is evaluated for each CARVER factor, enter the appropriate value into the matrix. Once all the potential targets have been evaluated, add the values for each potential target. The sums represent the relative desirability of each potential target; this constitutes a prioritized list of targets. Attack those targets with the highest totals first.

If additional men and/or munitions are available, allocate these resources to the remaining potential targets in descending numerical order. This allocation scheme will maximize the use of limited resources. The SIO can use the CARVER matrix to present operation planners with a variety of attack options. With the matrix he can discuss the strengths and weaknesses of each COA against the target. Having arrived at conclusions through the rigorous evaluation process, the SIO can comfortably defend his choices.

SITE RECONNAISSANCE

During target systems analysis, it is advantageous (in a permissive environment) to have a reconnaissance element perform a site survey. This reconnaissance can take place at the specific target site or at a similar site in a more accessible location.

Preparation for reconnaissance and analysis of an industrial establishment or other technically sophisticated complex is one of the more difficult missions for the ARSOF SIO. Reconnaissance could be done in support of DA missions or to assist a host nation to defend a potential target in a FID environment. Target analysis is a cooperative effort

between the operational element and intelligence personnel. This analysis seeks to answer PIR, IR, and SIR in the categories of CARVER.

Target analysis is the responsibility of the SIO, but a reconnaissance team can often be used to answer SIR that cannot be satisfied by any other means. Preparation for a site reconnaissance requires review and understanding of the following process. Site reconnaissance-

- Can be overt, covert, or clandestine.
- Can be conducted as part of a larger area assessment or a distinct mission activity.
- Can take place across the operational continuum.
- Is often more than a traditional reconnaissance and less than a full-scale analysis. Comprised of four cyclic steps.

SIO AND RECONNAISSANCE TEAM REVIEW

The first step is a joint SIO and reconnaissance team review of the commander's guidance and stated requirements. This sets out what is to be accomplished regarding the target. In a FID environment, this includes an evaluation of the threat to the target.

GATHER, ORGANIZE, AND EVALUATE INFORMATION

The second step is to gather, organize, and evaluate available information about the target; and to identify gaps in the data. Maps, photographs, flow charts, blueprints, diagrams, and other data are provided by the SIO and are examined, as appropriate, in light of the mission and the PIR and IR.

An initial CARVER report and targeting folder that highlights gaps in the data may be prepared at this step. The folder is used to develop a detailed collection and reconnaissance and surveillance (R&S) plan.

SURVEY THE TARGET

The third step is a survey of the target by the reconnaissance team. The team-

- Gathers information validating data gathered and conclusions drawn up to this step.
- Satisfies intelligence and information requirements.
- Gives the supported targeting analyst and/or operational element a "feel" for the target.

If the survey is overt and with the cooperation of personnel working at the site, the operational element follows these basic rules:

- Look for those things that would be important if the site had to be attacked in the future. Note layout, construction, location, and composition of key components, security, and communications.
- Dress neatly but not conspicuously.
- Find out what the key components of the site are likely to be by pre-visit research; check this against what you see or are told on the site. Note manufacturers and model numbers of key components. Find out which, if any, of these components are made by cast methods or are otherwise difficult to replace. Determine who the key personnel at the site are and what they do.
- Be conscious of where you are on site at all times in relation to key components; notice what type of machinery or equipment is in your immediate vicinity.
- Discern how the site is similar to other sites in its class. Note any significant differences.
- Take brief but good notes.
- Be polite and attentive when someone from the site is speaking. Direct questions, one at a time, to your host. Do not badger guides with persistent questions about subjects they are unsure about or do not want to discuss.
- Avoid making comments about perceived lax security.
- Do not wander away from the tour group or into restricted areas: such conduct will offend your host.
- Do not smoke where there is even a remote fire hazard or without the permission of your host.
- Ask for informational handouts, such as illustrated public relations literature, that provide technical information about the site. Be especially alert for flowcharts and layout diagrams.
- Do not volunteer any more information about yourself or your purpose than is necessary to accomplish your mission.
- Avoid immature discussions concerning sabotage or destruction of components onsite. Hosts are often concerned about security and, fearing widespread dissemination of their vulnerabilities, may prevent them from offering useful information or hosting return visits in the future.

- Get permission before taking any photographs.

COMPLETE THE RECONNAISSANCE REPORT

The fourth step is to complete the reconnaissance report and transmit it to the SIO. The following are minimum recommended data requirements for a reconnaissance report prepared by a reconnaissance team:

- R&S plan employed.
- Site, system, or complex layout diagram with north arrow and scale.
- During the CARVER assessment, the operational element normally focuses on accessibility, vulnerability, and recognizability.
- Map coverage of 1:250,000, 1:50,000, and 1:25,000 extending out from the target and including significant installations and activities noted by the team that are inside the AO and AI.
- High-angle overhead, oblique overhead, and ground photography of the target with annotations of installations, activities, and differences observed by the operational element.
- Site communications and electronics data can be gathered by an attached SOT-A.
- Locations of the nearest fixed-wing capable airstrip and fixed-wing capable instrumented airstrip; ground distances from each of the above airstrips to the target; and the nearest DZs LZs, and BLSs.
- Summary of local human and material resources available to support special operations.
- List of sources used.

SITE RECONNAISSANCE TRAINING

Since target analysis is the responsibility of the SIO, the SIO trains reconnaissance teams or surrogates in technical appreciation and CARVER analysis. When training, consider the following:

- Know the characteristics and lowest common denominators of the type target being surveyed. This is a strong base for assessing vulnerabilities.
- It is best to get verification from at least two different sources for every critical fact.

- It is important to know the separate capabilities of the attacking and defending forces.
- When doing a map or sketch reconnaissance of the target and its environment, it is helpful to inventory the area in a clockwise fashion, starting at the farthest point considered and moving in circles of decreasing size to the center of the target.
- Although persons working or living near the target can provide important information, one should be wary of the advice of local "experts." Often they have little or no training or experience in target evaluation, and may know less than they think they do.
- Assessments should focus on the CARVER elements separately, and as they interrelate.
- When conducting assessments, three heads (and three sets of eyes) are better than one or two. A second opinion is helpful, but a third will provide greater insight and prevent potential deadlock.
- It is helpful to do separate analyses of like sites, and then to compare the results.
- Where security considerations permit, CARVER reports should be signed and dated to fix responsibility, facilitate scheduled updating, and identify future reconnaissance team and analyst training requirements.
- The farther one is from the risks of an operation, the more feasible the operation appears and the more academic the intelligence requirements seem. It is important to integrate both planning staff and operators in the analysis process.

APPENDIX B

Survey to obtain data

Possible Survey

x_1 = Target type and name: _____
(if good data can be used here then this item can be used with TOPSIS or give it a value from 1-9 for importance where 9 is extremely and 1 is equal.
 x_2 = Target distance or measured importance to Center of Gravity _____
 x_3 = Target significance ratio: US/opponent _____
 y = Escalation due to destruction/damage: 1 if yes 0 if no

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APPENDIX C USER'S GUIDE (DRAFT)

DRAFT Methodology Red Line Analysis DRAFT

Model and Model Updates as Necessary

Part 1. Targeting data

Step 1. Using experts and the CARVER Matrix explained in FM 34-36 as a targeting guide for analysis, choose the target list and fill in the values as directed by the matrix CARVER., see Appendix A and B.

NOTE: CARVER can be substituted for by any targeting scheme with criteria.

Step 2. Using experts perform a pairwise comparison among the criteria list under CARVER:

CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, AND RECOGNIZABILITY FACTORS.

Step 2a. Prioritize these variables as to their importance in the mission.

Step 2b. Fill in the pairwise comparison worksheet, comparing each pair A to B, and indicate intensity values on the 1-9 scale.

Step 2c. Insure the consistency ratio , CR , value is less than 0.1. If not, then some of the values in step 2b were inconsistent. This means that it is violates $A > B$, $B > C$, so we conclude $A > C$ or violates the intensity levels. This is fixed by trial and error.

Step 3. Reorder the current target CARVER matrix so that the columns match the prioritized order used in Step 2. This could be ACVERR, for example.

Step 4. The TOPSIS completes it update of the ranking of the targets and these values will be used as the target variable x_1 in the red line model.

Part 2. Red line Analysis

Step 1. Choose the appropriate model template for either two or three independent variables.

Step 2. The variable x_1 is automatically populated into the logistics model to be used unless otherwise directed by the user. We note that two columns are available in the worksheet for x_1 but only one of these columns that include either TOPSIS values or user inputs will be used.

Step 3. Enter in the columns for x_2 (and x_3) the data for each target in column x_1 in the appropriate worksheet.

Step 4. Enter the binary y_i variable were 1 implies escalation and 0 implies no escalation.

Step 5. As directed in the sheet, the Excel *Solver* needs to be engaged. In the order as directed inside the template highlight the MLFs to be maximized and then appropriate coefficients and for each in order call the solver and click solve. The template worksheet is updated automatically. The user might examine the output for significance.

Step 6. If using the two variable worksheet template you may view the 3-dimensional plot otherwise with 3 independent variable there is no plot available because we cannot plot in 4 dimensions. However, if hold one variable constant, say x_3 , then for each constant value of x_3 , we can obtain a surface to view.

Step 7. You can now enter your predictive analysis portion where for each possible real target you enter the data for x_1, x_2, x_3 .

Step 7a. We note that you may need to run another TOPSIS_Pred with CARVER to obtain values of x_1 for your target list.

Step 7 b. CRITICAL :Using the slider bar select the probability level for the red line, r , $0 \leq r \leq 1$.

Step 7c. The model compares the predicted probability with your slider value. Any values greater than the red line value is indicated as ***above red line***.

Step 8. Interpret all results.

Modeling Decision Criteria

Step 1. Prioritize decision criteria in CARVER

CRITICALITY, ACCESSIBILITY, RECUPERABILITY, VULNERABILITY, EFFECT, AND RECOGNIZABILITY FACTORS

Step 2. Using Saaty's 9 point scale whose meaning is shown in Table 1. Compare the criteria and when a criteria is compared to itself it gets a 1. Only fill in the upper portion.

Table 1. Saaty's 9-Point Scale

Intensity of Importance in Pair-wise Comparisons	Definition
1	Equal Importance
3	Moderate Importance
5	Strong Importance
7	Very Strong Importance
9	Extreme Importance
2,4,6,8	For comparing between the above
Reciprocals of above	In comparison of elements i and j if i is 3 compared to j , then j is 1/3 compared to i .
Rationale	Force consistency; measure values available

For example:

	C	A	R	V	E	R
C	1	3	4	5	6	8
A		1	3	4	5	6
R			1	4	5	6
V				1	3	4
E					1	3
R						1

Enter letters for criteria in priority order in the first column and first row. In the upper portion fill in the values using the intensity scale in Table 1.

	1					

		1				
			1			
				1		
					1	
						1

Doing this makes the use of the decision criteria worksheet in the template easier.

Example by the numbers:

Given that we have 31 historical or survey inspired targets, $t_1, t_2, \dots, t_{31}$. We gather information on target type as x_1 which was suggest using TOPSIS to generate these values. Variables x_2 and x_3 might be any measurable component of the target such as distance, ratio of importance to each side or any other measurable variables. For each target with its associated attributes in x_2 and x_3 , determine if escalation is possible (1) or no escalation (0). This might be highly subjective unless historical information or data is assessable.

Before we enter the data, we obtain a priority of the CARVER elements.

Criticality
Effect
Recuperability
Recognizability
Accessibility
Vulnerability

We enter this list, in this order, into the TOPSIS_Model

Let's assume that in our matrix looks like this:

	C	E	R	R	A	V
C	1	2	3	5	5	6
E		1	2	4	5	5
R			1	3	4	4
R				1	2	3
A					1	1
V						1

We input this information into the worksheet page, DecisionWeight_Model.

Criterion Decision Tool (Analytical Hierarchy Process)

Data can only be input into yellow fields
Blue fields are automatic

Step 1. Verify the number of criterion being compared (pulled from the TOPSIS Tool): 6

Step 2. Input criterion names in approximate order of importance (1 is most important):

	Criterion Name	Comments (Not Required)
1	C	
2	E	
3	R	
4	R	
5	A	
6	V	

We input those values, in the areas marked in yellow.

Step 3. Compare the importance of each criterion element to all others as they relate to the overall project objective:

a) Use the dropdown menu to select A or B for the element you consider to be "More Important" as it relates to the others
b) Select an "Intensity" value (1-9) to describe the degree of importance of the "More Important" element over the other
c) Intensity value definitions are listed below the comparison table

	Element			
	A	B	More Important	Intensity (1-9)
1	C	E	A	2
2		R	A	3
3		R	A	5
4		A	A	5
5		V	A	6

We check the CR value on the worksheet to insure that it is less than 0.1.

Step 4. Final Criterion Weights (automatically transferred to TOPSIS Tool) and Consistency Ratio:

	Criterion Name	Weight
1	C	0.4168
2	E	0.2416
3	R	0.1499
4	R	0.0770
5	A	0.0607
6	V	0.0540
7	NA	0.0000
8	NA	0.0000

CR= 0.012 *Consistent <0.10*

Here we find the CR is 0.012 which is less than 0.1 so we are consistent and we can use the weights for the criteria. Since we prioritized the CARVER criteria our weights are in numerical order.

Next we take our data of our 31 targets and reorganize the columns to match our prioritized CARVER order as CERRAV.

C	A	R	V	E	R				C	E	R	R	A	V
5	1	5	4	6	2				5	6	5	2	1	4
4	9	4	4	7	3				4	7	4	3	9	4
6	5	2	6	8	2				6	8	2	2	5	6
5	9	2	1	6	3				5	6	2	3	9	1
5	3	4	2	4	1				5	4	4	1	3	2
1	2	8	5	9	3				1	9	8	3	2	5
8	9	1	8	2	1				8	2	1	1	9	8
4	2	8	7	9	6				4	9	8	6	2	7
6	7	4	7	5	1				6	5	4	1	7	7
7	3	3	6	6	4				7	6	3	4	3	6
6	5	3	6	1	8				6	1	3	8	5	6
1	1	7	1	7	1				1	7	7	1	1	1
1	5	2	7	5	7				1	5	2	7	5	7
3	9	1	2	4	4				3	4	1	4	9	2
7	4	4	1	9	2				7	9	4	2	4	1
3	7	8	2	4	3				3	4	8	3	7	2

We input these into our TOPSIS_Model template. The output as the TOPSIS values that do rank the targets from 1 to 31 in order of strategic or tactical importance based upon the criteria used.

Alternatives	C	E	R	R	A	V	NA	NA	Value	Rank	
1	5	6	5	2	1	4	0	0	0.307	29	T1
2	4	7	4	3	9	4	0	0	0.635	10	T2
3	6	8	2	2	5	6	0	0	0.522	16	T2
4	5	6	2	3	9	1	0	0	0.510	17	T3
5	5	4	4	1	3	2	0	0	0.281	31	T4
6	1	9	8	3	2	5	0	0	0.482	22	T5
7	8	2	1	1	9	8	0	0	0.638	9	T7
8	4	9	8	6	2	7	0	0	0.493	20	T8
9	6	5	4	1	7	7	0	0	0.634	11	T9
10	7	6	3	4	3	6	0	0	0.409	25	T10
11	6	1	3	8	5	6	0	0	0.501	19	T11
12	1	7	7	1	1	1	0	0	0.355	27	T12
13	1	5	2	7	5	7	0	0	0.657	6	T13

The ranks are listed under Rank. The model uses this in its building of the logistics regression model next. We also input the attributes of each target, x2 and x3 and the binary y value under the appropriate heading in the logistics worksheet. Currently the values of x1 are from the TOPSIS worksheet.

Y	x1	x2	x3	x4
0	0.307359	19.2	24.000	0.000
1	0.635242	24.1	22.000	0.000
0	0.522035	-7.1	19.000	0.000
1	0.510246	3.9	15.000	0.000
0	0.280965	4.5	15.000	0.000
1	0.482208	10.6	25.000	0.000
0	0.638221	-3	19.000	0.000
1	0.49337	16.2	19.000	0.000
1	0.634412	72.8	14.000	0.000
0	0.408865	28.7	16.000	0.000
0	0.501083	11.5	28.000	0.000
0	0.354648	56.3	30.000	0.000
1	0.656819	-0.5	25.000	0.000
1	0.562575	-1.3	21.000	0.000

...

For all 31 data elements.

The user tells the worksheet the number of data points and the number of variables used as well as whether or not to use the TOPSIS results as $x1$. These are highlighted in yellow.

Enter	Enter	Enter your own x1, enter 1 or from TOPSIS enter 2	Item #
Number of data rows	Number of		2
	31	parameters	
Constant Model		3	

Obtaining the correct model is a two part Solver process.

First, highlight cell B6, click Solver. We want to maximize this cell and the Changing cells is only cell A6.

bo	MLF (constant)
0	-21.4875626

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help Solve Close

Click on Solve.

The Solver found the solution.

MLF (constant)	0.459532327	-20.6903829
----------------	-------------	-------------

Next, again engage the Solver but this time maximize cell C8, by changing cells B8-B10.

Solver Parameters

Set Objective:

To: ☒ Max ☐ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Add

Change

Delete

Reset All

Load/Save

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method:

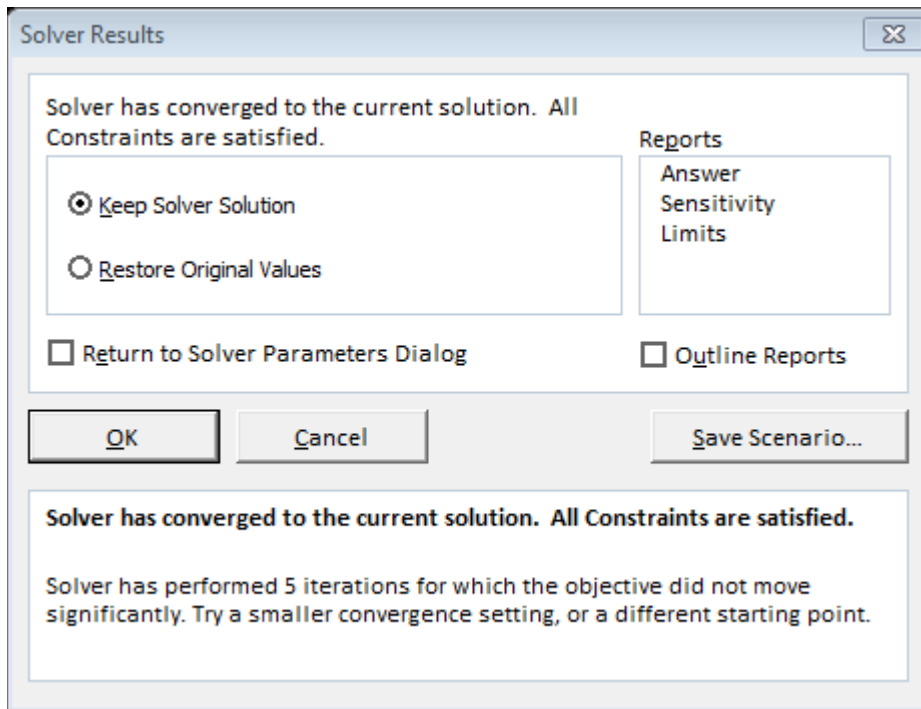
Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help Solve Close

We click Solve.



We have found the solution to the 2nd model.

Full Model		MLF Full Model
b0	-10.97441015	-10.08383818
b1	21.67322996	
b2	0.028868569	

This is our logistics model,

$$y = \frac{\exp(-10.9744 + 21.67323x_1 + 0.028869)}{1 + \exp(-10.9744 + 21.67323x_1 + 0.028869x_2)}$$

This model is used to calculate the *red line predictive* portion of the model. The y is no longer a {0,1} binary variable but a probability between [0,1]. The user is asked to set the bar for y later in the model.

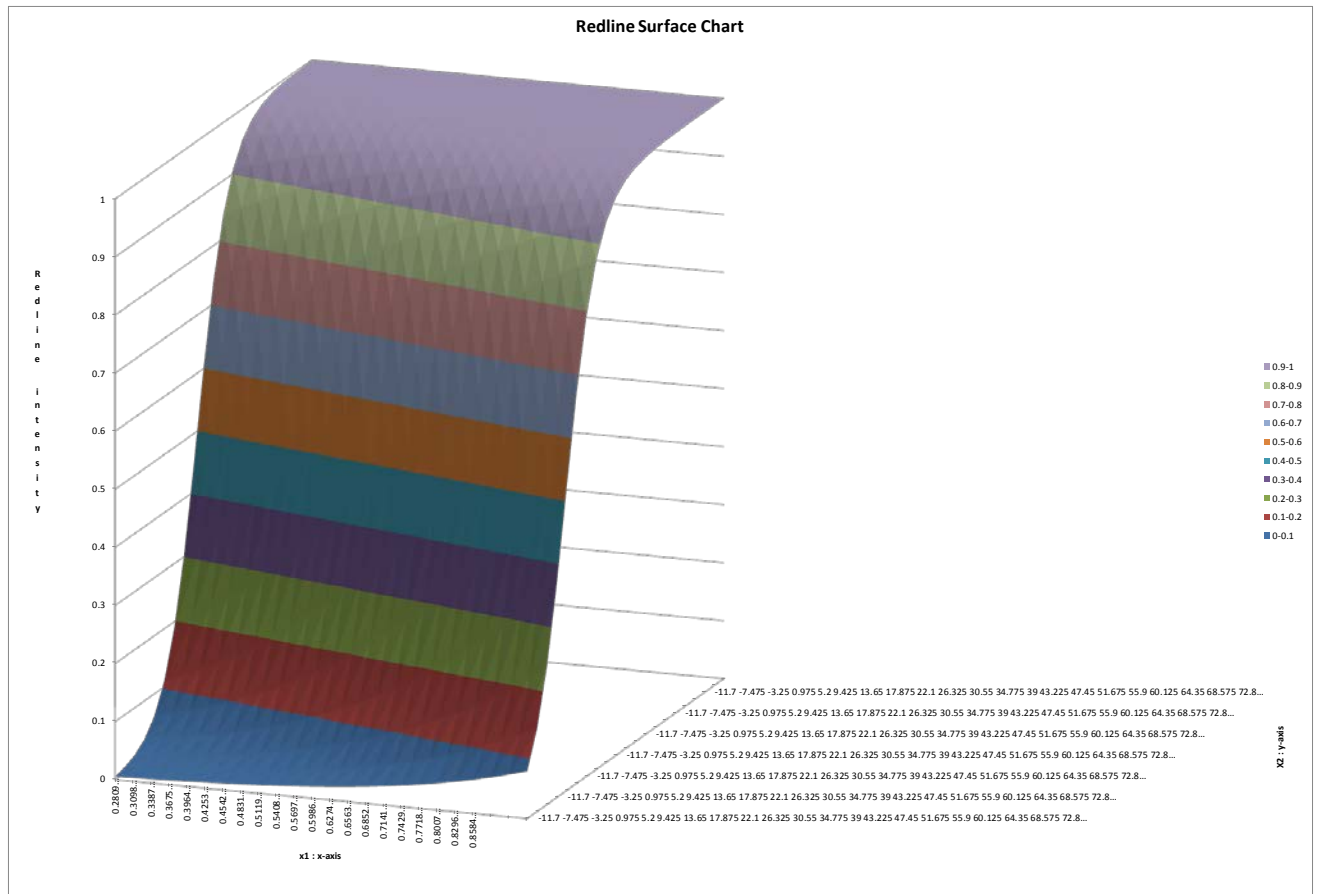
But how good is the model?

The statistics on the model are provided within the worksheet.

							ch
Parameters	Coeff	SE	Z-Stat	P-Value	Odds ratio		
b0	-10.9744	4.441572	-2.47084	0.019831812			
b1	21.67323	8.340388	2.598588	0.014764606	2.59E+09	3.87E-10	
b2	0.028869	0.040534	0.712202	0.482232336	1.029289	0.971544	
b3	0	0	NA	NA	1	1	
b4	0	0	NA	NA	1	1	
Log-Likelihood=		-10.0838					
Test of all slopes = to zero			41.38077	20.16767636			
	G=		21.21309				
	df		2				
	P-value		2.48E-05	good fit			

The model is a good fit and only the coefficient of x_2 is not significant. Since the model is good, we continue.

In this example, since we have only 2 independent variables (x_1, x_2) and a binary y , we can obtain a logistics surface found under the sheet, Chart 1.



We are now ready for Part 2, using the predictive portion of the model.

Part 2 Discussion.

In this portion of the model we only have information on x1, x2, x3. As a matter of fact, we suggest using the TOPSIS_PRED and DecsionWeight_PRED to generate the target list values. These worksheets are available. You do not need the same weights as before in the original data sets analysis although it might be helpful.

For sake of illustration, we change a value from a 5 to a 6.

	C	E	R	R	A	V
C	1	2	3	5	6	6
E		1	2	4	5	5
R			1	3	4	4
R				1	2	3
A					1	1
V						1

The CR is 0.007 and the weights are as shown

Step 4. Final Criterion Weights (automatically transferred to TOPSIS Tool) and Consistency Ratio:

	Criterion Name	Weight
1	C	0.4221
2	E	0.2416
3	R	0.1497
4	R	0.0771
5	A	0.0553
6	V	0.0543
7	NA	0.0000
8	NA	0.0000

CR= **0.007** Consistent <0.10

We then compute the TOPSIS ranks after we input our CARVER data in the order of CERRVA into the model and using these weights.

Step 5. Enter the criterion values for each alternative (accepts either numerical or categorical data) and view results:

Alternatives	C	E	R	R	A	V	NA	NA	TOPSIS Value	Final Rank	
1	9	3	3	8	7	6	0	0	0.597	3	Bulk Elec Power
2	7	3	7	4	3	8	0	0	0.545	4	Bulk Petroleum
3	3	5	3	7	7	3	0	0	0.624	1	Water Supply
4	3	4	7	2	2	2	0	0	0.352	7	Communication Sys
5	1	1	3	1	2	2	0	0	0.396	6	Air Transport
6	1	1	3	1	1	1	0	0	0.350	8	Ports and waterways
7	2	4	4	1	4	3	0	0	0.506	5	Rail Transport
8	1	5	2	8	2	8	0	0	0.606	2	Road Networks

The TOPSIS values then feed into our Predictive_Model sheet. We enter the corresponding x2 data with our calculated x1 values for our targets. We also must use the slider bar to set the probability levels of acceptance.

A	B	C	D	E	F	G	H
			b0	b1	b2		
			-10.9744102	21.67322996	0.028868569		
	Redline Probability			57			
			P(X>n)	0.57	User Inputs probability between [0,1]		
	User Inputs				Scroll Bar		
Do you have x1 data or want it inputed			by Topisis: 1 yes or 2 no		1		
			x1	x2		Model Calculation	Decision
		Bulk Elec Power	0.59705	16		0.918902206	Above redline
		Bulk Petroleum	0.54505	12		0.765834853	Above redline
		Water Supply	0.62413	5		0.936843421	Above redline
		Communication Sys	0.35165	3		0.036744168	Below redline
		Air Transport	0.3958	11		0.111199879	Below redline
		Ports and waterways	0.3496	4		0.03619513	Below redline
		Rail Transport	0.50559	5.5		0.535477788	Below redline
		Road Networks	0.60598	5		0.909156732	Above redline

In the example the probability of being a 1 versus a 0 (above the red line) is set at 0.57. Additionally, if we set it the probability at 0.85, we find no change in our predictions.

		b0	b1	b2	
		-10.9744102	21.67322996	0.028868569	
Redline Probability			85		
	P(X>n)		0.85	User Inputs probability between [0,1]	
User Inputs				Scroll Bar	
Do you have x1 data or want it inputed			by Topisis: 1 yes or 2 no		1
		x1	x2	Model Calculation	Decision
	Bulk Elec Power	0.59705	16	0.918902206	Above redline
	Bulk Petroleum	0.54505	12	0.765834853	Below redline
	Water Supply	0.62413	5	0.936843421	Above redline
	Communication Sys	0.35165	3	0.036744168	Below redline
	Air Transport	0.3958	11	0.111199879	Below redline
	Ports and waterways	0.3496	4	0.03619513	Below redline
	Rail Transport	0.50559	5.5	0.535477788	Below redline
	Road Networks	0.60598	5	0.909156732	Above redline

Looking at the values if we set our probability at 0.95 then no target would be above the red line.

The Three Variable Logistics' Model

If we choose to use the three variable Logistics' model, all processes are the same except:
 (1) No surface can be directly generated for viewing since we can only directly view in 2 or 3 dimensions. NOTE: if needed we can hold one of the variable x_1 , x_2 , or x_3 as a constant value and obtain a surface for the remaining variables. This is the same as taking a slice through the surface.

(2) We added an additional independent variable to the logistics process and thus require an additional b coefficient to the b_i coefficients.

We only provide the screenshots of the model here.

Entering the data: x_1 is the target type and can be entered directly into the worksheet labeled x_1 or accepted from the TOPSIS output.

	A	B	C	D
Enter		Enter	Enter your own x1, enter 1 or from TOPSIS enter 2	Item
Number of data rows		Number of		
	31	parameters		2
Constant Model		4		
b0		MLF (constant)	This number is b0,b1,b2 for 2 indepdent variables	
	-0.064538537	-21.47143077		
Full Model		MLF Full Model		
b0		-3.35004541	-18.14334801	
b1		6.841381769		
b2		0.030153631		
b3		-0.02565912		

Y	x1	x2	x3
0	0.381845	19.2	24.000
1	0.504357	24.1	22.000
0	0.449995	-7.1	19.000
1	0.44669	3.9	15.000
0	0.274323	4.5	15.000
1	0.579838	10.6	25.000
0	0.193153	-3	19.000
1	0.692072	16.2	19.000
1	0.317215	72.8	14.000
0	0.435951	28.7	16.000
0	0.489406	11.5	28.000
0	0.456532	56.3	30.000
1	0.669774	-0.5	25.000
1	0.461139	-1.3	21.000

The coefficients are found by engaging the Solver twice as directed in the worksheet,

Enter	Enter	Enter your own x1, enter 1 or from TOPSIS enter 2 It
Number of data rows	Number of	2
31	parameters	
Constant Model	4	
MLF (constant)	This number is b0,b1,b2 for 2 independent variables	
-0.064538537	-21.47143077	
Full Model	MLF Full Model	
b0	-3.35004541	-18.14334801
b1	6.841381769	
b2	0.030153631	
b3	-0.02565912	

Step 1. Enter number of data rows in cell a3

The model for this 3 variable case is

$$y = \frac{\exp(-3.35005 + 6.8414x_1 + 0.03015x_2 - 0.025659x_3)}{1 + \exp(-3.35005 + 6.8414x_1 + 0.03015x_2 - 0.025659x_3)}$$

Again, TOPSIS or direct input is used to create the prediction input data for x1 and we assume we have good data for x2, and x3 that correspond with each x1. The scroll bar is set at 0.57 for this run

	User Inputs				Scroll Bar	
	Do you have x1 data or want it inputed		by TOPSIS: 1 yes or 2 no			1
		<i>x1</i>	<i>x2</i>	<i>x3</i>	Model Calculation	Decision
	Bulk Elec Power	<i>0.597051617</i>	<i>16</i>	<i>25</i>	<i>0.640052664</i>	Above redline
	Bulk Petroleum	<i>0.545046921</i>	<i>12</i>	<i>23</i>	<i>0.537557241</i>	Below redline
	Water Supply	<i>0.624132466</i>	<i>5</i>	<i>32</i>	<i>0.562068045</i>	Below redline
	Communication Sys	<i>0.351653397</i>	<i>3</i>	<i>31</i>	<i>0.161214113</i>	Below redline
	Air Transport	<i>0.395802233</i>	<i>11</i>	<i>27</i>	<i>0.268289655</i>	Below redline
	Ports and waterways	<i>0.34960048</i>	<i>4</i>	<i>21</i>	<i>0.201568154</i>	Below redline
	Rail Transport	<i>0.505590679</i>	<i>5.5</i>	<i>24</i>	<i>0.415547043</i>	Below redline
	Road Networks	<i>0.605975628</i>	<i>5</i>	<i>22</i>	<i>0.594339165</i>	Above redline

Again we can adjust the scroll bar. If set at 0.70 then in this example all targets are below the **Red line**.

Author's note: This model is a draft and has yet to be soldier proofed.

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LIST OF REFERENCES

- (1) Gordon, M.R/ (2013). Aim of U.S. attack: restore a ‘Red Line’ that became blurred. NY Times.
<http://www.nytimes.com/2013/08/30/world/middleeast/aim-of-a-us-attack-on-syria-sharpening-a-blurred-red-line.htm>, accessed November 4, 2016.
- (2) NTI. What is a Red Line worth. <http://www.nti.org/gsn/article/what-red-line-worth/> Accessed November 4, 2016.
- (3) Steiner, J. (2016). Challenging the Red Line between Intelligence and Policy.
https://isd.georgetown.edu/sites/isd/files/Associates_Steiner_Challenging_Red_line.pdf Accessed November 4, 2016.
- (4) Intelligence on Iran, <http://www.iranintelligence.com/miloptiononly> Accessed November 4, 2016.
- (5) Department of the Army (2012), *Special Operations* (ARDP 3_05), 31 August 2012, Washington, DC, pp. V.
- (6) Department of the Army (2007), *Psychological Operations Process Tactics, Techniques, and Procedures* (FM 3-05.301), August 2007.
- (7) Department of the Army (1991). FM 34-36: Special Operations Forces Intelligence and Electronic Warfare Operations, Appendix D. 30 September 1991.
- (8) Gaijinass (March 11, 2010). [*CARVER Matrix: Tactical Target analysis*](#). gaijinass. Retrieved January 22, 2016.
- (9) Gaijinass (September 7, 2009). [*Use the CARVER Matrix for Management*](#). gaijinass. Retrieved January 22, 2016.
- (10) Saaty, Rozanne W. (1987) The analytic hierarchy process—what it is and how it is used. *Mathematical Modelling* 9(3), pp. 161-176.

- (11) Saaty. T. (1980). *The Analytic Hierarchy Process*. New York, NY: McGraw-Hill Book Company.
- (12) Fox, W. (2014) Chapter 221, TOPSIS in Business Analytics. *Encyclopedia of Business Analytics and Optimization*. IGI Global and Sage Publications. V(5), 281-291.
- (13) Fox, W. (2015), Chapter 4 Introduction to Multi-Attribute Decision Making in Business Analytics, Springer book on Advanced Business Analytics, edited by Fausto Pedro García Márquez and Benjamin Lev, Springer Publishers, pp. 55-92.
- (14) Fox, W. P. (2012). Mathematical Modeling of the Analytical Hierarchy Process Using Discrete Dynamical Systems in Decision Analysis, *Computers in Education Journal*, 3(3), 2012, 27-34.
- (15) Fox, W (2015). Chapter of *Mathematical Modeling in Risk Analysis*, IGI-Global book on Analyzing Risk through Probabilistic Modeling in Operations Research, edited by Dr. Dariusz Jacek Jakóbczak. Chapter 3, pp. 64-93
- (16) Fox, W. (2013). Mathematical Modeling with Logistics Regression Analysis in Excel, *Computer in Education Journal*, 4(3), pp 9-15.
- (17) Hosmer, D, S. Lemeshow, & R. Sturdivant. (2013). Applied Logistics Regression, 3rd Ed. John Wiley and Sons. Hoboken, NJ.
- (18) Mendenhall, W. & T. Sincich. (1996). A Second Course in Statistics Regression Analysis, 5th ed. Prentice Hall, Upper Saddle River, NJ. pp. 476-485.
- (19) Montgomery, D., E. Peck, & G. Vinning. (2006). Introduction to Linear Regression Analysis, 4th ed. John Wiley and Sons. Hoboken, NJ. pp. 428-448.
- (20) Fox, William P. (2011). Using the EXCEL Solver for Nonlinear Regression, *Computers in Education Journal (COED)*. October-December, 2(4), pp. 77-86.
- (21) Fox, William P. (2012). Issues and Importance of ‘Good’ Starting Points for Nonlinear regression for Mathematical Modeling with Maple: Basic Model Fitting to Make Predictions with Oscillating Data. *Journal of Computers in Mathematics and Science Teaching*. 31(1), pp. 1-16.

(22) Kleinbaum, D. & M. Klein (2010). Logistics Regression: A self-learning text. 3rd edition, Springer, New York: NY. Chapter 10, pp. 346-372.

(23) Alinezhad, A. & Amini, A. (2011). Sensitivity Analysis of TOPSIS technique: The results of change in the weight of one attribute on the final ranking of alternatives, *Journal of Optimization in Industrial Engineering*, 7(2011), 23-28.

(24) Fox, William P. (2012) Mathematical Modeling with Maple. Cengage Publishers. Boston, MA.

(25) Giordano, F. W. Fox, S. Horton, & M. Weir. (2013) A First Course in Mathematical Modeling, 5th ed. Cengage Publishing. Boston, MA.

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