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ANALYSIS OF PRICING MODELS IN THE DEFENSE INDUSTRY

TO SUPPORT COST PROJECTIONS

by

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ABSTRACT

The government runs a program to develop a technologically advanced weapon system. In the competition phase, the government provides initial funding to several defense contractors to develop system prototypes. Based on the demonstration of these prototypes, the government selects one defense contractor as the sole source to produce the final product. We develop a mathematical model to describe this process. By analyzing the model and conducting a numerical study, we find three main reasons why such a program often suffers delay and cost overrun. First, the selected contractor tends to be luckier than usual in the competition phase, so the government tends to overestimate its capability. Second, once a contractor becomes the sole source, their goal is to complete the scheduled tasks within each fiscal year on time, but not to deliver the final product as soon as possible. Third, the contractor may be motivated to exert extra effort during the competition phase in order to improve their chance of getting selected as the sole source, which may result in an overly optimistic estimation on program completion time. Based on a cost structure, our model offers recommendations on the optimal length of the competition phase and the number of contractors to invite, in order to minimize the program completion time and total cost.

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Executive Summary

The pricing models in the defense industry cannot rely on basic economic principles, such as supply and demand, since it involves advanced technology that is pertinent to national security. The cost estimation of developing a technologically advanced weapon system is typically done based on component cost, labor cost, inflation, cost to acquire new technology, and opinions of subject matter experts. Since not all necessary technologies are in place at the onset of the system development, there is a lot of uncertainty in the total program cost and completion time. If a technological hurdle cannot be overcome in time, the whole program may suffer substantial delay and cost overrun.

Besides technological uncertainty, developing a technologically advanced weapon system also involves a lot of political uncertainty. The budget needs to be approved on a yearly basis and sometimes a program may get cancelled. This uncertainty puts pressure on defense contractors to secure sole-source contracts when competing against other contractors. Once becoming a sole-source contractor, however, the contractor's main motivation is to complete the scheduled tasks on time in each fiscal year, but not to deliver the final product as soon as possible.

This study aims to develop a mathematical model to describe the interactions between defense contractors and the government, in order to estimate the cost and schedule for developing a technologically advanced weapon system. The research goal is twofold: (1) explain why the development of a new weapon system is often subject to cost overrun and schedule delay; and (2) identify market mechanisms to improve the efficiency of the bidding and contracting process in order to better manage risk.

To achieve these goals, we develop a mathematical model in which the government manages a program to develop a technologically advanced weapon system in two phases: the competition phase and the sole-source phase. Both phases consist of three steps, which are described below.

1. The government funds a few defense contractors to develop a prototype for a technologically advanced weapon system in the competition phase. At the end of the competition phase, the government selects a sole source to develop the final product.
2. Each contractor may or may not be motivated to exert extra effort in the competition phase in order to improve their chance of winning the sole-source contract.
3. At the end of the competition phase, each contractor demonstrates their product prototype. The quality of the prototype depends on the contractor's design capability and also their luck in overcoming technological hurdles.
4. Based on the prototype demonstration, the government selects a sole-source contractor to develop a final product. The program enters the sole-source phase.
5. The sole-source contractor continues to develop the weapon system. The progress may be affected by fiscal-year budget constraints, as well as the contractor's capability and luck in overcoming technological hurdles.
6. The government's payoff depends on the total program cost and program completion time. The government prefers to spend less money and complete the program sooner.

By using probabilistic modeling to capture the uncertainty of developing a technologically advanced weapon system, we are able to quantify the effect of several model parameters on the eventual program cost and completion time. After analyzing the model, we run a simulation study to gain insights into the entire process and identify three main reasons why such a program often suffers cost overrun and schedule delay.

1. The selected contractor tends to be luckier than usual in the competition phase, so the government tends to overestimate the contractor's capability.
2. Once a contractor becomes the sole source, their goal is to complete the scheduled tasks within each fiscal year on time, but not to deliver the final product as soon as possible.
3. The contractor may be motivated to exert extra effort during the competition phase in order to improve their chance of getting selected as the sole source, which may result in an overly optimistic estimation on program completion time.

Based on a cost structure, our model offers recommendation on the optimal length of the competition phase, and the number of contractors to invite, in order to minimize the program completion time and total cost.

Our model assumes that the government announces the length of the competition phase in advance, and selects one contractor at the conclusion of the competition phase. An alternative approach is to review each contractor's progress on a yearly basis and decide which contractors to fund for another year. In addition, if the government has some prior knowledge about each contractor's capability, then a Bayesian approach may produce a more reliable estimate on the program completion time. A separate study is needed to explore these issues.

1 Introduction

Because building an aviation system, such as FA-18, MH-60, and F-35 aircraft, involves advanced technology pertinent to national security, the pricing of such aviation systems primarily relies on estimating the cost to build the system, as opposed to basic economic principles such as supply and demand. There are only a handful of defense contractors that are capable of building such aviation systems—such as Boeing, Northrop Grumman, and Lockheed Martin. On the demand side, the only major buyer of these aviation systems is the United States Government. Whereas it is possible to sell these aviation systems to foreign governments, any such transactions need to be approved by the U.S. Government.

On the surface, the market resembles a monopsony, where there is only one buyer (the U.S. Government) in the market with several suppliers (defense contractors) offering similar products. Upon a closer look, however, the market also shares a lot of similarities with a monopoly, since once an aviation system is selected for production, the selected defense contractor becomes the only supplier. For instance, after the Lockheed Martin F-35 was selected over the Boeing X-32 in the Joint Strike Fighter program, Lockheed Martin enjoyed monopolistic power to manufacture the next-generation multipurpose fighter jets. As a consequence, the entire procurement process is vulnerable to many market inefficiencies shared by monopoly and monopsony.

This project aims to develop a mathematical model to describe the relationships among the stakeholders in the defense industry, focusing on the U.S. Government and major defense contractors. The research goal is twofold: (1) explain why the development and manufacturing of advanced aviation systems are often subject to program cost increases and delays; and (2) identify market mechanisms to improve the efficiency of the bidding and contracting process in order to better manage risk.

The rest of this report proceeds as follows. Section 2 reviews the relevant literature. Section 3 develops a mathematical model to describe the relationship between the government and major defense contractors. Section 4 analyzes the competition phase, while Section 5 analyzes the sole-source phase. Section 6 presents an extensive numerical study. Finally, Section 7 concludes the report.

2 Literature Review

Sapolsky, Gholz, and Talmadge (2013) provides a comprehensive overview of U.S. defense politics. It discusses America's security strategy, the political economy of defense and the weapons acquisition process, among others. It offers many interesting observations and their plausible explanations in the U.S. defense industry.

There are two main reasons why cost is difficult to project for major weapons systems acquisition:

1. Technological uncertainty: The Department of Defense (DoD) requests weapon systems that require future technology. When the contracts are awarded, typically not all the technologies required to build the systems are in place.
2. Political uncertainty: The U.S. budget needs to be approved on a yearly basis. Each year, the congress may decide whether to push forward a project, postpone it, or cancel it. This uncertainty puts pressure on defense contractors to win contracts against competition by offering favorable bids that may not be entirely realistic.

Sapolsky et al. (2013) further offer many observations to support their arguments.

1. The U.S. defense industry has production overcapacity. A lot of mergers and acquisitions happened in the late 90s; however, with fewer major contractors, the production capacity remains roughly the same. There are still six shipyards and more plants building military aircraft and armored vehicles. Defense contractors want to lobby for a line in the defense budget to keep the plan running, while congressional members have an interest in preserving employment in their respective districts. Keeping production of current weapon systems partially reduces the contractors' motivation and ability to invest in research and development.
2. The defense budget is reviewed on a yearly basis, and goes up and down through cycles. The armed services have incentive to push a weapon system into production before it is fully developed (such as B-1 bomber of the Air Force), when the defense budget allows. It is easier to get additional funding to fix a problem once the production has started, if the defense budget goes through a downturn in the following years.
3. Defense is like no other business in its forgiveness of cost overruns and time slippages. There is no immediate consequence to national security if the new systems are not deployed on schedule, as long as new threats have not emerged.
4. Defense projects depend on two different kinds of support: military requirements (severity of national security threats) and congressional votes (constituent interests and lobbying). When the threats are perceived less salient, lobbying and pork-barrel politics play a more important role in the political economy of defense.
5. The Navy uses smaller contractors to build a large number of small, inexpensive surface combatants. By letting smaller players into the defense business, the Navy gained more yards to feed: Adding new, small players is much easier than driving the big players away. The small players will learn that it is easier to lobby the government for follow-on projects, each promising billions of dollars in

future revenues, than it is to go hunting for low-cost commercial deals for which lower-cost overseas yards will complete. The result is the problem of production overcapacity.

6. Projects with foreign partners are harder to terminate. The Joint Strike Fighter has an extra wall around it because Britain, Australia, Canada, Italy, and several other allies have not only promised to buy the aircraft, but have contributed funds to its development.

Gholz and Sapolsky (1999) studied the U.S. defense industry in the late 1990s. During the Cold War, the U.S. defense buildup led many contractors to invest in huge production capacity, which was no longer needed after the Cold War ended in 1989. In the 1990s, some contractors were forced out of business and some were merged, but the production capacity remained roughly the same throughout this process. This overcapacity puts a lot of pressure on defense contractors to secure funding. If two projects are awarded to the same contractor, where the success of the first project has potential to attract additional funding, while the success of the second project does not, then it is conceivable that the contractor is motivated to exert more effort on the first project.

Besides aforementioned studies on the defense industry, Augustine (1997) discusses reshaping the defense industry by studying Lockheed Martin's survival story. Blair and Harrison (2010) present monopsony in law and economics. Guay and Callum (2002) study the Europe's defense industry and how it was affected by the changes in the U.S. defense industry.

There is a lot of work on cost estimation in the literature, such as Garvey (2000) and Stewart (1991). In particular, Mislick and Nussbaum (2015) discuss many methods that are applicable to projects managed by the DoD. However, cost estimation for long-term, high-technology, high-risk projects has not been addressed adequately in the literature. Brown, Grose, and Koyak (2006) study this specific problem, where the task durations and costs may increase over time. They use U.S. Army Future Combat Systems as the motivating application and compare different scheduling plans, based on the risk assessment of each task. While these earlier works assume that it is possible to objectively quantify the duration of each task by a probability distribution, our model requires the government to estimate them in real time, and accounts for how contractors might be motivated to affect this estimation.

3 The Models

This section develops a model for the development of a technologically advanced weapon system. We first present a descriptive model in Section 3.1, via an influence diagram, and then introduce assumptions on the descriptive model to produce a mathematical model in Section 3.2.

3.1 A Descriptive Model

We break the entire process of weapon system development into six steps, as shown in an influence diagram in Figure 1. In steps 1–3, the government funds several contractors to develop system prototypes. In steps 4–6, the government selects one contractor to further develop the system. The details of the process are explained as follows.

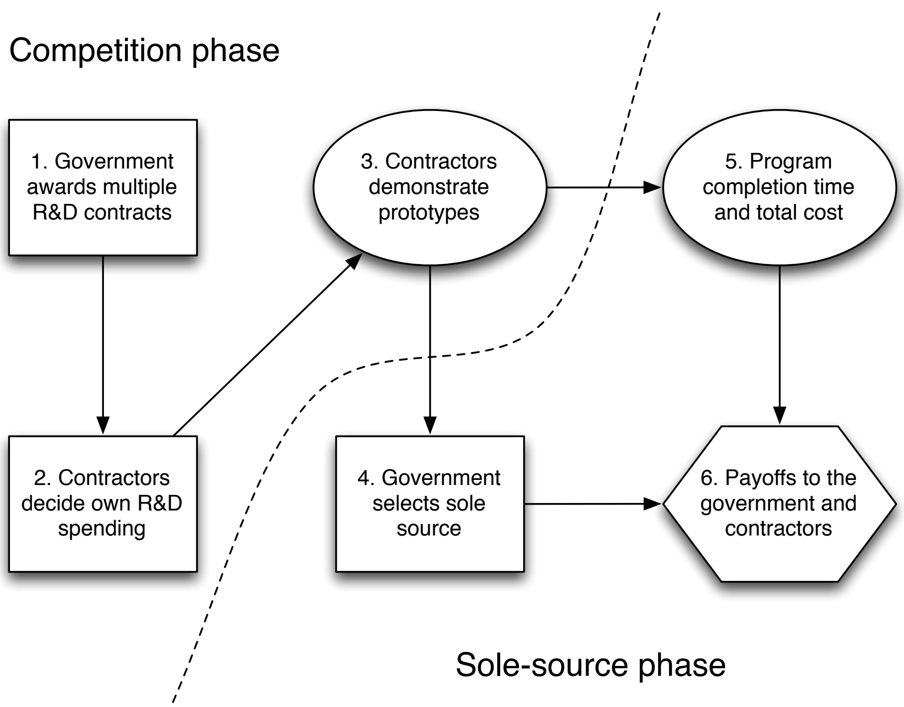

Figure 1: An influence diagram for new weapon systems development and acquisition process.

1. There are a few (typically two or three) defense contractors invited to develop a technologically advanced weapon system. The government provides funding to each contractor for research and development, and will select a sole source after a period of time (typically 3–7 years).
2. Each contractor may decide to add its own funds (or effort) to the development the system prototype, or they may be prohibited from doing so (such as the case of the Joint Strike Fighter program). The research funds—whether from the government or the contractor's own—directly affects the R&D results.

3. When the competition phase completes, each contractor is required to demonstrate their research results, perhaps via a prototype of the system. A contractor is more likely to produce a more promising prototype if the contractor is more capable than the other contractors (better design team), or if the contractor exerts more effort; but, there is no guarantee, since the research process involves a lot of technological risk.
4. Based on the prototype demonstration, the government (Congress and DoD) then selects a sole source to further develop the weapon system. Typically, the government selects the contractor whose prototype looks most promising and whose projected time to completion is the shortest.
5. At the time of sole source selection, the actual time to program completion and the actual total cost are random variables. After further development, the actual time to completion and total cost are revealed.
6. The payoff for the government depends on the total program cost and the total time to program completion. The government prefers to spend less money and complete the program sooner.

3.2 A Mathematical Model

We next introduce mathematical model assumptions for the descriptive model in Figure 1. The assumptions made at each node are summarized below.

1. A project to develop a technologically advanced weapon system includes many routine tasks whose time and cost are straightforward to estimate, as well as many research tasks that bear a lot of risk. To model the risky research tasks, we assume that a project consists of m *technical steps*, and a contractor has a maximal capable rate λ to complete these technical steps. By working at rate λ, the expected amount of time required to complete the next technical step is $1/\lambda$, but the actual amount of time follows an exponential distribution. Therefore, the time it takes to complete the project (all m technical steps) follows a gamma distribution, with shape parameter m and rate parameter λ. The parameters λ and m can be chosen to fit the assessment from subject matter experts at the onset of the project.

   The government's program to develop a technologically advanced weapon system consists of two phases: the *competition phase* and the *sole-source phase*. In the competition phase, the government decides on n, the number of contractors to fund (typically $n = 2$ or 3). The government does not know the ability of each contractor; in other words, the government does not know each contractor's actual maximal capable rate. In the competition phase, the government provides funds for these contractors, with the idea that only the best-performing contractor during the competition phase will be selected to receive continual funding to complete the project in the sole-source phase.
2. In the competition phase, each contractor has strong motivation to work at its maximal capable rate in order to maximize their chance of getting selected by the government for the sole-source phase. It is also conceivable that a contractor

will be motivated to exert extra effort to help improve their chance of getting selected. To model this potential behavior, we suppose that in the competition phase, each contractor works at a rate $(1+\alpha)\lambda$. The parameter $\alpha \geq 0$ models a contractor's motivation in the competition phase. In our numerical study, we will test $\alpha = 0\%, 5\%, 10\%$.

3. At the conclusion of the competition phase, each contractor is required to demonstrate their progress. Research and development is inherently a risky process. A contractor is more likely to deliver a better prototype, if they are more capable (larger maximal capable rate λ) or exerts more effort (larger extra effort α), but there is no guarantee. A less-capable contractor may get lucky and end up producing the best prototype. We assume that at the end of the competition phase, each contractor reveals the number of technical steps they completed during the competition phase.

4. At the conclusion of the competition phase, the government will examine the progress of each contractor. For instance, for the Joint Strike Fighter program, the government assessed Boeing and Lockheed Martin after five years (1996–2001) via prototype demonstration. By assessing how many technical steps each contractor has accomplished in the competition phase, the government estimates how much longer it will take for each contractor to complete the project. The government selects the project whose expected time to completion is the soonest, and commits to funding the project through its fruition. The program thus enters the sole-source phase.

5. In the sole-source phase, the contractor enters into a multiyear contract with the government. The idea is for the government to fund the project through its fruition; but, in reality, the government's budget is appropriated annually for each fiscal year that begins on 1 October and ends on 30 September the next year. Based on the demonstration of the contractor's prototype, the government and the contractor negotiate a contracting period of performance that specifies the statement of work in each year. In our model, we assume that the technical steps yet to be completed are divided across multiple fiscal years, based on the completion rate demonstrated by the contractor during the competition phase. In reality, after each fiscal year, the term of the contract can be renegotiated if there is a delay, and if the Congress does not approve the new budget, the government has the power to cancel the project.

   If a contractor gets selected for the sole-source phase, then the contractor need not always work at its maximal capable rate. In most government contracts, finishing the project ahead of schedule or under budget does not get rewarded. Instead, finishing ahead of time results in less work for the contractor and its employees. In addition, it may be perceived as having requested more money than actually needed, which may hurt the contractor's ability to negotiate a budget for other projects in the future. Consequently, the contractor's goal is to finish the project *right on time*, but not as soon as possible. A small delay is not ideal, but it is often tolerable, and in many cases can be a better outcome than finishing the project ahead of time from the contractor's standpoint. For these reasons, we assume that during the sole-source phase, the selected contractor will work at a rate such that they expect to complete all technical steps assigned to the current fiscal year at the end of the fiscal year, but no higher than their maximal capable rate. In other words, if during a fiscal year the project gets ahead of schedule, then the progress tends to slow down. If the project is behind schedule, then the contractor will work at their maximal capable rate in order to catch up with the schedule.

6. The government has two goals: (1) minimize the amount of money spent to have a completed project; and (2) minimize the time it takes to have a completed project. On one extreme, if the government sets the length of the competition phase to infinity, then the government funds all n projects until one completes, which will achieve goal (2), but not (1). On the other extreme, if the government sets the length of the competition phase to zero, then the government funds only one contractor from the onset, which is unlikely to achieve either (1) or (2), since the government does not benefit from the competition between contractors. By choosing the appropriate length of the competition phase, the government can learn about each contractor's design approach by observing what is accomplished in the competition phase, so as to improve the chance of selecting the best contractor design. The contractors are also motivated to work at their respective maximal capable rates in order to increase the chance of getting selected for the sole-source phase, which, in turn, shortens the program completion time.

4 The Sole-Source Phase

This section presents a model on how a project progresses after the government commits to funding the project through its fruition. The contractor that is awarded the project faces no competition. The project may be planned for multiple fiscal years, with a certain number of technical steps scheduled for each fiscal year.

4.1 One-Year Project

We first consider a project that is scheduled and funded within a single fiscal year, with the period of performance denoted by $t \in [0,1]$ year. Recall that in our model, the contractor can work at a maximal capable rate λ. Let m denote the number of technical steps required to complete the project.

As discussed earlier, a sole-source contractor's goal is to complete the project on time, but not ahead of schedule. To model this behavior, we assume that the contractor will work at his maximal capable rate λ, if such rate will result in an expected completion time later than the deadline. In other words, if the project falls behind schedule, then the contractor will work as hard as it can to catch up with the schedule. However, if the project gets ahead of schedule at any time point, then the contractor will work at a rate that gives the expected completion time exactly equal to the project deadline, so as to finish the project on time, but not ahead of time.

Suppose at a time point, the project still has m technical steps to go, with t time units left. If $m/\lambda > t$ (behind schedule), then the contractor works at an instantaneous rate equal to his maximal capable rate λ; if $m/\lambda \le t$ (ahead of schedule), then the contractor works at a reduced rate m/t so that the expected time to completion matches the deadline t, but is not ahead of it. The contractor changes the rate continuously until the project completes.

Write X for the time to complete the next technical step, when the project still has m technical steps to go, with t time units left. If $m/\lambda > t$, then X follows an exponential distribution with rate λ. If $m/\lambda < t$, then the instantaneous hazard rate (at completing the next technical step) of X is given by

$$r(x) = \begin{cases} \frac{m}{t-x}, & x < t - \frac{m}{\lambda}; \\ \lambda, & x > t - \frac{m}{\lambda}. \end{cases}$$

For $x < t - m/\lambda$, we can compute

$$\bar{F}(x) = \exp\left(-\int_0^x r(y)\,dy\right) = \exp\left(-\int_0^x \frac{m}{t-y}\,dy\right) = \left(1 - \frac{x}{t}\right)^m.$$

Because conditional on $X > t - m/\lambda$, the additional time to complete the next technical step, namely $X - (t - m/\lambda)$, follows an exponential distribution with rate λ, we can conclude that

$$\bar{F}(x) = \begin{cases} \left(1 - \frac{x}{t}\right)^m, & x < t - \frac{m}{\lambda}; \\ \left(\frac{m}{\lambda t}\right)^m e^{-\lambda(x-(t-m/\lambda))}, & x > t - \frac{m}{\lambda}. \end{cases} \tag{1}$$

For given λ, m, and t, we are interested in two random quantities: (1) the time it takes to complete the project, denoted by T, and (2) the number of technical steps that are yet completed by the deadline, denoted by N. We first present analytical solutions for two special cases and then discuss how to use Monte Carlo simulation to estimate these two quantities in general.

4.1.1 Special Case $m = 1$

Consider the special case where $m = 1$, so that there is just one more technical step left to complete the project. If $t < 1/\lambda$, the contractor will work at its maximal capable rate λ, so that the time to completion T follows an exponential distribution with rate λ. Therefore, $E[T] = 1/\lambda$, and $\mathrm{Var}(T) = 1/\lambda^2$; the expected delay is $E[T]-t = 1/\lambda-t$. In addition, the probability of completing the project on time is $P(N = 0) = P(T < t) = 1 - e^{-\lambda t}$, and the probability that there remains one technical step unresolved at the deadline is $P(N = 1) = e^{-\lambda t}$.

If $t > 1/\lambda$, then the distribution of the time needed to complete the final technical step, denoted by X, is given by Equation (1) with $m = 1$. In addition, since there is only $m = 1$ technical step left, by definition $T = X$. Letting $m = 1$ in Equation (1), we see that

$$P\left(X < t - \frac{1}{\lambda}\right) = 1 - \frac{1}{\lambda t}, \qquad P\left(X > t - \frac{1}{\lambda}\right) = \frac{1}{\lambda t}.$$

In addition, since $(X|X < t - 1/\lambda)$ follows a uniform distribution over $(0, t - 1/\lambda)$, and $(X|X > t - 1/\lambda)$ is distributed as the sum of $t - 1/\lambda$ and an exponential random variable having rate λ, we obtain

$$E\left[X|X < t - \frac{1}{\lambda}\right] = \frac{1}{2}\left(t - \frac{1}{\lambda}\right), \qquad E\left[X|X > t - \frac{1}{\lambda}\right] = \left(t - \frac{1}{\lambda}\right) + \frac{1}{\lambda},$$

$$\mathrm{Var}\left(X|X < t - \frac{1}{\lambda}\right) = \frac{(t - \frac{1}{\lambda})^2}{12}, \qquad \mathrm{Var}\left(X|X > t - \frac{1}{\lambda}\right) = \frac{1}{\lambda^2}.$$

By conditioning on whether $X < t - 1/\lambda$, we can compute

$$\begin{aligned} E[X] &= P\left(X < t - \frac{1}{\lambda}\right) E\left[X|X < t - \frac{1}{\lambda}\right] + P\left(X > t - \frac{1}{\lambda}\right) E\left[X|X > t - \frac{1}{\lambda}\right] \\ &= \left(1 - \frac{1}{\lambda t}\right) \cdot \frac{1}{2}\left(t - \frac{1}{\lambda}\right) + \frac{1}{\lambda t}\left(t - \frac{1}{\lambda} + \frac{1}{\lambda}\right) \\ &= \frac{t}{2}\left(1 + \frac{1}{\lambda^2 t^2}\right), \end{aligned}$$

and

$$\begin{aligned} \mathrm{Var}(X) = &\left(1 - \frac{1}{\lambda t}\right) \cdot \frac{(t - \frac{1}{\lambda})^2}{12} + \frac{1}{\lambda t} \cdot \frac{1}{\lambda^2} \\ &+ \left(1 - \frac{1}{\lambda t}\right) \cdot \frac{1}{4}\left(t - \frac{1}{\lambda}\right)^2 + \frac{1}{\lambda t} \cdot t^2 - \frac{t^2}{4}\left(1 + \frac{1}{\lambda^2 t^2}\right)^2 \\ = &\frac{t^2}{12}\left(1 + \frac{6}{\lambda^2 t^2} + \frac{8}{\lambda^3 t^3} - \frac{3}{\lambda^4 t^4}\right). \end{aligned}$$

The expected delay is therefore

$$E[X] - t = \frac{t}{2}\left(\frac{1}{\lambda^2 t^2} - 1\right),$$

which is negative.

The probability that the project will be completed by the deadline is

$$P(N = 0) = 1 - \frac{1}{\lambda t} e^{-1},$$

which follows from Equation (1) with $m = 1$ and $x = t$. In other words, with probability $P(N = 1) = e^{-1}/(\lambda t)$, there will be one technical step unresolved at the deadline.

4.1.2 Special Case $t < 1/\lambda$

Once the deadline is within $1/\lambda$ year, the contractor will work at its maximal capable rate λ for the remainder of the project. The time to complete m technical steps, namely T, follows a gamma distribution with shape parameter m and rate parameter λ, so $E[T] = m/\lambda$, and $\mathrm{Var}(T) = m/\lambda^2$.

Since the contractor works at the same rate λ for the remainder of the project, the time points at which the technical steps are completed constitute a Poisson process with rate λ—until all m technical steps are completed. The probability that the contractor will complete i technical steps in time t, for $i = 0, 1, \ldots, m-1$, is

$$e^{-\lambda t}\frac{(\lambda t)^i}{i!}.$$

With probability

$$1 - \sum_{j=0}^{m-1} e^{-\lambda t}\frac{(\lambda t)^j}{j!},$$

the contractor will complete all m technical steps within t time units (before the deadline). Therefore, we can conclude that

$$\begin{aligned} P(N = 0) &= 1 - \sum_{j=0}^{m-1} e^{-\lambda t}\frac{(\lambda t)^j}{j!}, \\ P(N = i) &= e^{-\lambda t}\frac{(\lambda t)^{m-i}}{(m-i)!}, \qquad i = 1, 2, \ldots, m. \end{aligned}$$

4.1.3 Simulation Method

Other than the two special cases just discussed, we need to use Monte Carlo simulation to estimate the two quantities of interest: (1) the time it takes to complete the project and (2) the number of technical steps still unresolved at the deadline. To simulate the project's progress, we first generate a random variable X, which is the time required to complete the next technical step, according to the distribution given in (1).

1. Generate $u \leftarrow U(0, 1)$.

2. If $u < (\frac{m}{\lambda t})^m$, then $X > t - m/\lambda$. Use an exponential distribution with rate λ to generate Y, and set

$$X \leftarrow t - \frac{m}{\lambda} + Y.$$

3. If $u > (\frac{m}{\lambda t})^m$, then $X < t - m/\lambda$. Set

$$X \leftarrow t(1 - u^{1/m}).$$

After generating X, we can compute the remaining time $t - X$, at which point the contractor completes one technical step, and the number of unresolved technical steps becomes $m - 1$. If $t - X < 1/\lambda$, or if $m - 1 = 1$, then the situation becomes one of the two special cases, so we can use the analytic solution as the simulation output. Otherwise, we can continue to generate the time it takes to complete the next technical step.

If $X > t$, then no technical step is completed before the deadline. Although we can use that as the simulation output, it is possible to make the simulation more efficient by using a variance reduction technique known as the conditional estimator. Specifically, we can move the time clock to $1/\lambda$ before the deadline and consider the situation, where there are still m technical steps yet to be resolved with $1/\lambda$ (fractional) year before deadline. Since the situation is identical to what is discussed in Section 4.1.2, we can use the analytical solution as the simulation output, as opposed to simulating the rest of the process.

Now, suppose that we have perform n independent simulation runs. Let μ_i denote the estimate of $E[T]$, and σ_i^2 the estimate of $\text{Var}(T)$, obtained in the ith simulation run, $i = 1, \ldots, n$. The estimate of $E[T]$ is

$$\bar{\mu} = \frac{\sum_{i=1}^{n} \mu_i}{n},$$

and the estimate of $\text{Var}(T)$ is

$$\frac{\sum_{i=1}^{n} \sigma_i^2}{n} + \frac{\sum_{i=1}^{n} (\mu_i - \bar{\mu})^2}{n - 1}.$$

4.2 Multiyear Project

This section develops a model for a project's progress if it is funded through multiple fiscal years. In a multiyear project, the statement of work typically specifies the number of technical steps that will be completed each year. Suppose that there are still m technical steps to be completed and the number of technical steps scheduled to be completed in each fiscal year is r, where r is a positive integer. The multiyear project is then scheduled for m/r years. If m/r is not an integer, then the project is scheduled for $\lfloor m/r \rfloor$ full fiscal years and an additional $m/r - \lfloor m/r \rfloor$ (fractional) year.

In the first year, the goal of the contractor is to complete r technical steps of the project. If it completes r technical steps early, there is no motivation for the contractor to continue working on technical steps originally planned for the next year. For the same reasons as discussed in Section 4.1, the contractor tends to slow down when they are ahead of schedule, so that they can meet the year-end goal as closely as possible. If the contractor does finish all the r technical steps allocated in this first year, then we assume that the contractor will work on the nontechnical aspects of the project for the remainder of the fiscal year, and resume working on technical steps at the beginning of the next fiscal year. In this model, the number of technical steps that will be completed within the first fiscal year is a random variable and we can use the model in Section 4.1 to determine its distribution.

If all r technical steps planned for the first fiscal year are completed on time, then the same process repeats for the second fiscal year for the next r technical steps. If only $b < r$ technical steps are completed in the first fiscal year, then the project falls behind. At the beginning of the second fiscal year, the project still requires another $m-b$ technical steps, and the projected completion time will be pushed back by $(r-b)/r$ year. Typically, the contractor can find some justification for a small delay, such as running into unexpected technical challenges, and revise the schedule. The goal of the second fiscal year is still to complete r technical steps, and the same process repeats after each fiscal year. The government may accept the new schedule, or it may decide to cancel the project altogether.

If, at the beginning of a fiscal year, the number of remaining technical steps drops to r or below, then the project enters its final fiscal year, which is precisely the case discussed in Section 4.1.

For given λ, m, and r, we are interested in two random variables: (1) the time it takes to complete the project, denoted by T, and (2) the number of technical steps that are yet completed by the original deadline (in m/r years), denoted by N. We discuss how to use Monte Carlo simulation to estimate the expected value and the variance of these two random variables.

4.2.1 Time Needed to Complete the Project

Let T_k denote the time it takes to complete the project at the beginning of a fiscal year, if there are still k technical steps left. If $k \leq r$, then $E[T_k]$ can be estimated by simulation, as discussed in Section 4.1. If $k > r$, then r technical steps will be planned for the next fiscal year. Write Z for the number of technical steps that will be completed in a year, if r technical steps are planned at the beginning of the year. The distribution of Z can be estimated by simulation, as discussed in Section 4.1.

By conditioning on the number of technical steps completed in the next fiscal year, we can write

$$E[T_k] = 1 + \sum_{i=0}^{r} P(Z = i) \cdot E[T_{k-i}].$$

Solving for $E[T_k]$ gives

$$E[T_k] = \frac{1 + \sum_{i=1}^{r} P(Z = i) \cdot E[T_{k-i}]}{1 - P(Z = 0)},$$

which can be used to compute $E[T_k]$ recursively, for $k = r+1, r+2, \ldots$.

We next describe how to estimate $\mathrm{Var}(T_k)$. If $k \leq r$, then $\mathrm{Var}(T_k)$ can be estimated by simulation, as discussed in Section 4.1. If $k > r$, then recall that r technical steps will be planned for the next fiscal year and we write Z for the number of technical steps that will be completed during the year. To compute $\text{Var}(T_k)$, we condition on Z and use the formula

$$\text{Var}(T_k) = E[\text{Var}(T_k|Z)] + \text{Var}(E[T_k|Z]). \tag{2}$$

First, note that

$$\text{Var}(T_k|Z = i) = \text{Var}(T_{k-i}),$$

since, at the beginning of the next fiscal year, there are still $k - i$ technical steps yet to be completed. Hence, we can write

$$\begin{aligned} E[\text{Var}(T_k|Z)] &= \sum_{i=0}^{r} \text{Var}(T_k|Z = i) \cdot P(Z = i) \\ &= \sum_{i=0}^{r} \text{Var}(T_{k-i}) \cdot P(Z = i) \\ &= \text{Var}(T_k) \cdot P(Z = 0) + \sum_{i=1}^{r} \text{Var}(T_{k-i}) \cdot P(Z = i). \end{aligned} \tag{3}$$

Second, by definition, we have that

$$\begin{aligned} \text{Var}(E[T_k|Z]) &= \sum_{i=0}^{r} (E[T_k|Z = i] - E[T_k])^2 \cdot P(Z = i) \\ &= \sum_{i=0}^{r} (1 + E[T_{k-i}] - E[T_k])^2 \cdot P(Z = i). \end{aligned} \tag{4}$$

Use Equations (3) and (4) in Equation (2), and solve for $\text{Var}(T_k)$ to obtain

$$\text{Var}(T_k) = \frac{\sum_{i=1}^{r} \text{Var}(T_{k-i}) \cdot P(Z = i) + \sum_{i=0}^{r} (1 + E[T_{k-i}] - E[T_k])^2 \cdot P(Z = i)}{1 - P(Z = 0)},$$

which can be used to compute $\text{Var}(T_k)$ recursively, for $k = r + 1, r + 2, \ldots$.

4.2.2 Progress at the Deadline

The second performance measure is the progress of the project at the original deadline. Let N denote the number of technical steps yet to be completed at the deadline. If $N = 0$, then the project completes on time; otherwise, the project does not get completed on time.

Recall that in a multiyear project, there are three parameters: the total number of technical steps that need to be completed m, the number of technical steps planned for each fiscal year r, and the contractor's maximal capable rate to complete technical steps λ. If m/r is an integer, then the deadline of the project falls at the end of a fiscal year. Recall that Z stands for the random number of technical steps that will be completed in each year, so the m/r-fold convolution of Z gives the distribution of the number of technical steps completed in m/r years, which, in turn, yields the distribution of N.

Now, suppose that m/r is not an integer. After $\lfloor m/r \rfloor$ fiscal years, we want to determine the progress of the project after another $m/r - \lfloor m/r \rfloor$ year. In other words, we need to determine the distribution of the number of technical steps completed by time $m/r - \lfloor m/r \rfloor$, given that d technical steps are planned for t year, where $t \geq m/r - \lfloor m/r \rfloor$. If the contractor completes r technical steps in each of the first $\lfloor m/r \rfloor$ years, then there is no delay, so $t = m/r - \lfloor m/r \rfloor$ and $d = m - r \times \lfloor m/r \rfloor$. If the project suffers any delay, then $t > m/r - \lfloor m/r \rfloor$. If the delay is long enough such that r or more technical steps are yet to be completed after $\lfloor m/r \rfloor$ years, then $t = 1$ and $d = r$.

Consider the situation, where d technical steps are planned for the next $t \leq 1$ year. Let N denote the number of technical steps yet to be completed at the time moment when there is still s year before the deadline. In the other, after the contractor works for $t - s$ year, the random variable N represents the number of technical steps yet to be completed at that point.

If $t \leq 1/\lambda$, then the contractor works at rate λ, so that the probability that i technical steps will be completed within the next $t - s$ year is

$$e^{-\lambda(t-s)} \frac{(\lambda(t-s))^i}{i!}, \qquad i = 0, 1, \ldots, d-1.$$

With probability

$$1 - \sum_{j=0}^{d-1} e^{-\lambda(t-s)} \frac{(\lambda(t-s))^j}{j!},$$

the contractor will complete all d steps in the next $t - s$ year. Therefore, we can conclude that

$$P(N = 0) = 1 - \sum_{j=0}^{d-1} e^{-\lambda(t-s)} \frac{(\lambda(t-s))^j}{j!},$$

$$P(N = i) = e^{-\lambda(t-s)} \frac{(\lambda(t-s))^{d-i}}{(d-i)!}, \qquad i = 1, 2, \ldots, d.$$

Now, suppose $t > 1/\lambda$. Consider the special case $d = 1$, so there is only one technical step left. The distribution of the time to complete the final technical step is given by Equation (1) with $m = 1$. If $s > 1/\lambda$, then

$$P(N = 1) = P\{\text{no completion}\} = 1 - \frac{t-s}{t} = \frac{s}{t}.$$

If $s < 1/\lambda$, then

$$P(N = 1) = P\{\text{no completion}\} = \frac{1}{\lambda t} e^{-\lambda(t-s-t+1/\lambda)} = \frac{1}{\lambda t} e^{-1+\lambda s}.$$

For $d \geq 2$, we need to use Monte Carlo simulation to estimate the distribution of N. First, use Equation (1) to generate the time to complete the next technical step and denote it by x. If $t - x < s$, then no technical step is completed, so $N = d$. If $t - x > s$, then it takes x year to complete one technical step. We can update $d \leftarrow d - 1$ and $t \leftarrow t - x$ and repeat the process until either $t \leq 1/\lambda$ or when $d = 1$; in which case, we can use the analytic solution in the special cases as the simulation output.

5 The Competition Phase

During the competition phase, the government funds n contractors (typically $n = 2$ or 3). Let λ_i denote the maximal capable rate of contractor i, $i = 1, \dots, n$. The government does not know the exact ability of each contractor; in other words, the government does not know the actual maximal capable rate of each contractor. At the end of the competition phase, the government will examine each contractor's progress, and use it to estimate the additional time required for each contractor to complete its project, assuming continual funding support.

Without any additional knowledge about each contractor's maximal capable rate, the most sensible way to estimate the project's completion time is to assume that the project will progress at the same pace in the future as in the past. That is, if the contractor completes d technical steps in the competition phase, which has length t years, then the average time to complete each technical step is t/d. Therefore, the estimated additional time required to complete another $m - d$ technical steps is $(m - d) \times (t/d) = t(m/d - 1)$. Thus, after the competition phase, the government selects the contractor with the smallest such value and terminates the funding support for the other contractors.

Once the government selects a contractor to fund in the sole-source phase, the two sides need to set up a multiyear contract, which specifies the tasks to be completed in each fiscal year. Since the contractor completed d technical steps in t years during the competition phase, we assume that the multiyear contract requires the contractor to complete $\lfloor d/t \rfloor$ technical steps in each fiscal year, where $\lfloor d/t \rfloor$ is the largest integer not exceeding d/t. As the program enters the sole-source phase, we can use the model in Section 4 to calculate the progress of the selected project.

6 Numerical Study

This section presents a set of numerical studies to demonstrate our models to shed lights on how the program completion time and total program cost are affected by different model parameters. Section 6.1 concerns the case without a competition phase, where a single contractor is selected at the very beginning of the program. Section 6.2 focuses on symmetric contractors in the sense that all contractors involved in the competition phase have the same maximal capable rate. The goal is to examine how the length of the competition phase affects the program completion time and cost. Section 6.3 studies the case where the contractors have different maximal capable rates. The longer the competition phase, the more funds are needed in the competition phase, but the better the chance that the government will be able to award the sole-source contract to the most-capable contractor.

6.1 The Case without Competition Phase

We first study a program that does not have a competition phase. In other words, the government selects one contractor to develop the new weapon system from the very beginning. We set the number of technical steps $m = 120$ and a nominal program duration to be 15 years. In other words, the goal is to complete eight technical steps each year. We use the model in Section 4 to estimate the total program completion time and the program's progress after 15 years.

In addition to the completion time of a project, we are also interested in the total cost to run the program. To do so, we use nominal cost projections provided by N98, under a nominal schedule of 15 years. The yearly costs to fund one contractor, from year 1 to year 15—assuming that the project proceeds as scheduled—are given by the following in million dollars:

$$c_1 = 38.33, \quad c_2 = 36.00, \quad c_3 = 52.00, \quad c_4 = 68.00, \quad c_5 = 84.00,$$
$$c_6 = 487.75, \quad c_7 = 992.19, \quad c_8 = 1513.74, \quad c_9 = 5467.07, \quad c_{10} = 8020.64,$$
$$c_{11} = 6349.81, \quad c_{12} = 7247.63, \quad c_{13} = 8313.25, \quad c_{14} = 8582.30, \quad c_{15} = 6973.17.$$

In reality, the project may not proceed as scheduled and may suffer delay. If the actual project completion time is x years, then we assume that the total cost will scale proportionally to x. In other words, we use

$$\frac{x}{15} \sum_{i=1}^{15} c_i$$

for the total program cost. Table 1 displays the simulation results by varying the maximal capable rate λ of the contractor, between 7 and 9 technical steps per year. For each case, the simulation results are based on 400 independent simulation runs and the standard error of the mean program completion time is within 0.04% of the estimate.

As seen in Table 1, the mean completion time decreases as λ increases, since the contractor becomes more capable. However, even if $\lambda = 9$, which is greater than 8—the number of scheduled technical steps in each year—the project still suffers an average

Table 1: The effect of the maximal capable rate λ on the project's progress, if the program selects a sole-source contractor from the very beginning. For each case, the results are based on 400 independent simulation runs, with the standard error of the mean program completion time within 0.04% of the estimate.

Maximal Capable Rate (per year)	Completion Time (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	Mean	Std	Mean	Std	Mean	Std
7	18.98	1.22	78.8	5.7	68.6	4.4
7.5	18.23	1.08	82.0	5.3	65.9	3.9
8	17.66	0.96	84.6	4.9	63.8	3.5
8.5	17.22	0.86	86.7	4.6	62.3	3.1
9	16.89	0.78	88.4	4.3	61.1	2.8

delay of $16.89 - 15 = 1.89$ years. The reason for this delay is that in each fiscal year, the contractor can complete, at most, eight technical steps as scheduled. If the contractor completes eight technical steps ahead of time, it does not have incentive (in some cases, it is even against the law) to work on the technical steps scheduled for the next fiscal year. In addition, if in the first six months of a fiscal year, the contractor has already completed 6 technical steps out of 8 scheduled in the fiscal year, then the contractor tends to slow down, since they are only motivated to complete the work *on time*, but not *ahead of time*. Consequently, in some fiscal years, the contractor completes fewer than 8 technical tasks and the delay is inevitable.

The next two columns display the progress of the project at the deadline of 15 years. With similar reasons, the progress increases, on average, as λ increases; however, even if $\lambda = 9$, the expected progress after 15 years is still less than 90%. The last two columns display the total cost to run the program with a sole-source contractor. The total program cost is higher if the contractor is less capable (smaller maximal capable rate λ).

6.2 The Case with Symmetric Contractors

This section presents the numerical study, where the n contractors involved in the competition phase have the same maximal capable rate λ. As was the case in Section 6.1, we set the number of technical steps $m = 120$ and a nominal program duration to be 15 years. Consequently, we use $\lambda = 8$ for each contractor. If a contractor works at the maximal capable rate $\lambda = 8$ at all time, then the time required to complete the project follows a gamma distribution (bell shape) with expected value $120/8 = 15$ years, and standard deviation $\sqrt{120/8^2} \approx 1.37$ years.

The government has two decision variables:

1. The number of contractors funded in the competition phase, denoted by n. We will consider $n = 2, 3, 4$.

2. The length of the competition phase in years, denoted by t. We will consider

$t = 3, 4, \ldots, 10$ years.

The government has two objective functions: minimize the expected time to program completion and minimize the total program cost. These two objectives, however, are competing, and there will be some trade-off between them. As we vary $n = 2, 3, 4$ and $t = 3, 4, \ldots, 10$, we run the model in Sections 4 and 5 to estimate the actual program completion time. In addition, we also estimate the progress at the projected program completion time, where the projected completion time is estimated based on the number of technical steps completed in the competition phase.

To compute the program total cost, we consider the cost in the competition phase and the cost in the sole-source phase separately. Recall from Section 6.1 that the nominal cost projections for years 1–15 are provided by N98, and we write c_i for the cost in year i, for $i = 1, 2, \ldots, 15$. In the competition phase, the funding cost is fixed. If the government funds n contractors in the competition phase for t years, then the total cost in the competition phase is

$$n \sum_{i=1}^{t} c_i.$$

In the sole-source phase, however, the actual cost depends on the program completion time. We estimate the actual cost by assuming that the government will provide continual funding until the sole-source contractor completes their project. The actual cost will be less if the project finishes ahead of time, or more if there is a delay. Based on a nominal program schedule of 15 years, we use

$$\frac{x - t}{15 - t} \times \sum_{i=t+1}^{15} c_i$$

to estimate the program cost in the sole-source phase, where x is the actual program completion time and t is the length of the competition phase. Consequently, we estimate the total program cost as

$$n \sum_{i=1}^{t} c_i + \frac{x - t}{15 - t} \times \sum_{i=t+1}^{15} c_i, \tag{5}$$

where n is the number of contractors involved in the competition phase, t the length of the competition phase, and x the actual project completion time by the selected contractor.

Table 2 displays the results, when we vary the number of contractors in the competition phase (namely n), and the length of the competition phase (namely t), based on 2,000 independent simulation runs. The standard error of the mean actual program completion time is less than 0.4% of the estimate. For a fixed n, there is no significant difference in the projected program completion time when t varies, since the projected program completion time is estimated by the assumption that the selected project will progress in the sole-source phase at the same pace as in the competition phase. However, the actual program completion time does depend on t. As seen in the next column, the longer the competition phase t, the sooner the entire program completes.

Table 2: The symmetric case, with each contractor's maximal capable rate $\lambda = 8$. A project has $m = 120$ technical steps, and each contractor works at 100% of their maximal capable rate ($\alpha = 0$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.4% of the estimate.

n	t	Average Completion Time (yrs)		Delay (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	(yrs)	Projected	Actual	Mean	Std	Mean	Std	Mean	Std
2	3	14.38	16.73	2.35	2.26	86.1	6.2	62.1	10.2
	4	14.24	16.34	2.10	2.05	87.6	5.4	61.0	10.1
	5	14.42	16.23	1.81	1.90	89.4	4.7	61.1	10.2
	6	14.61	16.17	1.56	1.82	91.0	4.2	61.9	10.8
	7	14.54	15.89	1.35	1.71	92.2	3.8	61.8	11.2
	8	14.47	15.60	1.14	1.65	93.4	3.4	61.9	12.0
	9	14.52	15.45	0.93	1.64	94.6	3.0	66.4	12.5
	10	14.48	15.21	0.73	1.52	95.7	2.7	72.6	11.4
3	3	13.47	15.97	2.50	1.79	84.8	6.2	58.9	8.1
	4	13.61	15.79	2.18	1.67	86.9	5.4	58.5	8.2
	5	13.77	15.66	1.89	1.65	88.7	4.8	58.3	8.9
	6	13.80	15.42	1.63	1.54	90.3	4.3	58.3	9.2
	7	13.95	15.33	1.37	1.49	91.8	3.8	59.9	9.8
	8	13.96	15.10	1.14	1.44	93.2	3.4	61.5	10.5
	9	13.99	14.91	0.92	1.41	94.5	3.1	71.0	10.7
	10	13.92	14.62	0.70	1.33	95.7	2.7	84.9	10.0
4	3	12.92	15.55	2.63	1.58	83.8	6.3	57.1	7.1
	4	13.20	15.45	2.24	1.52	86.3	5.4	57.0	7.5
	5	13.41	15.33	1.93	1.45	88.3	4.8	56.9	7.8
	6	13.55	15.19	1.64	1.41	90.1	4.3	57.6	8.4
	7	13.55	14.94	1.40	1.37	91.6	3.9	59.1	9.0
	8	13.65	14.79	1.15	1.30	93.1	3.5	62.5	9.5
	9	13.70	14.61	0.92	1.30	94.5	3.1	77.5	9.9
	10	13.72	14.42	0.69	1.23	95.8	2.7	100.1	9.2

The main reason is that during the competition phase, each contractor works at their maximal capable rate, but during the sole-source phase, the contractor works at a rate to meet the scheduled deadline, which often results in delays.

The next two columns display the delay, in terms of its expected value and standard deviation, assuming the selected project in the sole-source phase is funded through completion. The delay is the difference between the projected program completion time and the actual program completion time. The shorter the competition phase (t), the longer the delay, since there are more technical steps left for the selected contractor to complete in the sole-source phase. If we fix t, then the delay increases as n increases, which appears to be counterintuitive at the first glance. To understand this phenomenon, note that with more contractors in the competition phase, it becomes more likely for one contractor to get lucky and accomplish more than they would normally do, which leads the government to be overly optimistic about the contractor's capability. When the government overestimates the selected contractor's capability, the projected program completion time becomes unrealistically short; hence, the long delay.

The next two columns display the progress at the projected program completion time, as percentage of the technical steps completed compared with the total required $m = 120$. For instance, if the projected program completion time is 14.5 years, then we run the simulation to determine how many technical steps are actually completed in 14.5 years. If the contractor completes 111 technical steps in 14.5 years, then we say the project is $111/m = 111/120 = 92.5\%$ completed at the projected completion time. This statistic and the delay have a negative correlation. The more work that is completed at the projected completion time, the shorter the eventual delay.

The final two columns in Table 2 display the total program cost (in billions) until the sole-source contractor completes the development of the new weapon system, based on Equation (5). As seen in Table 2, for a fixed n, the total program cost increases, if t is too small or too large. If t is too small, then the government does not benefit much from the competition phase, when each contractor is motivated to work at their maximal capable rate. The program completion time lengthens because of the long sole-source phase, and therefore the total program cost. If t is too large, then the program cost also increases, since the government has to fund multiple contractors in the competition phase. As seen in Table 2, the optimal length of the competition phase t, which minimizes the total program cost, tends to lie between 4 and 6 years.

The government has two objective functions: a small program completion time and a small total program cost. Based on the results in Table 2, the efficient frontier contains six (n, t) combinations, namely $n = 4$ and $t = 5, \ldots, 10$. In other words, the government should fund four independent contractors in the competition phase (if possible) and then set the competition phase to be at least five years. However, since as t increases from 5 to 10, the expected program completion time decreases slowly, while the expected total program cost increases slowly for $t = 5, 6, 7$, but quickly for $t = 8, 9, 10$, a strong argument can be made that the competition phase should be set between 5 and 7 years. With $t = 5$ years, the program will cost less, but take longer to complete; with $t = 7$ years, the program will cost more, but take less time to complete. In practice, the choice of n may be limited to the state of the defense industry. For instance, perhaps there are only two defense contractors that are capable of developing the weapon system of interest, which will default $n = 2$. The recommendation would also change, if the yearly program costs c_i, $i = 1, \ldots, 15$ vary from a weapon system to another.

We next examine the possibility, if each contractor exerts more effort in the competition phase in order to increase their chances of getting selected as the sole source. In some cases, the contractors are explicitly prohibited to do so, but in some other cases such a practice is not inconceivable, since winning the sole-source contract often translates to work for the contractor's employees and huge profit. In our mathematical model, we assume that in the competition phase, a contractor works at a rate equal to

$\lambda(1+\alpha)$, where α models the extra effort. Table 3 displays the results for $\alpha = 0.05$, and Table 4 displays the results for $\alpha = 0.1$.

Table 3: The symmetric case, with each contractor's maximal capable rate $\lambda = 8$. A project has $m = 120$ technical steps and each contractor works at 105% of their maximal capable rate ($\alpha = 0.05$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.4% of the estimate.

n	t	Average Completion Time (yrs)		Delay (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	(yrs)	Projected	Actual	Mean	Std	Mean	Std	Mean	Std
2	3	13.48	16.01	2.53	1.99	84.6	6.4	58.9	9.0
	4	13.77	15.93	2.16	1.89	87.0	5.5	59.0	9.3
	5	13.80	15.68	1.88	1.73	88.8	4.8	58.2	9.3
	6	13.78	15.41	1.63	1.66	90.2	4.3	57.4	9.8
	7	13.74	15.12	1.39	1.64	91.7	3.9	56.8	10.8
	8	13.79	14.93	1.14	1.54	93.2	3.5	57.0	11.2
	9	13.76	14.67	0.91	1.51	94.5	3.1	60.5	11.5
	10	13.78	14.47	0.69	1.48	95.8	2.7	67.0	11.1
3	3	12.94	15.57	2.64	1.71	83.7	6.4	57.1	7.7
	4	12.98	15.28	2.30	1.61	85.8	5.6	56.0	7.9
	5	13.16	15.13	1.97	1.49	88.0	4.9	55.5	8.0
	6	13.09	14.78	1.69	1.41	89.7	4.4	54.5	8.4
	7	13.27	14.68	1.41	1.39	91.5	3.9	55.6	9.1
	8	13.33	14.48	1.15	1.35	93.0	3.5	57.0	9.8
	9	13.26	14.16	0.90	1.31	94.5	3.1	65.3	10.0
	10	13.31	13.97	0.66	1.26	95.9	2.7	80.0	9.4
4	3	12.31	15.11	2.80	1.44	82.5	6.4	55.1	6.5
	4	12.63	15.00	2.37	1.43	85.3	5.5	54.8	7.0
	5	12.67	14.72	2.05	1.38	87.3	4.9	53.5	7.5
	6	12.83	14.55	1.72	1.32	89.4	4.4	53.8	7.8
	7	12.91	14.33	1.43	1.30	91.2	3.9	55.1	8.5
	8	12.98	14.13	1.15	1.23	92.9	3.5	57.7	9.0
	9	13.01	13.90	0.89	1.22	94.5	3.1	72.1	9.2
	10	13.02	13.66	0.64	1.18	96.0	2.7	94.5	8.9

The same patterns observed in Table 2 are also observed in Tables 3 and 4. Across the three tables, we see that as α increases, both average projected completion time and average actual completion time decrease, because the contractors tend to accomplish more technical tasks in the competition phase. However, the average delay increases as α increases, since after seeing the contractors making great progress in the competition phase, the government becomes too optimistic and projects a unrealistic completion

Table 4: The symmetric case with each contractor's maximal capable rate $\lambda = 8$. A project has $m = 120$ technical steps, and each contractor works at 110% of its maximal capable rate ($\alpha = 0.1$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.4% of the estimate.

n	t (yrs)	Average Completion Time (yrs) Projected	Actual	Delay (yrs) Mean	Std	Progress at the Deadline (%) Mean	Std	Total Cost (billions) Mean	Std
2	3	12.89	15.56	2.67	1.85	83.5	6.6	56.9	8.3
	4	12.96	15.28	2.31	1.67	85.8	5.6	55.8	8.2
	5	13.07	15.06	1.99	1.68	87.8	5.0	54.8	9.1
	6	13.10	14.80	1.69	1.56	89.7	4.4	53.8	9.3
	7	13.15	14.56	1.41	1.51	91.4	3.9	53.1	9.9
	8	13.18	14.33	1.15	1.48	93.0	3.5	52.6	10.8
	9	13.16	14.05	0.89	1.44	94.5	3.1	55.7	10.9
	10	13.12	13.75	0.64	1.39	96.0	2.7	61.6	10.4
3	3	12.17	15.03	2.86	1.60	82.0	6.7	54.6	7.2
	4	12.38	14.80	2.43	1.47	84.8	5.7	53.6	7.2
	5	12.52	14.59	2.07	1.39	87.1	5.0	52.6	7.5
	6	12.60	14.35	1.75	1.33	89.2	4.4	51.9	7.9
	7	12.67	14.11	1.44	1.34	91.1	3.9	51.9	8.8
	8	12.65	13.81	1.15	1.30	92.9	3.5	52.1	9.5
	9	12.68	13.56	0.88	1.26	94.5	3.1	60.8	9.5
	10	12.68	13.29	0.60	1.22	96.2	2.6	74.9	9.2
4	3	11.76	14.74	2.97	1.40	81.1	6.6	53.4	6.3
	4	12.01	14.52	2.51	1.33	84.2	5.6	52.5	6.6
	5	12.24	14.35	2.11	1.35	86.8	5.0	51.6	7.3
	6	12.35	14.12	1.77	1.28	89.0	4.4	51.3	7.6
	7	12.33	13.79	1.45	1.25	90.9	3.9	51.5	8.2
	8	12.43	13.58	1.15	1.20	92.8	3.5	53.7	8.7
	9	12.47	13.33	0.86	1.18	94.6	3.1	67.8	8.9
	10	12.44	13.02	0.58	1.11	96.3	2.6	89.7	8.3

time. As a result, the progress at the projected deadline also drops.

As seen in the last two columns in Table 3 and 4, for a given number of contractors, the optimal length of the competition phase that minimizes the total program cost tends to lie between 5 and 8 years—longer than 4–6 years in the case $\alpha = 0$. Since the contractor exerts more effort in the competition phase, the government benefits more from the extra effort with a longer competition phase. Consequently, it is desirable for the government to choose a longer competition phase. In practice, however, it is often impossible for the government to verify whether the contractor exerts extra effort in the competition phase. In those situations, it is reasonable to choose a competition phase length that works well for different α values. Based on Tables 2 to 4, setting $t = 5$ or 6 appears to be sound policy choices.

6.3 The Case with Asymmetric Contractors

This section presents a numerical study, where the n contractors involved in the competition phase have different maximal capable rates. We adopt the same model parameters in Section 6.2 with $m = 120$, and vary $\alpha = 0, 0.05, 0.1$. For $n = 2$, we set $\lambda_1 = 7.5$ and $\lambda_2 = 8.5$. For $n = 3$, we add $\lambda_3 = 8$.

Table 5 displays the numerical results for $\alpha = 0$. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.3% of the estimate. If two or more contractors complete the same number of technical steps in the competition phase, then we assume that the government will select a contractor at random.

Table 5: The asymmetric case, with $\lambda_1 = 7.5$, $\lambda_2 = 8.5$, and $\lambda_3 = 8$ (if applicable). A project has $m = 120$ technical steps and each contractor works at 100% of their maximal capable rate ($\alpha = 0$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.3% of the estimate.

n	t	Average Completion Time (yrs)		Delay (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	(yrs)	Projected	Actual	Mean	Std	Mean	Std	Mean	Std
2	3	14.08	16.38	2.30	2.38	86.2	6.2	60.6	10.7
	4	14.08	16.09	2.01	2.10	88.0	5.4	59.8	10.3
	5	14.21	15.94	1.72	1.95	89.7	4.7	59.6	10.5
	6	14.35	15.82	1.47	1.92	91.3	4.2	59.9	11.4
	7	14.32	15.58	1.27	1.81	92.5	3.8	59.8	11.9
	8	14.26	15.31	1.05	1.74	93.8	3.4	59.8	12.7
	9	14.19	15.04	0.85	1.68	94.9	3.0	63.2	12.8
	10	14.22	14.87	0.65	1.59	96.0	2.7	70.0	11.9
3	3	13.24	15.71	2.47	1.84	84.8	6.3	57.7	8.3
	4	13.50	15.60	2.10	1.78	87.2	5.4	57.6	8.8
	5	13.67	15.48	1.80	1.69	89.1	4.8	57.4	9.1
	6	13.71	15.26	1.55	1.60	90.7	4.3	57.3	9.5
	7	13.78	15.08	1.30	1.51	92.2	3.9	58.3	9.9
	8	13.80	14.87	1.08	1.50	93.5	3.5	59.8	10.9
	9	13.77	14.62	0.85	1.45	94.8	3.1	68.8	11.0
	10	13.88	14.53	0.65	1.44	96.0	2.7	84.2	10.8

Intuitively, the longer the competition phase, the more likely the government will select the most-capable contractor—contractor 2 in our numerical study. This extra benefit makes it more attractive for the government to choose a longer competition phase—a larger t. As seen in Table 5, for $n = 2$, the optimal length that minimizes the total program cost is between $t = 4$ and 8 years. The program completion time still decreases in t, as was the case in Section 6.2. Given an opportunity, the government should involve $n = 3$ contractors, rather than just $n = 2$. In addition, setting $t = 5, 6, 7, 8$ remains an attractive choice to balance the program completion time and total cost.

Tables 6 and 7 display the numerical results for $\alpha = 0.05$ and $\alpha = 0.1$, respectively, for the asymmetric case. Similar qualitative observations can be made in these two tables. Of course, the larger the value of α, the more technical steps each contractor can complete in the competition phase, which drives down both program completion time and total cost. The average delay lengthens as α increases, since the government would be too optimistic to predict the project completion time by assuming the project will progress at the same pace in the sole-source phase, as was observed in the competition phase. Finally, the optimal length of the competition phase, which offers a great balance between the program completion time and total cost, remains to be between 5 and 8 years.

Table 6: The asymmetric case, with $\lambda_1 = 7.5$, $\lambda_2 = 8.5$, and $\lambda_3 = 8$ (if applicable). A project has $m = 120$ technical steps and each contractor works at 105% of their maximal capable rate ($\alpha = 0.05$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.3% of the estimate.

n	t	Average Completion Time (yrs)		Delay (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	(yrs)	Projected	Actual	Mean	Std	Mean	Std	Mean	Std
2	3	13.32	15.78	2.45	2.12	84.9	6.6	57.9	9.6
	4	13.54	15.62	2.09	1.97	87.3	5.6	57.5	9.7
	5	13.54	15.34	1.80	1.84	89.0	4.9	56.4	9.9
	6	13.63	15.15	1.52	1.77	90.7	4.3	55.9	10.5
	7	13.55	14.84	1.29	1.68	92.1	3.9	54.9	11.0
	8	13.60	14.66	1.05	1.60	93.6	3.5	55.0	11.6
	9	13.61	14.44	0.83	1.57	94.9	3.1	58.7	11.9
	10	13.52	14.13	0.61	1.50	96.2	2.7	64.5	11.3
3	3	12.66	15.26	2.60	1.78	83.7	6.6	55.6	8.0
	4	12.88	15.09	2.21	1.65	86.2	5.6	55.1	8.1
	5	13.08	14.96	1.89	1.61	88.4	5.0	54.6	8.7
	6	13.10	14.69	1.59	1.58	90.2	4.4	53.9	9.4
	7	13.08	14.42	1.34	1.47	91.8	3.9	53.9	9.6
	8	13.15	14.22	1.07	1.45	93.4	3.5	55.1	10.6
	9	13.15	13.98	0.83	1.38	94.8	3.1	64.0	10.5
	10	13.16	13.76	0.60	1.33	96.2	2.6	78.5	10.0

Table 7: The asymmetric case, with $\lambda_1 = 7.5$, $\lambda_2 = 8.5$, and $\lambda_3 = 8$ (if applicable). A project has $m = 120$ technical steps and each contractor works at 110% of their maximal capable rate ($\alpha = 0.1$) in the competition phase. For each case, the results are based on 2,000 independent simulation runs, with the standard error of the mean program completion time within 0.3% of the estimate.

n	t	Average Completion Time (yrs)		Delay (yrs)		Progress at the Deadline (%)		Total Cost (billions)	
	(yrs)	Projected	Actual	Mean	Std	Mean	Std	Mean	Std
2	3	12.79	15.36	2.57	1.98	83.9	6.7	56.0	8.9
	4	12.90	15.10	2.19	1.88	86.3	5.8	54.9	9.2
	5	12.88	14.76	1.88	1.75	88.3	5.0	53.2	9.4
	6	12.96	14.54	1.58	1.67	90.2	4.5	52.3	9.9
	7	12.93	14.25	1.32	1.62	91.8	4.0	51.1	10.6
	8	12.89	13.94	1.05	1.55	93.4	3.5	49.8	11.3
	9	12.93	13.73	0.80	1.50	95.0	3.1	53.3	11.4
	10	12.86	13.41	0.55	1.39	96.4	2.6	59.1	10.4
3	3	12.08	14.86	2.78	1.65	82.4	6.7	53.9	7.4
	4	12.29	14.62	2.33	1.55	85.3	5.7	52.8	7.6
	5	12.41	14.38	1.97	1.49	87.6	5.0	51.4	8.0
	6	12.49	14.13	1.65	1.43	89.7	4.5	50.6	8.5
	7	12.52	13.88	1.36	1.39	91.5	4.0	50.4	9.1
	8	12.59	13.67	1.08	1.38	93.2	3.5	51.1	10.0
	9	12.59	13.39	0.80	1.33	94.9	3.1	59.5	10.1
	10	12.56	13.11	0.54	1.27	96.5	2.6	73.6	9.5

7 Conclusions

This report uses mathematical modeling to study a government program that develops a technologically advanced weapon system. The government funds several contractors to develop system prototypes in the competition phase and then selects one contractor to develop the weapon system. The government can choose the number of contractors involved in the competition phase and the length of the competition phase, with the goal to minimize the program completion time and the total program cost.

Based on model analysis and numerical study, we identified three main reasons why such a program often suffers delay and cost overrun.

1. At the end of the competition phase, the contractor that gets selected tends to be the one that performs “luckier than usual.” Hence, the government tends to be overly optimistic about the selected contractor’s actual ability.
2. In the sole-source phase, the goal of the contractor is to finish scheduled tasks in each fiscal year on time, but not to deliver the final product as soon as possible. Whenever the contractor is ahead of schedule, the contractor tends to slow down in order to “meet” the yearly deadline. Since the R&D process involves a lot of risk, slowing down whenever the project is going well often results in an eventual delay and cost overrun.
3. It is conceivable that in order to improve their chance of winning the sole-source contract, a contractor is motivated to exert extra effort in the competition phase. If that is the case, then using the accomplishment in the competition phase to predict progress in the sole-source phase would produce an unrealistic estimation on the program completion time.

The numerical study is based on a nominal project schedule of 15 years, with the nominal yearly costs provided by N98. Using these numbers, we found that it is better for the government to involve more contractors in the competition phase, if available. The longer the competition phase, the shorter the program completion time; however, the total program cost tends to increase if the competition phase is either too short or too long. When the number of contractors in the competition phase is between 2 and 4, the optimal length of the competition phase typically lies between 5 and 8 years, which offers great balance between program completion time and total cost.

Our model assumes that the government announces the length of the competition phase in advance and selects one contractor at the conclusion of the competition phase. An alternative approach is to review each contractor’s progress on a yearly basis and decide which contractors to fund for another year. In addition, if the government has some prior knowledge about each contractor’s capability, then a Bayesian approach may produce a more reliable estimate on the program completion time. A separate study is needed to explore these issues.

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