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## **Sea Ice Koopman Mode Analysis**

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- The program goal was to apply Koopman Mode Analysis (KMA) to sea ice data sets to study their temporal and geographic behavior over multi-year time scales.
- Work in this period primarily consisted of prediction of future sea ice concentration values using Koopman reconstruction techniques.
  - Prediction was performed for varied input year ranges between 3 and 38 years, and for prediction years between 1982 and 2017, and for the entire data available for each hemisphere and regions within each hemisphere.
  - Results shown include prediction of spatial distribution of sea ice concentration, prediction of pixel-by-pixel deviation between predicted and actual results, and comparisons of the mean of the predicted and actual data for each region.

# Koopman reconstruction and prediction

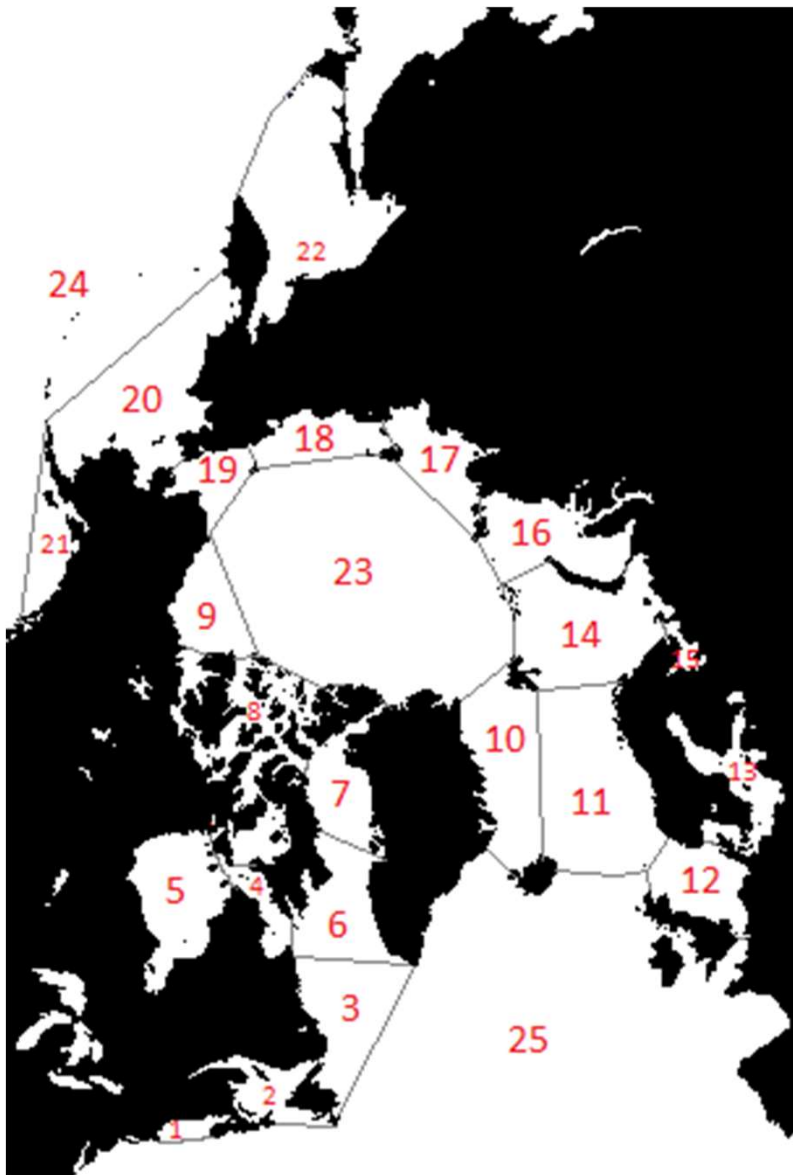
Reconstruction of the  $N_p$  sea ice concentration pixel values  $C$  at discrete time step  $k$  is performed using the Koopman eigenvalues  $\lambda_j$  and the Koopman modes  $v_j$  obtained from applying KMD to the concentration values over  $N$  time steps (months, in this case):

$$C_k = \sum_1^N \lambda_j^{k-1} v_j$$

Here, there are  $N$  Koopman eigenvalues and Koopman modes, where each Koopman eigenvalue is a single complex number and each Koopman mode has dimensions 1 by  $N_p$ .

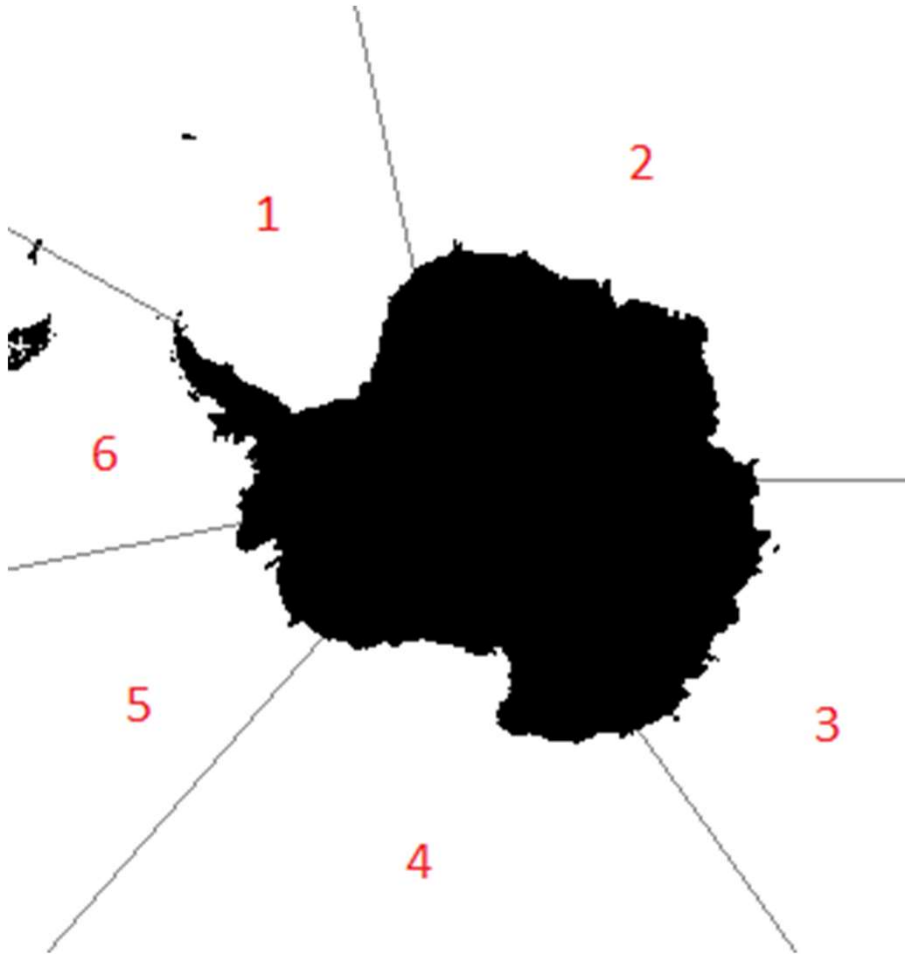
For  $1 \leq k \leq N$ ,  $C_k$  is termed a reconstruction of the  $k$ th time step in the original data  $C$ , as the Koopman eigenvalues and modes came from a decomposition of the observations over this time range and should simply reproduce the data used as input to the KMD. For  $k > N$ ,  $C_k$  is a prediction of the future behavior of the sea ice concentration for the (future)  $k$ th time step, based on the system dynamics deduced from the decomposition of earlier observations.

# Regions in northern hemisphere



1: Bay of Fundy, 2: Gulf of St Lawrence, 3: Labrador Sea, 4: Hudson Strait, 5: Hudson Bay, 6: Davis Strait, 7: Baffin Bay, 8: Northwestern Passages, 9: Beaufort Sea, 10: Greenland Sea, 11: Norwegian Sea, 12: North Sea, 13: Baltic Sea, 14: Barents Sea, 15: White Sea, 16: Kara Sea, 17: Laptev Sea, 18: East Siberian Sea, 19: Chukchi Sea, 20: Bering Sea, 21: Gulf of Alaska, 22: Sea of Okhotsk, 23: Arctic Ocean, 24: North Pacific Ocean, 25: North Atlantic Ocean.

# Regions in southern hemisphere

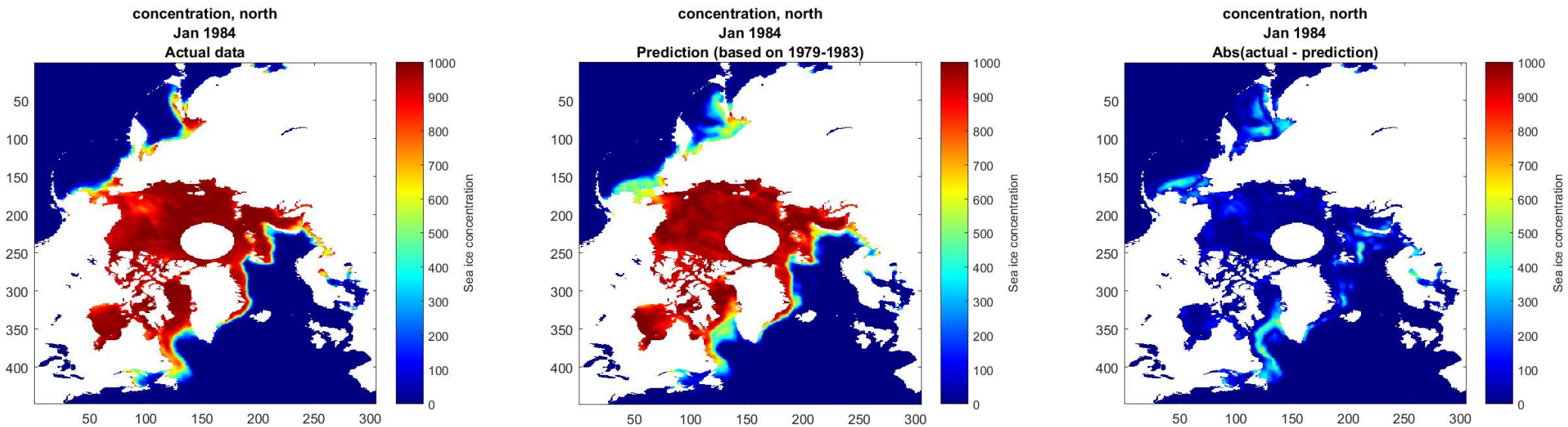


1: Weddell Sea, 2: Indian Ocean, 3: West Pacific Ocean, 4: Ross Sea, 5: Amundsen Sea, 6: Bellingshausen Sea.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### January 1984, 1 month after 1979-1983 input data

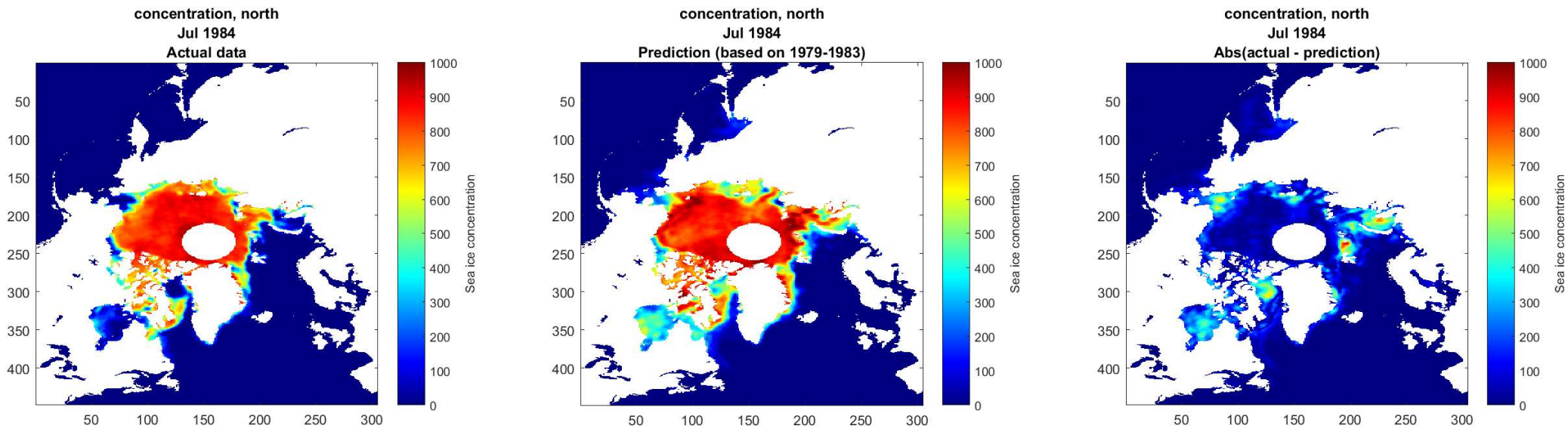


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### July 1984, 7 months after 1979-1983 input data

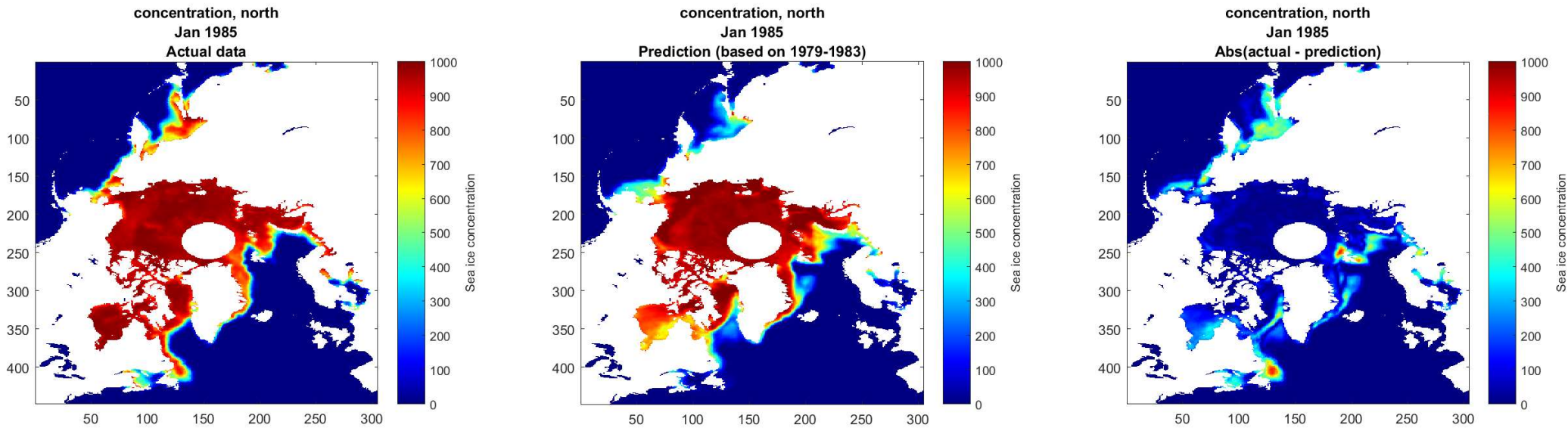


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### January 1985, 13 months after 1979-1983 input data

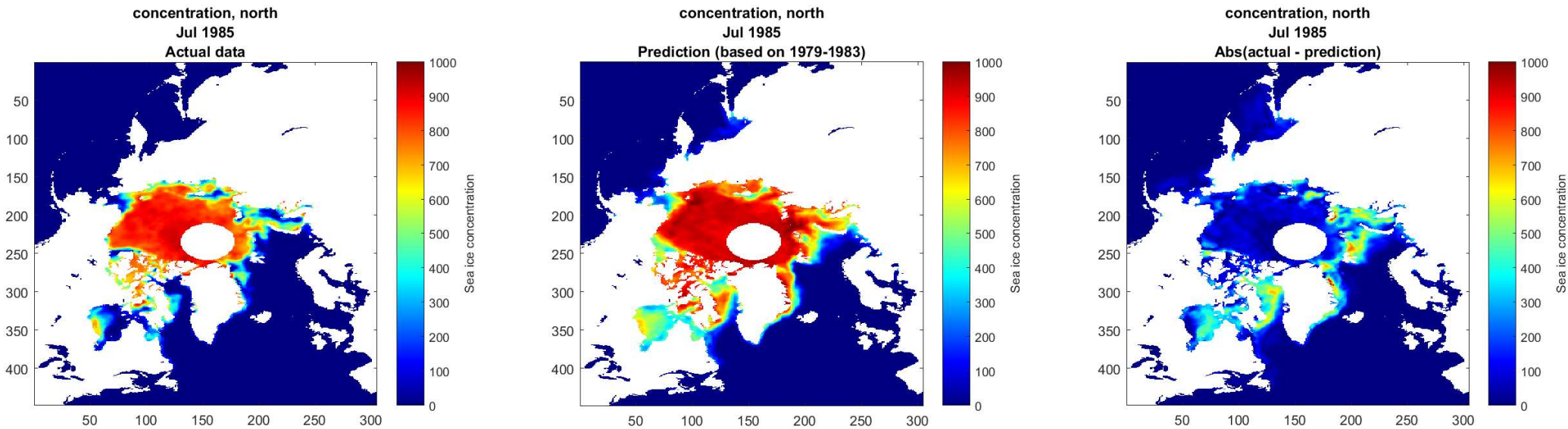


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### July 1985, 19 months after 1979-1983 input data

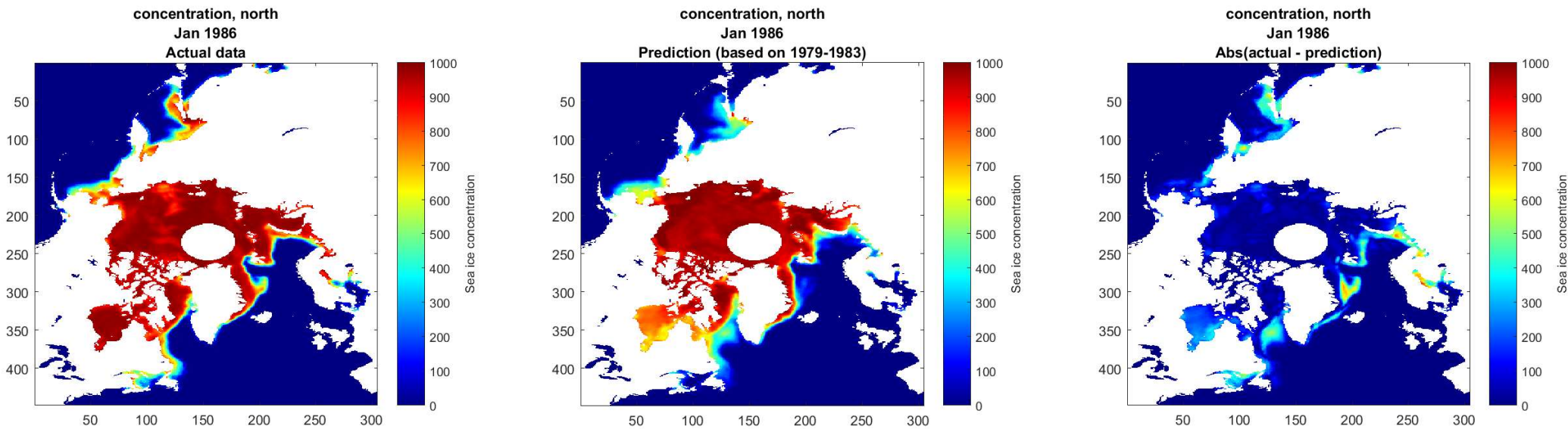


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### January 1986, 25 months after 1979-1983 input data

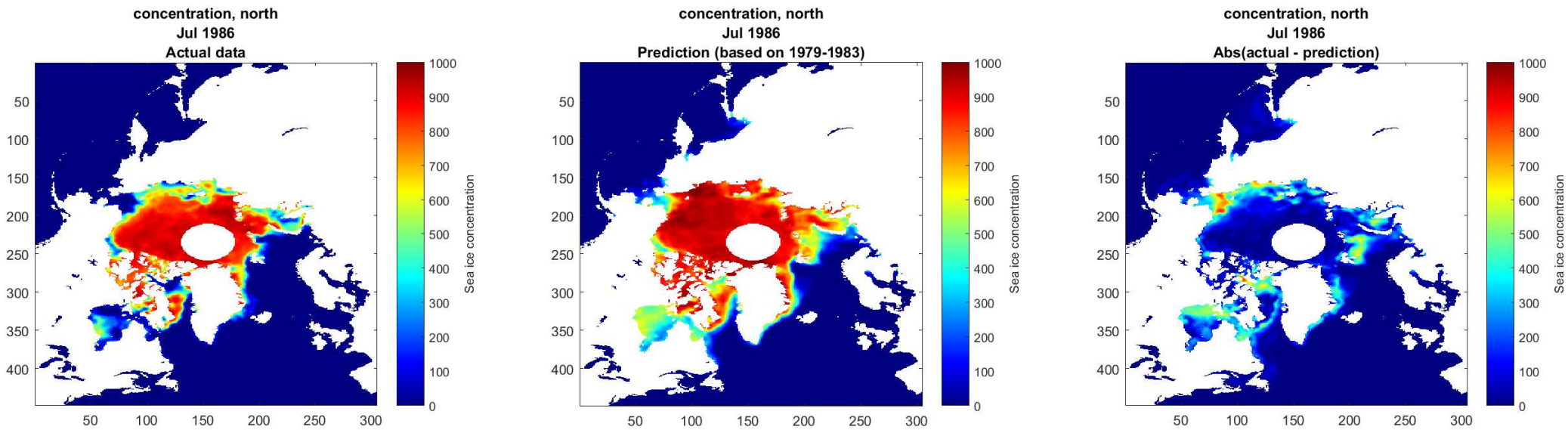


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### July 1986, 31 months after 1979-1983 input data

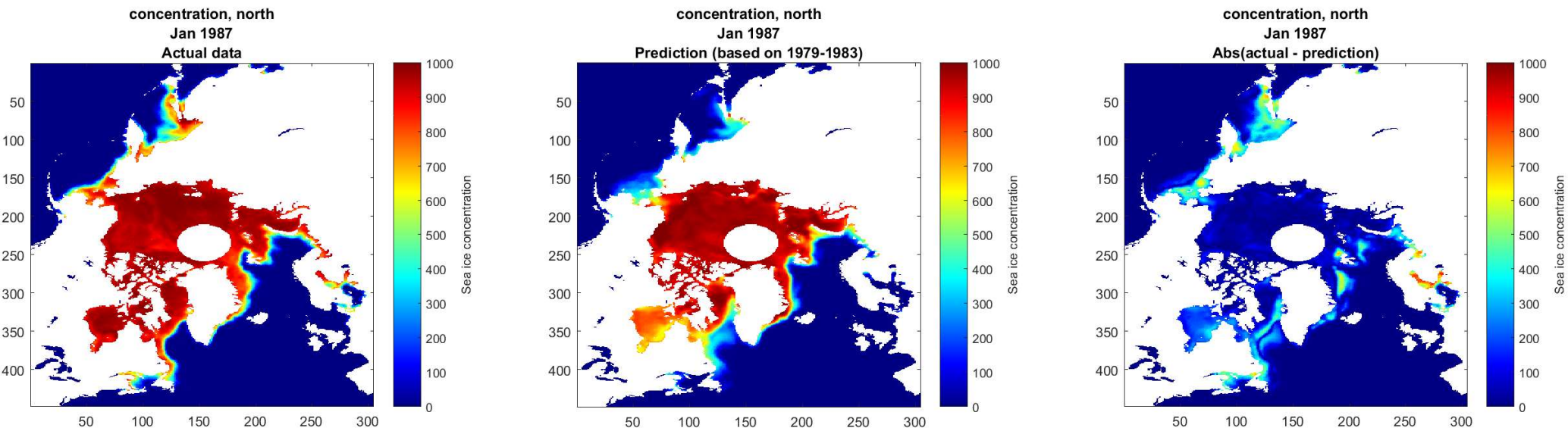


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### January 1987, 37 months after 1979-1983 input data

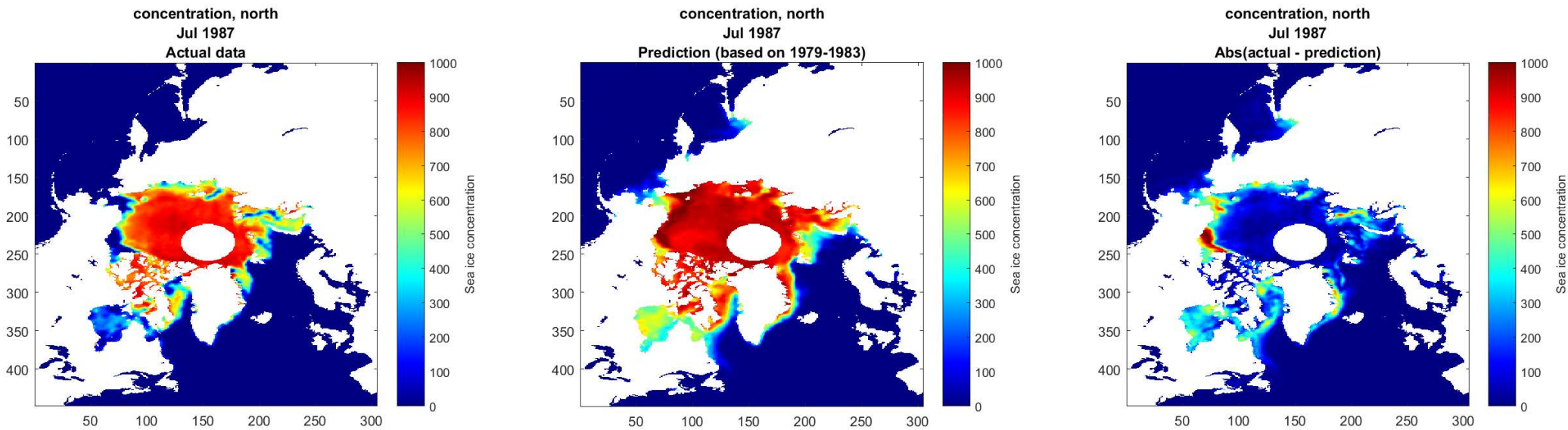


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# Sea ice concentration, northern hemisphere

## Prediction, 5 year windows

### July 1987, 43 months after 1979-1983 input data

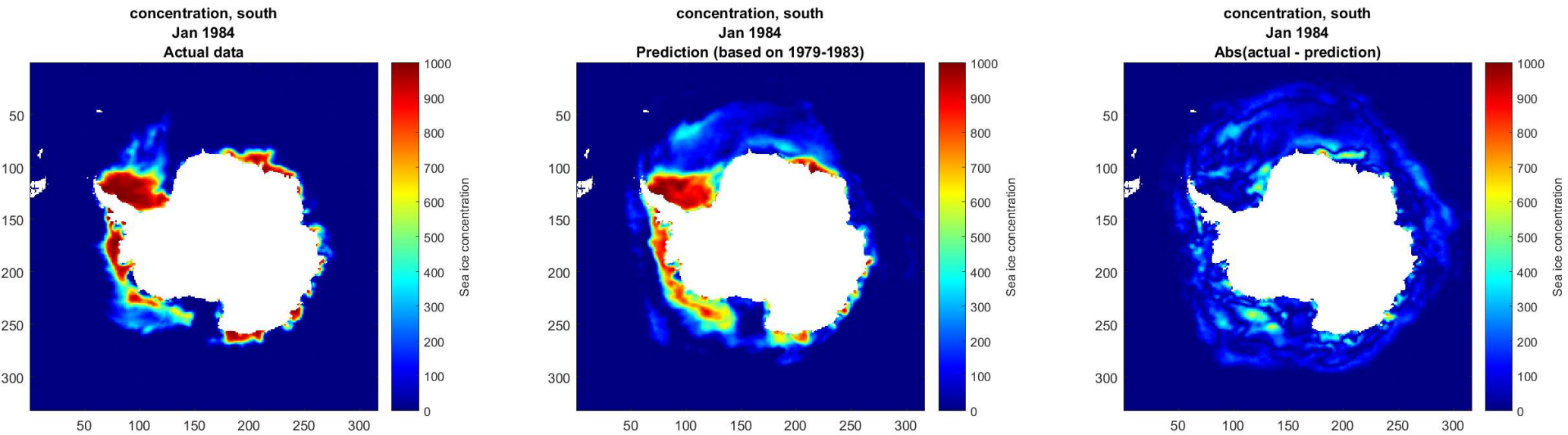


Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration.

# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

### January 1984, 1 month after 1979-1983 input data

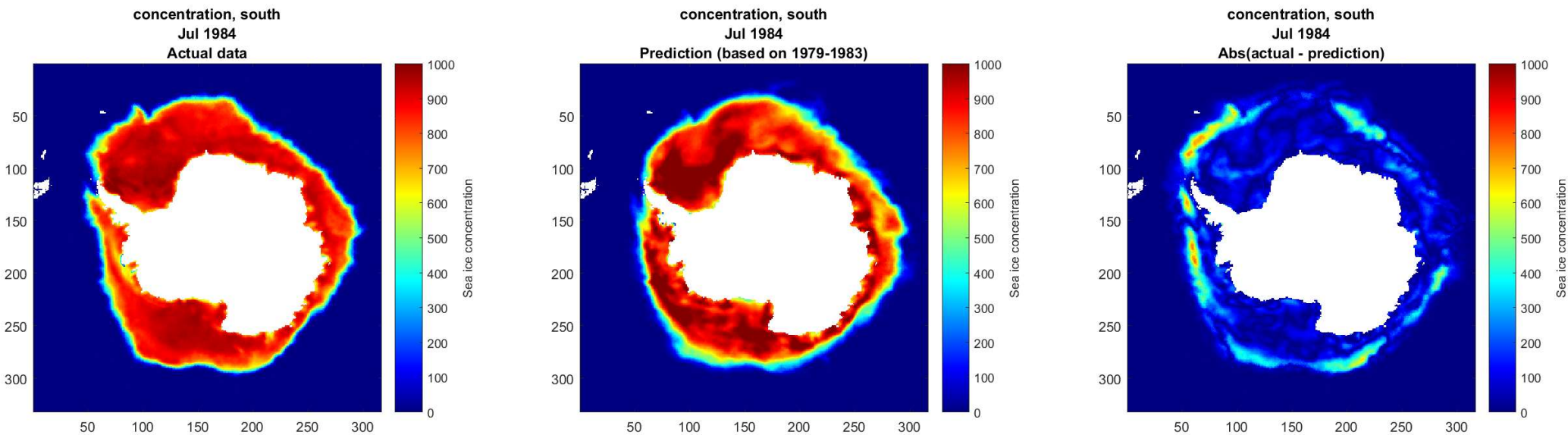


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

### July 1984, 7 months after 1979-1983 input data

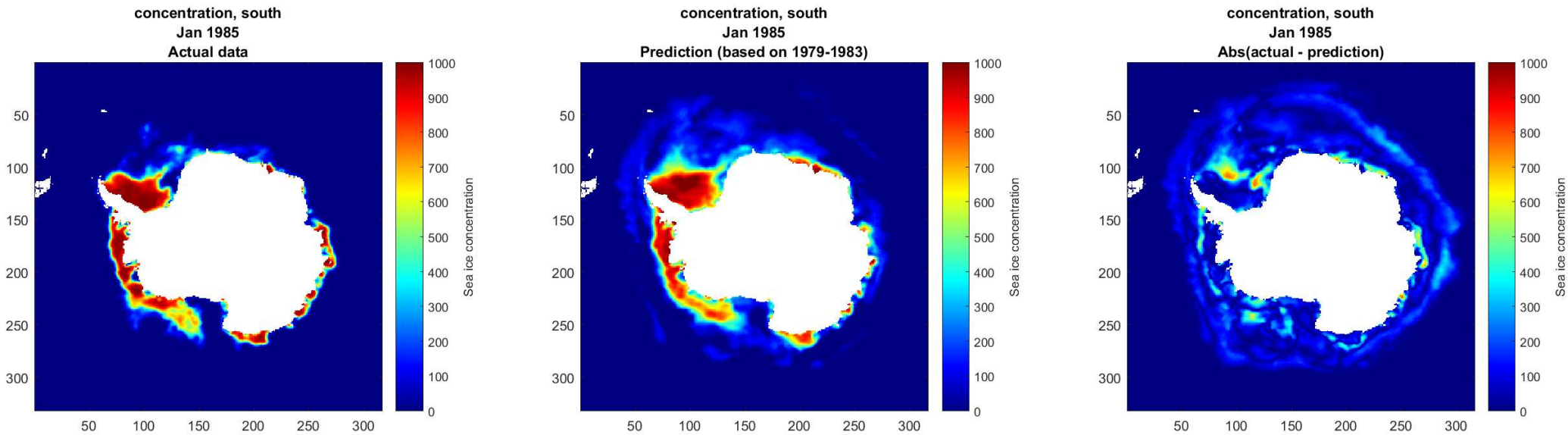


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

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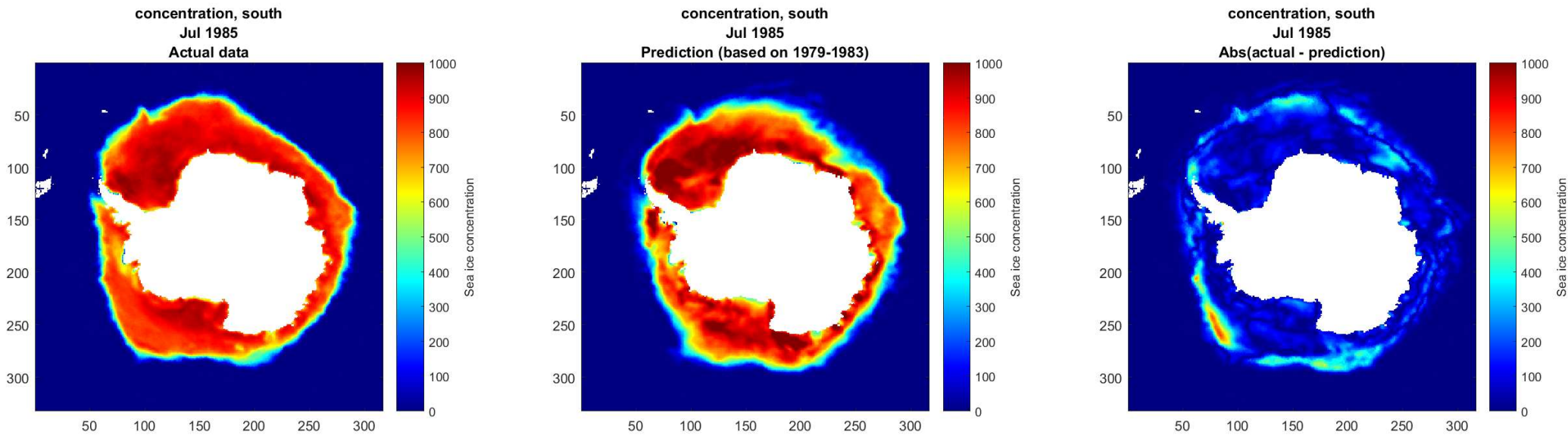


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

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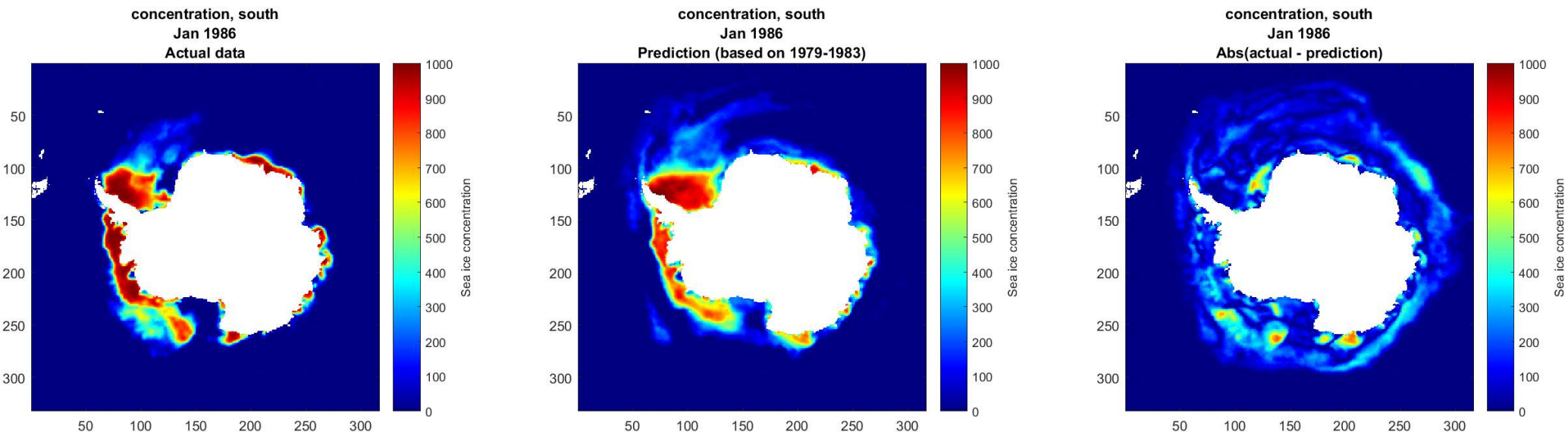


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

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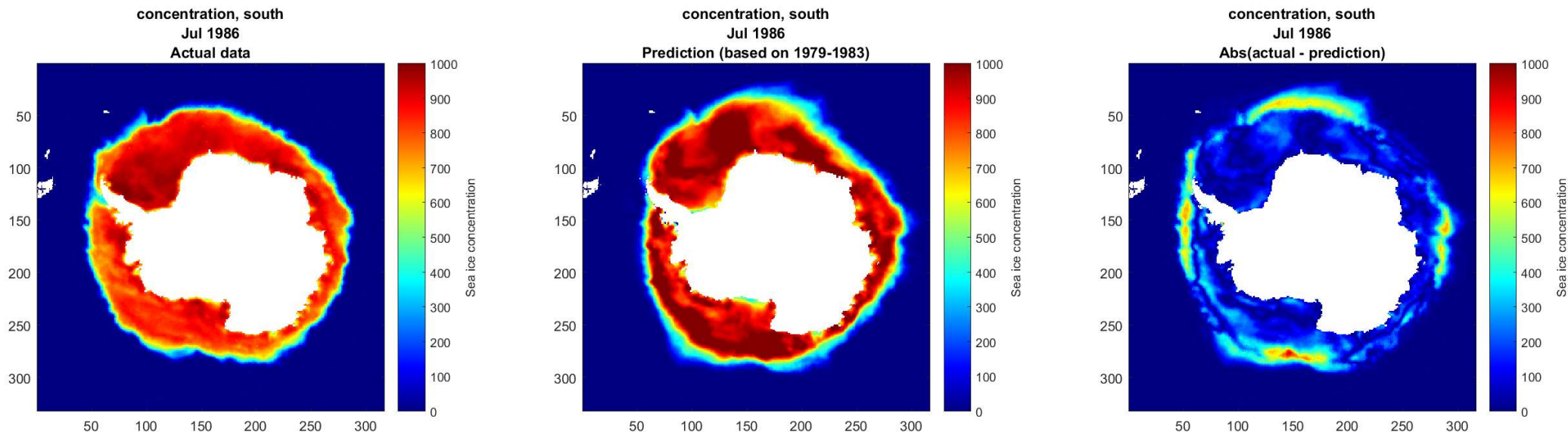


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

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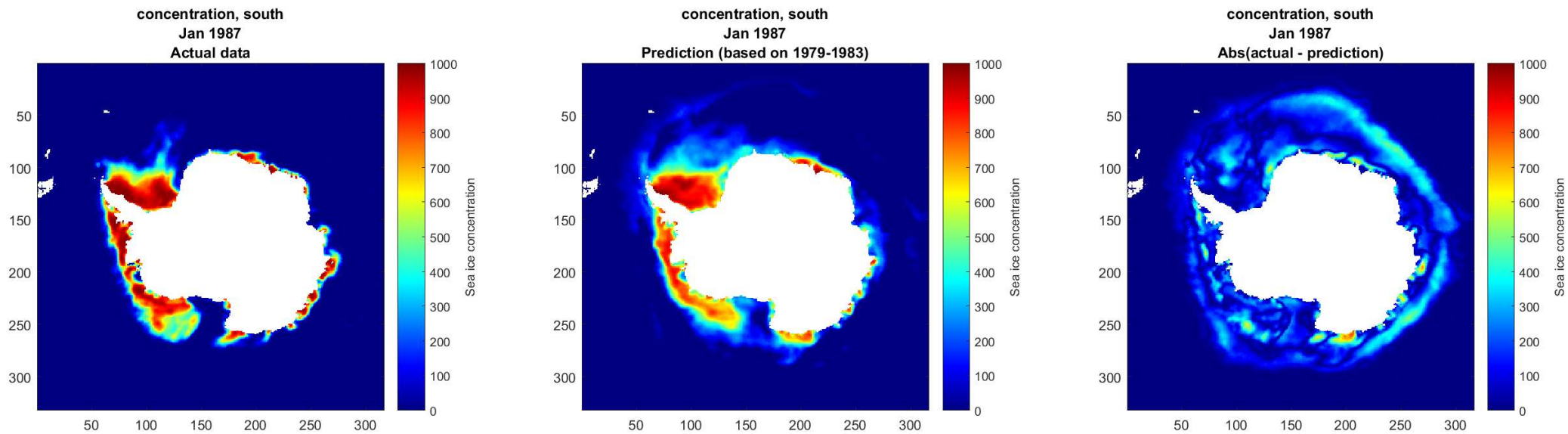


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

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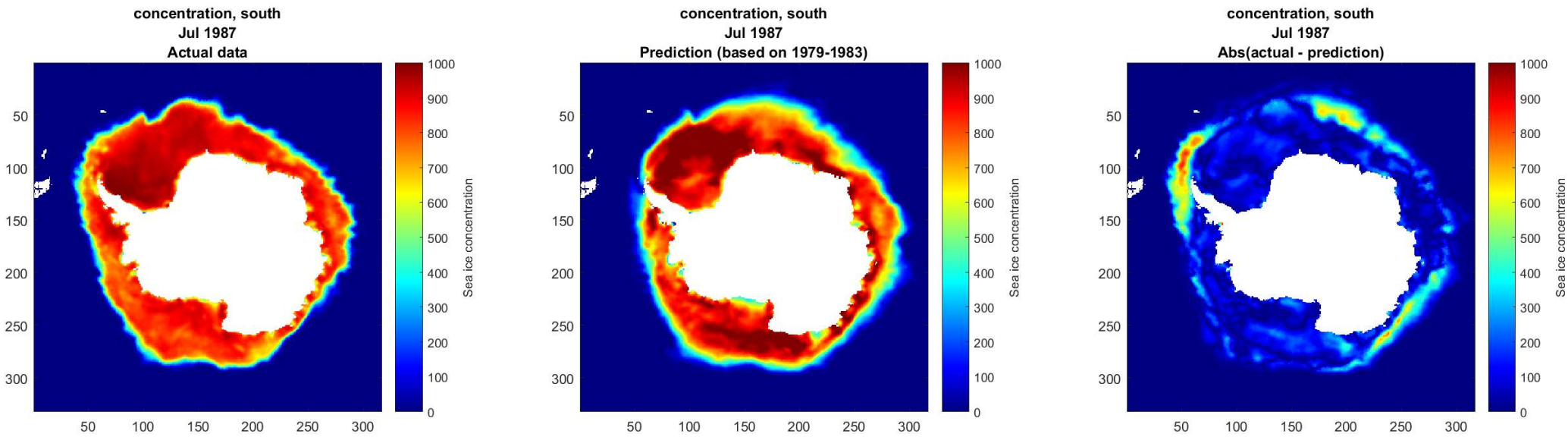


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# Sea ice concentration, southern hemisphere

## Prediction, 5 year windows

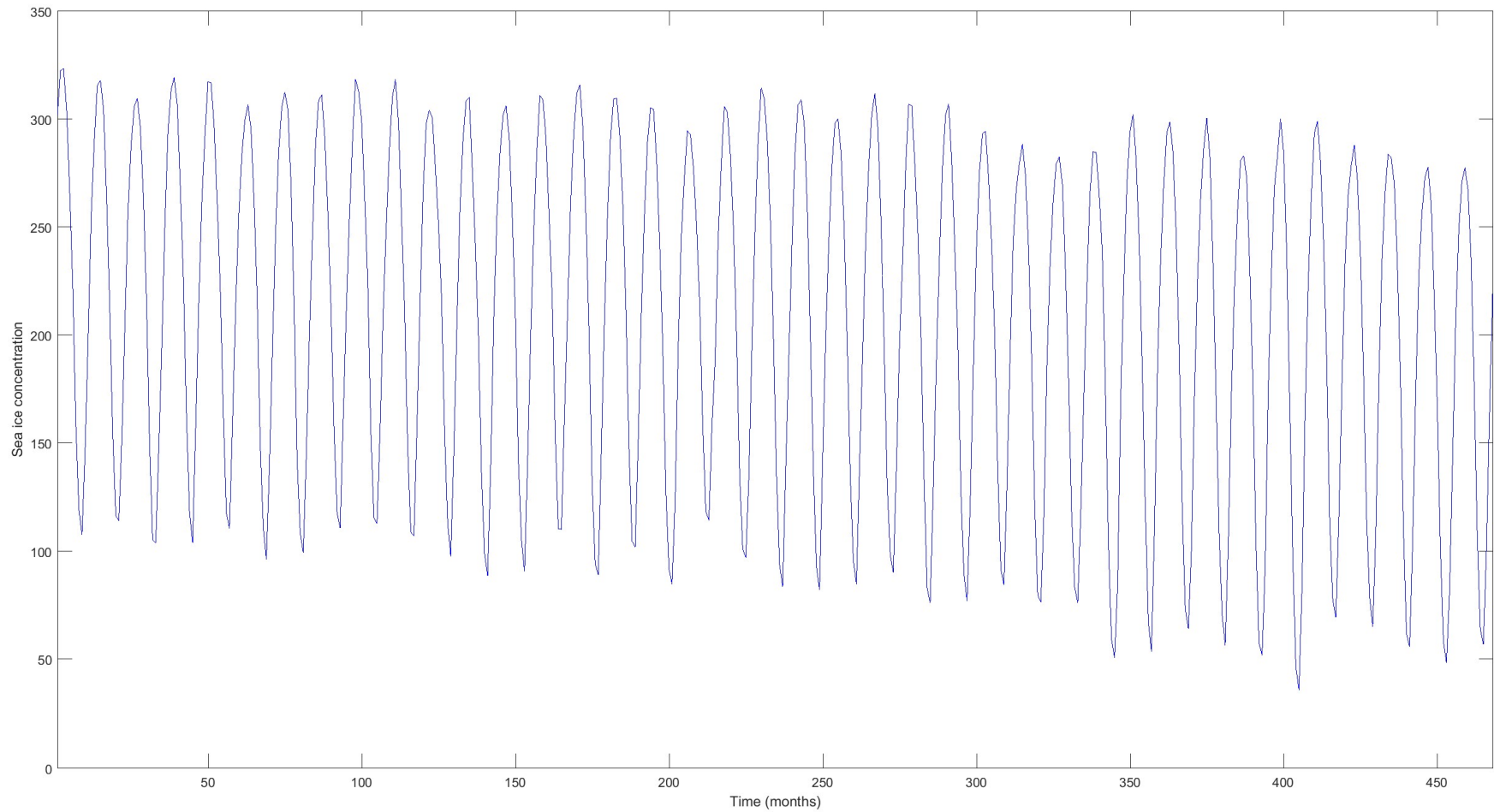
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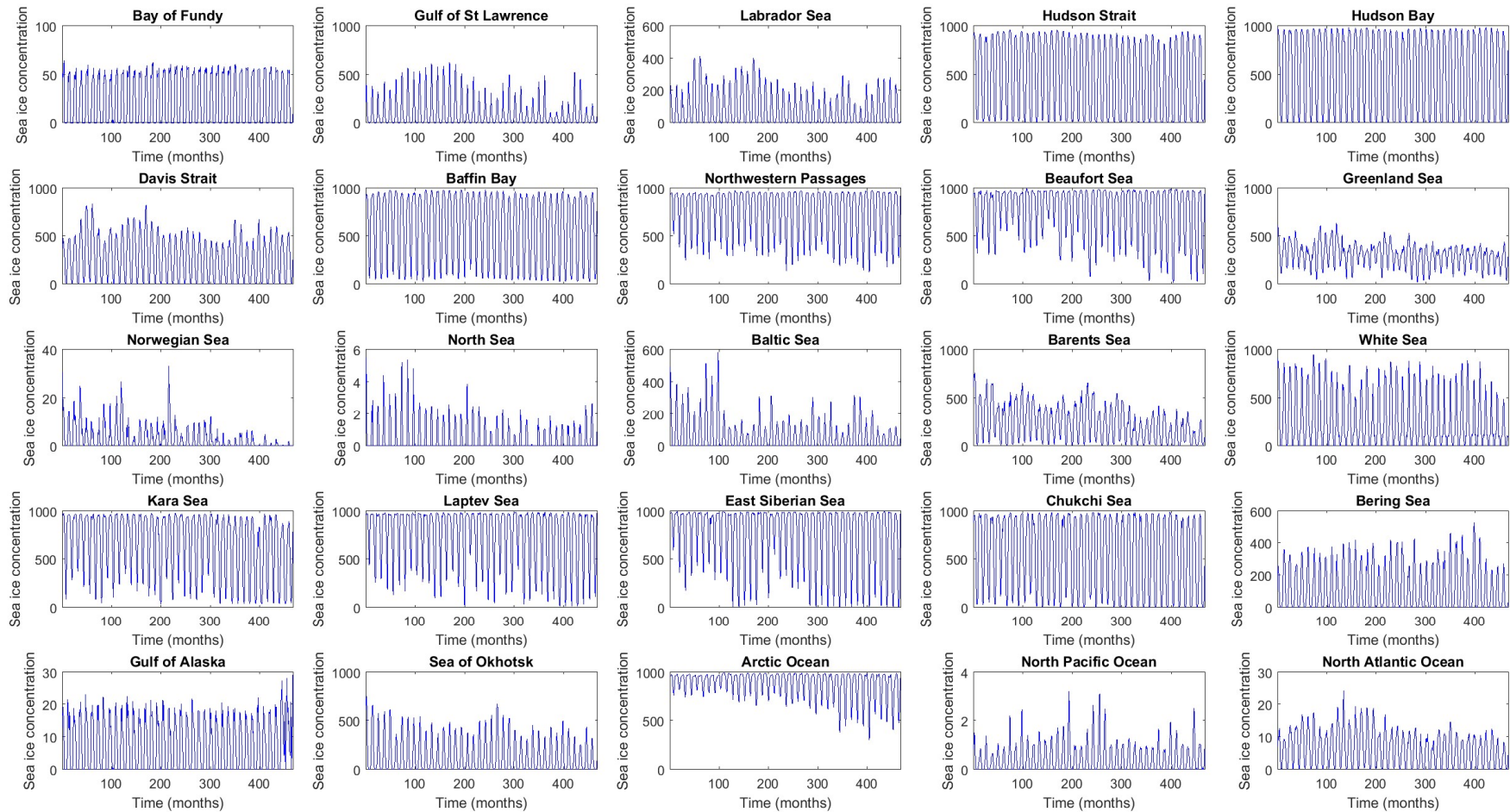
# Sea ice concentration, northern hemisphere

## Monthly average sea ice concentration for whole hemisphere



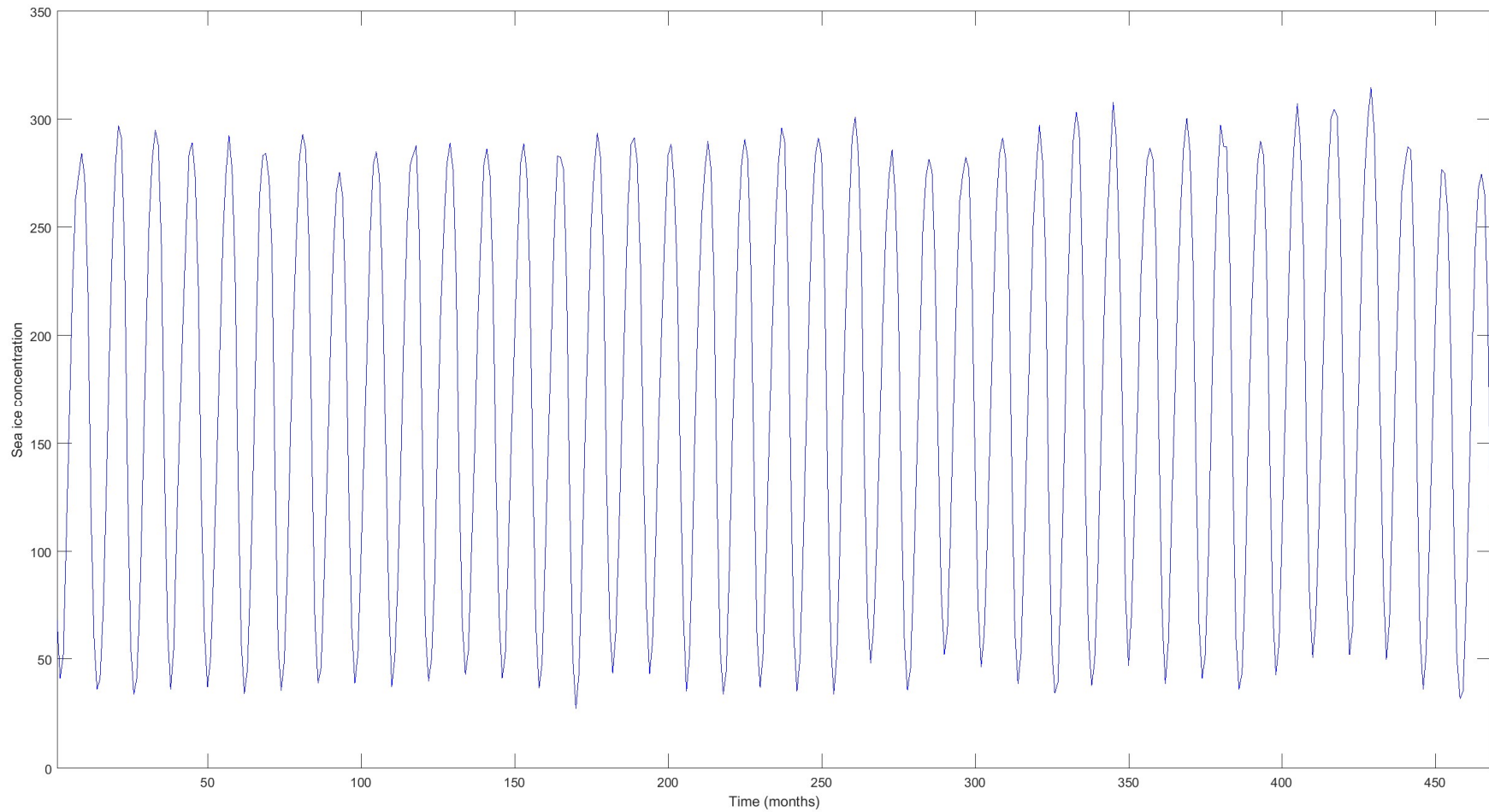
# Sea ice concentration, northern hemisphere

## Monthly average sea ice concentration for each region



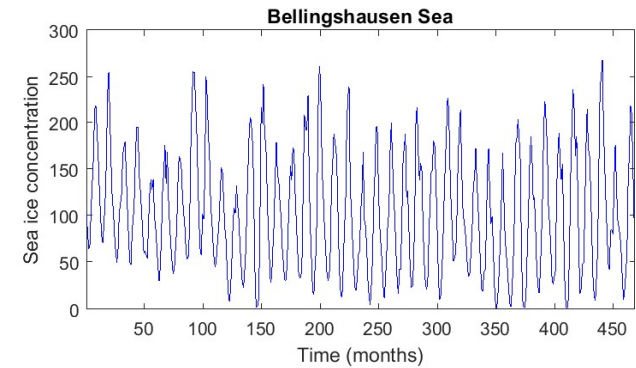
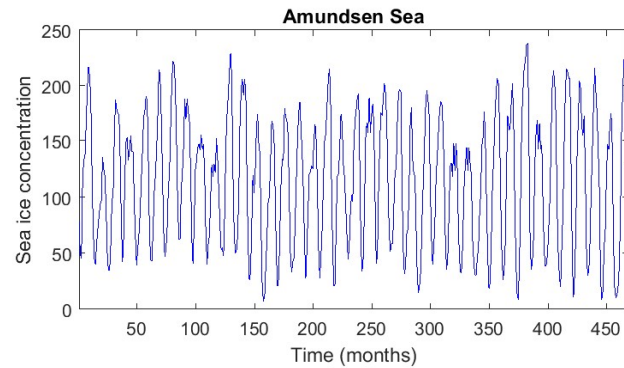
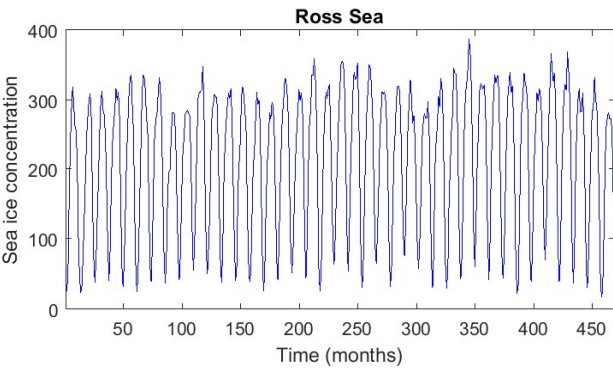
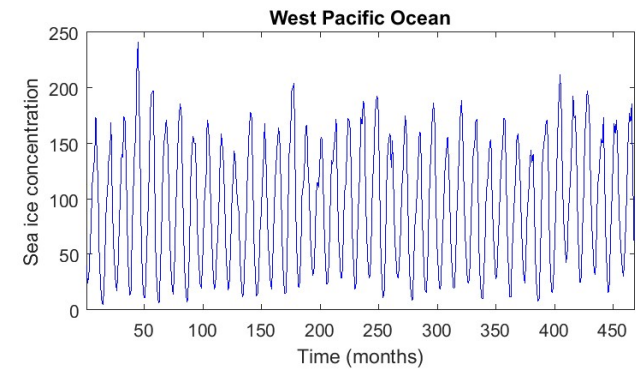
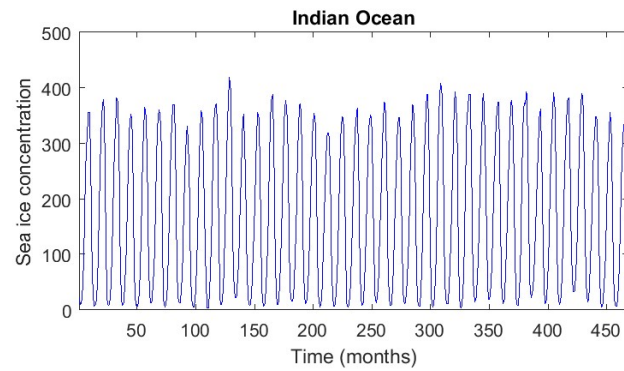
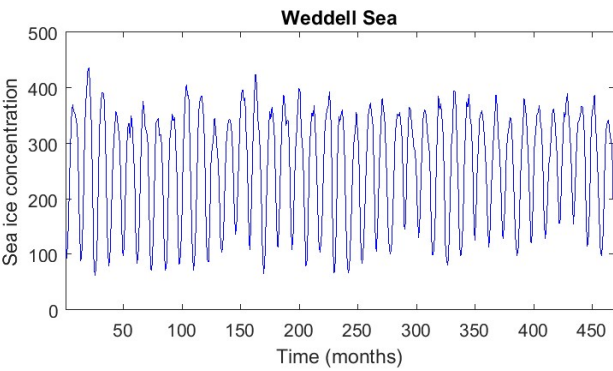
# Sea ice concentration, southern hemisphere

## Monthly average sea ice concentration for whole hemisphere



# Sea ice concentration, southern hemisphere

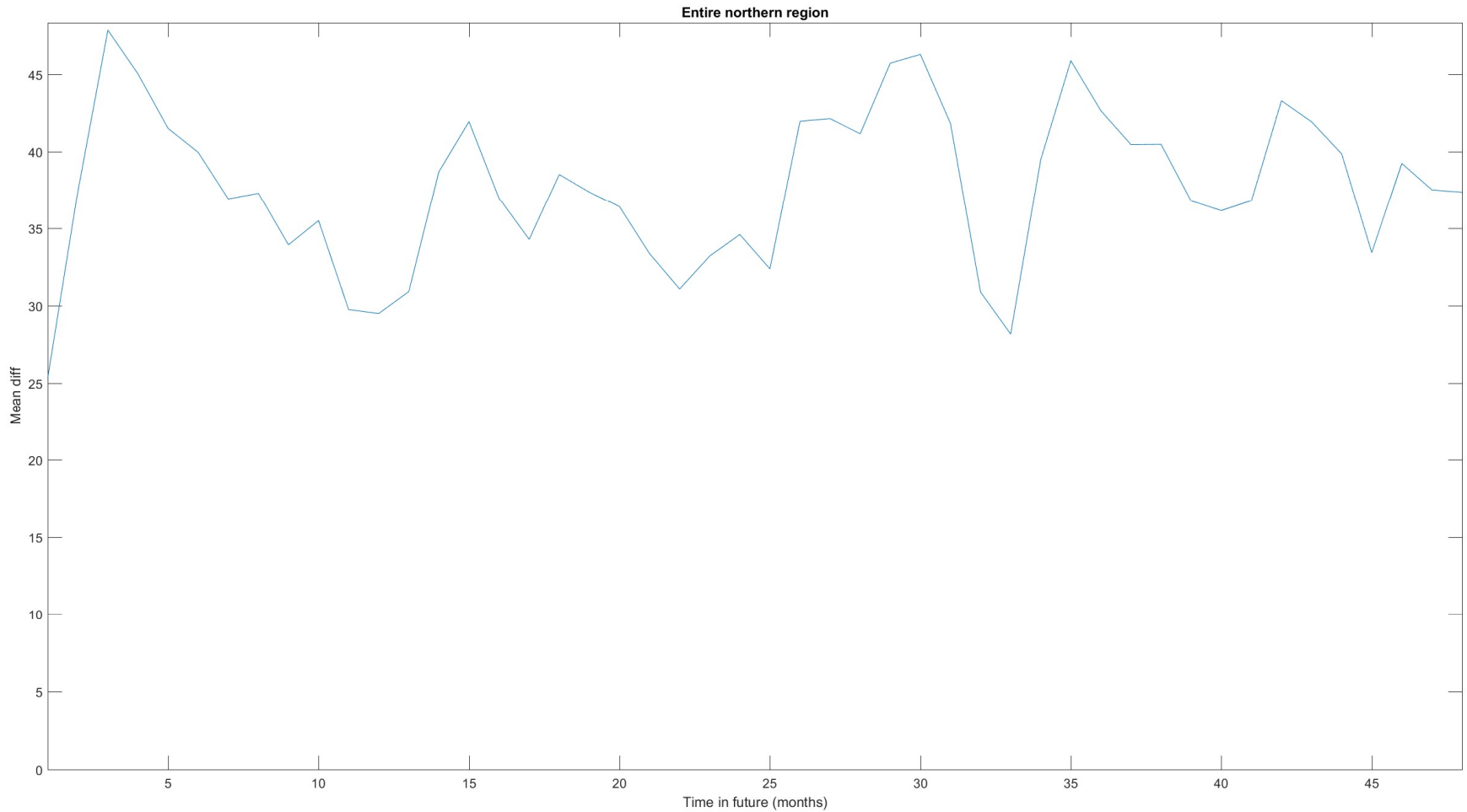
## Monthly average sea ice concentration for each region



# Sea ice concentration, northern hemisphere

Goodness of prediction results for whole hemisphere

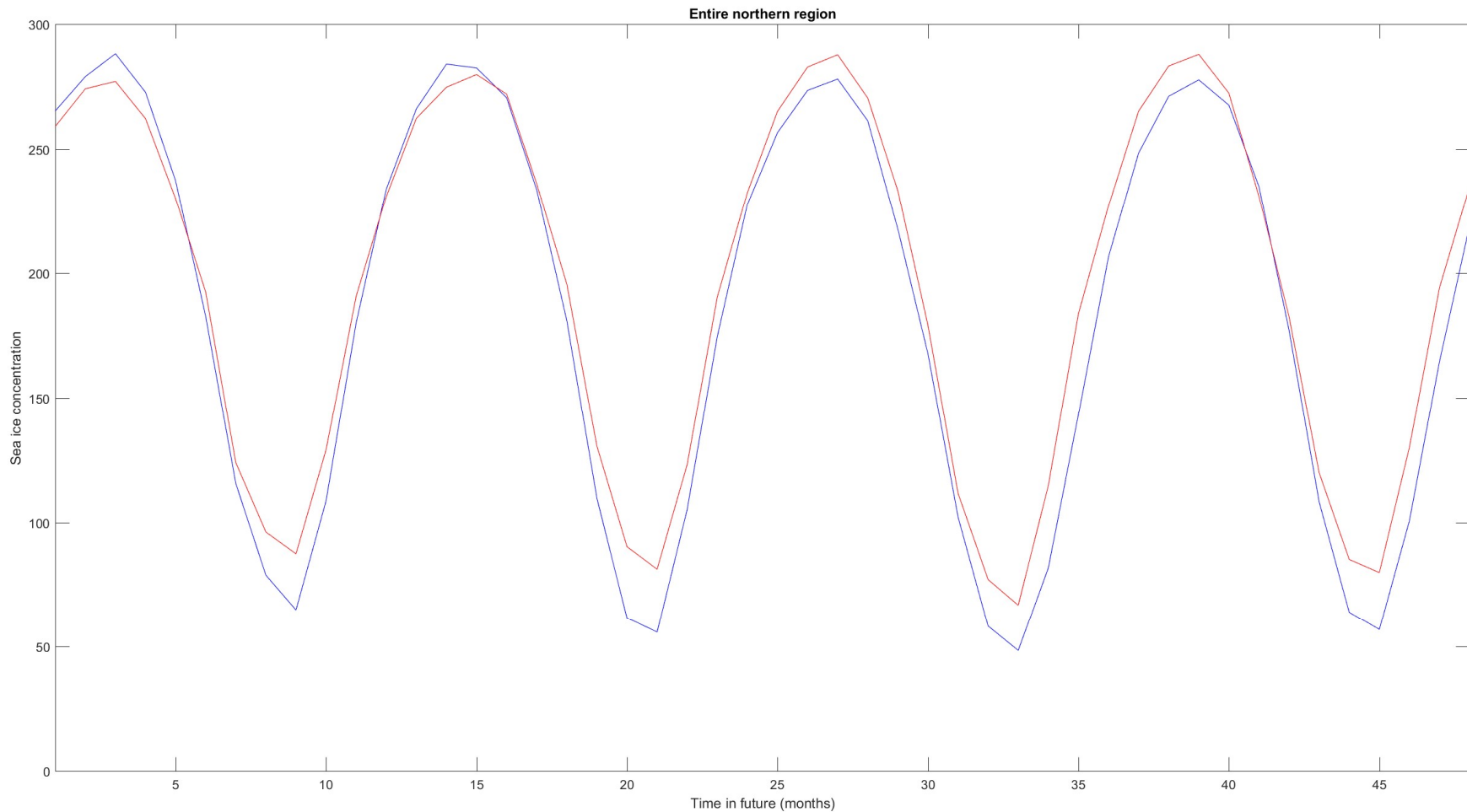
Sum of absolute value of difference between actual and prediction pixels



# Sea ice concentration, northern hemisphere

## Goodness of prediction results for whole hemisphere

### Comparison of actual and predicted values

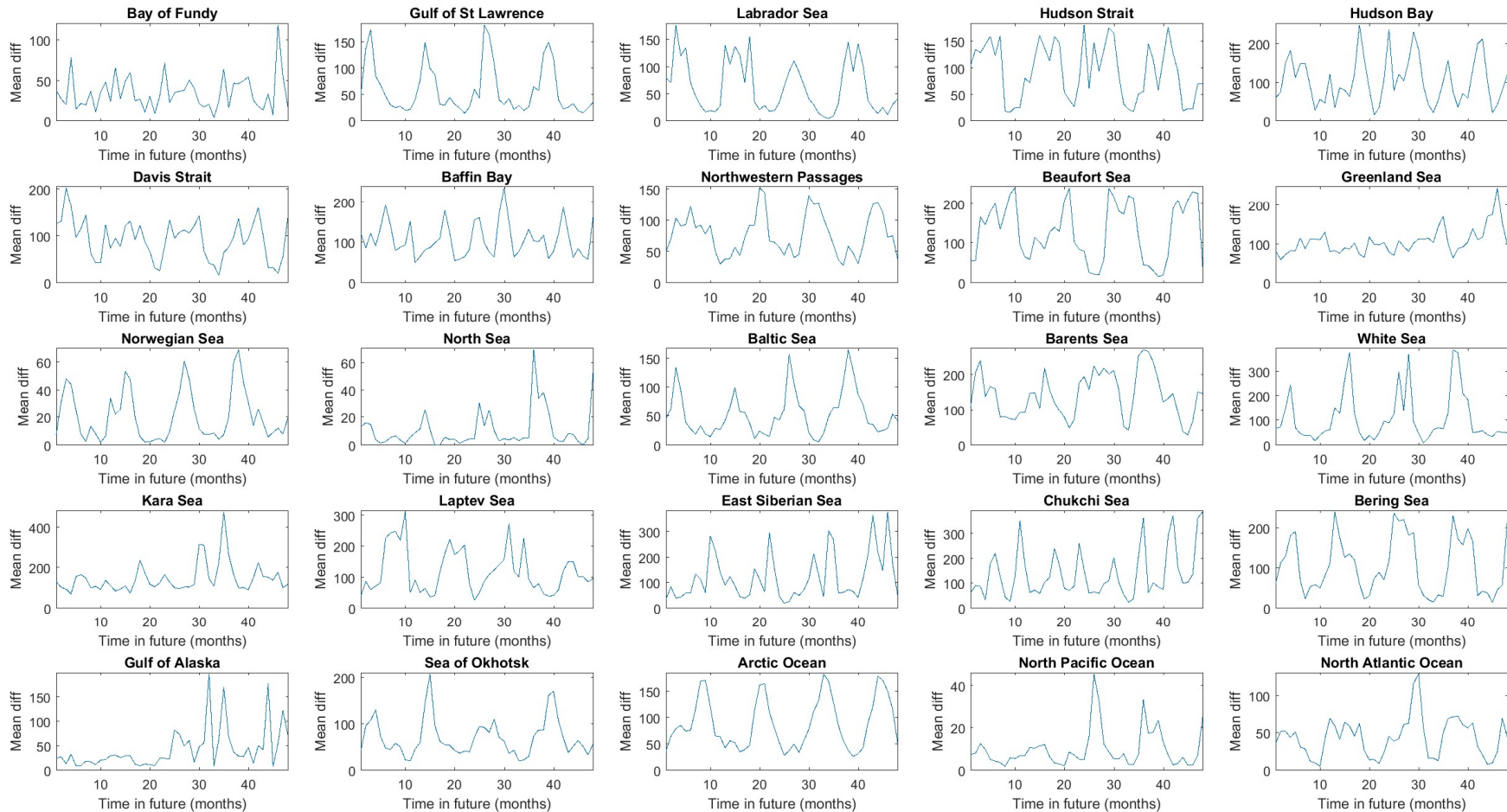


Blue: actual concentration data. Red: Predicted concentration. (1000=100%)

# Sea ice concentration, northern hemisphere

## Goodness of prediction results for each region

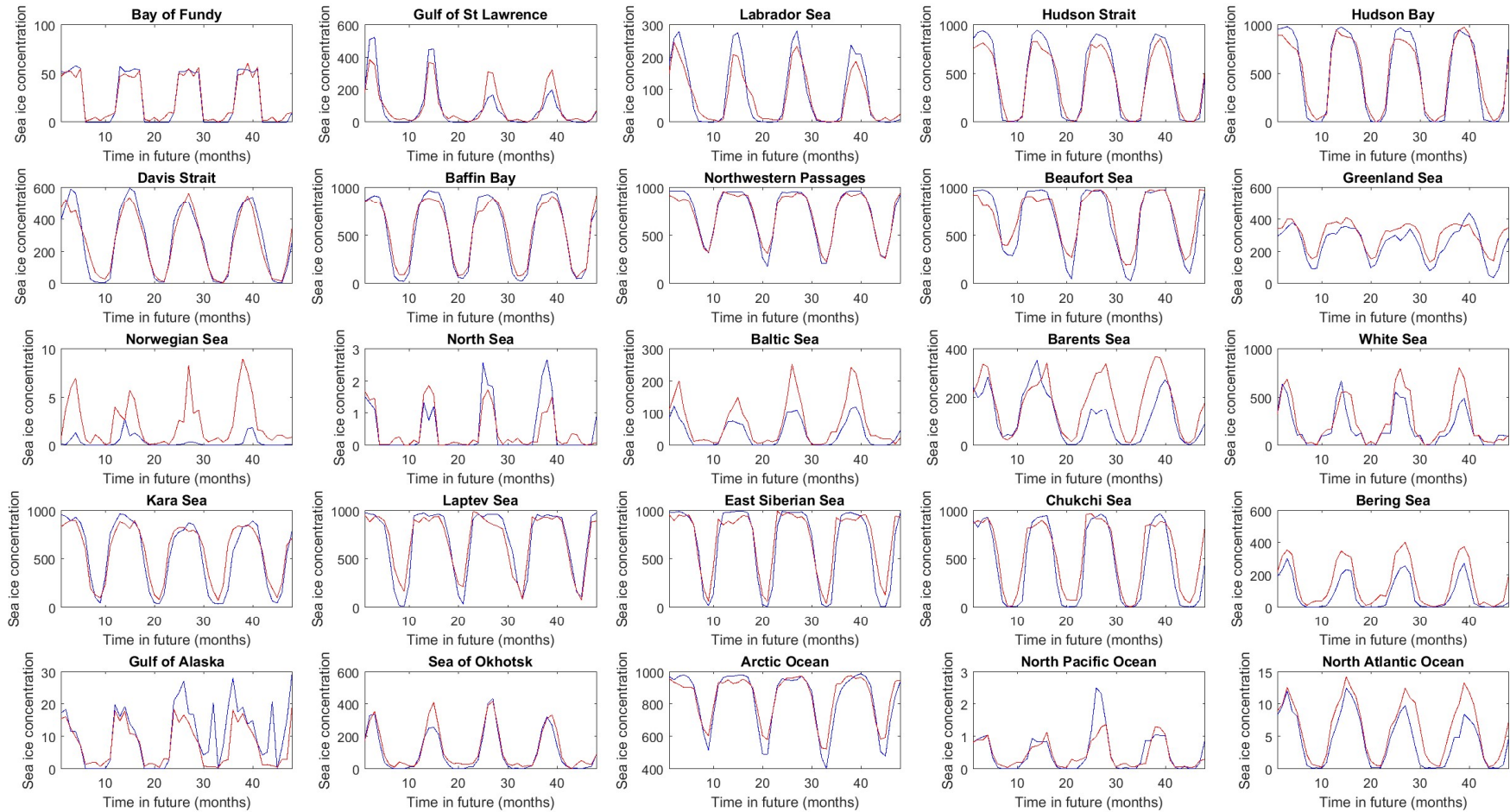
### Sum of absolute value of difference between actual and prediction pixels



# Sea ice concentration, northern hemisphere

## Goodness of prediction results for each region

### Comparison of actual and predicted values

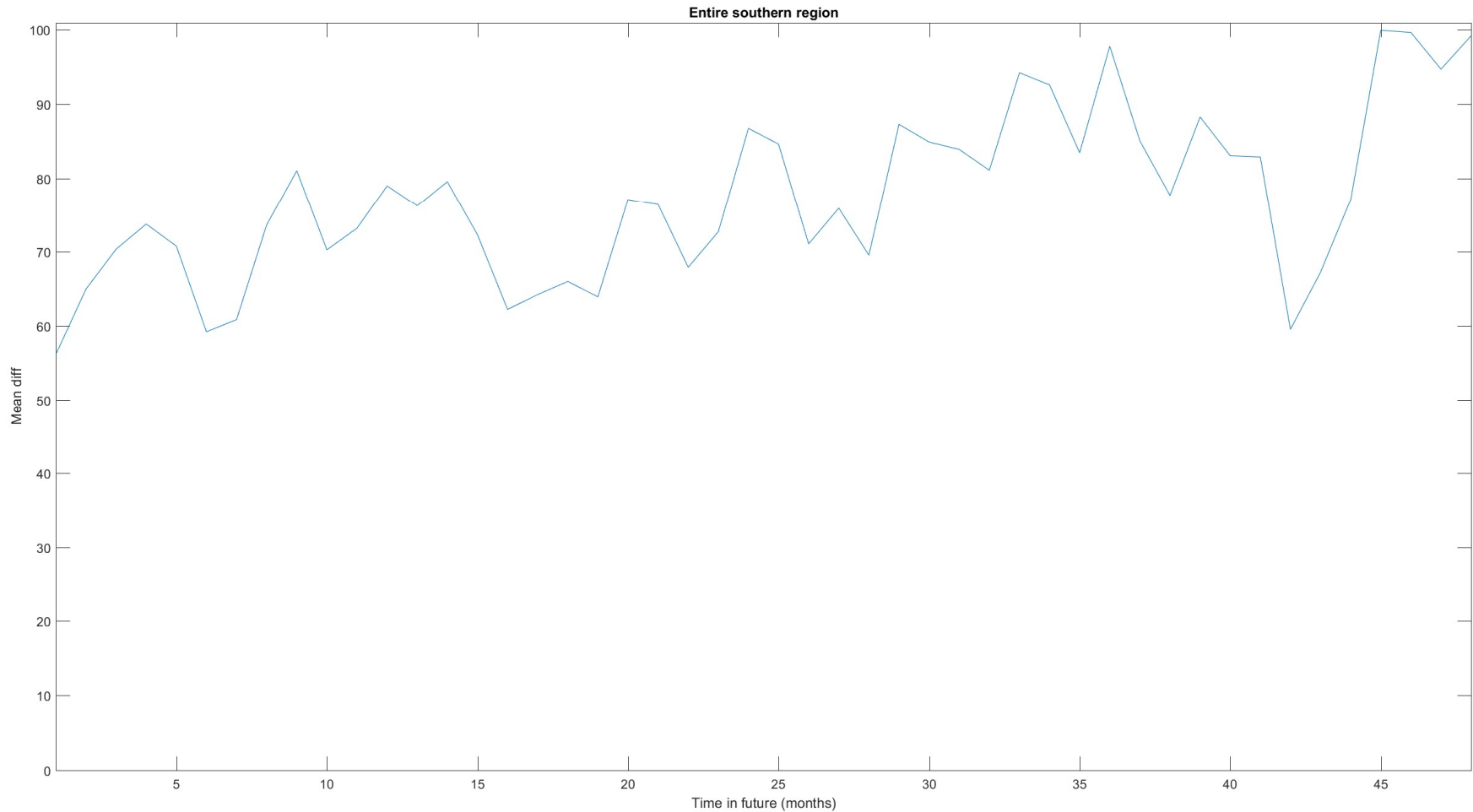


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# Sea ice concentration, southern hemisphere

## Goodness of prediction results for whole hemisphere

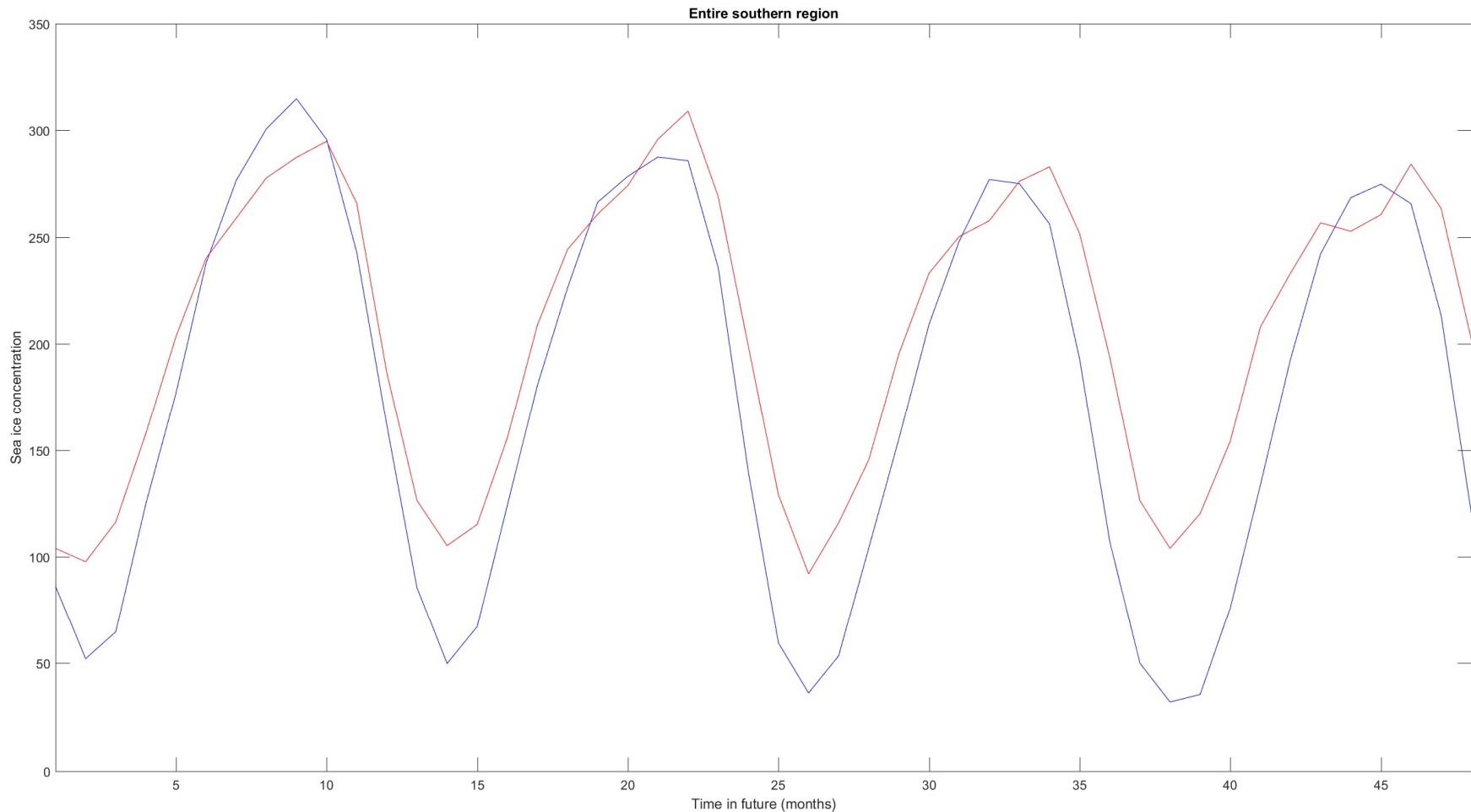
### Sum of absolute value of difference between actual and prediction pixels



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for whole hemisphere

### Comparison of actual and predicted values

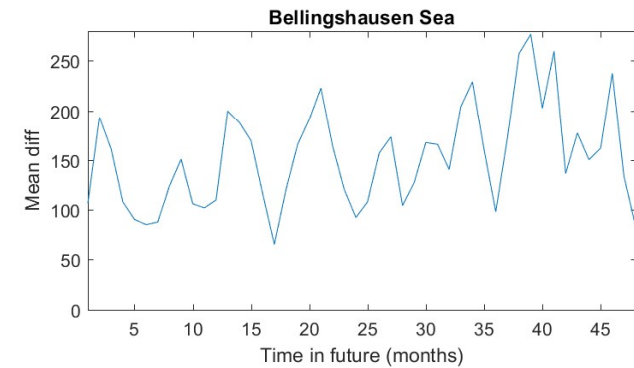
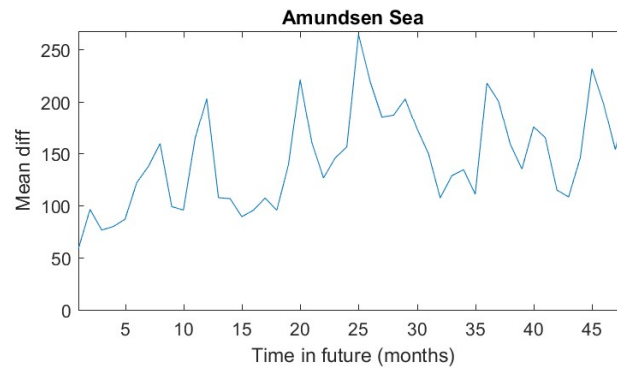
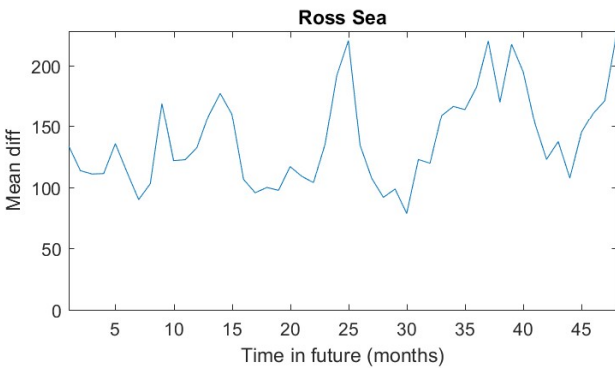
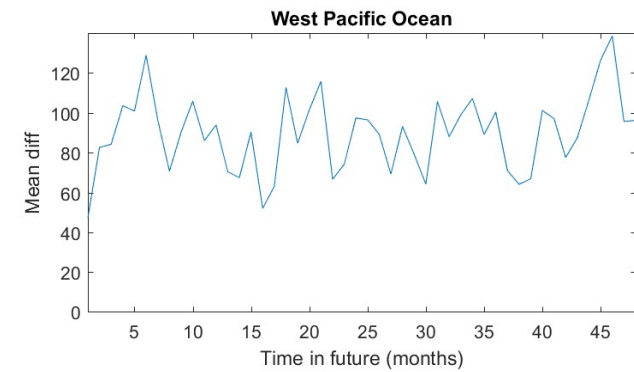
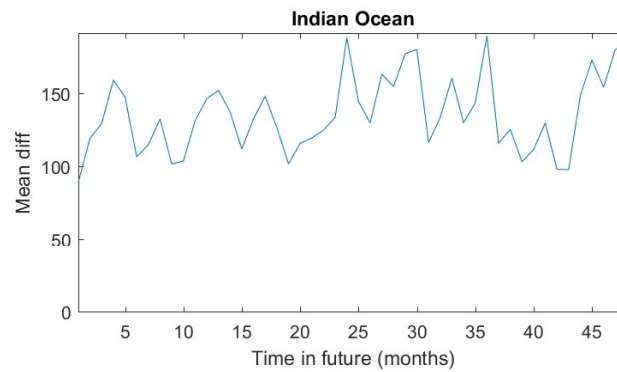
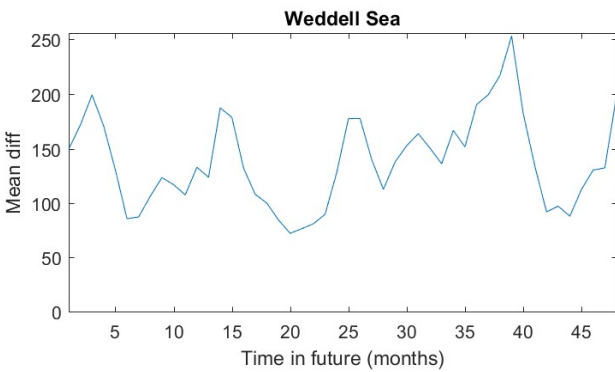


Blue: actual concentration data. Red: Predicted concentration. (1000=100%)

# Sea ice concentration, southern hemisphere

## Goodness of prediction results for each region

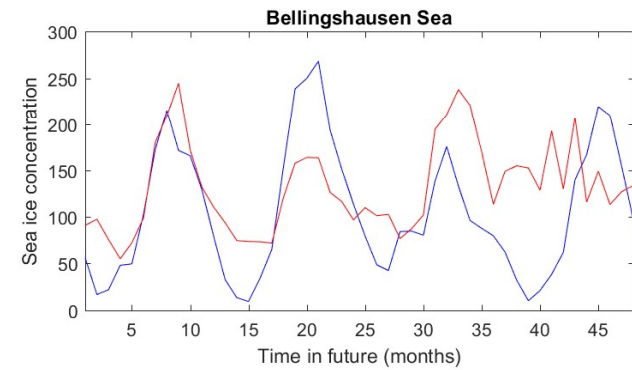
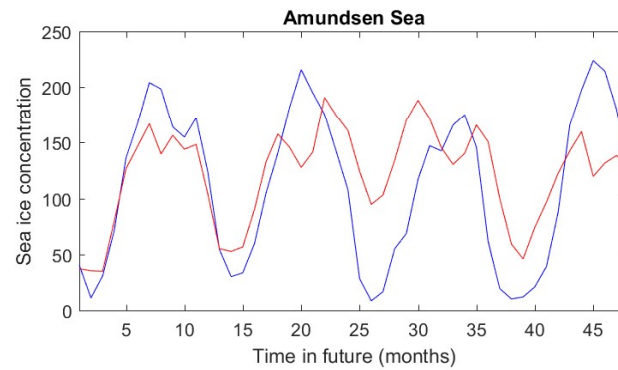
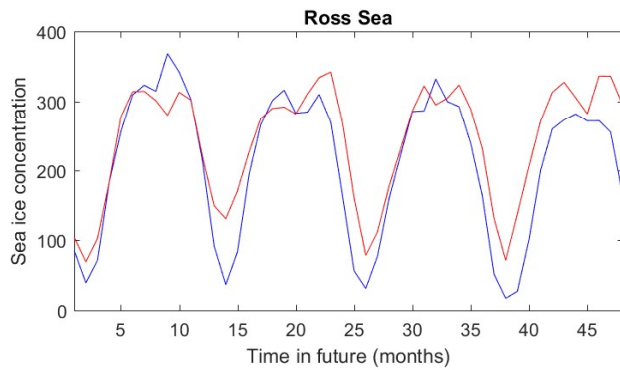
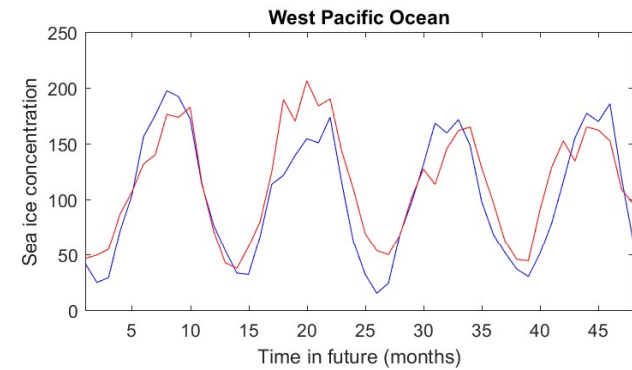
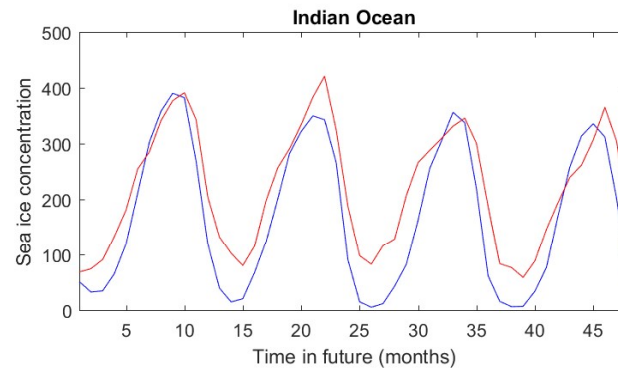
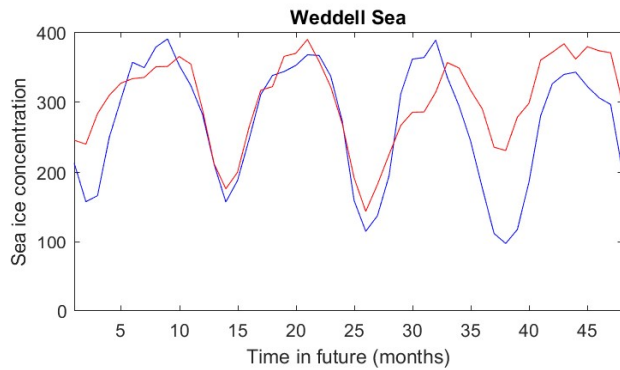
### Sum of absolute value of difference between actual and prediction pixels



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for each region

### Comparison of actual and predicted values

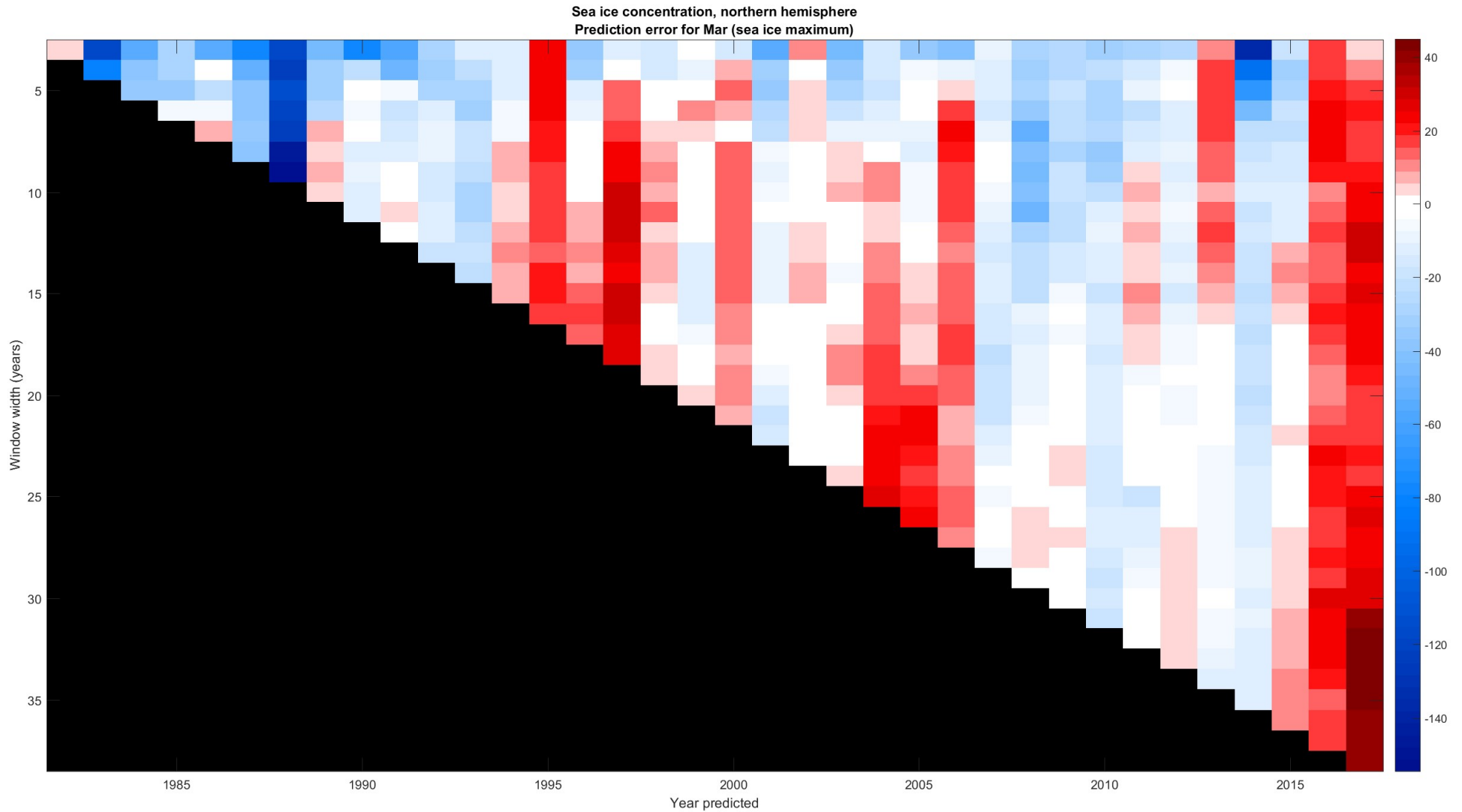


Blue: actual concentration data. Red: Predicted concentration. (1000=100%)

# Sea ice concentration, northern hemisphere

## Goodness of prediction results for whole hemisphere

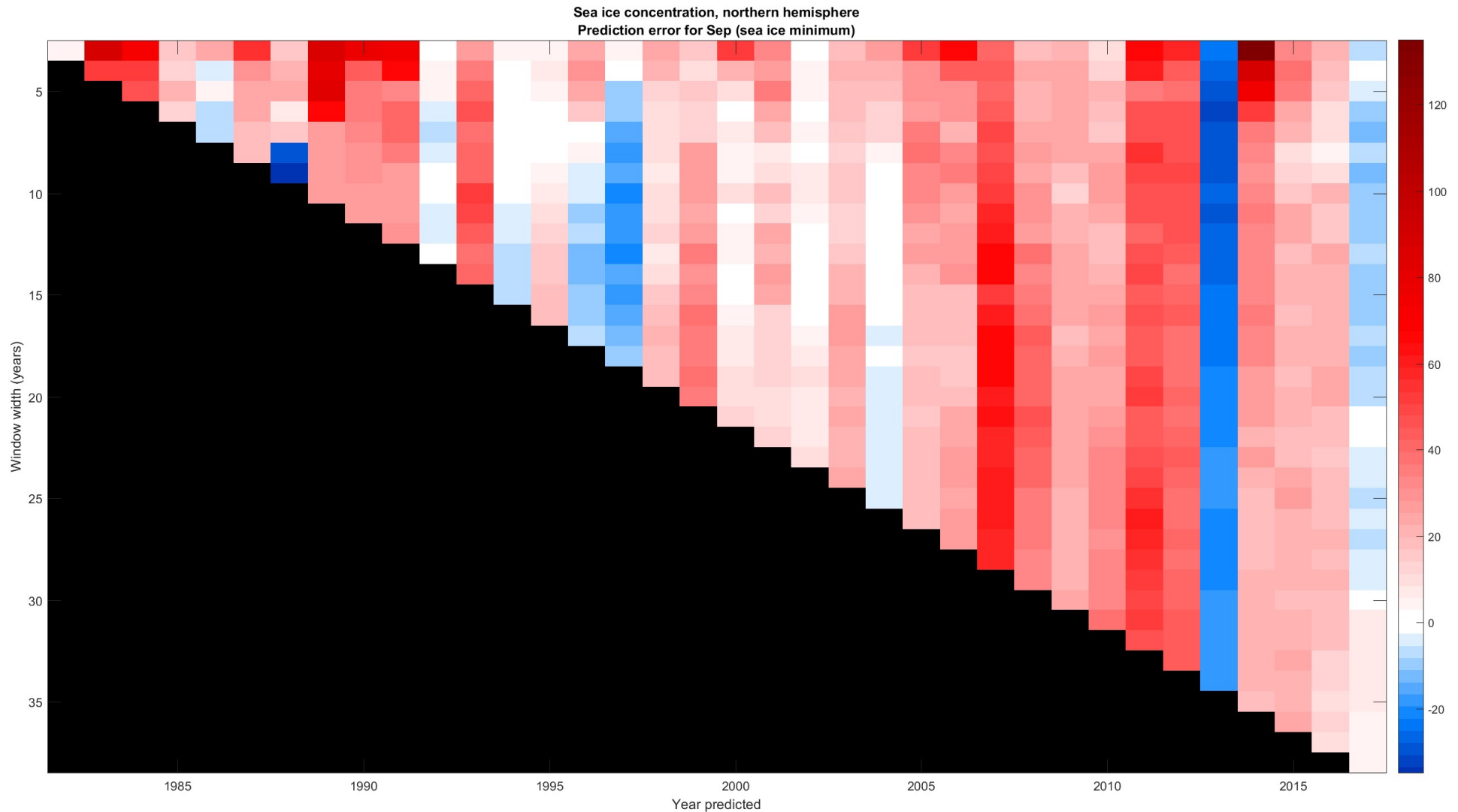
### Maximum sea ice concentration month (March)



# Sea ice concentration, northern hemisphere

## Goodness of prediction results for whole hemisphere

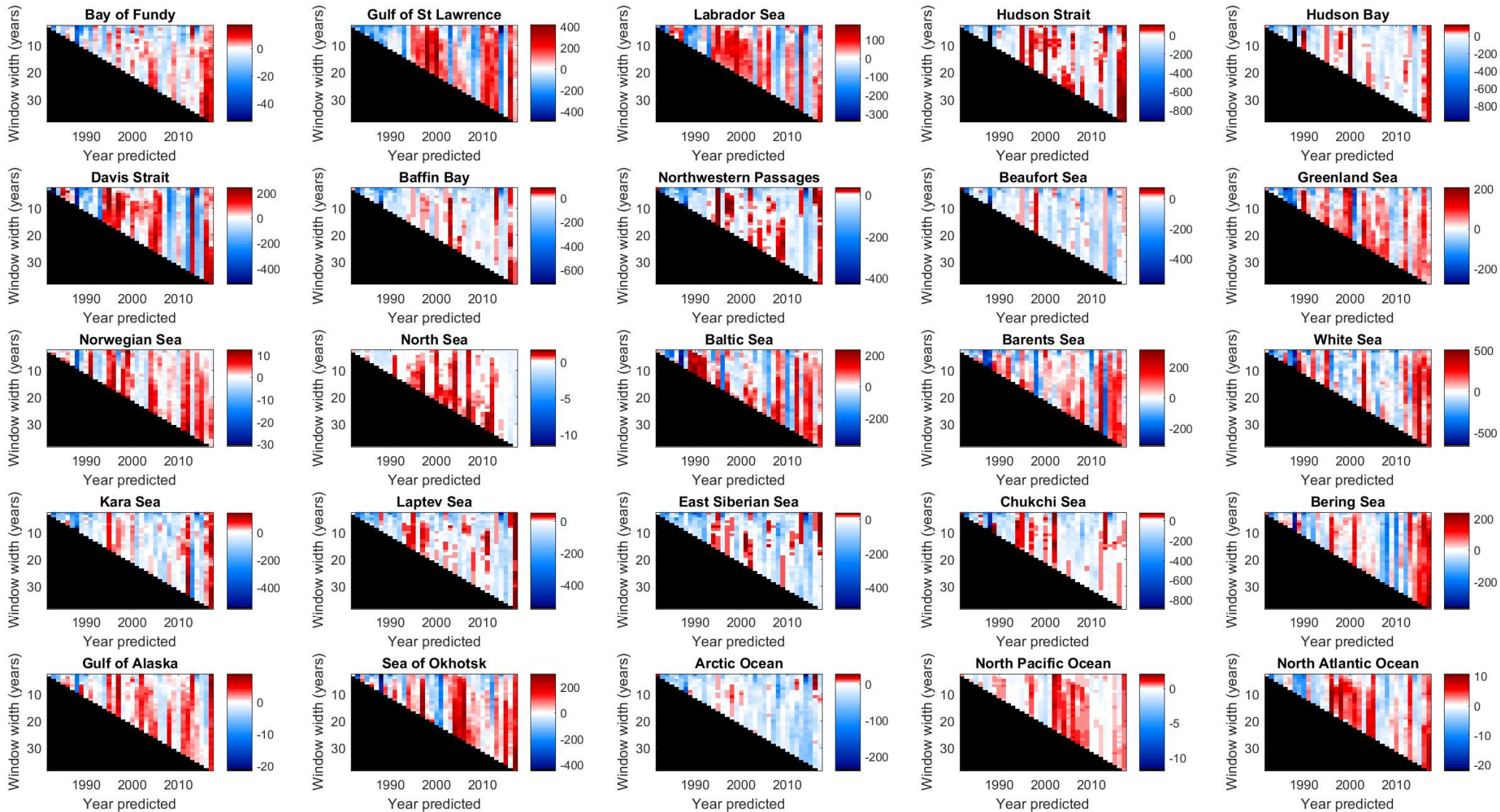
### Minimum sea ice concentration month (September)



# Sea ice concentration, northern hemisphere

## Goodness of prediction results for each region

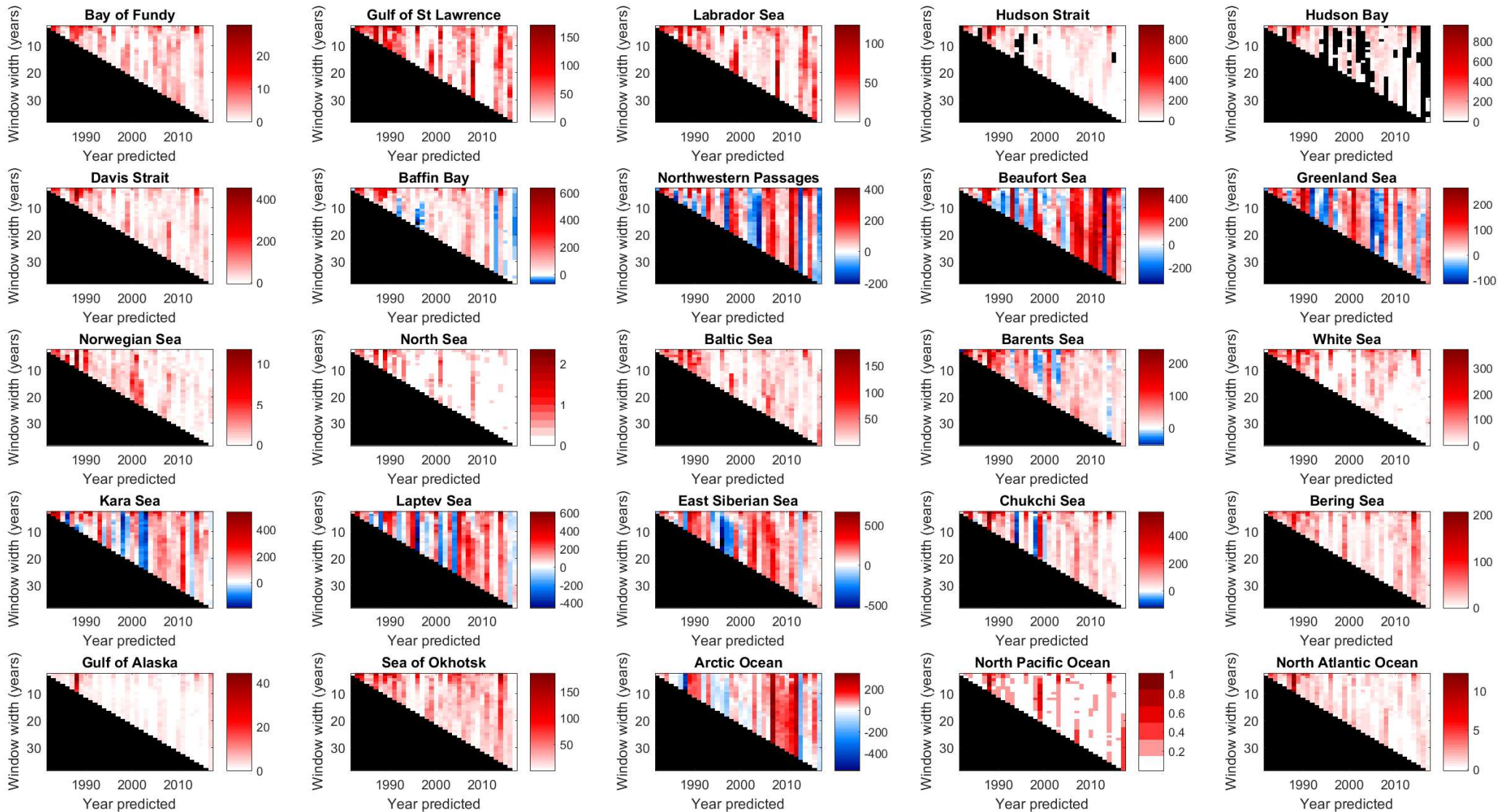
### Maximum sea ice concentration month (March)



# Sea ice concentration, northern hemisphere

## Goodness of prediction results for each region

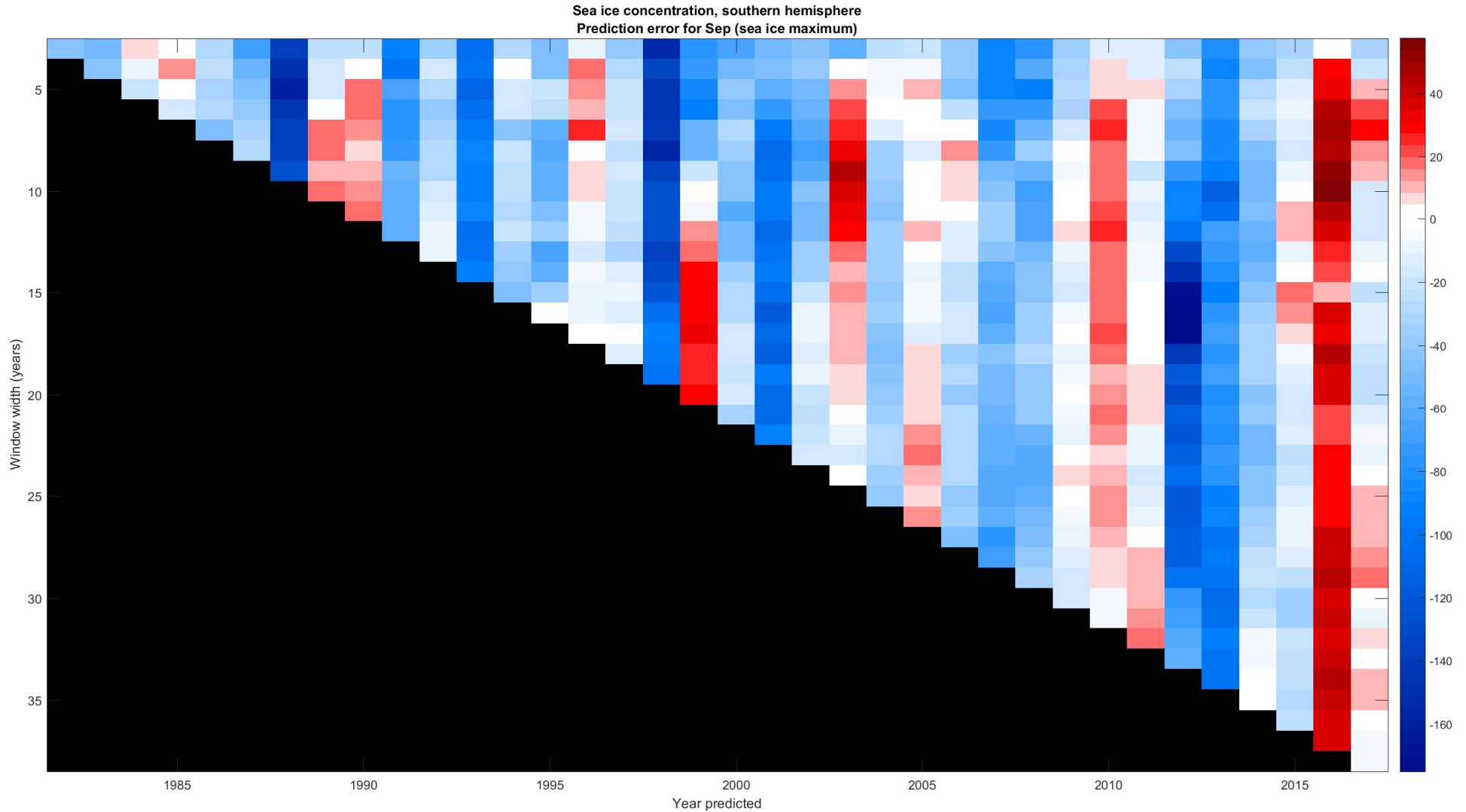
### Minimum sea ice concentration month (September)



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for whole hemisphere

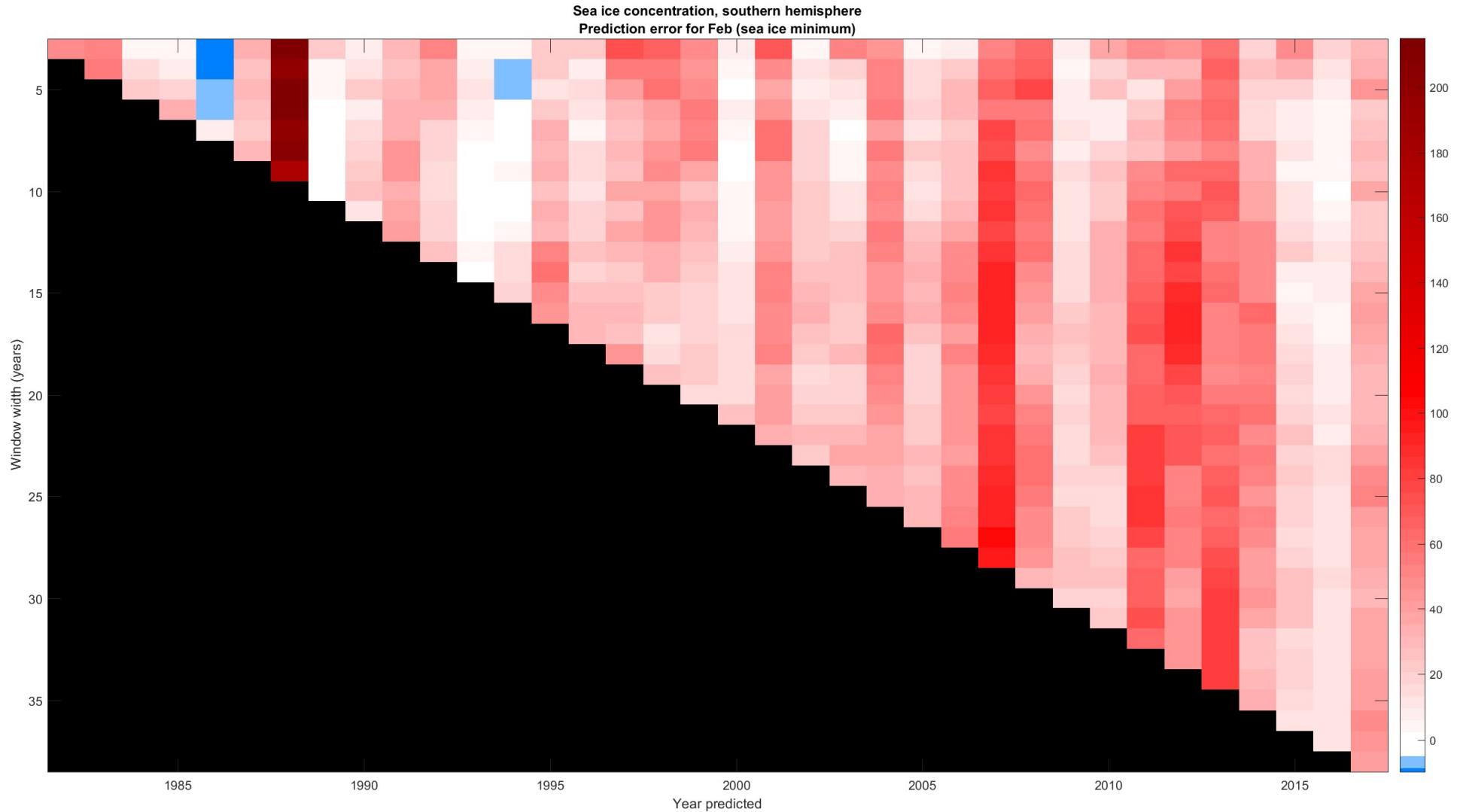
### Maximum sea ice concentration month (September)



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for whole hemisphere

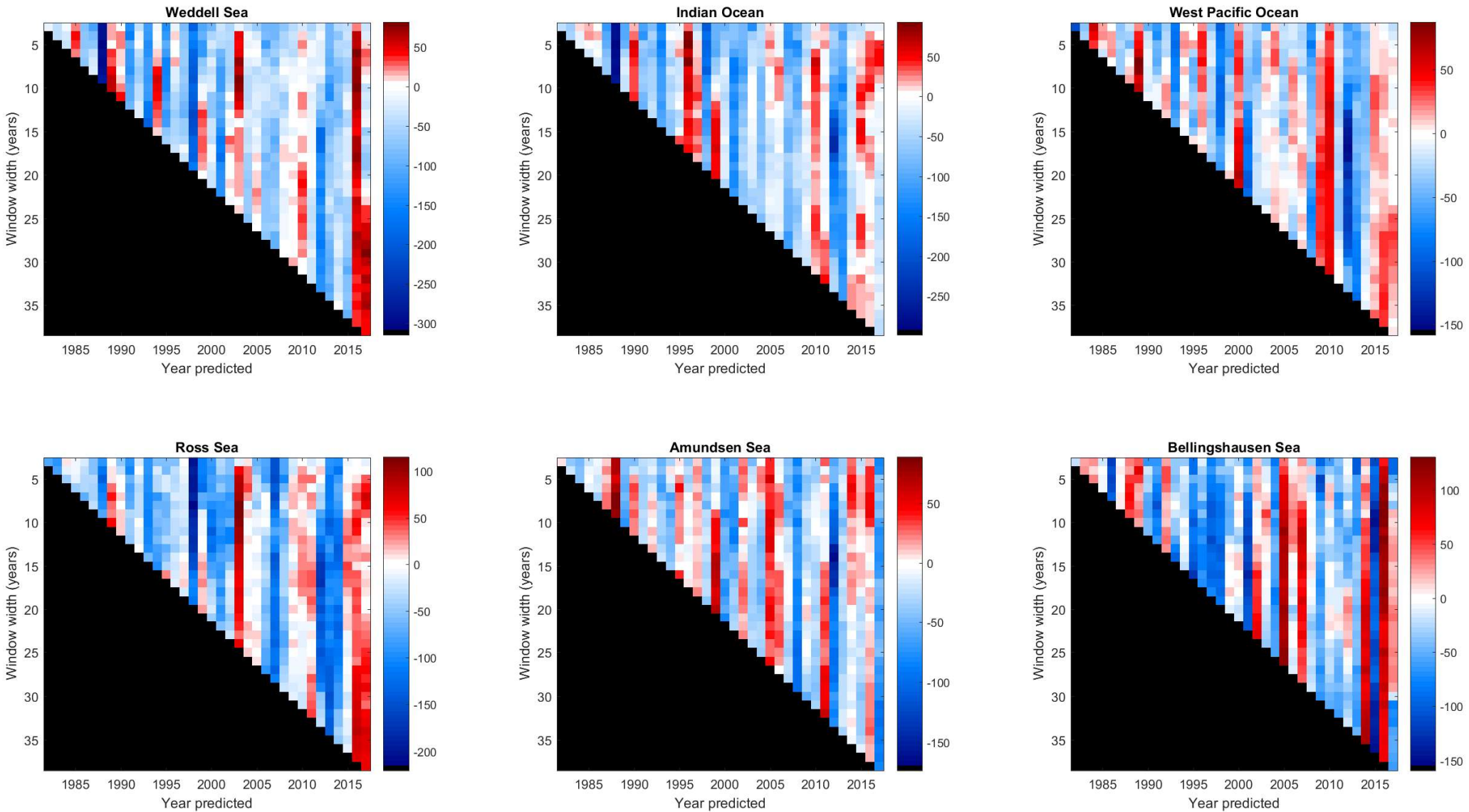
### Minimum sea ice concentration month (February)



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for each region

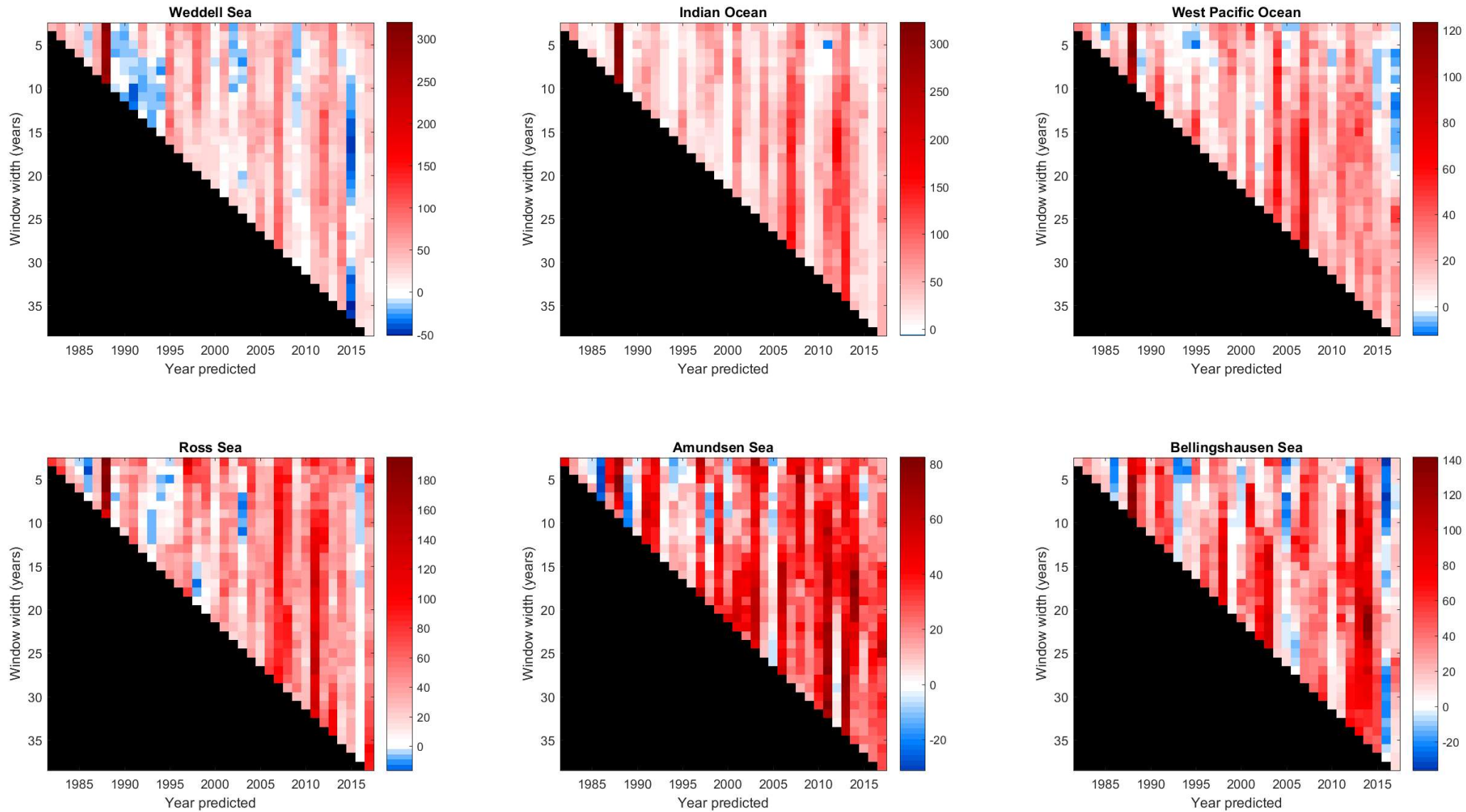
### Maximum sea ice concentration month (September)



# Sea ice concentration, southern hemisphere

## Goodness of prediction results for each region

### Minimum sea ice concentration month (February)



- General conclusions from the application of Koopman Mode decomposition and analysis techniques to sea ice data is that these techniques can reveal meaningful insights into the temporal and spatial dynamics of sea ice behavior.
- The results showed the consistency of the Koopman analysis technique as applied to sea ice concentration data from the northern and southern hemispheres, as well as the ability of the analysis to identify geographic regions undergoing long-term decreases in sea ice concentration.
- The results for the sea ice concentration and thickness data sets show the expected consistency in their dynamical behavior and also additional localized variations present in the thickness data that the bounded concentration data does not show.
- For prediction, KMD was found to produce relatively accurate predictions over several years of future sea ice concentration values for a range of multiyear data windows.
- The observation of windows over which prediction works better are of interest, since regions where shorter time prediction works better might indicate those that are experiencing more rapid change.
- Given the success of the current, purely data-driven methodology, adding a physical model and using the current methodology for data assimilation is an interesting prospect.