

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

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1. REPORT DATE 06 August 2018		2. REPORT TYPE Briefing Charts		3. DATES COVERED (From - To) 21 July 2018 - 30 August 2018	
4. TITLE AND SUBTITLE Improved Models and Understanding for Field-Reversed Configuration (FRC) Thrusters (Briefing Charts)				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Robert Martin, Eder Sousa, Robert Lilly, Michael Kapper				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER Q200	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Air Force Research Laboratory (AFMC) AFRL/RQRS 1 Ara Drive Edwards AFB, CA 93524-7013				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Research Laboratory (AFMC) AFRL/RQR 5 Pollux Drive Edwards AFB, CA 93524-7048				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) AFRL-RQ-ED-VG-2018-259	
12. DISTRIBUTION/AVAILABILITY STATEMENT Distribution Statement A: Approved for Public Release; Distribution is Unlimited. PA Clearance Number: 18476 Clearance Date: 30 July 2018.					
13. SUPPLEMENTARY NOTES For presentation at AFOSR Space Propulsion and Power Annual Review; Arlington, Virginia, USA; August 7-10, 2018. Prepared in collaboration with ERC, Inc. The U.S. Government is joint author of the work and has the right to use, modify, reproduce, release, perform, display, or disclose the work.					
14. ABSTRACT Viewgraph/Briefing Charts					
15. SUBJECT TERMS N/A					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			Robert Martin
Unclassified	Unclassified	Unclassified	SAR	79	19b. TELEPHONE NUMBER (Include area code) N/A

IMPROVED MODELS AND UNDERSTANDING FOR FIELD-REVERSED CONFIGURATION (FRC) THRUSTERS

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EDWARDS AIR FORCE BASE, CA USA



U.S. AIR FORCE

AFOSR Space Propulsion and Power Review, 2018
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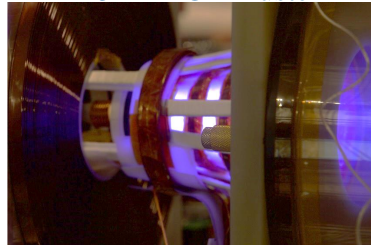
- 1 INTRODUCTION & CHALLENGES
- 2 MODEL ENHANCEMENTS
- 3 LEARNING IN PARAMETER SPACE
- 4 CONCLUSION & FUTURE DIRECTIONS



Field-Reversed Configuration:

- Concept from Fusion Energy
 - Challenge Scaling Down for Propulsion

RP3X FRC Thruster

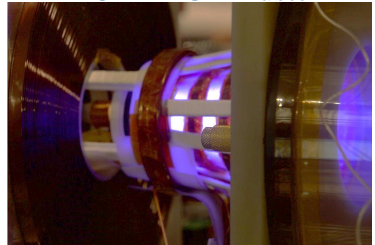




Field-Reversed Configuration:

- Concept from Fusion Energy
 - Challenge Scaling Down for Propulsion
- Electrodeless (+ Limits Erosion)

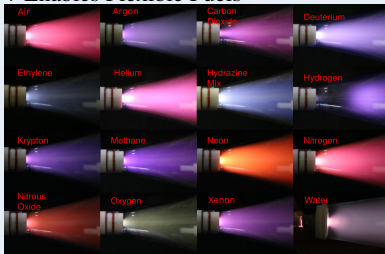
RP3X FRC Thruster





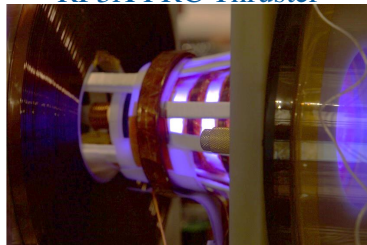
Field-Reversed Configuration:

- Concept from Fusion Energy
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- Electrodeless (+ Limits Erosion)
- Acceleration Mechanism $J \times B$
+ Enables Flexible Fuels



Pancotti, et al, "Adaptive Electric Propulsion for ISRU Missions", 20th Adv. Space Prop., 11/2014

RP3X FRC Thruster

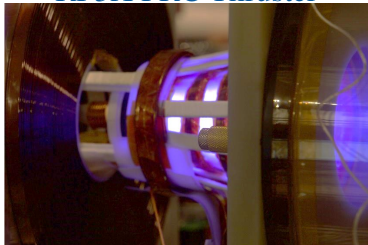




Field-Reversed Configuration:

- Concept from Fusion Energy
 - Challenge Scaling Down for Propulsion
- Electrodeless (+ Limits Erosion)
- Acceleration Mechanism $J \times B$
 - + Enables Flexible Fuels
- Pulsed Operation
 - + Tunable Thrust/ISP
 - Complex Coupled Dynamics
 - High Dimensional Parameter Space

RP3X FRC Thruster



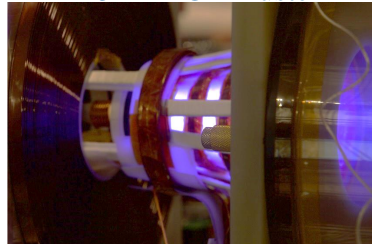


Field-Reversed Configuration:

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 - Complex Coupled Dynamics
 - High Dimensional Parameter Space
- Common Challenges/Benefits for EM
 - + MPD/PIT/PPT

Complex Devices to Design

RP3X FRC Thruster





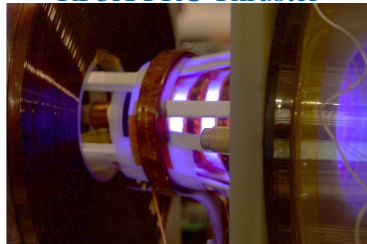
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 - Complex Coupled Dynamics
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- Common Challenges/Benefits for EM
 - + MPD/PIT/PPT
 - + Similar Tools Required

Complex Devices to Design

Pose Significant Modeling Challenge

RP3X FRC Thruster

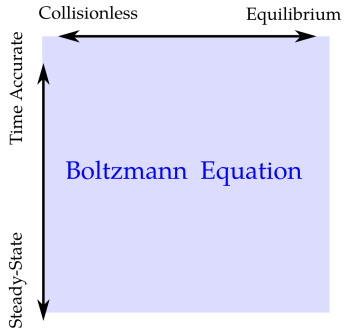




The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t} \right)_{coll}$$

Range of Validity

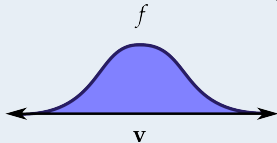




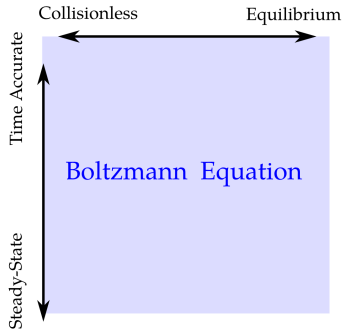
TOOL DEVELOPMENT SPANNING REGIMES

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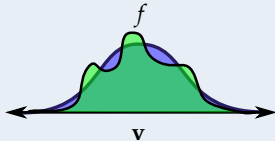
Range of Validity



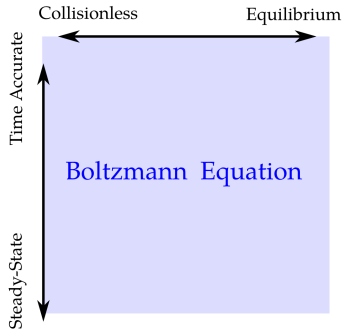


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Range of Validity





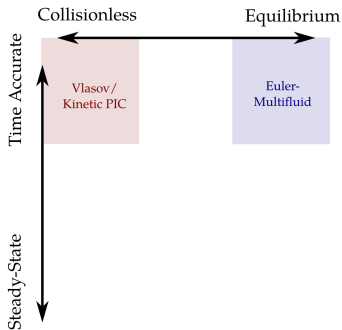
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- Collisionality Limits:

$$(\text{Vlasov}) 0 \leftarrow \left(\frac{\partial f_s}{\partial t} \right)_{coll} \rightarrow \infty (\text{Euler})$$

Range of Validity





The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

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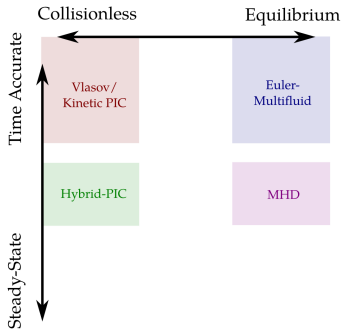
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- Ohm's Law:

$$(m_e \ll m_i) \rightarrow \frac{\partial f_e}{\partial t} \equiv 0$$

Range of Validity





The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

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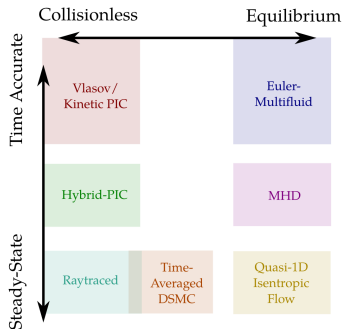
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- Steady-State Solutions:

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Range of Validity





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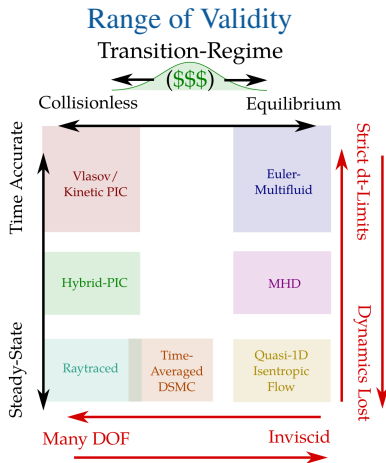
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$$(m_e \ll m_i) \rightarrow \frac{\partial f_e}{\partial t} \equiv 0$$

- Steady-State Solutions:

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Limits Exchange Accuracy for Cost





The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

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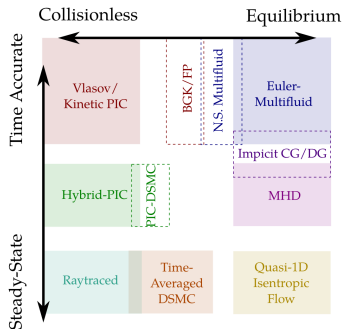
$$(m_e \ll m_i) \rightarrow \frac{\partial f_e}{\partial t} \equiv 0$$

- Steady-State Solutions:

$$\frac{\partial}{\partial t} \equiv 0$$

Limits Exchange Accuracy for Cost
Transition Incurs Costs to Limit

Range of Validity





The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

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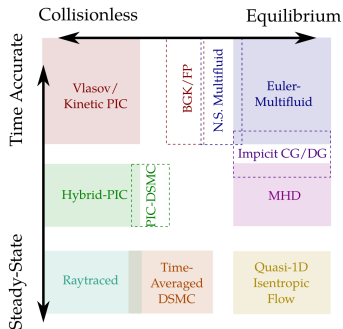
Limits Exchange Accuracy for Cost

Transition Incurs Costs to Limit

$$(\mathbf{v} = 0 \neq \mathbf{v} \rightarrow 0)$$

But Relaxes/Removes Assumptions

Range of Validity





TOOL DEVELOPMENT SPANNING REGIMES



The Boltzmann Equation for $f(\mathbf{x}, \mathbf{v}; t)$:

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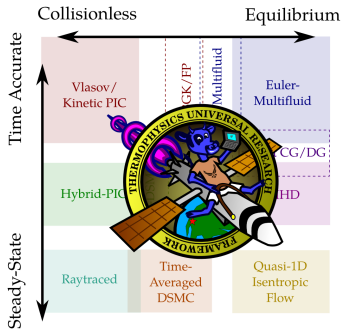
- Steady-State Solutions:

$$\frac{\partial}{\partial t} \equiv 0$$

Limits Exchange Accuracy for Cost
Transition Incurs Costs to Limit

TURF: For Exploring Efficient Methods
Minimizing Cost/Accuracy & Redundancy

Range of Validity

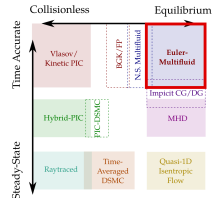
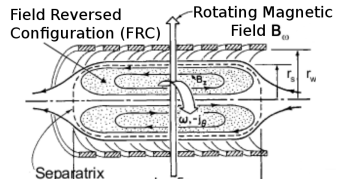




Apollo Formation Simulations:

- RMF $\rightarrow j_{\theta} \rightarrow B_z$ -Reversal

DG FRC-Formation

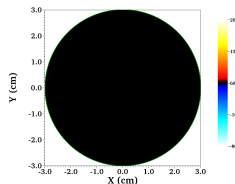
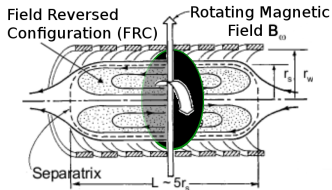




Apollo Formation Simulations:

- RMF $\rightarrow j_{\theta} \rightarrow B_z$ -Reversal
- Initially Conditions:
Fully Ionized Xe Gas
 $n_i=n_e=10^{20}/m^3$
60G Bias (Z) + 60G RMF @ 1MHz

DG FRC-Formation



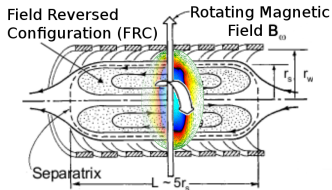
$t=0\omega$



Apollo Formation Simulations:

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60G Bias (Z) + 60G RMF @ 1MHz
- Strong Reversal B_z Reversal by 180°

DG FRC-Formation



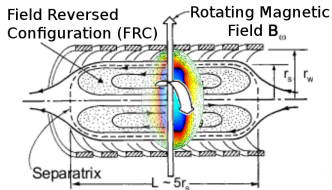
$t=0-0.5\omega$



Apollo Formation Simulations:

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60G Bias (Z) + 60G RMF @ 1MHz
- Strong Reversal B_z Reversal by 180°
- Reversal Weakens, 180° - 360°

DG FRC-Formation



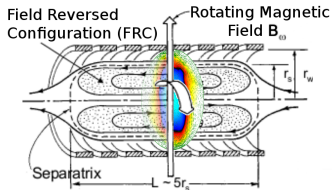
$$t=0.5-1.0\omega$$



Apollo Formation Simulations:

- RMF $\rightarrow j_\theta \rightarrow B_z$ -Reversal
- Initially Conditions:
Fully Ionized Xe Gas
 $n_i=n_e=10^{20}/m^3$
60G Bias (Z) + 60G RMF @ 1MHz
- Strong Reversal B_z Reversal by 180°
- Reversal Weakens, 180° - 360°
- Cycle Repeats
(But Lower Amplitude)

DG FRC-Formation



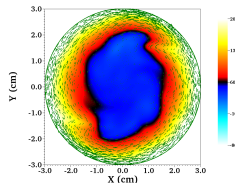
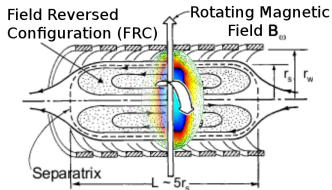
$$t=1.0-2.0\omega$$



Apollo Formation Simulations:

- RMF $\rightarrow j_{\theta} \rightarrow B_z$ -Reversal
- Initially Conditions:
Fully Ionized Xe Gas
 $n_i=n_e=10^{20}/m^3$
60G Bias (Z) + 60G RMF @ 1MHz
- Strong Reversal B_z Reversal by 180°
- Reversal Weakens, 180° - 360°
- Cycle Repeats
(But Lower Amplitude)
- Parameter Space is being Explored

DG FRC-Formation



$$t=1.0-2.0\omega$$

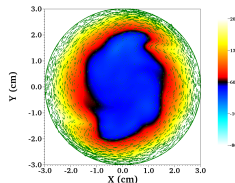
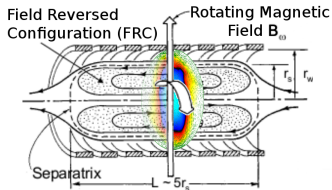


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Also Needs Experimental Validation!

DG FRC-Formation



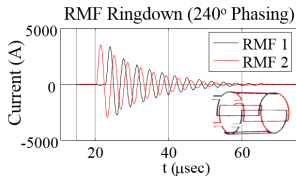
$$t=1.0-2.0\omega$$



High Fidelity Validation:

- FRCs are Dynamic Pulsed Devices

DG FRC-Formation



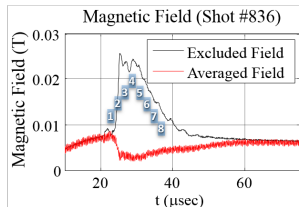
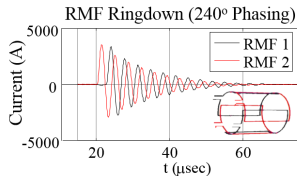
MSNW Ar FRC Formation



High Fidelity Validation:

- FRCs are Dynamic Pulsed Devices
- Ext. Probes avoid Disruption ...but Mostly Limited View

DG FRC-Formation



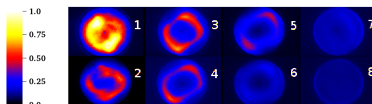
MSNW Ar FRC Formation



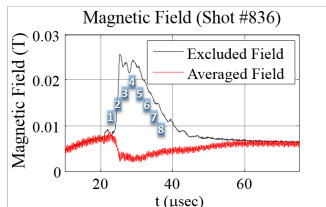
High Fidelity Validation:

- FRCs are Dynamic Pulsed Devices
- Ext. Probes avoid Disruption
...but Mostly Limited View
- Fast Camera:
Absolute Magnitudes Difficult
Relatively High Dimensional

DG FRC-Formation



(False Color Intensity - MSNW Fast Camera Studies)



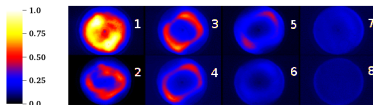
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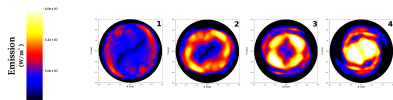
High Fidelity Validation:

- FRCs are Dynamic Pulsed Devices
- Ext. Probes avoid Disruption ...but Mostly Limited View
- Fast Camera:
Absolute Magnitudes Difficult
Relatively High Dimensional
- Emission from IonMix (n_e, T_e) Table
- Plotted at Equivalent ω

DG FRC-Formation



(False Color Intensity - MSNW Fast Camera Studies)



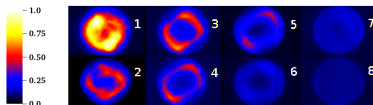
(Apollo Multifluid Simulation Emission Intensity)



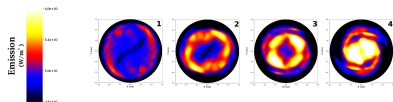
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- Plotted at Equivalent ω
- Similar Structures in 2 & 3
Despite Different Conditions

DG FRC-Formation



(False Color Intensity - MSNW Fast Camera Studies)



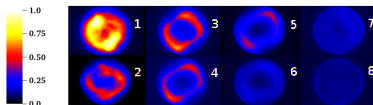
(Apollo Multifluid Simulation Emission Intensity)



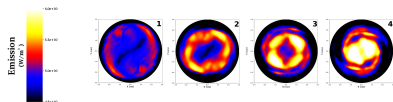
High Fidelity Validation:

- FRCs are Dynamic Pulsed Devices
- Ext. Probes avoid Disruption ...but Mostly Limited View
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Absolute Magnitudes Difficult
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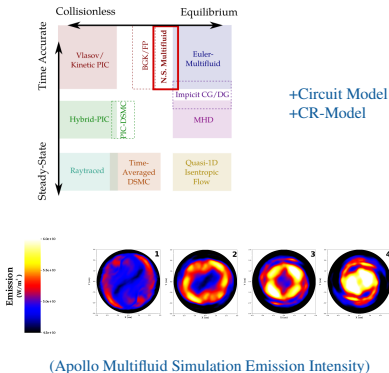


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Model Needs Additional Physics

DG FRC-Formation





AFRL Model Development/Project Synergies:

Model	Need	Platform	Project	Progress	Demo
Circuit	BC/Energy Cons.	TURF	1 & 2	Runtime Arbitrary ODE B.C.s	Lorenz DSMC Inflow
Radiative	Physics/Energy Cons.	R&D	3	Full Level-Grouped Ar Model	Abrantes PhD (UCLA 2018)
Multi-Fluid	Physics/Stability	Apollo	4	N.S./Braginskii	Poiselle/Taylor Couette
Multi-Fluid	Physics/Stability	TURF/R&D	4	Hybrid Entropy/Energy Cons.	Einfeldt/Step/Cylinder/Jet
Kinetic	Charge Separation	TURF	1 & 4	Boltzmann- n_e vs. $\langle n_e \rangle$	Quasi-1D HET

Projects:

1. In-House AFRL 6.2 Research
2. AFOSR Comp Math Lab-Task (New), PO: Fahroo
3. AFOSR Plasma/Electro-energetics Lab-Task, PO: Marshall
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N.S. MULTIFLUID DEFINITIONS

$$\partial_t \mathbf{Q} + \nabla \cdot \mathbf{F}^{Hyp} + \nabla \cdot \mathbf{F}^{Para} = \mathbf{S}^{EM} + \mathbf{S}^{ie}$$

\mathbf{Q}	\mathbf{F}^{Hyp}	\mathbf{F}^{Para}	\mathbf{S}^{EM}	\mathbf{S}^{ie}
$\left[\begin{array}{c} \rho_e \\ \rho_e \mathbf{u}_e \\ \mathcal{E}_e \\ \rho_i \\ \rho_i \mathbf{u}_i \\ \mathcal{E}_i \end{array} \right]$	$\left[\begin{array}{c} \rho_e \mathbf{u}_e \\ \rho_e \mathbf{u}_e \mathbf{u}_e + p_e \mathbf{I} \\ \mathbf{u}_e \cdot (\mathcal{E}_e + p_e) \mathbf{I} \\ \rho_i \mathbf{u}_i \\ \rho_i \mathbf{u}_i \mathbf{u}_i + p_i \mathbf{I} \\ \mathbf{u}_i \cdot (\mathcal{E}_i + p_i) \mathbf{I} \end{array} \right]$	$\left[\begin{array}{c} 0 \\ \mathbf{\Pi}_e \\ \mathbf{u}_e \cdot \mathbf{\Pi}_e + \mathbf{q}_e \\ 0 \\ \mathbf{\Pi}_i \\ \mathbf{u}_i \cdot \mathbf{\Pi}_i + \mathbf{q}_i \end{array} \right]$	$\left[\begin{array}{c} 0 \\ q_e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) \\ q_e n_e \mathbf{u}_e \mathbf{E} \\ 0 \\ q_i n_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) \\ q_i n_i \mathbf{u}_i \mathbf{E} \end{array} \right]$	$\left[\begin{array}{c} 0 \\ -\mathbf{R}_{ie} \\ \mathbf{u}_e \cdot \mathbf{R}_{ie} + \mathbf{Q}_{ie}^e \\ 0 \\ -\mathbf{R}_{ei} \\ \mathbf{u}_i \cdot \mathbf{R}_{ei} + \mathbf{Q}_{ei}^i \end{array} \right]$

The EOS used here: $p = nk_B T$. Energy $\mathcal{E} = \frac{p}{\gamma-1} + \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u}$.



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Progress:

Classic (Newtonian) and Braginskii Formulations Implemented



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ρ_e	$\rho_e \mathbf{u}_e$	0	0	0
$\rho_e \mathbf{u}_e$	$\rho_e \mathbf{u}_e \mathbf{u}_e + p_e \mathbf{I}$	$\mathbf{\Pi}_e$	$q_e n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$	$-\mathbf{R}_{ie}$
\mathcal{E}_e	$\mathbf{u}_e \cdot (\mathcal{E}_e + p_e) \mathbf{I}$	$\mathbf{u}_e \cdot \mathbf{\Pi}_e + \mathbf{q}_e$	$q_e n_e \mathbf{u}_e \mathbf{E}$	$\mathbf{u}_e \cdot \mathbf{R}_{ie} + \mathbf{Q}_{ie}^e$
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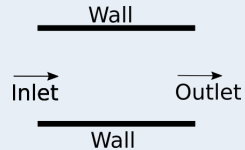
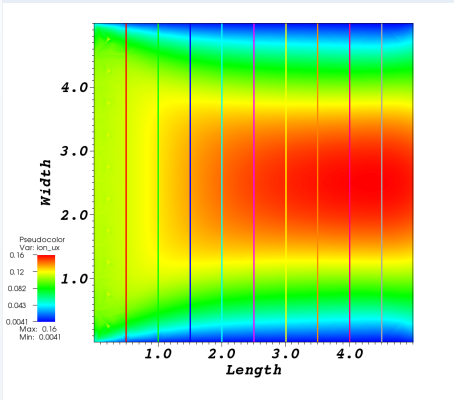
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Validation Cases:

Newtonian	Braginskii
Channel Flow: ✓	Hartman: (Pending)
Cylindrical Couette: ✓	Magneto-Rotational Instability: (Pending)



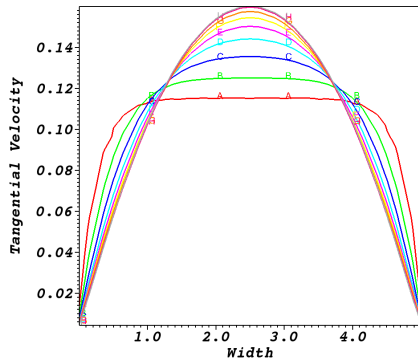
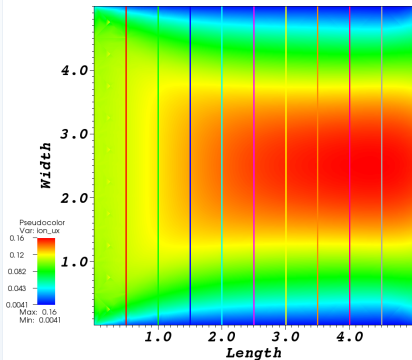
CHANNEL FLOW RESULTS



- A solid, no slip, boundary is enforced at the walls
- The input boundary uses a fixed Dirichlet condition. The flow is low subsonic (0.10 Mach).
- The exit using a von Neumann boundary condition on all variables except for pressure, which is set to the inlet condition.
- The chosen Reynolds number is approximately 30.



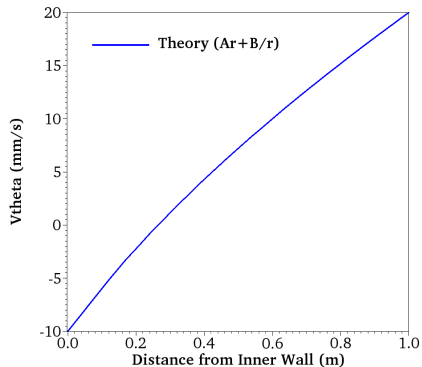
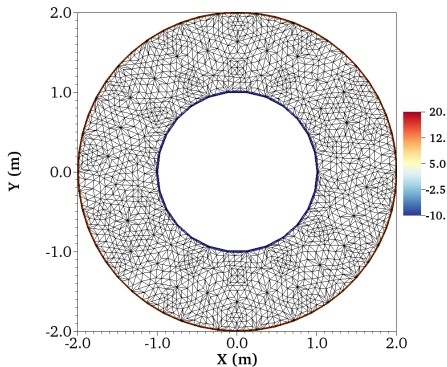
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TAYLOR COUETTE RESULTS



- Counter rotating solid, no slip, boundary is enforced at the inner/outer walls
- Reynolds number independent solution is rapidly established
- Good Agreement with Analytic Solution Attained



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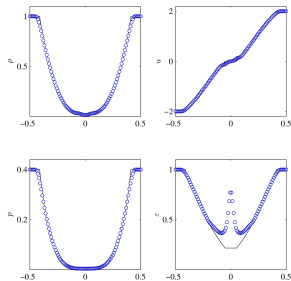
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Entropy Formulation:

- Strong Rarefactions often break Fluid Codes (i.e. Negative Pressures/Artificial Heating)
- Low- β Numerical Instabilities in Apollo (Low- T_e)

Einfeldt 1-2-3 : Energy



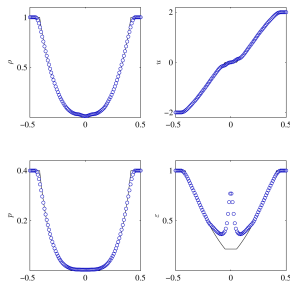
Solution to Einfeldt's 1-2-3 problem as obtained with conservation of energy. (Kapper PhD, OSU, '09)



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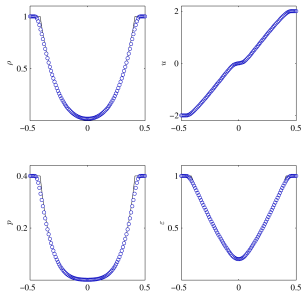
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Einfeldt 1-2-3 : Entropy



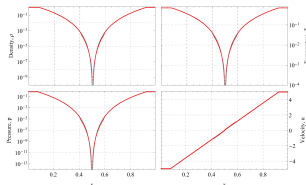
Solution to Einfeldt's 1-2-3 problem as obtained with conservation of entropy. (Kapper PhD, OSU, '09)



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Einfeldt Vacuum Limit : Entropy



Solution to Einfeldt's Vacuum Limit (Mach 5, $\gamma=7/5$) problem as obtained with conservation of entropy and new linearization. (Kapper 18)

$$T/T_0=10^{-4}$$

$$\rho/\rho_0=10^{-9}$$

$$P/P_0=10^{-13}$$



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- Apollo $Q \rightarrow$ Electron Entropy Cons. (Sousa, TBD)

$$\begin{bmatrix} Q \\ \rho_e \\ \rho_e \mathbf{u}_e \\ \mathcal{E}_e \\ \rho_i \\ \rho_i \mathbf{u}_i \\ \mathcal{E}_i \end{bmatrix} \rightarrow \begin{bmatrix} Q \\ \rho_e \\ \rho_e \mathbf{u}_e \\ \rho_e s_e \\ \rho_i \\ \rho_i \mathbf{u}_i \\ \mathcal{E}_i \end{bmatrix}$$

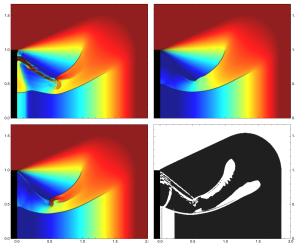
$$s_e = P_e / \rho_e^\gamma$$



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Linde-Roe Corner Expansion



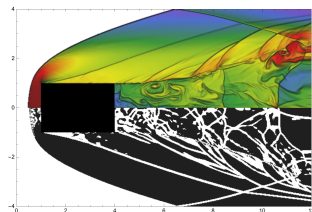
Top Left: Energy Conservation
Top Right: Entropy Conservation
Bottom Left: Hybrid Solution
Bottom Right: Hybrid Compression Flag
(Kapper '18)



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Mach 10.2 Cylinder



Plot of $\log T$ plus simulated Schlieren (top) and Hybrid Switch (bottom)

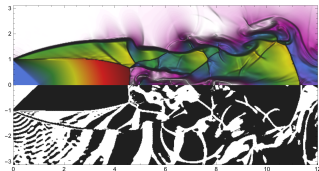
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Mach 1.5 Underexpanded Jet



Plot of Mach plus simulated Schlieren (top) and Hybrid Switch (bottom)

(Kapper '18)



Complex Design Space for Pulsed EM:

- Device Geometry: Length/Diameter/Pitch Angle/etc.
- Gas Flow: Mass Flow/Injection/Mixture/Rate/Density/Pre-Ionization
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15+ Dimensional Design Space...
Finding 'Good' Designs like a Needle in a Haystack!



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THEORY: RMF PENETRATION

RMF Characterized by Dimensionless Parameters:

- $\lambda = R/\delta$, where $\delta = (2\eta/\mu_o\omega)^{1/2}$ is the classical skin depth
- $\gamma = \omega_{ce}/\nu_{ei}$, where $\nu_{ei} = \eta(ne^2/m_e)$ is the electron-ion collision frequency

These parameter can be expressed in terms of RMF intensity, frequency and the plasma resistivity:

$$\bullet \lambda = R \left(\frac{\mu_o\omega}{2\eta} \right)^{1/2} \quad \gamma = \frac{1}{e} \left(\frac{B_\omega}{n\eta} \right)$$

Theory: Hugrass (Aust. J. Phys. 38, 157 1985)

Semi-Emperical: Milroy (PoP 6, 2771 1999)

(RMF penetration in a fixed ion plasma column)

Question: Does this Relationships hold in MFPM?

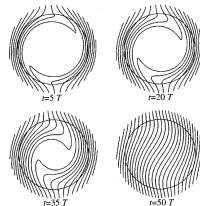


FIG. 1. Evolution of magnetic field lines as the RMF penetrates a plasma column.

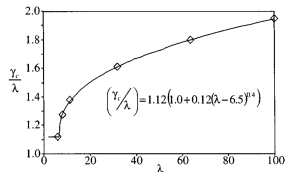


FIG. 4. Critical value of γ vs λ .

Milroy, PoP 6, 2771 (1999)



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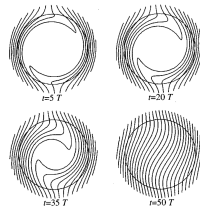


FIG. 1. Evolution of magnetic field lines as the RMF penetrates a plasma column.

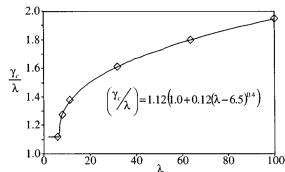


FIG. 4. Critical value of γ vs λ .

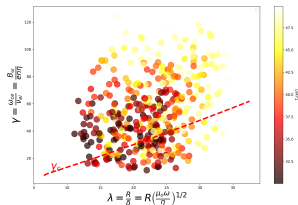
Milroy, PoP 6, 2771 (1999)



Machine Learning on MFPM Data:

- Randomly Explored (ω , B_ω , T_e) with MFPM
- Points Plotted w.r.t. (λ , γ) from Theory

MFPM Simulated Dataset



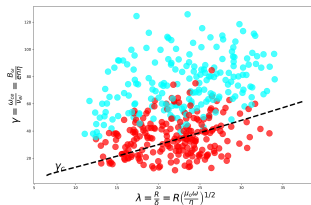
- Density, $n = 1 \times 10^{19} m^{-3}$
- $2\pi \times 10^5 Hz < \omega < 10\pi \times 10^5 Hz$
- $5G < B_\omega < 30G$
- Electron temperature, $30eV < T_e < 50eV$



Machine Learning on MFPM Data:

- Randomly Explored (ω , B_ω , T_e) with MFPM
- Points Plotted w.r.t. (λ , γ) from Theory
- Theory: Necessary but Not Sufficient

Observed B_z -Reversal



Confusion Matrix

-	Pred No	Pred Yes
Actual No	23	21
Actual Yes	0	36

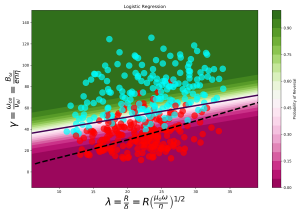


LEARNING: RMF PENETRATION

Machine Learning on MFPM Data:

- Randomly Explored (ω, B_ω, T_e) with MFPM
- Points Plotted w.r.t. (λ, γ) from Theory
- Theory: Necessary but Not Sufficient
- Classical Classifiers Explored
- Fewer False Positives & Higher γ_c

Predicted B_z -Reversal Logistic Regression



Confusion Matrix

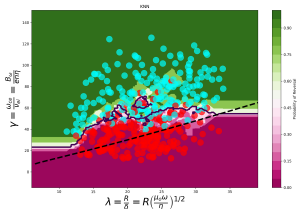
	Pred No	Pred Yes
Actual No	39	5
Actual Yes	4	32



Machine Learning on MFPM Data:

- Randomly Explored (ω, B_ω, T_e) with MFPM
- Points Plotted w.r.t. (λ, γ) from Theory
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- Classical Classifiers Explored
- Fewer False Positives & Higher γ_c
- Balancing Complexity vs. Accuracy?

Predicted B_z -Reversal K-Nearest Neighbors



Confusion Matrix

	Pred No	Pred Yes
Actual No	38	6
Actual Yes	5	31

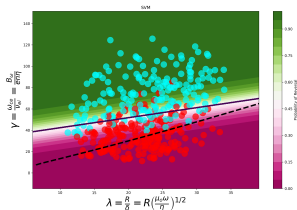


LEARNING: RMF PENETRATION

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Predicted B_z -Reversal Support Vector Machines



Confusion Matrix

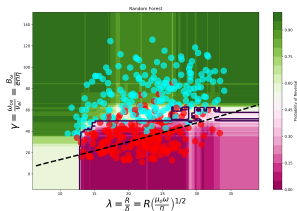
-	Pred No	Pred Yes
Actual No	39	5
Actual Yes	4	32



Machine Learning on MFPM Data:

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- Balancing Complexity vs. Accuracy?

Predicted B_z -Reversal Random Forest



Confusion Matrix

-	Pred No	Pred Yes
Actual No	37	7
Actual Yes	3	33

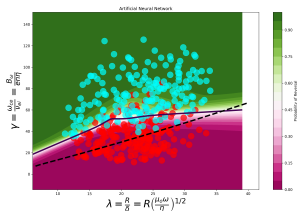


LEARNING: RMF PENETRATION

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- Balancing Complexity vs. Accuracy?
- Artificial Neural Network (ANN) with Stochastic Gradient Descent
- Better Classification w/o Extreme Complexity

Predicted B_z -Reversal ANN



Confusion Matrix

-	Pred No	Pred Yes
Actual No	39	5
Actual Yes	2	34



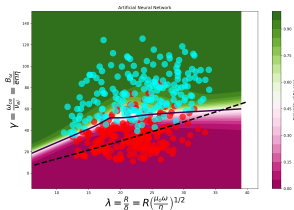
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Open Questions:

- Classification Space (λ , γ) Right for MFPM?
- Relevant Dimensions beyond 2D?
- Data Requirements in 3+D Classification?
- Do Experiments Exhibit Similar Trends?

Predicted B_z -Reversal ANN



Confusion Matrix

	Pred No	Pred Yes
Actual No	39	5
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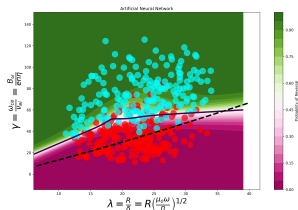
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ACTIVE SUBSPACE FOR REVERSAL?

Curse of Dimensionality:

- Optimization Cost Exponential w/ Dim





ACTIVE SUBSPACE FOR REVERSAL?

Curse of Dimensionality:

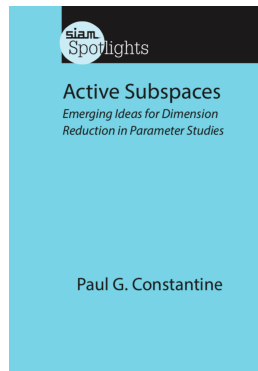
- Optimization Cost Exponential w/ Dim
- Not all Dimensions Created Equal!





Curse of Dimensionality:

- Optimization Cost Exponential w/ Dim
- Not all Dimensions Created Equal!





Curse of Dimensionality:

- Optimization Cost Exponential w/ Dim
- Not all Dimensions Created Equal!
- First Step: Identify Dominant Dimensions



ACTIVE SUBSPACES

GOAL

Make intractable high-dimensional parameter studies tractable by discovering and exploiting low-dimensional structure.

DEFINE

First $n < m$ eigenvectors of $\int \nabla f \nabla f^T \rho dx$

DISCOVER

First $n < m$ eigenvectors of $\frac{1}{N} \sum_{i=1}^N \nabla f_i \nabla f_i^T$

APPROXIMATION

$$f(x) \approx g(W^T x)$$

INTEGRATION

$$\int f(x) \rho dx$$

OPTIMIZATION

minimize $f(x)$



<http://activesubspaces.org>

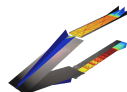
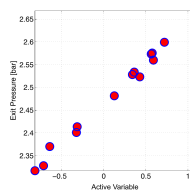


Curse of Dimensionality:

- Optimization Cost Exponential w/ Dim
- Not all Dimensions Created Equal!
- First Step: Identify Dominant Dimensions
- Often Works in Complex Looking Problems (Dominant Low-D Structure is Common)



Active subspaces helped us optimize an expensive scramjet model.



- Multiphysics model of hypersonic scramjet
- 7 input parameters
- No gradients
- Noisy function evaluations
- 2 hours per evaluation

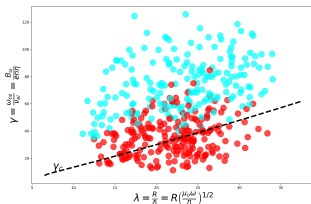
DIMENSION REDUCTION: 7 to 1

<http://activesubspaces.org>



Curse of Dimensionality:

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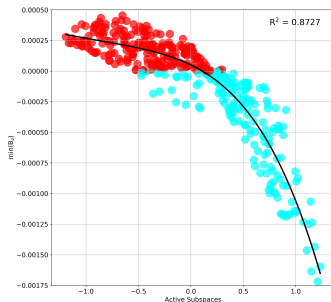
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Curse of Dimensionality:

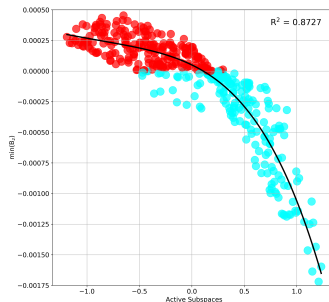
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- Active Subspace for $\text{Min}(B_z)$ in (ω, B_ω, T_e)





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 - Result Model Artifact?
 - Exp. has Many More Dimensions



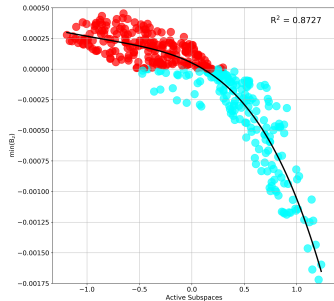


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EP Exp Produce Data Faster than Simulation

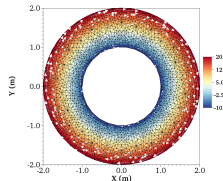
Active Subspaces in the Experimental Loop?





AFRL Pulsed EP Progress:

- **Enhanced Models**
 - Viscous Multi-Fluid
 - Hybrid Entropy/Energy Conservation
- **Improved Understanding**
 - Classifiers & Machine Learning
 - Reduced Dimensionality



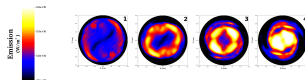
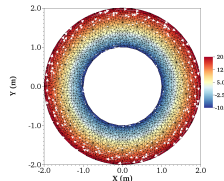


AFRL Pulsed EP Progress:

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Near Term Goals:

- **Integrate Models**
 - Porting DG MFPM to TURF
 - Verify Braginskii Implementation
 - Circuit BCs
 - Radiation Losses
- **Re-Engage with Experimental Community**





AFRL Pulsed EP Progress:

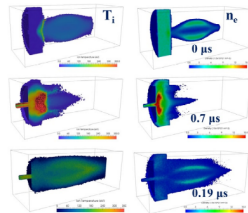
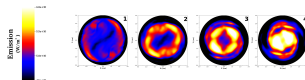
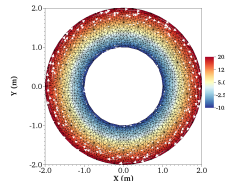
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Near Term Goals:

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 - Verify Braginskii Implementation
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Long Term Goals:

- Cross-Verify MFPM/Kinetic Assumptions
- Quantify Extrapolative Model Power
- Optimize Design of Pulsed EP in High-D
- Actively Control Pulsed EP Systems



HYPERS Simulation Results for MSX
Omelchenko, *Phys Rev E*. **92**, 023105, 2015



The Braginskii equations:

- Implementation of Parabolic Flux: $\mathbf{\Pi}_\alpha$, and \mathbf{q}_α (Viscosity & Heat Flux)
- The viscosity begins with the classic Newtonian tensor (1).

$$\overleftrightarrow{\mathbf{W}} = \nabla \mathbf{u} + \nabla \mathbf{u}^T - \left(\frac{2}{3} \nabla \cdot \mathbf{u} \right) \overleftrightarrow{\mathbf{I}} \quad (1)$$

- $\overleftrightarrow{\mathbf{W}}$ is rotated into the magnetic field coordinate frame.
- $\overleftrightarrow{\mathbf{W}}_r$ is then decomposed into parallel, perpendicular, and gyroviscous \mathscr{W}_i
- Each of \mathscr{W}_i components is multiplied by a scalar η_i .

Finally,

$$\mathbf{\Pi}_\alpha = rot_{lab} \left(\sum_{i=0}^{N=4} \eta_i \mathscr{W}_i \right) \quad (2)$$