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THE UNITED STATES AIR FORCE
MANUAL SPACE NAVIGATION EXPERIMENT ON SKYLAB
(DOD/NASA SKYLAB EXPERIMENT T-002)

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The purpose of this report is to present the results of the USAF manual space navigation experiment flown on SKYLAB in 1973. The manual space navigation problem is defined as "onboard orbit determination independent of primary system sensors, computers, and electrical power." The word "manual" implies obtaining input data with hand-held optical instruments. USAF objectives

Block 20, Abstract, continued:

were twofold: to define the quality of manual navigation data typical of low earth orbit, and to test a data processing scheme developed for the Air Force in the late sixties. Two hand-held instruments were used: a space sextant and a space stadimeter. The sextant was used to measure star-horizon angles and the stadimeter was used to measure altitude. Analysis of data obtained with these instruments indicates that no sharply defined horizon reference exists for low altitude manual space navigation. Mean horizon height is different for each instrument and must be treated as an uncertain bias in the data. Star-horizon data is roughly twice as precise as altitude data. The top of the airglow is the best reference for star-horizon measurements. Characteristics of manual space navigation data invalidate the assumptions involved in the simple data processing scheme developed for the Air Force. Current minicomputer technology makes it possible to mechanize more sophisticated techniques within reasonable mass and volume limits. Least-squares differential correction (LSDC) can be used to obtain navigation solutions accurate to within 7 km and 7 mps. Altitude data is not required. The authors recommend the development of a new hand-held sextant which incorporates a minicomputer for automatic LSDC processing. Critical to improved performance is increased measurement precision.

PREFACE

This report presents the results of the USAF manual space navigation experiment flown on SKYLAB in 1973. This experiment was part of a joint DOD/NASA effort labeled T-002. Results of the NASA portion are being reported separately.(13)[1] The USAF portion was initiated by the Navigation Division, USAF Academy, in 1972 under the sponsorship of the Space and Missile Systems Organization, Air Force Systems Command, through Research Contract XRO 72-1. The Navigation Division produced a Preliminary Report which is included as Appendix A of this report. USAF Principal Investigator responsibilities were transferred to the Department of Astronautics and Computer Science, USAF Academy, in the summer of 1974.

A major feature of this report is the presentation of a data processing technique which produces consistently good results with typical manual navigation data as the only input. Although the technique is well known, to the authors' knowledge this is the first time that it has been successfully applied in this context. This is also the first time that typical measurement errors and an uncertain earth horizon have been successfully handled in any manual navigation scheme.

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[1] Randle, R.J. Results of Skylab Experiment T-002, Manual Navigation Sightings. NASA TND-(In Process), NASA Ames Research Center, California, 1975.

The authors wish to thank Colonel Thomas C. Brandt, USAF, Commander, 1st Aerospace Control Squadron, Aerospace Defense Command, for his cooperation in allowing Captain Ferguson to collaborate in this research while a member of his command. In addition to encouraging Captain Ferguson's participation, Colonel Brandt authorized use of the computational facilities at the North American Air Defense Command Cheyenne Mountain Complex in support of this project. The authors would also like to thank Colonel John P. Wittry, USAF, Professor and Head, Department of Astronautics and Computer Science, USAF Academy, for allowing Major Walsh to pursue this research, and for authorizing use of the USAF Academy computer facility where most of the data reduction and software development was accomplished. Special thanks go to Lieutenant Colonel Stanley Powers, USAF, formerly of the Navigation Division, USAF Academy, for making the transition of Principal Investigator responsibility as smooth as possible, and for granting permission to include his Preliminary Report.

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LIST OF SYMBOLS

Symbol	Definition
A	Matrix of partial derivatives (Nx6, Nx7, or Nx8)
B	Matrix of data residuals (Nx1)
N	Number of observations
X	State correction matrix (6x1, 7x1, or 8x1)
A^T	Transpose of matrix A (6xN, 7xN, or 8xN)
$(A^TA)^{-1}$	Inverse of matrix A^TA (6x6, 7x7, or 8x8)
W	Diagonal weighting matrix (NxN)
$(A^TWA)^{-1}$	Variance-covariance matrix (6x6, 7x7, or 8x8)

I. INTRODUCTION

The Problem

The manual space navigation problem is defined as "onboard orbit determination independent of primary system sensors, computers, and electrical power."(7)[1] This definition is taken to mean the determination of any six independent parameters which specify uniquely the geometric qualities of an orbit, or trajectory, and its orientation in inertial space. The word "manual" implies obtaining input data with hand-held optical instruments.

Background

The U.S. Air Force has had an active interest in manual space navigation since the early sixties. Manned military missions seemed inevitable even in those early years of our nation's space program. It was realized that such missions would require backup navigation systems capable of performing independent of communication with the ground and primary onboard sensors, computers, and electrical power. These systems would allow a military crew to complete a high priority mission and return safely to earth despite serious system malfunctions.

No manned military space missions have been flown to date. The first such missions were to have been accomplished by the

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[1] Horrigan, R.C. and Walsh, R.C. "Manual Astronaut Navigation." Unpublished Masters Thesis, AFIT, Wright-Patterson AFB, Ohio, June 1969.

USAF Manned Orbiting Laboratory (MOL) in the early seventies. That program was cancelled in 1969. Although this was a setback for the manned USAF space program, it was not the final word. The coming Space Shuttle Program is a joint USAF/NASA venture. Over 100 military missions have been scheduled for the decade of the eighties. The backup navigational capability envisioned over a decade ago seems even more logical in light of today's costs and international political climate.

SKYLAB experiment T-002 had two basic objectives. NASA was interested in the ability of the astronaut to perform navigational measurements under the conditions of long term weightlessness. Their primary interest was in "midcourse" type measurements.(13)[1] The Air Force was interested in defining the quality of manual navigation data typical of low earth orbit, and in testing a data processing scheme developed by the Universal Technology Corporation under contract to the Air Force Avionics Laboratory.(16)[2] Two hand-held instruments were used: a space sextant and a space stadimeter. The sextant was similar to those flown previously on Gemini missions, and was used to measure the angle between two known stars, a known star and the moon's limb, and a known star and the

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[1] Randle, R.J. Results of Skylab Experiment T-002, Manual Navigation Sightings. NASA TND-(In Process), NASA Ames Research Center, California, 1975.

[2] Silva, R.M. and J.G. Mills, Analytical Development of Optimum Astronaut Procedures for Use of the Air Force Space Navigation System in the Manual Mode. Universal Technology Corporation Technical Report No. AFAL-TR-69-14, September 1969.

earth's horizon. The latter measurement type was accomplished for the Air Force. The stadimeter was used to measure altitude above the earth's surface. This measurement was done solely in support of the Air Force part of the experiment. (The Universal Technology data processing scheme uses star-horizon angles and height as input.) A complete description of the sextant and stadimeter, along with the design of the inflight experiment, a brief history of manual space navigation, and a preliminary analysis of the data obtained is contained in Appendix A, which is a Preliminary Report written by the Navigation Division, USAF Academy.

Purpose

The purpose of this paper is to finalize the work initiated by the Navigation Division. This report includes a refined analysis of the data obtained on SKYLAB and the presentation of a new approach to obtaining a navigation solution from this data. The Universal Technology scheme was unsuccessful under operational conditions. The authors point out the factors which led to this failure and, at the same time, suggest a path for future development in this area.

Scope

This report is not intended to be the last word on the subject of manual space navigation. The authors believe that they have found a solution to the manual space navigation problem, but it is by no means the only solution. Only low altitude, low eccentricity SKYLAB type orbits have been thoroughly investigated. Higher altitudes and eccentricities have only been briefly touched upon. No attempt has been made to construct an operational system based upon the findings of this research. The authors believe, however, that this system is within the realm of existing technology and that all that stands between this report and an operational system is adequate funding.

Organization

The second section of this report deals with the results of analyses done on data obtained with the sextant and stadimeter. These analyses were done for the Preliminary Report, but have been re-accomplished using precise NASA ephemeris data which became available only recently. A comparison is made between the preliminary results and the results obtained with this more precise information. Characteristics of the earth horizon are also discussed in this section. The third section of this report deals with a data processing scheme found successful with typical manual navigation data. The algorithm is presented, along with results obtained in processing data under various conditions. Section four is a list of conclusions drawn as a result of this research; section five presents the authors' recommendations.

II. RESULTS OF DATA ANALYSIS

Hand-held Space Sextant

Table 1 presents the results of standard statistical analysis applied to the star-horizon data acquired on SKYLAB II on the night side of the orbit. (To be consistent with the Preliminary Report, the second and third manned SKYLAB missions will be referred to as SKYLAB II and SKYLAB III. In some documents, these are referred to as SKYLAB III and SKYLAB IV.) The only difference between this analysis and that done for the Preliminary Report is the use of recently received NASA precision ephemeris data. Comparison of Table 1 and the preliminary analysis shows little change in the overall mean of the computed minus observed angles for each of the three horizon references used. It should be noted that these mean angles are, in effect, biases in the sextant data caused by sighting to horizons above the true surface of the earth. (The computed angles are referenced to the earth's surface. See Figure 2, Appendix A.) Smaller standard deviations from the means of the individual sighting periods result when the precision ephemeris is used. The overall standard deviations, not calculated in the Preliminary Report, reflect the precision achieved for each set of sighting periods and indicate the relative precision of the measurements from one horizon to another. These standard deviations are meaningful in that they give

TABLE 1

RESULTS OF ANALYSIS - SEXTANT STAR TO EARTH HORIZON OBSERVATIONS

1 TYPE HORIZON	NUMBER of SIGHTING PERIODS	TOTAL MEASUREMENTS	2 MEAN ANGLE COMP-OBS (DEG)	3 STD DEV RANGE (DEG)	4 OVERALL STD DEV (DEG)
E	8	129	.816	.204 .099	.211
G	5	50	.226	.106 .049	.113
L	3	43	2.40	.174 .025	.120

1. TYPE HORIZON: E - Base of Airglow Horizon
   G - Apparent Earth Horizon
   L - Top of Airglow Horizon
2. OVERALL MEAN OF THE COMPUTED MINUS OBSERVED ANGLES FOR THIS HORIZON
3. RANGE OF STANDARD DEVIATIONS FOR ALL SIGHTING PERIODS (BASED UPON INDIVIDUAL SIGHTING PERIOD MEANS)
4. OVERALL STANDARD DEVIATION FOR THIS HORIZON (BASED UPON OVERALL MEAN FOR THIS HORIZON)

quantitative information on how well the sextant/astronaut combination performs over a long time span. This information can be used to estimate the error in subsequent navigation solutions. (A discussion of this will follow in the next section.)

Hand-held Space Stadimeter

Analysis of altitude data acquired on SKYLAB II results in a mean height error (height measured minus height above an oblate earth) of -7.76nm (-14.37km) with a standard deviation of 4.74nm (8.78km) for the 73 measurements taken on the day side of the orbit. The 15 measurements taken on the night side display a mean height error of -53.83nm (-99.69km) with a standard deviation of 4.99nm (9.24km). Eighty-six stadimeter sightings were made on SKYLAB III. All but 31 of these were discounted due to the presence of a plexiglass shield which should have been removed from the observation window. The 31 day side measurements made with the shield removed display a mean height error of -12.50nm (-23.15km) with a standard deviation of 4.06nm (7.52km). Again, the mean measurement errors reflect bias in the data caused by sighting to horizon levels above the true surface of the earth. (Negative mean height errors result from adopting the "measured minus computed" convention.)

Horizon Characteristics

Data obtained with the sextant and stadimeter can be used to derive the characteristics, i.e., the mean height and variability, of the observed horizon. This derivation is accomplished directly in the case of the stadimetric observations. Mean height error and standard deviation can be thought of as the mean horizon sighted and its variability. For example, on SKYLAB II the 73 day side measurements imply a mean horizon height of 14.37 km with a variability (standard deviation) of 8.78km. It must be emphasized that these figures apply only to the stadimeter and the astronaut performing the measurement task. Different instrument/astronaut combinations produce different results. This fact can be seen by comparing the stadimetric results obtained on SKYLAB II with those obtained on SKYLAB III. (Major Lousma did all the T-002 sighting on SKYLAB II, and Lieutenant Colonel Pogue did all the sighting on SKYLAB III.) It can be concluded that although the variability of the horizon sighted with a given instrument (i.e., sextant or stadimeter) might be relatively stable due to similar astronaut ability and training (e.g., 8.78km versus 7.52km), the mean height can vary considerably due to personal bias and horizon lighting conditions (14.37km versus 23.15km). This is an important point to be considered in the design of a data processing scheme.

TABLE 2

RESULTS OF ANALYSIS - HORIZON CHARACTERISTICS DERIVED FROM SEXTANT DATA

TYPE HORIZON	1 MEAN HORIZON SIGHTED (KM)	2 STANDARD DEVIATION RANGE (KM)	3 OVERALL STANDARD DEVIATION (KM)
E	33.53	8.26 4.01	8.43
G	9.35	4.37 2.04	4.67
L	95.32	6.51 0.89	4.54

1. MEAN HORIZON SIGHTED: MEAN ALTITUDE ABOVE EARTH
2. RANGE OF STANDARD DEVIATIONS FOR ALL SIGHTING PERIODS (BASED UPON INDIVIDUAL SIGHTING PERIOD MEANS)
3. OVERALL STANDARD DEVIATION FOR THIS HORIZON

Table 2 presents the results obtained in deriving horizon characteristics from the sextant data obtained on SKYLAB II. (No sextant measurements were made on SKYLAB III.) There is little change in the mean horizon heights from what was presented in the Preliminary Report. Again, the standard deviations are smaller when the precision ephemeris is used. Astronaut comments and the results of this analysis point to the top of the so-called "airglow," or bright band surrounding the earth, as the optimum reference for this kind of manual navigation measurement. The astronauts found this horizon easiest to use throughout the experiment. It should be noted that the standard deviation for this type measurement and horizon is roughly half the standard deviation obtained from analysis of the SKYLAB II stadimetric data (4.54km versus 8.78km), and that the mean horizon height can vary considerably between the sextant and stadimeter.

Conclusions

Several things may be concluded from this analysis:

1. There is no sharply defined horizon reference for manual space navigation sightings made in low altitude earth orbit.
2. Generally, neither mean horizon height nor variability are known in an absolute sense. Mean horizon height is different for each instrument and must be treated as an uncertain

bias in the data obtained with each instrument, while the standard deviation, or variability, is more certain, but still a function of the training and skill of the user.

3. Sextant star-horizon measurements appear to be more precise than stadimeter height measurements. In fact, this analysis shows the sextant measurements to be almost twice as precise as the stadimeter measurements.
4. The top of the airglow is the best reference for star-horizon measurements. This reference implies a mean horizon height of about 95km and measurement precision on the order of .1 degree.

These conclusions must weigh heavily in the selection of a data processing algorithm.

III. DATA PROCESSING

Background

Manual space navigation has always implied autonomous, low mass, low volume systems designed to fulfill a backup role in manned spaceflight. During the 1960s, low mass and low volume meant data processing with little or no help from independent digital computers. This fact led manual space navigation researchers to discard all but the simplest orbit determination techniques due to stringent restrictions on computational capability. The emphasis was on manual solution via prepared tables and charts, not unlike air navigation.

In 1969, the Universal Technology Corporation, under contract to the Air Force Avionics Laboratory, developed a manual data processing scheme involving three stadimetric height measurements and four sextant measurements.(16)[1] This scheme was tested on SKYLAB for the first time.

The SKYLAB Operational Test

One of the objectives of T-002 was to test the manual data processing scheme developed for the Air Force. Data was taken by the SKYLAB crew to be processed by experimenters on the ground. The lack of precision in the stadimeter data was immediately apparent, however, and this

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[1] Silva, R.M. and J.G. Mills, Analytical Development of Optimum Astronaut Procedures for Use of the Air Force Space Navigation System in the Manual Mode. Universal Technology Corporation Technical Report No. AFAL-TR-69-14, September 1969.

effort was terminated since the method relies heavily upon precise height information. This scheme, like several others developed in the 1960s, suffers from inability to handle typical manual space navigation measurement errors. There was, of course, no way to foresee this problem. Not enough was known about the characteristics of the earth's horizon. T-002 has shown that manual space navigation measurements cannot be made with the precision assumed in these early schemes due to the indistinct nature of this horizon.

The SKYLAB experience forces a complete re-evaluation of manual space navigation data processing.

A New Approach

The 1970s will probably be called the decade of the "minicomputer explosion." This new technology is made to order for application in manual space navigation. Requirements for low mass and low volume no longer restrict orbit determination to the techniques of the 1960s. The manual space navigation problem may be approached with the knowledge that virtually any data processing scheme found effective can be mechanized within reasonable mass and volume limits. The problem has become one of software development, with streamlining a secondary consideration. With this fact in mind, the authors offer a solution to the data processing problem. This solution is compatible with the conclusions drawn in Section II.

Least-squares Differential Correction (LSDC)

Introduction. LSDC is the workhorse of USAF ground-based orbit determination. It is employed by the Air Force Satellite Control Facility, Air Force Systems Command, to keep track of all U.S. military satellites, and by the North American Air Defense Command Space Detection and Tracking System to fulfill its mission of cataloguing all man-made objects in earth orbit.

In applying LSDC to the manual space navigation problem, one must realize that ground-based orbit determination and manual space navigation differ in two respects. First, measurements are made from the orbiting vehicle instead of tracking sites on the surface of the earth. This may be an obvious fact, but its implications are not so obvious. This is a major advantage for manual space navigation because measurements can be made at will, subject only to the constraint of star visibility. (This constraint implies making star-horizon measurements on the night side of the orbit.) Ground-based tracking is limited by line-of-sight between the antenna and the orbiting vehicle. This is a much more severe restriction upon data acquisition. Second, data must be processed onboard the spacecraft, which implies mass and volume limitations discussed earlier. The objective of both manual space navigation and ground-based orbit determination is exactly the same: to determine the "state" of the vehicle orbit at some "epoch," or reference time. LSDC is a logical processing scheme to apply to manual space navigation because:

1. It is a statistical process which can handle random measurement errors. Extreme measurement precision is not assumed or required.
2. There is no theoretical limitation on the number of observations which may be processed. (The minimum number required is equal to the dimension of the state being estimated.) This feature permits the observer to take advantage of his position in orbit and allows the statistical process to effectively average out random measurement errors.
3. Data from sensors of different precision can be weighted to reflect this difference.
4. A single measurement type, such as the star-horizon angle, suffices.
5. Uncertain biases, such as the altitude of the horizon reference, can be estimated as part of the state determination.
6. Knowledge of the precision of the data can be used to obtain an estimate of the precision of the resulting navigation solution.

These features make LSDC a much more realistic manual space navigation processing scheme than any developed to date.

LSDC Algorithm. Least-squares differential correction is an iterative, statistical curve-fitting process. Basic to this process is use of "residuals," or differences between computed and observed measurement data, to drive the orbit elements at epoch to values which minimize these differences in a least-squares sense. LSDC may be thought of as iterative solution of a set of linearized equations in as many unknowns as the dimension of the state being estimated. If this state involves only the classical orbit elements -- or, alternately, position and velocity at epoch -- then the state dimension is six. It then takes at least six measurements to arrive at six equations in six unknowns. Although this minimum number allows solution for the orbit state, it does not take advantage of the real power of LSDC in averaging out random errors in measurement data. To take advantage of this averaging process, the system must be overdetermined. This means using more than six measurements, and hence more than six equations. There is no theoretical limit to the number of measurements which may be used in LSDC, and strictly speaking the process assumes an infinite number with randomly distributed errors. In a practical sense, however, the number of measurements used is constrained by the availability of data and the physical limitations of the processing system.

The easiest way to handle large systems of equations is through use of matrix algebra. Given N measurements, let B represent an Nx1 column matrix of residuals obtained by computing the measurement data with some initial estimate of the orbit state and then taking the algebraic differences between the observed and computed values. Then, let A be an Nx6 matrix of partial derivatives obtained by differentiating the known functional relationships between the measurements and the orbit elements and evaluating these partials with the initial state values, assuming for the moment that the state dimension is six. Finally, let X be a 6x1 column matrix of "deltas," or corrections to be applied to the orbit elements. In matrix notation, this system of N equations in six unknowns (the correction terms) becomes:

$$B = AX.$$

It is necessary to solve for the X matrix to make corrections to the initial estimate, so the A matrix must be inverted, and this inverse must pre-multiply both sides of the equation. Difficulties arise if the A matrix is not square, which occurs if more than six measurements are used. This can be handled by first taking the transpose of the A matrix, pre-multiplying both sides of the equation by A^T, and then pre-multiplying both sides by $(A^TA)^{-1}$ thusly:

$$A^TB = A^TAX$$

$$(A^TA)^{-1} A^TB = (A^TA)^{-1} A^TAX$$

$$\text{or,} \quad X = (A^TA)^{-1} A^TB \, .$$

Now, corrections are made to the initial estimate of the state, and the process is repeated until the square root of the mean of the squared residuals (RMS) is minimized. Data of different quality (reflecting differences in sensor precision) can be weighted by inserting an NxN diagonal matrix involving known measurement standard deviations. If this matrix is labeled W:

$$X = (A^TWA)^{-1} A^TWB \, .$$

Matrix $(A^TWA)^{-1}$ is significant in that it is a statistical variance-covariance matrix. The terms on the main diagonal are the error variances expected in the output elements. (The square root of the variance is the standard deviation.) The authors' LSDC program uses the following classical orbit elements: longitude of the ascending node, argument of perigee, inclination, mean anomaly at epoch, mean motion, and eccentricity. The variances expected in these elements are read in the above order from upper left to lower right along the main diagonal. Units are radians-squared for all angles, and (revs/day)-squared for the mean motion. Input measurement standard deviations are obtained from the data analysis presented in the previous section.

Although data can be weighted to reflect differences in sensor precision, LSDC does not require more than one measurement type. Star-horizon angles are a function of all six orbit elements, so this measurement type suffices, at least mathematically, in the LSDC process. This is not true of altitude.

An integral part of the LSDC algorithm is the force model used to describe the dynamics of the situation. Low altitude earth orbits are perturbed from restricted two-body motion by the irregular shape of the geopotential field and atmospheric drag. In the SKYLAB orbit, these perturbations are too significant to ignore, even over relatively short time spans. For this reason, the J2 and J3 harmonics and drag are included in an analytic formulation adapted from NORAD Space Defense Center software.(6)[1] The ephemeris generator remains Keplerian except for this modification.

The LSDC algorithm developed for this project occupies approximately 5,000 words of core memory. An operational version would probably occupy about half of this, or 2,500 words. (Many of the convenience items included for ease of analysis could be removed for streamlining.)

For more information on LSDC, interested readers are directed to references 1, 2, 3, 5, 9, and 10, listed on page 105.

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[1] Hilton, C.G. and J.R. Kuhlman. Mathematical Models for the Space Defense Center. Aeronutronic Publication No. U-3871, Philco-Ford Corporation, Aeronutronic Division, Newport Beach, California, 1966.

Horizon Estimation. Uncertain horizon height biases can be estimated along with the classical element set. This estimation is possible because star-horizon and altitude measurements are functions of the height of the horizon reference used. These two measurement types involve different height biases, so two biases must be estimated. The authors' LSDC program is designed to handle single bias estimation when sextant data alone is used, or dual bias estimation if both sextant and stadimeter data is used. The seventh variance on the main diagonal of the variance-covariance matrix relates to the sextant horizon bias, and the eighth relates to the stadimeter horizon bias. Units for both are kilometers-squared. Obviously, if one or both of these biases is to be estimated, an initial estimate must be input along with initial values for the other six elements of the state. For star-horizon data, this initial estimate may be obtained from knowledge of the horizon level selected, e.g., if the top of the airglow is used, a good initial estimate for the horizon bias is 95 kilometers. Initial estimates for stadimeter height biases may be made based upon lighting conditions (i.e., day side or night side of the orbit) and known personal biases in using this instrument.

The Effects of Star Number, Geometry, and Measurement Sequencing. The stars selected for sextant star-horizon measurements were Dabih, Fomalhaut, Achernar, and Diphda. Each sighting period was conducted with one of these four stars. There were no sighting periods involving multiple star measurements. Stadimeter sightings were made primarily on the day side of the orbit, whereas sextant sightings were made on the night side to facilitate star acquisition. The measurement schedule conformed to the requirements of the Universal Technology data processing scheme mentioned previously.

The authors' first attempts at data processing using LSDC were made with sextant data involving a single star. These attempts were unsuccessful. Single star sextant data caused the A matrix to be singular. Singular matrices have no inverse, so it is impossible to carry out the required solution for the X matrix. Physically, this indicates an indeterminate condition caused by the fact that an infinite number of orbits can yield the observed data. This singularity problem led the authors to combine data from sighting periods involving two stars. Unfortunately, suitable sighting periods with the preferred L type horizon (top of the airglow) were not available. Instead, data sets had to be constructed using sighting periods involving the less distinct E type horizon. (Sextant standard deviations for the L and E type horizons are approximately .1 degrees and .2 degrees, respectively.) The inclusion of a second star solved the singularity problem. However, output position and velocity at epoch were poor in quality when compared with the NASA precision ephemeris. (See Appendix B, Run #1)

It was noted that the four stars used in T-002 are fairly close to each other on the celestial sphere. (This was required due to constraints on vehicle maneuvering and the relatively narrow field of view afforded by the observation window.) Since using one star results in a singularity, two closely spaced stars can cause this condition to be approached, especially if the observations are noisy. It was decided to attempt to improve the quality of the navigation solution by generating data using the star Betelgeuse, which is well separated from the original four, and substituting this data at selected times in the observation span. The original noise distribution was unchanged. Drastic improvement was seen in the output position and velocity estimates. Further experimentation led to the following conclusions:

1. Three stars, well separated in space, are required. (Two stars result in significant degradation in the output, and four stars make no significant improvement.)
2. The measurements should be sequenced in a cyclic fashion, e.g., star 1, star 2, star 3, then repeat. (Non-cyclic measurement schedules degrade the output.)

Run #2 in Appendix B demonstrates the effectiveness of improving relative star geometry and using a cyclic measurement schedule. The stars used are Fomalhaut, Dabih, and Betelgeuse. The noise distributions in Run #2 and Run #1 are identical.

Position error is reduced from 304 km to 6 km, and velocity error is reduced from 444 mps to 14 mps. The same starting elements are used in both runs. Run #1 takes eight iterations, whereas Run #2 takes four.

Further experimentation was done with simulated L type horizon data, since this horizon provides a better reference for sextant measurements. A random number generator was used to produce star-horizon data displaying the same characteristics as the data obtained on SKYLAB II. Average output position and velocity errors within 10 km and 10 mps were achieved for 20 minute observation spans involving 15 to 20 measurements spaced roughly one minute apart. More will be said about this experimentation later in this section.

Processing Sextant and Stadimeter Data. Since no stadimeter measurements were made in conjunction with sextant measurements, it was necessary to generate simulated height data for times within the sextant data spans. (Perhaps it should be mentioned that all simulated data were based upon precision ephemeris information. The modified two-body ephemeris generator was not used for this purpose.) Again, the simulated data displayed the same characteristics as data obtained in flight. Typical height data were substituted for star-horizon data at selected times in the observation span. In Run #3, Appendix B, it is seen that this substitution results in position and velocity errors larger than those seen in Run #2 (33 km and 24 mps versus

6 km and 14 mps). This degradation indicates a disadvantage in using height data in lieu of sextant data. Exact height data was substituted with similar results. This outcome can be explained by examining the elements of the A matrix. (This matrix is displayed to the right of the final residual column at the end of each run.) The first six numbers in each row are the partial derivatives of the observation with respect to the orbit elements. The order is the same as that stated for the variance-covariance matrix. The seventh number is the partial derivative of the observation with respect to the sextant horizon bias, and the eighth number is the partial derivative of the observation with respect to the stadimeter horizon bias. In each of the first six columns, the partials involving height measurements are significantly smaller than the partials involving star-horizon measurements. Since the partial derivatives are really sensitivity coefficients, this indicates a weaker relationship between height and the orbit state than between the star-horizon angle and the orbit state. At first it was thought that this situation might be unique to near-circular orbits, so higher eccentricities were investigated. Space Shuttle missions involving manned Tug excursions to synchronous altitude will require transfer ellipses with eccentricities in the neighborhood of .7. Trajectories of this type were generated, and the same set of partial derivatives was examined. Although the height partials were slightly larger

than those seen previously, they were still consistently smaller than the star-horizon partials. The conclusion to be drawn is that height data should not be used if star-horizon data is available. If height data is desired for some reason on a high eccentricity transfer ellipse, it can be obtained with the sextant by measuring the subtended angle of the earth once an altitude of 2500 nautical miles is exceeded. (This takes about 20 minutes on a Hohmann transfer to synchronous altitude.) The hand-held space stadimeter is not necessary, or even desirable, in manual space navigation using LSDC processing.

The Effect of Measurement Time Span. As stated earlier in this section, average position and velocity errors within 10 km and 10 mps were achieved using simulated L type horizon data and 20 minute time spans involving 15 to 20 star-horizon measurements spaced approximately one minute apart. Increasing the volume of data within this time span had no significant effect on these errors. (Up to 140 observations were processed.) It was decided to increase the measurement time span with hopes of reducing the errors in output position and velocity. The requirement that the vehicle be on the dark side of the orbit restricts the measurement time span to a maximum of about 30 minutes on low altitude, low eccentricity orbits. Increasing the time span to 30 minutes resulted in a reduction of position and velocity errors by about 30 percent.

It was possible to achieve average position and velocity errors within 7 km and 7 mps with 25 to 31 star-horizon observations spaced approximately one minute apart. Run #4 in Appendix B demonstrates this improvement. This run involves 31 observations over a 30 minute time span. The stars used are Dabih, Achernar, and Betelgeuse. Output position and velocity errors are 1.9 km and 5.7 mps.

It can be concluded that by increasing the measurement time span from 20 to 30 minutes, or about 30 percent, the output position and velocity errors are reduced by about the same percentage. Further increases in the time span are difficult to justify due to the dark side restriction.

Convergence Characteristics. Since an initial estimate of the orbit state must be made to start the LSDC process, the effect of errors in this initial state must be considered. Knowledge of the orbit state may be very limited, especially if vehicle maneuvering takes place prior to the use of the manual space navigation system. With this fact in mind, the authors made several LSDC runs with gross errors in the input state estimate. The results indicated that large input errors could be tolerated with no degradation in the output. The only penalty imposed was the requirement for one or two more iterations for convergence to be achieved. Run #4 in Appendix B demonstrates the convergence characteristics of the authors' LSDC algorithm. This run involves input errors of 10 degrees in longitude of the ascending node, 9 degrees in inclination, and 10 minutes in period. The resulting position and velocity errors in the input state are 1646 km and 2119 mps. The input estimate of the horizon bias is 90 km. Convergence is achieved in five iterations. As stated previously, the output position and velocity errors are 1.9 km and 5.7 mps. The authors found that it was very difficult to force divergence. Errors in the input period in excess of 12 minutes caused the algorithm to converge on poor solutions, reflected by star-horizon residuals of the same algebraic sign, but divergence occurred only if large period errors were combined with out-of-plane errors in excess of 15 degrees.

The convergence characteristics demonstrated by the LSDC algorithm in processing star-horizon data eliminate the need for pre-processing the data to obtain an initial state estimate, even if sizable orbit changes have been made between state updates. A very rough initial estimate suffices. If this estimate is too rough, the algorithm responds by converging on a solution that produces residuals of the same algebraic sign. This is a direct indication to the astronaut-navigator that he must re-initialize the algorithm with a new initial estimate of the orbit state. The authors have found that poor solutions causing residuals of the same algebraic sign are usually good enough to serve as this new initial estimate.

IV. CONCLUSIONS

The major conclusions drawn as a result of this research are summarized as follows:

A. There is no sharply defined horizon reference for manual space navigation sightings made in low altitude earth orbit.

B. Mean horizon height is a function of lighting conditions, the instrument/astronaut combination, and, for star-horizon measurements, the horizon level selected. Mean horizon height must be treated as an uncertain bias in the data obtained with each instrument.

C. The top of the airglow is the most distinct horizon reference for sextant star-horizon measurements.

D. Stadimetric height data is roughly half as precise as sextant star-horizon data acquired with the top of the airglow as the horizon reference.

E. Simple data processing schemes developed prior to SKYLAB cannot handle typical manual navigation measurement errors.

F. Current minicomputer technology eliminates past restrictions on manual space navigation data processing techniques.

G. Least-squares differential correction (LSDC) can be employed to average out random measurement errors in typical manual space navigation data. Uncertain horizon biases can be estimated along with the six elements of the orbit state.

H. Altitude data is not required if LSDC processing is used. Thus, the stadimeter may be eliminated as a manual space navigation sensor.

I. Although no sharply defined horizon reference exists, navigation solutions involving position and velocity errors within 7 kilometers and 7 meters per second can be achieved using LSDC and star-horizon data if:

    1. The top of the airglow is consistently used as the horizon reference.
    2. Three well separated stars are selected.
    3. A cyclic measurement schedule is employed.
    4. Twenty-five to thirty measurements are made over a thirty minute time span on the dark side of the orbit.
    5. A rough initial estimate of the orbit state is available.

V. RECOMMENDATIONS

SKYLAB experiment T-002 has established the feasibility of obtaining reasonably accurate navigation solutions from sextant star-horizon data obtained in low altitude earth orbit. The key to this feasibility lies in processing the data statistically to average out measurement errors. Least-squares differential correction is recommended by the authors because of its simplicity and adaptability to the manual space navigation problem. The LSDC algorithm makes sense operationally because it requires minimal information for initialization. A manual space navigation system employing LSDC could be "cold started" with a very rough initial estimate of the orbit state. Once the orbit is defined, action can be taken to guide the spacecraft to a safe re-entry, or to continue the assigned mission. The authors see the first of these goals as the most critical in an emergency situation requiring the use of a backup navigation system. If mission continuation is required, the orbit state can be periodically updated by repeating the process of data acquisition and reduction. It may be that LSDC is not the optimum choice of algorithms in this case. Data filtering via dynamic processors, such as the Kalman filter, might be more efficient. Complete re-initialization is not required, but more information must be saved from one update to the next. The Kalman filter is discussed in references 1, 5, and 9, listed on page 105.

If serious consideration is given to developing a manual space navigation system, the authors recommend the design of a new hand-held space sextant which incorporates a minicomputer for automatic data processing. The astronaut could enter the coordinates of the stars to be used and the initial state estimate, and then simply input the star-horizon data by depressing an appropriate switch or button located on the sextant when star-horizon superposition is achieved. Self-timing circuitry could be used to automatically "time tag" the input observations. The instrument could be battery powered, and could be designed to weigh in the neighborhood of five pounds. (The current space sextant weighs 6.25 pounds.) Of critical importance in the design of the minicomputer is data word size. Orbit determination via LSDC requires substantial data word size to maintain accuracy. During this research, the output of identical LSDC runs made on two different computers was compared to assess the effect of data word size. One machine employed a 48 bit data word, whereas the other employed a 32 bit word. Degradation in the output of the computer using the smaller word was significant. (About .3 percent in position and velocity.) The authors recommend using no less than 48 bits. More than 48 bits are not required. (The NORAD Space Defense Center uses a 48 bit data word in single precision.)

The key to obtaining accurate navigation solutions via the space sextant and LSDC processing lies in the precision obtainable in making star-horizon measurements. If this precision can be increased, the resulting navigation solutions will be more accurate. T-002 has shown that the sextant/astronaut combination is capable of star-horizon measurement precision on the order of .1 degrees (360 arc-seconds) in low altitude earth orbit. This precision might be increased through training and experience with this instrument. The sextant/astronaut combination is capable of measurement precision on the order of 10 arc-seconds. This performance was demonstrated in the NASA portion of T-002, which involved star-star and star-moon limb measurements.(13)[1] Such extreme precision cannot be expected in making star-horizon measurements due to the indistinct nature of the horizon reference, but the results of the NASA portion of the experiment are encouraging and indicate that increased measurement precision might be possible if a concerted effort is made towards this goal. The authors recommend further study of star-horizon data acquisition with this in mind.

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[1] Randle, R.J. Results of Skylab Experiment T-002, Manual Navigation Sightings. NASA TND-(In Process), NASA Ames Research Center, California, 1975.

APPENDIX A

Preliminary Results from SKYLAB Experiment T-002, Manual Navigation

Prepared by Lieutenant Colonel Stanley W. Powers, USAF

(Formerly of the Navigation Division, USAF Academy)

(This is an exact reproduction of the report.)

PRELIMINARY RESULTS FROM SKYLAB EXPERIMENT T002, MANUAL NAVIGATION

by

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Navigation Division
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ABSTRACT

The Skylab Experiment T002, Manual Space Navigation Sightings was successfully completed on Skylab missions II and III. Performing the experiment inflight were Major Jack R. Lousma, Pilot SL-II, and Lt Colonel William Pogue, pilot SL-III. The purpose of T002 was to investigate the feasibility of a manual navigation system for use in back-up navigation applications and to investigate the capability of man to make the necessary celestial observations in terms of accuracy and precision under the conditions of long-term space flight. The experiment, co-sponsored by NASA Ames Research Center and the United States Air Force, involves the use of a handheld space stadimeter for measuring altitude as a function of earth horizon curvature and a handheld space sextant for measuring space observable angles such as star to earth horizon and star to moon.

This study provides an analysis of the Air Force functional objectives--the sextant star-horizon measurements and stadimeter altitude measurements. Included are the preliminary accuracy and precision data, horizon characteristics, and instrument performance. During Skylab II, 378 sextant sightings and 88 stadimeter sightings were made. These data resulted in definitive horizon altitude determinations as well as an evaluation of the ranging performance of the space stadimeter.

Star to earth horizon sextant sightings yielded mean observable horizon heights at 9.63 km for the apparent earth, 33.34 km for the dark to light transition layer at the base of the airglow horizon, and 95.13 km for the top of the airglow horizon. Standard deviation for these horizons ranged from approximately 8.5 to 2.2 km during the sighting periods. The mean spacecraft height error for stadimeter sightings was 6.77 nm with standard deviations in the range of 2.28 to 6.45 nm.

This study was conducted by the Navigation Division, United States Air Force Academy, under sponsorship of the Space and Missile Systems Organization, Air Force Systems Command.

INTRODUCTION

The joint Department of Defense (USAF)/NASA experiment T002 for Skylab is designated "Manual Navigation Sightings." It is, in fact, much more than that. The basic purpose of the T002 experiment was to investigate man, the navigator, and the feasibility of a manual space navigation system (MSNS). Both NASA and the USAF are concerned with the capability of man in space to make celestial observations--to measure angles--in terms of both accuracy and precision and the utility of these data for both orbital and midcourse navigation.

The NASA objective is concerned with midcourse type measurements for updating and midcourse correction of interplanetary trajectories as well as man's ability to measure accurately such space observable angles as star to moon and moon angular diameter. These data were obtained during the Skylab T002 experiment and are reported separately by Mr. Robert J. Randle of Ames Research Center, the NASA Principal Investigator. (13)

The Air Force objective in T002 was to perform an operational check of the MSNS consisting of a handheld sextant, handheld stadimeter for range or altitude, computation system, and, of course, the astronaut navigator.

Manual space navigation entails two processes--obtaining inputs from measurements and determining the desired navigational information from the inputs. Manual space navigation is distinguished from other methods of navigation by the manner in which the inputs are obtained. That is, a man measures appropriate angles associated with the navigation targets, such as star-to-earth horizon, and visually acquires, aligns, and reads the results from precision handheld optical instruments. (Figures 3, 4) Navigational solutions can be obtained through graphical computations or independent computer.

Manual navigation sightings using the NASA space sextant (5, 8) and the USAF space stadimeter (9) were made during Skylab II by pilot Jack R. Lousma, Major, USMC. Sightings pertaining to the USAF objectives totaled 378 star-to-earth sextant measurements, and 88 spacecraft altitude measurements using the stadimeter. Preliminary accuracy, precision, and predicted results from these data will be discussed.

Rationale for Studying Manual Space Navigation

The Air Force has a continuing role in space as long as well established Air Force missions can be advantageously supported. The Air Force will participate to some extent in the Space Shuttle Program, and possibly future space station programs. Manual navigation is an attractive candidate for enhancing the reliability of space operations and, for some navigation tasks, may prove superior to contemplated alternatives. In addition, it seems safe to predict that through this study, much will be learned about the complex relations of man in space, the visual characteristics of the space observables, and the optical instruments which will be applicable to related systems.

The outstanding feature of a MSNS is autonomy, or independence of spacecraft and ground support. Second is reliability, a feature which is always essential. This is especially true of manned space operations. A back-up navigation system such as the MSNS, which can perform independently of major failures in other spacecraft subsystems or loss of communications with the ground, is inherently valuable. Finally, an autonomous system could reduce the costly support of a ground tracking network.

A MSNS is lightweight and small. The Air Force stadimeter weighs 4.38 lbs. and occupies 0.13 cu. ft., the NASA sextant, 6.25 lbs. and 0.18 cu. ft. The instruments are relatively inexpensive since they are simple, primarily optical-mechanical instruments with no electronics. While the MSNS is inherently reliable because man is at the heart of the system, the trend in manned space flight is to free the astronauts by performing such tasks as navigation automatically. A crewman's time will remain critical in future missions but perhaps less so than before with the advent of specialized pilot, navigator, scientist crew positions. Nevertheless, for the foreseeable future, a MSNS is generally more appropriate for back-up than for primary navigation.

GENERAL PLAN, BACKGROUND, AND THEORY

General Plan

The general plan of the Air Force objectives in Experiment T002 was to flight-test a manual space navigation system on Skylab missions II and III. The USAF has developed an orbital navigation scheme which uses as observables range above the earth surface and sextant measured angles between selected stars and the earth horizon. Range is measured using an Air Force developed stadimeter; the NASA developed sextant is used for the star-earth horizon measurements. The data observed and the GMT of the sightings are used to compute orbital parameters (12).

The experiment as planned included the following sighting periods: two stadimeter periods, three sextant periods, and five combined stadimeter and sextant periods to be conducted in an operational sequence. (2) Data were gathered on the space observables, the operational suitability of the instruments, and the measurement accuracies of both the instruments and astronaut navigator.

Background

An item of key concern to all system designers is reliability. Many forms of redundancy are used, duplicate and even triplicate systems are commonplace to achieve reliability. Superior reliability is achieved where redundant systems are completely independent of common power systems or communication links.

This philosophy led the Air Force Avionics Laboratory (AFAL) to investigate techniques of manual space navigation. A study program was begun in 1961 by Kollsman Instrument Corporation (3, 4) to examine the feasibility of a manual approach to space navigation. Attention was focused on a new navigation system for spacecraft in low altitude, low eccentricity, earth orbits. Manual solution was made possible by dividing the problem into two parts--orbit geometry parameters and orbit orientation parameters. Several computation techniques were investigated resulting in procedures, graphs, tables, and computational aids to mechanize a manual positioning technique (6). An extensive sextant and stadimeter design effort was performed which included analyses of optical accuracy, mechanical limitations, readout and timing accuracy, and operational use and limitations.

These study efforts resulted in the development of two space navigation instruments (5, 7, 8, 9). Because the navigation system was intended to be self-contained, autonomous, and capable of manual inflight solution and because of the limited development at that time of miniaturized electronic computers, efforts began to be explicitly oriented to orbit navigation through sight reduction procedures, using graphs and tables (1, 10, 14, 16).

Toward the end of 1963, manufacturing of hardware began for the Air Force Avionics Laboratory (AFAL) experiment D9 for Gemini. Kollsman built one version of the sextant which was used on Gemini IV and VII, another version which was used on Gemini VI and X, and two versions of the stadimeter, neither of which were used in flight (11).

On Gemini IV there was a loss of timing data that prevented the experimenters from deriving any useful navigation information. Gemini VII went much better. The 5777 Angstrom green line was not visible as planned; instead, sextant sightings were made to the natural air glow horizon which was determined to be about 14.9 nm above the reference earth spheroid. Post flight computations compared to ground tracking data yielded navigation errors of the same order of magnitude as generally accepted ground track errors (11, 24). A modified sextant was used on Gemini VI in an experiment sponsored by NASA wherein extremely precise angles were measured between the star Sirius and the Agena vehicle. The sextant was again used on Gemini X to measure star to horizon angles. These sightings confirmed the accuracy of the instrument and the apparent horizon altitude reported in the Gemini VII D-9 experiment.

Because of design changes required of the stadimeter, an operational check of the stadimeter was not completed. An improved stadimeter and the sextant were scheduled to be used in AFAL experiment D009, Simple Navigation, on Apollo but this experiment, as others, was cancelled from Apollo following the tragic fire.

In 1969, sponsorship of the D009 experiment was transferred to the Air Force Space and Missile Systems Organization and was assigned to Captain Cary Hunter. Subsequently, both the Air Force experiment D009, Simple Navigation, and NASA Ames Research Center experiment T002, Manual Navigation Sightings, requested assignment to the Skylab program. Because of their similarity in hardware and spacecraft interfaces, these two experiments were joined as a single experiment for Skylab entitled T002B, Manual Navigation Sightings. Mr. Robert J. Randle of NASA Ames Research Center is the Principal Investigator for the NASA portion of the experiment and the author, representing the Navigation Division at the USAF Academy is the Co-Principal Investigator for the Air Force objectives.

Theory

At least six independent parameters must be formed to uniquely describe an elliptical orbit. The choice of parameters are limited since they must be associated directly with and, easily derived from, observable phenomena.

Early studies (3) rejected an operational scheme which attempted to handle the 6 x 6 matrix computation of independent parameters. Instead the problem of parameter determination was divided into two simpler 3 x 3 matrices. The first matrix is independent, requiring only time and an optical range measurement as inputs. The second requires the output of the first, time, and star-horizon celestial measurements as inputs (15).

The output of the first 3 x 3 matrix is a set of "geometric" parameters which describe the size and shape of the orbit. These are then used in the solution of the second matrix for the "orientation" parameters of the orbit. Both sets of parameters are defined below:

Geometric Parameters

e = eccentricity of orbit
τ = period of orbit in minutes
t_o = time from perigee to first stadimeter measurement

Orientation Parameters

i = inclination of orbit plane
Ω = right ascension of ascending node
ψ_Ω = true anomaly of ascending node

These parameters can be expressed as a function of the observable phenomena as depicted in Figures 1 and 2.

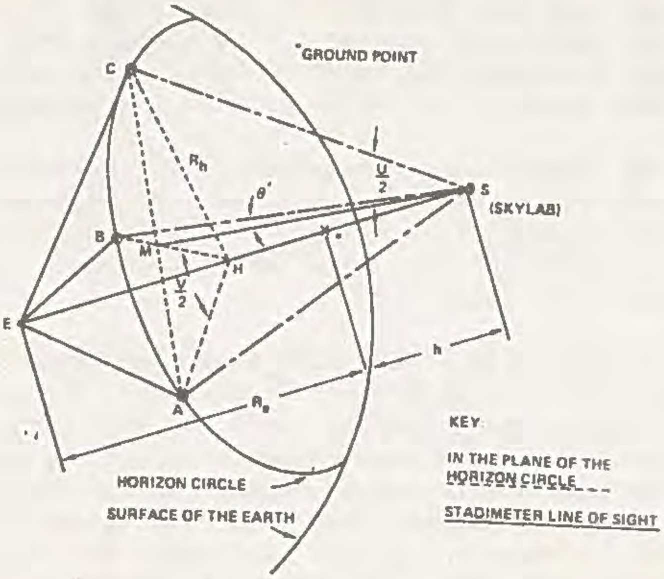

Figure 1. Geometry of the Stadimeter Observation (11:74)

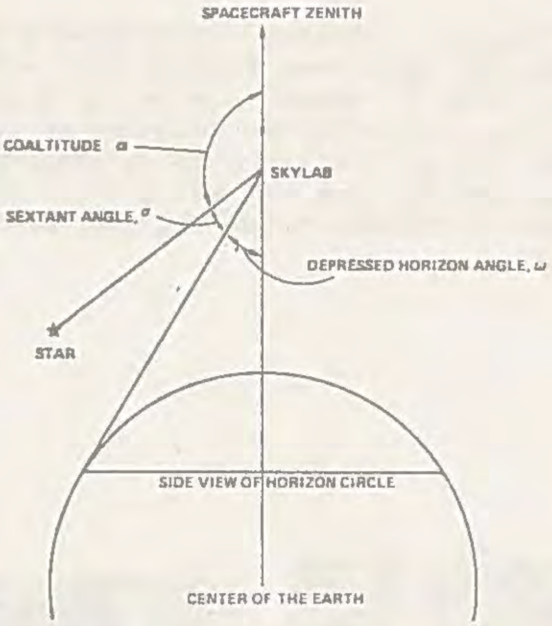

Figure 2. Geometry of the Sextant Observation (11:104)

The position fixing procedure for T002 as planned is fully supported by worksheets, tables, and graphs for manual solution. Development of these manual solution techniques have been fully described in references 3, 6, 10, 14, 16.

Data reduction for T002 is being conducted post mission since experimental time constraints did not allow for this in flight. Preliminary analysis of data has revealed an inadequacy in the stadimeter determined altitudes that will likely result in major revisions to the theoretical approach. Accurate altitudes are required in the above referenced solution schemes; any inaccuracies are seriously propagated from the orbit geometric parameter determination to the orientation parameter solution. For these reasons, further study is ongoing to develop direct matrix solution by the differential correction method which can be handled by current state-of-the art miniaturized computers. This development is under the direction of Captain Richard C. Walsh of the Department of Astronautics and Computer Science, USAF Academy.

EQUIPMENT

Equipment for T002 included the sextant, stadimeter, collapsible hood to shield the wardroom window from internal reflection, spare batteries, storage locker cushion, and a battery assembly transfer case.

The sextant was developed for NASA by Kollsman, Figure 3, and incorporates two lines of sight (two separate telescopes sharing a common eyepiece). Through a series of optics and controls the image from each line of sight is erect and, when superimposed on the focal plane, establishes the angular separation between the targets. Sextant characteristics are listed in Table 1.

The stadimeter, Figure 4, developed by Kollsman for the Air Force incorporates three separate telescopes which share a common eyepiece. Two lines of sight are fixed 65° apart. The center line of sight is capable of scanning along a perpendicular bisector of the plane of the two fixed fields. The angular readout is the separation between the chord of the center arc and the line connecting the centers of the outer segments, an angle directly related to the altitude of the spacecraft above the curved horizon.

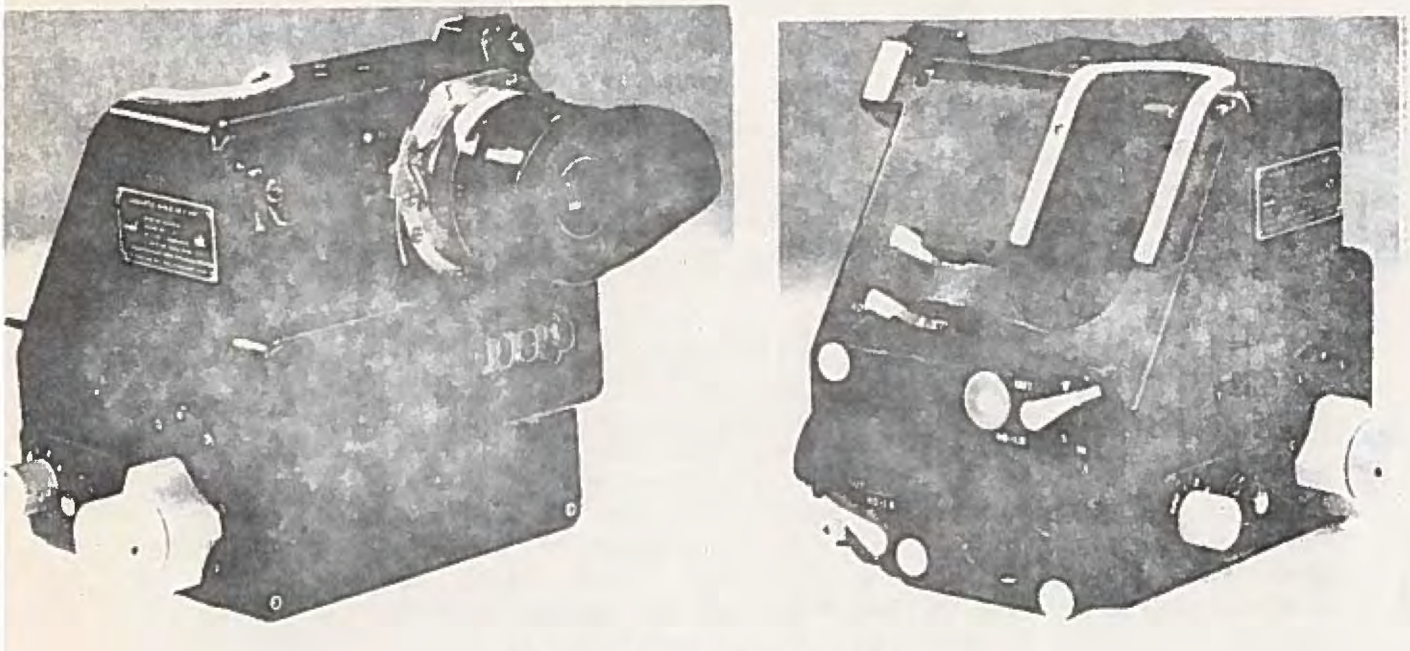

Figure 3. Space Sextant

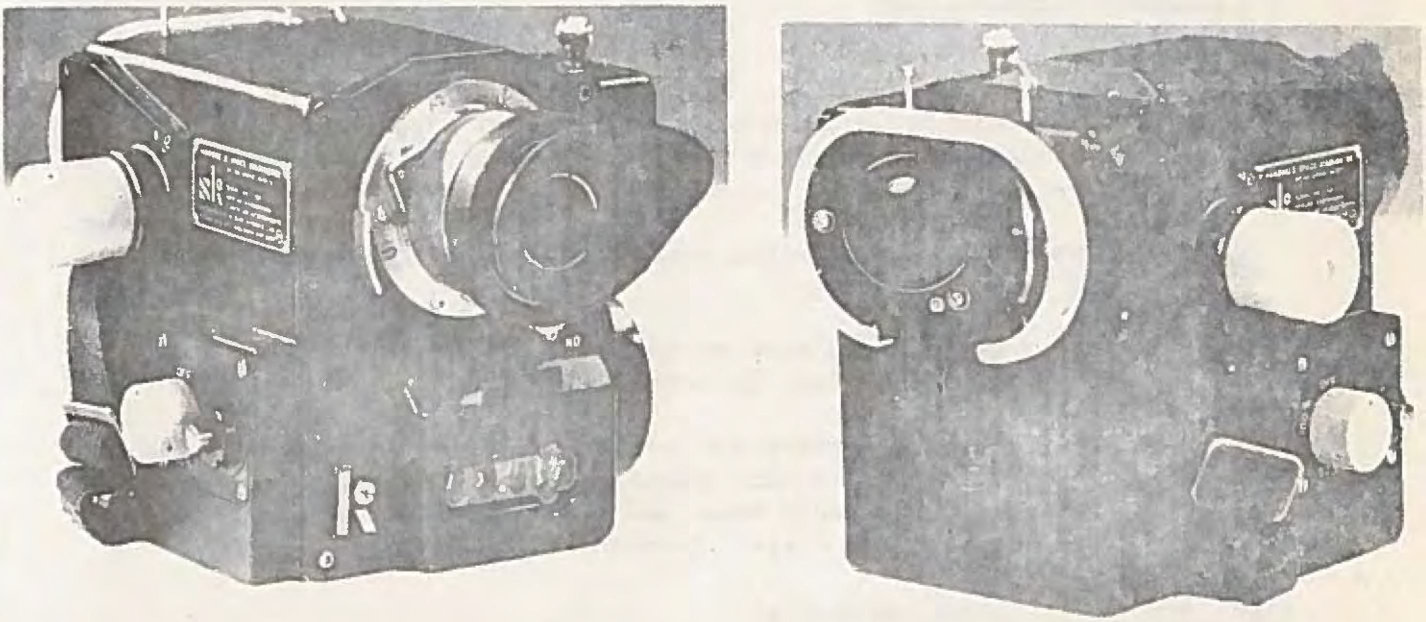

Figure 4. Space Stadimeter

Table 1
Sextant and Stadimeter Characteristics

Parameter	Sextant Value	Sextant Unit	Stadimeter Value	Stadimeter Unit
Magnification	8	power	3.4	power
Field of View	7	deg	$2^o x 8^o$	deg
			(scanning line of sight)	
			$3^o x 8^o$	deg
			(fixed lines of sight)	
Resolution	7	sec	15	sec
Scanning Range	0-76	deg	20	deg
Weight	6.25	lbs	4.37	lbs
Data Readout	nearest .001	deg	nearest .001	deg

THE SKYLAB EXPERIMENT T002

Mission as Planned

The T002 experiment consists of six functional objectives (FO), the last three of which constitute the Air Force portion. Functional objectives as planned are:

- FO-4: two stadimeter sighting periods using the earth sunlit horizon, 15 marks (sightings).
- FO-5: three sextant sighting periods measuring star-earth night air glow horizon angles, 10 marks.
- FO-6: five combined sextant and stadimeter sighting periods consisting of three 15 minute spaced day orbit stadimeter sighting periods, three marks each, and night orbit sextant star-earth sightings on two stars, 20 marks each star (2).

Experiment as Conducted, Skylab II

Table 2 provides a summary of the functional objectives as conducted on Skylab II wherein all but one FO-6 was completed. The T002 experiment was also conducted on Skylab III by pilot William Pogue, Lt Colonel, USAF. The Skylab III data, consisting only of stadimeter height measurements, are not included in this preliminary analysis.

Table 2
Summary of Functional Objectives

Day of Year	Scheduled Flight Objective	Accomplished Flight Objective	Number of Marks	Body Observed
226	6	5	44	Fomalhaut Archenar
226	6	4	15	Earth
		4	14	Earth
233	6	6	32	Diphda,Dabih
			9	Earth
234	6	5	10	Dabih
		4	9	Earth
237	6	6	34	Fomalhaut Dabih
			9	Earth
240	4	4	15	Earth (Night)
241	6	6	32	Fomalhaut Dabih
			12	Earth
242	5	5	72	Dabih
242	6	6	60	Fomalhaut Dabih
			11	Earth

During Skylab II astronaut Lousma devoted considerable time to T002 and demonstrated excellent initiative in attempting to complete all experiment requirements. More than the scheduled number of marks were taken; and in one case, a scientifically valuable stadimeter period was conducted at night. Major Lousma also provided a wealth of verbal data in the form of comments pertaining to the instrument operation, star identification, pacing, horizon characteristics, and other operational considerations. His qualitative performance, as the data show, was superior in all aspects of the experiment.

Some difficulties were encountered in conducting the sightings as scheduled. The principal difficulty in operating the sextant was in star identification/acquisition and horizon definition. The sextant gathers more light than the human eye and also magnifies the star field. More stars are seen and, with a 7° field of view, star identification is at first slow but improves rapidly with practice. The indefinite earth horizon is also troublesome. Major Lousma made the majority of his star-earth horizon sightings to what he defined as "the transition layer where the dark earth meets the white base of the airglow." This particular horizon is not well defined and may vary as a function of moon illumination and position in orbit relative to the sunlight terminators. On the other hand, the top of the airglow horizon was sighted and was described as relatively distinct and always observable. This judgment is confirmed by the analysis of observations to this horizon.

The tendency to "float" in the zero-g environment increased the difficulty in operating the sextant and stadimeter. This required the astronaut to use various methods to stabilize himself while sighting.

Because of variable day-earth horizon characteristics, performance of the stadimeter was degraded. The accuracy obtained being less than the design criteria. Operation of the stadimeter requires alignment of three segments of the earth horizon; the outer two segments are separated by 65 degrees (See Figure 1). Levels of illumination, cloud layering, and other horizon characteristics vary over the 65° degree coverage of the stadimeter. This causes the horizon segments to appear somewhat different and are thus difficult to match up with accuracy. Sightings to the night airglow horizon or sightings made near orbital noon decrease this problem significantly.

EXPERIMENTAL RESULTS

Method

Data from the sextant and stadimeter are discussed separately. Analysis in this preliminary report is limited to a determination of the accuracy and precision of the measurements rather than the resultant navigational solution.

Each stadimeter observation was converted to a measured spacecraft altitude by a computer programmed with the stadimeter equations (11). These altitudes are compared to ground tracking data with resultant means and standard deviations (S.D.) computed.

Each sextant observation was compared to a computed star-earth angle. The computed angle was derived from known spacecraft coordinates, star positions, times, and assumed earth radii (See Figure 2). Height of the sighted earth horizon and precision of the measurements were then derived.

Sextant and Stadimeter Data

Sextant data are evaluated by sighting period and by type of horizon used. Of the 378 sextant sightings made, 222 were consistently made to the horizon levels as defined in Table 3. The remaining observations were made to various less well defined horizon levels and are not incorporated in this analysis.

Table 3
Analysis of Sextant Observations - Star to Earth Horizon

Type Horizon[1]	Sighting Periods	Total Measurements	Mean Horizon Sighted (km)[2]	St. Dev. Range (km)[3]	Mean Angle Comp-Obs (Deg)[4]	St. Dev. Range (Deg)[3]
E	8	129	33.34	8.52 4.07	0.811	0.211 0.101
G	5	50	9.36	6.23 2.17	0.226	0.152 0.052
L	3	43	95.13	6.53 0.91	2.40	0.175 0.026

1. Type Horizon: E - Base of airglow horizon
   G - Apparent earth horizon
   L - Top of airglow horizon
2. Mean Horizon Sighted: Mean altitude above earth
3. Range of standard deviations computed for each sighting period
4. Computed angle (star-earth horizon) minus observed angle

A total of 129 star-earth horizon sightings were made using dark to light transition layer at the base of the airglow horizon (E) as a reference. The mean altitude of this horizon is 33.34 nm with a range of standard deviations during the eight sighting periods of 8.52 to 4.07 km (Table 3). Sighting precision was lowest for this reference horizon; standard deviations of the computed minus observed angles ranged from 0.211 to 0.101 degrees.

Fifty observations were made using the apparent earth (G) as a reference. The mean altitude of this horizon is 9.63 km. Precision of these observations is slightly better than the E horizon observations--S.D. was 0.152 to 0.052 degrees in five periods. Astronaut comments indicate this is a difficult horizon to use.

Forty-three observations were made using the top of the airglow (L) as a reference. The mean altitude of this horizon is 95.13 km; S.D. range during the three periods was 6.23 to 2.17 km (Table 3). Sighting precision to this horizon was generally superior; S.D. of the computed minus observed angles ranged from 0.175 to 0.026 degrees. This horizon was evaluated by the astronaut as distinct, always available at night, and the easiest to use horizon.

Table 4
Analysis of Stadimeter Observations

Horizon	Measurements	Mean Height Error* $h_m - h_o$	Standard Deviation $h_m - h_o$
Day			
Earth	28	7.12nm	4.44nm
"	3	2.69	3.51
"	9	10.18	6.45
"	9	4.96	2.28
"	12	6.99	3.38
"	12	5.58	2.68
Day Earth Total	73	Weighted Mean 6.77	
Night Earth	15	-40.28	5.78

*$h_m - h_o$: height measured minus height above oblate earth

Eighty-eight stadimeter sightings were made, 15 of which were made during the night portion of the orbit (Table 2). Sightings were planned to be conducted during the entire 40 to 45 minutes of daylight, but the indistinct quality of the sunlit earth horizon limited sighting periods to within 10 to 15 minutes of orbital noon. Sightings attempted near orbital sunrise or sunset were progressively inaccurate.

The mean height error for daylight sightings is 6.77 nm; S.D. ranged from 2.28 to 6.45 nm (Table 4). Sightings made at night are negatively biased--the mean height error is -40.28 nm. The measured height is lower than the actual height. Precision of the night sightings is not significantly different from the day sightings--5.78 nm (S.D.). Bias in the measured height is to be expected and can be handled in any computation scheme.

The problem is lack of precision, i.e., variability. The 40.28 nm bias obtained for the night observations is a result of sighting to the higher airglow level thus measuring an apparent lower orbital height.

CONCLUSIONS

Experiment T002 Manual Navigation Sightings was successfully completed on Skylab II through the expert performance of Astronaut Major Jack R. Lousma. All flight objectives were conducted, the space stadimeter was tested for the first time, and operational type data were gathered for the development of a backup manual space navigation system. During fifteen sighting periods 378 sextant and 88 stadimeter observations were made. Sextant sightings to the top of the stable airglow horizon yielded a mean 95.13 km height with a measurement precision in the order of 0.15° to 0.03° standard deviation. These data compare favorably in precision to previous Gemini experience (15), wherein orbital parameter determination and along-track position determination approximated the accepted uncertainty in the ground track record.

Stadimeter accuracy for height determination was less than anticipated. For day sightings a mean 6.77 nm height error with a range in precision from 2.28 to 6.45 nm S.D. was obtained. Sightings to the night horizon yielded a negative 40.28 nm (5.78 nm S.D.) error. These accuracies may prove acceptable when applied to a least squares matrix solution with appropriate weights assigned to sextant and stadimeter observations. Optimum stadimeter accuracy is obtained near orbital noon or at night because of horizon lighting variability near the sunlight terminator.

T002 was designed to test both man and the space observables in search for a simple, reliable navigation system. The performance of Major Lousma in the sighting tasks and the accuracies obtained in this preliminary post-flight analysis reveals that both tasks were accomplished successfully. Further post-flight analysis of the T002 data will investigate fully the various options for graphical or minicomputer navigational solutions.

ACKNOWLEDGEMENT

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APPENDIX B

LSDC RUNS 1-4

This appendix contains four LSDC runs referenced in the body of this report.

The observation code used in these runs is:

Star Number	Observation Type
1	Height (kilometers)
2	Fomalhaut-Earth Horizon (degrees)
3	Achernar-Earth Horizon (degrees)
5	Dabih-Earth Horizon (degrees)
6	Betelgeuse-Earth Horizon (degrees)

The Geocentric-Equatorial star coordinates used are:

Star	Right Ascension	Declination
Fomalhaut	343.7232 degrees	-29.8888 degrees
Achernar	23.9639 degrees	-57.4904 degrees
Dabih	304.5509 degrees	-14.9407 degrees
Betelgeuse	88.0 degrees	7.0 degrees

(Approximate values for the position of Betelgeuse were used to generate simulated data.)

SUMMARY OF LSDC RUN #1

```
EPOCH:  YEAR=1973
         DAY=242
          UT=18:59:37.99

NASA PRECISION EPHEMERIS STATE
(GEOCENTRIC-EQUATORIAL COORDINATES):

     X=4871.907 km      XDOT=5.056 km/sec

     Y=-2185.958 km     YDOT=4.606 km/sec

     Z=4227.094 km      ZDOT=-3.426 km/sec

INPUT DATA:  13 STAR-HORIZON OBS
             STARS 2 AND 5
             E TYPE HORIZON
             ACTUAL FLIGHT DATA

MEASUREMENT SCHEDULE:  NON-CYCLIC

INPUT STATE ESTIMATE:

     X=4817.1684 km     XDOT=5.077818 km/sec

     Y=-2100.7027 km    YDOT=4.664743 km/sec

     Z=4110.6079 km     ZDOT=-3.484950 km/sec

     SEXTANT HORIZON BIAS=35.0 km  INPUT SIGMA=0.2 degrees

     POSITION ERROR=154.3824 km

     VELOCITY ERROR=86.03 meters/sec

CONVERGENCE TOLERANCE=0.2

NUMBER OF ITERATIONS=8

OUTPUT STATE SOLUTION:

     X=5091.0127 km     XDOT=4.823578 km/sec

     Y=-2074.4472 km    YDOT=4.943377 km/sec

     Z=4048.6319 km     ZDOT=-3.255593 km/sec

     SEXTANT HORIZON BIAS=34.99 km

     POSITION ERROR=303.7938 km

     VELOCITY ERROR=443.7 meters/sec
```

SUMMARY OF LSDC RUN #2

EPOCH: YEAR=1973
DAY=242
UT=18:59:37.99

NASA PRECISION EPHEMERIS STATE
(GEOCENTRIC-EQUATORIAL COORDINATES):

X=4871.907 km   XDOT=5.056 km/sec

Y=-2185.958 km   YDOT=4.606 km/sec

Z=4227.094 km   ZDOT=-3.426 km/sec

INPUT DATA: 13 STAR-HORIZON OBS
STARS 2, 5, AND 6
E TYPE HORIZON
ACTUAL FLIGHT DATA AUGMENTED WITH STAR 6 DATA

MEASUREMENT SCHEDULE: CYCLIC

INPUT STATE ESTIMATE:

X=4817.1684 km   XDOT=5.077818 km/sec

Y=-2100.7027 km   YDOT=4.664743 km/sec

Z=4110.6079 km   ZDOT=-3.484950 km/sec

SEXTANT HORIZON BIAS=35.0 km INPUT SIGMA=0.2 degrees

POSITION ERROR=154.3824 km

VELOCITY ERROR=86.03 meters/sec

CONVERGENCE TOLERANCE=0.2

NUMBER OF ITERATIONS=4

OUTPUT STATE SOLUTION:

X=4872.2560 km   XDOT=5.058423 km/sec

Y=-2185.3886 km   YDOT=4.594329 km/sec

Z=4233.0813 km   ZDOT=-3.433274 km/sec

SEXTANT HORIZON BIAS=35.01 km

POSITION ERROR=6.024432 km

VELOCITY ERROR=13.96 meters/sec

SUMMARY OF LSDC RUN #3

EPOCH: YEAR=1973
DAY=242
UT=18:59:37.99

NASA PRECISION EPHEMERIS STATE
(GEOCENTRIC-EQUATORIAL COORDINATES):

X=4871.907 km	XDOT=5.056 km/sec
Y=-2185.958 km	YDOT=4.606 km/sec
Z=4227.094 km	ZDOT=-3.426 km/sec

INPUT DATA: 9 STAR-HORIZON OBS, 4 HEIGHT OBS
STARS 2, 5, AND 6
E TYPE HORIZON
ACTUAL FLIGHT DATA AUGMENTED WITH STAR 6 DATA
AND HEIGHT DATA

MEASUREMENT SCHEDULE: CYCLIC

INPUT STATE ESTIMATE:

X=4817.1684 km	XDOT=5.077818 km/sec
Y=-2100.7027 km	YDOT=4.664743 km/sec
Z=4110.6079 km	ZDOT=-3.484950 km/sec

SEXTANT HORIZON BIAS=35.0 km INPUT SIGMA=0.2 degrees

STADIMETER HORIZON BIAS=5.0 km INPUT SIGMA=4.0 km

POSITION ERROR=154.3824 km

VELOCITY ERROR=86.03 meters/sec

CONVERGENCE TOLERANCE=0.2

NUMBER OF ITERATIONS=4

OUTPUT STATE SOLUTION:

X=4870.4354 km	XDOT=5.074632 km/sec
Y=-2161.8408 km	YDOT=4.591636 km/sec
Z=4204.0312 km	ZDOT=-3.423520 km/sec

SEXTANT HORIZON BIAS=35.37 km

STADIMETER HORIZON BIAS=4.65 km

POSITION ERROR=33.40206 km

VELOCITY ERROR=23.66 meters/sec

SUMMARY OF LSDC RUN #4

EPOCH: YEAR=1973
DAY=242
UT=18:58:23.00

NORAD PRECISION EPHEMERIS STATE
(GEOCENTRIC-EQUATORIAL COORDINATES):

X=4478.9243 km	XDOT=5.481984 km/sec
Y=-2499.6806 km	YDOT=4.412064 km/sec
Z=4478.6951 km	ZDOT=-3.001309 km/sec

INPUT DATA: 31 STAR-HORIZON OBS
STARS 3, 5, AND 6
L TYPE HORIZON
SIMULATED DATA

MEASUREMENT SCHEDULE: CYCLIC

INPUT STATE ESTIMATE:

X=5889.2182 km	XDOT=3.782586 km/sec
Y=-1929.6326 km	YDOT=5.668651 km/sec
Z=3848.9428 km	ZDOT=-2.854421 km/sec

SEXTANT HORIZON BIAS=90.00 km INPUT SIGMA=0.1 degrees

POSITION ERROR=1646.3510 km

VELOCITY ERROR=2118.618 meters/sec

CONVERGENCE TOLERANCE=0.2

NUMBER OF ITERATIONS=5

OUTPUT STATE SOLUTION:

X=4478.9670 km	XDOT=5.479903 km/sec
Y=-2500.7114 km	YDOT=4.407491 km/sec
Z=4480.2421 km	ZDOT=-3.004022 km/sec

SEXTANT HORIZON BIAS=93.77 km

POSITION ERROR=1.8594572 km

VELOCITY ERROR=5.709926 meters/sec

LSDC RUN #1

```
INPUT ORBITAL ELEMENTS
EPOCH(YDDD.DD...)     MEAN ANOM   RT ASC    ARG PER      ECC        INC      PERIOD
     3242.79141168    97.4931    197.4720    28.1995  0.0065843  49.9928    90.27226

                                 PREDICTION EPHEMERIS
  YY DDD HH MM SS.SS          X             Y              Z              XDOT           YDOT          ZDOT

  3 242 18 59 37.99      4817.1684     -2100.7027      4110.6079       5.077819       4.664743      -3.484950


   POINT      T-TZERO    STAR        OB
            HH MM SS.SS            (PRED)   RESIDUAL
     1     19  8 39.00     5      39.70790     3.677  -9.0E-01 -3.1E-01 -2.9E-01 -3.0E-01 -1.5E-01  1.9E 00 -5.2E-04
     2     19  9 40.00     5      38.45798     3.328  -9.2E-01 -3.4E-01 -2.2E-01 -3.2E-01 -1.5E-01  2.0E 00 -5.2E-04
     3     19 10 33.00     5      37.25824     3.778  -9.4E-01 -3.6E-01 -1.7E-01 -3.4E-01 -1.6E-01  2.1E 00 -5.2E-04
     4     19 11 29.00     5      35.97429     3.693  -9.5E-01 -3.8E-01 -1.0E-01 -3.6E-01 -1.6E-01  2.3E 00 -5.1E-04
     5     19 12 31.00     5      34.43397     3.406  -9.6E-01 -4.0E-01 -3.9E-02 -3.8E-01 -1.6E-01  2.4E 00 -5.1E-04
     6     19 13 12.00     5      33.37013     3.830  -9.6E-01 -4.1E-01  4.5E-03 -4.0E-01 -1.6E-01  2.5E 00 -5.1E-04
     7     19 14 45.00     2      63.49371     2.981  -7.7E-01 -1.0E-02  1.1E-01 -2.6E-03 -1.4E-01  3.0E 00 -5.1E-04
     8     19 15 55.00     2      63.29179     3.025  -8.1E-01 -9.4E-02  1.9E-01 -8.7E-02 -1.4E-01  3.0E 00 -5.1E-04
     9     19 16 57.00     2      62.78859     3.403  -5.4E-01 -1.7E-01  2.6E-01 -1.6E-01 -1.5E-01  3.1E 00 -5.1E-04
    10     19 18 27.00     5      24.27630     4.127  -9.3E-01 -4.7E-01  3.2E-01 -4.6E-01 -1.7E-01  3.1E 00 -5.1E-04
    11     19 19 25.00     5      22.47529     4.004  -9.2E-01 -4.8E-01  3.7E-01 -4.7E-01 -1.8E-01  3.1E 00 -5.0E-04
    12     19 20 19.00     5      20.77717     3.987  -9.0E-01 -4.9E-01  4.2E-01 -4.9E-01 -1.8E-01  3.2E 00 -5.0E-04
    13     19 21 12.00     5      19.09541     4.334  -8.9E-01 -4.8E-01  4.6E-01 -4.9E-01 -1.9E-01  3.3E 00 -5.0E-04
```

```
VARIANCE-COVARIANCE MATRIX


      4,381E-03 -7,304E-03 -3,638E-03  3,594E-03  9,339E-03  1,709E-03 -2,165E-04  0,000E 00

     -7,304E-03  4,940E-01 -9,609E-03 -4,980E-01  1,286E-02  2,717E-04  2,220E-04  0,000E 00

     -3,638E-03 -9,609E-03  4,211E-03  1,369E-02 -1,708E-02 -1,931E-03 -3,475E-05  0,000E 00

      3,594E-03 -4,980E-01  1,369E-02  5,063E-01 -3,260E-02 -2,240E-03  7,316E-05  0,000E 00

      9,339E-03  1,286E-02 -1,708E-02 -3,260E-02  1,284E-01  9,035E-03 -2,836E-03  0,000E 00

      1,709E-03  2,717E-04 -1,931E-03 -2,240E-03  9,035E-03  9,413E-04 -1,274E-05  0,000E 00

     -2,165E-04  2,220E-04 -3,475E-05  7,316E-05 -2,836E-03 -1,274E-05  1,000E 00  0,000E 00

     -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                    P           E         RT ASC       ARG PER        I         MEAN ANOM      BIAS 1       BIAS 2

  ELEMENTS      90,272251    0,006584   197,472000    28,199500   49,992800    97,493100   35,000000    0,000000
  DELTAS         4,327204   -0,017229    -0,096954     1,738870    1,261516    -0,163207   -0,002403    0,000000

                PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS        X            Y             Z           XDOT          YDOT          ZDOT

  0 242 18 59 37,99    4998,2890    -2024,6351    4267,9550     4,919406      4,563303     -3,596471

    OLD RMS =    100000,000000   NEW RMS =     17,91561   CHANGE IN RMS =       0,999821
```

```
VARIANCE-COVARIANCE MATRIX


      3.129E-03 -3.798E-04 -3.080E-03 -2.476E-03  7.048E-03  1.542E-03 -1.836E-04  0.000E 00

     -3.798E-04  4.996E-01  1.386E-03 -4.986E-01 -1.100E-02 -8.663E-04  1.341E-04  0.000E 00

     -3.080E-03  1.386E-03  4.015E-03  2.902E-03 -1.992E-02 -2.196E-03 -4.899E-05  0.000E 00

     -2.476E-03 -4.986E-01  2.902E-03  5.026E-01 -1.539E-02 -1.576E-03  1.048E-04  0.000E 00

      7.048E-03 -1.100E-02 -1.992E-02 -1.539E-02  1.909E-01  1.258E-02 -2.324E-03  0.000E 00

      1.542E-03 -8.663E-04 -2.196E-03 -1.576E-03  1.258E-02  1.235E-03  1.637E-06  0.000E 00

     -1.836E-04  1.341E-04 -4.899E-05  1.048E-04 -2.324E-03  1.637E-06  1.000E 00  0.000E 00

     -0.000E 00 -0.000E 00 -0.000E 00 -0.000E 00 -0.000E 00 -0.000E 00 -0.000E 00  1.000E 00


                  P            E         RT ASC       ARG PER        I          MEAN ANOM      BIAS 1       BIAS 2
 ELEMENTS      94.599484    0.000001   197.375046    29.938370    51.254316    97.329893    34.997597    0.000000
 DELTAS        -0.776225    0.010918     2.173558    -0.222496    -2.568800    -1.938265    -0.000838    0.000000

              PREDICTION EPHEMERIS
YY DDD HH MM SS.SS         X            Y             Z           XDOT         YDOT         ZDOT

 0 242 15 59 37.99     5044.0595   -2071.9421    4141.5390     4.848347     4.342063    -3.345586

   OLD RMS =        17.915615   NEW RMS =       6.07456   CHANGE IN RMS =         0.660935
```

```
VARIANCE-COVARIANCE MATRIX


 3,247E-03 -8,943E-03 -2,901E-03  6,024E-03  7,427E-03  1,567E-03 -1,966E-04  0,000E 00

-8,943E-03  4,879E-01 -1,213E-02 -4,959E-01  2,260E-02  9,062E-04  2,516E-04  0,000E 00

-2,901E-03 -1,213E-02  4,720E-03  1,731E-02 -2,469E-02 -2,468E-03 -5,361E-05  0,000E 00

 6,024E-03 -4,959E-01  1,731E-02  5,099E-01 -5,554E-02 -3,775E-03  3,065E-06  0,000E 00

 7,427E-03  2,260E-02 -2,469E-02 -5,554E-02  2,333E-01  1,508E-02 -2,409E-03  0,000E 00

 1,567E-03  9,062E-04 -2,468E-03 -3,775E-03  1,508E-02  1,388E-03 -5,765E-06  0,000E 00

-1,966E-04  2,516E-04 -5,361E-05  3,064E-06 -2,409E-03 -5,765E-06  1,000E 00  0,000E 00

-0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                 P            E         RT ASC       ARG PER         I        MEAN ANOM      BIAS 1       BIAS 2

ELEMENTS     93,623260     0,010919   199,548604    29,715874    48,685516    95,391625    34,996759     0,000000
DELTAS        2,243843    -0,034184    -2,339415     2,821329     3,411524     1,099014    -0,001165     0,000000

                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS        X             Y             Z          XDOT         YDOT        ZDOT

0 242 15 59 37,99     5161,2796    -1872,2205     4258,4445     4,752567     4,539960   -3,704173

  OLD RMS =          6,074562   NEW RMS =       0,93887   CHANGE IN RMS =        0,845442
```

```
VARIANCE-COVARIANCE MATRIX


  2,142E-03 -5,564E-06 -2,132E-03 -1,819E-03  2,871E-03  1,053E-03 -1,740E-04  0,000E 00

 -5,568E-06  4,996E-01  1,111E-03 -4,958E-01 -1,125E-02 -7,660E-04  1,240E-04  0,000E 00

 -2,132E-03  1,111E-03  3,353E-03  2,555E-03 -1,893E-02 -1,915E-03 -5,127E-05  0,000E 00

 -1,819E-03 -4,958E-01  2,585E-03  5,025E-01 -1,626E-02 -1,474E-03  9,513E-05  0,000E 00

  2,871E-03 -1,125E-02 -1,893E-02 -1,626E-02  2,176E-01  1,291E-02 -2,175E-03  0,000E 00

  1,053E-03 -7,660E-04 -1,915E-03 -1,474E-03  1,291E-02  1,138E-03  6,083E-06  0,000E 00

 -1,740E-04  1,240E-04 -5,127E-05  9,513E-05 -2,175E-03  6,083E-06  1,000E 00  0,000E 00

 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                 P            E          RT ASC        ARG PER          I          MEAN ANOM        BIAS 1         BIAS 2

 ELEMENTS       96,067103     0,000001   197,209190     32,537203     52,097340     96,490642     34,995593      0,000000
 DELTAS          1,325546    -0,038375    -0,360924      0,821291      1,320067      0,851713     -0,000470      0,000000


                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS     X              Y              Z              XDOT           YDOT           ZDOT

 0 242 15 59 37,99     5295,3194     -1706,2363     4268,1324      4,620368       4,461448      -3,948810

   DC DIVERGING, OLD RMS = 9,38872E-01    NEW RMS = 1,29297E 01

   OLD RMS =          0,938872    NEW RMS =       12,92973    CHANGE IN RMS =        12,771560
```

```
VARIANCE-COVARIANCE MATRIX


  1,699E-03 -3,758E-05 -1,772E-03 -1,509E-03  2,948E-03  9,496E-04 -1,550E-04  0,000E 00

 -3,758E-05  5,001E-01  1,620E-03 -4,983E-01 -1,885E-02 -1,236E-03  9,879E-05  0,000E 00

 -1,772E-03  1,620E-03  3,512E-03  2,697E-03 -2,615E-02 -2,276E-03 -7,867E-05  0,000E 00

 -1,509E-03 -4,983E-01  2,697E-03  5,025E-01 -2,190E-02 -1,740E-03  6,549E-05  0,000E 00

  2,948E-03 -1,885E-02 -2,615E-02 -2,190E-02  3,295E-01  1,968E-02 -1,792E-03  0,000E 00

  9,496E-04 -1,236E-03 -2,276E-03 -1,740E-03  1,968E-02  1,535E-03  2,938E-05  0,000E 00

 -1,550E-04  9,879E-05 -7,867E-05  6,549E-05 -1,792E-03  2,938E-05  1,000E 00  0,000E 00

 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                      P           E         RT ASC      ARG PER        I         MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS        97,392649     0,000001   196,848266   33,358494   53,417407   97,342356   34,995123    0,000000
 DELTAS          -0,849280    -0,022795     1,315052   -0,399045   -1,879469   -1,374652   -0,000189    0,000000


                PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS        X           Y            Z           XDOT        YDOT        ZDOT

 0 242 15 59 37,99    5214,3083   -1839,8988    4245,8443    4,667129    4,642105   -3,721166


  DC DIVERGING, OLD RMS = 1,29297E 01   NEW RMS = 1,99961E 01


  OLD RMS =         12,929728   NEW RMS =       19,99606   CHANGE IN RMS =         0,546519
```

```
VARIANCE-COVARIANCE MATRIX


   2,220E-03 -2,052E-04 -2,431E-03 -1,889E-03  4,478E-03  1,211E-03 -1,657E-04  0,000E 00

  -2,052E-04  5,000E-01  1,726E-03 -4,984E-01 -1,577E-02 -1,119E-03  1,147E-04  0,000E 00

  -2,431E-03  1,726E-03  4,402E-03  3,114E-03 -2,661E-02 -2,523E-03 -7,077E-05  0,000E 00

  -1,889E-03 -4,984E-01  3,114E-03  5,027E-01 -2,072E-02 -1,793E-03  8,288E-05  0,000E 00

   4,478E-03 -1,577E-02 -2,661E-02 -2,072E-02  2,833E-01  1,754E-02 -2,028E-03  0,000E 00

   1,211E-03 -1,119E-03 -2,523E-03 -1,793E-03  1,754E-02  1,493E-03  1,612E-05  0,000E 00

  -1,657E-04  1,147E-04 -7,077E-05  8,288E-05 -2,028E-03  1,612E-05  1,000E 00  0,000E 00

  -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                  P            E          RT ASC       ARG PER         I         MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS     96,543369     0,000001    198,163318    32,959449    51,537938    95,967674    34,994935    0,000000
 DELTAS        1,252006    -0,042450     -1,162463     1,164324     2,299429     0,953427    -0,000590    0,000000


                  PREDICTION EPHEMERIS
YY DDD HH MM SS,SS        X              Y              Z            XDOT          YDOT          ZDOT

 0 242 18 59 37,99     5334,3067     -1639,9673     4279,6940     4.572994      4,450962     -3,994286

   OLD RMS =        19,996057   NEW RMS =       11,30619   CHANGE IN RMS =         0,434579
```

```
VARIANCE-COVARIANCE MATRIX


   2,278E-03 -2,933E-04 -2,716E-03 -2,186E-03  7,010E-03  1,478E-03 -1,559E-04  0,000E 00

  -2,933E-04  5,000E-01  1,822E-03 -4,982E-01 -1,733E-02 -1,282E-03  1,079E-04  0,000E 00

  -2,716E-03  1,822E-03  4,857E-03  3,675E-03 -2,978E-02 -2,968E-03 -6,324E-05  0,000E 00

  -2,186E-03 -4,982E-01  3,675E-03  5,032E-01 -2,447E-02 -2,240E-03  7,539E-05  0,000E 00

   7,010E-03 -1,733E-02 -2,978E-02 -2,447E-02  3,092E-01  2,070E-02 -1,943E-03  0,000E 00

   1,478E-03 -1,282E-03 -2,968E-03 -2,240E-03  2,070E-02  1,868E-03  1,855E-05  0,000E 00

  -1,559E-04  1,079E-04 -6,324E-05  7,539E-05 -1,943E-03  1,855E-05  1,000E 00  0,000E 00

  -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                  P            E          RT ASC      ARG PER         I        MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS     97,825375     0,000001   197,000854   34,123772    53,837366    96,951101    34,994345    0,000000
 DELTAS       -0,010351    -0,043479     0,040863    0,065946    -0,309281    -0,052862    -0,000034    0,000000

                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS        X            Y            Z           XDOT         YDOT         ZDOT

 0 242 18 59 37,99    5342,8454   -1657,1179    4261,5827    4,562344     4,475366    -3,979670

   DC DIVERGING, OLD RMS = 1,13062E 01   NEW RMS = 2,13208E 01


   OLD RMS =        11,306157   NEW RMS =      21,32079   CHANGE IN RMS =        0,885763
```

```
VARIANCE-COVARIANCE MATRIX


  2,160E-03 -8,946E-05 -2,214E-03 -1,724E-03  1,862E-03  1,126E-03 -1,625E-04  0,000E 00

 -8,946E-05  4,997E-01  1,224E-03 -4,987E-01 -1,195E-02 -8,798E-04  1,136E-04  0,000E 00

 -2,214E-03  1,224E-03  3,398E-03  2,430E-03 -1,685E-02 -1,991E-03 -5,066E-05  0,000E 00

 -1,724E-03 -4,987E-01  2,430E-03  5,022E-01 -1,392E-02 -1,418E-03  9,643E-05  0,000E 00

  1,862E-03 -1,195E-02 -1,685E-02 -1,392E-02  2,034E-01  1,224E-02 -2,047E-03  0,000E 00

  1,126E-03 -8,798E-04 -1,991E-03 -1,418E-03  1,224E-02  1,218E-03  9,848E-06  0,000E 00

 -1,625E-04  1,136E-04 -5,066E-05  8,643E-05 -2,047E-03  9,845E-06  1,000E 00  0,000E 00

 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                  P            E          RT ASC       ARG PER          I         MEAN ANOM       BIAS 1        BIAS 2

 ELEMENTS     97,815024     0,000001   197,041717    34,189718    53,528096    96,898239    34,994311     0,000000
 DELTAS       -4,436088     0,021554     3,126101    -2,556565    -5,971130    -4,483017     0,000567     0,000000

                     PREDICTION EPHEMERIS
YY DDD HH MM SS,SS          X              Y              Z            XDOT          YDOT          ZDOT

 0 242 18 59 37,99      5091,0127     -2074,4472     4048,6319     4,923578     4,943377     -3,255593

   OLD RMS =        21,320792   NEW RMS =        20,96079   CHANGE IN RMS =         0,021575
```

```
 POINT     T-TZERO    STAR          OB
          HH MM SS.SS              (PRED)  RESIDUAL
   1     19  8 39,00    5       43,63733   -0,252  -9,0E-01 -3,6E-01 -3,1E-01 -3,1E-01 -1,2E-01  1,0E 00 -3,9E-04
   2     19  9 40,00    5       42,37033   -0,084  -9,2E-01 -3,5E-01 -2,4E-01 -3,4E-01 -1,2E-01  1,1E 00 -3,9E-04
   3     19 10 33,00    5       41,19809   -0,132  -9,3E-01 -4,0E-01 -1,9E-01 -3,5E-01 -1,2E-01  1,2E 00 -3,8E-04
   4     19 11 29,00    5       39,89321   -0,226  -9,5E-01 -4,2E-01 -1,3E-01 -3,7E-01 -1,3E-01  1,3E 00 -3,8E-04
   5     19 12 31,00    5       38,37592   -0,536  -9,6E-01 -4,4E-01 -7,3E-02 -3,9E-01 -1,3E-01  1,4E 00 -3,8E-04
   6     19 13 12,00    5       37,33417   -0,134  -9,6E-01 -4,5E-01 -3,3E-02 -4,0E-01 -1,3E-01  1,5E 00 -3,8E-04
   7     19 14 45,00    2       66,86635   -0,391  -7,6E-01 -3,2E-02  6,3E-02 -7,5E-03 -1,0E-01  2,0E 00 -3,7E-04
   8     19 15 55,00    2       66,67298   -0,356  -8,0E-01 -1,1E-01  1,4E-01 -8,1E-02 -1,1E-01  2,1E 00 -3,7E-04
   9     19 16 57,00    2       66,22862   -0,032  -8,3E-01 -1,7E-01  2,0E-01 -1,4E-01 -1,1E-01  2,1E 00 -3,7E-04
  10     19 18 27,00    5       28,54238   -0,139  -9,5E-01 -4,9E-01  2,6E-01 -4,6E-01 -1,4E-01  2,0E 00 -3,7E-04
  11     19 19 25,00    5       26,81262   -0,334  -9,3E-01 -5,0E-01  3,1E-01 -4,7E-01 -1,4E-01  2,1E 00 -3,7E-04
  12     19 20 19,00    5       25,18234   -0,418  -9,2E-01 -5,0E-01  3,6E-01 -4,7E-01 -1,4E-01  2,2E 00 -3,7E-04
  13     19 21 12,00    5       23,56757   -0,139  -9,0E-01 -5,0E-01  4,0E-01 -4,8E-01 -1,5E-01  2,2E 00 -3,7E-04

***CONVERGED***
```

LSDC RUN #2

```
INPUT ORBITAL ELEMENTS
EPOCH(YDDD.DD...)    MEAN ANOM   RT ASC    ARG PER     ECC        INC      PERIOD
     3242.79141188   97.4931    197.4720   28.1995  0.0065843  49.9928   90.27228


                           PREDICTION EPHEMERIS
  YY DDD HH MM SS.SS              X                Y                Z              XDOT           YDOT           ZDOT

   3 242 18 59 37.99        4817.1684       -2100.7027        4110.6079        5.077818       4.664743      -3.484950


   POINT     T-TZERO    STAR         OB
          HH MM SS.SS              (PRED)   RESIDUAL
     1    19  8 39.00     5       39.70790     3.677   -9.0E-01  -3.1E-01  -2.9E-01  -3.0E-01  -1.5E-01   1.9E 00  -5.2E-04
     2    19  9 40.00     6       28.32468     2.269    9.8E-01   5.9E-01   1.9E-01   6.0E-01  -1.1E-01   3.2E 00  -5.2E-04
     3    19 10 33.00     2       61.00380     2.602   -5.5E-01   2.8E-01  -1.7E-01   2.9E-01  -1.2E-01   2.9E 00  -5.2E-04
     4    19 11 29.00     5       35.97429     3.693   -9.5E-01  -3.8E-01  -1.0E-01  -3.6E-01  -1.6E-01   2.3E 00  -5.1E-04
     5    19 12 31.00     6       34.88689     1.546    9.9E-01   5.5E-01   3.5E-02   5.6E-01  -1.1E-01   3.4E 00  -5.1E-04
     6    19 13 12.00     2       63.15140     2.744   -7.0E-01   1.0E-01   4.2E-03   1.1E-01  -1.3E-01   3.0E 00  -5.1E-04
     7    19 14 45.00     5       30.83855     3.951   -9.7E-01  -4.4E-01   1.0E-01  -4.2E-01  -1.6E-01   2.7E 00  -5.1E-04
     8    19 15 55.00     6       41.96896     1.771    9.7E-01   4.8E-01  -1.7E-01   4.8E-01  -1.0E-01   3.4E 00  -5.1E-04
     9    19 16 57.00     2       62.78859     3.408   -5.4E-01  -1.7E-01   2.6E-01  -1.6E-01  -1.5E-01   3.1E 00  -5.1E-04
    10    19 18 27.00     5       24.27630     4.127   -9.3E-01  -4.7E-01   3.2E-01  -4.6E-01  -1.7E-01   3.1E 00  -5.1E-04
    11    19 19 25.00     6       48.00413     1.794    8.9E-01   3.8E-01  -3.9E-01   3.8E-01  -1.0E-01   3.3E 00  -5.0E-04
    12    19 20 19.00     2       59.20520     3.649   -8.7E-01  -3.7E-01   4.4E-01  -3.7E-01  -1.7E-01   3.2E 00  -5.0E-04
    13    19 21 12.00     5       19.09541     4.334   -9.3E-01  -4.9E-01   4.6E-01  -4.3E-01  -1.8E-01   3.3E 00  -5.0E-04
```

```
VARIANCE-COVARIANCE MATRIX


     9,253E-06 -3,357E-04  4,697E-09  3,191E-04  1,321E-05  5,342E-07  4,024E-06  0,000E 00

    -3,357E-04  2,260E-01  5,974E-04 -2,197E-01 -8,166E-02 -3,301E-03 -1,274E-03  0,000E 00

     4,697E-09  5,974E-04  2,522E-05 -5,603E-04 -3,863E-04 -1,715E-05 -1,136E-04  0,000E 00

     3,191E-04 -2,197E-01 -5,603E-04  2,137E-01  7,901E-02  3,190E-03  1,419E-03  0,000E 00

     1,321E-05 -8,166E-02 -3,863E-04  7,901E-02  3,554E-02  1,467E-03 -2,930E-03  0,000E 00

     5,342E-07 -3,301E-03 -1,715E-05  3,190E-03  1,467E-03  6,092E-05  3,287E-05  0,000E 00

     4,024E-06 -1,274E-03 -1,136E-04  1,419E-03 -2,930E-03  3,287E-05  9,999E-01  0,000E 00

    -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                    P            E          RT ASC       ARG PER         I          MEAN ANOM       BIAS 1       BIAS 2

 ELEMENTS       90,272261     0,006584    197,472000    23,199500     49,992500     97,493100     35,000000     0,000000
 DELTAS          2,773117    -0,004797     -0,056703    -0,373741      0,145570      0,723153      0,003441     0,000000

                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS          X              Y              Z             XDOT           YDOT          ZDOT

 0 242 16 59 37,99      4852,1003     -2156,9350      4238,2407       5,073158       4,589212      -3,423940

   OLD RMS =     100000,000000   NEW RMS =       15,33565   CHANGE IN RMS =          0,999847
```

```
VARIANCE-COVARIANCE MATRIX


   8,154E-06  4,501E-05  8,058E-08 -5,151E-05 -9,979E-05 -3,879E-06  6,396E-06  0,000E 00

   4.501E-05  4,671E-01  4,467E-04 -4,639E-01 -4,662E-02 -1,957E-03  3,449E-05  0,000E 00

   8,058E-08  4,467E-04  2,592E-05 -4,179E-04 -2,554E-04 -1,258E-05 -1,090E-04  0,000E 00

  -5,151E-05 -4,639E-01 -4,179E-04  4,609E-01  4.573E-02  1,916E-03  1,306E-04  0,000E 00

  -9,979E-05 -4,662E-02 -2,554E-04  4,573E-02  1,217E-02  5,486E-04 -3,213E-03  0,000E 00

  -3,879E-06 -1,957E-03 -1,258E-05  1,916E-03  5,486E-04  2,507E-05  1,822E-05  0,000E 00

   6,396E-06  3,449E-05 -1,090E-04  1,306E-04 -3,213E-03  1,822E-05  9,999E-01  0,000E 00

  -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                    P            E          RT ASC       ARG PER        I         MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS       93,045398     0,001787   197,415297    27,320759    50,138370    98,216253    35,003441     0,000000
 DELTAS          0,225639    -0,000397    -0,025639     0,147726    -0,056096     0,052240     0,003753     0,000000

                    PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS           X             Y              Z             XDOT          YDOT          ZDOT

 0 242 15 59 37,99       4872,1293    -2155,4097      4233,1062      5,058530      4,594329      3.433320

   OLD RMS =         15,335945   NEW RMS =        1,58542   CHANGE IN RMS =         0,896620
```

```
VARIANCE-COVARIANCE MATRIX


   8,250E-06  9,368E-05  1,334E-07 -9,995E-05 -1,037E-04 -4,055E-06  6,335E-06  0,000E 00

   9,368E-05  4,783E-01  3,880E-04 -4,758E-01 -3,773E-02 -1,598E-03  1,495E-04  0,000E 00

   1,334E-07  3,880E-04  2,587E-05 -3,600E-04 -2,441E-04 -1,216E-05 -1,092E-04  0,000E 00

  -9,995E-05 -4,758E-01 -3,600E-04  4,734E-01  3,696E-02  1,560E-03  1,575E-05  0,000E 00

  -1,037E-04 -3,773E-02 -2,441E-04  3,696E-02  1,060E-02  4,851E-04 -3,214E-03  0,000E 00

  -4,055E-06 -1,598E-03 -1,216E-05  1,560E-03  4,851E-04  2,257E-05  1,785E-05  0,000E 00

   6,335E-06  1,495E-04 -1,092E-04  1,575E-05 -3,214E-03  1,785E-05  9,999E-01  0,000E 00

  -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                  P             E          RT ASC       ARG PER         I          MEAN ANOM      BIAS 1        BIAS 2

 ELEMENTS     93,271237      0,001391    197,389655    27,468455    50,032283    98,265493    35,007195     0,000000
 DELTAS        0,000356      0,000002      0,000113    -0,256935    -0,000400     0,257603     0,003867     0,000000


                  PREDICTION EPHEMERIS
YY DDD HH MM SS.SS          X             Y              Z             XDOT          YDOT          ZDOT

 0 242 15 59 37,99     4872,2658    -2155,3995     4233,0927      5,058396      4,594349     -3,433328

   OLD RMS =          1,585415   NEW RMS =        0,70219   CHANGE IN RMS =        0,557092
```

```
VARIANCE-COVARIANCE MATRIX


  5,242E-06  9,307E-05  1,252E-07 -9,930E-05 -1,043E-04 -4,075E-06  6,345E-06  0,000E 00

  9,307E-05  4,775E-01  3,904E-04 -4,753E-01 -3,810E-02 -1,616E-03  1,502E-04  0,000E 00

  1,252E-07  3,904E-04  2,591E-05 -3,620E-04 -2,473E-04 -1,232E-05 -1,092E-04  0,000E 00

 -9,930E-05 -4,753E-01 -3,620E-04  4,729E-01  3,731E-02  1,578E-03  1,464E-05  0,000E 00

 -1,043E-04 -3,810E-02 -2,473E-04  3,731E-02  1,033E-02  4,955E-04 -3,209E-03  0,000E 00

 -4,075E-06 -1,616E-03 -1,232E-05  1,578E-03  4,955E-04  2,298E-05  1,805E-05  0,000E 00

  6,345E-06  1,502E-04 -1,092E-04  1,464E-05 -3,209E-03  1,805E-05  9,999E-01  0,000E 00

 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                 P            E          RT ASC       ARG PER        I        MEAN ANOM      BIAS 1       BIAS 2

ELEMENTS     93,272094     0,001393   197,389771   27,211550   50,091583   98,526096   35,011062    0,000000
DELTAS       -0,001375     0,000009     0,000126   -0,241461   -0,000040    0,240453    0 003907    0,000000

                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS      X             Y             Z            XDOT          YDOT          ZDOT

0 242 18 59 37,99   4572,2560    -2155,3886    4233,0813     5,058423      4,594329     -3,433274

  OLD RMS =         0,702193    NEW RMS =       0,65572    CHANGE IN RMS =        0,066186
```

```
POINT     T-TZERO    STAR         DB
        HH MM SS,SS              (PRED)   RESIDUAL
  1     19  8 39,00    5        43,39093   -0,006  -8,9E-01 -3,0E-01 -3,1E-01 -3,0E-01 -1,3E-01  1,5E 00 -4,3E-04
  2     19  9 40,00    6        30,40862    0,185   9,8E-01  5,9E-01  2,1E-01  5,9E-01 -9,4E-02  2,8E 00 -4,3E-04
  3     19 10 33,00    2        63,58851    0,017  -5,2E-01  3,1E-01 -2,0E-01  3,1E-01 -1,0E-01  2,5E 00 -4,3E-04
  4     19 11 29,00    5        39,74656   -0,080  -9,4E-01 -3,7E-01 -1,3E-01 -3,7E-01 -1,4E-01  1,8E 00 -4,3E-04
  5     19 12 31,00    6        36,73563   -0,303   9,9E-01  5,6E-01  6,3E-02  5,6E-01 -8,8E-02  2,9E 00 -4,3E-04
  6     19 13 12,00    2        65,92520   -0,030  -6,7E-01  1,4E-01 -3,0E-02  1,4E-01 -1,1E-01  2,6E 00 -4,3E-04
  7     19 14 45,00    5        34,74719    0,043  -9,7E-01 -4,3E-01  6,8E-02 -4,2E-01 -1,5E-01  2,2E 00 -4,3E-04
  8     19 15 55,00    6        43,67236    0,068   9,8E-01  5,0E-01 -1,3E-01  5,0E-01 -8,4E-02  2,9E 00 -4,3E-04
  9     19 16 57,00    2        66,07088    0,126  -8,2E-01 -1,2E-01  2,2E-01 -1,2E-01 -1,3E-01  2,6E 00 -4,3E-04
 10     19 18 27,00    5        28,36735    0,036  -9,4E-01 -4,7E-01  2,8E-01 -4,7E-01 -1,6E-01  2,6E 00 -4,3E-04
 11     19 19 25,00    6        49,74802    0,050   9,1E-01  4,0E-01 -3,5E-01  4,0E-01 -8,5E-02  2,9E 00 -4,3E-04
 12     19 20 19,00    2        63,06360   -0,210  -8,7E-01 -3,4E-01  4,1E-01 -3,4E-01 -1,5E-01  2,7E 00 -4,3E-04
 13     19 21 12,00    5        23,33329    0,096  -9,0E-01 -4,8E-01  4,3E-01 -4,8E-01 -1,6E-01  2,3E 00 -4,3E-04

***CONVERGED***
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LSDC RUN #3

```
INPUT ORBITAL ELEMENTS
EPOCH(YDDD.DD...)    MEAN ANOM   RT ASC    ARG PER      ECC        INC      PERIOD
     3242.79141188   97.4931   197.4720   28,1995  0,0065843  49,9928   90,27228

                        PREDICTION EPHEMERIS
  YY DDD HH MM SS,SS            X                Y                Z              XDOT             YDOT             ZDOT

   3 242 18 59 37,99       4817,1654      -2100,7027       4110,6079       5,077818        4,664743        -3,484950


    POINT      T-TZERO    STAR          OB
            HH MM SS,SS                (PRED)   RESIDUAL
      1     19  8 39,00    1       311,40029  115,983     0,0E 00  8,6E-04 -2,3E-04  5,8E-03 -4,4E-02  7,3E-01  0,0E 00  1,6E-04
      2     19  9 40,00    6        28,32465    2,269     9,8E-01  5,9E-01  1,9E-01  6,0E-01 -1,1E-01  3,2E 00 -5,2E-04 -0,0E 00
      3     19 10 33,00    2        61,00380    2,602    -5,5E-01  2,8E-01 -1,7E-01  2,9E-01 -1,2E-01  2,9E 00 -5,2E-04 -0,0E 00
      4     19 11 29,00    5        35,97429    3,693    -9,5E-01 -3,8E-01 -1,0E-01 -3,6E-01 -1,6E-01  2,3E 00 -5,1E-04 -0,0E 00
      5     19 12 31,00    1       320,21723  115,726     0,0E 00  9,0E-04 -3,5E-05  4,4E-03 -4,4E-02  9,0E-01  0,0E 00  1,6E-04
      6     19 13 12,00    2        63,15140    2,744    -7,0E-01  1,0E-01  4,2E-03  1,1E-01 -1,3E-01  3,0E 00 -5,1E-04 -0,0E 00
      7     19 14 45,00    5        30,83855    3,951    -9,7E-01 -4,4E-01  1,0E-01 -4,2E-01 -1,6E-01  2,7E 00 -5,1E-04 -0,0E 00
      8     19 15 55,00    6        41,96896    1,771     9,7E-01  4,8E-01 -1,7E-01  4,8E-01 -1,0E-01  3,4E 00 -5,1E-04 -0,0E 00
      9     19 16 57,00    1       327,01051  112,732     0,0E 00  8,7E-04  1,9E-04  2,4E-03 -4,4E-02  1,0E 00  0,0E 00  1,6E-04
     10     19 18 27,00    5        24,27630    4,127    -9,3E-01 -4,7E-01  3,2E-01 -4,6E-01 -1,7E-01  3,1E 00 -5,1E-04 -0,0E 00
     11     19 19 25,00    6        48,00413    1,794     8,9E-01  3,8E-01 -3,9E-01  3,8E-01 -1,0E-01  3,3E 00 -5,0E-04 -0,0E 00
     12     19 20 19,00    2        59,20520    3,649    -8,7E-01 -3,7E-01  4,4E-01 -3,7E-01 -1,7E-01  3,2E 00 -5,0E-04 -0,0E 00
     13     19 21 12,00    1       329,65488  112,928     0,0E 00  7,7E-04  3,9E-04  3,4E-04 -4,4E-02  1,0E 00  0,0E 00  1,6E-04
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VARIANCE-COVARIANCE MATRIX


  7,630E-06 -1,794E-04 -3,046E-07  1,642E-04  3,090E-05  1,352E-06 -3,508E-04 -3,502E-04

 -1,794E-04  1,139E-01  1,903E-04 -1,108E-01 -4,267E-02 -1,732E-03 -8,654E-03 -8,054E-03

 -3,046E-07  1,903E-04  3,092E-05 -1,746E-04 -7,689E-05 -3,379E-06  9,780E-05  1,975E-04

  1,642E-04 -1,108E-01 -1,746E-04  1,079E-01  4,139E-02  1,679E-03  9,002E-03  8,222E-03

  3,090E-05 -4,267E-02 -7,689E-05  4,139E-02  1,823E-02  7,519E-04  4,616E-03  7,851E-03

  1,352E-06 -1,732E-03 -3,379E-06  1,679E-03  7,519E-04  3,115E-05  2,538E-04  2,372E-04

 -3,508E-04 -8,654E-03  9,780E-05  9,002E-03  4,616E-03  2,538E-04  9,286E-01 -6,883E-02

 -3,502E-04 -8,054E-03  1,975E-04  8,222E-03  7,851E-03  2,372E-04 -6,883E-02  9,336E-01


                  P            E          RT ASC      ARG PER         I         MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS     90,272281     0,006584   197,472000    28,199500    49,992800    97,493100    35,000000    -5,000000
 DELTAS        2,525439    -0,002042     0,167637    -8,486241     0,001799     7,847477     0,650053     0,651636

                  PREDICTION EPHEMERIS
YY DDD HH MM SS.SS          X              Y              Z             XDOT          YDOT          ZDOT

 0 242 15 59 37,99     4845,4150     -2189,4783      4235,6761      5,089947      4,602495     -3,388691

   OLD RMS =     100000,000000    NEW RMS =       19,64694    CHANGE IN RMS =          0,999804
```

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VARIANCE-COVARIANCE MATRIX


    6.765E-06 -1,685E-04 -1,191E-06  1,546E-04  2,889E-05  1,452E-06 -3,488E-04 -3,466E-04

   -1,685E-04  1,390E-01  2,186E-04 -1,358E-01 -4,485E-02 -1,934E-03 -4,079E-03 -3,890E-03

   -1,191E-06  2,186E-04  3,216E-05 -2,016E-04 -9,702E-05 -4,482E-06  1,131E-04  2,046E-04

    1,546E-04 -1,358E-01 -2,016E-04  1,328E-01  4,352E-02  1,875E-03  4,499E-03  4,125E-03

    2,889E-05 -4,485E-02 -9,702E-05  4,352E-02  1,871E-02  8,191E-04  3,604E-03  6,733E-03

    1,452E-06 -1,934E-03 -4,482E-06  1,875E-03  8,191E-04  3,599E-05  2,063E-04  1,906E-04

   -3,488E-04 -4,079E-03  1,131E-04  4,499E-03  3,604E-03  2,063E-04  9.431E-01 -5,366E-02

   -3,466E-04 -3,890E-03  2,046E-04  4,125E-03  6,733E-03  1.906E-04 -5.366E-02  9,493E-01


                     P            E          RT ASC       ARG PER         I         MEAN ANOM       BIAS 1        BIAS 2

 ELEMENTS       92,797719     0,004542    197,639637    19,713259    49,994599    105,340577    35,680053    -4.348364
 DELTAS         -0,396345     0,003164     -0,273270    -7,074237     0,035317      7,335240    -0,167440    -0,164490

                 PREDICTION EPHEMERIS
YY DDD HH MM SS,SS          X            Y              Z             XDOT          YDOT          ZDOT

 0 242 18 59 37.99     4872,9389    -2163,3818     4205,9715      5,070827      4,589714     -3,426394

   OLD RMS =        19,646937    NEW RMS =        1,44662    CHANGE IN RMS =        0,926369
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VARIANCE-COVARIANCE MATRIX


     6,964E-06 -1,184E-04 -1,146E-06  1,016E-04  6,946E-05  3.336E-06 -3.338E-04 -3.313E-04

    -1,184E-04  2,512E-02  8,546E-05 -2,384E-02 -1,765E-02 -7,807E-04 -2,820E-04 -2,758E-04

    -1,146E-06  8,546E-05  3,251E-05 -6.633E-05 -1,238E-04 -5,751E-06  1,060E-04  1,988E-04

     1,016E-04 -2,384E-02 -6,633E-05  2,271E-02  1,592E-02  7,028E-04  5,618E-04  3,815E-04

     6,946E-05 -1,765E-02 -1,238E-04  1,592E-02  2,372E-02  1,060E-03  5,336E-03  8,323E-03

     3,336E-06 -7,807E-04 -5,751E-06  7,028E-04  1,060E-03  4,750E-05  2,845E-04  2,620E-04

    -3,338E-04 -2,820E-04  1,060E-04  5,618E-04  5,336E-03  2,845E-04  9,435E-01 -5,336E-02

    -3,313E-04 -2,758E-04  1,988E-04  3,815E-04  8,323E-03  2,620E-04 -5,336E-02  9,496E-01


                       P            E        RT ASC      ARG PER        I        MEAN ANOM      BIAS 1       BIAS 2
  ELEMENTS         92,401371     0,007726  197,366367   12,639022   50,079916  112,675817    35,512613    -4,512854
  DELTAS           -0,072288     0,000557    0,018333    2,128853   -0,005142   -2,205382    -0,139115    -0,137401

                    PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS      X             Y              Z            XDOT          YDOT          ZDOT
  0 242 18 59 37,99   4870,5121    -2161,2091     4203,3917      5,074805      4,591415     -3,423914
   OLD RMS =         1,446619   NEW RMS =       0,54829   CHANGE IN RMS =         0,620984
```

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VARIANCE-COVARIANCE MATRIX


  6,968E-06 -1,246E-04 -1,123E-06  1,080E-04  6,647E-05  3,168E-06 -3,353E-04 -3,327E-04

 -1,246E-04  2,906E-02  1,046E-04 -2,753E-02 -2,091E-02 -9,192E-04 -1,392E-03 -1,293E-03

 -1,123E-06  1,046E-04  3,251E-05 -8,561E-05 -1,224E-04 -5,666E-06  1,066E-04  1,994E-04

  1,080E-04 -2,753E-02 -8,561E-05  2,616E-02  1,920E-02  8,426E-04  1,684E-03  1,410E-03

  6,647E-05 -2,091E-02 -1,224E-04  1,920E-02  2,346E-02  1,043E-03  5,202E-03  8,200E-03

  3,168E-06 -9,192E-04 -5,666E-06  8,426E-04  1,043E-03  4.648E-05  2.781E-04  2.559E-04

 -3,353E-04 -1,392E-03  1,066E-04  1,684E-03  5,202E-03  2.781E-04  9,434E-01 -5,344E-02

 -3,327E-04 -1,293E-03  1,994E-04  1,410E-03  8,200E-03  2,559E-04 -5,344E-02  9,495E-01


                     P          E        RT ASC       ARG PER        I       MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS       92,329083   0,008283   197,384700   14,767874   50,074774   110,470435   35,373498   -4,650255
 DELTAS          0,016346  -0,000107     0,003359   -0,001003   -0,002239     0,004149   -0,124620   -0,123666

                  PREDICTION EPHEMERIS
YY DDD HH MM SS,SS        X            Y             Z            XDOT          YDOT          ZDOT

 0 242 18 59 37,99    4870,4354   -2161,8408    4204,0312     5,074632      4,591636     -3,423520

   OLD RMS =         0,548292   NEW RMS =        0,45641   CHANGE IN RMS =         0,167590
```

```
POINT     T-TZERO     STAR        OB
        HH MM SS,SS             (PRED)    RESIDUAL
   1    19  8 39,00     1     423,65142    -1,268   0,0E 00   8,5E-04  -2,5E-04   5,7E-03  -4,5E-02   9,8E-01   0,0E 00   1,6E-04
   2    19  9 40,00     6      30,44729     0,147   9,8E-01   5,9E-01   2,1E-01   5,9E-01  -9,4E-02   3,0E 00  -4,3E-04  -0,0E 00
   3    19 10 33,00     2      63,58882     0,017  -5,2E-01   3,0E-01  -1,9E-01   3,1E-01  -1,0E-01   2,7E 00  -4,3E-04  -0,0E 00
   4    19 11 29,00     5      39,60903     0,058  -9,4E-01  -3,7E-01  -1,3E-01  -3,6E-01  -1,4E-01   2,2E 00  -4,3E-04  -0,0E 00
   5    19 12 31,00     1     436,57157    -0,629   0,0E 00   9,0E-04  -5,7E-05   3,7E-03  -4,5E-02   1,0E 00   0,0E 00   1,6E-04
   6    19 13 12,00     2      65,94142    -0,046  -6,8E-01   1,4E-01  -2,6E-02   1,4E-01  -1,1E-01   2,7E 00  -4,3E-04  -0,0E 00
   7    19 14 45,00     5      34,66957     0,120  -9,7E-01  -4,3E-01   7,1E-02  -4,2E-01  -1,5E-01   2,5E 00  -4,3E-04  -0,0E 00
   8    19 15 55,00     6      43,77476    -0,035   9,8E-01   4,9E-01  -1,4E-01   4,9E-01  -9,3E-02   2,8E 00  -4,3E-04  -0,0E 00
   9    19 16 57,00     1     441,29901    -1,556   0,0E 00   8,8E-04   1,7E-04   1,2E-03  -4,6E-02   1,1E 00   0,0E 00   1,6E-04
  10    19 18 27,00     5      28,30733     0,096  -9,4E-01  -4,7E-01   2,8E-01  -4,6E-01  -1,6E-01   2,8E 00  -4,3E-04  -0,0E 00
  11    19 19 25,00     6      49,80132    -0,003   9,1E-01   4,0E-01  -3,5E-01   3,9E-01  -8,4E-02   2,6E 00  -4,3E-04  -0,0E 00
  12    19 20 19,00     2      63,00684    -0,153  -8,7E-01  -3,4E-01   4,1E-01  -3,4E-01  -1,5E-01   2,8E 00  -4,3E-04  -0,0E 00
  13    19 21 12,00     1     441,11153     1,471   0,0E 00   7,9E-04   3,6E-04  -1,4E-03  -4,6E-02   1,0E 00   0,0E 00   1,6E-04

***CONVERGED***
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LSDC RUN #4

```
INPUT ORBITAL ELEMENTS
EPOCH(YDDD.DD...)     MEAN ANOM    RT ASC    ARG PER      ECC          INC      PERIOD
    3242.79054398    97.4931    207.4720    28.1995  0.0065843  40.9928  103.21293


                             PREDICTION EPHEMERIS
 YY DDD  HH  MM  SS.SS           X                 Y                 Z              XDOT             YDOT              ZDOT

  3 242 18 58 23.00        5889.2182       -1929.6326        3848.9428         3.752586          5.665651         -2.854421



  POINT      T-TZERO    STAR         OB
           HH MM  SS.SS            (PRED)    RESIDUAL
     1    18 58 23.00     6       17.75363   -14.742    8.2E-01   7.4E-01   5.4E-01   7.5E-01  -9.2E-02   1.7E 00  -3.0E-04
     2    18 59 23.00     5       57.64958   -10.607   -6.3E-01  -3.1E-01  -7.3E-01  -2.9E-01  -9.3E-02  -2.1E-01  -3.0E-04
     3    19  0 23.00     3       26.75193   -20.205    2.7E-01   7.5E-01  -4.8E-01   7.6E-01  -8.5E-02   2.0E 00  -3.0E-04
     4    19  1 23.00     6       25.66265   -16.033    9.8E-01   7.5E-01   4.5E-01   7.6E-01  -8.1E-02   2.0E 00  -3.0E-04
     5    19  2 23.00     5       54.12285    -7.644   -7.6E-01  -3.9E-01  -5.9E-01  -3.8E-01  -9.8E-02  -8.0E-03  -2.9E-04
     6    19  3 23.00     3       34.72725   -18.155    2.1E-01   7.5E-01  -3.9E-01   7.6E-01  -7.4E-02   2.2E 00  -2.9E-04
     7    19  4 23.00     6       33.62294   -16.833    9.3E-01   7.5E-01   3.6E-01   7.6E-01  -7.1E-02   2.2E 00  -2.9E-04
     8    19  5 23.00     5       49.79943    -5.298   -9.6E-01  -4.6E-01  -4.3E-01  -4.5E-01  -1.0E-01   2.4E-01  -2.9E-04
     9    19  6 23.00     3       42.57228   -15.742    1.5E-01   7.3E-01  -3.0E-01   7.4E-01  -6.5E-02   2.3E 00  -2.9E-04
    10    19  7 23.00     6       41.46749   -17.567    9.7E-01   7.4E-01   2.6E-01   7.4E-01  -6.1E-02   2.3E 00  -2.9E-04
    11    19  8 23.00     5       44.87134    -3.035   -9.2E-01  -5.1E-01  -2.8E-01  -4.9E-01  -1.1E-01   5.4E-01  -2.9E-04
    12    19  9 23.00     3       50.15637   -13.344    8.1E-02   7.0E-01  -1.9E-01   7.1E-01  -5.6E-02   2.3E 00  -2.9E-04
    13    19 10 23.00     6       49.06944   -18.193    9.9E-01   7.1E-01   1.4E-01   7.1E-01  -5.3E-02   2.3E 00  -2.9E-04
    14    19 11 23.00     5       39.51140    -1.546   -9.5E-01  -5.4E-01  -1.2E-01  -5.3E-01  -1.2E-01   5.5E-01  -2.9E-04
    15    19 12 23.00     3       57.31921   -11.031    1.7E-03   6.6E-01  -6.6E-02   6.6E-01  -5.0E-02   2.3E 00  -2.9E-04
    16    19 13 23.00     6       56.27505   -15.957    9.9E-01   6.7E-01   1.9E-02   6.6E-01  -4.6E-02   2.3E 00  -2.9E-04
    17    19 14 23.00     5       33.87159    -0.651   -9.7E-01  -5.6E-01   2.9E-02  -5.5E-01  -1.3E-01   1.2E 00  -2.9E-04
    18    19 15 23.00     3       63.84917    -8.033   -8.8E-02   5.9E-01   7.6E-02   5.8E-01  -4.6E-02   2.2E 00  -2.9E-04
    19    19 16 23.00     6       62.88309   -19.734    9.7E-01   6.0E-01  -1.2E-01   5.9E-01  -4.3E-02   2.1E 00  -2.9E-04
    20    19 17 23.00     5       28.08784     0.227   -9.6E-01  -5.7E-01   1.8E-01  -5.6E-01  -1.4E-01   1.5E 00  -2.9E-04
    21    19 18 23.00     3       69.46101    -4.927   -1.9E-01   4.8E-01   2.4E-01   4.8E-01  -4.7E-02   2.0E 00  -2.9E-04
    22    19 19 23.00     6       68.62216   -20.366    9.1E-01   5.0E-01  -2.8E-01   5.0E-01  -4.4E-02   2.0E 00  -2.9E-04
    23    19 20 23.00     5       22.25754     0.442   -9.2E-01  -5.6E-01   3.2E-01  -5.5E-01  -1.4E-01   1.7E 00  -2.9E-04
    24    19 21 23.00     3       73.78650    -2.050   -3.0E-01   3.4E-01   4.1E-01   3.4E-01  -5.5E-02   1.9E 00  -2.9E-04
    25    19 22 23.00     6       73.13834   -20.751    8.1E-01   3.7E-01  -4.6E-01   3.6E-01  -5.2E-02   1.8E 00  -2.9E-04
    26    19 23 23.00     5       16.59804     0.619   -8.7E-01  -5.4E-01   4.6E-01  -5.3E-01  -1.5E-01   1.9E 00  -2.9E-04
    27    19 24 23.00     3       76.41327     0.334   -4.0E-01   1.6E-01   5.9E-01   1.6E-01  -7.1E-02   1.8E 00  -2.9E-04
    28    19 25 23.00     6       76.02377   -21.132    6.4E-01   1.9E-01  -6.3E-01   1.8E-01  -6.7E-02   1.7E 00  -2.9E-04
    29    19 26 23.00     5       11.15479     0.396   -7.9E-01  -5.1E-01   5.9E-01  -5.0E-01  -1.5E-01   2.1E 00  -2.9E-04
    30    19 27 23.00     3       77.00779     1.485   -4.8E-01  -4.3E-02   7.3E-01  -4.7E-02  -9.5E-02   1.8E 00  -2.9E-04
    31    19 28 23.00     6       76.92548   -21.189    4.1E-01  -1.1E-02  -7.7E-01  -1.6E-02  -9.1E-02   1.7E 00  -2.9E-04
```

```
VARIANCE-COVARIANCE MATRIX


      7,287E-07 -1,864E-05  8,971E-08  1,772E-05  1,131E-06  1,164E-07 -2,782E-05  0,000E 00

     -1,864E-05  1,097E-02  1,294E-05 -1,080E-02 -2,310E-03 -1,083E-04  5,236E-03  0,000E 00

      8,971E-08  1,294E-05  6,631E-07 -1,253E-05 -5,668E-06 -3,149E-07 -6,507E-05  0,000E 00

      1,772E-05 -1,080E-02 -1,253E-05  1,062E-02  2,263E-03  1,058E-04 -5,050E-03  0,000E 00

      1,131E-06 -2,310E-03 -5,668E-06  2,263E-03  6,271E-04  3,006E-05 -3,250E-03  0,000E 00

      1,164E-07 -1,083E-04 -3,149E-07  1,058E-04  3,006E-05  1,526E-06 -5,730E-06  0,000E 00

     -2,782E-05  5,236E-03 -6,507E-05 -5,050E-03 -3,250E-03 -5,730E-06  9,827E-01  0,000E 00

     -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                     P            E          RT ASC       ARG PER         I          MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS       103,212929     0,006584    207,472000    28,199500     40,992800     97,493100    90,000000     0,000000
 DELTAS         -12,170271    -0,003041     -8,742889   -77,757167     10,863981     71,372276     3,491536     0,000000

                      PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS            X              Y              Z            XDOT          YDOT          ZDOT

 0 242 18 58 23,00       4289,4423     -2365,0147      4609,5123      5.595684      4,353995     -2,962741

   OLD RMS =    100000,000000   NEW RMS =      130,89202   CHANGE IN RMS =         0,995691
```

```
VARIANCE-COVARIANCE MATRIX


     4,373E-07  1,693E-05  4,496E-08 -1,749E-05  1,272E-06  7,789E-08 -9,728E-06  0,000E 00

     1,693E-05  1,845E-02 -4,607E-06 -1,860E-02  1,682E-03  5,639E-05  1,090E-02  0,000E 00

     4,496E-08 -4,607E-06  4,463E-07  4,569E-06 -1,175E-08  1,303E-08 -4,084E-05  0,000E 00

    -1,749E-05 -1,860E-02  4,569E-06  1,874E-02 -1,695E-03 -5,690E-05 -1,075E-02  0,000E 00

     1,272E-06  1,682E-03 -1,175E-08 -1,695E-03  1,776E-04  5,699E-06 -2,602E-03  0,000E 00

     7,789E-08  5,639E-05  1,303E-08 -5,690E-05  5,699E-06  2,199E-07  1,416E-05  0,000E 00

    -9,728E-06  1,090E-02 -4,084E-05 -1,075E-02 -2,602E-03  1,416E-05  9,827E-01  0,000E 00

    -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                   P            E          RT ASC      ARG PER        I        MEAN ANOM      BIAS 1       BIAS 2

 ELEMENTS      91,042658     0,003543    195,729111   -49,557667   51,856781  165,865376   93,491536    0,000000
 DELTAS         1,397840    -0,001885     -0,366110    12,354925   -1,753997  -11,014213    0,196405    0,000000


                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS        X            Y            Z           XDOT         YDOT         ZDOT

 0 242 18 58 23,00    4464,5917   -2503,0630    4487,6360     5,487498     4,401231    -2,997213

   OLD RMS =       130,892022   NEW RMS =      29,86857   CHANGE IN RMS =        0,771809
```

```
VARIANCE-COVARIANCE MATRIX


      4,854E-07  4,382E-05  4,500E-08 -4,451E-05  2,287E-06  1,397E-07 -5,174E-06  0,000E 00

      4,382E-05  6,866E-02 -5,241E-06 -6,899E-02  3,870E-03  1,585E-04  2.405E-02  0,000E 00

      4,500E-08 -5,241E-06  4,576E-07  5,191E-06  3,226E-07  2,153E-08 -4,272E-05  0,000E 00

     -4,451E-05 -6,899E-02  5,191E-06  6,932E-02 -3,889E-03 -1,594E-04 -2,392E-02  0,000E 00

      2,287E-06  3,870E-03  3,226E-07 -3,889E-03  2,585E-04  1,036E-05 -2,328E-03  0,000E 00

      1,397E-07  1,585E-04  2,153E-08 -1,594E-04  1,036E-05  4,615E-07  2,824E-05  0,000E 00

     -5,174E-06  2,405E-02 -4,272E-05 -2,392E-02 -2,328E-03  2,824E-05  9,853E-01  0,000E 00

     -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                    P              E           RT ASC       ARG PER          I         MEAN ANOM      BIAS 1        BIAS 2

  ELEMENTS      92,940497      0,001658    197,863001    -37,172742    50,102784    157,851163     93,687941      0,000000
  DELTAS         0,177780     -0,000418     -0,010783     23,328572    -0,044560    -23,262753      0,056310      0,000000

                        PREDICTION EPHEMERIS
  YY DDD HH MM SS,SS          X             Y              Z              XDOT          YDOT          ZDOT

  0 242 18 58 23,00      4476,0419     -2502,2184     4482,9996      5.480707      4,406501     -3,002436

    OLD RMS =          29,868575    NEW RMS =        3,00528    CHANGE IN RMS =        0,899383
```

```
VARIANCE-COVARIANCE MATRIX

   4.660E-07  1,161E-05  3.559E-08 -1,222E-05  1,359E-06  1,118E-07 -1,053E-05  0,000E 00

   1,161E-05  3,455E-02 -1,837E-05 -3,472E-02  2,057E-03  9,783E-05  2,010E-02  0,000E 00

   3,559E-08 -1,837E-05  4,587E-07  1,838E-05 -4,934E-07 -2,543E-08 -4,455E-05  0,000E 00

  -1,222E-05 -3,472E-02  1,838E-05  3,490E-02 -2,076E-03 -9,881E-05 -1,995E-02  0,000E 00

   1,359E-06  2,057E-03 -4,934E-07 -2,076E-03  2,489E-04  1,168E-05 -2,632E-03  0,000E 00

   1,118E-07  9,783E-05 -2,543E-08 -9,881E-05  1,168E-05  5,969E-07  2,619E-05  0,000E 00

  -1,053E-05  2,010E-02 -4,455E-05 -1,995E-02 -2,632E-03  2,619E-05  9,837E-01  0,000E 00

  -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                  P              E           RT ASC        ARG PER          I          MEAN ANOM       BIAS 1        BIAS 2

ELEMENTS      93,118278      0,001239     197,852219    -13,844170     50,058224    134,558410     93,744251      0,000000
DELTAS        -0,007349      0,000263      -0,004352     12,124223     -0,006127    -12,132537      0,026786      0,000000

                PREDICTION EPHEMERIS
YY DDD HH MM SS,SS           X              Y              Z            XDOT          YDOT          ZDOT

0 242 18 55 23,00       4478,5253     -2500,2734     4480,4072      5,480230      4,407459     -3,003642

  OLD RMS =         3,005282   NEW RMS =        1,03684   CHANGE IN RMS =        0,654994
```

```
VARIANCE-COVARIANCE MATRIX


     4,576E-07 -5,579E-06  3,581E-08  5,024E-06  6,695E-07  7,773E-08 -1,426E-05  0,000E 00

    -5,579E-06  1,393E-02 -1,434E-05 -1,395E-02  3,089E-04  1,448E-05  1,351E-02  0,000E 00

     3,581E-08 -1,434E-05  4,639E-07  1,439E-05 -8,494E-07 -4,914E-08 -4,559E-05  0,000E 00

     5,024E-06 -1,395E-02  1,439E-05  1,398E-02 -3,251E-04 -1,537E-05 -1,334E-02  0,000E 00

     6,695E-07  3,089E-04 -8,494E-07 -3,251E-04  2,189E-04  1,056E-05 -2,857E-03  0,000E 00

     7.773E-08  1,448E-05 -4,914E-08 -1,537E-05  1,056E-05  5,585E-07  1,979E-05  0,000E 00

    -1,426E-05  1,351E-02 -4,559E-05 -1,334E-02 -2,857E-03  1,979E-05  9,822E-01  0,000E 00

    -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00 -0,000E 00  1,000E 00


                       P            E         RT ASC       ARG PER         I        MEAN ANOM       BIAS 1        BIAS 2

  ELEMENTS         93,110925     0,001502   197,847867    -1,719947    50,052098   122,455872    93,771037     0,000000
  DELTAS            0,011614    -0,000054    -0,004177     0,176640    -0,003274    -0,167892     0,007481     0,000000

                 PREDICTION EPHEMERIS
 YY DDD HH MM SS,SS        X             Y             Z            XDOT          YDOT          ZDOT

 0 242 18 58 23,00     4475,9670    -2500,7114     4430,2421     5,479903      4,407491     -3,004022

   OLD RMS =          1,036640    NEW RMS =         0,87213    CHANGE IN RMS =          0,158860
```

```
POINT     T-TZERO     STAR        DB
        HH MM SS,SS             (PRED)   RESIDUAL
   1    18 58 23,00     6      3,00558     0,006    6,9E-01   5,7E-01   7,1E-01   5,7E-01  -1,3E-01   2,6E 00 -4,7E-04
   2    18 59 23,00     5     47,01996     0,023   -4,4E-01   3,4E-03  -8,2E-01   6,3E-03  -1,3E-01   1,8E 00 -4,7E-04
   3    19  0 23,00     3      6,55301    -0,006    3,1E-01   8,6E-01  -4,0E-01   8,7E-01  -1,3E-01   3,3E 00 -4,7E-04
   4    19  1 23,00     6      9,71415    -0,084    8,0E-01   6,0E-01   5,9E-01   6,0E-01  -1,2E-01   3,0E 00 -4,7E-04
   5    19  2 23,00     5     46,45963     0,019   -6,2E-01  -1,1E-01  -6,9E-01  -1,0E-01  -1,3E-01   2,1E 00 -4,7E-04
   6    19  3 23,00     3     16,63836    -0,066    2,6E-01   8,7E-01  -3,2E-01   8,7E-01  -1,1E-01   3,5E 00 -4,7E-04
   7    19  4 23,00     6     16,75932     0,031    8,9E-01   6,1E-01   4,6E-01   6,1E-01  -1,2E-01   3,2E 00 -4,7E-04
   8    19  5 23,00     5     44,65546    -0,154   -7,7E-01  -2,1E-01  -5,2E-01  -2,1E-01  -1,4E-01   2,4E 00 -4,7E-04
   9    19  6 23,00     3     26,72991     0,100    2,1E-01   8,7E-01  -2,4E-01   8,7E-01  -1,0E-01   3,5E 00 -4,7E-04
  10    19  7 23,00     6     23,82974     0,070    9,4E-01   6,1E-01   3,3E-01   6,1E-01  -1,1E-01   3.3E 00 -4,7E-04
  11    19  8 23,00     5     41.73733     0,099   -8,8E-01  -3,0E-01  -3,3E-01  -3,0E-01  -1,4E-01   2,7E 00 -4,7E-04
  12    19  9 23,00     3     36,71977     0,092    1,6E-01   8,5E-01  -1,5E-01   8,5E-01  -9.0E-02   3,4E 00 -4,7E-04
  13    19 10 23,00     6     30,74013     0,136    9,8E-01   5,8E-01   1,8E-01   5,8E-01  -1,0E-01   3,2E 00 -4,7E-04
  14    19 11 23,00     5     37,88822     0,077   -9,4E-01  -3,7E-01  -1,4E-01  -3,7E-01  -1,5E-01   2,9E 00 -4,7E-04
  15    19 12 23,00     3     46,47779    -0,190    9,5E-02   8,3E-01  -5,1E-02   8,3E-01  -8,1E-02   3,1E 00 -4,7E-04
  16    19 13 23,00     6     37,29244     0,026    9,9E-01   5,5E-01   1,6E-02   5,4E-01  -9,5E-02   3,0E 00 -4,7E-04
  17    19 14 23,00     5     33,31179    -0,121   -9,7E-01  -4,2E-01   4,3E-02  -4,2E-01  -1,6E-01   3,1E 00 -4,7E-04
  18    19 15 23,00     3     55,81388    -0,048    2,2E-02   7,8E-01   7,0E-02   7,8E-01  -7,3E-02   2,8E 00 -4,7E-04
  19    19 16 23,00     6     43,26028    -0,161    9,7E-01   4,8E-01  -1,6E-01   4.8E-01  -9,3E-02   2,8E 00 -4,7E-04
  20    19 17 23,00     5     28,21028     0,105   -9,5E-01  -4,6E-01   2,2E-01  -4,6E-01  -1,7E-01   3,2E 00 -4,7E-04
  21    19 18 23,00     3     64,41263     0,121   -7,6E-02   7,0E-01   2,2E-01   6,9E-01  -7,0E-02   2,4E 00 -4,7E-04
  22    19 19 23.00     6     48,37820    -0,122    9,1E-01   4,0E-01  -3,4E-01   4,0E-01  -9,4E-02   2,5E 00 -4,7E-04
  23    19 20 23,00     5     22,77494    -0,045   -9,1E-01  -4,8E-01   3,8E-01  -4,8E-01  -1,3E-01   3,1E 00 -4,7E-04
  24    19 21 23,00     3     71,71721     0,020   -2,0E-01   5,5E-01   4,1E-01   5,5E-01  -7.5E-02   2,0E 00 -4,7E-04
  25    19 22 23,00     6     52,34577     0,041    7,9E-01   2,9E-01  -5,3E-01   2,8E-01  -1,0E-01   2,1E 00 -4,7E-04
  26    19 23 23,00     5     17,18615     0,031   -8,4E-01  -4,8E-01   5,3E-01  -4,8E-01  -1,8E-01   3,0E 00 -4,7E-04
  27    19 24 23,00     3     76,79163    -0,045   -3,5E-01   3,1E-01   6,3E-01   3,0E-01  -9,6E-02   1,8E 00 -4,7E-04
  28    19 25 23,00     6     54,86039     0,032    6,3E-01   1,5E-01  -7,0E-01   1,5E-01  -1,1E-01   1,9E 00 -4,7E-04
  29    19 26 23,00     5     11,61872    -0,068   -7,5E-01  -4,7E-01   6,7E-01  -4,7E-01  -1,9E-01   2,7E 00 -4,7E-04
  30    19 27 23,00     3     78,46031     0,033   -4,8E-01  -2,5E-02   8,0E-01  -3,0E-02  -1,3E-01   1,8E 00 -4,7E-04
  31    19 28 23,00     6     55,68628     0,050    4,2E-01  -1,3E-03  -8,3E-01  -5,9E-03  -1,3E-01   1,6E 00 -4,7E-04


***CONVERGED***
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APPENDIX C

FORTRAN LSDC PROGRAM

This Appendix contains the FORTRAN LSDC program generated for this study. Key variables in this program are:

A. Main Program

1. Matrices

A	Partial derivative matrix at each time interval
AI	Entire partial derivative matrix
ATA	A^TWA matrix
ATB	A^TWB matrix
WRES	WB matrix

2. Vectors

DEC	Star declinations
DX	State vector corrections
OB	Star-horizon angle and height observations
RA	Star right ascensions
TIME	Observation times, days from 1 Jan 70
X,XOUT	State vector of orbital elements

3. Variables

CH	Change in RMS
CONV	Convergence criteria
COS1, COS2, COS3	Star direction cosines
DEL, RES	Observed minus predicted delta
DELP	Delta period, minutes
DELSP, DELOU, DELN	Star-horizon angle, predicted

DELX	Perturbation applied to compute partial derivatives
EPOCH	Epoch of elements in days relative to 1 Jan 70
ISTAR	Star number (1=height ob, 2-6=star ob)
ITYP	Type of correction (0=6 elements and both biases, 1=6 elements and one bias, 2=6 elements only)
KP	Number of obs
NS	Number of states being corrected
OLDX	Unperturbed state
RADUS	Satellite radius in KM
RMSN	New RMS
RMSO	Old RMS
W1, W2, W	Observation weights

B. Ephemeris subroutine (KEPH)

1. Vectors

X	Input orbital elements
Y	Output position and velocity. Position in unit vector or KM, velocity in KM/sec

2. Variables

AO	Semi-major axis at epoch, KM
ANEW	Semi-major axis at time of interest (t_i), KM
AXN	Eccentricity x cosine of argument of perigee
AYN	Eccentricity x sine of argument of perigee
BIGU	Mean argument of latitude at t_i (argument of perigee + mean anomaly), radians
DELTAT	Time from epoch to t_i, seconds
ECCO	Eccentricity at epoch

ECC	Eccentricity at t_i after drag perturbation
ECCL	Eccentricity at t_i after long period perturbation
ISW	Indicator for position output units
LO	Mean orbital longitude at epoch (mean anomaly + right ascension + argument of perigee), radians
LS	Mean orbital longitude at t_i with short period perturbations
LD	LS plus drag perturbations
LL	Long perturbation effect on LD
LT	Mean orbital longitude at t_i, radians
MO	Mean anomaly at epoch, radians
NNEW	Mean motion at t_i, radians/sec
NO	Mean motion at epoch, radians/sec
NODOT	Rate of change of mean motion due to atmospheric drag, radians/sec/sec
PL	Semi-latus rectum at t_i, KM
PO	Semi-latus rectum at epoch, KM
QO	Perigee radius at epoch, KM
RAP	Argument of perigee at epoch, radians
RAPDOT	Rate of change of argument of perigee, radians/sec
RK	Radius at t_i, KM
RINC	Inclination at epoch, radians
RINCK	Inclination at t_i, radians
RRA	Right ascension of ascending node, radians
RRADOT	Rate of change of right ascension, radians/sec
SMALLU	True argument of latitude at t_i (argument of perigee + true anomaly), radians
U	Mu of the earth, KM^3/sec^2

```
      DIMENSION ATA(8,[illegible]),TIME(200),[illegible](200),A([illegible]),L(8),M(8),ATR(8),
     1 REF(200),I(200,2),X(6),D(6),XOUT(8),RA(4),DEC(4),I[illegible]AR(200),
     2  I[illegible]AE(200),SEC[illegible]([illegible]),DUME(8)
      DATA RAD,AE,E/.01745[illegible],6378.160,1.0/
    1 D[illegible]SQ=1.0E5
      DMSM=6.0
      N=0
      IC=-1
      T=1.0
C
C--------------------------------  READ ELEMENTS  -----------------------
C
C          THE STATE VECTOR X(I) = RT ASC, ARG PER, INC, MEAN ANOMALY,
C          MEAN MOTION, AND ECC.
C          UNITS ARE CONVERTED FROM DEGREES TO RADIANS. MEAN MOTION UNITS
C          REMAIN IN REVS/DAY
C
C-----------------------------------------------------------------------
      READ(5,100)        IYR,IDAY,EPOCH,X(4),X(1),X(2),X(6),X(3),X(5),B1,B
     12,W1,W2,CONV,IOB
  100 FORMAT(         30X,I1,I3,F9.0,1X,F8.4,2(1X,F7.4)/6X,F8.0,F8.4,1X,F11
     1.8,5F5.0,I1)
      IF(IYR .LE. 0) GO TO 84
      PER=1440.0/X(5)
      II=EPOCH*1.0E8
      WRITE(6,200)       IYR,IDAY,II,X(4),X(1),X(2),X(6),X(3),PER
  200 FORMAT(1X,22HINPUT ORBITAL ELEMENTS/1X,17HEPOCH(YDDD.DD...),4X,9HM
     1EAN ANOM,3X,6HRT ASC,3X,7HARG PER,5X,3HECC,7X,3HINC,6X,6HPERIOD/7
     2X,I1,I3,1H.,I8,2X,F8.4,3X,F8.4,2X,F8.4,2X,F9.7,1X,F8.4,1X,F10.5)
      W1=1.0/(W1*RAD)
      W2=1.0/(W2/AE)
      I=1
      IH=EPOCH*24.0
      MN=EPOCH*1440.0-IH*60
      SEC=EPOCH*86400.0-MN*60-IH*3600
```

```
      EPOCH=365*IYR+IYR/4+IDAY+EPOCH1
      ITYP1=0
      ITYP2=0
      DO 96 J=1,6
      READ(5,130) RA(J),DEC(J)
  130 FORMAT(2F10.0)
      RA(J)=RA(J)*RAD
      DEC(J)=DEC(J)*RAD
      IF(J .LE. 4) X(J)=X(J)*RAD
   96 CONTINUE
      CALL KEPH(X,0.0,1,DX,RADUS)
      WRITE(6,240)
      WRITE(6,250)IYR,IDAY,IH,MM,SEC,(DX(J),J=1,6)
      WRITE(6,210)
  210 FORMAT(////4X,5HPOINT,4X,'T-TZERO',3X,4HSTAR,7X,2HOR,7X,'STAR-HOR
     1 HEIGHT'/11X,'HH MM SS.SS',10X,6H(PRED),5X,'RESIDUAL RESIDUAL'/)




C
C----------READ OBSERVATION AND STORE STAR NO IN OB WORD ----------------
C
    2 READ(5,110)ISTAR,IYR,IDOB,II,IJ,DELM,OBUF
  110 FORMAT(I1,I2,I3,2I2,F5.0,5F10.0)
      OB(I)=OBUF(IOB)
```

```
      IF(ISTAR)62,62,63
   63 IYR=IYR-70
      IHRAR(I)=II
      IMNAR(I)=IJ
      SECAR(I)=DELY
      IF(ISTAR .GT. 1) ITYP1=1
      IF(ISTAR .EQ. 1) ITYP2=1
      PY=II
      MY=IJ
      TIME(I)=(IYR*365+IYR/4+IDAR+(PY/24.0)+(MY/1440.0)-EPOCH)*86400.0 +
     1DELY
      OB(I)=OB(I)+ISTAR*1000
      I=I+1
      GO TO 2
   62 N=I
      KP=I-1
      I=1
      ITYP=ITYP1-ITYP2
      IF(B1 .EQ. 0.0 .AND. B2 .EQ. 0.0) ITYP=2
      NS=8-ABS(ITYP)
      IF(ITYP)54,55,55
   54 X(7)=B2
      GO TO 5
   55 X(7)=B1
      X(8)=B2
C
C           KEPH IS THE ANALYTIC EPHEMERIS ROUTINE -----------------------
    5 CALL KEPH(X,TIME(I),0,DX,RADUS)
C
C----------DECIDE WHICH DIRECTION COSINES TO USE FOR STAR ---------------
C
       ISTAR=OB(I)/1000.0
      IF(ISTAR-1)30,30,11
   11 COS2=COS(DEC(ISTAR-1))
      COS1=COS2*COS(RA(ISTAR-1))
      COS2=COS2*SIN(RA(ISTAR-1))
      COS3=SIN(DEC(ISTAR-1))
```

```
C
C----------DELSP IS THE PREDICTED STAR - HORIZON ANGLE IN RADIANS -------
C
      DELSP=ARCOS(-COS1*DX(1)-COS2*DX(2)-COS3*DX(3))-ARSIN((RE+(X(7)/AE
     1))/RADUS)
      DEL=(OB(I)-ISTAR*1000        )*RAD-DELSP
      RES=DEL/RAD
      DELOU=DELSP/RAD
      RADUS=RADUS*AE
      W=W1
      DO 511 LL=1,3
  511 DX(LL)=DX(LL)*RADUS
      IF(IC)48,43,48
   48 WRITE(6,220)I,IHRAR(I),IMNAR(I),SECAR(I),ISTAR,DELOU,RES,(DX(LL),
     1LL=1,3)
  220 FORMAT(4X,I3,4X,    I2,1X,I2,1X,F5.2,2X,I2,3X,F10.5,1X,F9.3,10X,4(1
     1X,F9.3))
      IF(IC)43,43,49
   30 RES=OB(I)-ISTAR*1000-X(NS)
      IF(ITYP .EQ. 2) RES=OB(I)-ISTAR*1000
      DEL=RES/AE-RADUS+RE
      RES=DEL*AE
      DELSP=RADUS-RE
      DELOU=DELSP*AE
      W=W2
      DO 512 LL=1,3
  512 DX(LL)=DX(LL)*RADUS*AE
      IF(IC)51,43,51
   51 WRITE(6,225)I,IHRAR(I),IMNAR(I),SECAR(I),ISTAR,DELOU,RES,(DX(LL),
     1LL=1,3)
  225 FORMAT(4X,I3,4X,    I2,1X,I2,1X,F5.2,2X,I2,3X,F10.5,10X,F9.3, 1X,4(
     11X,F9.3))
      IF(IC)43,43,49
C
C----------THE NEXT SECTION (DOWN TO 72) CALCULATES PARTIALS
C           BY NUMERICAL MEANS.  CHANGES TO THE ELEMENTS REFLECT
C           IN CHANGES IN THE PREDICTED OB.  THE PARTIAL IS THEN
C           DELTA OB/DELTA ELEMENT. -----------------------------------
```

```
C
 63 DO 77 IP=1,NS
    OLDX=X(IP)
    DELX=.001*X(IP)
    IF(DELX .EQ. 0.0) DELX=0.001
    X(IP)=X(IP)+DELX
    CALL KEPH(X,TIME(I),0,DX,PR)
    IF(ISTAR-1)73,74,73
 73 DEL =ARCOS(-COS1*DX(1)-COS2*DX(2)-COS3*DX(3))-ARSIN((RE+X(7)/AE)
   1/PR)
    GO TO 75
 74 DELM=PR-RE +X(NS)/AE
    IF(ITYP .EQ. 2) DELM=PR-RF
 75 A(IP)=(DELM-DELSP)/DELX
 77 X(IP)=OLDX
C----------ACCUMULATE A AND RESIDUAL MATRICES ----------------------------
    DO 3 II=1,NS
  3 AT(I,II)=A(II)*W
    WRES(I)=DEL*W
    RMSW=RMSW+DEL*DEL*W*W
 42 I=I+1
    IF(I-N)5,66,66
C
C----------AT END OF LAST OR PROCESSING, CALCULATE ATA, ATA
C          INVERSE (COVARIANCE) AND ATR MATRICES -------------------------
C
 66 IF(IC)67,67,50
 67 DO 601 II=1,8
    ATR(II)=0.0
    DO 601 IJ=1,8
    ATA(II,IJ)=0.0
601 ATA(II,II)=1.0
    DO 70 II=1,NS
    DO 70 IJ=1,NS
```

```
      DO 79 IK=1,KP
   79 ATA(II,IJ)=AI(IK,II)*AI(IK,IJ)  +ATA(II,IJ)
      IC=0
C
C---------MINV IS THE MATRIX INVERSION SUBROUTINE ---------------------
C
      CALL MINV(ATA,8,     L,M)
      DO 89 IJ=1,NS
      DO 89 IK=1,KP
   89 ATB(IJ)=AI(IK,IJ)*WRES(IK)  +ATB(IJ)
      WRITE(6,510)
  510 FORMAT(//1X,'VARIANCE-COVARIANCE MATRIX'///)
      DO 505 IJ=1,NS
  505 WRITE(6,520)(ATA(IJ,IK),IK=1,8)
  520 FORMAT(6X,8(E10.3,1X))
C
C----------CORRECTIONS TO THE ELEMENTS ARE THEN (ATA)-1(ATB) ------------
      DO 4 IJ=1,NS
      DX(IJ)=0.0
      DO 4 K=1,NS
    4 DX(IJ)=ATA(IJ,K)*ATB(K)+DX(IJ)
      DO 106 K=1,4
  106 XOUT(K)=X(K)/RAD
      PER=1440.0/X(5)
      DELP=1440.0/(X(5)+DX(5))-PER
      WRITE(6,300)PER ,X(6),(XOUT(K),K=1,4),X(7),X(8),DELP,DX(6),(XOUT(I
     1K),IK=5,8),DX(7),DX(8)
  300 FORMAT(//20X,1HP,13X,1HE,9X,'RT ASC',7X,'ARG PER', 8X,1HI, 9X,'MEA
     1N ANOM',          7X,'BIAS 1',7X,'BIAS 2'//3X,8HELEMENTS,3X,8(F
     212.6,1X)/3X,6HDELTAS,5X,8(F12.6,1X))
      DO 45 K=1,NS
   45 X(K)=X(K)+DX(K)
      IF(X(6) .LT. 0.0) X(6)=0.0
      IF(X(6) .GT. 1.0 .OR. PER .LT. 85.0) STOP
      WRITE(6,240)
      CALL KEPH(X,0.0,1,DX,RADUS)
      WRITE(6,250)IYR,IDAY,IH,IM,SEC,(DX(K),K=1,6)
```

```
C
C---------THE CHANGE IN RMS DETERMINES CONVERGENCE --------------------
C
      XNN=N-1
      RMSN=SQRT(RMSN/XNN)
      IF(RMSO-RMSN)82,83,83
   82 WRITE(6,115)RMSO,RMSN
  115 FORMAT(//5X,'DC DIVERGING,  OLD RMS = ',E12.5,'  NEW RMS = ',E12.5
     1//)
   83 CH=ABS(RMSN-RMSO)/RMSO
      WRITE(6.120)RMSO,RMSN.CH
  120 FORMAT(/5X,'OLD RMS = ',F12.3,'  NEW RMS = ',F12.3,' CHANGE IN RMS
     1 = ',F15.6/)
      RMSO=RMSN
      RMSN=0.0
C
C---------NO CONVERGENCE - APPLY CORRECTIONS AND TRY AGAIN ------------
C
      IF(CH-CONV )  84.84.85
   85 I=1
      GO TO 5
   84 IC=1
      I=1
      WRITE(6,210)
      GO TO 5
   50 WRITE(6,230)
  230 FORMAT(///5X,'*** CONVERGED ***'////)
      WRITE(6,240)
  240 FORMAT(///20X,'PREDICTION EPHEMERIS'/'  YY DDD HH MM SS.SS',9X,1HX
     1,13X,1HY,13X,1HZ,11X,4HXDOT,10X,4HYDOT,10X,4HZDOT/)
  250 FORMAT(2X.I2,1X,I3.2(1X,I2),1X,F5.2.3(3X.F11.4),3(3X.F11.6))
   90 READ(5.110)IP,IYR,IDOP,II.IJ.DELN
      IF(IYR .EQ. 0) GO TO 1
      IF(IYR .LT. 0) STOP
   47 IK=IYR-70
      RX=II
```

```
PY=IJ
DELT=(IK*365+IK/4+IDOB+(PX/24.0)+(PY/1440.0)-EPOCH)*86400.0+DELN
CALL KEPH(X,DELT,1,DX,RADUS)
WRITE(6,250)IYR,IDOB,II,IJ,DELN,(DX(J),J=1,6)
GO TO 99
END

    SUBROUTINE KEPH(X,DELTAT,ISW,Y,RK)
    DIMENSION X(8),Y(6)
    REAL J2,J3,MO,NO,NODOT,MBARO,LO,LS,LD,NNEW,LL,LT
    U=398603.2
    AE=6378.165
    J2=0.0010823
    J3=-2.55E-6
    TWOPI=6.283185308
    RINC=X(3)
    SINC=SIN(RINC)
    CINC=COS(RINC)
    MO=X(4)
    SMO=SIN(MO)
    CMO=COS(MO)
    RRA=X(1)
    SRRA=SIN(RRA)
    CRRA=COS(RRA)
    RAP=X(2)
    SRAP=SIN(RAP)
    CRAP=COS(RAP)
    NO=X(5)*TWOPI/86400.0
    NODOT=0.0001*TWOPI/(86400.0*86400.0)
    ECCO=X(6)
    C=1.-ECCO**2
    D=1.-ECCO
    AO=(U/(NO**2))**(1./3.)
    ABARO=AO
    NBARO=NO
    AXNO=ECCO*CRAP
    AYNO=ECCO*SRAP
```

```
      PO=ABARO*C
      QO=AO*D
      LO=MO+PRA+RAP
      PART=J2/((PO/AE)*(PO/AE))
      PART2=5.*SINC*SINC
      PART4=(3.+5.*CINC)/(1.+CINC)
      RAPDOT=(3./4.)*PART*NBARO*(4.-PART2)
      RRADOT=0.-1.5*PART*NBARO*CINC
      LS=LO+(NBARO+RAPDOT+RRADOT)*DELTAT
      WS=RAPDOT*DELTAT
      CWS=COS(WS)
      SWS=SIN(WS)
      WSO=RAP+WS
      OSO=PRA+RRADOT*DELTAT
      LD=LS+(NODOT/2.0)*DELTAT*DELTAT
      NNEW=NBARO+NODOT*DELTAT
      F3=(NBARO/NNEW)**(2./3.)
      ANEW=ABARO*F3
      IF(ANEW.GE.QO)GO TO 120
      ECC=0.
      GO TO 130
  120 ECC=1.-QO/ANEW
  130 PNEW=ANEW*(1.-ECC**2)
      AYNS=(ECC/ECCO)*(AXNO*CWS-AYNO*SWS)
      AYNS=(ECC/ECCO)*(AXNO*SWS+AYNO*CWS)
      PART3=(J3*AE)/(J2*PNEW)
      LL=0.-0.25*PART3*AXNS*SINC*PART4
      AYNL=0.-0.5*PART3*SINC
      IF(DELTAT.NE.0.0) GO TO 132
      LL=0.
      AYNL=0.
  132 LT=LD+LL
      AXN=AYNS
      AYN=AYNS+AYNL
      PIOU=LT-OSO
```

```
      BIGU=AMOD(BIGU,TWOPI)
      EW1=BIGU
      DO 140 K=1,20
      EW2=BIGU+AYN*SIN(EW1)-AYN*COS(EW1)
      IF(ABS(EW2-EW1).LT..0000001)GO TO 150
      EW1=EW2
  140 CONTINUE
  150 C1=COS(EW2)
      C2=SIN(EW2)
      ECE=AXN*C1+AYN*C2
      ESE=AXN*C2-AYN*C1
      ECCL=SQRT(AXN**2+AYN**2)
      PL=ANEW*(1.-ECCL**2)
      E2=1.-ECE
      R=ANEW*E2
      RDOT=(SQRT(U*ANEW)/R)*ESE
      RVDOT=SQRT(U*PL)/R
      PART5=ESE/(1.+SQRT(1.-ECCL**2))
      CU=(ANEW/R)*(C1-AXN+AYN*PART5)
      SU=(ANEW/R)*(C2-AYN-AXN*PART5)
      SMALLU=ATAN2(SU,CU)
      CTWOU=COS(SMALLU+SMALLU)
      STWOU=SIN(SMALLU+SMALLU)
      PART6=J2*(AE/PL)**2
      PART7=7.*SINC*SINC
      DELR=0.25*PART6*SINC*SINC*CTWOU
      DELU=0.-(1./8.)*PART6*(6.-PART7)*STWOU
      DELRRA=0.75*PART6*CINC*STWOU
      DELINC=0.75*PART6*SINC*CINC*CTWOU
      IF(DELTAT .NE. 0.0) GO TO 153
      DELR=0.
      DELU=0.
      DELRRA=0.
      DELINC=0.
  153 RK=R+DELR
      UK=SMALLU+DELU
```

```
      RRAK=OSO+DELRPA
      RINCK=RINC+DELINC
      SUK=SIN(UK)
      CUK=COS(UK)

      SRAK=SIN(RRAK)
      CRAK=COS(RRAK)
      SINCK=SIN(RINCK)
      CINCK=COS(RINCK)
      XDOT1=RDOT*(CUK*CRAK-SUK*SRAK*CINCK)
      XDOT2=-RVDOT*(CRAK*SUK+CUK*SRAK*CINCK)
      YDOT1=RDOT*(CUK*SRAK+SUK*CRAK*CINCK)
      YDOT2=-RVDOT*(SRAK*SUK-CUK*CRAK*CINCK)
      ZDOT1=RDOT*SUK*SINCK
      ZDOT2=RVDOT*CUK*SINCK
      Y(1)=CUK*CRAK-SUK*SRAK*CINCK
      Y(2)=CUK*SRAK+SUK*CRAK*CINCK
      Y(3)=SINCK*SUK
      Y(4)=XDOT1+XDOT2
      Y(5)=YDOT1+YDOT2
      Y(6)=ZDOT1+ZDOT2
      IF(ISK .NE. 0) GO TO 160
      RK=RK/AE
      RETURN
  160 DO 161 I=1,3
  161 Y(I)=Y(I)*RK
      RETURN
      END
```

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