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OPTIMAL DESIGN OF RESILIENT CAPACITATED NETWORKS

**Alper Atamturk
REGENTS OF THE UNIVERSITY OF CALIFORNIA**

**06/26/2018
Final Report**

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14. ABSTRACT
The performed research led to the develop mathematical models and effective computational methods for building resilience into physical and cyber-physical such networks in an optimal way that allowed not only identifying critical vulnerabilities in our infrastructure networks, but also for efficient allocation of scarce resources to address those vulnerabilities. The research also led to major advances in the dual problem of optimal persistent interdiction of an enemy network against an adversary who can shore up the links and when link capacities are probabilistic.

15. SUBJECT TERMS
Resilient network design, persistent network interdiction, nonlinear discrete optimization

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NSSEFF Final Performance Report

Grant Title

Optimal Design of Resilient Capacitated Networks

Principle Investigator

Alper Atamturk

Grant

FA9550-10-1-0168

Reporting Period

Final: May 1, 2010 – December, 2017

Scientific and Technical Activities and Findings

The NSEEFF funding has been tremendously helpful for PI's research program on resilient networks. Over the seven years, the funding supported five Ph.D. students, three postdoc fellows, and three visiting scholars. The research led to fundamental advances in the understanding of the combinatorial structure of networks subject to uncertain to capacity as well the counterpart problem of persistent interdiction of with probabilistic capacities. The research effort led to 30 published or submitted papers in major archival journals listed in the subsequent sections.

Budget

\$2,319,201 (reduced from original \$2,913,584)

Personnel

- Mark Velednitsky (PhD student)
- Birce Tezel (PhD candidate - graduated in Winter 2017)
- Andres Gómez (PhD candidate - graduated in Summer 2017)
- Chen Chen (PhD candidate - graduated in Summer 2015)
- Avinash Bhardwaj (PhD candidate - graduated in Summer 2015)
- Sarah Drewes (Postdoc fellow - completed in Summer 2011)
- Hyemin Jeon (Postdoc fellow - completed in Winter 2017)
- Ramtin Madani (Postdoc fellow - completed in Winter 2016)

- Laurent Muller (Visiting scholar - completed in Winter 2012)
- Henrik Friberg (Visiting scholar - completed in Summer 2014)
- Özge Şafak (Visiting scholar - completed in Winter 2017)
- Alper Atamturk (PI)

Andres Gómez successfully defended his thesis in the summer of 2017 and joined University of Pittsburgh faculty as an assistant professor. Birce Tezel defended her thesis in the winter of 2017 and joined Facebook as an optimization researcher. Chen Chen successfully completed their PhD theses in the summer of 2015. After a postdoc year at Columbia University he joined Ohio State University as an assistant professor. Avinash Bhardwaj is currently an postdoc fellow at UCL, Belgium and will join IIT Bombay as assistant professor in the Fall. Ramtin Madani completed his postdoc position and joined UT Arlington as an assistant professor. Hyemin Jeon completed her postdoc position in winter 2017 and joined BNSF Railroad as an optimization researcher. Ramtin Madani completed his postdoc position and joined UT Arlington as an assistant professor.

Publications

All papers are available for download at <http://ieor.berkeley.edu/~atamturk>

1. A. Atamtürk and O. Günlük “Mingling: Mixed Integer Rounding with Bounds,” *Mathematical Programming* 123, 315-338, 2010. (June)
2. M. S. Aktürk, A. Atamtürk, and S. Gürel, “Parallel Match-Up Scheduling with Manufacturing Cost Considerations,” *Journal of Scheduling* 10, 95-110, 2010. (February)
3. A. Atamtürk and V. Narayanan, “Lifting for Conic Mixed-Integer Programming,” *Mathematical Programming* 126, 351-363, 2011. (February)
4. S. Ahmed and A. Atamtürk, “Maximizing a Class of Submodular Utility Functions,” *Mathematical Programming* 128, 149-169, 2011. (June)
5. T. Siau, A. Cunha, A. Atamtürk, I-C Hsu, J. Pouliot, K. Goldberg, “IPIP: A New Approach to Inverse Planning for HDR Brachytherapy by Directly Optimizing Dosimetric Indices,” *Medical Physics* 38, 4045-4051, 2011. (July)
6. A. Atamtürk and K. Kianfar, “n-step Mingling Inequalities: Facets for Mixed-Integer Knapsack Sets,” *Mathematical Programming* 132, 79-98, 2012. (April)
7. A. Atamtürk, G. Berenguer, and Z.-J. M. Shen, “A Conic Integer Programming Approach to Stochastic Joint Location-Inventory Problems,” *Operations Research* 60, 366-381, 2012. (March–April)
8. T. Siau, A. Cunha, D. Berenson, A. Atamtürk, I-C. Hsu, J. Pouliot, K. Goldberg “NPIP: A Skew Line Needle Configuration Optimization System for HDR Brachytherapy,” *Medical Physics* 39, 4339-4346, 2012 (July).

9. A. Atamtürk, L. F. Muller, and D. Pisinger, “Separation and Extension of Cover Inequalities for Conic Quadratic Knapsack Constraints with Generalized Upper Bounds,” *INFORMS J. of Computing* 25, 420-431, 2013 (Summer)
10. S. Aktürk, A. Atamtürk and S. Gürel, “Aircraft Rescheduling with Cruise Speed Control,” *Operations Research* 62, 829-845, 2014. (July–August)
11. A. Atamtürk and A. Bhardwaj “Supermodular Covering Knapsack Polytope,” *Discrete Optimization* 18, 74-86, 2015. (November)
12. C. Chen, A. Atamtürk and S. S Oren “Bound Tightening for Alternating Current Optimal Power Flow Instances with Duality Gap,” *IEEE Transactions on Power Systems* 31, 3729-3736, 2015. (September)
13. P. Damci, S. Küçükyavuz, D. Rajan and A. Atamtürk, “A Polyhedral Study of Production Ramping,” *Mathematical Programming* 158, 175-205, 2016. (July)
14. A. Atamtürk, A. Gomez and S. Küçükyavuz, “Three-Partition Inequalities for Constant Capacity Fixed-Charge Networks,” *Networks* 67, 299-315, 2016. (March)
15. A. Atamtürk and A. Gomez “Maximizing a Class of Utility Functions over the Vertices of a Polytope,” *Operations Research* 65, 433-445, 2017 (March-April)
16. C. Chen, A. Atamtürk and S. S Oren “A Spatial Branch-and-Cut Algorithm for Non-convex QCQP with Bounded Complex Variables,” *Mathematical Programming* 165, 549–577, 2017. (October)
17. S. Fattahi, M. Ashraphijuo, J. Lavaei and A. Atamtürk “Conic Relaxations of the Unit Commitment Problems,” *Energy* 134, 1079-1095, 2017. (September)
18. A. Atamtürk, B. Tezel and S. Küçükyavuz, “Path Cover and Path Pack Inequalities for the Capacitated Fixed-Charge Network Flow Problem,” *SIAM Journal on Optimization* 27, 1943-1976, 2017. (September)
19. A. Atamtürk and A. Bhardwaj “Network Design with Probabilistic Capacities,” *Networks* 71, 16–30 2018. (January)
20. A. Atamtürk and O. Günlük, “On Capacity Models for Network Design,” *Operations Research Letters* 46, 414–417, 2018. (July)
21. A. Sen, A. Atamtürk and P. Kaminsky, “A Conic Integer Programming Approach to Constrained Assortment Optimization under the Mixed Multinomial Logit Model,” forthcoming in *Operations Research*.
22. Alper Atamtürk and Andres Gomez, “Strong Formulations for Quadratic Optimization with M-matrices and Indicator Variables,” forthcoming in *Mathematical Programming*.
23. A. Atamturk and H. Jeon, “Lifted Polymatroid Inequalities for Mean-Risk Optimization with Indicator Variables,” submitted to *INFORMS J. on Computing*.

24. A. Atamturk and A. Gomez, “Simplex QP-based Methods for Minimizing a Conic Quadratic Function over Polyhedra,” submitted to *Mathematical Programming Computation*.
25. R. Madani, A. Atamturk and A. Davoudi, “A Scalable Semidefinite Relaxation Approach to Grid Scheduling,” submitted to *Proceedings of the National Academy of Sciences*.
26. A. Atamtürk, C. Deck and H. Jeon, “Successive Quadratic Upper-Bounding for Discrete Mean-Risk Minimization and Network Interdiction,” submitted to *Journal of Global Optimization*.
27. S. Fattahi, J. Lavaei and A. Atamtürk, “A Bound Strengthening Method for Optimal Transmission Switching in Power Systems,” submitted to *IEEE Transactions on Power Systems*.
28. Ozge Safak, Alper Atamturk and M. Selim Akturk, “Accommodating New Flights into an Existing Airline Flight Schedule,” submitted to *Transportation Science*.
29. A. Atamtürk and O. Günlük, “Multi-Commodity Multi-Facility Network Design,” submitted to *Annals of Operations Research*.
30. A. Atamturk and A. Gomez, “Submodularity in conic quadratic mixed 0-1 optimization,” submitted to *Operations Research*.

Conference Proceedings

N/A

Presentations

N/A

Inventions or Patent Disclosures

N/A

Sabbatical or other professional development

N/A

Awards and Honors

N/A

Accomplishments

- The challenge in optimization over probabilistic constraints on cuts of a network has been to identify the right set of small number of constraints to perform the optimization over as it is practically impossible to include all of the exponentially many constraints. The task required decomposing the overall optimization problem into two subproblems: (a) a relaxed problem with fewer constraints and (b) a separation problem to identify missing constraints for the relaxed problem. Both of these problems are still nonlinear integer optimization problems, but practically easier to solve than the overall. Using them iteratively could potentially lead to efficient solutions. However, our initial attempts were still not computationally viable as many hours of computing were required even for small instances. Through better understanding of the structure of the problems and tangent hyperplane approximations, we have made major breakthroughs in modeling both problems in a convenient way and solving them in a manner of minutes.
- Cover and pack inequalities and their extensions have been one of the most effective tools for solving linear MIPs. A natural investigation is consider their use in nonlinear MIPs. However, doing so requires solving nonlinear integer separation and lifting problems, which are quite challenging. We have made significant advances in understanding their structure and put this understanding into algorithmic use. In two separate studies we have shown that a judicious application of special purpose separation and lifting heuristics leads to major computational improvements. In the first year we focused on cover inequalities, whereas in the second year we developed pack inequalities and their extensions for probabilistic constraints.
- Unlike probabilistic constraints on cuts of the network, probabilistic constraints on paths of a network cannot be directly modeled using conic quadratic constraints. The nonlinear constraints used for modeling involve power functions. Optimizing over binary variables involving power functions has been a major challenge for addressing probabilistic constraints on paths.

Our earlier research exploited super and submodular functional properties together with superadditive lifting for their effective solution in special cases. Recently, we have considered approximating the power functions using a series of conic quadratic functions. The approach turned out to be not only more general, but also much more effective practically. This appears to be another breakthrough that have led to orders of magnitude computational speed up in our ability to solve the corresponding problems. We expect the results to be of significant use in interdiction operations on networks.

- Even if probabilistic constraints are modeled with convex constraints, their separation problem turns out to be a combinatorial problem with a non-convex objective. Finding strong bounds from effective convexifications of the separation problem is thus becomes a very important task for computational solution of combinatorial problems with probabilistic constraints. We have recently devised new convexifications based on SOS (sum of squares) reformulations and their strengthening. Preliminary computations indicate the effectiveness of these new relaxations.

- Minimization of a conic quadratic objective over polyhedra arise in parametric value-at-risk minimization, portfolio optimization, and robust optimization with ellipsoidal objective uncertainty; and they can be solved by polynomial interior point algorithms for conic quadratic optimization. However, interior point algorithms are not well-suited for branch-and-bound algorithms for the discrete counterparts of these problems due to the lack of effective warm starts necessary for the efficient solution of convex relaxations repeatedly at the nodes of the search tree.

In order to overcome this shortcoming, we reformulate the problem using the perspective of the quadratic function. The perspective reformulation lends itself to simple coordinate descent and bisection algorithms utilizing the simplex method for quadratic programming, which makes the solution methods amenable to warm starts and suitable for branch-and-bound algorithms. We test the simplex-based quadratic programming algorithms to solve convex as well as discrete instances and compare them with the state-of-the-art approaches. The computational experiments indicate that the proposed algorithms scale much better than interior point algorithms and return higher precision solutions. In our experiments, for large convex instances, they provide up to 22x speed-up. For smaller discrete instances, the speed-up is about 13x over a barrier-based branch-and-bound algorithm and 6x over the LP-based branch-and-bound algorithm with extended formulations.

- Quadratic optimization with indicator variables and an M-matrix, i.e., a PSD matrix with non-positive off-diagonal entries, arises in image segmentation and portfolio optimization with transaction costs, as well as a substructure of general quadratic optimization problems. We prove, under mild assumptions, that the minimization problem is solvable in polynomial time by showing its equivalence to a submodular minimization problem. To strengthen the formulation, we decompose the quadratic function into a sum of simple quadratic functions with at most two indicator variables each, and provide the convex-hull descriptions of these sets. We also describe strong conic quadratic valid inequalities. Preliminary computational experiments indicate that the proposed inequalities can substantially improve the strength of the continuous relaxations with respect to the standard perspective reformulation.
- The advances in conic optimization have led to its increased utilization for modeling data uncertainty. In particular, conic mean-risk optimization gained prominence in probabilistic and robust optimization. Whereas the corresponding conic models are solved efficiently over convex sets, their discrete counterparts are intractable. In this paper, we give a highly effective successive quadratic upper-bounding procedure for discrete mean-risk minimization problems. The procedure is based on a reformulation of the mean-risk problem through the perspective of its convex quadratic term. Computational experiments conducted on the network interdiction problem with stochastic capacities show that the proposed approach yields solutions within 1-2% of optimality in a small fraction of the time required by exact search algorithms. We demonstrate the value of the proposed approach for constructing efficient frontiers of flow-at-risk vs. interdiction cost for varying confidence levels.

International Collaborations

International visiting scholars and postdocs:

- Henrik Friberg (Denmark)
- Laurent Müller (Denmark)
- Sarah Drewes (Germany)
- Birce Tezel (Turkey)

Interactions with DoD

N/A

Other contributions

N/A

Change in AFORS program manager, if any

N/A

Extensions granted or milestones slipped, if any

N/A