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Report Title

Final Report: \$16.6k Research Instrumentation in support of non-contact ultrasonic imaging STIR

ABSTRACT

Ultrasound provides an excellent method of imaging objects embedded within a medium having high water content. For example, at a frequency of 2 MHz, the imaging resolution is on the order of a millimeter. The dramatic mismatch of acoustic impedance between air and water, however, typically necessitates good acoustic contact between the transducer and the medium to be studied. This mismatch causes the transmitted intensity to drop by a factor of one thousand from that incident on the boundary. This difficulty in directly coupling acoustic energy into the medium via an air gap is the fundamental reason why non-contact ultrasound imaging remains infeasible. We propose to overcome this long standing problem by developing new methods of coupling into the medium at standoff. In particular, we believe that the acoustic nonlinearity of both the air and the medium may yield a range of effects in the vicinity of the surface permitting an efficient transmission of ultrasound from the air into the medium. Experimentally, we have conducted this study by directing commercially available sources of 45 and 96 kHz ultrasound beams at the air-water free surface and searching for anomalous behavior.

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Contents

Contents	1
Abstract	2
List of Illustrations and Figures	2
Statement of Problem Studied	2
Scientific Progress and Accomplishments	3
Theoretical Introduction	3
Impedance Mismatch	3
Westervelt Equation	4
Nonlinear Reflection off of a Curved Surface	6
Radiation Pressure	7
Viscoelastic Liquids and Shear Waves.....	8
Experimental Approach	8
Configuration of the Hypersound Speakers.....	8
Observation of Surface Deformation	10
Mhz Ultrasound Projected Out of Water.....	11
Bibliography	13

Abstract

Ultrasound provides an excellent method of imaging objects embedded within a medium having high water content. For example, at a frequency of 2 MHz, the imaging resolution is on the order of a millimeter. The dramatic mismatch of acoustic impedance between air and water, however, typically necessitates good acoustic contact between the transducer and the medium to be studied. This mismatch causes the transmitted intensity to drop by a factor of one thousand from that incident on the boundary. This difficulty in directly coupling acoustic energy into the medium via an air gap is the fundamental reason why non-contact ultrasound imaging remains infeasible. We propose to overcome this long standing problem by developing new methods of coupling into the medium at standoff. In particular, we believe that the acoustic nonlinearity of both the air and the medium may yield a range of effects in the vicinity of the surface permitting an efficient transmission of ultrasound from the air into the medium. Experimentally, we have conducted this study by directing commercially available sources of 45 and 96 kHz ultrasound beams at the air-water free surface and searching for anomalous behavior.

List of Illustrations and Figures

- Figure 1: Schematic drawing of parametric speakers
- Figure 2: Resonant condition for parametric sound
- Figure 3: Diagram of nonlinear interaction of evanescent waves
- Figure 4: Refraction through a curved surface's effect on nonlinear interaction
- Figure 5: Experimental apparatus
- Figure 6: Beam shape
- Figure 7: Parametric sound generation vs. difference frequency
- Figure 8: Schematic of apparatus to view surface deformation
- Figure 9: Surface deformation due to airborne radiation pressure
- Figure 10: Surface deformation due to waterborne radiation pressure
- Figure 11: Waterborne parametric sound transmitted through free surface

Statement of Problem Studied

The focus of this ARO STIR was to identify wave modes or nonlinear behaviors unique to an acoustic boundary which offer means to increase acoustic energy transmission across dramatic impedance mismatches. The primary goal of this project was to overcome the acoustic impedance mismatch by matching the sum and difference phase velocity of sound beams in air to a propagating wave in water.

We set out to conduct this search on three fronts:

- 1) Find a method that enables mode conversion directly between longitudinal modes at opposing angles in air to a longitudinal mode in water.
- 2) Probe transverse surface waves caused by high amplitude sound.
- 3) Examine what nonlinear coupling might be possible if the requirements of the fluid under study were relaxed to include propagating viscoelastic waves.

These directions of study were inspired by processes in elastic solids that cause conversion between transverse waves [1,2] and longitudinal waves upon reflection, by theoretical descriptions of wave attenuation and harmonic generation in dielectrics via nonlinear coupling processes such as the Landau-

Rumer process [3], and by the wide-reaching success of manipulating non-linear optical phenomenon. [4]

Scientific Progress and Accomplishments

- 1) Observed free-surface deformation potentially due to the radiation pressure of an air-based ultrasound transducer.
- 2) Observed that the surface deformation caused by opposed transducers producing different frequency sound can induce surface wave motion with wave speed dependent on the difference in frequency.
- 3) Demonstrated that parametric sound generated under water can be produced with large enough intensity to be heard in air.

Theoretical Introduction

Impedance Mismatch

When a sound wave in air with pressure amplitude P_i impacts the air-water interface, both a reflected and transmitted wave are created. These three waves must simultaneously satisfy the acoustic wave equation

$$\frac{\partial^2 P_1}{\partial t^2} = c^2 \frac{\partial^2 P_1}{\partial x^2} \quad (1)$$

and two boundary conditions at the free surface: $P_i + P_r = P_t$ and $u_i + u_r = u_t$ where u represents the fluid velocity. For plane waves, the acoustic impedance r is defined as $r = \frac{P}{u} = \rho c$ where ρ is the density of the fluid and c is the speed of sound. The two boundary conditions can then be reduced to give:

$$r_1 \frac{(P_i + P_r)}{P_i - P_r} = r_2 \quad (2)$$

and the reflected and transmitted pressure amplitudes are solved to be

$$P_r = P_i R = P_i \frac{r_2 - r_1}{r_2 + r_1} \quad (3)$$

$$P_t = P_i T = P_i \frac{2r_2/r_1}{r_2/r_1 + 1} \quad (4)$$

For the boundary between air and water $\frac{r_2}{r_1} \approx 3500$, so the intensity reflected and intensity transmitted are approximately

$$I_r = \frac{\tilde{P}_r^2}{2\rho_{\text{air}}c_{\text{air}}} \approx .999I_i \quad (5)$$

$$I_t = \frac{\tilde{P}_t^2}{2\rho_{\text{water}}c_{\text{water}}} \approx .001I_i$$

(6)

The impedance mismatch between air and water reduces the transmitted intensity by a factor of 1000. Similarly, if the remaining I_t reflects off of an object underwater and heads back toward the surface the intensity transmitted back into the air loses another factor of 1000 [5]. This loss of signal severely reduces the practicality of stand-off ultrasound technologies that rely on linear acoustics. For this reason medical ultrasound imaging systems require the familiar ultrasound gel.

Westervelt Equation

All wave propagation in real ponderable media is nonlinear for a fundamental reason: it is required by the Galilean transformation. In the case of water and air Galilean relativity leads to a convective derivative that appears in the Euler fluid equation [6]:

$$\frac{\partial v}{\partial t} + (v \cdot \nabla) \cdot v = -\frac{\nabla P}{\rho} \quad (7)$$

To a next order approximation, this convective term leads to a nonlinear forcing term in the acoustic wave equation. [5,7]

$$\frac{\partial^2 \rho_2}{\partial t^2} - c^2 \frac{\partial^2 \rho_2}{\partial x^2} = \frac{\partial^2}{\partial x^2} (G) \rho_1^2 \quad (8)$$

Which has a solution in terms of the pressure which takes the form:

$$P_{\text{mix}} \propto x * (\omega - \omega') \frac{1}{c_0^3 \rho_0} \left(1 + \frac{B}{A}\right) P_1 P_2 \quad (9)$$

Note that the amount of energy converted to the mixed frequency increases with the distance along which the two original sound beams propagate (x). Also note that both increasing density and speed of sound hinders the rate of conversion. Both of these forebode that generating sound in this manner in the bulk of water will prove difficult. Figure 1 outlines how parametric speakers use this solution to the nonlinear wave equation to produce audible sound.

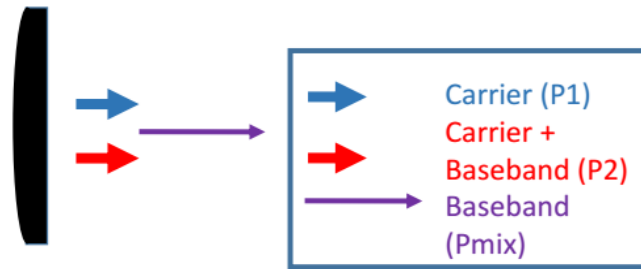


Figure 1: Schematic drawing of parametric speakers generating audible sound from modulated ultrasound

This example provides a demonstration of how nonlinear effects in acoustics give rise to the generation of both harmonics and mixed frequency terms. The simplicity of this example, however, limits the sort of mixing that can happen. Because the dispersion relation is always of the form $\omega = ck$, the most efficient wave mixing always occurs when the two sound waves propagate collinearly. Stated another way: the phase velocity of the mixed wave only matches the phase velocity of sound in the medium if the wave vectors (k_1, k_2) are collinear. This simple calculation is shown below for a few difference frequencies:

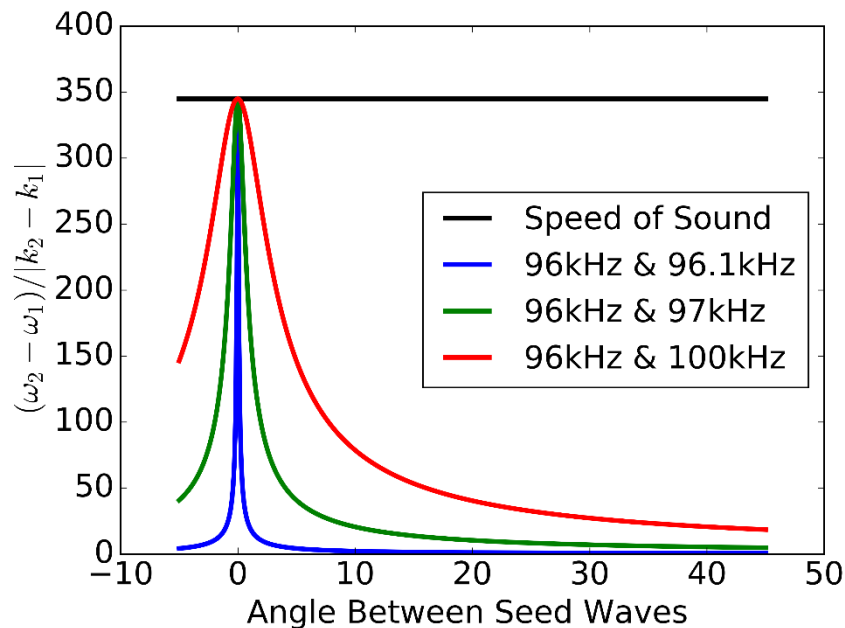


Figure 2: Examination of the nonlinear resonance condition comparing the phase velocity of the forcing term in Westervelt equation vs. relative angle of the initial waves.

Other wave types such as a gravity-capillary waves which appear on a babbling brook have a more complicated dispersion relation that allows wave mixing between non-collinear waves. [8]

One of our initial strategies to both increase the generation of sound at the water surface and to distinguish it from air-generated sound was to take advantage of the production of evanescent waves when the sound interacts with the surface above the critical angle. Two incoming waves above the critical angle, but at mutually different angles both propagate along the water parallel to the surface. The idea was to produce a funnel effect that forced two non-collinear waves to become collinear and subsequently only generate difference frequencies as they travelled along the surface. Despite the reduced efficiency of difference frequency sound produced in the air due to the varied angles, this configuration proved difficult to measure because of the amount of sound at the difference frequency still produced in the air.

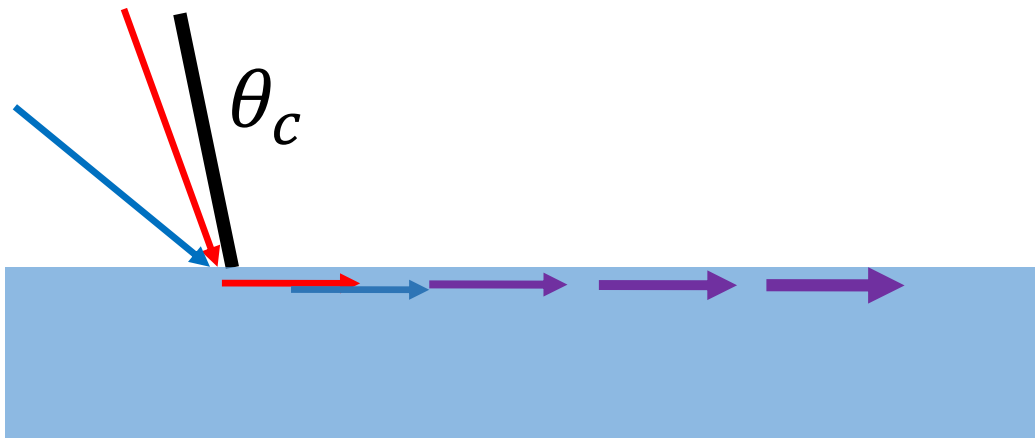


Figure 3: Two sound waves incident above critical angle on water surface producing nonlinear difference term along surface via evanescent waves.

Nonlinear Reflection off of a Curved Surface

Using the collinearity condition required to generate sum and difference frequencies discussed above, we call attention to another avenue to separate sound generated in air from sound generated in liquid. Two incoming rays incident upon a curved surface may reflect collinearly. At a different curvature, this same reasoning could cause the transmitted waves to propagate collinearly as shown in Figure 4. This effect has the interesting property that its resolution capability is limited by the wavelength of the ultrasound (λ_1, λ_2), but the result can be detected by a transducer sensitive to much longer wavelengths ($\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$).

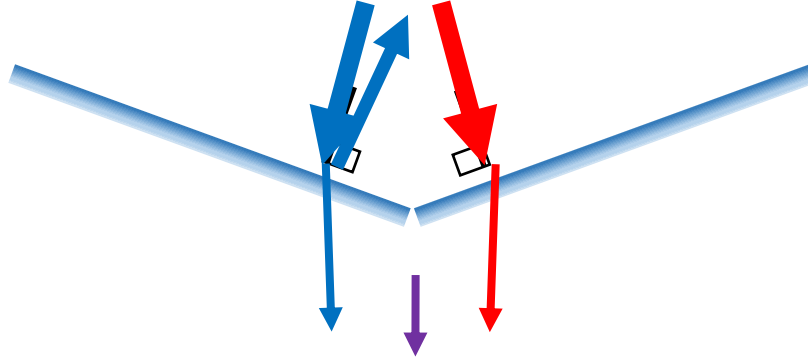


Figure 4: Refracted waves closer to collinear than incident due to curvature of water surface.

Radiation Pressure

The oscillatory pressure of ultrasound changes direction so rapidly that large scale deformations at a surface typically do not have time to form. The oscillations of sound, however, can provide a DC pressure component known as the radiation pressure. Following Lord Rayleigh's work on the rectified downward force on a pendulum, Brillouin studied an application of Boltzmann-Ehrenfest theory of adiabatic invariance that can motivate the source of acoustic radiation pressure. Simply stated, the differential of the product of the kinetic energy (E) and oscillation period τ of a sound wave in a slowly varying cavity is zero. Following Beyer [9]

$$\delta(E\tau) = 0 \quad (10)$$

Which gives

$$\frac{\delta\tau}{\tau} = -\frac{\delta E}{E} \quad (11)$$

Knowing that the period of oscillation of the lowest standing wave in a cavity of cross sectional area A and length l is

$$\tau = \frac{2l}{c} \quad (12)$$

And from the first law of thermodynamics: $dE = TdS - PdV$, we can arrive at a relation of the radiation pressure in a gas acting against the walls of the cavity is:

$$P = -\frac{\partial E}{\partial V} = \frac{E}{\tau} \frac{\partial \tau}{\partial V} = \frac{E}{V} \left(1 + \frac{\rho}{c} \frac{\partial c}{\partial \rho} \right) \quad (13)$$

Simply stated, the radiation pressure due to an oscillating acoustic wave is proportional to the energy density in that medium.

Because the radiation pressure increases with energy density, the free surface boundary between air and water serves as an excellent place to observe its effects. As stated above for sound reflecting off of the air-water interface from the air, the energy density of the incident and reflected waves are around 1000x that of the transmitted wave. This provides a DC pressure imbalance that the surface compensates for by deforming.

We suggest that this surface deformation, having been caused by sound traveling through air may provide a platform to inject acoustic energy into water in the following ways:

- 1) Low frequency sound with significant amplitude caused by the surface deformation might be injected into the water.
- 2) The deformation itself may cause diffraction-like patterns that could be used to focus ultrasound from a different source.

Viscoelastic Liquids and Shear Waves

When a disturbance oscillates faster than the relaxation time of a liquid, the liquid can support propagating shear waves. For a typical liquid (i.e. water), this frequency exceeds 10^{15} Hz – far outside the limits of typical acoustics. The addition of small amounts of alginates or polyacrylamide to water can decrease that frequency to .1 Hz. [10] Thus, these materials support shear waves with typical acoustic frequencies. The phase velocity of these shear waves, however, decreases with the increase in this relaxation time as [11]:

$$c_s = \sqrt{\frac{\eta}{\tau\rho}}$$

Previous research that indicated the possibility of increased acoustic energy transmission across gas-solid boundary due to the coupling of sound to Rayleigh wave modes required that the solid's shear wave velocity exceeds the speed of sound in the gas [2]. The simultaneous presence of propagating shear waves at acoustic frequencies and a shear velocity greater than 343 m/s narrow the space of materials to study. Paraffin, for example, has been demonstrated to have a tunable speed of sound and a shear wave speed across the solid-liquid phase transition. [12] Similar materials might provide a platform to study transmission across impedance mismatches.

Experimental Approach

Configuration of the Hypersound Speakers

Experimentally, the recent consumer availability of parametric speakers that rely on bulk acoustic nonlinearities to produce music incentivizes a new attempt at breaching the impedance mismatch between air and water. These sources of ultrasound provide well collimated beams with high enough amplitudes (135dB) to generate sum and difference frequencies with significant amplitude (around 90dB). Our initial experimental study was to point the parametric speakers at the air-water free surface in order to measure the transmission from air to water. Figure 5 shows the layout of the experiment.

Parametric Array speakers to generate 115 Pa (135dB) 96kHz ultrasound and up to 90dB audible sound.

1/8" mic to measure ultrasound levels

Hydrophone to measure transmission up to 170kHz

Signal Processing and Acquisition

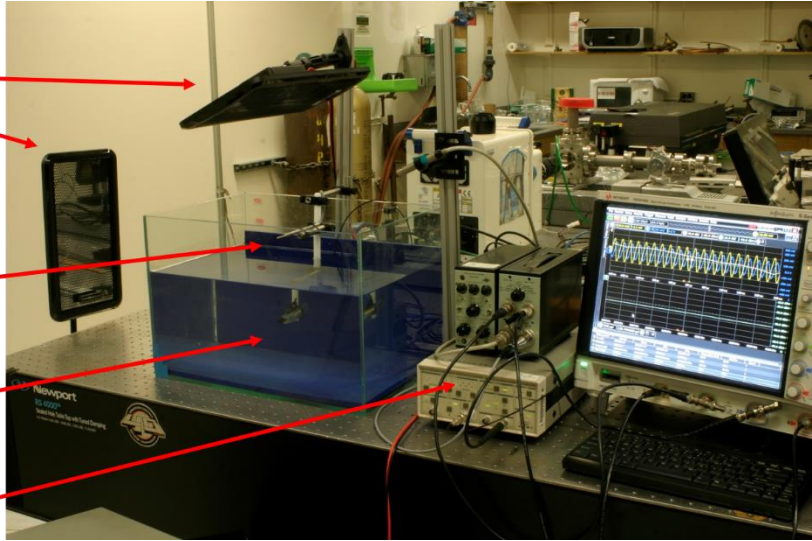


Figure 5: Experimental arrangement to probe ultrasound transmission into water.

The ability of these speakers to produce beam-like ultrasound made them particularly amenable to fish tank scale experiments and a useful tool to study effects washed-out by plane-wave sources. Figure 6 shows a characterization of the beam when compared to a calculation of the propagation of a 96 kHz beam through a 2" slit.

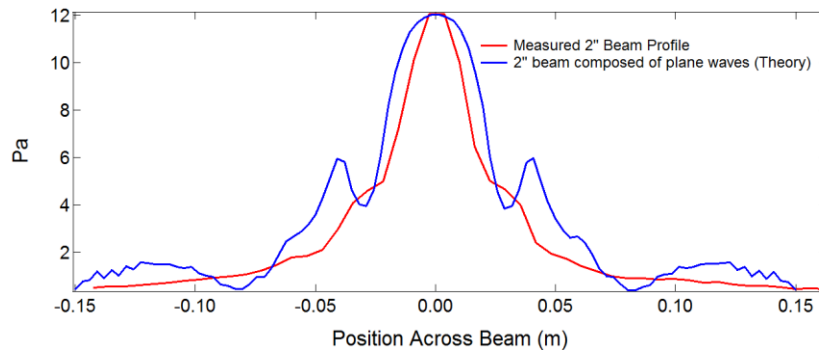


Figure 6: Transverse profile of a beam produced by parametric speakers

Our initial experiments were to observe the dependencies of the generation of difference terms via the Westervelt equation. These measurements served as a baseline for what transmission occurs without tapping into finite amplitude effects. It was essential to confirm that the bulk nonlinear generation in air would overshadow signal similarly generated at the surface and in the liquid.

In particular, it is necessary to mention that the efficiency of the generation of the difference frequency in the bulk for a given liquid increases with the amplitude of the difference. Higher frequencies are produced more efficiently than lower as shown both above in equation (9) and by data shown in Figure 7.

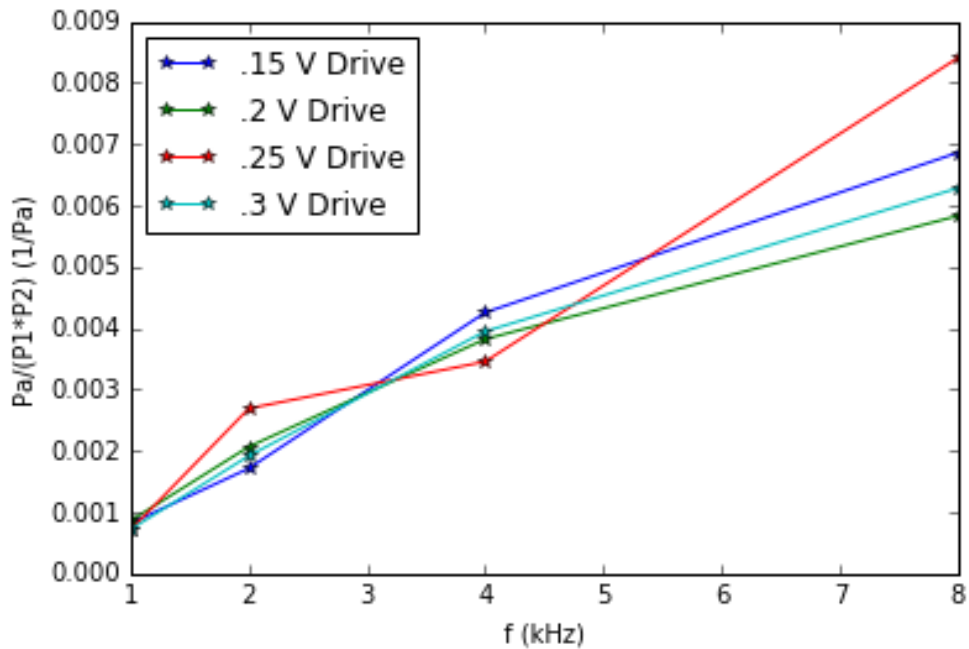


Figure 7: Relative efficiency of the generation of difference frequencies.

Observation of Surface Deformation

The acoustic energy of the HSS speakers is capable of producing a visible surface deformation on the free-surface. We have not yet measured precisely the height of the deformation, but suspect that it to be around 10 μ m. We suspect the deformation is caused by the radiation pressure described above. Here is a schematic of the experimental setup.

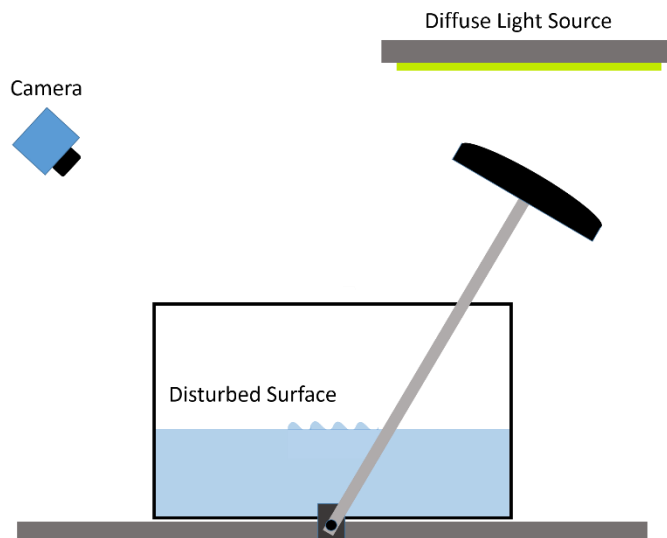


Figure 8: Experimental arrangement used to image small surface deformations caused by radiation pressure.

The disturbances shown in Figure 9 have a wavelength that is around 1cm, which is close to the wavelength of the ultrasound and increases as the angle of incidence decreases.

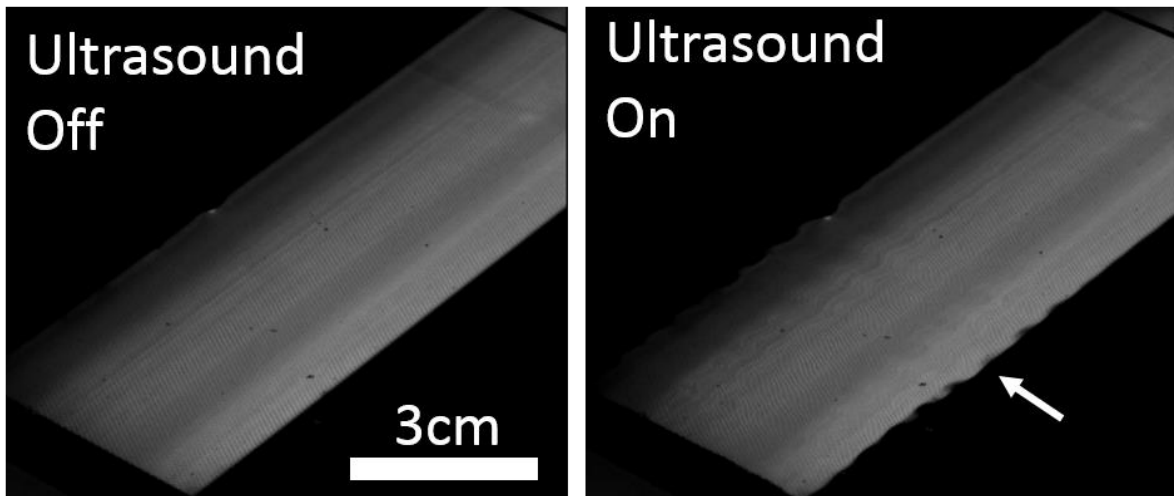


Figure 9: Observation radiation pressure disturbed surface due to higher acoustic energy in the air.

A particularly interesting effect that suggests the potential for introducing higher amplitude, but lower frequency acoustic motion into the water is that when two counter propagating ultrasound beams with different frequencies impact the surface the wave tends to move toward the lower frequency beam. As the difference in frequency increases the rate of surface motion increases as well. This demonstrates a direct method for sound to couple to surface water wave motion which has a much lower phase velocity.

Mhz Ultrasound Projected Out of Water

The availability of underwater MHz ultrasound devices with amplitudes above 10^5 Pa provides an additional method to cause surface wave motion. Medical ultrasound transducers provide enough energy to deform the free surface by several mm and cause capillary wave motion dramatic enough to produce micro droplets in the form of mist as shown in Figure 10. This effect has been well researched and supported many technologies including humidifiers and atomizers. [4]

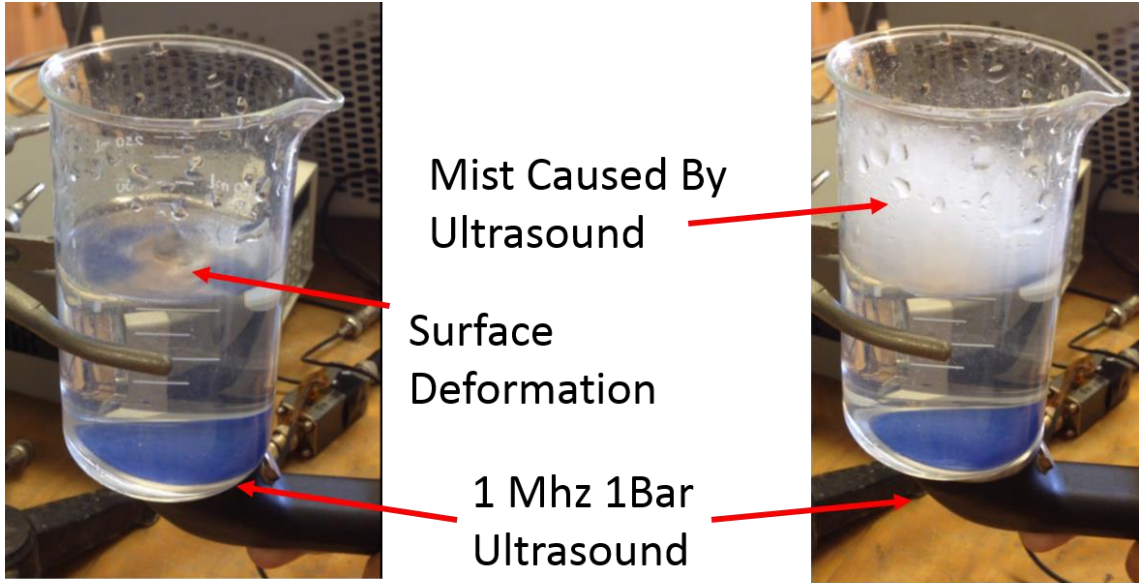


Figure 10: *Mhz Ultrasound at pressures around $10^5 Pa$ deforming the free surface and causing the production of $\sim 10\mu m$ droplets.*

The density and speed of sound, according to eq. (9), reduce the production of nonlinearly generated sound in water compared to air by a factor of more than 10^4 , but the amplitude of sound produced by transducers as shown above more than compensate for this reduction. By amplitude modulating Mhz ultrasound produced underwater at frequencies ranging between 1 and 10 kHz, it was possible to produce audible parametric sound that was nearly 60dB above the water's surface.

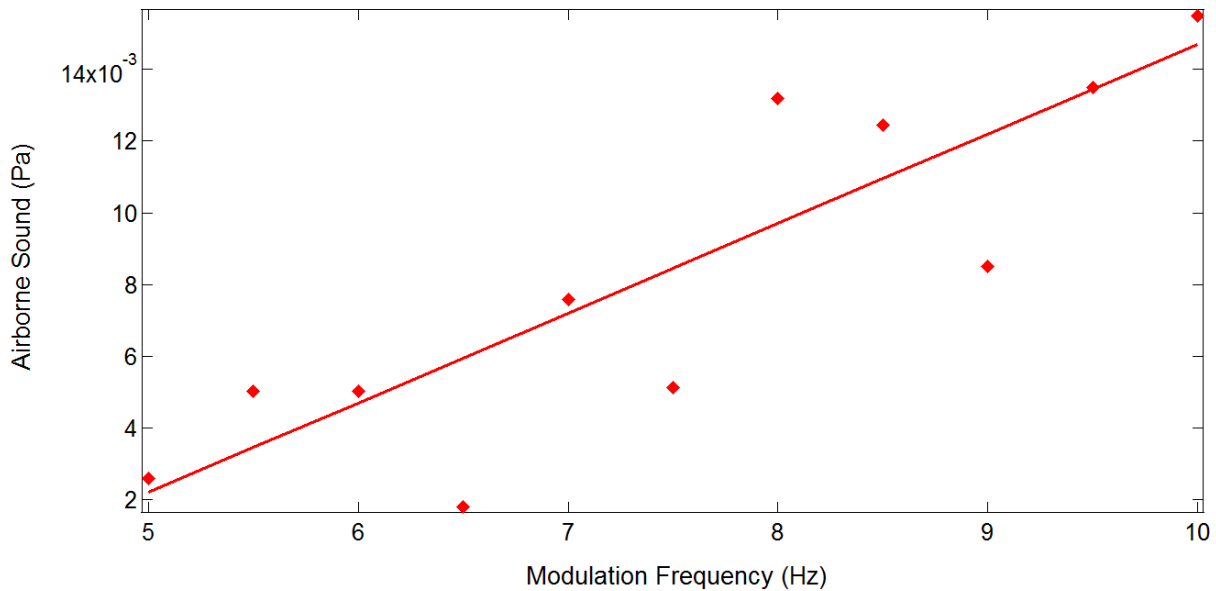


Figure 11: *Airborne sound produced by modulating Mhz ultrasound underwater.*

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