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GAME-THEORETICAL TOOLS FOR THE DESIGN OF DECENTRALIZED CONTROL ALGORITHMS FOR HYBRID SYSTEMS WITH UNCERTAINTY

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14. ABSTRACT
To accomplish our goal, we formulate hybrid dynamical games that permit modeling, analysis, and design of algorithms for multiple agents with hybrid dynamics. We define a framework for modeling, analysis, and control design for games involving dynamical systems that combine continuous and discrete dynamics. Also, we apply hybrid dynamical game analysis and control synthesis tools to game theoretic problems with corresponding dynamics in the context of multi-agent networked systems.

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1.0 SUMMARY

Emerging technologies permitting cooperation of multiple autonomous systems are suitable not only for the accomplishment of defense missions but also for the establishment of the needed infrastructure for civilian use, ranging from daily-use activities such as the internet and weather services to sporadic events, such as local emergencies and disasters. However, the presence of uncertainty in the environment, disruptions caused by other users, and potential failures impose significant challenges to the design of such systems. In particular, they impose stringent requirements on the coordination and communication algorithms used for the control of multi-agent systems, such as fast reaction times, adaptation capabilities, and robustness margins to uncertainty. *We believe that such requirements can be met by algorithms for multi-agent systems designed with game theoretical tools that explicitly take into account the presence of uncertainty as well as the combination of continuous and discrete dynamics emerging from the interconnection between physical variables, abrupt changes in the algorithms, and associated digital systems.* In this project, we propose to generate new tools for the design of decentralized algorithms that make control decisions in a decentralized and robust manner to accomplish a common task.

2.0 INTRODUCTION

Our goal in this project is to generate tools for the design of decentralized and robust algorithms for underactuated networked agents in environments with disruptions, uncertainties, and adversarial actions. Game theoretical tools for the solution of differential games allow us to formulate and solve problems that capture individual and collective payoffs as well as the capabilities and effect of each of the agents (or players) participating in the game. On the other hand, hybrid systems theory allows for the modeling and analysis of dynamical systems with intertwined continuous and discrete dynamics. To accomplish our goal, we formulate hybrid dynamical games that permit modeling, analysis, and design of algorithms for multiple agents with hybrid dynamics.

We refer to the games to study in this project as *hybrid dynamical games*. Among the main benefits is the capability of our proposed tools to handle hybrid dynamics, which, though they play a key role in the dynamics of the systems, are typically neglected in most differential game formulations. Robustness is a key property obtained from applying our hybrid control methods to nonlinear systems, which, in our project, will provide quantifiable robustness margins to uncertainty and unmodeled dynamics.

3.0 METHODS, ASSUMPTIONS, AND PROCEDURES

In this project, we propose a framework for the study of hybrid dynamical games, i.e., games that involve systems with combined continuous and discrete dynamics. Stability and robustness properties of this class of hybrid systems have recently been studied by the Principal Investigator and his coauthors, but the study of optimality in a game setting is a widely open research area. The framework to generate in this project will provide a formal synthesis method for optimal policies solving hybrid games with quantifiable robustness margins to uncertainty and unmodeled dynamics. In the proposed framework, the agents (or players) will exhibit continuous and discrete dynamics, either to their own natural behavior, the structure of the network, or the family of algorithms to be employed for estimation, control, and coordination. The fact that our framework will be able to handle hybrid dynamics will be highly useful in studying realistic problems, which do include combined continuous and discrete behavior.

The results obtained from this project are summarized in the following section. Our main assumptions include the following:

- The dynamics of the systems to model can be captured by a combination of differential equations (or inclusions) and difference equations, possibly with constraints.
- The state of the systems is the quantity measured to make the decisions.
- The communication structure is fixed over time.

The tools employed to derive our results include modeling and control theoretical tools for hybrid dynamical systems. In particular, modeling of the systems as hybrid dynamical systems in [1] and [2]; Lyapunov methods for such systems in [3]; and simulation techniques in [4].

4.0 RESULTS AND DISCUSSION

A framework for modeling and analysis of the dynamical properties of multiple agents with control inputs that are to be designed to optimize a cost/agreement function that depends on the state variables of the agents has been developed. Of particular interest is the capability to model both continuous and discrete (i.e., hybrid) behavior of the agents, perhaps due to the hybrid dynamics of their state variables, the type of network over which they interact, or of the algorithms used for coordination, control, and estimation. Selected published results leading to the new framework are summarized as follows:

- In the paper, “Robust Asymptotic Stabilization of Hybrid Systems using Control Lyapunov Functions,” the authors propose tools for the study of robust stabilizability and the design of robustly stabilizing feedback laws for a wide class of hybrid systems given in terms of hybrid inclusions with inputs and disturbances. They introduce notions of robust uniform global stabilizability and stabilization that capture the case when disturbances can be fully rejected, practically rejected, and when they induce a residual set that can be stabilized. Robust control Lyapunov functions are employed to determine when stabilizing static state-feedback laws are available and also to synthesize robustly stabilizing feedback laws with minimum pointwise norm. Sufficient conditions on the data of the hybrid system as well as on the control Lyapunov function are proposed for the said properties to hold. An example illustrates the results throughout the paper. For details, see [5].
- In the paper, “A Computationally Tractable Implementation of Pointwise Minimum Norm State-Feedback Laws for Hybrid Systems,” the authors propose a computationally tractable implementation of state-feedback laws for hybrid systems given by differential equations capturing the continuous dynamics or flows, and by difference equations capturing the discrete dynamics or jumps. By exploiting the availability of a control Lyapunov function, along with other properties of the system, they show that pointwise minimum norm control

laws can be implemented in a sample-and-hold fashion, with events triggered by timers, to render a desired compact set semiglobally and practically asymptotically stable. Examples illustrate the results. For details, see [6].

- In the paper, “On the Optimality of Lyapunov-based Feedback Laws for Constrained Difference Inclusions,” state-feedback optimal control and cost evaluation problems for constrained difference inclusions are considered. Sufficient conditions, in the form of Lyapunov-like inequalities, are provided to derive an upper bound on the cost associated with the solution to a constrained difference inclusion with respect to a given cost functional. Under additional sufficient conditions, we determine the cost exactly without computing solutions. The proposed approach is extended to study an optimal control problem for discrete-time systems with constraints. In this setting, sufficient conditions for closed-loop optimality are given in terms of a constrained steady-state-like Hamilton-Jacobi-Bellman equation. Applications and examples of the proposed results are presented. For details, see [7].
- In the paper, “Cost Evaluation for Hybrid Inclusions: A Lyapunov Approach,” cost evaluation problems for hybrid inclusions are studied. Sufficient conditions, in the form of Lyapunov-like inequalities, are provided to derive an upper bound on the cost associated with the solution to a hybrid inclusion with respect to a hybrid cost functional. Under additional sufficient conditions, we determine the cost exactly without computing solutions. Constructive results are proposed to solve cost evaluation problems in some relevant applications. Numerical examples are presented. For details, see [8].

5.0 CONCLUSIONS

For a wide class of hybrid systems given in terms of hybrid inclusions with inputs and disturbances, we presented a framework and results to design stabilizing state-feedback controllers. Our results allow to constructively design the feedback, even under the presence of general disturbances, in particular, those generated by adversaries. When a control Lyapunov function is available and the required conditions hold, a state-feedback law with pointwise minimum norm can be constructed to asymptotically stabilize a compact set with robustness to disturbances. A remarkable feature of this controller construction is that it guarantees w-robust asymptotic stability of the closed-loop system for any admissible disturbance taking values from (the w components of) points in the flow set or jump set. Such disturbances can indeed be large, unlike the disturbances allowed in our previous nominal robustness results, and, as a difference to input-to-state stability-based results, at times can be fully rejected.

The implementation of the proposed feedback laws requires careful treatment to allow for computation in realistic systems. In particular, the computations involved in determining the minimizers in the state-feedback laws require a nonzero amount of time to terminate. A sample-and-hold or event-triggered implementation of such laws would require variables that trigger the computation events, allow the computations to terminate, and upon termination of the computations, update the inputs to the hybrid system under control.

Our results in [6] suggest that, as long as the time for the computations to terminate can be made sufficiently small, it is possible to implement such laws while preserving the stability properties semiglobally and practically. Handling the challenges in performing such computations is part of current research efforts.

The proposed state-feedback law with pointwise minimum norm are expected to also induce an optimality property of the closed-loop system. Using inverse optimality ideas, the robust stabilization problem solved in [5] was recast as a two-player zero-sum hybrid dynamical game. To determine the value of the cost for each solution, we generated results to provide estimates and, under additional assumptions, provide the exact value of the cost; see our recent work in [7] and [8]. Under appropriate assumptions, we conjecture that the proposed control law suboptimally solves such hybrid game with a meaningful cost function.

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