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"RECTANGULAR HEAT SINK DESIGN AND OPTIMIZATION CODE" MANUAL

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14. ABSTRACT

This paper is a manual on how to use the "Rectangular Heat Sink Design and Optimization Code," a program that can be used to design and optimize heat sinks with vertically oriented rectangular fins. The program, based in Microsoft Excel, calculates the optimal fin spacing and efficiency, a whole heat sink efficiency, heat dissipation, and the maximum heat sink temperature based on the input parameters (fin length, fin height, fin thickness, fin material, maximum allowable temperature, and ambient temperature).

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INTRODUCTION

One important consideration of electronic equipment design is the effective dissipation of the heat produced by the electronic components. Thermal management is necessary to avoid overheating the apparatus since overheating can degrade reliability and cause premature failure. Continuous advances in the electronic industry have produced more sensitive components, which has caused proper heat management to become even more crucial. Heat is dissipated from the electronic devices by being transferred to the external ambient by all heat transfer modes (conduction, convection, and radiation). The natural convection and radiation modes of heat transfer are commonly applied cooling techniques for electronic equipment of low to moderate power density, such as computer chips, electronics, and telecommunication boxes. The main advantages of natural convection are high reliability, low noise, and low power consumption.

Heat sinks are devices that are used to enhance heat dissipation from hot surfaces to cooler air. Typically, the fins of heat sinks are oriented in a way to permit a natural convection air draft to flow upward through rectangular U-channels. Heat sink design goals may vary, but the main concern, most commonly, is the optimization of the natural convection heat sink. The optimization of the heat sink has become an essential practice for electronic equipment design. The procedure for the optimization of a heat sink consisting of an array of straight, vertical, and rectangular fins was formulated in a DTIC report (ref. 1).

The procedure mentioned in reference 1 was coded into Microsoft Excel. Excel was used for its ability to quickly manipulate large amounts of data into orderly spreadsheets and charts. Excel makes it easy to store a vast amount of data, perform numerical calculations, and follow trends on corresponding charts. For the type of heat sink under consideration, it allows the user to immediately see the resulting heat sink efficiency, given a combination of input parameters, and determine an optimum heat sink design.

The program, “Rectangular Heat Sink Design and Optimization Code,” is used for design and optimization of the vertical oriented rectangular heat sink. It calculates the optimal fin spacing and efficiency, a whole heat sink efficiency, heat dissipation, and the maximum heat sink temperature based on the input parameters (fin length, fin height, fin thickness, fin material, maximum allowable temperature, and ambient temperature). One of the advantages of this Excel code is the ability to display new results with any alteration of the input parameters. As an example, the code user could fix the heat generated by the electronic component and then change the fin parameters until the total heat transferred from the heat sink matches the heat generated. From that moment, the heat sink temperature can be immediately read as well as the corresponding optimal fin spacing. Detailed examples are included further on in this report as a guide to the reader and to demonstrate the versatility of the code.

This report is essentially an attempt to put together a documentation for this code in one place for the convenience of the user. The author would appreciate any feedback from the user that can help improve future documentation of this program.

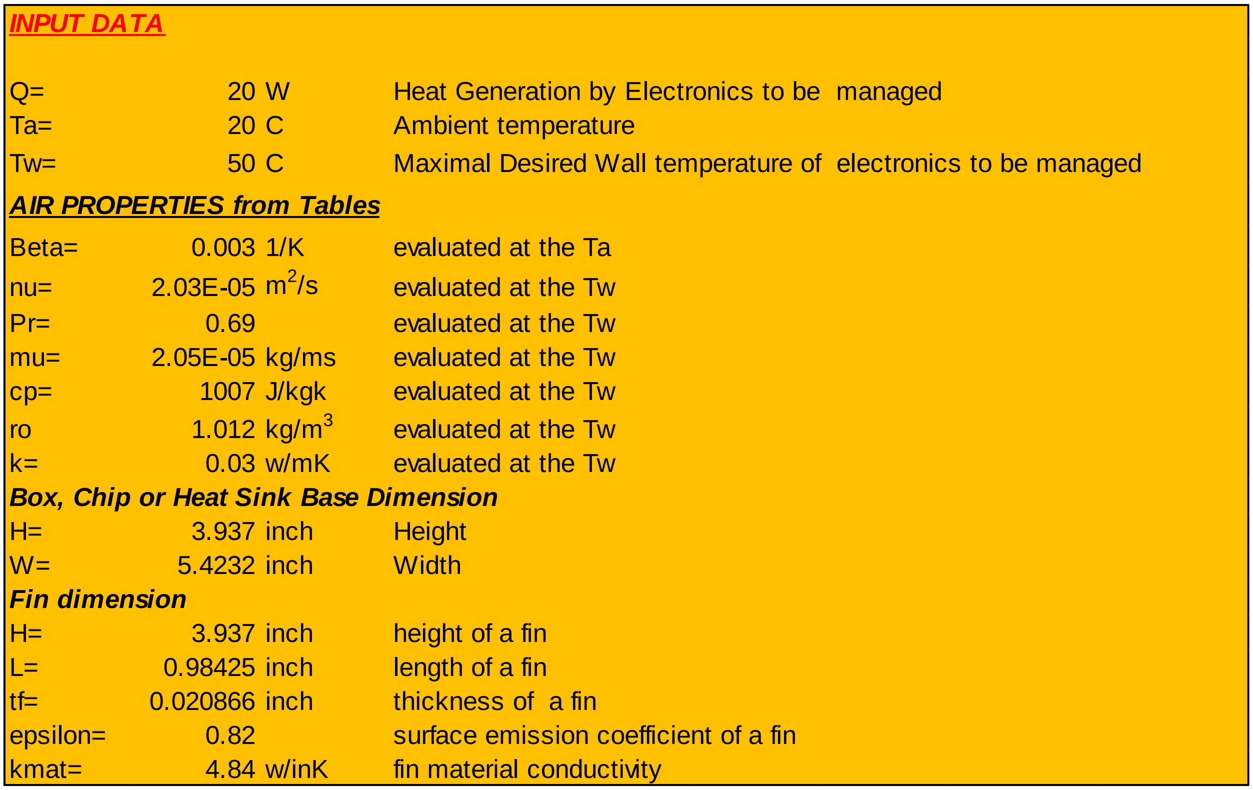
DESCRIPTION OF THE CODE

As mentioned previously, a procedure for the design and optimization of a naturally cooled heat sink with rectangular fins was coded into Microsoft Excel. This spreadsheet represents the coded procedure outlined in reference 1. This procedure includes the calculation of a convective heat transfer coefficient for natural convection (ref. 2) as well as the radiative heat transfer coefficient for the U-channels outlined in references 3 and 4.

INPUT DATA

The input data consists of a series of cells enclosed in an orange colored box. In table 1, an example of the input “deck” is given. The input is categorized into four groups: electronic component details and ambient temperature, air properties from tables, heat sink base dimensions, and fin dimensions.

Table 1
Input data for the present code



INPUT DATA			
Q=	20	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	50	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	3.937	inch	Height
W=	5.4232	inch	Width
Fin dimension			
H=	3.937	inch	height of a fin
L=	0.98425	inch	length of a fin
tf=	0.020866	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

In the first three cells, the user is required to define the amount of heat generated by the electronics under consideration, the ambient temperature, and the maximum allowed temperature of the heat sink (or of the electronics in contact with the heat sink). The next seven cells are reserved for the air properties obtained from air tables. Except for the volume expansion coefficient (Beta, β), which is evaluated at the ambient air temperature (Ta), all other air properties [i.e., Nusselt number (Nu), Prandtl number (Pr), kinematic viscosity (Mu, μ), specific heat (Cp), density (rho, ρ) and thermal conductivity (k)] are evaluated at the wall temperature (Tw). The box, chip, or heat sink’s base height (H) and width dimensions (W) are the next required input. Finally, the fin dimensions are also required. Fin dimensions consist of its height (H, generally the same as heat sink base height), length (L), and thickness [tf (fig. 1)] as well as the surface emission coefficient of the fins (epsilon, ε) and the fin material conductivity (kmat).

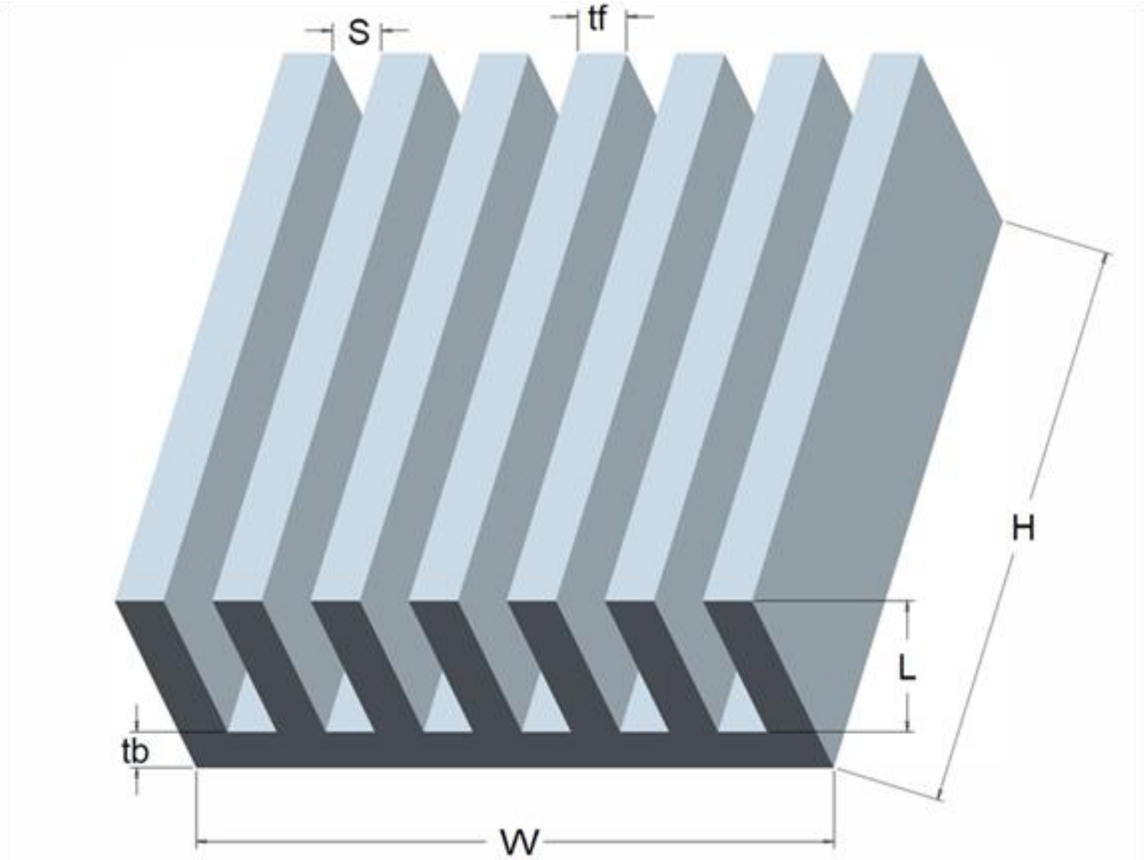


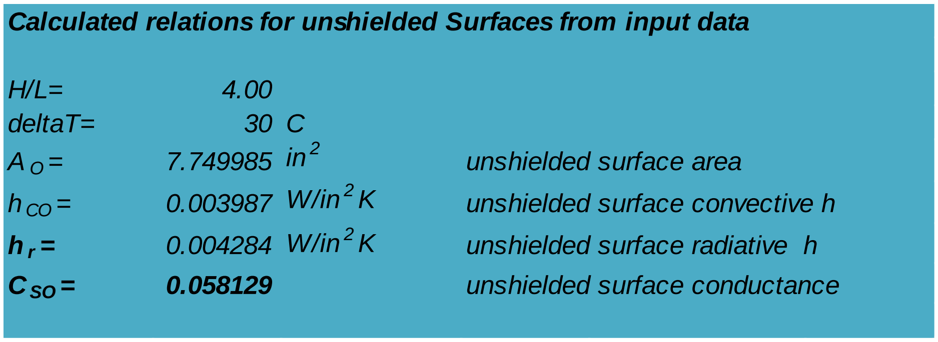
Figure 1
Heat sink and fin dimension parameters

The only additional requirement for this section is using units consistent with what is shown in table 1. Note that the units are a mix of metric and U.S. customary units.

CALCULATED RELATIONS FOR UNSHIELDED SURFACES FROM INPUT DATA

To obtain more realistic results for the heat sink dissipation and temperature, the outer surfaces of the heat sink must be considered separately. Table 2 represents calculated relations for those unshielded surfaces.

Table 2
Calculated relations for unshielded surfaces



Calculated relations for unshielded Surfaces from input data

H/L=	*4.00*		
deltaT=	*30*	*C*	
A_O =	*7.749985*	in^2	*unshielded surface area*
h_{CO} =	*0.003987*	$W/in^2 K$	*unshielded surface convective h*
$\boldsymbol{h_r}$ **=**	*0.004284*	$W/in^2 K$	*unshielded surface radiative h*
$\boldsymbol{C_{SO}}$ **=**	***0.058129***		*unshielded surface conductance*

The heat transfer coefficient equations used for the calculation of the properties of the unshielded surfaces, resulting from the input data, are presented in the following equations. The theoretical background for these correlations can be found in reference 5.

$$h_{co} = 0.0024\left(\frac{\Delta T}{H}\right)^{0.25} \tag{1}$$

$$h_r = 3.657x10^{-11}\,\frac{(T_w + 273)^4 - (T_a + 273)^4}{T_w - T_a} \tag{2}$$

$$h_o = h_{co} + \varepsilon h_r \tag{3}$$

$$C_{so} = h_o A_o \tag{4}$$

Where h_0 is the total (convective and radiative) heat transfer coefficient for the outer fins.

CALCULATED CONVECTIVE CORRELATIONS FOR U-CHANNELS

The Nusselt number (Nu, the ratio of convective to conductive heat transfer across a boundary) for the U-channels formed by the rectangular vertical fins is calculated using a modified Elenbass's equation as proposed by Van de Pol and Tierney (ref. 5). The following equations are used to calculate the Nusselt number for the U-channels.

$$Nu_r = \frac{Ra^*}{\Psi}\left\{1 - e^{\left[-\Psi\left(\frac{0.5}{Ra^*}\right)^{3/4}\right]}\right\} \tag{5}$$

$$\Psi = \frac{24(1 - 0.483e^{-\frac{0.17}{a}})}{\left\{\left[1 + \frac{a}{2}\right]\left[1 + \left(1 - e^{-0.83a}\right)\left(9.14a^{\frac{1}{2}}e^{VS} - 0.61\right)\right]\right\}^3} \tag{6}$$

$$Ra^* = \frac{r}{H}Gr_r \Pr \tag{7}$$

$$r = \frac{2LS}{(2L + S)} \tag{8}$$

$$a = \frac{S}{L} \tag{9}$$

$$V = -11.8(in^{-1}) \tag{10}$$

Where:

- Ra is the Rayleigh number, a dimensionless number used to calculate nature convection
- Gr is the Grashof number, a dimensionless number used to approximate the ratio of the buoyancy to the viscous force acting on a fluid
- Psi (ψ) is the linear thermal transmittance
- r is the ratio of area to perimeter of the U-channel
- S is the spacing between fins
- a is the ratio of spacing to fin length
- V is the numerical constant obtained experimentally (ref. 3)

Values of the Nusselt number together with the values obtained from equations 5 through 9 are shown in tabular form in table 3 as a function of the number of fins.

Table 3
Calculated Nusselt number and correlations as a function of fin number

# of Fins	**S(inch)**	**r(inch)**	**a**	**Ra***	**ψ**	**Nu_r**
2	5.3815	1.4413	5.4676	26552.6089	3.9327	7.5860
3	2.6803	1.1350	2.7232	10209.7802	10.6621	5.9584
4	1.7799	0.9347	1.8084	4697.3269	13.3851	4.8882
5	1.3297	0.7936	1.3510	2440.9918	14.3530	4.1285
6	1.0596	0.6888	1.0766	1385.2678	14.8033	3.5581
7	0.8795	0.6079	0.8936	840.3511	15.0931	3.1111
8	0.7509	0.5436	0.7629	537.1249	15.3191	2.7488
9	0.6544	0.4911	0.6649	358.0498	15.5025	2.4469
10	0.5794	0.4476	0.5887	247.0671	15.6450	2.1899
11	0.5194	0.4109	0.5277	175.4833	15.7464	1.9670
12	0.4703	0.3796	0.4778	127.7344	15.8092	1.7709
13	0.4293	0.3525	0.4362	94.9582	15.8391	1.5963
14	0.3947	0.3288	0.4010	71.8962	15.8435	1.4394
15	0.3650	0.3079	0.3709	55.3154	15.8300	1.2974
16	0.3393	0.2894	0.3447	43.1661	15.8058	1.1680
17	0.3168	0.2729	0.3218	34.1129	15.7769	1.0498
18	0.2969	0.2580	0.3017	27.2647	15.7483	0.9415
19	0.2793	0.2446	0.2837	22.0141	15.7235	0.8421
20	0.2635	0.2324	0.2677	17.9392	15.7051	0.7510
21	0.2493	0.2212	0.2532	14.7415	15.6948	0.6676
22	0.2364	0.2110	0.2402	12.2068	15.6936	0.5915
23	0.2247	0.2017	0.2283	10.1791	15.7020	0.5223
24	0.2140	0.1930	0.2174	8.5431	15.7202	0.4597
25	0.2042	0.1850	0.2075	7.2129	15.7479	0.4035
26	0.1952	0.1776	0.1984	6.1235	15.7848	0.3531

Note: Ra* refers to the Rayleigh number.

One of the most important correlations in table 3 is spacing, S. As it will be seen in worked examples at the end of this report, optimal spacing is one of the most important variables in heat sink design and optimization. Once the optimal number of fins is reached, any other increase in the number of fins will lead to a reduction in the heat transfer from the fins and ultimately lead to an increase of heat sink's temperature.

Once the Nusselt number is obtained, the convective heat transfer coefficient (h_{ci}) for the U-channels, fin efficiency (η_{fin}), heat sink efficiency (η_{sink}), conductance of the inter-fin passages (C_{ci}), and effective conductance of the inter-fin passage (C_{cieta}) can be calculated using equations 11 through 15.

$$h_{ci} = \frac{Nu_r k}{r} \tag{11}$$

$$\eta_{fin} = \sqrt{\frac{kHt_f}{2h_{ci}HL^2}} \tanh \sqrt{\frac{2h_{ci}HL^2}{kHt_f}} \tag{12}$$

$$A_i = WH\left[1 + \frac{2(N-1)}{W}L\right] \tag{13}$$

$$C_{ci} = \eta h_{ci} A_i \quad C_{cieta} = \eta C_{ci} \tag{14}$$

$$\eta_{\sin k} = 1 - A_i \frac{1 - \eta_{fin}}{H_{box} W_{box}} \tag{15}$$

Where Ai is the interior area determined by the number of fins, N.

Obtained results are sorted in tabular form in table 4 as a function of the numbers of fins.

Table 4
Convective heat transfer and conductance results as a function of number of fins

# of Fins	**hci(W/in^{2}C)**	**Ai**	**Cci (W/K)**	η_{fin}	η_{sink}	***Ccieta (W/K)***
2	0.00401	29.10	**0.12**	0.975	0.966	***0.11***
3	0.00400	36.85	**0.15**	0.975	0.957	***0.14***
4	0.00398	44.60	**0.18**	0.975	0.948	***0.17***
5	0.00396	52.35	**0.21**	0.975	0.940	***0.20***
6	0.00394	60.10	**0.24**	0.976	0.931	***0.23***
7	0.00390	67.85	**0.26**	0.976	0.923	***0.26***
8	0.00385	75.60	**0.29**	0.976	0.915	***0.28***
9	0.00380	83.35	**0.32**	0.976	0.908	***0.31***
10	0.00373	91.10	**0.34**	0.977	0.901	***0.33***
11	0.00365	98.85	**0.36**	0.977	0.895	***0.35***
12	0.00356	106.60	**0.38**	0.978	0.890	***0.37***
13	0.00345	114.35	**0.39**	0.978	0.885	***0.39***
14	0.00334	122.10	**0.41**	0.979	0.881	***0.40***
15	0.00321	129.85	**0.42**	0.980	0.878	***0.41***
16	0.00308	137.60	**0.42**	0.981	0.876	***0.42***
17	0.00293	145.35	**0.43**	0.982	0.875	***0.42***
18	0.00278	153.10	**0.43**	0.983	0.875	***0.42***
19	0.00262	160.85	**0.42**	0.984	0.876	***0.42***
20	0.00246	168.60	**0.42**	0.985	0.878	***0.41***
21	0.00230	176.35	**0.41**	0.986	0.881	***0.40***
22	0.00214	184.10	**0.39**	0.987	0.884	***0.39***
23	0.00197	191.85	**0.38**	0.988	0.888	***0.37***
24	0.00181	199.60	**0.36**	0.989	0.893	***0.36***
25	0.00166	207.35	**0.34**	0.990	0.898	***0.34***
26	0.00152	215.10	**0.33**	0.990	0.904	***0.32***

CALCULATED RADIATIVE CORRELATIONS FOR U-CHANNELS

The procedure for calculating the radiative heat transfer coefficient for the U-channels is outlined in the book by Ellison (ref. 3). It is assumed that all surfaces of the heat sink are gray and reflective with the base and fins at a uniform temperature. Figure 2 shows a single U-channel with numbered interior surfaces. Numerals 1, 3, and 4 identify the heat sink surfaces while numerals 2, 5, and 6 refer to the nonreflecting ambient air. A simplified thermal network for a single U-channel is shown in figure 3.

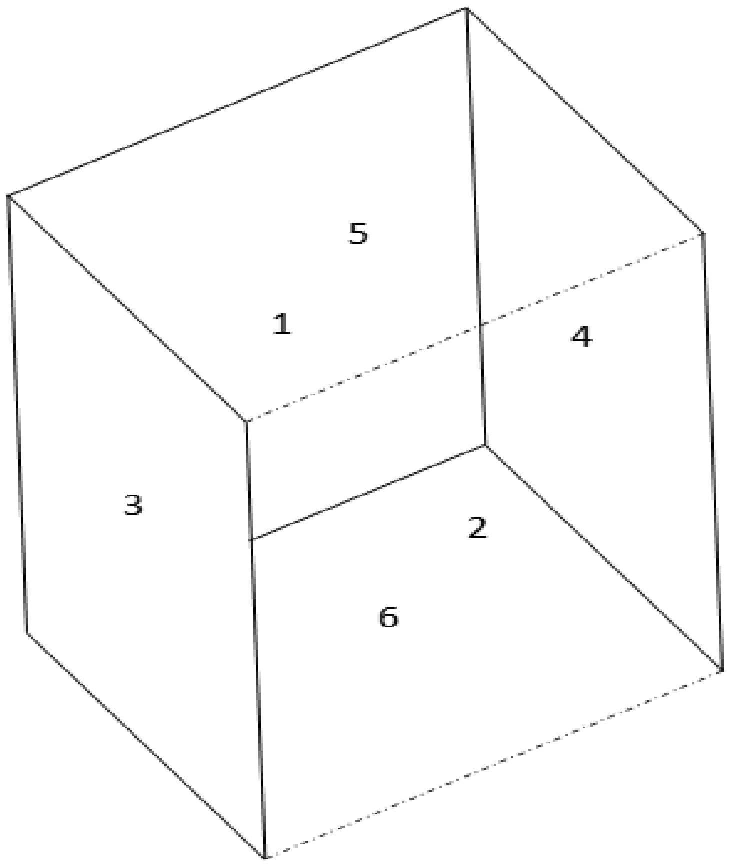


Figure 2
U-channel interior surface identification

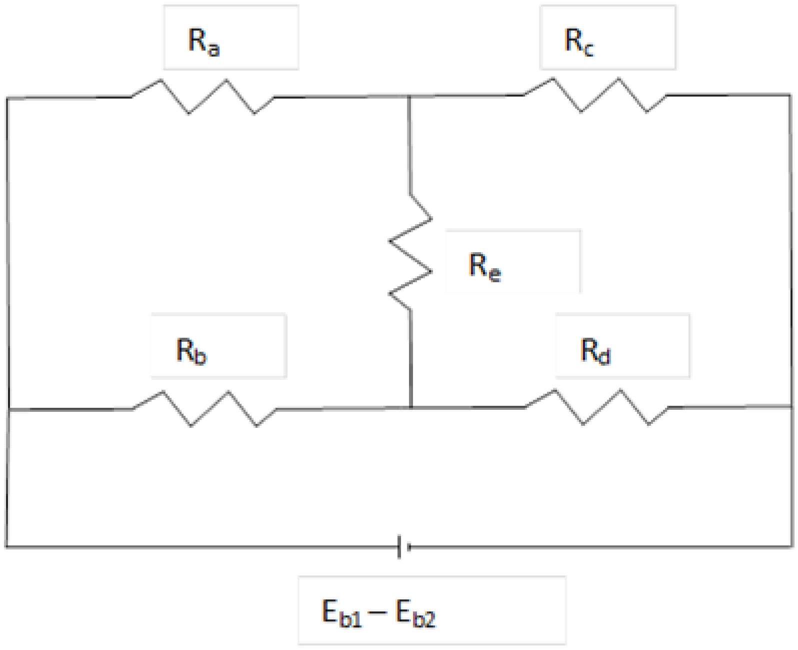


Figure 3
Simplified thermal network for U-channel radiation

The correlations between net shape factor (F) for the gray body U-channels, overall conductance for the U-channel (C_{NET}), and resistances in the simplified thermal network are given in equations 16 through 22.

$$F = \frac{2C_{NET}}{\left[H(S+2L)\right]} \tag{16}$$

$$C_{NET} = \frac{\left[(R_a + R_b + R_e)(R_c + R_d + R_e) - R_e{}^2\right]}{\left\{\begin{array}{l}(R_b + R_d)\left[(R_a + R_b + R_e)(R_c + R_d + R_e) - R_e{}^2\right] - \\ R_b\left[R_b(R_c + R_d + R_e) + R_e R_d\right] - R_d\left[R_d(R_a + R_b + R_e) + R_b R_e\right]\end{array}\right\}} \tag{17}$$

$$R_a = \frac{1-\varepsilon}{\varepsilon A_3} \tag{18}$$

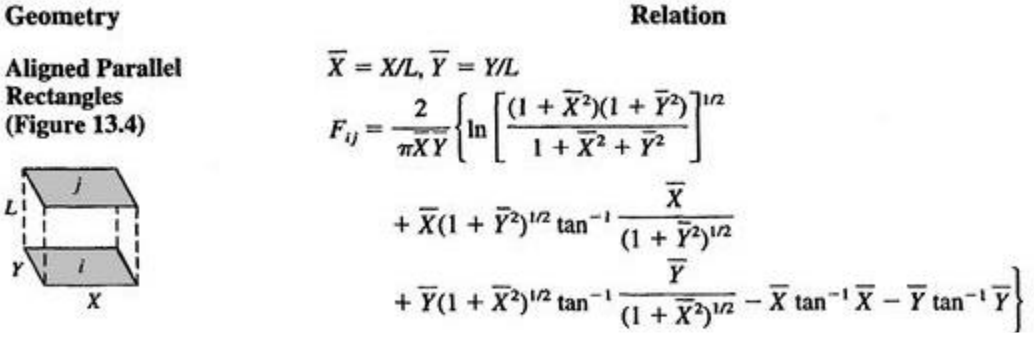
$$R_b = \frac{2(1-\varepsilon)}{\varepsilon A_1} \tag{19}$$

$$R_c = \frac{1}{A_1 F_{13} + 2A_3 F_{35}} \tag{20}$$

$$R_d = \frac{2}{A_1 F_{12} + 2A_1 F_{15}} \tag{21}$$

$$R_e = \frac{1}{A_1 F_{13}} \tag{22}$$

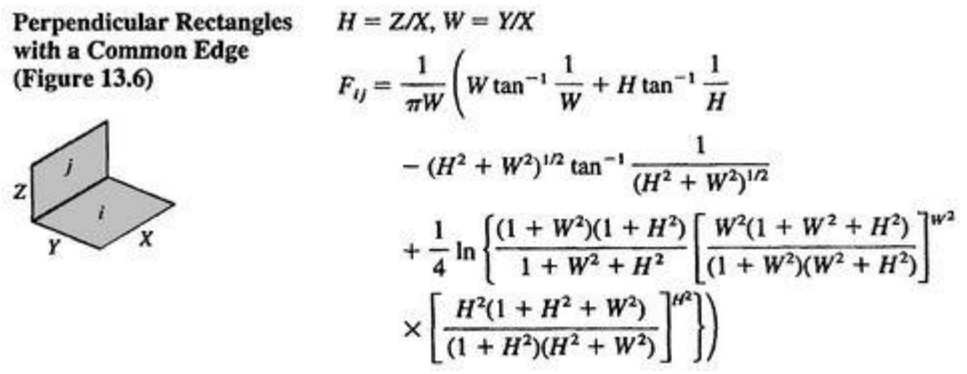
The view (shape or configuration) factors F_{12}, F_{13}, F_{15}, and F_{35} are computed using the appropriate parallel and perpendicular formulae for two rectangles. These formulae are given in figures 4 and 5. Calculated values for the correlations and the resulting F shape factors (Fshape in the last column) are shown in table 5.



Geometry	Relation
Aligned Parallel Rectangles (Figure 13.4)	$\bar{X} = X/L, \bar{Y} = Y/L$

$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

Figure 4
Formulae for view factors of parallel rectangles



Perpendicular Rectangles with a Common Edge (Figure 13.6)

$H = Z/X,\ W = Y/X$

$$F_{ij} = \frac{1}{\pi W}\left(W \tan^{-1}\frac{1}{W} + H \tan^{-1}\frac{1}{H} - (H^2 + W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$$

Figure 5
Formulae for view factors of two perpendicular rectangles

Table 5
Values of radiative correlations for U-channels

# of Fins	x	y	x	y	z	x	y	z	x	y	z	F1-2	F1-3	F1-5	F3-5	A1	A3	Ra	Rb	Rc	Rd	Re	Cnet	Fshape
2	5.4675824	4	0.25	1.366896	1.930904	0.182896	0.731585	0.568667	5.467582	4	45.89446	0.67007	0.068631	0.096334	0.106557	21.18684	3.874992	0.056648	0.020722	0.438617	0.109417	0.687721	9.705894	0.67
3	2.7231913	4	0.25	0.680798	0.525986	0.367216	1.468865	2.292412	2.723191	4	23.41577	0.570515	0.129531	0.085212	0.092467	10.55235	3.874992	0.056648	0.041604	0.479968	0.255799	0.731608	5.227406	0.57
4	1.8083942	4	0.25	0.452099	0.266893	0.552977	2.211907	5.198316	1.808394	4	19.27029	0.486843	0.180261	0.076318	0.080548	7.007514	3.874992	0.056648	0.062651	0.529823	0.446314	0.791653	3.670672	0.50
5	1.3509957	4	0.25	0.337749	0.176574	0.740195	2.960779	9.314102	1.350996	4	17.82519	0.418671	0.221523	0.069142	0.071174	5.235098	3.874992	0.056648	0.083862	0.584355	0.685939	0.862297	2.859519	0.44
6	1.0765566	4	0.25	0.269139	0.134936	0.928888	3.71555	14.66815	1.076557	4	17.15897	0.363703	0.254899	0.06325	0.063718	4.171648	3.874992	0.056648	0.10524	0.642195	0.978017	0.940427	2.354316	0.39
7	0.8935972	4	0.25	0.223399	0.112407	1.119072	4.47629	21.2895	0.893597	4	16.79852	0.319328	0.282005	0.058331	0.057661	3.462682	3.874992	0.056648	0.126787	0.702562	1.324772	1.024071	2.006208	0.36
8	0.7629119	4	0.25	0.190728	0.098877	1.310767	5.24307	29.20789	0.762912	4	16.58203	0.283241	0.304218	0.054162	0.052642	2.956278	3.874992	0.056648	0.148506	0.764922	1.727752	1.111912	1.750238	0.33
9	0.6648979	4	0.25	0.166224	0.090131	1.50399	6.015961	38.45378	0.664898	4	16.44209	0.253594	0.322622	0.050582	0.048413	2.576474	3.874992	0.056648	0.170397	0.828895	2.188131	1.203042	1.553292	0.30
10	0.5886648	4	0.25	0.147166	0.084158	1.69876	6.795038	49.05833	0.588665	4	16.34653	0.228962	0.338047	0.047472	0.0448	2.281072	3.874992	0.056648	0.192464	0.894209	2.706903	1.296832	1.396616	0.28
11	0.5276783	4	0.25	0.13192	0.079903	1.895094	7.580376	61.05348	0.527678	4	16.27844	0.208267	0.351123	0.044743	0.041675	2.044749	3.874992	0.056648	0.214708	0.960669	3.284986	1.392836	1.268736	0.26
12	0.4777803	4	0.25	0.119445	0.076767	2.093012	8.372049	74.4719	0.47778	4	16.22827	0.190692	0.362326	0.042328	0.038945	1.851395	3.874992	0.056648	0.237132	1.028135	3.923279	1.490738	1.162217	0.24
13	0.4361986	4	0.25	0.10905	0.074392	2.292534	9.170134	89.34707	0.436199	4	16.19027	0.175618	0.372018	0.040173	0.036539	1.690266	3.874992	0.056648	0.259737	1.096504	4.622697	1.590307	1.072013	0.23
14	0.4010141	4	0.25	0.100254	0.072551	2.493678	9.974711	105.7133	0.401014	4	16.16081	0.162569	0.380478	0.038238	0.034402	1.553927	3.874992	0.056648	0.282526	1.165704	5.384186	1.691377	0.99457	0.21
15	0.370856	4	0.25	0.092714	0.071096	2.696465	10.78586	123.6057	0.370856	4	16.13753	0.151179	0.387922	0.036489	0.032491	1.437064	3.874992	0.056648	0.305501	1.23568	6.208732	1.793823	0.927311	0.20
16	0.3447189	4	0.25	0.08618	0.069927	2.900914	11.60366	143.0601	0.344719	4	16.11883	0.14116	0.394521	0.034899	0.030771	1.335783	3.874992	0.056648	0.328664	1.306394	7.097363	1.897554	0.868318	0.19
17	0.321849	4	0.25	0.080462	0.068974	3.107047	12.42819	164.1136	0.321849	4	16.10359	0.132287	0.400409	0.033447	0.029214	1.247162	3.874992	0.056648	0.352019	1.377818	8.051153	2.002502	0.816129	0.18
18	0.3016696	4	0.25	0.075417	0.068188	3.314884	13.25954	186.8038	0.30167	4	16.091	0.124378	0.405696	0.032115	0.027799	1.168968	3.874992	0.056648	0.375566	1.449931	9.071223	2.108614	0.769613	0.17
19	0.2837325	4	0.25	0.070933	0.067532	3.524447	14.09779	211.1694	0.283732	4	16.0805	0.117289	0.410468	0.030888	0.026507	1.099461	3.874992	0.056648	0.399309	1.522719	10.15874	2.215855	0.727881	0.16
20	0.2676834	4	0.25	0.066921	0.066978	3.735757	14.94303	237.2499	0.267683	4	16.07165	0.1109	0.414797	0.029753	0.025322	1.037271	3.874992	0.056648	0.423249	1.596174	11.31491	2.324195	0.690219	0.16
21	0.2532392	4	0.25	0.06331	0.066508	3.948835	15.79534	265.0861	0.253239	4	16.06413	0.105115	0.418742	0.028701	0.024231	0.9813	3.874992	0.056648	0.447391	1.670289	12.54099	2.433614	0.656053	0.15
22	0.2401707	4	0.25	0.060043	0.066105	4.163705	16.65482	294.7195	0.240171	4	16.05768	0.099853	0.422353	0.027721	0.023223	0.93066	3.874992	0.056648	0.471735	1.745061	13.83828	2.544099	0.62491	0.14
23	0.2282902	4	0.25	0.057073	0.065757	4.380389	17.52156	326.1928	0.22829	4	16.05212	0.095048	0.42567	0.026806	0.02229	0.884623	3.874992	0.056648	0.496284	1.82049	15.20813	2.65564	0.596403	0.14
24	0.2174428	4	0.25	0.054361	0.065455	4.59891	18.39564	359.5496	0.217443	4	16.04728	0.090644	0.428728	0.02595	0.021423	0.842589	3.874992	0.056648	0.521042	1.896577	16.65193	2.768231	0.570205	0.13
25	0.2074994	4	0.25	0.051875	0.065191	4.819292	19.27717	394.8348	0.207499	4	16.04306	0.086592	0.431557	0.025147	0.020615	0.804058	3.874992	0.056648	0.546011	1.973324	18.17111	2.881871	0.546045	0.13
26	0.1983514	4	0.25	0.049588	0.064959	5.041558	20.16623	432.0942	0.198351	4	16.03934	0.082854	0.434181	0.024392	0.01986	0.76861	3.874992	0.056648	0.571193	2.050734	19.76715	2.996559	0.523691	0.12

Once the shape factor is obtained, the radiation conductance (C_{ri}), overall conductance for the inter-fin passages (C_i), overall conductance for the heat sink (C_s), overall resistance for the heat sink (R_s), heat sink dissipation (Q_s), and the sink's base temperature (T_s) can be calculated using equations 23 through 28. Obtained results are sorted and shown in table 6 as a function of the numbers of fins.

$$C_{ri} = 3.657x10^{-11} FA_i \frac{(T_s + 273)^4 - (T_a + 273)^4}{T_s - T_a} \tag{23}$$

$$C_i = C_{ci} + C_{ri} \tag{24}$$

$$C_s = C_i + C_{so} \tag{25}$$

$$R_s = \frac{1}{C_s} \tag{26}$$

$$Q_s = \frac{\Delta T}{R_s} \tag{27}$$

$$T_s = T_a + R_s Q \tag{28}$$

Table 6
Values for conductance and heat sink resistance, dissipation, and base temperature as a function of the number of fins

# *of Fins*	**Fshapefac**	**Cri**	***Ci***	***Cs***	**Rs**	***Qs***	**Ts**
2	0.671	**0.08**	***0.20***	***0.26***	3.9020	***7.69***	**98.0**
3	0.571	**0.09**	***0.24***	***0.29***	3.4124	***8.79***	**88.2**
4	0.497	**0.10**	***0.27***	***0.33***	3.0511	***9.83***	**81.0**
5	0.440	**0.10**	***0.30***	***0.36***	2.7716	***10.82***	**75.4**
6	0.394	**0.10**	***0.34***	***0.39***	2.5488	***11.77***	**71.0**
7	0.357	**0.10**	***0.37***	***0.42***	2.3680	***12.67***	**67.4**
8	0.326	**0.11**	***0.39***	***0.45***	2.2195	***13.52***	**64.4**
9	0.300	**0.11**	***0.42***	***0.48***	2.0968	***14.31***	**61.9**
10	0.278	**0.11**	***0.44***	***0.50***	1.9955	***15.03***	**59.9**
11	0.259	**0.11**	***0.47***	***0.52***	1.9122	***15.69***	**58.2**
12	0.242	**0.11**	***0.48***	***0.54***	1.8444	***16.27***	**56.9**
13	0.227	**0.11**	***0.50***	***0.56***	1.7903	***16.76***	**55.8**
14	0.213	**0.11**	***0.51***	***0.57***	1.7483	***17.16***	**55.0**
15	0.201	**0.11**	***0.52***	***0.58***	1.7177	***17.47***	**54.4**
16	0.191	**0.11**	***0.53***	***0.59***	1.6975	***17.67***	**53.9**
17	0.181	**0.11**	***0.54***	***0.59***	1.6872	***17.78***	**53.7**
18	0.172	**0.11**	***0.54***	***0.59***	1.6866	***17.79***	**53.7**
19	0.164	**0.11**	***0.53***	***0.59***	1.6954	***17.69***	**53.9**
20	0.157	**0.11**	***0.53***	***0.58***	1.7135	***17.51***	**54.3**
21	0.150	**0.11**	***0.52***	***0.57***	1.7409	***17.23***	**54.8**
22	0.144	**0.11**	***0.51***	***0.56***	1.7775	***16.88***	**55.5**
23	0.138	**0.11**	***0.49***	***0.55***	1.8232	***16.45***	**56.5**
24	0.132	**0.11**	***0.48***	***0.53***	1.8780	***15.97***	**57.6**
25	0.127	**0.11**	***0.46***	***0.52***	1.9415	***15.45***	**58.8**
26	0.123	**0.11**	***0.44***	***0.50***	2.0134	***14.90***	**60.3**

CODE VALIDATION

Validation of the code was done by comparing results obtained by using the code to the results of a published paper (ref. 6). In reference 6, three different fin materials were examined in a heat sink model without an enclosure, and the optimal number of fins and the amount of heat dissipated was determined. The following parameters were used for the heat sink and fins, which was subsequently used as input for the code:

Geometry:

- Equipment box
  - Height Hs=3.94 in.
  - Width Ws=5.42 in.

- Fins
  - Height Hf=0.98 in.
  - Length Lf= 3.94 in.

Fin thickness and conductivity vary depending on the heat sink material as shown in table 7 [used in Aihara's paper (ref. 6)]. Table 8 gives a comparison between the present analysis and the results obtained from Aihara (ref. 6) for the number of fins and the heat dissipation by the fins.

Table 7
Fin material properties used for code validation

Fin material	Thickness of the fin, t_f (in.)	Fin conductivity k(W/in.-K)
Aluminum	0.021	4.84
Steel	0.046	1.625
Porcelain	0.094	0.267

Note: Ambient temperature: T_a= 200°C, fin temperature: T_f=500°C, and fin emissivity: ε_e=0.82.

Table 8
Comparison of results for optimal number of fins (ref. 6)

Material of	**No. of fins**		**Q dissipation/fin,W**	
heat sink	**Aiharas**	**Present**	**Aiharas**	**Present**
Aluminum	***19***	***18***	***17.9***	***17.7***
Steel	***17***	***16***	***16.8***	***16.6***
Porcelain	***15***	***14***	***15.1***	***14.6***

Table 8 shows a good agreement with Aiharas, and the differences are actually due to the method that the present work used to account for fin efficiency in the calculation procedure.

WORKED EXAMPLES

Use of the "Rectangular Heat Sink Design and Optimization Code" is demonstrated with two worked examples. The first example deals with optimizing the design of a heat sink for a power transistor by finding the ideal number of fins and the fin thickness. In the second example, a heat sink for electric package cooling is designed to meet the heat dissipation requirements.

Heat Sink for a Power Transistor

In this first example, a commercial aluminum heat sink for a power transistor is analyzed. The heat sinks are black anodized, and, accordingly, their surface emissivity is assumed to be $\varepsilon e = 0.82$ on the basis of experimental measurements.

Several cases of optimization are attempted of various heat sinks with the same external dimensions and the same temperature conditions as the commercial ones. The external dimension of the power transistor and the heat sinks, as well as the temperature conditions, are as follows.

Power Transistor Dimensions:

- H = 2.756 in.
- W = 3.8484 in.

Temperature Conditions:

- Ta = 20°C – ambient temperature
- Tw = 50°C – maximum desired wall temperature of power transistor

Fin Dimension:

- H = 2.756 in.
- L = 0.98425 in.

For the commercial heat sink, the thickness of the fin is given as tf = 0.06889 in. The code will be used to optimize the heat sink by optimizing the fin's thickness and number of fins but keeping the external dimensions the same.

In the first step, the commercial heat sink will be analyzed to obtain the optimal number of the fins given certain conditions (temperature conditions and external dimensions). The input data deck populated properly with the commercial sink's data is given in table 9. The heat generation cell (Q) can be approximated for the first pass in the code as 10 W. The code immediately calculates the parameters for the unshielded surfaces from the input data, as shown in table 10. At the same time, the code calculates the net shape factor, F, for the gray body, U-channel interior (table 11) as a function of the number of fins.

Table 9
Input data deck for the commercial heat sink

INPUT DATA			
Q=	10	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	50	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	2.756	inch	Height
W=	3.8484	inch	Width
Fin dimension			
H=	2.756	inch	height of a fin
L=	0.98425	inch	length of a fin
tf=	0.06889	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

Table 10
Unshielded surface relations

Calculated relations for unshielded Surfaces from input data			
H/L=	*2.80*		
deltaT=	*30*	*C*	
A_O=	*5.425186*	in^2	*unshielded surface area*
h_{CO}=	*0.004359*	$W/in^2 K$	*unshielded surface convective h*
h_r=	*0.004284*	$W/in^2 K$	*unshielded surface radiative h*
C_{SO}=	***0.042709***		*unshielded surface conductance*

Table 11
Calculated radiative correlations for commercial sink

# of Fins	x	y	x	y	z	x	y	z	x	y	z	F1-2	F1-3	F1-5	F3-5	A1	A3	Ra	Rb	Rc	Rd	Re	Cnet	Fshape
2	3.769997	2.800102	0.35713	1.346379	1.940278	0.265252	0.742733	0.622011	3.769997	2.800102	22.05345	0.568191	0.09061	0.125295	0.132003	10.22647	2.712593	0.080923	0.04293	0.608733	0.238856	1.079195	4.999817	0.64
3	1.850003	2.800102	0.35713	0.660691	0.564055	0.54054	1.513566	2.583066	1.850003	2.800102	11.26308	0.45111	0.167512	0.106933	0.111418	5.018304	2.712593	0.080923	0.087485	0.692	0.599331	1.189589	2.750168	0.53
4	1.210004	2.800102	0.35713	0.432129	0.314277	0.826443	2.314125	6.038185	1.210004	2.800102	9.304679	0.360835	0.226824	0.092758	0.094194	3.282249	2.712593	0.080923	0.133757	0.796485	1.115287	1.343194	1.940476	0.45
5	0.890005	2.800102	0.35713	0.317847	0.228569	1.123589	3.146164	11.1608	0.890005	2.800102	8.632678	0.293687	0.27145	0.081706	0.080861	2.414222	2.712593	0.080923	0.181849	0.914052	1.81235	1.525923	1.506559	0.38
6	0.698006	2.800102	0.35713	0.249279	0.189682	1.432653	4.011575	18.14523	0.698006	2.800102	8.327781	0.243698	0.305259	0.072893	0.070434	1.893405	2.712593	0.080923	0.23187	1.041562	2.712054	1.730169	1.230585	0.34
7	0.570006	2.800102	0.35713	0.203566	0.168981	1.754368	4.912408	27.20956	0.570006	2.800102	8.165476	0.205864	0.331365	0.065703	0.0621	1.546194	2.712593	0.080923	0.283939	1.177502	3.835198	1.951774	1.037423	0.30
8	0.478578	2.800102	0.35713	0.170914	0.156753	2.089525	5.850883	38.59895	0.478578	2.800102	8.069605	0.176611	0.351974	0.059721	0.055296	1.298186	2.712593	0.080923	0.338183	1.321142	5.203848	2.18853	0.893677	0.27
9	0.410006	2.800102	0.35713	0.146426	0.148982	2.438987	6.82941	52.5895	0.410006	2.800102	8.008674	0.153499	0.368594	0.054656	0.049641	1.11218	2.712593	0.080923	0.394742	1.472202	6.842425	2.439363	0.782041	0.24
10	0.356673	2.800102	0.35713	0.127379	0.143767	2.803687	7.850609	69.49273	0.356673	2.800102	7.967785	0.134872	0.382258	0.050305	0.044866	0.967509	2.712593	0.080923	0.453768	1.630676	8.778384	2.703884	0.692567	0.22
11	0.314007	2.800102	0.35713	0.112141	0.140117	3.184646	8.917333	89.66081	0.314007	2.800102	7.939169	0.119591	0.393684	0.04652	0.040779	0.851772	2.712593	0.080923	0.515425	1.79674	11.04277	2.982146	0.619096	0.20
12	0.279098	2.800102	0.35713	0.099674	0.137477	3.582976	10.0327	113.4927	0.279098	2.800102	7.918464	0.106858	0.40338	0.043191	0.037242	0.757078	2.712593	0.080923	0.579893	1.970699	13.67074	3.2745	0.557592	0.18
13	0.250007	2.800102	0.35713	0.089285	0.135514	3.999892	11.2001	141.4414	0.250007	2.800102	7.903072	0.0961	0.411714	0.040236	0.034149	0.678167	2.712593	0.080923	0.64737	2.15296	16.7022	3.581521	0.50529	0.17
14	0.225391	2.800102	0.35713	0.080494	0.134021	4.436725	12.42328	174.0225	0.225391	2.800102	7.89137	0.086903	0.418959	0.037589	0.031422	0.611395	2.712593	0.080923	0.71807	2.344016	20.18248	3.903967	0.460227	0.15
15	0.204293	2.800102	0.35713	0.072959	0.132865	4.89494	13.70633	211.8239	0.204293	2.800102	7.882304	0.078955	0.425319	0.035203	0.028998	0.554163	2.712593	0.080923	0.79223	2.544441	24.16316	4.242753	0.420971	0.14
16	0.186007	2.800102	0.35713	0.066429	0.131955	5.376143	15.05375	255.5182	0.186007	2.800102	7.875168	0.072024	0.430951	0.033037	0.026829	0.504561	2.712593	0.080923	0.870111	2.754888	28.70304	4.598948	0.386447	0.13
17	0.170007	2.800102	0.35713	0.060715	0.131228	5.882111	16.47051	305.8769	0.170007	2.800102	7.869471	0.065928	0.435976	0.03106	0.024876	0.46116	2.712593	0.080923	0.952001	2.976095	33.86929	4.973773	0.355834	0.12
18	0.155889	2.800102	0.35713	0.055673	0.130641	6.414805	17.96211	363.787	0.155889	2.800102	7.86487	0.060527	0.440491	0.029246	0.023108	0.422864	2.712593	0.080923	1.038215	3.208886	39.73887	5.368611	0.328493	0.11
19	0.14334	2.800102	0.35713	0.051191	0.130162	6.976401	19.53463	430.272	0.14334	2.800102	7.861115	0.05571	0.444572	0.027573	0.0215	0.388824	2.712593	0.080923	1.129108	3.454184	46.40016	5.785014	0.303917	0.10
20	0.132112	2.800102	0.35713	0.047181	0.129768	7.569315	21.19485	506.5163	0.132112	2.800102	7.858023	0.051389	0.448282	0.026023	0.020031	0.358367	2.712593	0.080923	1.225069	3.71302	53.95497	6.22473	0.281701	0.10
21	0.122007	2.800102	0.35713	0.043572	0.12944	8.196244	22.95031	593.8953	0.122007	2.800102	7.855455	0.047491	0.451671	0.024583	0.018683	0.330956	2.712593	0.080923	1.326535	3.986549	62.52096	6.689716	0.261515	0.09
22	0.112864	2.800102	0.35713	0.040307	0.129166	8.860199	24.80946	694.0123	0.112864	2.800102	7.853307	0.043958	0.454782	0.023239	0.017442	0.306155	2.712593	0.080923	1.433995	4.276062	72.23456	7.182172	0.243091	0.08
23	0.104553	2.800102	0.35713	0.037339	0.128936	9.564563	26.78175	808.7428	0.104553	2.800102	7.8515	0.04074	0.457648	0.021982	0.016295	0.283609	2.712593	0.080923	1.547993	4.583015	83.25461	7.704576	0.226203	0.08
24	0.096964	2.800102	0.35713	0.034629	0.128741	10.31314	28.87783	940.2901	0.096964	2.800102	7.849971	0.037799	0.4603	0.020801	0.015232	0.263023	2.712593	0.080923	1.669148	4.909043	95.76668	8.25972	0.210665	0.07
25	0.090007	2.800102	0.35713	0.032144	0.128575	11.11022	31.10975	1091.254	0.090007	2.800102	7.84867	0.035099	0.462762	0.019689	0.014244	0.244153	2.712593	0.080923	1.798154	5.255998	109.9886	8.850764	0.19632	0.07
26	0.083607	2.800102	0.35713	0.029859	0.128433	11.96069	33.49115	1264.715	0.083607	2.800102	7.847559	0.032613	0.465055	0.018639	0.013322	0.226792	2.712593	0.080923	1.935799	5.625978	126.1771	9.481297	0.183032	0.06
27	0.0777	2.800102	0.35713	0.027749	0.128312	12.87009	36.03756	1464.345	0.0777	2.800102	7.846606	0.030317	0.467197	0.017645	0.012461	0.210767	2.712593	0.080923	2.082982	6.021377	144.6365	10.15541	0.170688	0.06
28	0.072229	2.800102	0.35713	0.025795	0.128207	13.84477	38.76675	1694.538	0.072229	2.800102	7.845786	0.028189	0.469203	0.016702	0.011655	0.195929	2.712593	0.080923	2.24073	6.444926	165.729	10.87777	0.15919	0.06
29	0.06715	2.800102	0.35713	0.023981	0.128117	14.89201	41.69913	1960.59	0.06715	2.800102	7.845078	0.026211	0.471089	0.015806	0.010898	0.182151	2.712593	0.080923	2.410223	6.89977	189.8883	11.65376	0.148453	0.05
30	0.062421	2.800102	0.35713	0.022292	0.128039	16.02023	44.85828	2268.913	0.062421	2.800102	7.844465	0.024369	0.472864	0.014952	0.010186	0.169323	2.712593	0.080923	2.592823	7.389537	217.6372	12.48959	0.138401	0.05
31	0.058007	2.800102	0.35713	0.020716	0.127971	17.23922	48.27155	2627.333	0.058007	2.800102	7.843934	0.022649	0.474539	0.014136	0.009515	0.15735	2.712593	0.080923	2.790111	7.918437	249.6094	13.39247	0.128972	0.05
32	0.053878	2.800102	0.35713	0.019242	0.127912	18.56036	51.9709	3045.461	0.053878	2.800102	7.843472	0.02104	0.476124	0.013356	0.008881	0.14615	2.712593	0.080923	3.003934	8.491389	286.5788	14.37082	0.120108	0.04
33	0.050007	2.800102	0.35713	0.017859	0.127861	19.99708	55.99385	3535.195	0.050007	2.800102	7.84307	0.01953	0.477627	0.012608	0.008282	0.135649	2.712593	0.080923	3.236463	9.11417	329.4974	15.43453	0.111759	0.04

Finally, the spacing, heat transfer coefficients, sink surface temperature, conductance, and correlations used in the calculations can be read from the table as a function of the number of fins, as shown in table 12.

Table 12
Results for spacing, heat transfer, surface temperature, etc., as a function of number of fins

# of Fins	S(inch)	r(inch)	a	Ra*	PSI	Nur	hci(W/in^{2}C)	Ai	Cci (W/K)	η_{fin}	η_{sink}	Ccieta (W/K)	L/S	Fshapefac	Cri	Ci	Cs	Rs	Q	Ts
2	3.7106	1.2862	3.7700	24054.3645	7.4362	7.3965	0.00438	16.03	0.07	0.992	0.987	0.07	0.27	0.639	0.04	0.11	0.16	6.4000	4.69	84.0
3	1.8209	0.9459	1.8500	7036.8352	13.2809	5.4180	0.00436	21.46	0.09	0.992	0.983	0.09	0.54	0.527	0.05	0.14	0.18	5.4351	5.52	74.4
4	1.1909	0.7420	1.2100	2664.7550	14.5918	4.2225	0.00434	26.88	0.12	0.992	0.979	0.12	0.83	0.446	0.05	0.17	0.21	4.7702	6.29	67.7
5	0.8760	0.6062	0.8900	1187.1556	15.0990	3.4139	0.00429	32.31	0.14	0.992	0.975	0.14	1.12	0.384	0.05	0.19	0.23	4.2845	7.00	62.8
6	0.6870	0.5093	0.6980	591.2843	15.4396	2.8227	0.00422	37.73	0.16	0.992	0.971	0.16	1.43	0.336	0.05	0.21	0.26	3.9195	7.65	59.2
7	0.5610	0.4366	0.5700	319.3840	15.6779	2.3648	0.00413	43.16	0.18	0.992	0.968	0.18	1.75	0.298	0.06	0.23	0.27	3.6437	8.23	56.4
8	0.4710	0.3801	0.4786	183.4558	15.8084	1.9944	0.00400	48.58	0.19	0.992	0.965	0.19	2.09	0.266	0.06	0.25	0.29	3.4388	8.72	54.4
9	0.4035	0.3349	0.4100	110.5660	15.8444	1.6847	0.00383	54.01	0.21	0.993	0.963	0.21	2.44	0.239	0.06	0.26	0.30	3.2942	9.11	52.9
10	0.3511	0.2979	0.3567	69.2495	15.8182	1.4191	0.00363	59.43	0.22	0.993	0.961	0.21	2.80	0.217	0.06	0.27	0.31	3.2041	9.36	52.0
11	0.3091	0.2671	0.3140	44.7537	15.7660	1.1874	0.00339	64.86	0.22	0.993	0.960	0.22	3.18	0.197	0.05	0.27	0.32	3.1668	9.47	51.7
12	0.2747	0.2411	0.2791	29.6828	15.7176	0.9829	0.00311	70.28	0.22	0.994	0.960	0.22	3.58	0.180	0.05	0.27	0.31	3.1839	9.42	51.8
13	0.2461	0.2187	0.2500	20.1189	15.6936	0.8019	0.00279	75.71	0.21	0.995	0.962	0.21	4.00	0.166	0.05	0.26	0.31	3.2595	9.20	52.6
14	0.2218	0.1994	0.2254	13.8885	15.7058	0.6428	0.00246	81.13	0.20	0.995	0.964	0.20	4.44	0.152	0.05	0.25	0.29	3.4002	8.82	54.0
15	0.2011	0.1824	0.2043	9.7379	15.7595	0.5050	0.00211	86.56	0.18	0.996	0.967	0.18	4.89	0.141	0.05	0.23	0.28	3.6132	8.30	56.1
16	0.1831	0.1675	0.1860	6.9190	15.8557	0.3885	0.00177	91.98	0.16	0.997	0.970	0.16	5.38	0.130	0.05	0.21	0.26	3.9049	7.68	59.0
17	0.1673	0.1542	0.1700	4.9723	15.9926	0.2930	0.00145	97.41	0.14	0.997	0.974	0.14	5.88	0.121	0.05	0.19	0.23	4.2771	7.01	62.8
18	0.1534	0.1423	0.1559	3.6082	16.1670	0.2175	0.00116	102.83	0.12	0.998	0.978	0.12	6.41	0.112	0.05	0.17	0.21	4.7242	6.35	67.2
19	0.1411	0.1316	0.1433	2.6402	16.3747	0.1598	0.00092	108.26	0.10	0.998	0.982	0.10	6.98	0.105	0.05	0.15	0.19	5.2318	5.73	72.3
20	0.1300	0.1220	0.1321	1.9457	16.6113	0.1168	0.00073	113.68	0.08	0.999	0.985	0.08	7.57	0.097	0.05	0.13	0.17	5.7798	5.19	77.8
21	0.1201	0.1132	0.1220	1.4424	16.8724	0.0854	0.00058	119.11	0.07	0.999	0.988	0.07	8.20	0.091	0.05	0.11	0.16	6.3483	4.73	83.5
22	0.1111	0.1052	0.1129	1.0747	17.1535	0.0626	0.00045	124.54	0.06	0.999	0.990	0.06	8.86	0.085	0.05	0.10	0.14	6.9227	4.33	89.2
23	0.1029	0.0978	0.1046	0.8040	17.4506	0.0461	0.00036	129.96	0.05	0.999	0.991	0.05	9.56	0.079	0.04	0.09	0.13	7.4933	4.00	94.9
24	0.0954	0.0910	0.0970	0.6034	17.7596	0.0340	0.00028	135.39	0.04	0.999	0.993	0.04	10.31	0.074	0.04	0.08	0.12	8.0543	3.72	100.5
25	0.0886	0.0848	0.0900	0.4540	18.0768	0.0251	0.00023	140.81	0.03	1.000	0.994	0.03	11.11	0.069	0.04	0.07	0.12	8.6013	3.49	106.0
26	0.0823	0.0790	0.0836	0.3422	18.3989	0.0186	0.00018	146.24	0.03	1.000	0.995	0.03	11.96	0.065	0.04	0.07	0.11	9.1314	3.29	111.3
27	0.0765	0.0736	0.0777	0.2582	18.7226	0.0138	0.00014	151.66	0.02	1.000	0.996	0.02	12.87	0.061	0.04	0.06	0.10	9.6428	3.11	116.4
28	0.0711	0.0686	0.0722	0.1948	19.0451	0.0102	0.00011	157.09	0.02	1.000	0.997	0.02	13.84	0.057	0.04	0.06	0.10	10.1349	2.96	121.3
29	0.0661	0.0639	0.0672	0.1470	19.3639	0.0076	0.00009	162.51	0.01	1.000	0.997	0.01	14.89	0.053	0.04	0.05	0.09	10.6079	2.83	126.1
30	0.0614	0.0596	0.0624	0.1108	19.6765	0.0056	0.00007	167.94	0.01	1.000	0.998	0.01	16.02	0.049	0.04	0.05	0.09	11.0626	2.71	130.6
31	0.0571	0.0555	0.0580	0.0833	19.9811	0.0042	0.00006	173.36	0.01	1.000	0.998	0.01	17.24	0.046	0.03	0.04	0.09	11.5006	2.61	135.0
32	0.0530	0.0516	0.0539	0.0625	20.2759	0.0031	0.00005	178.79	0.01	1.000	0.999	0.01	18.56	0.043	0.03	0.04	0.08	11.9238	2.52	139.2
33	0.0492	0.0480	0.0500	0.0467	20.5597	0.0023	0.00004	184.21	0.01	1.000	0.999	0.01	20.00	0.040	0.03	0.04	0.08	12.3342	2.43	143.3

Also, directly from the charts, a graph can be used to find the optimal number of fins that corresponds to the minimum temperature of the base of the sink (fig. 6) or the maximum heat transfer, or conductance, from the heat sink (fig. 7). From the charts or tables, the optimal spacing, maximal heat transfer, and lowest temperature attainable with the considered heat sink can be directly read, as shown in table 13.

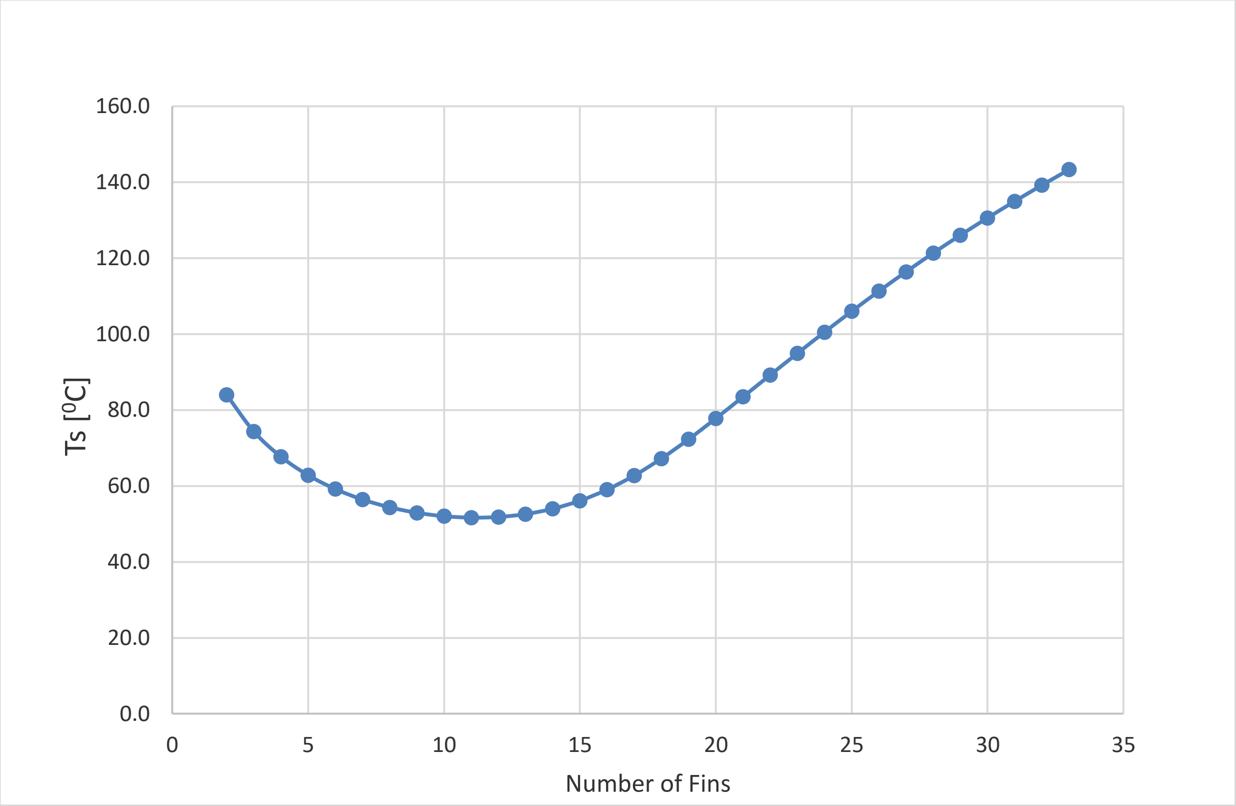


Figure 6
Temperature at the base of the sink as a function of the number of fins

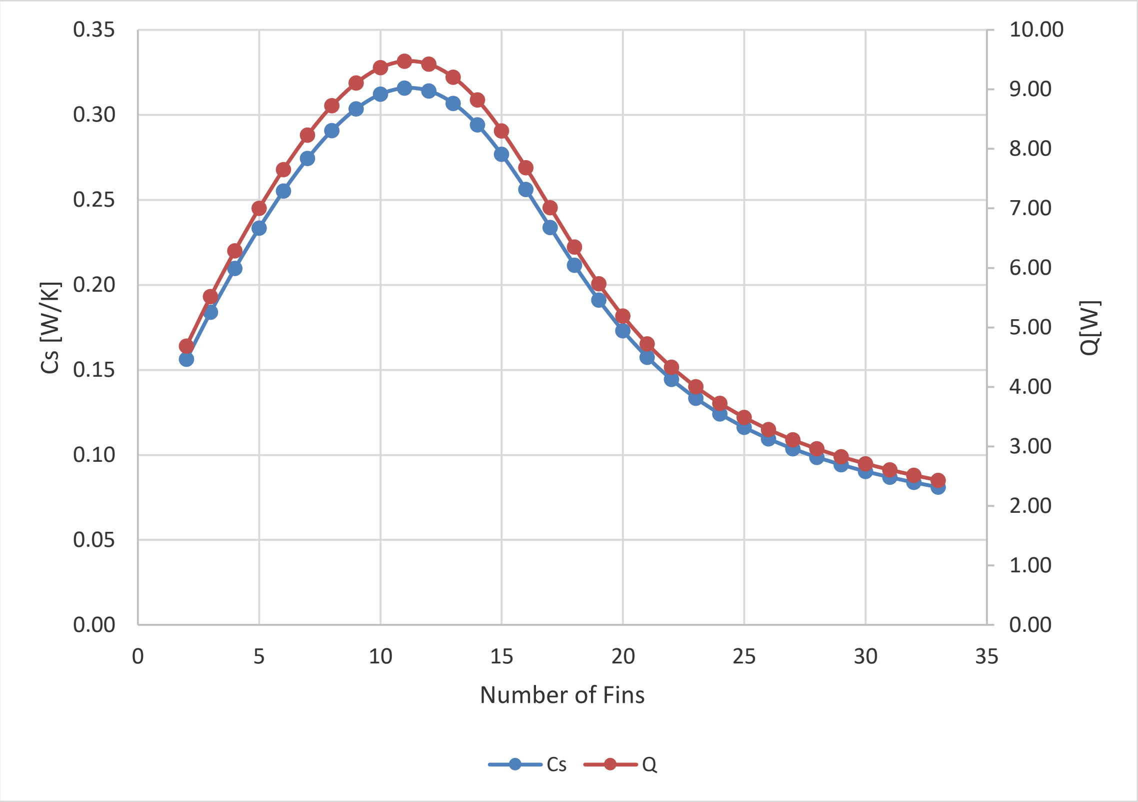


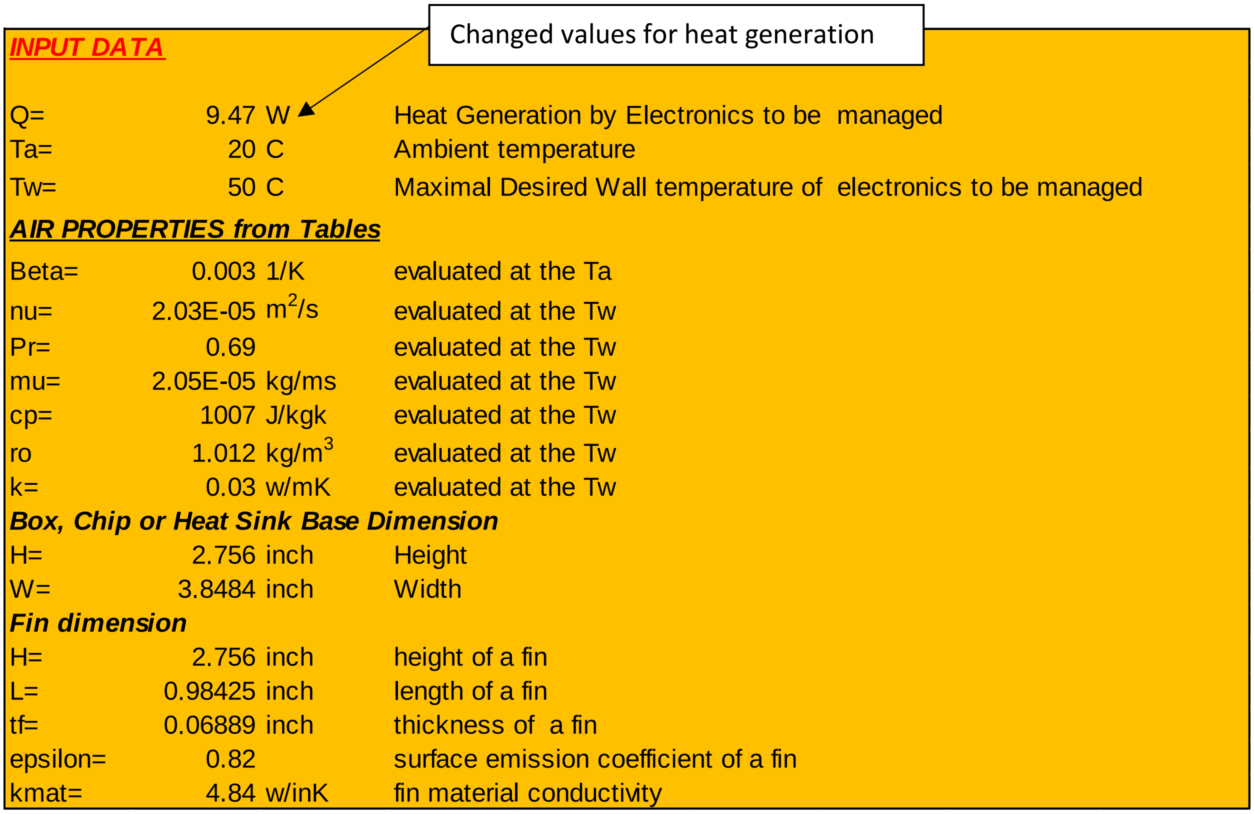
Figure 7
Conductance and heat transfer by the heat sink as a function of the number of fins

Table 13
Spacing, S, Heat transfer, Q, and surface temperature as a function of the number of fins

# of Fins	**S(inch)**	***Q [W]***	**Ts [^{0}C]**
8	**0.4710**	***8.72***	**54.4**
9	**0.4035**	***9.11***	**52.9**
10	**0.3511**	***9.36***	**52.0**
11	**0.3091**	***9.47***	**51.7**
12	**0.2747**	***9.42***	**51.8**
13	**0.2461**	***9.20***	**52.6**
14	**0.2218**	***8.82***	**54.0**
15	**0.2011**	***8.30***	**56.1**
16	**0.1831**	***7.68***	**59.0**

It can be seen that the optimal spacing in the considered commercial heat sink is achieved with 11 fins. In this case, the heat sink is able to dissipate 9.47 W of heat with the sink base temperature reaching 51.7°C. However, this temperature is a bit higher than the maximum allowed temperature (50°C). Subsequently, the maximum calculated heat dissipation can be used as input into the code (Q=9.47 W), and change the assumed Q in the input deck (table 14) with this value to see that the sink base temperature does not go beyond the maximum allowed wall temperature (table 15).

Table 14
Changing the heat generation cell with the calculated value of heat able to be dissipated



Changed values for heat generation

INPUT DATA			
Q=	9.47	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	50	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	2.756	inch	Height
W=	3.8484	inch	Width
Fin dimension			
H=	2.756	inch	height of a fin
L=	0.98425	inch	length of a fin
tf=	0.06889	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

Table 15
Recalculated surface temperature does not go over allowed maximum temperature

No. of fins	S (in.)	Q (W)	Ts (°C)
8	0.4710	**8.72**	**52.6**
9	0.4035	**9.11**	**51.2**
10	0.3511	**9.36**	**50.3**
11	0.3091	**9.47**	**50.0**
12	0.2747	**9.42**	**50.2**
13	0.2461	**9.20**	**50.9**
14	0.2218	**8.82**	**52.2**
15	0.2011	**8.30**	**54.2**
16	0.1831	**7.68**	**57.0**

Recalculated temperature

In the next step, another case of optimization is attempted with a heat sink with the same external dimensions and temperature condition as the commercial sink. The main parameter under consideration this time is the thickness of the fins. In this variation, the sink will be expected to remove 10 W of heat. The first attempt was made with a fin thickness of 0.045 in., and the following input deck is produced, as shown in table 16.

Table 16
Input deck for optimization of heat sink with fin thickness of 0.045 in.

INPUT DATA			
Q=	10	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	50	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	2.756	inch	Height
W=	3.8484	inch	Width
Fin dimension			
H=	2.756	inch	height of a fin
L=	0.98425	inch	length of a fin
tf=	0.045	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

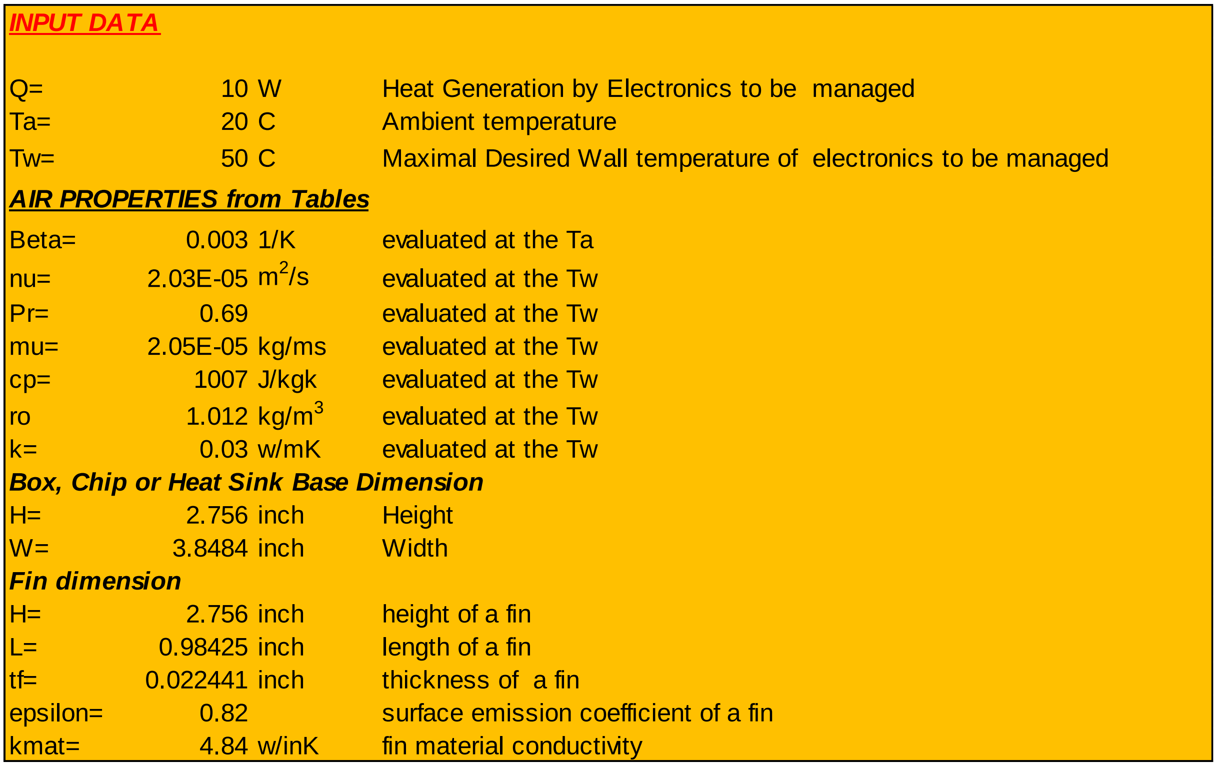
Results from the Excel sheet immediately show that this fin thickness does not allow the required amount of heat from the power transistor to be removed. As shown in table 17, Q always remains below 10 W, regardless of the number of fins.

Table 17
Fin spacing, heat removed, and temperature of the sink base for a fin thickness of 0.045 in.

# of Fins	S(inch)	Q [W]	Ts[^{0}C]
2	3.7584	4.68	84.1
3	1.8567	5.52	74.4
4	1.2228	6.29	67.7
5	0.9059	7.02	62.7
6	0.7157	7.69	59.0
7	0.5889	8.30	56.2
8	0.4983	8.83	54.0
9	0.4304	9.28	52.3
10	0.3776	9.63	51.2
11	0.3353	9.86	50.4
12	0.3008	9.97	50.1
13	0.2720	9.96	50.1
14	0.2476	9.80	50.6
15	0.2267	9.53	51.5
16	0.2086	9.14	52.8
17	0.1927	8.66	54.6
18	0.1787	8.13	56.9

To satisfy the criterion of the amount of heat to be removed by the heat sink (Q=10 W), the thickness of the fin should be decreased. The next attempt was made with a fin thickness of 0.022441 in., which is the smallest fin thickness for this kind of heat sink that can be manufactured without a danger of the fins bending. The new input deck is shown in table 18 with the results shown in table 19.

Table 18
New input deck for fin thickness of 0.022441 in.



INPUT DATA			
Q=	10	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	50	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	2.756	inch	Height
W=	3.8484	inch	Width
Fin dimension			
H=	2.756	inch	height of a fin
L=	0.98425	inch	length of a fin
tf=	0.022441	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

Table 19
New results for fin thickness of 0.022441 in.

# of Fins	S(inch)	Q [W]	Ts[^{0}C]
2	3.8035	4.66	84.4
3	1.8905	5.49	74.6
4	1.2529	6.27	67.9
5	0.9340	7.00	62.9
6	0.7428	7.68	59.1
7	0.6152	8.31	56.1
8	0.5241	8.87	53.8
9	0.4558	9.37	52.0
10	0.4027	9.78	50.7
11	0.3602	10.11	49.7
12	0.3254	10.35	49.0
13	0.2964	10.48	48.6
14	0.2719	10.51	48.5
15	0.2508	10.43	48.8
16	0.2326	10.26	49.2
17	0.2167	10.00	50.0
18	0.2026	9.66	51.0

This new considered thickness allows the removal of 10.51 W of heat with the heat sink temperature of 48.5°C and optimal fin spacing of 0.2719 in. for 14 fins in a heat sink. The temperature variation with the number of the fins and the fin spacing are shown in figures 8 and 9. Conductance and the heat transfer by the heat sink as a function of the number of fins and fin spacing is shown in figures 10 and 11.

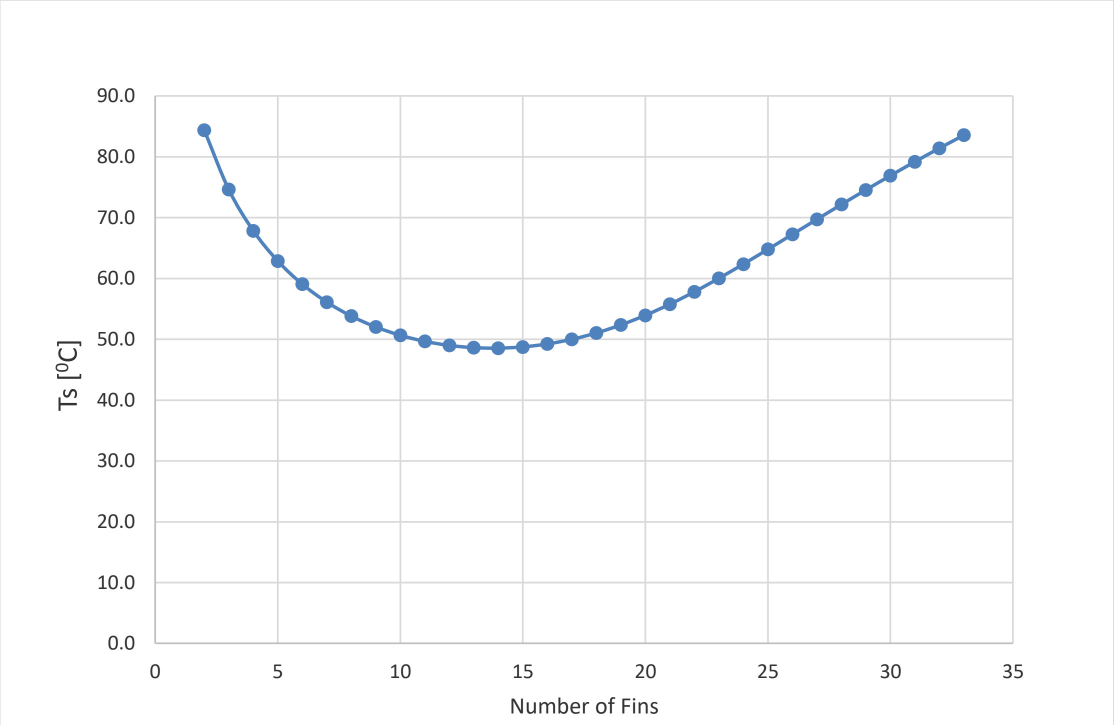


Figure 8
Temperature at the base of the heat sink for fin thickness of 0.0022441 in.

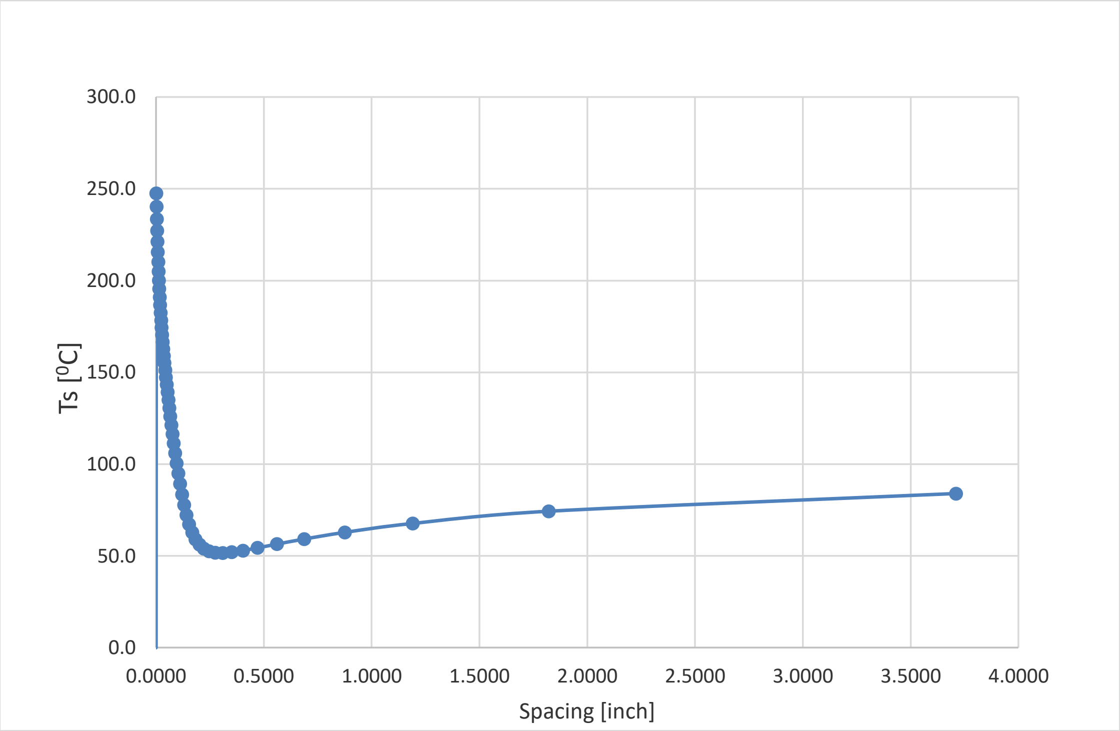


Figure 9
Temperature at the base of the heat sink as a function of spacing

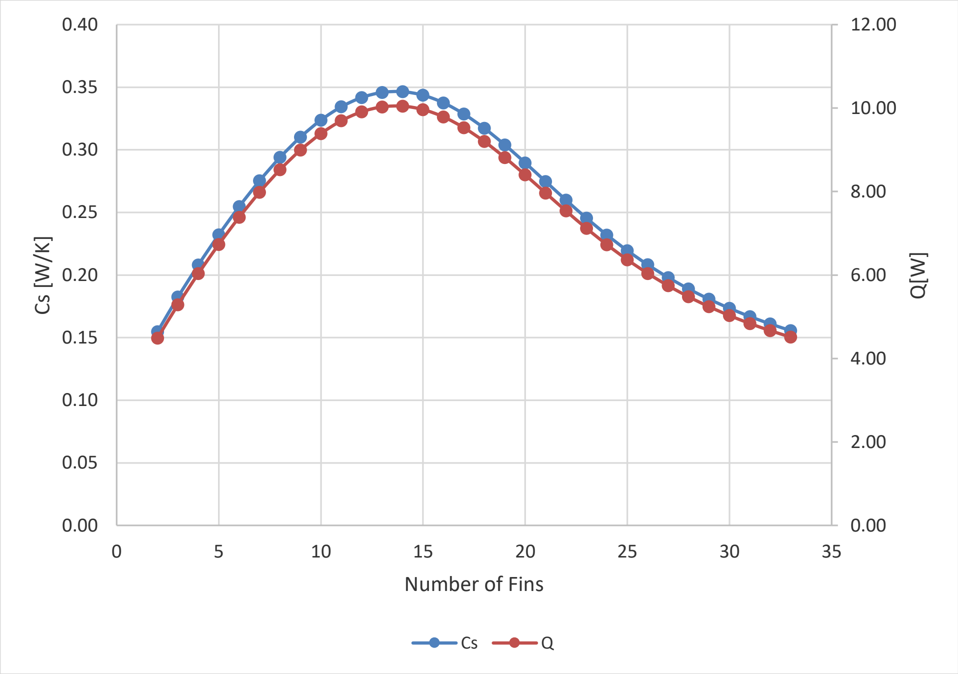


Figure 10
Conductance and heat transfer by the heat sink as a function of number of fins

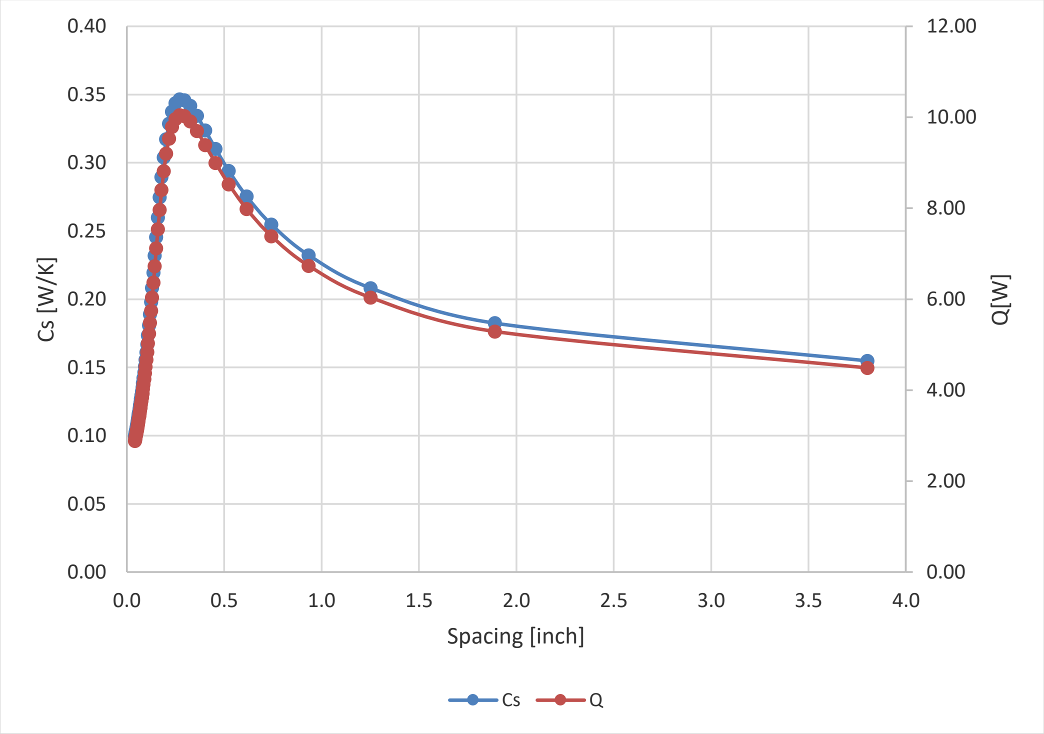


Figure 11
Conductance and heat transfer by the heat sink as a function of spacing between fins

The optimum-designed aluminum heat sink shows a relatively small increase in the performance, but the fin thickness is almost one third of the commercial ones. It could be said that the commercial heat sink has almost the maximum performance, but its fin weight can be reduced by one half, which could mean a reduction in the material cost.

Electric Package Cooling

A heat sink for an electric package cooling is to be designed to maintain the base temperature below 100°C. The base cold plate has a width of W=50 mm and length of H=30 mm. The ambient temperature is Ta=20°C. The heat sink material is aluminum with a material conductivity of k=4.84 W/inK.

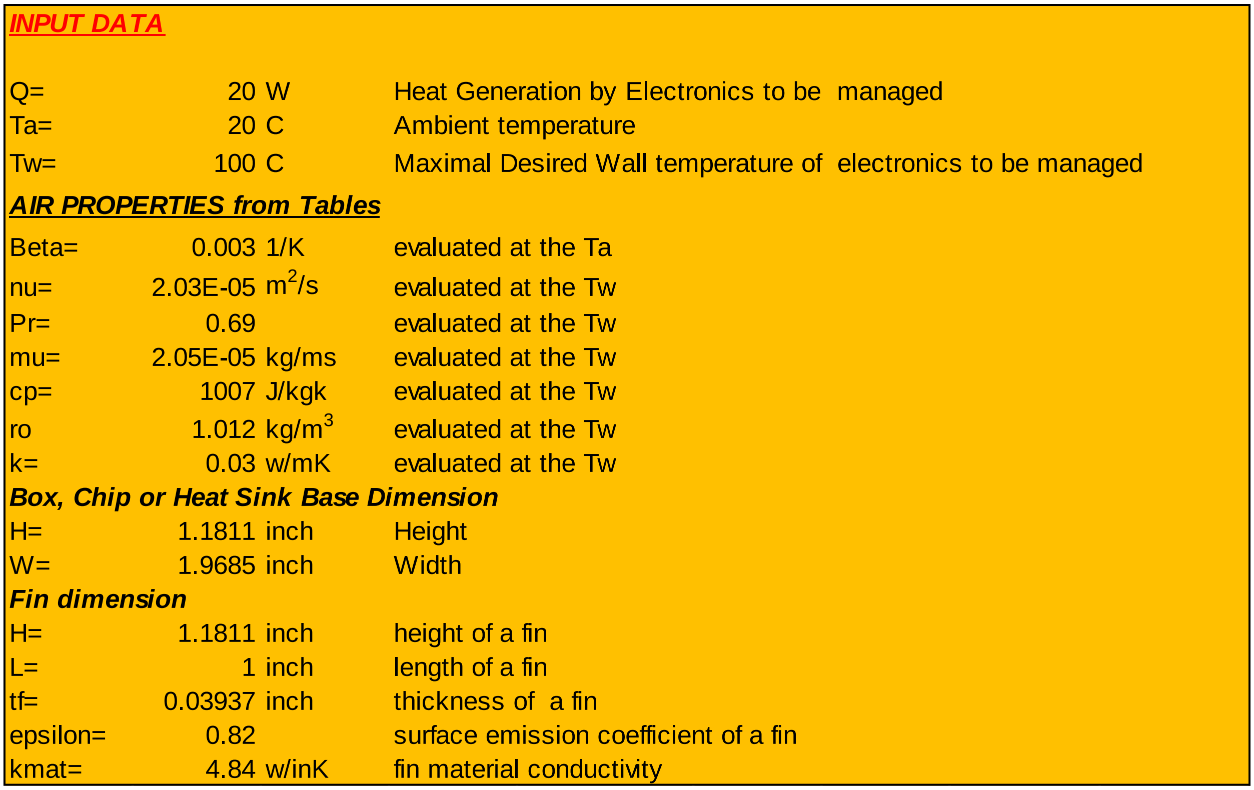
Next, determine the optimum dimensions for the heat sink providing the number of the fins, fin thickness, fin spacing, fin length, heat dissipation, overall efficiency, and thermal resistance. The power generated by the electric box is 20 W, and the fin's surface emissivity is ε_e=0.82.

The solution is based off of the assumption that the dimensions of the base cold plate are identical to the electrical box, and the dimensions need to be inputted in inches. The height of the fin, H, has the same measurement as the length of the box. Also, the ambient temperature, Ta=20°C, and the maximum allowed wall temperature of the electrical box of Tw=100°C (this is previously shown as the base temperature) needs to be inputted. The input deck for this problem is shown in table 20, and the calculated relations for unshielded surfaces are calculated from the previously shown values in table 20 and shown in table 21.

The fin dimensions are assumed to be:

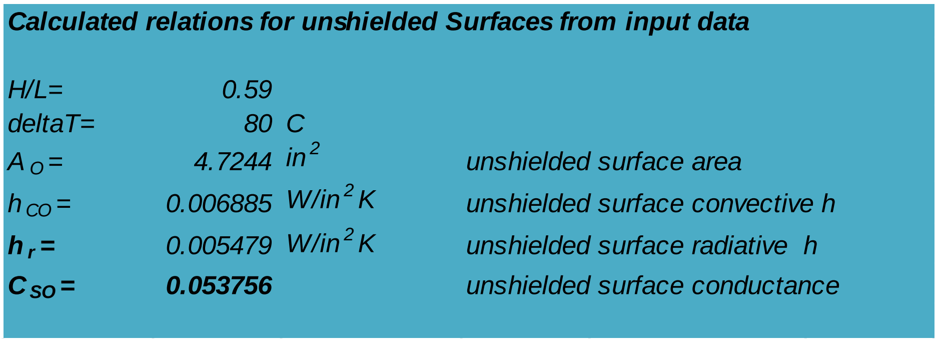
- Fin length: L=25.4 mm=1 in.
- Fin thickness: tf=1 mm= 0.0394 in. (assumed thickness)

Table 20
Input deck for electric package cooling



INPUT DATA			
Q=	20	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	100	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	1.1811	inch	Height
W=	1.9685	inch	Width
Fin dimension			
H=	1.1811	inch	height of a fin
L=	1	inch	length of a fin
tf=	0.03937	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

Table 21
Calculated relations for un-shielded surfaces for Electric package cooling



Calculated relations for unshielded Surfaces from input data			
H/L=	*0.59*		
deltaT=	*80*	*C*	
A_O=	*4.7244*	in^2	*unshielded surface area*
h_{CO}=	*0.006885*	W/in^2K	*unshielded surface convective h*
$\mathbf{h_r}$**=**	*0.005479*	W/in^2K	*unshielded surface radiative h*
$\mathbf{C_{SO}}$**=**	***0.053756***		*unshielded surface conductance*

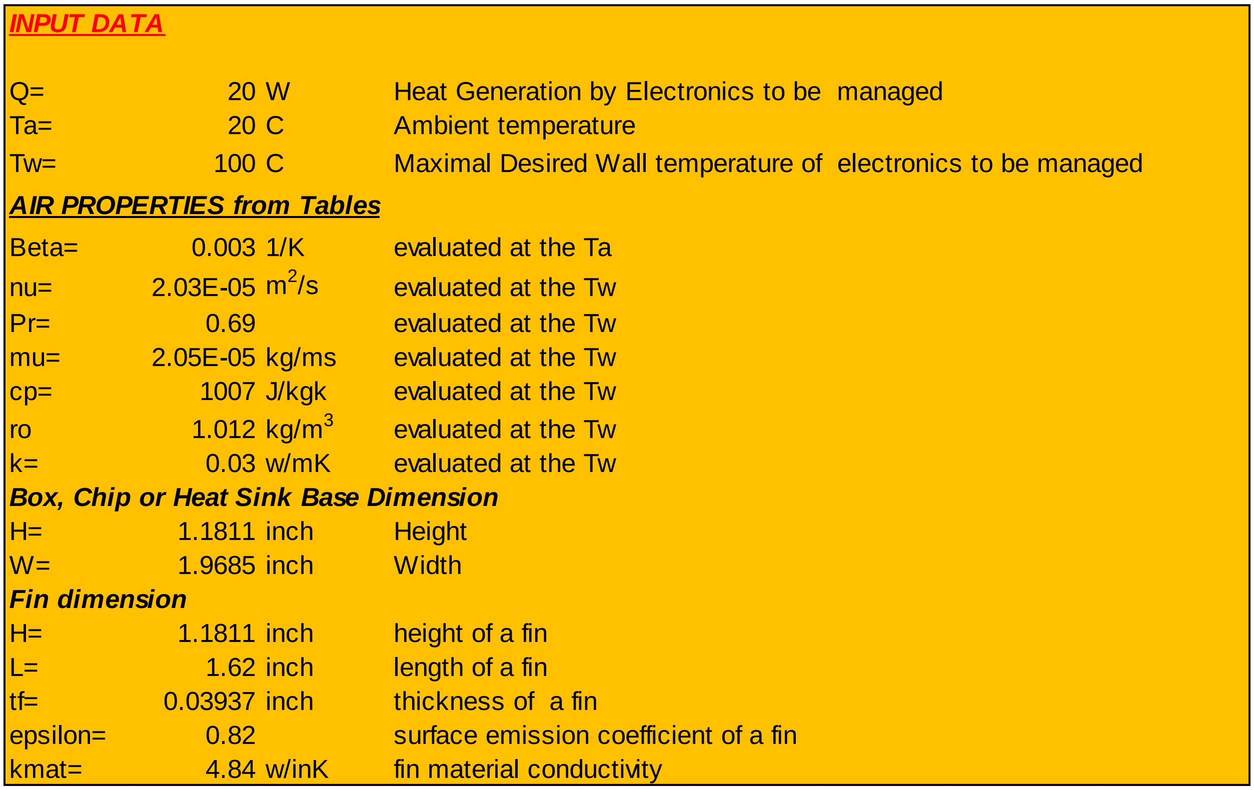
Now, the results can be immediately read in the Excel sheet from all the required parameters stated in the problem statement. Table 22 shows the calculated results for the first pass of the heat sink design and optimization. As can be seen in table 22, the fins, with the length and thickness assumed previously, are not capable of maintaining the temperature of the considered electrical box under 100°C. The lowest temperature that can be achieved with the proposed sink is 138.4°C, as shown in the eighth column of table 22.

Table 22
Results obtained from the problem statement

# of Fins	S(inch)	r(inch)	η_{sink}	Cs	Rs	Q	Ts
2	1.8898	0.9717	0.953	0.07	13.4821	5.93	289.6
3	0.9252	0.6326	0.929	0.09	10.7090	7.47	234.2
4	0.6037	0.4637	0.906	0.11	9.0225	8.87	200.5
5	0.4429	0.3626	0.884	0.13	7.8907	10.14	177.8
6	0.3465	0.2953	0.864	0.14	7.0962	11.27	161.9
7	0.2822	0.2473	0.846	0.15	6.5374	12.24	150.7
8	0.2362	0.2113	0.833	0.16	6.1654	12.98	143.3
9	0.2018	0.1833	0.824	0.17	5.9598	13.42	139.2
10	0.1750	0.1609	0.822	0.17	5.9185	13.52	138.4
11	0.1535	0.1426	0.827	0.17	6.0527	13.22	141.1
12	0.1360	0.1273	0.840	0.16	6.3827	12.53	147.7
13	0.1214	0.1144	0.858	0.14	6.9306	11.54	158.6
14	0.1090	0.1034	0.879	0.13	7.7071	10.38	174.1
15	0.0984	0.0938	0.901	0.12	8.6939	9.20	193.9
16	0.0892	0.0854	0.920	0.10	9.8360	8.13	216.7
17	0.0812	0.0780	0.936	0.09	11.0575	7.23	241.1
18	0.0741	0.0715	0.949	0.08	12.2933	6.51	265.9
19	0.0678	0.0656	0.959	0.07	13.5042	5.92	290.1
20	0.0622	0.0603	0.967	0.07	14.6701	5.45	313.4
21	0.0571	0.0555	0.974	0.06	15.7808	5.07	335.6
22	0.0525	0.0512	0.979	0.06	16.8308	4.75	356.6
23	0.0483	0.0472	0.983	0.06	17.8189	4.49	376.4
24	0.0445	0.0435	0.986	0.05	18.7465	4.27	394.9
25	0.0410	0.0402	0.989	0.05	19.6172	4.08	412.3

This temperature corresponds to a heat sink with 10 fins, with a spacing of 0.175 in., and an efficiency of 82.2. The heat sink designed and optimized with the assumed dimensions is capable of dissipating only 13.52 W, which is 30% less than required amount of 20 W. To improve the heat sink under consideration so it will be able to dissipate 20 W of heat and keep the box temperature under 100°C, it is necessary to change either the box's dimensions or the fin's dimensions. In general, it is usually always easier to change the fin's dimensions, particularly the length of the fin. In the case under consideration, the length of the fins was increased, iteratively, until the temperature of the wall was under required value. A new input deck with the L=1.62 in. is shown in table 23, while the resulting output is presented in table 24.

Table 23
New input deck with a new length of fin



INPUT DATA			
Q=	20	W	Heat Generation by Electronics to be managed
Ta=	20	C	Ambient temperature
Tw=	100	C	Maximal Desired Wall temperature of electronics to be managed
AIR PROPERTIES from Tables			
Beta=	0.003	1/K	evaluated at the Ta
nu=	2.03E-05	m^2/s	evaluated at the Tw
Pr=	0.69		evaluated at the Tw
mu=	2.05E-05	kg/ms	evaluated at the Tw
cp=	1007	J/kgk	evaluated at the Tw
ro	1.012	kg/m^3	evaluated at the Tw
k=	0.03	w/mK	evaluated at the Tw
Box, Chip or Heat Sink Base Dimension			
H=	1.1811	inch	Height
W=	1.9685	inch	Width
Fin dimension			
H=	1.1811	inch	height of a fin
L=	1.62	inch	length of a fin
tf=	0.03937	inch	thickness of a fin
epsilon=	0.82		surface emission coefficient of a fin
kmat=	4.84	w/inK	fin material conductivity

Table 24
Required output with optimal spacing, efficiency, Q, Tw

# of Fins	S(inch)	r(inch)	ηsink	Cs	Rs	Q	Ts
2	1.8898	1.1936	0.844	0.10	9.6408	8.30	212.8
3	0.9252	0.7197	0.747	0.13	7.4382	10.76	168.8
4	0.6037	0.5089	0.652	0.16	6.1707	12.96	143.4
5	0.4429	0.3896	0.562	0.19	5.3429	14.97	126.9
6	0.3465	0.3130	0.479	0.21	4.7725	16.76	115.4
7	0.2822	0.2595	0.407	0.23	4.3793	18.27	107.6
8	0.2362	0.2202	0.353	0.24	4.1254	19.39	102.5
9	0.2018	0.1899	0.321	0.25	3.9945	20.03	99.9
10	0.1750	0.1660	0.317	0.25	3.9839	20.08	99.7
11	0.1535	0.1466	0.343	0.24	4.0996	19.51	102.0
12	0.1360	0.1305	0.396	0.23	4.3527	18.38	107.1
13	0.1214	0.1170	0.470	0.21	4.7525	16.83	115.0
14	0.1090	0.1055	0.552	0.19	5.2967	15.10	125.9
15	0.0984	0.0955	0.632	0.17	5.9621	13.42	139.2
16	0.0892	0.0868	0.702	0.15	6.7057	11.93	154.1
17	0.0812	0.0792	0.760	0.13	7.4807	10.69	169.6
18	0.0741	0.0725	0.807	0.12	8.2536	9.69	185.1
19	0.0678	0.0664	0.844	0.11	9.0061	8.88	200.1
20	0.0622	0.0610	0.873	0.10	9.7292	8.22	214.6
21	0.0571	0.0561	0.897	0.10	10.4181	7.68	228.4
22	0.0525	0.0517	0.916	0.09	11.0703	7.23	241.4
23	0.0483	0.0476	0.932	0.09	11.6851	6.85	253.7
24	0.0445	0.0439	0.944	0.08	12.2633	6.52	265.3
25	0.0410	0.0405	0.955	0.08	12.8067	6.25	276.1

Increasing the length of the fin to L=1.61 in. and recalculating the results allowed the possibility to redesign and optimize the heat sink to meet the required specifications. As it can be seen in table 24, the optimal spacing is 0.175 in. for a heat sink with 10 fins. The wall temperature is 99.7°C, which is lower than the maximum temperature allowed of 100°C. This heat sink is capable of dissipating 20.08 W of heat to ambient, which satisfies the problem statement of 20 W.

In the end, it should be mentioned that the redesign of the heat sink could have been approached in a different way and still lead to the required results. For example, the fin's height, width, or thickness could have been manipulated separately or together and still achieve the stated requirements.

CONCLUSIONS

The "Rectangular Heat Sink Design and Optimization Code" is a robust code for design and optimization of rectangular fin heat sinks. It is capable of finding the optimal spacing between fins, maximum heat dissipation, and minimal wall temperature on an electronics device for an assumed fin geometry. Also, by allowing easy manipulation of the fin's parameters, this code allows a user to find heat sink fin designs that satisfy the required heat dissipation or maximum allowed temperature at the wall of an electronics package. The code should be used together with the DTIC report, "Heat Sink Design and Optimization," where the user can also find additional code capabilities useful in the design and optimization of rectangular fin heat sinks.

REFERENCES

1. Mehmedagic, I. and Krug, J., “Heat Sink Design and Optimization,” Technical Report ARMET-TR-15036, U.S. Army ARDEC, Picatinny Arsenal, NJ, December 2015.
2. Van de Pol, D.W. and Tierney, J.K., “Fee Convection Heat Transfer from Vertical Fin Arrays,” IEEE Transactions on Parts, Hybrids, and Packaging, vol. PHP-10, no. 4, pp. 267-271, December1974.
3. Ellison, G.N., “Thermal Computation for Electronic Equipment,” Van Nostrand Reinhold Company, New York, 1984.
4. Mills, A.F., “Heat Transfer,” 2nd edition, Prentice Hall, NJ, 1999.
5. Elenbass, W., “Dissipation of Heat by Free Convection,” Parts I and II, Philips Research Report, vol. 3, N.V. Philips, Gloeilampenfabrieken, Eindhoven, Netherlands, pp. 338-360, 450-465, 1948.
6. Aihara, T. and Maruyama, S., “Optimum Design of Natural Cooling Heat Sinks with Vertical Rectangular Fin Arrays,” International Symposium on Cooling Techniques for Electronic Equipment, Honolulu, Hawaii, 1987.

NOMENCLATURE

A_i	Inter-fin surface area (in.^{2})
A_o	Unshielded surface area (in.^{2})
a	Ratio of spacing to fin length
β	Thermal expansion coefficient (1/K)
C_{ci}	Conductance, inter-fin passages
C_{cieta}	Conductance, effective, inter-fin passage
C_{NET}	Conductance, overall for U-channel
C_{so}	Conductance, unshielded surface area
Cp	Specific heat (J/kJ-K)
ε	Surface emission coefficient of fin
η_{fin}	Fin efficiency
η_{sink}	Sink efficiency
F	Shape factor for gray body U-channels
G	Grashof number
H	Height of fin (in.)
H_{box}	Height of box, chip, or heat sink base (in.)
h_{ci}	Heat transfer coefficient, convective, inter-fin passages (W/in.^{2}-K)
h_{co}	Heat transfer coefficient, convective, unshielded surface (W/in.^{2}-K)
h_o	Heat transfer coefficient, total (convective and radiative) heat transfer coefficient for the outer fins (W/in.^{2}-K)
h_r	Heat transfer coefficient, radiative, unshielded surface (W/in.^{2}-K)
k_{air}	Thermal conductivity of air (W/m-K)
k_{mat}	Thermal conductivity of fin material (W/m-K)
L	Length of fin (in.)
Mu (μ)	Kinematic viscosity (kg/ms)
N	Number of fins
Nu	Nusselt number
Pr	Prandtl number
Psi (ψ)	Linear thermal transmittance
Q	Heat dissipation (W)
r	Thermal resistance
Ra	Rayleigh number
Rho (ρ)	Density (kg/m^3)
S	Space between fins (in.)
ΔT	Temperature difference (°C)
Ta	Ambient temperature (°C)
tf	Thickness of fin (in.)
T_w	Maximum desired wall temperature for electronics enclosure (°C)
V	=-11.8(in.$^{-1}$) constant value parameter existing in Elenbass's equation
W	Width of box, chip, or heat sink base (in.)

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