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The Effect of Noise on the Optimal Code Transmittance for Compressive Sensing

by Michael Don

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1. Introduction

The US Army Research Laboratory has been investigating compressive sensing (CS) to increase measurement resolution or measurement speed.¹⁻⁴ One important parameter of a CS architecture is code transmittance. Many CS systems perform well in simulation with low transmittance codes.⁵ These low levels are deceptive, however, since they result in low signal levels that are more susceptible to noise. Accounting for imaging noise, however, cannot be accurately modeled by simple Gaussian noise. A more complex noise model includes both read and shot noise.⁶ Read noise is caused by the system hardware and is independent of signal level. Shot noise is based in the light itself and is related to signal level. This note explores the effect of these noise sources on optimal code transmittance.

2. Reducing CS Complexity by Reducing Code Transmittance

A simple binary-coded CS scheme employs 50% blocked and 50% unblocked cells, resulting in a light transmittance of 0.5.⁷ Compressive sensing can perform well using less transmittance, however, and reducing the percentage of unblock cells significantly reduces recovery algorithm complexity. One simple method of reducing the complexity of a system using a 2-D code is to only measure one row at a time. Figure 1 shows the sensing matrix of an 8×8 code with the active row changed after every fourth measurement, resulting in a block diagonal matrix structure. Each row of the sensing matrix represents a single measurement and is a vectorized version of the entire code. In this example, the active code rows have a 0.5 transmittance and 32 measurements out of a total 64 pixels give a compression of 0.5.

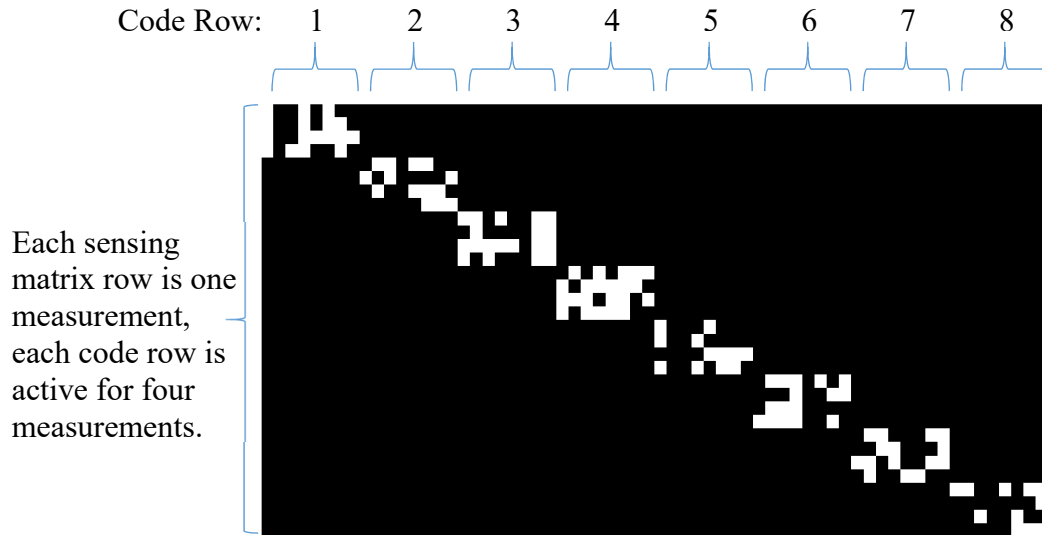


Fig. 1 The sensing matrix of an 8×8 code, with only one row of the code active at a time, resulting in a block diagonal structure. Each row of the sensing matrix is a vectorized version of the code, representing one measurement. The columns of the sensing matrix corresponding to the rows of the code are labeled. Each row of the code is active during four measurements, resulting in 4×8 submatrix blocks.

Figure 2 shows simulation results using a sensing matrix similar to that shown in Fig. 1 using a 128×128 code, 0.25 compression, and the active row changed at every 128 measurements. Smaller images are generally less sparse than larger images; therefore, for simulation purposes it is advantageous to increase the sparsity of the test image. The sparse image shown has the 10% largest wavelet coefficients of the original image, with the rest set to zero. The image was recovered with a peak signal-to-noise ratio (PSNR) of 29.5 dB using the gradient projection for sparse reconstruction algorithm.⁸

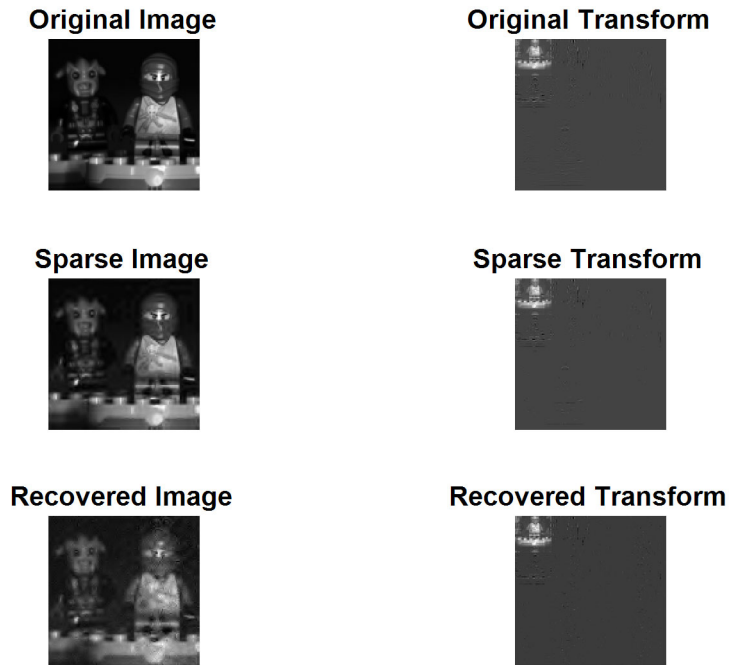


Fig. 2 An example CS simulation using low code transmittance. The sparse transform contains the largest 10% of the coefficients of the original transform. The sparse image was measured using only one active row of the code per measurement. The recovered image has a PSNR of 29.5 dB using a compression ratio of 0.25.

3. Optimal Code Transmittance with Gaussian Noise

The transmittance of the code can be further controlled by adjusting the transmittance of each active row. For low noise levels, a lower transmittance value outperforms higher transmittance. Using the setup from Fig. 2, Fig. 3 shows the PSNR versus transmittance of the active code row for various added Gaussian noise levels with standard deviation σ . For low noise levels, the optimal transmittance is around 0.2. As the noise increases, the optimal transmittance increases. This is because higher transmittance leads to higher signal measurement values, increasing the signal-to-noise ratio (SNR).

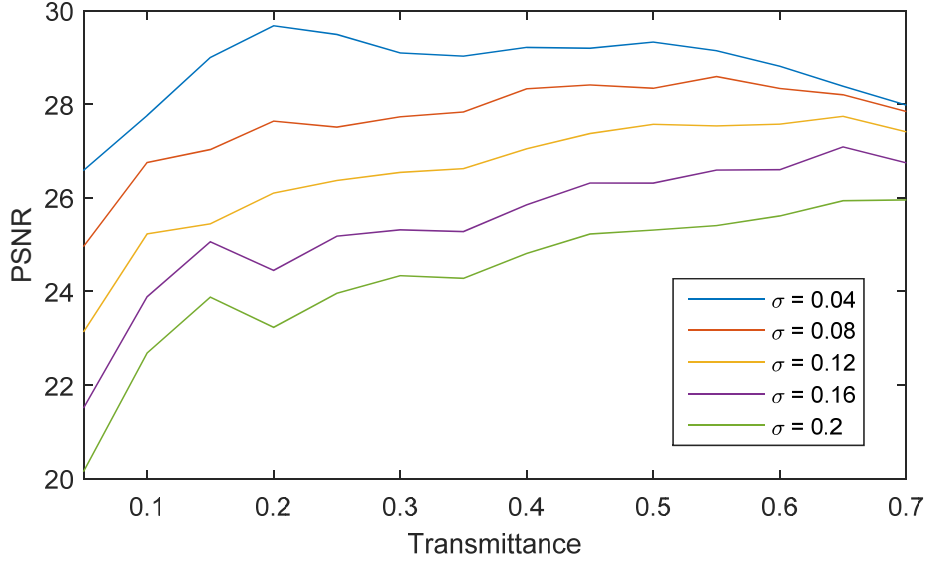


Fig. 3 PSNR vs. transmittance of the active code row for various noise levels. At low noise levels the optimal transmittance is low. As the noise increases, the optimal transmittance increases because higher transmittance leads to a higher SNR, which benefits CS recovery.

4. A More Complex Imaging Noise Model

Adding Gaussian noise to the measurement is a very simple noise model. In a more sophisticated image noise model, the noise is a combination of read noise and shot noise.⁶ The read noise R is a fixed amount of noise caused by the camera electronics and other hardware variations. The shot noise is caused by variations in the light itself and is related to the signal level S by a gain factor g . The total noise is given by

$$N = \sqrt{R^2 + S/g}. \quad (1)$$

For example, a 12-bit imaging system with a shot noise of 2.5 bits and a gain of 11.6 would have an SNR related to signal level as shown in Fig. 4. At low signal levels, the read noise dominates the SNR. At around a signal level of 70 bits, a knee in the curve indicates where the shot noise begins to dominate the read noise.

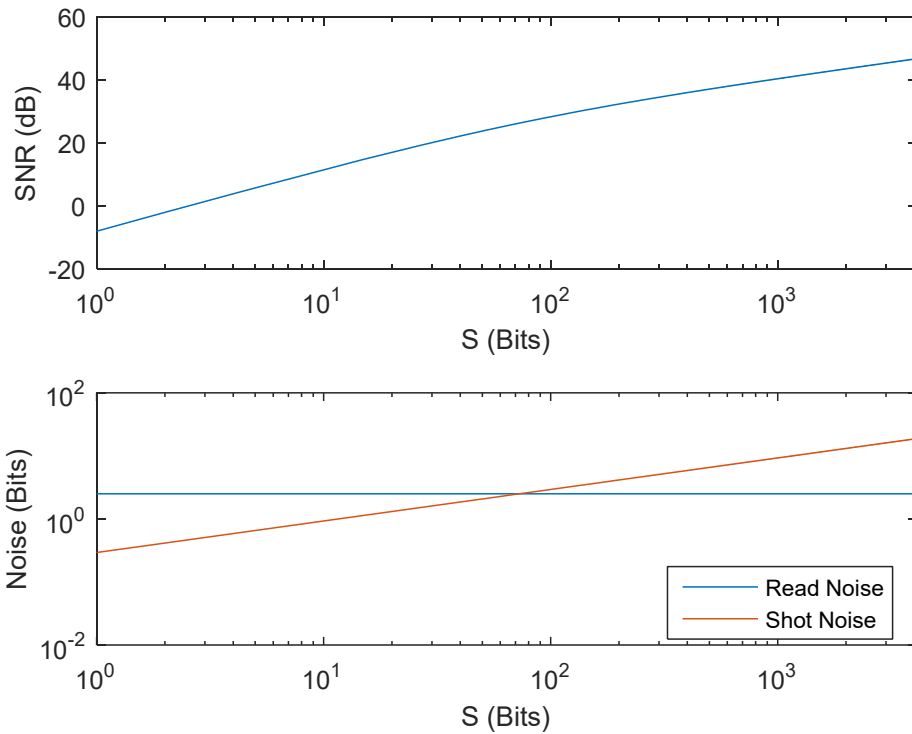


Fig. 4 SNR vs. signal level for a noise model including read and shot noise (top) and the read and shot noise displayed separately on a log–log scale (bottom). Note that the knee in the SNR curve occurs where the shot noise overtakes the read noise.

5. Optimal Code Transmittance with Read and Shot Noise

In the first noise model, a fixed noise level independent of measurement amplitude was used, simulating read noise without any shot noise. Before using a more realistic model that combines read and shot noise, we examine the effect of adding shot noise alone. Figure 5 shows the PSNR versus the transmittance of the active row for different levels of shot noise. In the simple noise model, the SNR increased as the signal level increased; therefore, higher noise levels benefited from a higher transmittance. Now that the noise increases with signal level, this effect is eliminated, and the optimal transmittance stays at about the same value.

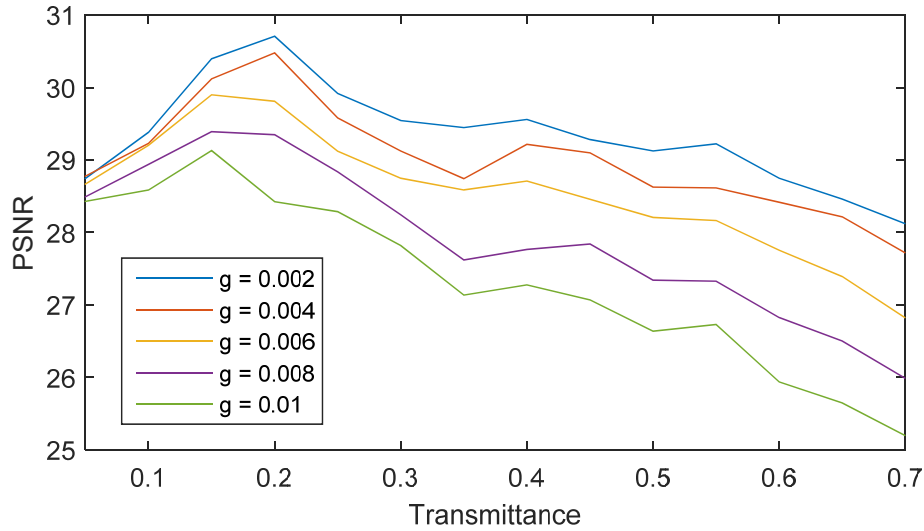


Fig. 5 PSNR vs. the transmittance of the active row for different amounts of shot noise. Since the shot noise depends on signal level, increasing the noise does not increase the optimal transmittance.

When both read and shot noise are present, we would expect to see something in between Figs. 3 and 5, such as the optimal transmittance level increasing with the noise level, but not as fast as the case of read noise alone. In Fig. 6, the standard deviations of the noise from Fig. 3 were combined with the gains of the shot noise in Fig. 5 but in reverse order to simulate a shift from a majority of shot noise in the first signal to a majority of read noise in the last signal. As predicted, the optimal transmittance increases slower here than when using read noise alone.

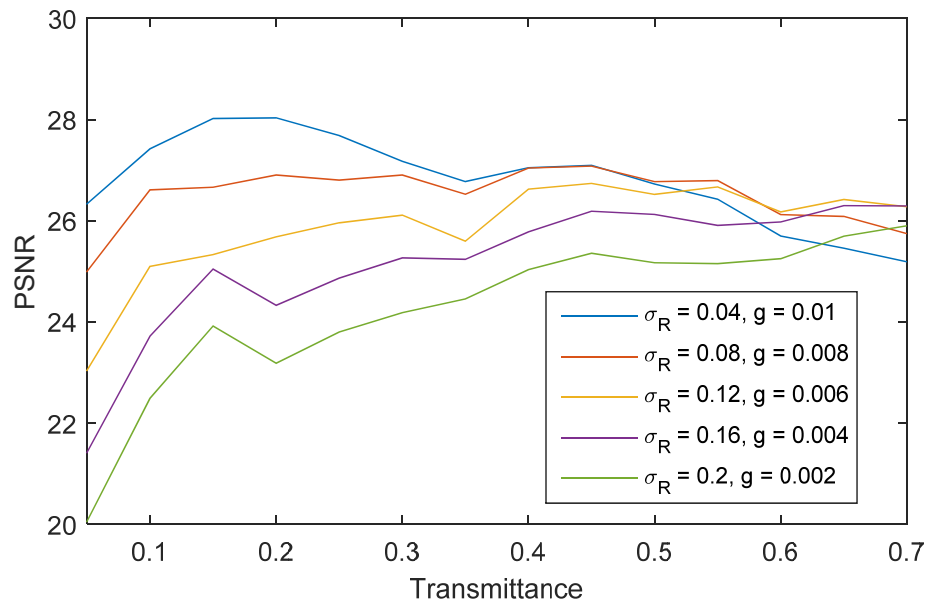


Fig. 6 PSNR vs. transmittance of the active code row with various noise levels using both read and shot noise. The optimal transmittance increases with the noise level, but not as quickly as with read noise alone, as illustrated in Fig. 3.

These results assume that camera settings remain the same as the transmittance changes. In reality, camera International Standards Organization (ISO) setting and exposure time can be changed to help increase SNR when using a code with lower transmittance. This adds more complexities, however, as increasing the ISO and exposure time itself has effects on the noise. These considerations are out of the scope of this note but are worthwhile avenues of future research.

6. Conclusion

This note has considered optimal code transmittance using realistic noise models that include both read and shot noise. The more the noise is dominated by shot noise, the more the optimal transmittance will remain at the ideal, noise-free level. The more the read noise dominates, the more the optimal transmittance will increase.

7. References

1. Don ML, Fu C, Arce GR. Compressive imaging via a rotating coded aperture. *Applied Optics*. 2017;56(3):B142–B153.
2. Fu C, Don ML, Arce GR. Compressive spectral imaging via polar coded aperture. *IEEE Transactions on Computational Imaging*. 2017;3(3):408–420.
3. Fu C, Don ML, Arce GR. Optimization and experimental demonstration of a compressive polar coded aperture spectral imager. *IEEE Transactions on Computational Imaging*. 2018. In publishing.
4. Boseman C. *Leading the study in mechanisms of DNA cell transformation: a practical approach*. New York (NY): Springer; 2017.
5. Cuadros AP, Peitsch C, Arguello H, Arce GR. Coded aperture optimization for compressive X-ray tomosynthesis. *Optics Express*. 2015;23(25):32788–32802.
6. Martinec E. *Noise, dynamic range and bit depth in digital SLRs*. Chicago (IL): University of Chicago; c2008 [accessed 2018 Sep]. <http://theory.uchicago.edu/~ejm/pix/20d/tests/noise/>.
7. Foucart S, Rauhut H. *A mathematical introduction to compressive sensing*. Vol. 1, no. 3. Basel (Switzerland): Birkhäuser; 2013.
8. Figueiredo M, Nowak RD, Wright SJ. Gradient projection for sparse reconstruction: application to compressed sensing and other inverse problems. *IEEE J of Selected Topics in Signal Processing*. 2007;1:586597.

List of Symbols, Abbreviations, and Acronyms

2-D	two-dimensional
ARL	US Army Research Laboratory
CS	compressive sensing
ISO	International Standards Organization
PSNR	peak signal-to-noise ratio
SNR	signal-to-noise ratio

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