



# Development of a Load-Agnostic Structural Light-Weighting Design Optimization Methodology

Dr. Ravi Thyagarajan, Ph. D. and Kumar B Kulkarni

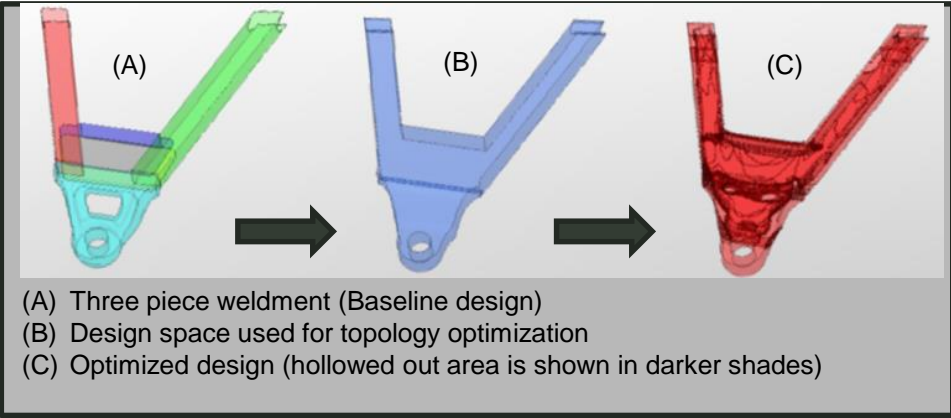
Name of the Presenter: Dr. Ravi Thyagarajan, Ph.D.

Rank/Title of Presenter: Senior Technical Expert

Organization of Presenter: US Army, Tank Automotive Research, Development and Engineering Center



# DEVELOPMENT OF A LOAD-AGNOSTIC STRUCTURAL LIGHT-WEIGHTING DESIGN OPTIMIZATION METHODOLOGY



## Innovation

- Ability to perform topology optimization on over-designed subsystems without information on actual service loads
- Internally generated criteria for design constraints based on existing designs
- Multiplicative impact of optimization methodology and Additive Manufacturing of spares/sustainment <sup>4</sup>

Start/finish date: October 1, 2017 – September 30, 2019 (2 year effort)

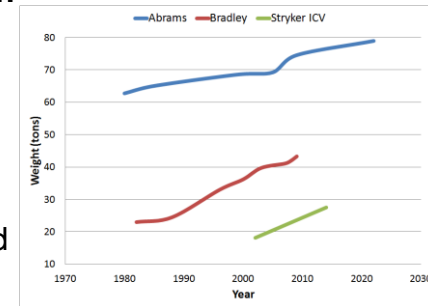
(\$M)	PE	Project	FY
0.08 (0.35 FTE)	61101	91A	18
0.08 (0.35 FTE)	61101	91A	19

## Problems/Questions

- Rapid insertion of new technologies needs quick turnarounds in designs of components and sub-systems. These are often over-designed to compensate for lack of good sub-system level performance targets.
- Lack of component-level service loads hampers systematic light-weighting efforts. How to perform a part optimization without the knowledge of service loads?

## Theoretical Basis for Research

- Systems are over-designed with large factors of safety, good candidates for weight reduction.
- Topology Optimization methods such as SIMP (Solid Isotropic Material with Penalization)<sup>7,8</sup>
- Mathematical algorithms such as Gradient-based searches/programming



Weight growth in US Army combat vehicles

## Relevance to Org Mission

- To improve overall protected and expeditionary mobility utilizing multiple approaches to light-weighting [Value Stream 1, LoE 1.3]
- Supports Army's Lightweight Combat Vehicle S&T Campaign (LCVSTC): Value Stream 3, LoE 3.1, 3.4
- Key enabler to leverage Additive Manufacturing/3D Printing (key Army initiative) for Readiness

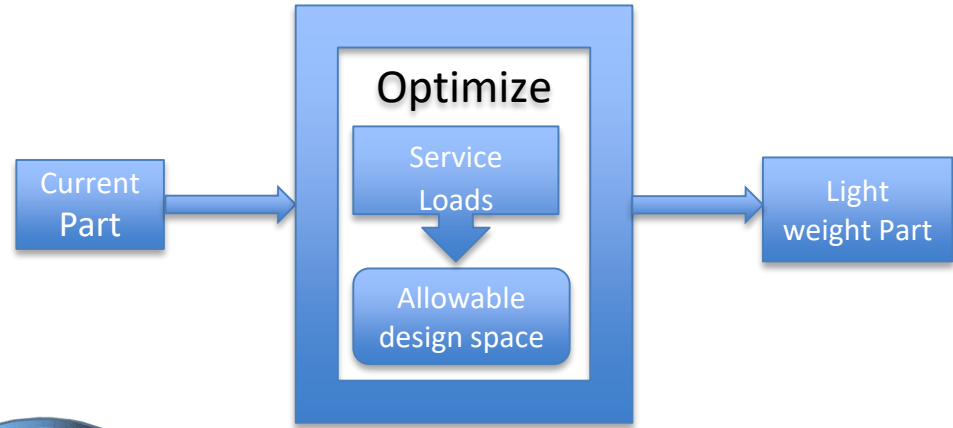
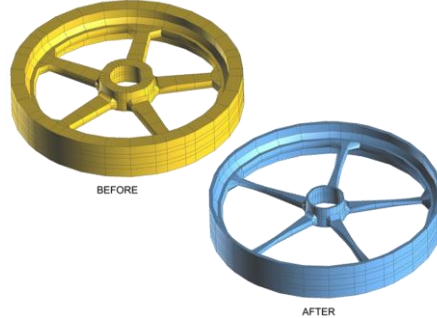


# MOTIVATION / PRIOR RESEARCH

Image Courtesy: Altair's Solid Thinking



Image Courtesy: Light Rider motorcycle

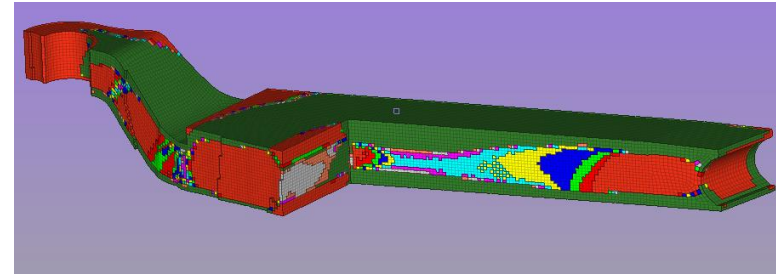


- Extensive research on successful implementation of topology optimization of structures subjected to known service loads<sup>1,2,3,6</sup>
- For military vehicles:
  - Key performance criteria for military vehicles are typically prescribed for a full system. Target cascading methodology to lower levels is not well established (unlike in aerospace industry). Service loads by lower structural components are not readily available.
  - This typically results in over design, adding weight, a significant drawback to Assured Mobility
  - In addition, there is often “dead weight” where the weight does not add any “value”
- Significant efforts are now being dedicated to measuring loads in key components using full vehicle tests, however this takes time and money.
  - Development of a service-load-agnostic method of component design optimization would thus be greatly beneficial, and is a niche research need for the Army.



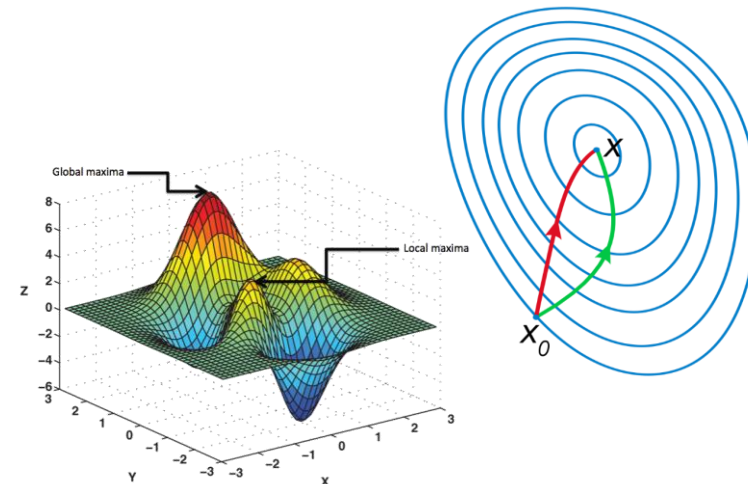
# OVERALL TECHNICAL APPROACH

- The key strategy of the approach is to use “normalized” design target criteria based on the current baseline component itself, these are:
  - Stiffness: Overall component compliances
  - Strength: Peak von mises stresses to unit loads
  - Other: First few natural frequencies of the global modes of deformation
- There are two potential avenues for weight reduction:
  - Removal of “dead weight” without loss of performance
  - “Significant” weight reductions for “small” accepted reductions in performance; Efficiency =  $(\Delta\text{weight} / \Delta\text{perf})$
- SIMP method employed<sup>7,8</sup> (Solid Isotropic Material with Penalization for intermediate densities)
  - Finite element-based
  - Material interpolation, Power Law or “density” method
  - Higher the penalization, more the algorithmic penalty for use of non-binary densities
  - Used by most COTS (we use Optistruct®)
- Gradient-based optimization search methods
  - Steepest descent
  - Newton’s method using curvature



SIMP method:

- Red (Low Density – remove)
- Green (High Density – keep)
- Other colors (Intermediate Density – penalty applied)



Gradient “steepest” descent (green) and Newton's method using curvature (red)



# PROBLEM FORMULATION

Topology optimization problem can be written in the general form<sup>7</sup> is stated as;

Minimize  $\Theta_o(d)$  ;      Subjected to       $\Theta_i(d) \leq 0$ ;    &     $\underline{d}_j \leq d_j \leq \bar{d}_j$

Where  $\Theta_o$  is the objective function

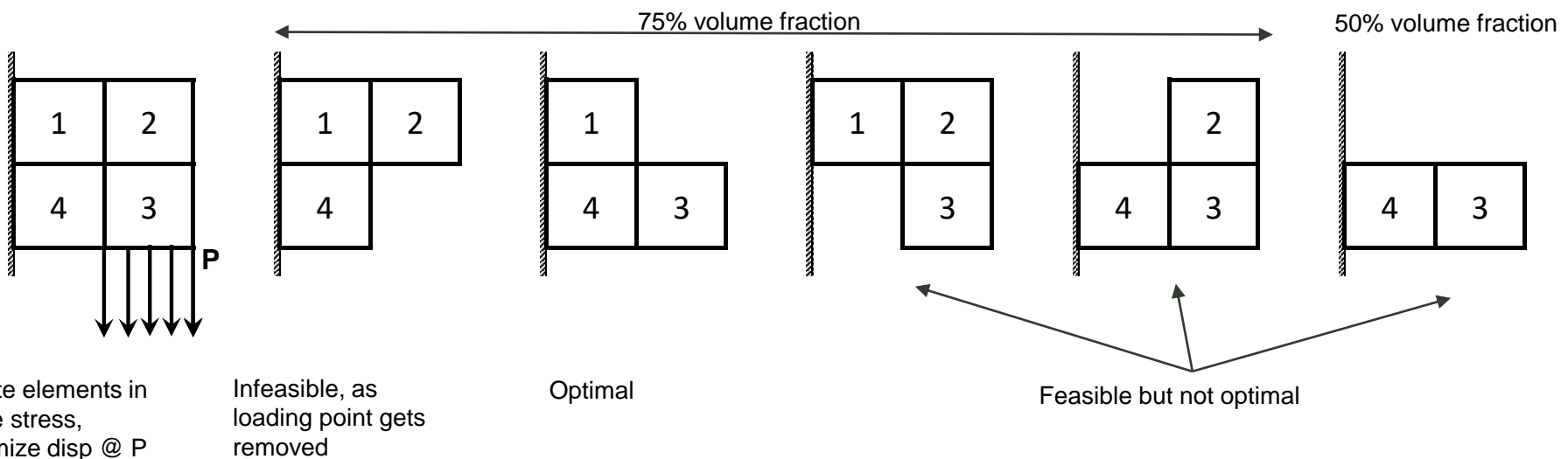
$\Theta_i$  (for  $i=1,nc$ ) are the inequality or equality constraints (material responses) that the solution must satisfy

$d_j$  (for  $j=1,nd$ ) are the design variables (material distribution densities) bounded by  $\underline{d}_j$  and  $\bar{d}_j$

The structural response is solved using the finite element method within the design domain.

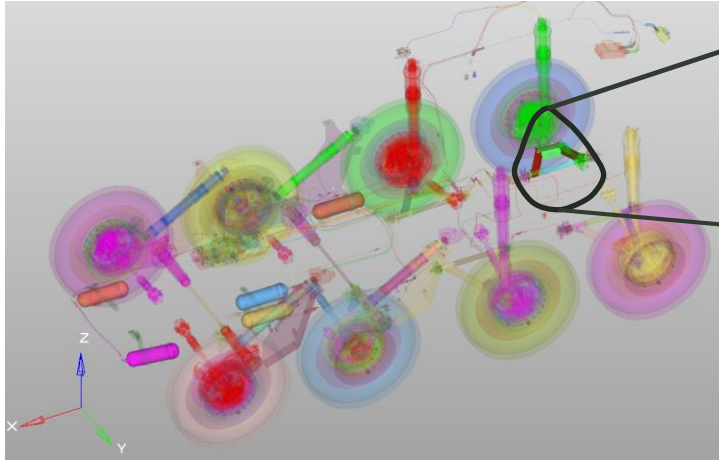
Interpolation of material distribution densities is performed in a continuous manner using the SIMP method which interpolates the material modulus within each discrete element of the design space.

An example of generalized shape optimization involving the optimal distribution of a single material within a given domain<sup>8</sup> is shown below.

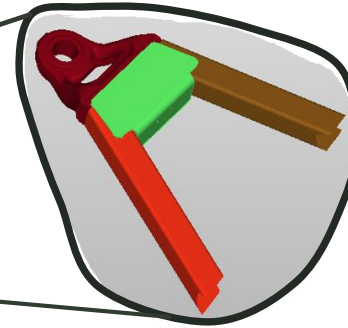




# EXAMPLE: STRYKER CONTROL ARM

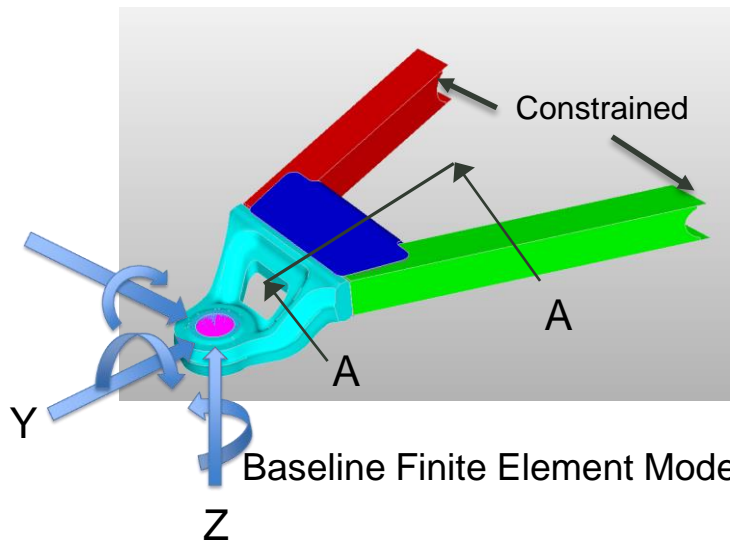


Suspension system of Stryker

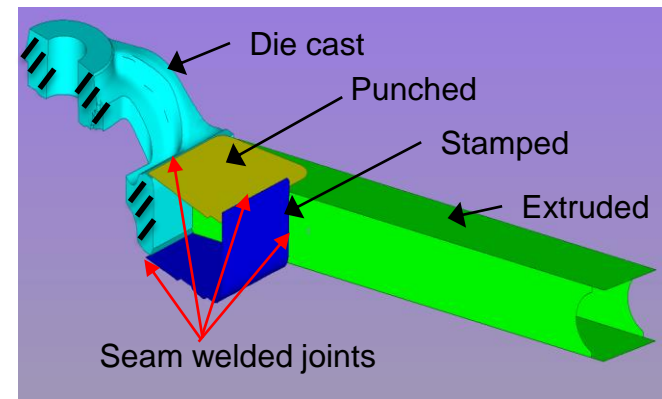


Control arm assembly

Performance targets are set by performing structural analysis using unit loads.



Baseline Finite Element Model



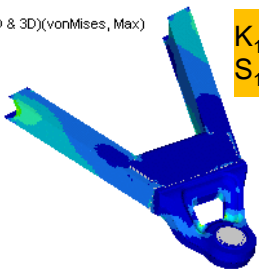
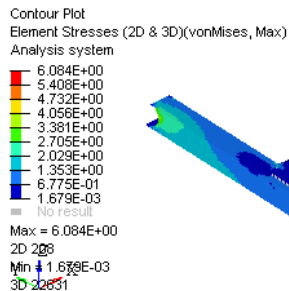
Section AA



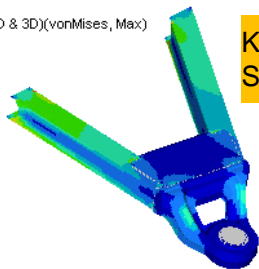
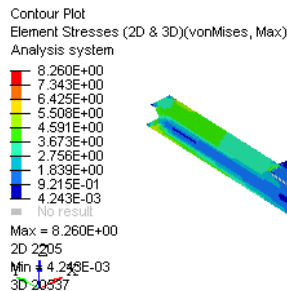
# PERFORMANCE TARGETS (CONSTRAINTS)

Stiffness and Strength

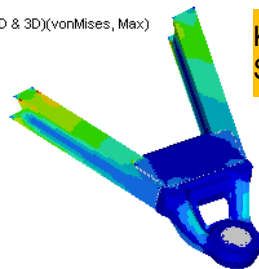
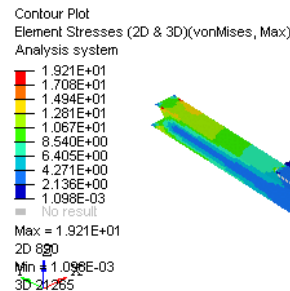
Modal Frequencies



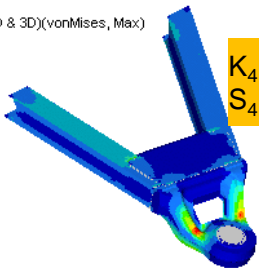
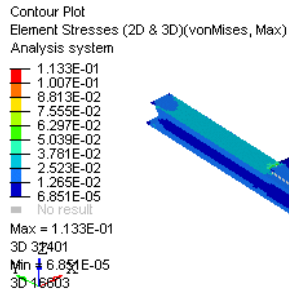
$K_1$ : 114 MN/m  
 $S_1$ : 6.08 MPa



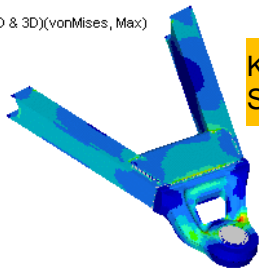
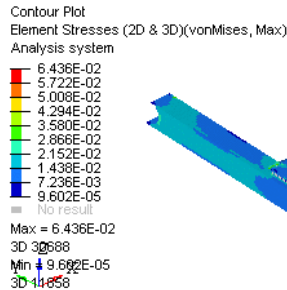
$K_2$ : 26.8 MN/m  
 $S_2$ : 8.26 MPa



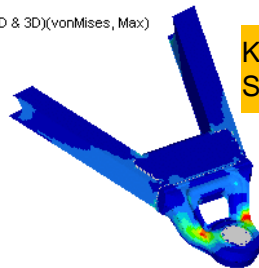
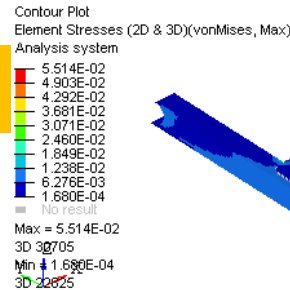
$K_3$ : 5.31 MN/m  
 $S_3$ : 19.21 MPa



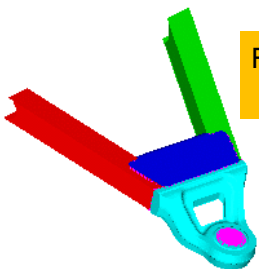
$K_4$ : 5.2 MN-m/deg  
 $S_4$ : 11.3 MPa



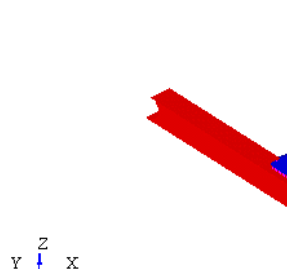
$K_5$ : 13 MN-m/deg  
 $S_5$ : 644 MPa



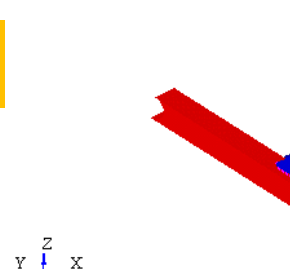
$K_6$ : 27 MN-m/deg  
 $S_6$ : 551 MPa



$F_1$ : Vert bending  
129 Hz



$F_2$ : Bending & twist  
506 Hz



$F_3$ : Twist  
621 Hz

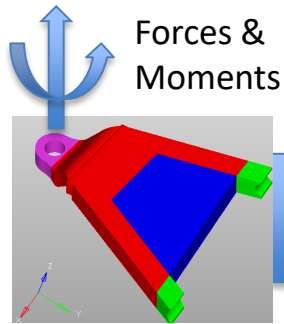
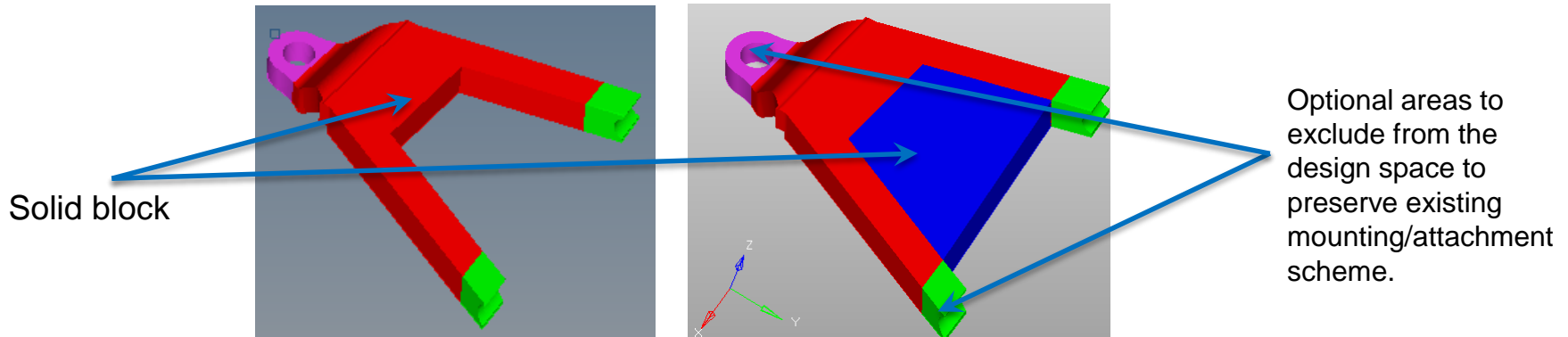
Stiffness ( $K_i$ ) and vonmises stress ( $S_i$ ) for  $i=1..6$  in all six directions including the first three natural frequencies,  $F_j$  ( $j=1..3$ ) are set as constraints.



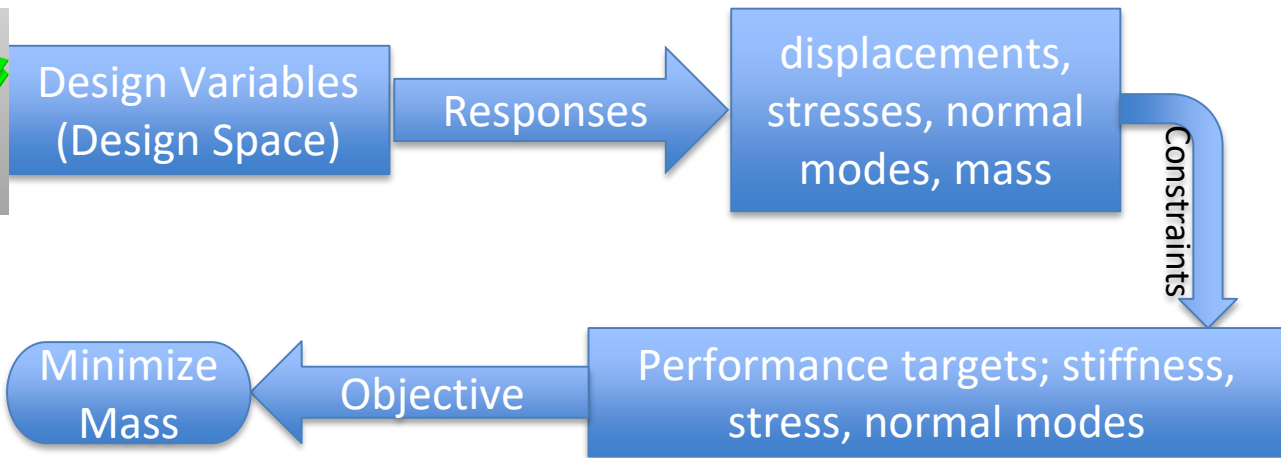
# TOPOLOGY OPTIMIZATION APPROACH

Design Space 1 -  
Bounded by the existing  
package space

Design Space 2 -  
Bounded by the feasible  
packaging space



Forces &  
Moments



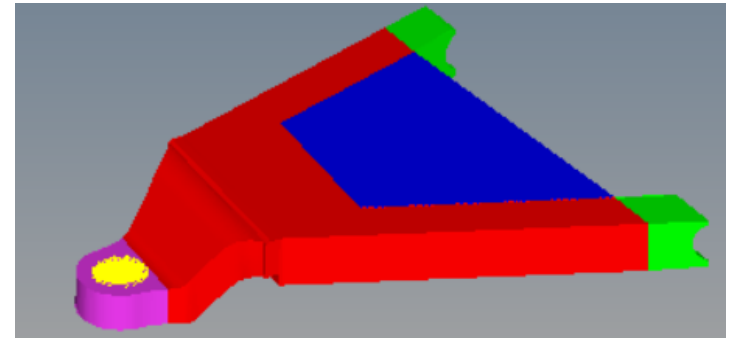
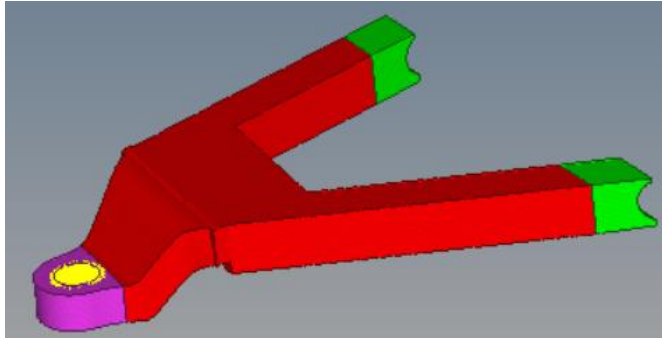


# RESULTS: OPTIMIZED DESIGN SPACE(S)

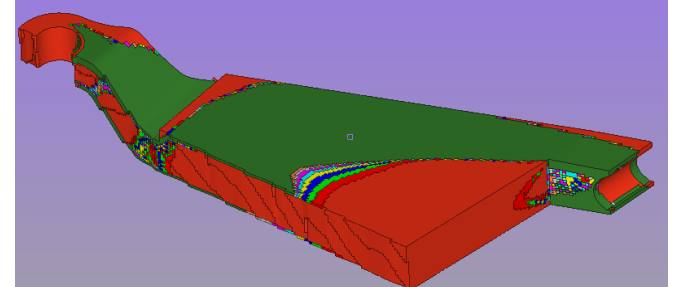
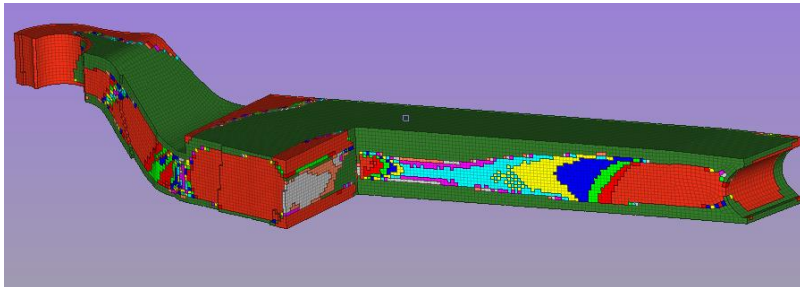
Design Space 1

Design Space 2

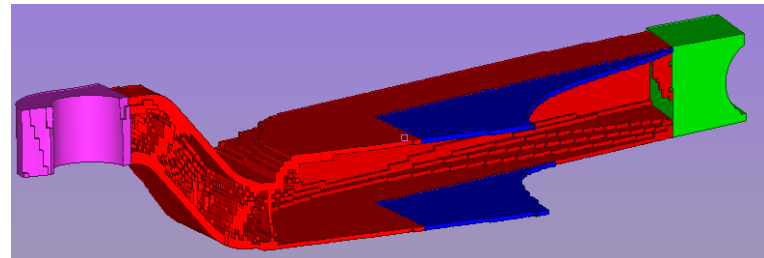
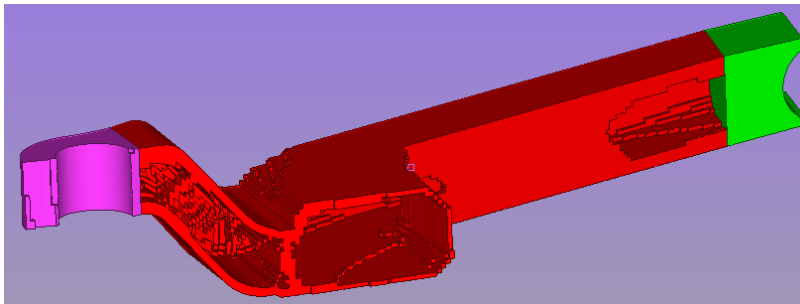
Design Spaces from which optimization is initiated



Contours based on element densities. Green > 0.9, Red < 0.1, other colors intermediate.



“Rough” designs created based on retaining element densities > 0.25. Ends unchanged to maintain assembly interfaces.





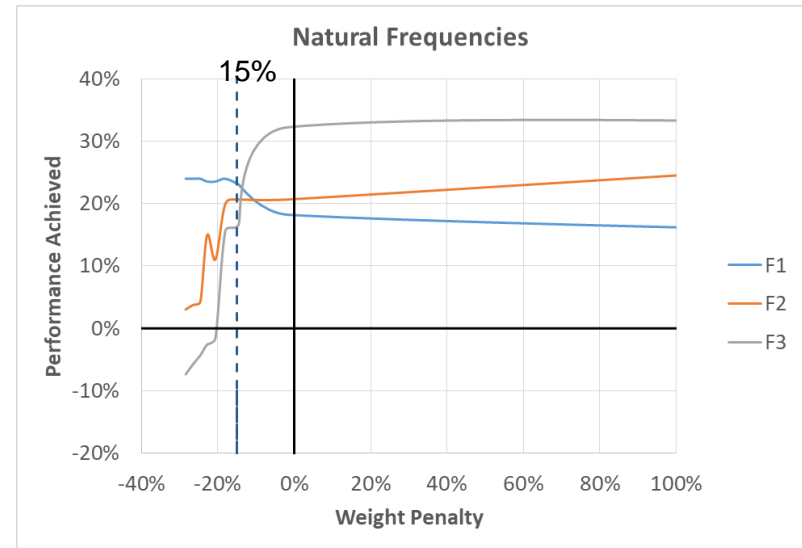
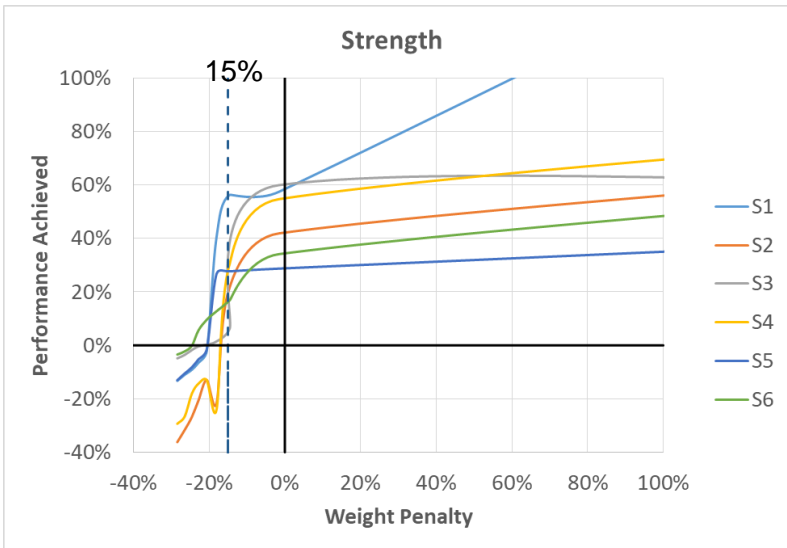
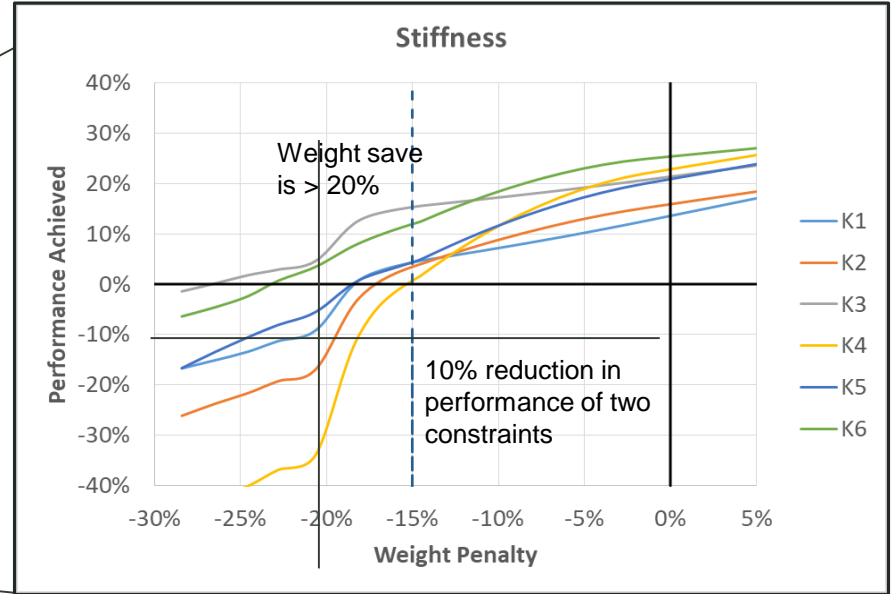
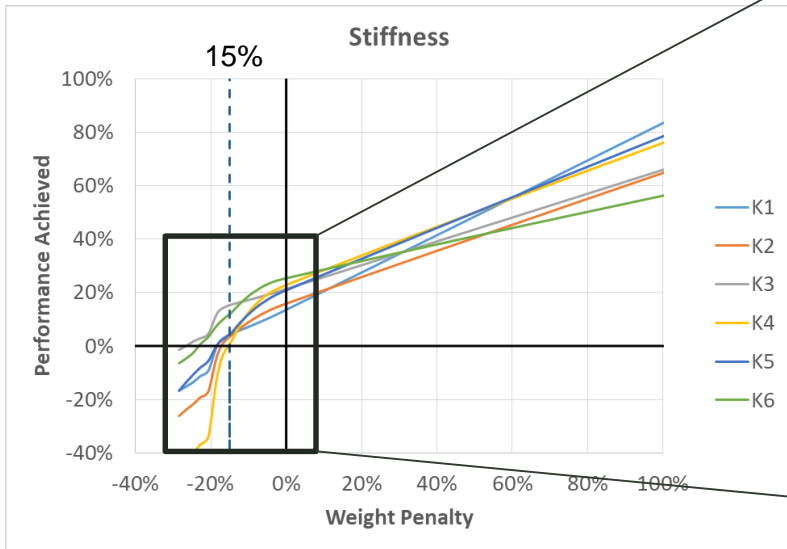
# RESULTS

Response Quantity				Baseline	DS - 1	%Δ	DS - 2	%Δ
<b>1</b>	<b>Mass, kg</b>			<b>18.3</b>	<b>17.14</b>	<b>7% ▲</b>	<b>15.97</b>	<b>15% ▲</b>
2	<b>F o r c e s</b>	Compliance, mm/kN	X	8.75E-03	7.97E-03	10% ▲	8.36E-03	5% ▲
3			Y	3.73E-02	3.46E-02	8% ▲	3.59E-02	4% ▲
4			Z	1.88E-01	1.64E-01	15% ▲	1.63E-01	16% ▲
5		Max Von Mises Stress, MPa	X	6.08	4.21	44% ▲	3.89	56% ▲
6			Y	8.26	6.20	33% ▲	6.79	22% ▲
7			Z	19.21	11.91	61% ▲	18.08	6% ▲
8		<b>M o m e n t s</b>	Compliance, rad/Nm	X	3.39E-06	2.95E-06	15% ▲	3.34E-06
9	Y			1.34E-06	1.19E-06	13% ▲	1.28E-06	5% ▲
10	Z			6.39E-07	5.53E-07	16% ▲	5.69E-07	12% ▲
11	Max Von Mises Stress, MPa		X	1.13E-01	8.23E-02	37% ▲	8.67E-02	30% ▲
12			Y	6.44E-02	5.09E-02	26% ▲	5.04E-02	28% ▲
13			Z	5.51E-02	4.50E-02	23% ▲	4.72E-02	17% ▲
	<b>Natural Frequencies, Hz</b>	Mode Shape	Mode #					
14		Vertical	#1	129	154	16% ▲	168	23% ▲
15		Lateral + twist	#2	506	570	11% ▲	638	21% ▲
16		Lateral + twist out of phase	#3	621	667	7% ▲	744	17% ▲
17		Vertical 2nd order	#4	761	807	6% ▲	802	5% ▲
18		Lateral higher order	#5	1218	1254	3% ▲	1352	10% ▲
19		Complex	#6	1469	1422	-3% ▼	1554	5% ▲

▲	Better. Lower compliance, lower stress, higher modal frequency
▼	Worse. Higher compliance, higher stress, lower modal frequency



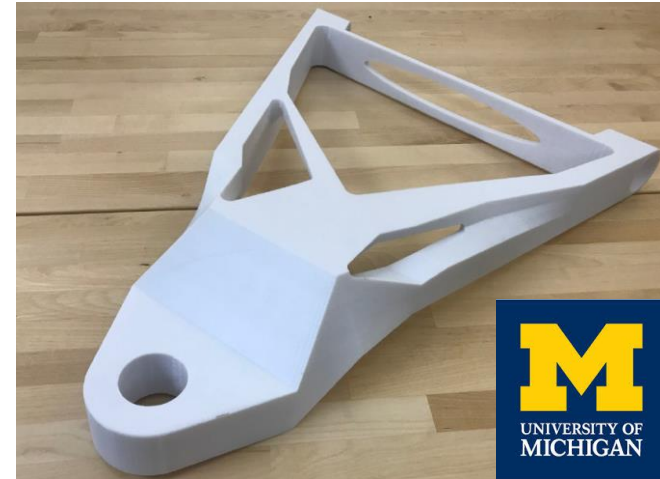
# RESULTS: DESIGN SPACE (#2) EXPLORATION





# CONCLUSIONS / FUTURE WORK

- A proof of concept modeling and simulation strategy has been demonstrated, identifying sizable weight save opportunities based on load-agnostic optimization methodology.
- The chosen design upon topology optimization yielded 15% weight, exceeded targets for stiffness (+10% avg) & strength (+32% avg) and first three natural frequencies (+16% avg)
  - In an independent study<sup>5</sup>, a Senior Capstone project at University of Michigan, Ann Arbor, realized a 17.5% weight savings in the same component, but by employing known service loads.



Future research will focus on three main areas:

1. Constraint perturbation (relaxed by -10% → more stringent by +10%) to establish the Efficiency ( $\Delta\text{weight} / \Delta\text{perf}$ ) relationship between weight save and constraints.
2. Determine the feasibility of this approach to develop blended solid and lattice structures to exploit additional weight savings (such structures are highly 3D-printer friendly).
3. Validate this methodology on a different component/sub-system. Characterize identifiable characteristics of a part that enable weight savings with minimal loss of performance.



GYROID



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# QUALIFICATIONS

**Dr. Ravi Thyagarajan** received his B.Tech. degree in Mechanical Engineering from the Indian Institute of Technology-Madras, India, and his M.S. and Ph.D. degrees in Applied Mechanics from the California Institute of Technology, Pasadena. He has three patents and has written over 80 technical papers and is Co-organizer of SAE Congress sessions as well as a Journal Editor for the SAE Journals. He is a past recipient of the Forest R McFarland Award from SAE for outstanding contributions to SAE. He received the 2010 and 2012 Army Materiel Command Systems Analysis awards. In 2013 and 2015, he was coauthor for the Best Paper Awards at the GVSETS Symposium. He was accepted in 2012 to the TARDEC Research Reviewer Board as a Senior Technical Specialist, served a three-year developmental assignment as the TARDEC Deputy Chief Scientist and currently is the Senior Technical Expert (STE) in Materials/Product Lifecycle Engineering.

In 2017, Dr. Thyagarajan was presented the Department of the Army Commander's Award for Civilian Service and in 2018, Dr. Thyagarajan was elected as a Fellow of the Society of Automotive Engineers (SAE).

**Mr. Kumar B Kulkarni** received his Bachelor of Engineering degree in Mechanical engineering from Bangalore University, India, and M.S. degree in Mechanical Engineering from the University of Toledo, Ohio. He has a wide ranging automotive product development experience (>20 years) using Computer Aided Engineering tools while working at Ford Motor Company and Visteon Corporation as a Technical Specialist in Automotive Interiors Engineering. He has three patents and has several technical papers published at SAE conferences. He has been working in the Analytics group within TARDEC since 2011, performing underbody blast simulation analysis and developing/evaluating countermeasures and mitigation technologies to improve soldier protection and survivability. His current areas of interests include structural optimization, mobility and autonomy of ground system vehicles.



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**Back up slides**



## RESPONSES TO QUESTIONS

What related research has been or is being performed elsewhere?

In the commercial auto industry and academia, and elsewhere within the US Army, topology optimization methodology is used to reduce weight when the actual service loads on a component are readily available. (Slides 2,3)

How does the project reflect an appropriate research niche that the Army is best suited to address?

The current advances in the area of Additive Manufacturing enable manufacture of complex free-forms that naturally occur after topology optimization with significant weight savings. Topology optimization multiplied with additive manufacturing offers significant light-weighting opportunities, often even with improved structural performance.

Development of a service-load-agnostic method of component design optimization is a niche research need for the Army

What is the research approach?

The approach is broadly based on combining the SIMP (Solid Isotropic Material with Penalization for intermediate densities) method, with Gradient-based optimization algorithms capable of handling large number of continuous variables and multiple constraints. (Slides 4,5,10)

What are the research hypotheses?

Under the assumption of over-designed structures with significant dead weight, which is largely true for military ground vehicles, the hypothesis is to obtain suitable performance targets from the existing baseline component itself as design criteria for the optimization problem. These are service load-agnostic and based on compliances, strength and modal properties of the current part. (Slides 6,7)



## RESPONSES TO QUESTIONS

What methods of data collection (laboratory experiment, modeling, field experiment, survey, etc.) are being used and why?

Modeling and numerical simulation techniques using modified finite element based topology optimization tools which are widely available. Once the models are generated, simulations are performed on DoD's High Performance Computers.

What specific equipment or tools are being employed in the research?

See above. Specifically, a commercially available software called Optistruct from Altair Inc. is used in this research project. Simulations are performed utilizing DoD Supercomputing Resource Centers (DSRCs)

What analyses of data are being applied to determine results, and why?

We use standard data analytics and post-processing of finite element analysis (FEA) results. The resulting "raw" shapes from the topology optimization are then refined to create feasible designs, and then performance-validated against the baseline design to ensure success.

What are the results to date?

Optimized designs have not only met but *exceeded* targets for stiffness (+10% avg) & strength (+32% avg) in all directions and first three natural frequencies (+16% avg), set by the original design. At the same time, we have achieved a 15% reduction in weight for these designs.

How do the results fit with expectations and with results found previously or by others?

From a general viewpoint, these findings are similar to those efforts conducted with knowledge of service loads. A comparison of this very same component at University of Michigan [7] yielded similar weight savings.

However, there are no known research efforts being done elsewhere without knowledge of loads, so no direct comparison can be made.



## RESPONSES TO QUESTIONS

What conclusions have you drawn from the results?

A proof of concept modeling and simulation strategy has been shown on a component design, demonstrating feasibility and identifying potential weight save opportunities (Slide 16).

What future research direction do you intend to pursue?

1. Perform constraint perturbations (+/-10%) and investigate relationship between weight savings and constraints.
2. Determine the feasibility of this approach to develop blended solid and lattice structures to exploit additional weight savings (such structures are highly 3D-printer friendly).
3. Validate this methodology on a different component/sub-system. Characterize identifiable characteristics of a part that enable weight savings with minimal loss of performance.

Are there any points on which you are challenged and are seeking assistance?

None at this time.

Are there collaborators involved in the project? If so, what are their roles and contributions and that of the RDEC researcher?

None, other than the two listed PIs

How much of your time is allocated to the project?

30% for PI#1, 5% for PI#2

Is the research limited by resources of staff, equipment, opportunities to publish, or other factors?

Not at this time



## RESPONSES TO QUESTIONS

Who reviews the technical quality of your research? With whom do you discuss the progress of your research?

1. At the time that the ILIR project is selected for initial or continued funding, the reviewers also provide feedback to the proposal, which are then incorporated to the extent possible.
2. The Office of Chief Scientist, US Army, TARDEC and the Senior Technical Experts (STEs) from the various tech areas in TARDEC perform a mid-year review of all ILIRs
3. At the end of every year, we write a Final report, this is reviewed both internally by STEs and by an external academic reviewer/SME. Based on that feedback, we modify the Report as well as future research direction.
4. The Final report is then submitted to RDECOM and then onto ASA/ALT, and any feedback from those reviews are also considered, if provided.

We also discuss progress informally with our colleagues, STEs, etc., often using them as sounding boards rather than as formal stage gate reviewers.

What publications and/or presentations have emerged from the research?

A technical paper is being submitted for the 2019 Ground Vehicle Systems Engineering and Technology Symposium (GVSETS)



# RESULTS: DESIGN SPACE (#1) EXPLORATION

Response Quantity			Baseline	Mat density > 90%	%Δ	> 80%	%Δ	> 70%	%Δ	> 60%	%Δ	
<b>1</b>	<b>Mass, kg</b>		<b>18.3</b>	<b>15.43</b>	<b>19%▲</b>	<b>15.63</b>	<b>17%▲</b>	<b>15.89</b>	<b>15%▲</b>	<b>16.13</b>	<b>13%▲</b>	
2	<b>F o r c e s</b>	Compliance, mm/kN	X	8.75E-03	1.11E-02	-21%▼	1.07E-02	-18%▼	1.03E-02	-15%▼	1.00E-02	-13%▼
3			Y	3.73E-02	4.94E-02	-24%▼	4.76E-02	-22%▼	4.58E-02	-19%▼	4.43E-02	-16%▼
4			Z	1.88E-01	1.88E-01	0%▲	1.84E-01	2%▲	1.81E-01	4%▲	1.78E-01	6%▲
5		Max Von Mises Stress, MPa	X	6.08	5.20	17%▲	5.78	5%▲	6.06	0%▲	5.78	5%▲
6			Y	8.26	7.47	11%▲	9.29	-11%▼	10.00	-17%▼	9.29	-11%▼
7			Z	19.21	12.38	55%▲	12.97	48%▲	14.24	35%▲	12.97	48%▲
8	<b>M o m e n t s</b>	Compliance, rad/Nm	X	3.39E-06	5.92E-06	-43%▼	5.59E-06	-39%▼	5.30E-06	-36%▼	5.04E-06	-33%▼
9			Y	1.34E-06	1.78E-06	-25%▼	1.68E-06	-20%▼	1.50E-06	-11%▼	1.46E-06	-8%▼
10			Z	6.39E-07	7.22E-07	-11%▼	6.96E-07	-8%▼	6.48E-07	-1%▼	6.35E-07	1%▲
11		Max Von Mises Stress, MPa	X	1.13E-01	1.03E-01	10%▲	1.33E-01	-15%▼	1.41E-01	-20%▼	1.33E-01	-15%▼
12			Y	6.44E-02	5.70E-02	13%▲	6.39E-02	1%▲	6.70E-02	-4%▼	6.39E-02	1%▲
13			Z	5.51E-02	4.90E-02	13%▲	5.02E-02	10%▲	5.06E-02	9%▲	5.02E-02	10%▲
	<b>N a t u r a l F r e q u e n c i e s, H z</b>	Mode Shape	Mode #									
14		Vertical	#1	129	166	22%▲	167	23%▲	166	22%▲	166	22%▲
15		Lateral + twist	#2	506	550	8%▲	566	11%▲	576	12%▲	583	13%▲
16		Lateral + twist out of phase	#3	621	604	-3%▼	607	-2%▼	611	-2%▼	612	-1%▼
17		Vertical 2nd order	#4	761	722	-5%▼	727	-5%▼	732	-4%▼	732	-4%▼
18		Lateral higher order	#5	1218	1126	-8%▼	1062	-15%▼	1044	-17%▼	1058	-15%▼
19		Complex	#6	1469	1421	-3%▼	1491	1%▲	1342	-9%▼	1354	-8%▼

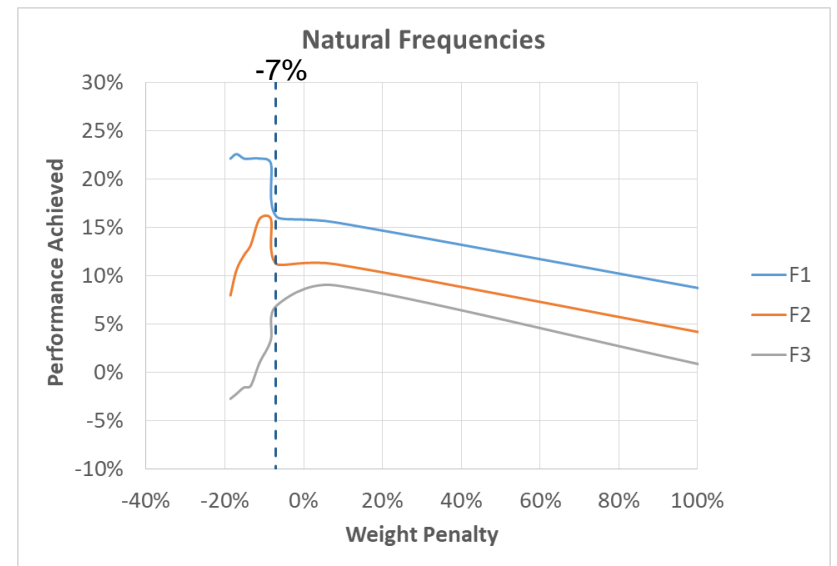
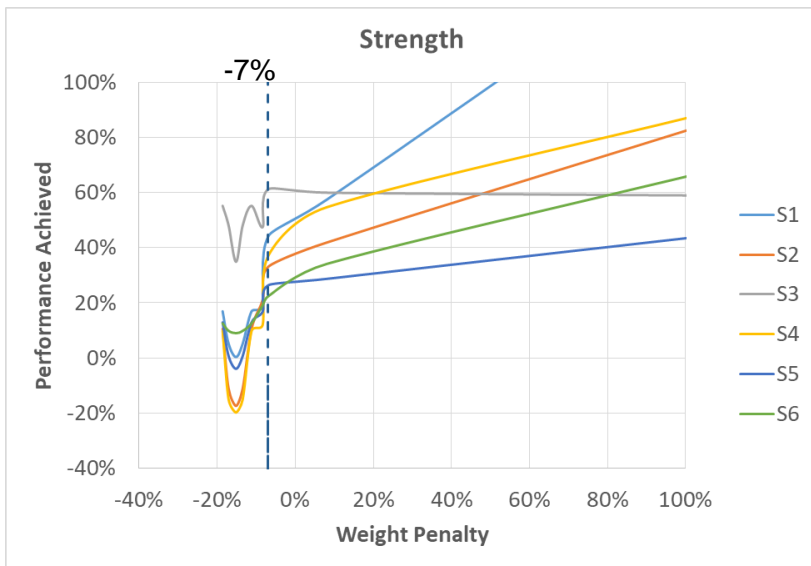
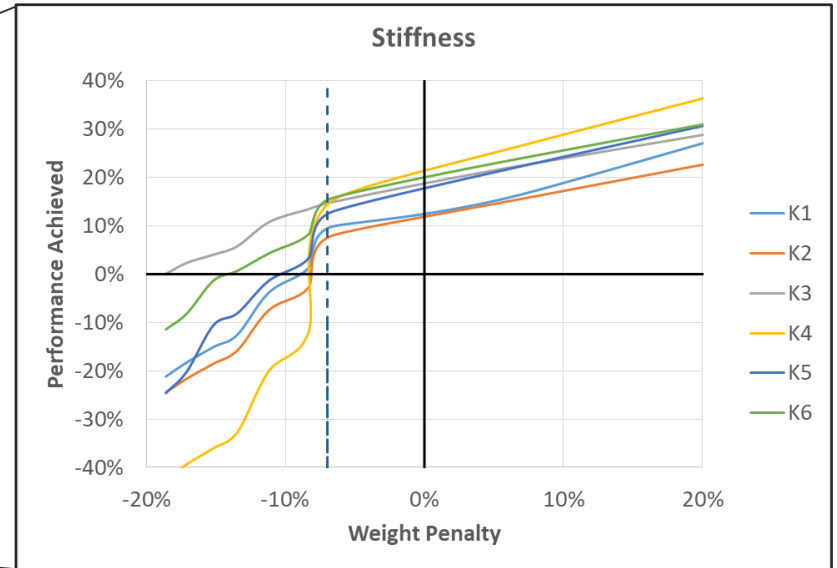
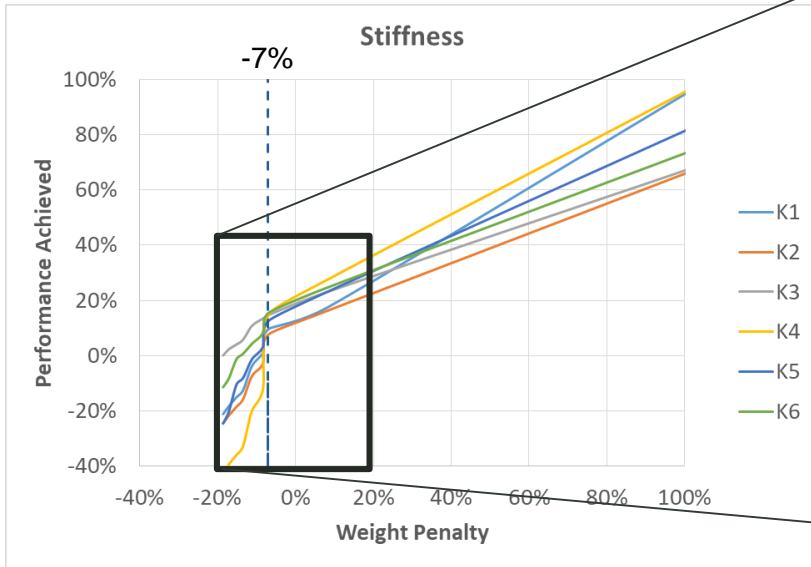


# RESULTS: DESIGN SPACE (#1) EXPLORATION (CONTINUED)

Response Quantity		Baseline	> 50%	%Δ	> 40%		> 25% (FINAL)	%Δ	> 10%	%Δ	> 0%	%Δ		
1	<b>Mass, kg</b>	<b>18.3</b>	<b>16.46</b>	<b>11%▲</b>	<b>16.89</b>	<b>8%▲</b>	<b>17.14</b>	<b>7%▲</b>	<b>19.93</b>	<b>-8%▼</b>	<b>37.59</b>	<b>-105%▼</b>		
2	<b>F o r c e s</b>	Compliance, mm/kN	X	8.75E-03	9.09E-03	-4%▼	8.64E-03	1%▲	7.97E-03	10%▲	7.45E-03	17%▲	4.39E-03	50%▲
3			Y	3.73E-02	4.03E-02	-7%▼	3.84E-02	-3%▼	3.46E-02	8%▲	3.21E-02	16%▲	2.21E-02	41%▲
4			Z	1.88E-01	1.70E-01	11%▲	1.66E-01	13%▲	1.64E-01	15%▲	1.53E-01	23%▲	1.11E-01	41%▲
5		Max Von Mises Stress, MPa	X	6.08	5.20	17%▲	5.12	19%▲	4.21	44%▲	3.86	58%▲	2.40	61%▲
6			Y	8.26	7.47	11%▲	6.82	21%▲	6.20	33%▲	5.82	42%▲	4.47	46%▲
7			Z	19.21	12.38	55%▲	13.03	47%▲	11.91	61%▲	12.01	60%▲	12.09	37%▲
8		<b>M o m e n t s</b>	Compliance, rad/Nm	X	3.39E-06	4.24E-06	-20%▼	3.86E-06	-12%▼	2.95E-06	15%▲	2.66E-06	27%▲	1.70E-06
9	Y			1.34E-06	1.36E-06	-1%▼	1.30E-06	3%▲	1.19E-06	13%▲	1.09E-06	23%▲	7.26E-07	46%▲
10	Z			6.39E-07	6.13E-07	4%▲	5.91E-07	8%▲	5.53E-07	16%▲	5.13E-07	25%▲	3.63E-07	43%▲
11	Max Von Mises Stress, MPa		X	1.13E-01	1.03E-01	10%▲	1.01E-01	12%▲	8.23E-02	37%▲	7.31E-02	55%▲	5.99E-02	47%▲
12			Y	6.44E-02	5.70E-02	13%▲	5.51E-02	17%▲	5.09E-02	26%▲	5.00E-02	29%▲	4.46E-02	31%▲
13			Z	5.51E-02	4.90E-02	13%▲	4.63E-02	19%▲	4.50E-02	23%▲	4.11E-02	34%▲	3.29E-02	40%▲
	<b>N a t u r a l F r e q u e n c i e s, H z</b>	Mode Shape	Mode #											
14		Vertical	#1	129	166	22%▲	165	22%▲	154	16%▲	153	16%▲	141	8%▲
15		Lateral + twist	#2	506	602	16%▲	602	16%▲	570	11%▲	570	11%▲	526	4%▲
16		Lateral + twist out of phase	#3	621	627	1%▲	642	3%▲	667	7%▲	682	9%▲	623	0%▲
17		Vertical 2nd order	#4	761	757	-1%▼	766	1%▲	807	6%▲	826	8%▲	616	-24%▼
18		Lateral higher order	#5	1218	1383	12%▲	1156	-5%▼	1254	3%▲	1310	7%▲	1489	18%▲
19		Complex	#6	1469	1664	12%▲	1395	-5%▼	1422	-3%▼	1495	2%▲	1716	14%▲



# RESULTS: DESIGN SPACE (#1) EXPLORATION





# RESULTS: DESIGN SPACE (#2) EXPLORATION

Response Quantity		Baseline	Mat density > 90%	%Δ	> 80%	%Δ	> 70%	%Δ	> 60%	%Δ	
<b>1</b>	<b>Mass, kg</b>	<b>18.3</b>	<b>14.25</b>	<b>28%▲</b>	<b>14.47</b>	<b>26%▲</b>	<b>14.69</b>	<b>25%▲</b>	<b>14.9</b>	<b>23%▲</b>	
2	Forces Compliance, mm/kN	X	8.75E-03	1.05E-02	-17%▼	1.03E-02	-15%▼	1.01E-02	-13%▼	9.86E-03	-11%▼
3		Y	3.73E-02	5.05E-02	-26%▼	4.89E-02	-24%▼	4.76E-02	-22%▼	4.62E-02	-19%▼
4		Z	1.88E-01	1.91E-01	-1%▼	1.88E-01	0%▲	1.85E-01	2%▲	1.83E-01	3%▲
5	Max Von Mises Stress, MPa	X	6.08	7.01	-13%▼	6.83	-11%▼	6.69	-9%▼	6.50	-6%▼
6		Y	8.26	12.94	-36%▼	12.08	-32%▼	11.28	-27%▼	10.40	-21%▼
7		Z	19.21	20.19	-5%▼	19.92	-4%▼	19.55	-2%▼	19.28	-0%▼
8	Moment Compliance, rad/Nm	X	3.39E-06	6.13E-06	-45%▼	5.89E-06	-42%▼	5.66E-06	-40%▼	5.37E-06	-37%▼
9		Y	1.34E-06	1.61E-06	-17%▼	1.55E-06	-13%▼	1.50E-06	-11%▼	1.46E-06	-8%▼
10		Z	6.39E-07	6.83E-07	-6%▼	6.70E-07	-5%▼	6.55E-07	-2%▼	6.35E-07	1%▲
11	Max Von Mises Stress, MPa	X	1.13E-01	1.60E-01	-29%▼	1.54E-01	-27%▼	1.38E-01	-18%▼	1.32E-01	-14%▼
12		Y	6.44E-02	7.40E-02	-13%▼	7.19E-02	-10%▼	7.01E-02	-8%▼	6.79E-02	-5%▼
13		Z	5.51E-02	5.71E-02	-3%▼	5.64E-02	-2%▼	5.52E-02	-0%▼	5.22E-02	6%▲
	Natural Frequencies, Hz	Mode Shape	Mode #								
14	Vertical	#1	129	170	24%▲	170	24%▲	170	24%▲	169	24%▲
15	Lateral + twist	#2	506	522	3%▲	526	4%▲	529	4%▲	595	15%▲
16	Lateral + twist out of phase	#3	621	578	-7%▼	587	-6%▼	595	-4%▼	605	-3%▼
17	Vertical 2nd order	#4	761	660	-15%▼	657	-16%▼	670	-14%▼	674	-13%▼
18	Lateral higher order	#5	1218	737	-65%▼	740	-65%▼	743	-64%▼	746	-63%▼
19	Complex	#6	1469	1158	-27%▼	1168	-26%▼	1176	-25%▼	1183	-24%▼



# RESULTS: DESIGN SPACE (#2) EXPLORATION (CONTINUED)

Response Quantity		Baseline	> 50%	%Δ	> 40%		> 25% (FINAL)	%Δ	> 10%	%Δ	> 0%	%Δ		
<b>1</b>	<b>Mass, kg</b>	<b>18.3</b>	<b>15.17</b>	<b>21%▲</b>	<b>15.5</b>	<b>18%▲</b>	<b>15.97</b>	<b>15%▲</b>	<b>18.04</b>	<b>1%▲</b>	<b>65.89</b>	<b>-260%▼</b>		
2	Forces Compliance, mm/kN	X 8.75E-03	9.63E-03	-9%▼	8.67E-03	1%▲	8.36E-03	5%▲	7.77E-03	13%▲	2.96E-03	195%▲		
3		Y 3.73E-02	4.49E-02	-17%▼	3.83E-02	-3%▼	3.59E-02	4%▲	3.24E-02	15%▲	1.54E-02	142%▲		
4		Z 1.88E-01	1.80E-01	5%▲	1.67E-01	13%▲	1.63E-01	16%▲	1.56E-01	21%▲	7.93E-02	137%▲		
5	Max Von Mises Stress, MPa	X 6.08	6.21	-2%▼	4.33	40%▲	3.89	56%▲	3.86	58%▲	1.79	240%▲		
6		Y 8.26	9.49	-13%▼	10.55	-22%▼	6.79	22%▲	5.82	42%▲	4.72	75%▲		
7		Z 19.21	19.15	0%▲	18.93	1%▲	18.08	6%▲	12.01	60%▲	12.16	58%▲		
8	Moment Compliance, rad/Nm	X 3.39E-06	5.13E-06	-34%▼	3.76E-06	-10%▼	3.34E-06	2%▲	2.78E-06	22%▲	1.31E-06	159%▲		
9		Y 1.34E-06	1.42E-06	-6%▼	1.33E-06	1%▲	1.28E-06	5%▲	1.12E-06	20%▲	4.98E-07	169%▲		
10		Z 6.39E-07	6.18E-07	3%▲	5.91E-07	8%▲	5.69E-07	12%▲	5.12E-07	25%▲	3.13E-07	104%▲		
11	Max Von Mises Stress, MPa	X 1.13E-01	1.30E-01	-13%▼	1.49E-01	-24%▼	8.67E-02	30%▲	7.31E-02	55%▲	5.99E-02	89%▲		
12		Y 6.44E-02	6.53E-02	-1%▼	5.08E-02	27%▲	5.04E-02	28%▲	5.00E-02	29%▲	4.44E-02	45%▲		
13		Z 5.51E-02	5.03E-02	10%▲	4.89E-02	13%▲	4.72E-02	17%▲	4.11E-02	34%▲	3.28E-02	68%▲		
	Natural Frequencies, Hz	Mode Shape	Mode #											
14		Vertical	#1	129	169	24%▲	170	24%▲	168	23%▲	158	18%▲	150	14%▲
15		Lateral + twist	#2	506	569	11%▲	632	20%▲	638	21%▲	638	21%▲	730	31%▲
16		Lateral + twist out of phase	#3	621	611	-2%▼	735	16%▲	744	17%▲	916	32%▲	916	32%▲
17		Vertical 2nd order	#4	761	677	-12%▼	790	4%▲	802	5%▲	1070	29%▲	1070	29%▲
18		Lateral higher order	#5	1218	1061	-15%▼	1069	-14%▼	1352	10%▲	1348	10%▲	1348	10%▲
19	Complex	#6	1469	1190	-23%▼	1194	-23%▼	1554	5%▲	1535	4%▲	1535	4%▲	



# RESULTS: DESIGN SPACE (#2) EXPLORATION

