

REPORT DOCUMENTATION PAGE

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HIGH INFORMATION BANDWIDTH ADAPTIVE HYBRID KINETIC SIMULATION

Robert Martin

IN-SPACE PROPULSION BRANCH,
AIR FORCE RESEARCH LABORATORY,
EDWARDS AIR FORCE BASE, CA USA

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U.S. AIR FORCE





Analog NTSC Video



333x525x30fps, ≈ 3 bytes/pixel*
Raw Bandwidth: 126MBPs
Transmitted in 4.2MHz AM Band

Digital Netflix Stream



1920x1080x30fps, 3 bytes/pixel
Raw Bandwidth: 1,500MBPs?
Bandwidth on Network: <5MBPs!

300x Efficiency Improvement?
Analog \rightarrow Compressed Digital

Kinetic Simulations \approx Uncompressed Digital

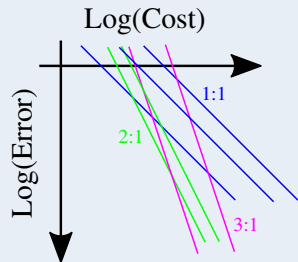
* Analog Equivalent to 'True' Color



Efficient Simulations:

- Many Methods 'Converge'

Efficient Methods

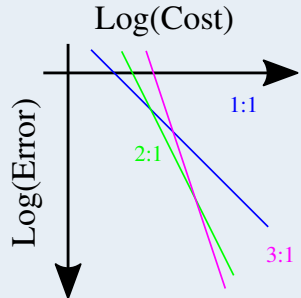




Efficient Simulations:

- Many Methods 'Converge'
- Some Methods are Better than Others
(*But This is Error Metric Dependent!)

Efficient Methods

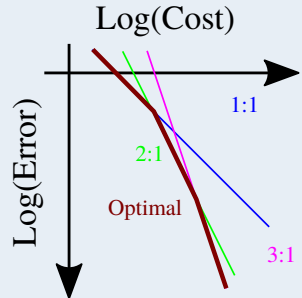




Efficient Simulations:

- Many Methods ‘Converge’
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- Higher Order not Necessarily ‘Better’

Efficient Methods

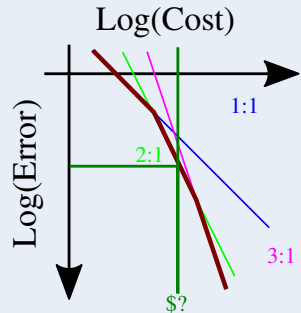




Efficient Simulations:

- Many Methods ‘Converge’
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- Higher Order not Necessarily ‘Better’
- ‘Best’ May Depend on Budget (p-Adaptivity)

Efficient Methods

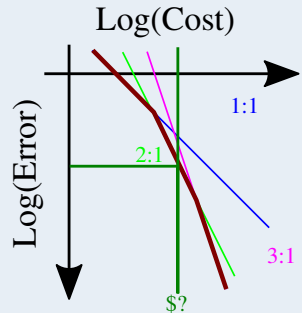




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Efficient Methods



Adaptivity \approx Lossy Encoding?

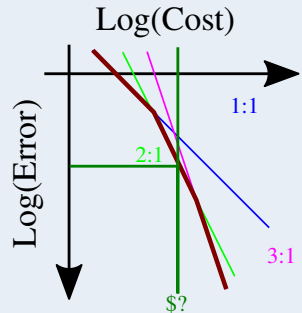


Efficient Simulations:

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- ‘Best’ May Depend on Budget (p-Adaptivity)
- Error May be Distributed Non-Uniformly (Need for h-Adaptivity Too)
- Adaptivity Requires *a Priori* Error Estimates

Efficient Algorithms \approx Efficient Encoding?

Efficient Methods



Adaptivity \approx Lossy Encoding?



- 1 INTRODUCTORY EXAMPLE: OCTREE PARTICLE RESAMPLING
- 2 ENTROPY: SIGNAL IN THE NOISE
- 3 ADAPTIVE QUADRATURES
- 4 CONCLUSION



IMPORTANCE OF KINETIC METHODS

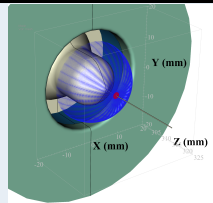
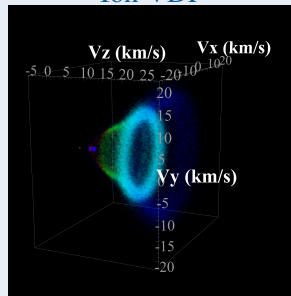
Nonequilibrium in Spacecraft Propulsion:

- Hall Thruster Plume Expansion
- Discharge and Breakdown in FRC
- Collisional Radiative Cooling/Ionization

Common Features in Spacecraft Flows:

- Relevant Densities Spanning Many Orders of Magnitude — 6+
- Transitions from Collisional to Collisionless
- Tiny Early e^- or Radical Populations Critical to Induction Delay
- Many types of Inelastic Collisions with Unknown Effects on Distribution Shapes

Ion VDF



Sampled 1cm Downstream
On Thruster Axis



Nonequilibrium in Spacecraft Propulsion:

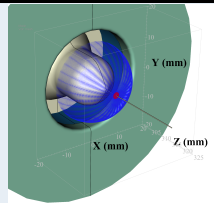
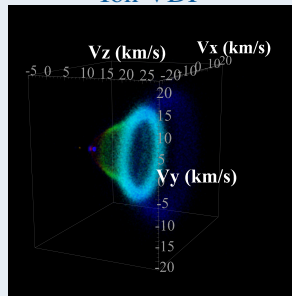
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Need Low Noise & High Dynamic Range
Kinetic Algorithms

Ion VDF

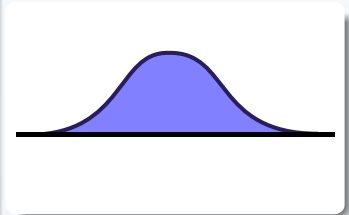


Sampled 1cm Downstream
On Thruster Axis



Continuum to Discrete Representation:

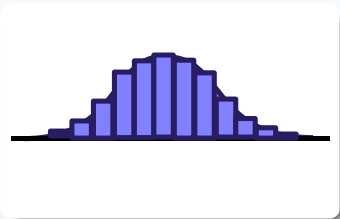
- Many Particles \rightsquigarrow Continuous Distribution





Continuum to Discrete Representation:

- Many Particles \rightsquigarrow Continuous Distribution
- Discretized VDF Yields Vlasov
But Collision Integral Still a Problem





Continuum to Discrete Representation:

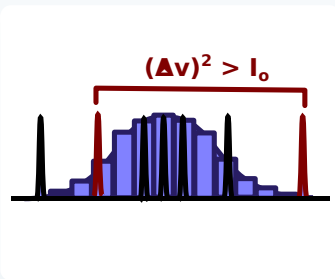
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- Particle Methods VDF to Delta Function Set
- Collisions between Discrete Velocities





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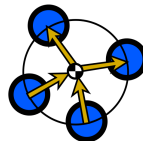




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Variable Weight “All-or-Nothing” Collisions?

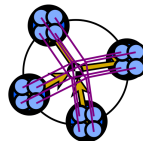




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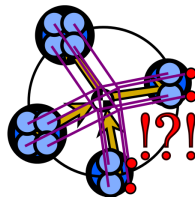
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Variable Weight “All-or-Nothing” Collisions?

Physically Inconsistent!

(Mixing Violates Momentum/Energy Conservation)

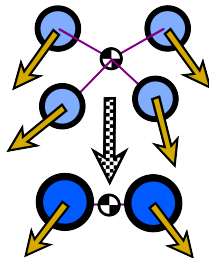




Merge to Pair \rightarrow DOF for Conservation:

- $(n+2):2$ yields Exact Mass, Momentum, and Kinetic Energy Conservation
- Applied Spatially also Shown to Conserve Electrostatic Energy
- Though Energy Conserving, Still Thermalizes VDF

(AFOSR Review 2006)



$$w_m = \sum_i^{(n+2)} w_i$$

$$\bar{\vec{v}} = \left(\sum_i^{(n+2)} w_i \vec{v}_i \right) / w_m$$

$$\bar{V}^2 = \left(\sum_i^{(n+2)} w_i (\vec{v}_i - \bar{\vec{v}})^2 \right) / w_m$$

$$w_{(a/b)} = w_m / 2$$

$$\vec{v}_{(a/b)} = \bar{\vec{v}} \pm \hat{\mathcal{R}} \sqrt{\bar{V}^2}$$

$$\left(\text{Similarly: } \vec{x}_{(a/b)} = \bar{\vec{x}} \pm \hat{\mathcal{R}} \sqrt{\bar{X}^2} \right)$$



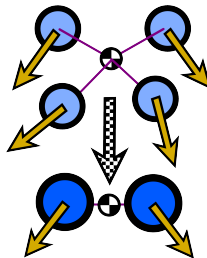
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(AFOSR Review 2006)

Selection of Near Neighbors in VDF Limits Thermalization

(\approx Near Neighbor Pairs in 2:1 Merges that Limit Numerical Cooling)



$$w_m = \sum_i^{(n+2)} w_i$$

$$\bar{\vec{v}} = \left(\sum_i^{(n+2)} w_i \vec{v}_i \right) / w_m$$

$$\bar{v}^2 = \left(\sum_i^{(n+2)} w_i (\vec{v}_i - \bar{\vec{v}})^2 \right) / w_m$$

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Phase-Space Decomposition

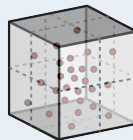
- Given a Set of Particles...





Phase-Space Decomposition

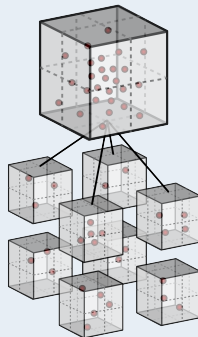
- Given a Set of Particles...
- Particles Binned in Octants





Phase-Space Decomposition

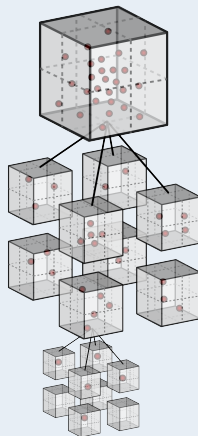
- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided





Phase-Space Decomposition

- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided
- Recursion Halted at 1-Particle/Bin or Other Criteria such as Bin-Density



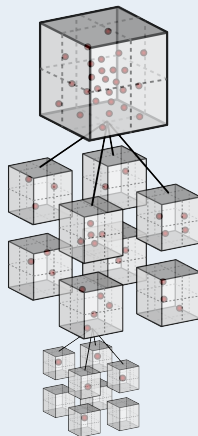


Phase-Space Decomposition

- Given a Set of Particles...
- Particles Binned in Octants
- Octants Recursively Sub-Divided
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Restricts Phase-Space Diffusion to
Within Local Bins

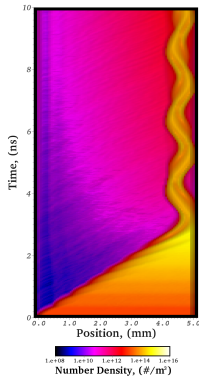
(Entropy, $\sum n \log(n)$, Constant within Octree Adaptive Quadrature)



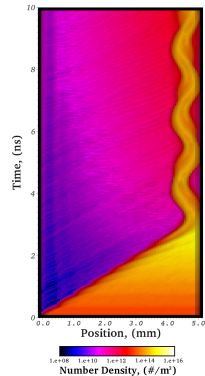


250V DC-Diode Test Case:

- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot



Control

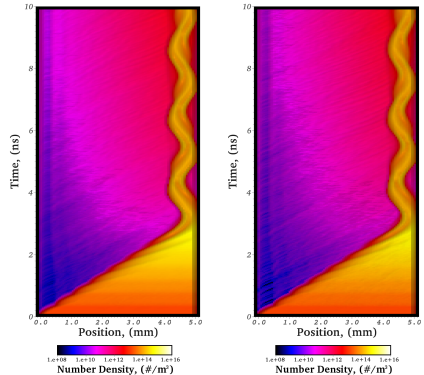


Merged



250V DC-Diode Test Case:

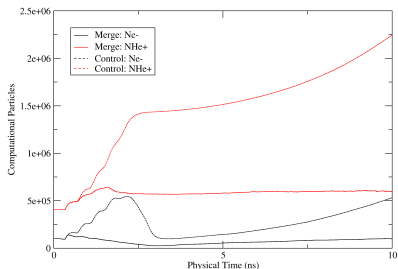
- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode → Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode





250V DC-Diode Test Case:

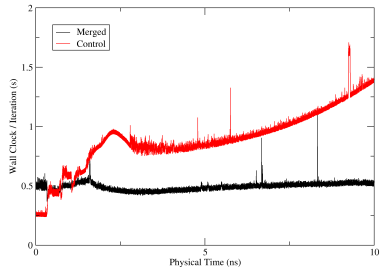
- Full 3D Electrostatic-PIC
- Averaged to 1D XT-Plot
- 250V Cathode → Anode
- MCC-Ionization Collisions
- Secondary Emission at Cathode
- Weak Chain-Branching (Marginal on Paschen Curve)





250V DC-Diode Test Case:

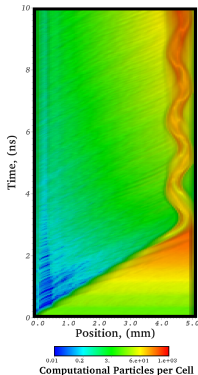
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- Negligible Merge Overhead





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- Negligible Merge Overhead
- Control: Parts/Cell \propto Density



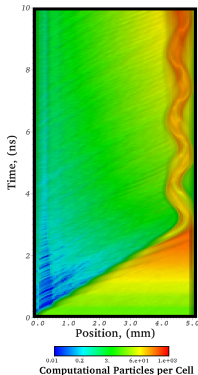
Control

Merged

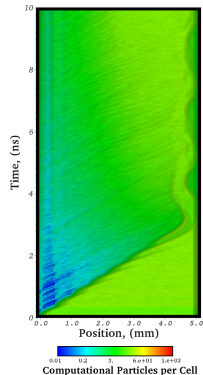


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- Negligible Merge Overhead
- Control: Parts/Cell \propto Density
- Merge: Parts/Cell **Reduced**



Control

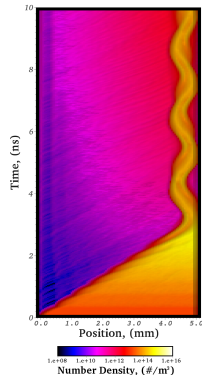
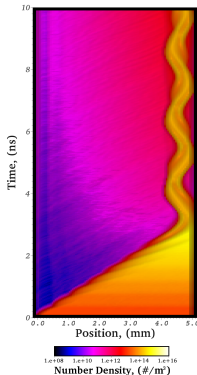


Merged



250V DC-Diode Test Case:

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- Secondary Emission at Cathode
- Weak Chain-Branching (Marginal on Paschen Curve)
- Negligible Merge Overhead
- Control: Parts/Cell \propto Density
- Merge: Parts/Cell **Reduced**
- Despite Identical Densities





1KV DC-Diode Test Case:

- Voltage increased to 1KV
- Otherwise Identical to 250V

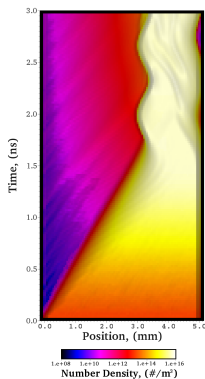
Control

Merged



1KV DC-Diode Test Case:

- Voltage increased to 1KV
- Otherwise Identical to 250V
- Much Stronger Ionization



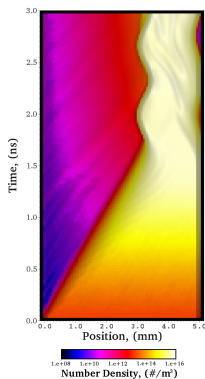
Control

Merged

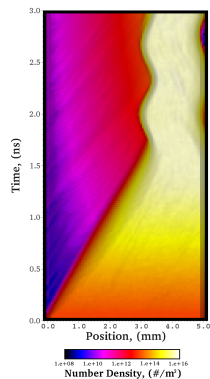


1KV DC-Diode Test Case:

- Voltage increased to 1KV
- Otherwise Identical to 250V
- Much Stronger Ionization
- Major Features Captured



Control

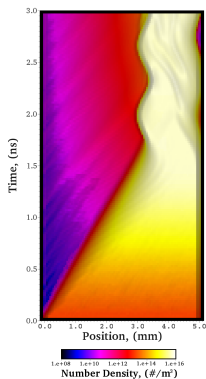


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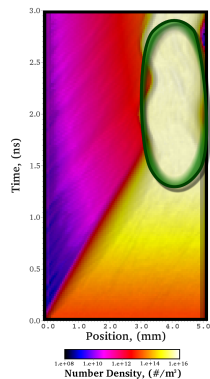


1KV DC-Diode Test Case:

- Voltage increased to 1KV
- Otherwise Identical to 250V
- Much Stronger Ionization
- Major Features Captured
- Some Features Lost...



Control

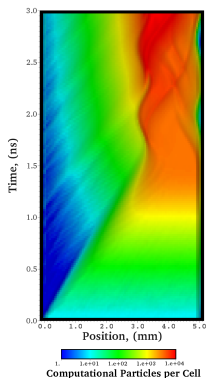


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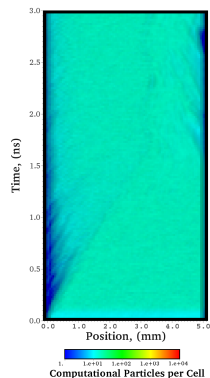


1KV DC-Diode Test Case:

- Voltage increased to 1KV
- Otherwise Identical to 250V
- Much Stronger Ionization
- Major Features Captured
- Some Features Lost...
- Might be Captured by Increasing Target #/cell?



Control

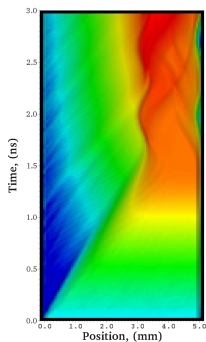


Merged

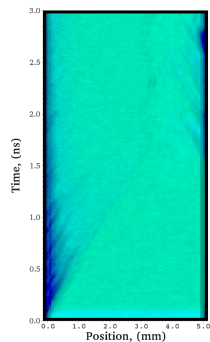


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- 26x Speedup



Control
Run: 6hr12min



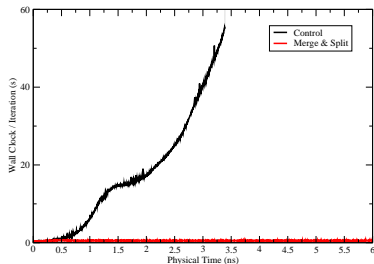
Merged
Run: 14min

Merged Particles → Compressed Solution
What has been Lost?



1KV DC-Diode Test Case:

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- 26x Speedup
- Control Halted Mem>15GB

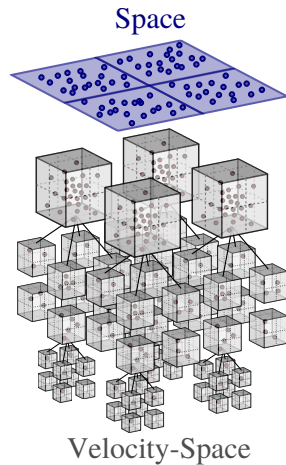


Merged Particles → Compressed Solution
What has been Lost?



Octree Merge:

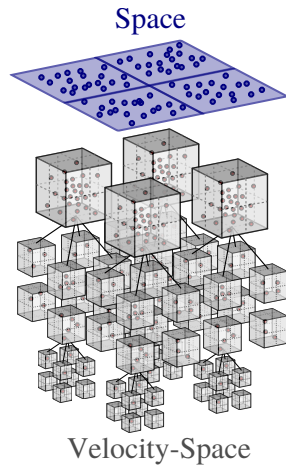
- Uses Spatially Localized Particles (i.e. Space Cells)





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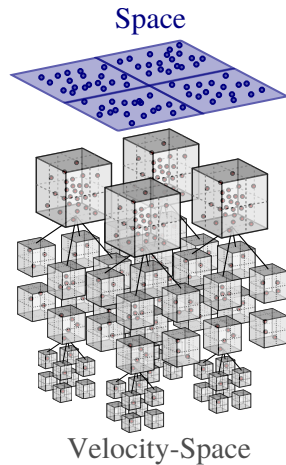
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Critical Collisional Invariants
Random Walks w/o Conservation





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Error is Excess Diffusion (Collisions?)

Entropy Error vs. Bin Count Uniform Bins

$$\left\| \int f \ln f \, dv - \sum f_i \ln f_i \right\|$$

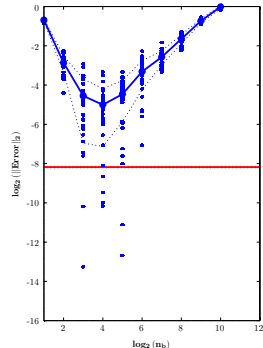


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2^8 Particles

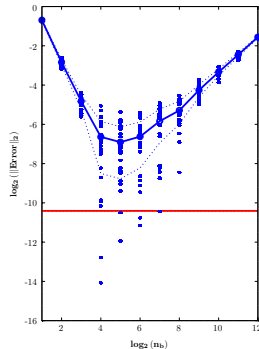


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2^{12} Particles

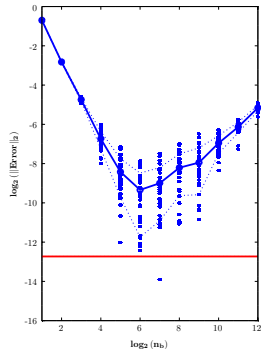


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2^{16} Particles



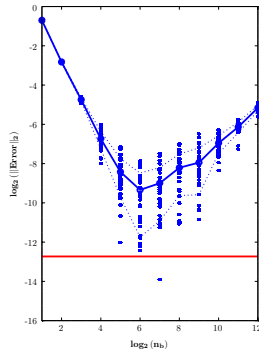
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Improve Quadrature → Conserve Entropy
Improve Performance w/ Fewer N_p ?

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2^{16} Particles



Extrapolating Communication Revolution:

- Pre-Shannon Communication:
More Power → Faster/Further/More Accurate



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Extrapolating Communication Revolution:

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122 Peta-FLOPS,
8.8MW Power
(Top500.org)



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- Ideas formed Basis of Information Theory

Communication in the Presence of Noise*

CLAUDE E. SHANNON†, MEMBER, IRE

Summary—A method is developed for representing any communication system geometrically. Messages and the corresponding signals are points in two “function spaces” and the modulation process is a mapping of one space into the other. Using this representation, a number of results in communication theory are derived: necessary expansion and compression of bandwidth and the threshold effect. Bandwidths are found for the maximum rate of transmission of binary digits and for the maximum rate of transmission of binary digits over a noisy channel.

I. INTRODUCTION

GENERAL COMMUNICATIONS system is shown schematically in Fig. 1. It consists essentially of five elements:

1. a **source**, which selects one message to be transmitted to the receiver out of all messages



Fig. 1—General communication system.

“The fundamental problem of communication is that of reproducing at one point, either exactly or approximately, a message selected at another point.” -Claude Shannon

$$C = W \log_2 \left(1 + \frac{P}{N} \right)$$



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Synopsis—A method is developed for representing any communication system generally. Messages and the corresponding signals are placed in two “classical spaces” and the modulation process is a mapping of one space into the other. Using this representation, a number of results in communication theory are obtained: necessary expansion and compression of bandwidth and the threshold effect. Paradoxes are found for the maximum rate of transmission of binary digital data.



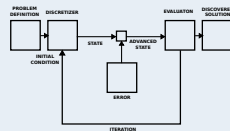
Fig. 1—General communication system.

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A GENERAL COMMUNICATIONS system is shown schematically in Fig. 1. It consists essentially of five elements.

1. *A* is information source. The source selects one message to be transmitted to the receiver from the set of possible

Simulation in the Presence of Error



General simulation system.



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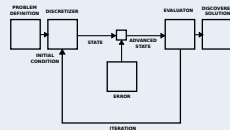
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Efficient Simulation:
Best Approximation in Finite Capacity

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Summary—A method is developed for representing any communication system generatically. Messages and the corresponding signals are placed in two “function spaces” and the modulation process is a mapping of one space into the other. Using this representation, a number of results in communication theory are deduced: necessary expansion and compression of bandwidth and the threshold effect. Paradoxes are found for the maximum rate of transmission of binary digital data.



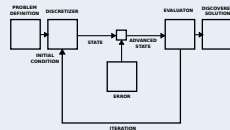
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Compression:

- Goal of Kinetic Simulations:
Know $f(x, v; t)$ from $t=0 \rightarrow t_f$
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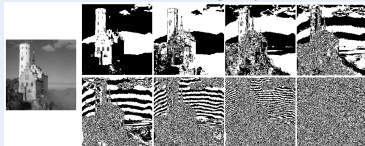
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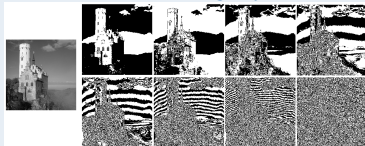
https://commons.wikimedia.org/wiki/File:Lichtenstein_bitplanes.png



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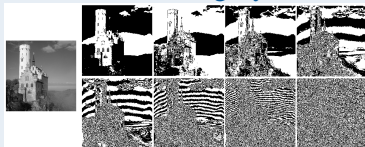
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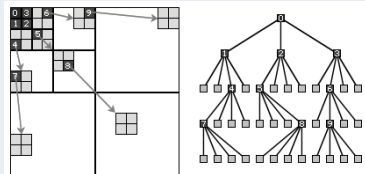
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Entropy Encoded EZW

https://deaksoftware.com.au/articles/wavelet_compression/

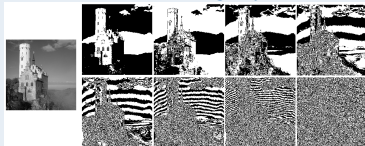


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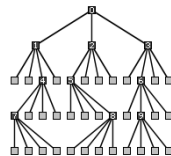
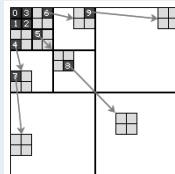
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- Kinetic Simulations?
Collisions \approx Diffusion \rightarrow Low-Pass Filter

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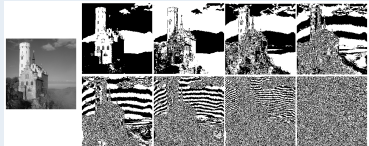


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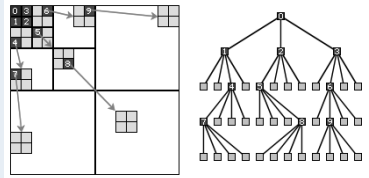
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Good Quadratures: Equal Information/DOF

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Alternatives to Continuum vs. Discrete:

- Particles and Continuum both Solve Boltzmann Eq:

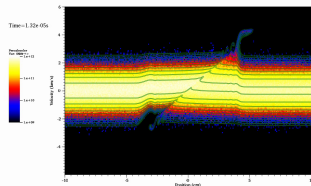
$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + a \cdot \nabla_v f = \left\{ \frac{\partial f}{\partial t} \right\}_{Coll.}$$



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$$\frac{\partial f}{\partial t} + v \cdot \nabla_x f + a \cdot \nabla_v f = 0$$
- Compare for 1D1V **Collisionless** Shock
Vlasov Results better & Faster
Does not Scale to High-D

PIC vs. Vlasov



Electrostatic Collisionless Shock
 $N_L = 1.25e15/m^3$, $N_R = 1e15/m^3$,
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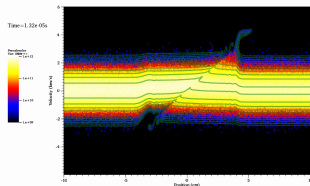


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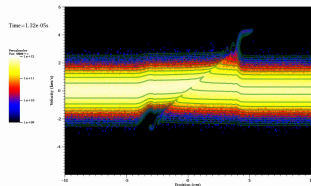


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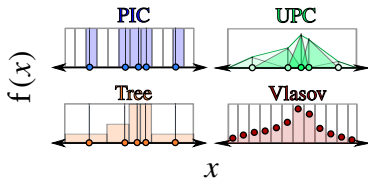


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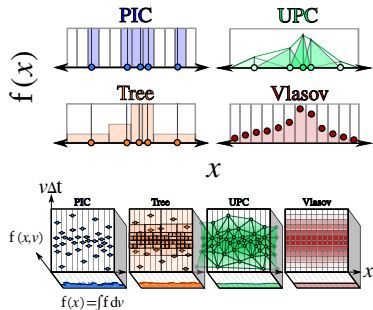




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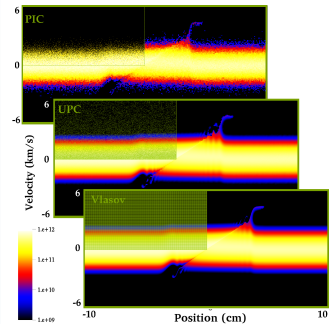




“Unstructured Particle Continuum Method”:

- Tested on Collisionless Shock

Collisionless Shock



PIC: 1.4e6 Particles

UPC: 2.74e5 Nodes

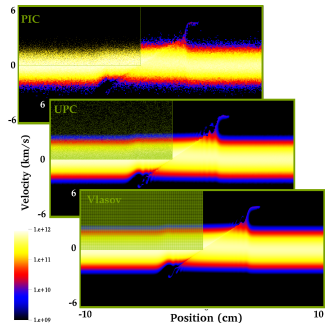
Vlasov: 1.2e6 Cells



“Unstructured Particle Continuum Method”:

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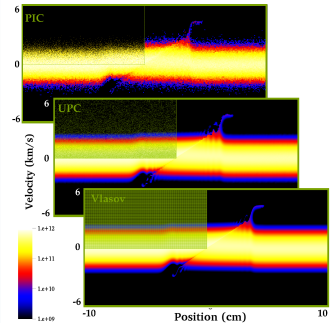
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Collisionless Shock



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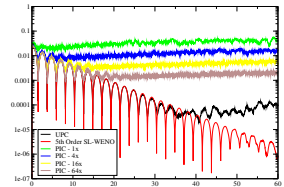
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“Unstructured Particle Continuum Method”:

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- Promising Preliminary Results

Weak Landau Damping



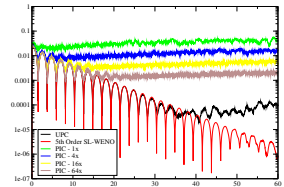
Random Particles
(Not Quiet-Start)



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- Extension Beyond 1D1V Challenging
Requires Delaunay Triangulation per Step
 $O(N^{\lfloor (d+1)/2 \rfloor})$

Weak Landau Damping



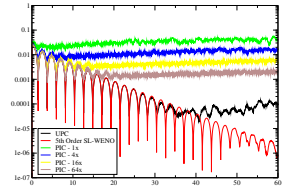
Random Particles
(Not Quiet-Start)



“Unstructured Particle Continuum Method”:

- Tested on Collisionless Shock
- Hybrid between Particle and Vlasov
- Exchanges “Noise” for Quadrature Error via Linear Splines
- Promising Preliminary Results
- Extension Beyond 1D1V Challenging
Requires Delaunay Triangulation per Step
 $O(N^{\lfloor (d+1)/2 \rfloor})$
- Tree Methods like 0^{th} Order UPC?
Better Scaling than $O(N^3)$?

Weak Landau Damping

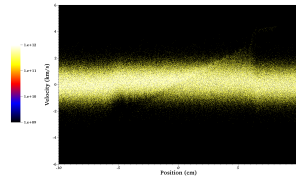
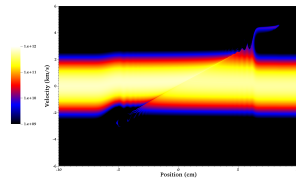


Random Particles
(Not Quiet-Start)



“Unsplit XV-Quadrees”:

- Tested on Collisionless Shock

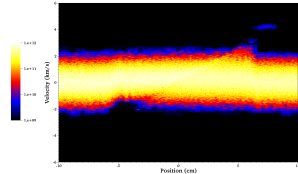
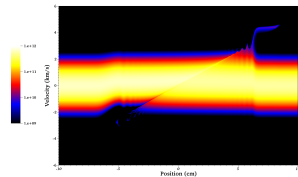


Vlasov: 2.88e6 Cells
PIC: 1.2e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)

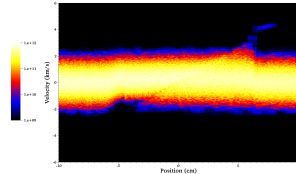
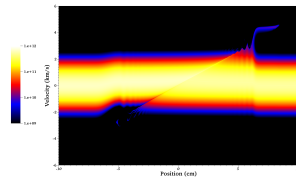


Vlasov: 2.88e6 Cells
QuadTree: 1.2e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p

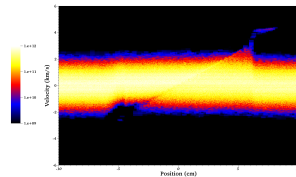
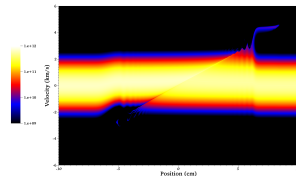


Vlasov: 2.88e6 Cells
QuadTree: 2.4e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p

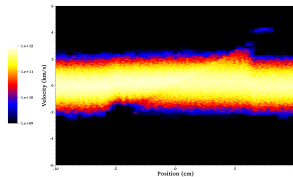
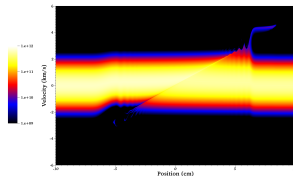


Vlasov: 2.88e6 Cells
QuadTree: 4.8e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p
- Need to Balance Coarseness with Noise (Coarse, low- f Cells Contribute Little Error)

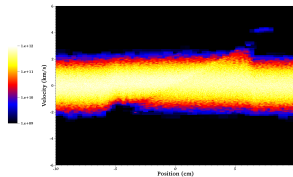
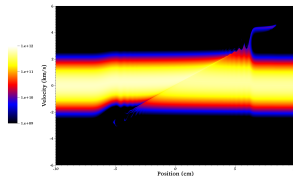


Vlasov: 2.88e6 Cells
QuadTree: 1.2e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p
- Need to Balance Coarseness with Noise (Coarse, low- f Cells Contribute Little Error)
- Built by Morton Coded Sort
 $O(N \log N) \ll O(N^3)$

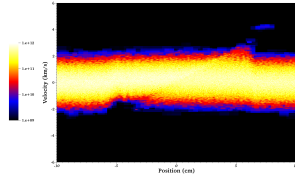
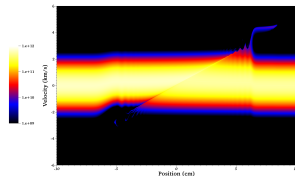


Vlasov: 2.88e6 Cells
QuadTree: 1.2e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p
- Need to Balance Coarseness with Noise (Coarse, low- f Cells Contribute Little Error)
- Built by Morton Coded Sort
 $O(N \log N) \ll O(N^3)$
- Clear Extensibility to 3D3V

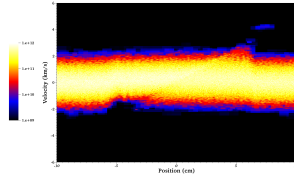
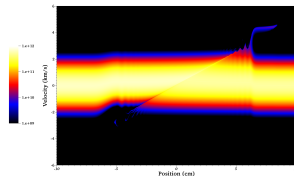


Vlasov: 2.88e6 Cells
QuadTree: 1.2e6 Particles



“Unsplit XV-Quadtrees”:

- Tested on Collisionless Shock
- Removes Phase Space Holes (Cause of Noise Entropy Error Growth)
- Cells Automatically Refine with N_p
- Need to Balance Coarseness with Noise (Coarse, low- f Cells Contribute Little Error)
- Built by Morton Coded Sort
 $O(N \log N) \ll O(N^3)$
- Clear Extensibility to 3D3V
- More Analysis Needed to Balance Quadrature Error vs. Noise



Vlasov: 2.88e6 Cells
QuadTree: 1.2e6 Particles

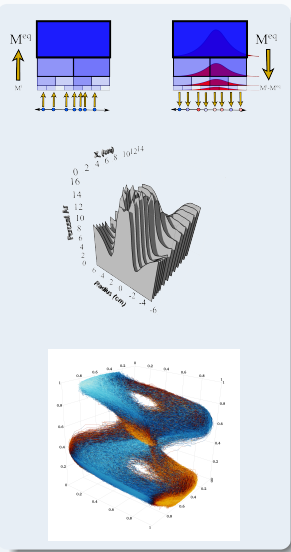


Early Results:

- Sample Density \rightarrow Optimal Resolution
- Lossy Compression & Info. Theory \rightarrow Simulations
- UPC Method Trades Noise for Quadrature Error
- UPC Outperforms PIC but Costs Scale with Dim
- Density Aware Unsplit Trees Potentially Balance Errors

Future Efforts:

- δf Extension to Trees
- Extension of Collision Models for δf
- Higher Dimensional Problems
- Dynamic Systems Approach to Improve SNR





END



Thank You

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Questions?