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PAPER

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E-mail: natashal@buffalo.edu**Keywords:** waveguides, couplers, and arrays, wave propagation, transmission and absorption, charge conjugation, parity, time reversal, and other discrete symmetries

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**Abstract**

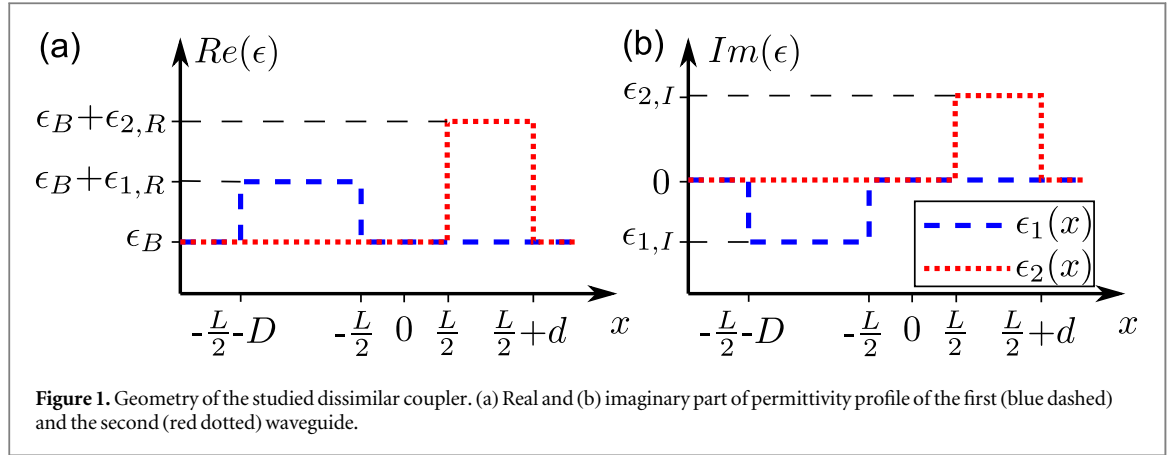
Despite the benefits which the directional couplers based on parity-time symmetric systems may offer to the field of integrated optics, the realization of such couplers relies on rather strict design constraints on the waveguide parameters. Here, we analytically and numerically investigate directional couplers built of dissimilar waveguides that do not fulfill the parity-time symmetry. Nevertheless, we demonstrate that for properly designed parameters, at least one mode of such couplers shows a behavior similar to the one observed in parity-time symmetric systems. We find an analytical condition relating the gain and loss that enables such a behavior. Moreover, if the individual waveguides composing the dissimilar coupler are designed such that the propagation constants of their modes are identical, the behavior of both super-modes supported by the coupler resembles that of the parity-time symmetric systems.

1. Introduction

Directional optical couplers have attracted a lot of attention in the fields of integrated and fiber optics owing to their versatile applications, including spectral filters, optical switches, signal multiplexers and modulators, power dividers, and optical cross-connects. Performance of directional couplers as a function of waveguide size, spacing, refractive index profiles, and length has been studied in detail in [1]. Moreover, directional optical couplers with gain, loss, and nonlinear effects have been analyzed and demonstrated experimentally [2–5].

Recently, studies of parity-time symmetric [6–8] (\mathcal{PT} -symmetric) directional couplers attracted a lot of attention [9–22]. \mathcal{PT} -symmetric systems belong to a more general class of pseudo-Hermitian systems [23, 24], which, despite of being non-Hermitian, may still possess real eigenvalues. In optics, \mathcal{PT} symmetry relies on a proper modulation of gain and loss, and was predicted to enable unconventional switching and memory functionalities. In particular, it was shown that a \mathcal{PT} -symmetric phase transition can occur from a full \mathcal{PT} -symmetric regime (both modes with real propagation constants) to a broken \mathcal{PT} -symmetric regime, where one mode is amplified while the other is attenuated [10, 12]. Change from full to broken \mathcal{PT} regime occurs with the increase of gain and loss in the system at a specific transition point. Both linear [9–15, 21, 22] and nonlinear [25–31] \mathcal{PT} -symmetric waveguide systems were analyzed and many remarkable phenomena were discovered, such as suppression of time reversal [32] and unidirectional nonlinear \mathcal{PT} -symmetric couplers [33].

Until now, a majority of studies focused on the systems strictly fulfilling the \mathcal{PT} symmetry condition: $\epsilon(x) = \epsilon^*(-x)$, implying waveguides with the same size, symmetric distribution of the real part of permittivity, and anti-symmetric gain/loss profile [10, 15]. Although, it is noteworthy that detailed analytical studies of a more general class of directional couplers with gain and loss have been reported in 1986 [34]. Following these studies, in this paper, we analytically and numerically investigate couplers with dissimilar cores and find a \mathcal{PT} -like regime of wave propagation in such systems. In particular, we show that even a dissimilar coupler with complex permittivity profile, provided that gain and loss are properly balanced, supports a mode with a real propagation constant. In such a coupler, the transition point that separates the regime in which a particular mode nearly conserves the energy from that, in which it experiences gain or loss is analytically identified. For



values of gain and loss below this point, the mode experiences a very low attenuation, negligible at the sub-millimeter propagation distances typical for integrated optical systems. Above this point, the mode experiences gain that grows rapidly with the increase of the magnitude of the imaginary part of permittivity.

2. Methods

The permittivity profile of the one-dimensional linear directional coupler studied here is given by

$$\epsilon(x) = \epsilon_1(x) + \epsilon_2(x) - \epsilon_B \quad (1)$$

and is shown in figure 1. Here, ϵ_B denotes the value of a relative background permittivity, and $\epsilon_1(x)$ and $\epsilon_2(x)$ are the complex permittivity profiles of the first waveguide and second waveguide, respectively. The opto-geometric parameters of the two waveguides are indicated in figure 1. One of the waveguides introduces loss to the system ($\epsilon_{1,I} < 0$), while the other one provides gain ($\epsilon_{2,I} > 0$).

In order to study the light propagation in the system described by equation (1), we solve the scalar wave equation for the electric field E :

$$\nabla^2 E(x, z) + k_0^2 \epsilon(x) E(x, z) = 0, \quad (2)$$

where $k_0 = 2\pi/\lambda$ denotes the free-space wavevector and λ is the free-space wavelength of light. The operator $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$ denotes the 2D Laplacian and the wave propagates in the z -direction. Equation (2) is obtained from Maxwell's equations under the assumption that both the structure and the field distributions are invariant along the y -direction.

First, we analyze equation (2) using the coupled mode theory [32]. In this case, the electric field in the coupler can be written as a sum of the fields of the individual waveguides $E(x, z) = \sum_{i=1}^2 \tilde{A}_i(z) \psi_i(x)$, where ψ_i denotes the transverse field distribution of the (fundamental) mode in the isolated waveguide described by the permittivity distribution $\Re[\epsilon_i(x)]$. The fast varying envelope of the corresponding mode is denoted by $\tilde{A}_i(z)$.

Using the slowly varying envelope approximation (SVEA) $\tilde{A}_i(z) = A_i(z) e^{-j\beta_i z}$ and neglecting small overlap terms, equation (2) can be transformed into coupled mode equations (CMEs) for a directional coupler with gain and loss:

$$j \frac{d\tilde{A}_i}{dz} = (\beta_i + C_i + j\gamma_i) \tilde{A}_i + \kappa_i \tilde{A}_{3-i}, \quad (3)$$

where $i \in \{1, 2\}$, and γ_i (γ_2) is the loss (gain) coefficient. C_i and κ_i denote the self-coupling and coupling coefficients, respectively. These coefficients are calculated as:

$$C_i = N_i \epsilon_{3-i,R} \langle \psi_i | \psi_i \rangle_{\text{WG}(3-i)}, \quad (4a)$$

$$\gamma_i = N_i \epsilon_{i,I} \langle \psi_i | \psi_i \rangle_{\text{WG}i}, \quad (4b)$$

$$\kappa_i = N_i \epsilon_{i,R} \langle \psi_{3-i} | \psi_i \rangle_{\text{WG}i}. \quad (4c)$$

Here $N_i = k_0^2 / [2\beta_i \int_{-\infty}^{\infty} \psi_i(x) \psi_i^*(x) dx]$, and $\langle f | g \rangle_{\text{WG}i} = \int_{\text{WG}i} f(x) g^*(x) dx$. The integral limits $\text{WG}i$ denote integration over the cross-section of the i th waveguide.

Dynamical properties of the dissimilar coupler can be found analytically from equation (3). To this end, we assume that the propagation constants of the two modes are nearly the same $\beta_1 \approx \beta_2 \approx \beta$, such that the solution can be written as $\tilde{\mathbf{A}} = \mathbf{A} e^{-j\beta z}$, where $\tilde{\mathbf{A}} = [\tilde{A}_1, \tilde{A}_2]^T$ and $\mathbf{A} = [A_1, A_2]^T$. This allows us to rewrite equation (3) in a matrix form $H\mathbf{A} = \beta\mathbf{A}$, where the Hamiltonian is given by

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \beta_1 + C_1 + j\gamma_1 & \kappa_1 \\ \kappa_2 & \beta_2 + C_2 + j\gamma_2 \end{pmatrix}. \quad (5)$$

The eigenvalues β correspond to the propagation constant of the coupler modes. Nontrivial solutions of equation (3) exist only if $\det(H - \beta\mathbf{I}) = 0$. This condition allows us to find the propagation constants of the coupler modes to be:

$$\beta_{\pm} = (h_{11} + h_{22} \pm \sqrt{(h_{11} + h_{22})^2 - 4 \det(H)})/2. \quad (6)$$

The dynamic properties of the studied coupler are determined by equation (6) and depend on the coupling strength, self-coupling coefficients, and gain and loss coefficients [35]. In the general case, the propagation constants given by equation (6) are complex. However, if the parameters of the waveguides are carefully chosen, the imaginary part of one of the propagation constants can be eliminated. Indeed, the condition for a purely real propagation constant can be obtained from equation (6):

$$\Im m(\beta_{\pm}) = [(\gamma_1 + \gamma_2) \pm q]/2 = 0, \quad (7)$$

where $q = \text{sgn}(b)/[(\sqrt{a^2 + b^2} - a)/2]^{1/2}$ and

$$a = \{(\beta_1 + \beta_2 + C_1 + C_2)^2 - (\gamma_1 + \gamma_2)^2\} + 4\kappa_1\kappa_2 - 4[(\beta_1 + C_1)(\beta_2 + C_2) - \gamma_1\gamma_2], \quad (8a)$$

$$b = 2(\beta_1 + \beta_2 + C_1 + C_2)(\gamma_1 + \gamma_2) - 4[(\beta_1 + C_1)\gamma_2 + (\beta_2 + C_2)\gamma_1]. \quad (8b)$$

The choice of the + (−) sign in equation (7) corresponds to the lossless fundamental (second-order) mode. The system can be optimized for only one mode that conserves its energy during the propagation at a time. Here, we optimize the system in such a way that the fundamental mode is lossless, and therefore we choose the + sign in all further computations.

For fixed geometry and real permittivity profile of the coupler, equations (7) and (4b) yield a relation between gain $\epsilon_{2,I}$ and loss $\epsilon_{1,I}$ for which we obtain a mode with a real propagation constant in a non- \mathcal{PT} -symmetric system built of dissimilar couplers. Below, we present examples of such systems and compare their behavior to \mathcal{PT} -symmetric couplers with similar parameters.

3. Results

As mentioned in the abstract, fabrication of a coupler strictly fulfilling the \mathcal{PT} -symmetric condition is challenging from the practical viewpoint. The symmetry can be broken by: (i) geometrical imperfections leading to waveguides with different thickness, (ii) dissimilar refractive index profiles resulting, for instance, from different doping agents or additional layers providing gain and loss, or (iii) a mixture of these effects. Usually these imperfections lead to small modifications of opto-geometric parameters of the coupler and can be easily corrected using the methods described in section 2. In order to show the applicability of our method in a general case, we study waveguides with simultaneous large perturbations of both geometric and optical parameters, and demonstrate the effectiveness of our method even in this overemphasized case.

All the couplers studied here can be described by three different properties. Despite the fact that the couplers are built of waveguides with different profiles of real part of permittivity, they can have either different or equal propagation constants β_1 and β_2 . Additionally, the super-mode with the real propagation constant can be located below or above the avoided exceptional point, leading to different propagation dynamics. Finally, the ratio between the gain and loss amplitudes $M = \epsilon_{2,I}/|\epsilon_{1,I}|$ (called thereafter the adjustment factor) can be equal to or different than 1. In this section, we present a few examples of dissimilar directional couplers characterized by different combinations of the properties described above.

3.1. Different propagation constants $\beta_1 \neq \beta_2$ and lossless mode below the avoided exceptional point

The parameters of our dissimilar coupler are the following (see figure 1 for definitions): $\epsilon_B = 3$, $\epsilon_{1,R} = 0.25$, $\epsilon_{1,I} = -0.02$, $D = 0.7 \mu\text{m}$, $\epsilon_{2,R} = 0.35$, $d = 0.5 \mu\text{m}$ and the separation between waveguides is $L = 0.2 \mu\text{m}$. At the wavelength of light used ($\lambda = 0.8 \mu\text{m}$) both waveguides support only one mode. To complete the design, the value of gain has to be chosen according to equation (7). This results in $\epsilon_{2,I} = -M\epsilon_{1,I}$, where M denotes an adjustment factor we introduce in order to balance the effective gain and loss in the system and achieve a real propagation constant of the fundamental mode. This analytical CME-based approach yields $M = 0.4729$.

In order to confirm the validity of the analytical approach, we study the modal properties of our dissimilar coupler using a numerical finite difference (FD) method in which the modes are found by solving a z -independent Helmholtz equation obtained from equation (2). This method allows for an alternative and more physically intuitive way of finding the adjustment factor. In this approach, we require the product of the absolute value of the imaginary part of the permittivity and the power propagating in each of the waveguides to be the same, which results in

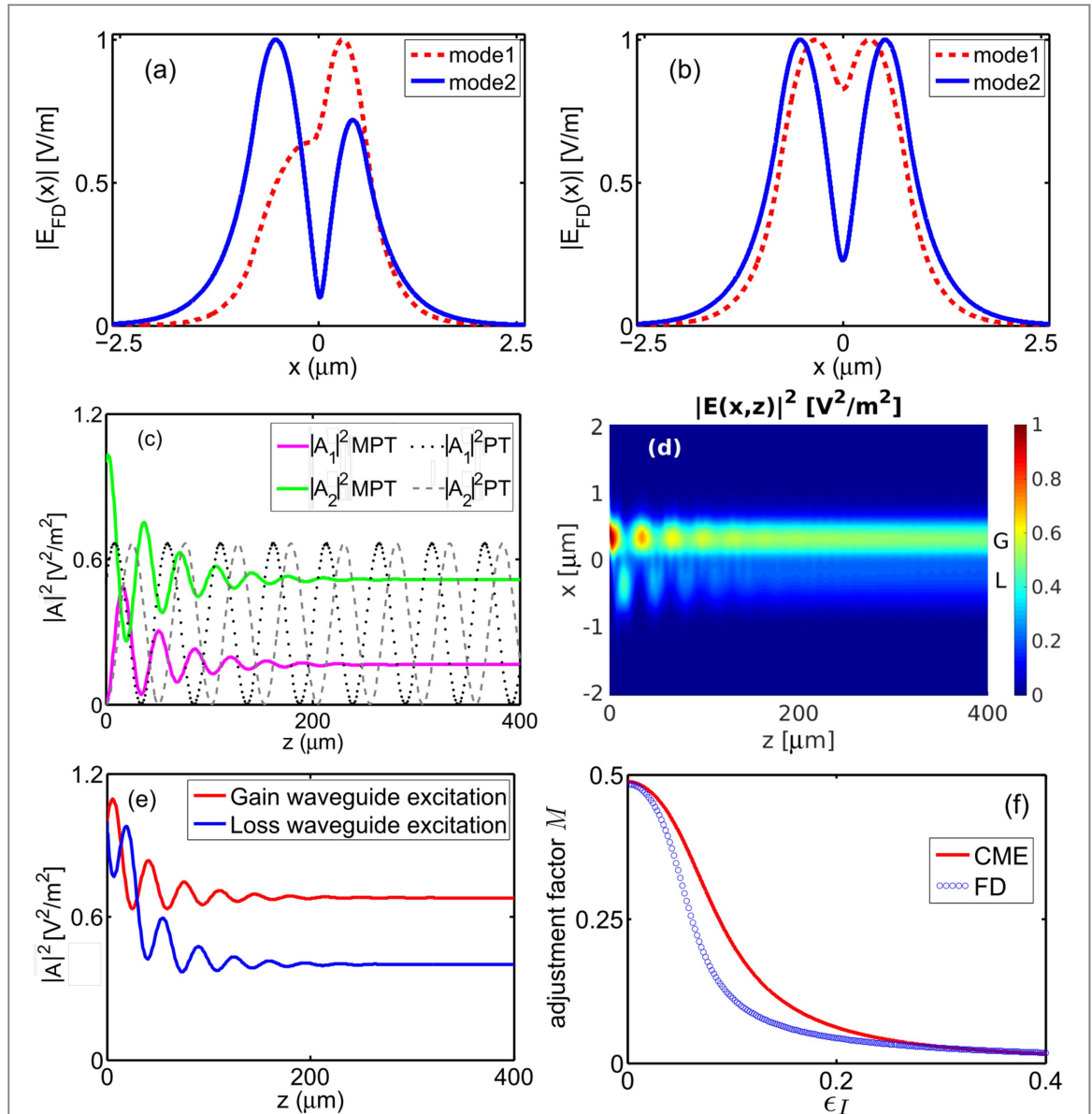
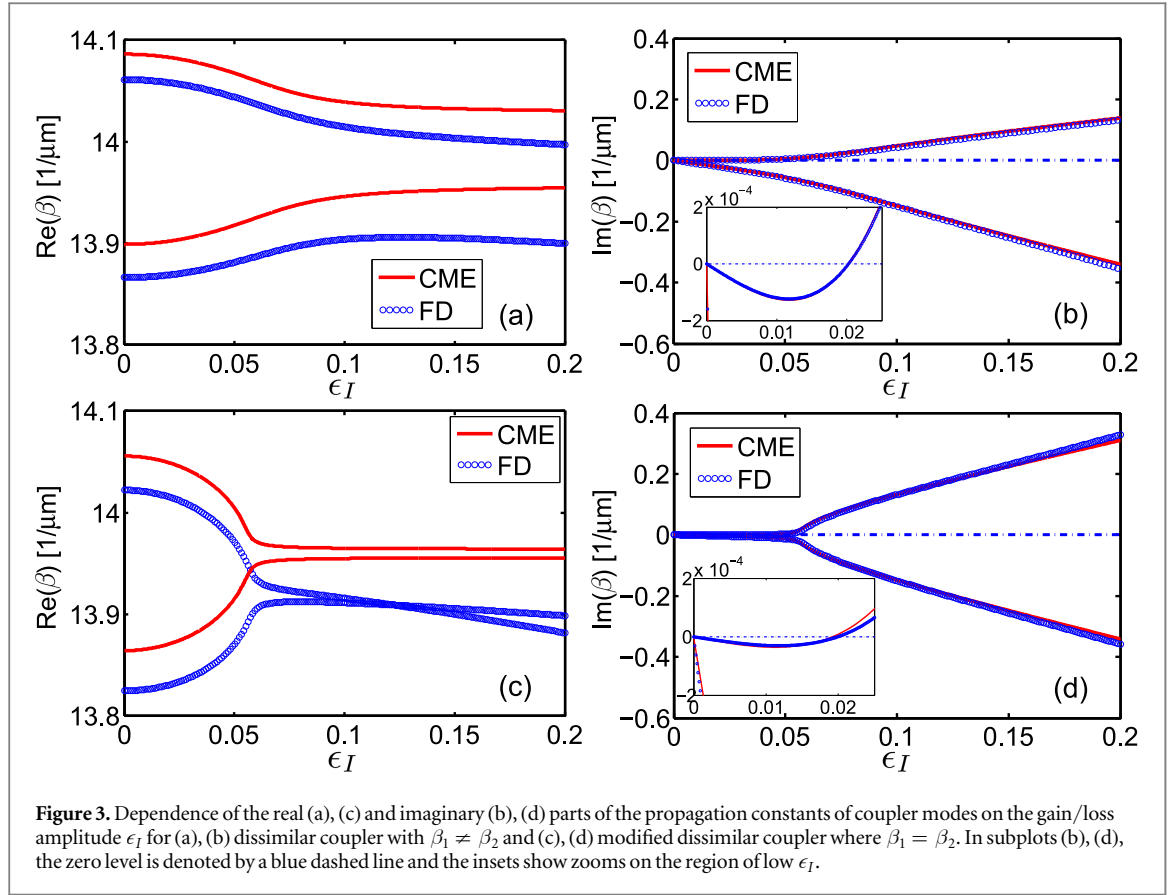


Figure 2. (a), (b) Mode profiles in (a) the dissimilar coupler system and (b) the \mathcal{PT} -symmetric system calculated by using the FD method (red dashed curve—fundamental mode and blue solid curve—second order mode). (c) Evolution of the slowly varying amplitudes $|A_1|^2$ (loss waveguide) and $|A_2|^2$ (gain waveguide) in the dissimilar coupler (solid color curves) and the \mathcal{PT} -symmetric coupler (gray curves) computed using equation (3). (d) Evolution of the electric field $|E(x, z)|^2$ obtained from FFT-BPM (G and L denote the gain and loss waveguides, respectively) in the dissimilar coupler. (e) Sum of the slowly varying intensities $|A_1|^2 + |A_2|^2$ in the dissimilar coupler for gain or loss waveguide excitation. (f) Relation between the adjustment factor M and ϵ_I under the condition of real propagation constants of the fundamental mode.

$$M_{\text{FD}} = \frac{|\epsilon_{2,I}|}{|\epsilon_{1,I}|} = \frac{\int_{\text{WG1}} |E_{\text{FD}}(x)|^2 dx}{\int_{\text{WG2}} |E_{\text{FD}}(x)|^2 dx}, \quad (9)$$

where $E_{\text{FD}}(x)$ denotes the electric field distribution of the coupler mode found using the FD method and presented in figure 2(a). Calculations using this approach result in $M_{\text{FD}} = 0.4703$ for the fundamental mode, which is in good agreement with the result obtained using the analytical method. In a conventional \mathcal{PT} -symmetric coupler, the adjustment factor $M = 1$, due to the symmetry of the system (see mode profiles in figure 2(b)). Nevertheless, the adjustment factor $M = 1$ does not imply that the system is \mathcal{PT} -symmetric, as it can be seen in section 3.3. In the following, all the results are obtained using the CME method and FD method. They are generally in a very good agreement; and therefore, we will only comment on the origin of any differences observed in the results.

For the optimized parameters of the coupler, it supports two modes shown in figure 2(a): (i) the fundamental mode with the propagation constant $\beta_+ = 14.08 \mu\text{m}^{-1}$ and (ii) the second-order mode with $\beta_- = (13.90 - j0.02) \mu\text{m}^{-1}$. As intended by the design, the fundamental mode has a real propagation constant,



while the second-order mode experiences loss. The lossless mode is located below the avoided exceptional point as it can be seen in figures 3(a) and (b).

Below, we focus our attentions on perfectly balanced systems with the optimal values of the adjustment parameter M leading to purely real propagation constant β_+ . Deviations from this optimal value of M result in an imperfect gain/loss balance leading to complex values of β_+ . Depending on the sign of $\Im m(\beta_+)$, the corresponding mode will experience loss or gain. The magnitude of M deviation from its optimal value will determine the rate of light intensity changes.

Let us now study the light propagation in a dissimilar coupler. The evolution of the slowly varying envelopes A_i is described by the solutions of equation (3). This solution can be found in an analytical form [36]:

$$A_1(z) = A_0 \left[\cos(\kappa z) + \frac{j\delta}{\kappa} \sin(\kappa z) \right], \quad (10a)$$

$$A_2(z) = j \frac{A_0 \kappa_2}{\kappa} \sin(\kappa z), \quad (10b)$$

where A_0 is the initial amplitude of the input in the first waveguide, and

$$\kappa = \sqrt{(\kappa_1 \kappa_2 + \delta^2)}, \quad (11a)$$

$$\delta = \frac{\beta_1 + C_1 + j\gamma_1 - (\beta_2 + C_2 + j\gamma_2)}{2}. \quad (11b)$$

We also solve equation (3) numerically, using the fourth-order Runge–Kutta algorithm [37], and obtain perfect agreement with the analytical solution. The evolution of the slowly varying envelopes A_i in our coupler is shown in figure 2(c). Here, we launch the light in the gain waveguide only. This excitation light overlaps with both the fundamental and the second-order coupler modes (shown in figure 2(a)) and excites them both. These modes propagate in the coupler and their interference results in oscillations of light intensity in the waveguides. The period of the interference pattern can be estimated as $2\pi/\Re e(\beta_+ - \beta_-) \approx 35 \mu\text{m}$. Similar oscillations are present in standard \mathcal{PT} -symmetric systems (here with parameters $\epsilon_{1,R} = \epsilon_{2,R} = 0.25$, $\epsilon_{1,I} = -\epsilon_{2,I} = -0.02$, and $D = d = 0.7 \mu\text{m}$) upon a single waveguide excitation (see gray curves in figure 2(c)). In the full \mathcal{PT} -symmetric regime, these oscillations continue throughout the entire propagation as both modes have real propagation constants. On the contrary, in the system of dissimilar couplers, one mode experiences loss and decays at a characteristic length of $L_D = 1/\Im m(\beta_-) \approx 50 \mu\text{m}$. After several L_D 's, this lossy mode vanishes and for $z > 200 \mu\text{m}$, we observe a stable propagation of the lossless fundamental mode, without the oscillations

induced by interference. The dynamical behavior of the system predicted from CMEs is confirmed by numerical solution of a $(1 + 1)D$ Schrödinger-like equation (obtained using the SVEA in equation (2)) using fast-Fourier transform beam-propagation method (FFT-BPM) [38] (see figure 2(d)).

A very similar behavior is obtained if the loss waveguide is excited instead of the gain one. The only difference is the lower total intensity reached after the stabilization phase, as shown in figure 2(e). After the stabilization the field remains confined in the fundamental mode, which means that the intensity ratio between the two waveguides is the same, regardless of which waveguide is excited.

Let us now investigate the ratio between the gain and loss $M = |\epsilon_{2,I}|/|\epsilon_{1,I}|$ required for the propagation constant of the fundamental mode to be purely real in the system of dissimilar couplers with a fixed optogeometric parameters but various amplitudes of the imaginary part of the permittivity ϵ_I ($\epsilon_{1,I} = -\epsilon_I$, $\epsilon_{2,I} = M\epsilon_I$). Figure 2(f) shows the dependence of adjustment factor M on ϵ_I calculated by CME method and the FD method. For the parameters chosen here, the maximum value of adjustment factor $M = 0.49$ is obtained when the imaginary part of permittivity approaches 0. With the increase of ϵ_I the adjustment factor M decreases rapidly. This means that the field distribution of the fundamental coupler mode becomes more asymmetric (more localized in the gain waveguide) and less gain is needed in order to support stable propagation of this mode. The differences between the results of the CME and the FD methods are caused by the fact that FD method, as opposed to the CME method, takes into account the imaginary parts of the permittivity in calculations of the mode profiles.

\mathcal{PT} -symmetric systems exist in two regimes depending on the amplitude of gain and loss ϵ_I : full \mathcal{PT} -symmetric regime for low values of ϵ_I and broken \mathcal{PT} -symmetric regime for ϵ_I above the threshold value corresponding to the exceptional point. With the increase of ϵ_I , the two real propagation constants of lossless modes of the full \mathcal{PT} -symmetric system become closer and coalesce at the exceptional point (see for example, figure 2 in [12]). Above this point two modes with complex conjugate propagation constants exist.

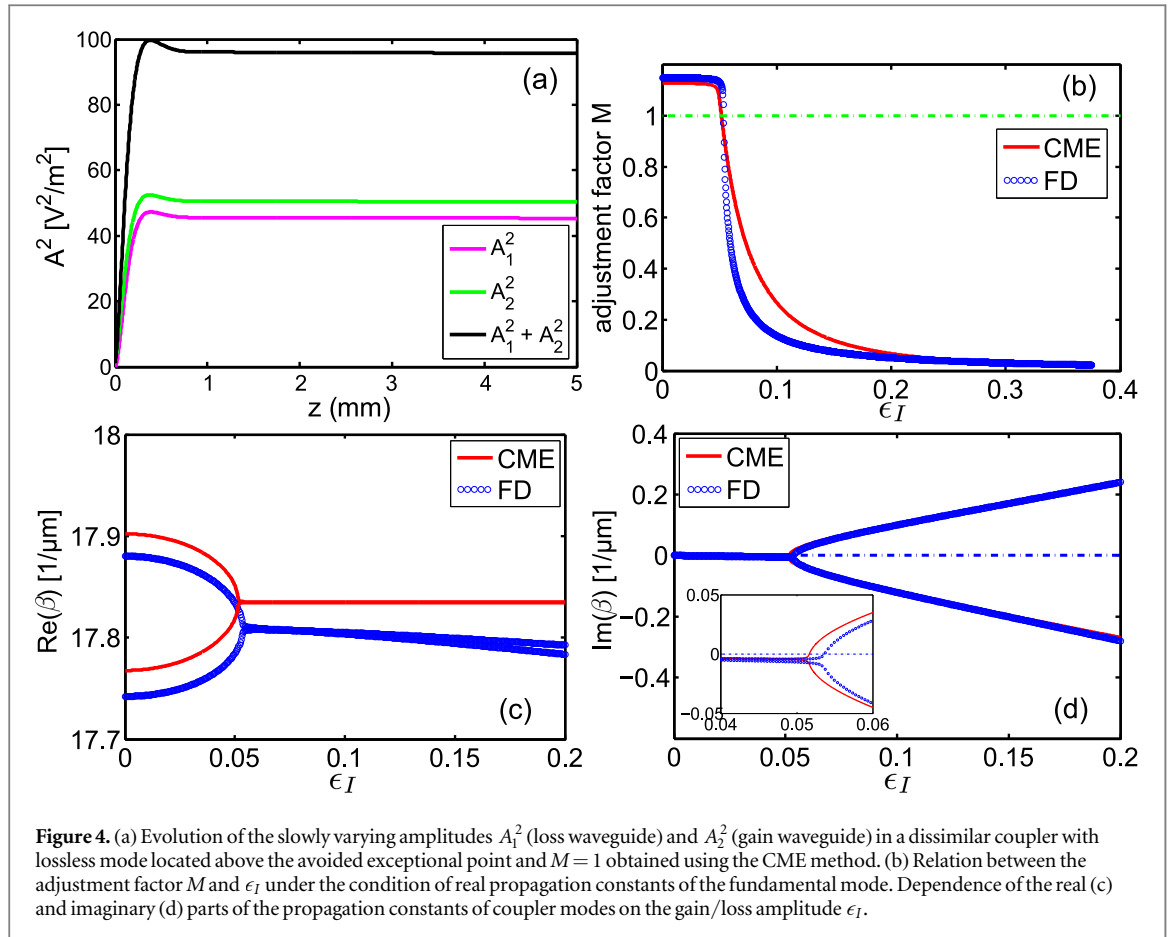
In the dissimilar couplers, the exceptional point cannot be clearly identified and an avoided exceptional point is present in such systems. Nevertheless, here we show that the value of gain and loss in a system of dissimilar couplers can be optimized in such a way that at least one of the modes of the system has a real propagation constant. This mode can be located below or above the avoided exceptional point depending on the coupler design. Figures 3(a) and (b) show the evolution of the propagation constants of the modes of our coupler as a function of gain/loss amplitude ϵ_I . The imaginary part of permittivity is given by $\epsilon_{1,I} = -\epsilon_I$, $\epsilon_{2,I} = M\epsilon_I$, and M chosen in such a way that $\Im m(\beta_+) = 0$ at $\epsilon_I = 0.02$.

Figure 3(b) shows that the imaginary part of the propagation constant of the fundamental mode $\Im m(\beta_+)$ remains small for $\epsilon_I < 0.05$. Above this value, $\Im m(\beta_+)$ starts to grow rapidly indicating that the mode experiences gain. As seen from the inset in figure 3(b) at $\epsilon_I = 0.02$ the propagation constant $\Im m(\beta_+) = 0$, as intended by the design. For $\epsilon_I < 0.02$, the fundamental mode experiences loss with the decay rate [39] $\alpha = 20\Im m(\beta_+)/\ln(10) \leq 0.4 \text{ dB mm}^{-1}$, which is negligible in integrated systems where the typical propagation length is of the order of hundreds of micrometers. The behavior of the propagation constant of the fundamental mode of a dissimilar coupler is similar to the one observed in \mathcal{PT} -symmetric systems. On the contrary, the second order mode for which the system was not optimized, experiences loss for all values of ϵ_I .

Similar to the \mathcal{PT} -symmetric system, the real parts of the propagation constants of a dissimilar coupler system become closer to each other with the increase of ϵ_I . Although here they do not merge into one curve, due to the asymmetry of the waveguides.

3.2. Equal propagation constants $\beta_1 = \beta_2$ and lossless mode below the avoided exceptional point

The system of dissimilar couplers discussed above was designed so that the modes of isolated waveguides building the coupler have different propagation constants $\beta_1 \neq \beta_2$. Here, we show that for dissimilar couplers ($\Re e[\epsilon(x)] \neq \Re e[\epsilon(-x)]$) build of waveguides where $\beta_1 = \beta_2$ the diagrams $\beta_{\pm}(\epsilon_I)$ resemble the standard diagrams for \mathcal{PT} -symmetric systems. Figures 3(c) and (d) show the $\beta_{\pm}(\epsilon_I)$ diagrams for the system with the width of the gain waveguide ($d = 0.401 \mu\text{m}$) chosen in such a way that $\beta_1 = \beta_2$. We observe that the real parts of the propagation constants are much closer than in the case of the original system of dissimilar couplers discussed above, even though they still do not merge to a single value as in the case of the \mathcal{PT} -symmetric systems. Moreover, the imaginary parts of the propagation constants of both coupler modes show behavior similar to a \mathcal{PT} -symmetric system, as they remains close to zero for $\epsilon_I < 0.05$. Nevertheless, as seen in the inset of figure 3(d), this system allows for lossless propagation of only one (fundamental) mode. The relative difference between the real parts of propagation constants computed using CME and FD methods is better than 1%; although, in modified system of dissimilar couplers with $\beta_1 = \beta_2$, we observe that the plots of real parts of the coupler mode propagation constants obtained using the FD method cross at $\epsilon_I \approx 0.12$. This interesting phenomenon will be discussed elsewhere.



3.3. Different propagation constants $\beta_1 \neq \beta_2$ and lossless mode above the avoided exceptional point

Finally, we design another system of dissimilar waveguides by modifying the optical parameters of the system described in section 3.1 while keeping the same geometrical parameters: $\epsilon_B = 5$, $\epsilon_{1,R} = 0.25$, $\epsilon_{1,I} = -\epsilon_{2,I} = -0.053$, $\epsilon_{2,R} = 0.3$. For these parameters, the modes of the two isolated waveguides have different propagation constants $\beta_1 \neq \beta_2$. However, in contrast to the previous designs, the mode with the real propagation constant is located above the avoided exceptional point, as it can be seen in the inset of figure 4(d). This fact has a crucial influence on the dynamics of the light propagation in the system. In the system studied in section 3.1, the lossless mode was located below the avoided exceptional point, where the difference in between the real parts of the propagation constants $\Re(\beta_+ - \beta_-)$ was relatively big leading to comparable values of the length at which modes exchange energy and the decay length L_D . In the case when the lossless mode is located above the avoided exceptional point, the situation is different. The propagation constants of the coupler modes are $\beta_+ = 17.8375 \mu\text{m}^{-1}$ and $\beta_- = (17.8322 - j0.0085) \mu\text{m}^{-1}$ leading to the energy exchange length $2\pi/\Re(\beta_+ - \beta_-) \approx 1.2 \text{ mm}$ and the decay length $L_D = 1/\Im(\beta_-) \approx 0.12 \text{ mm}$. Here, the decay length is 10 times shorter than the typical interference length between the modes. As a result, we observe only one oscillation in the propagation dynamics presented in figure 4(a).

The dynamics presented in figure 4(a) is interesting also from another point of view. In contrast to the case presented in section 3.1 where the propagation in the coupler resulted in loss of total light intensity, here we observe a 90-fold increase in the total amplitude of transmitted light.

The system of dissimilar couplers described in this section was designed to have a lossless mode for the value of the adjustment parameter $M=1$. From the definition of the adjustment parameter M given by equation (9), this requires the gain and loss coefficients to be equal: $\epsilon_{1,I} = -\epsilon_{2,I}$, and implies that the fractions of the energy of the fundamental mode propagating in each of the waveguides building the coupler are equal. The dependence of the adjustment parameter M on the loss and gain amplitudes ϵ_I for the system studied here is presented in figure 4(b). For $\epsilon_I \lesssim 0.05$, the value of M is almost constant. This means that for low values of ϵ_I the profile of the fundamental mode of the coupler remains unchanged. The rapid changes in mode profile for values of ϵ_I between 0.05 and 0.15 are reflected by changes in the adjustment factor M in this range of ϵ_I . Here, we have analyzed the behavior of the system of dissimilar couplers for which $M=1$. This shows that \mathcal{PT} -symmetry is related with the adjustment factor $M=1$ by logical implication (\mathcal{PT} -symmetry $\Rightarrow M=1$) and not by the logical equivalence (\mathcal{PT} -symmetry $\not\Rightarrow M=1$).

4. Conclusions

In conclusion, we have shown that dissimilar waveguide couplers, whose permittivity distribution does not fulfill the \mathcal{PT} condition, can support a mode with a real propagation constant. In such dissimilar non-Hermitian couplers, a proper choice of the ratio between gain and loss results in a stable mode propagation and energy conservation. Moreover, the ratio between the light intensity in the gain and loss waveguide at the output of the coupler does not depend on the type of the input field. Another feature distinguishing our dissimilar coupler systems from conventional \mathcal{PT} -symmetric couplers is the presence of a point corresponding to a single specifically chosen pair of gain and loss values where only one of the coupler modes has a real propagation constant. Below this point, the chosen mode experiences loss that is negligible at the typical propagation distances in integrated optics. For gain and loss values above this point, this mode experiences gain, similar to the \mathcal{PT} -symmetric system.

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