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14. ABSTRACT
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# RPPR Final Report

## as of 31-Jul-2018

Agency Code:

Proposal Number: 54107NS

Agreement Number: W911NF-09-1-0254

### INVESTIGATOR(S):

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**Report Date:** 30-Nov-2015

Date Received: 13-Jun-2018

**Final Report** for Period Beginning 12-May-2009 and Ending 31-Aug-2015

**Title:** Computational modelling of Cooperative Phenomena and Specific Heats in quasi-2D Vortex Filaments and Fully - 3D macroscopic flows - Research Areas 3.2 Computational Math and 3.5 Cooperative Systems

**Begin Performance Period:** 12-May-2009

**End Performance Period:** 31-Aug-2015

**Report Term:** 0-Other

Submitted By: Chjan Lim

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**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

**STEM Degrees:** 2

**STEM Participants:** 2

**Major Goals:** The major aims of this project are to first solve a simple equilibrium statistical mechanics model exactly for symmetry breaking phase transitions to super and sub rotating flows on rotating planets, and applications to solar system planetary atmospheres such as the enigmatic Venusian atmosphere, and second, to solve problems arising from social opinion dynamics on networks.

**Accomplishments:** Both major goals have been accomplished.

The first was done with a simple but physically realistic model of barotropic flows on a rotating sphere which is essentially a long range Kac's spherical model, and unlike previous expectations, was solvable exactly and found to support a phase transition to super rotation when the solid planet rotates slowly enough and the initial kinetic energy of the mixed barotropic flow is high enough (at negative temperatures).

The second was solved through the development of new mathematical methods for the exact solutions of social network models such as the nonequilibrium voter and naming game models. Such exact solutions are available only for the complete network but can be approximately extended to realistic social networks including small worlds and other clustered inhomogeneous networks. Our main results support a tipping point of around 9 percent of diehards for the minority opinion to dominate relatively quickly the majority opinion.

**Training Opportunities:** Nothing to Report

**Results Dissemination:** [79] W. Zhang, Chjan Lim, G. Korniss, B. Szymanski, "Opinion Dynamics and Social Influencing in Random Geometric Graphs", Scientific Reports 4, 5568 (2014) and doi: 10.1038/srep05568

[80] A. M. Thompson, B. Szymanski, C. Lim "Propensity and Stubbornness in the Naming Games - Tipping Fractions of Mavens", Phys Rev E 90, 042809, oct 15 2014

[81] W. Zhang and Chjan Lim, "Social Networks – a new preferential attachment procedure based on relevance and importance, submitted 2014.

[82] W. Pickering and C. Lim, "Solution of the Voter Models by Spectral Analysis", Physical Review E (Vol.91, No. 1):

DOI: 10.1103/PhysRevE.91.012812

<http://link.aps.org/doi/10.1103/PhysRevE.91.012812>

**RPPR Final Report**  
as of 31-Jul-2018

**Honors and Awards:** Nothing to Report

**Protocol Activity Status:**

**Technology Transfer:** Nothing to Report

**PARTICIPANTS:**

**Participant Type:** PD/PI

**Participant:** Chjan Lim

**Person Months Worked:** 15.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**Participant Type:** Graduate Student (research assistant)

**Participant:** Weituo Zhang

**Person Months Worked:** 15.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**ARTICLES:**

**Publication Type:** Journal Article

Peer Reviewed: Y

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**Journal:** Phys. Rev. E (accepted)

Publication Identifier Type:

Publication Identifier:

Volume: 84

Issue: 0

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Publication Location:

**Article Title:** Social consensus through the influence of committed minorities

**Authors:**

**Keywords:** tipping point, committed minority, naming games, consensus times

**Abstract:** The drive towards consensus in opinion or behavior in a social system is dramatically affected by the commitment and influenceability of individuals in the population, and the networks that bind them together. In this Letter, we show how the prevailing majority opinion in a population can be rapidly reversed by a small fraction  $p$  of randomly distributed agents who consistently proselytize the opposing opinion and are immune to influence. We call such agents committed agents. Specifically, we show that when the committed minority fraction grows beyond a critical value  $p_c \approx 0.0979$ , there is a super-exponential decrease in the time taken for the entire population to adopt the opinion of the committed set. We prove this by analyzing mean field equations for the particular model of influence dynamics considered here, and support our arguments using extensive simulations of the dynamics on complete graphs. Furthermore, using a quasi-stationary approximation, we show that for  $p < p_c$

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## RPPR Final Report as of 31-Jul-2018

**Publication Type:** Journal Article      Peer Reviewed: Y      **Publication Status:** 1-Published

**Journal:** INTERNET MATH

Publication Identifier Type:

Publication Identifier:

Volume: 0

Issue: 0

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Date Submitted:

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Publication Location:

**Article Title:** The Concentration and Stability of the Community Detecting Functions on Random Networks

**Authors:**

**Keywords:** Community Detection, Robustness of Algorithms, Large Deviation Principles, Spin-Glass

**Abstract:** We propose a general form of community detecting functions for finding the communities or the optimal partition of a random network, and examine the concentration and stability of the function values using the bounded difference martingale method. We derive LDP inequalities for both the general case and several specific community detecting functions: modularity, graph bipartitioning and q-Potts community structure. We also discuss the concentration and stability of community detecting functions on different types of random networks: the sparse and non-sparse networks and some examples such as ER and CL networks.

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**Journal:** Phys Review E

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Volume: 86

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Date Submitted:

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Publication Location:

**Article Title:** Analytic Treatment of Tipping Points for Social Consensus in Large Random Networks

**Authors:**

**Keywords:** Tipping points, committed minority, Naming games, large random networks

**Abstract:** We introduce a homogeneous pair approximation to the Naming Game (NG) model by deriving a six-dimensional ODE for the two-word Naming Game. Our ODE reveals the change in dynamical behavior of the Naming Game as a function of the average degree  $\langle k \rangle$  of an uncorrelated network. This result is in good agreement with the numerical results. We also analyze the extended NG model that allows for presence of committed nodes and show that there is a shift of the tipping point for social consensus in sparse networks.

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**Journal:** Scientific Reports

Publication Identifier Type: DOI

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Publication Location:

**Article Title:** Opinion Dynamics and Influencing on Random Geometric Graphs

**Authors:**

**Keywords:** Opinion Dynamics on Random Geographic Networks

**Abstract:** Naming game, the prototypical case of opinion dynamics, on 2-dimensional random geometric graph is investigated. Study of this model helps to understand the spatial distribution and propagation of social opinions. A main feature of this model is the automatic emergence of spatial structure called opinion domains which are geographic regimes sharing the same opinion with clear boundaries. We provide the mean field equation for this dynamics and discuss several properties of the equation such as the stationary solutions and two-time-scale separation. We find the governing rule of the opinion domain evolution that the opinion domain boundary propagates at the speed proportional to its inverse curvature. Finally we investigate the impact of committed agents on opinion domains and find the scaling of consensus time.

**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

Acknowledged Federal Support:



Nothing to report