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Circuit Models of Pyroelectric Charger

by Michael Grinfeld and Pavel Grinfeld

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14. ABSTRACT In the framework of electrical engineering notions, we suggest a model of a pyroelectric charger. The pyroelectric charger (also called pyroelectric sandwich) includes both metal conductors and a dielectric pyroelectric crystal. Depending on the scope of applications, the charger can be modeled on different levels of complexity, ranging from basic schematic models, adopted in electrical engineering, to extremely sophisticated models, incorporating contemporary condensed matter physics. In this report, we focus on the models adopted in electrical engineering. These models are based mostly on circuit theory and ordinary differential equations. We analyze two extreme regimes: the quasi-static regime, in which the polarization does not change, and the explosive regime, in which the polarization abruptly changes or completely disappears. More elaborate models, based on using partial differential equations, will be analyzed in future reports.					
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1. Introduction

Pyroelectricity is a group of phenomena known for millennia; its presentation can be found in a multitude of textbooks and monographs.¹⁻³ As indicated by the ancient Greek root name of these phenomena, they are characterized by the appearance of electrical events under the action of fire. During the long history of developments of the physical science and technology based on pyroelectricity, there appeared various methods of generating pyroelectric phenomena not directly associated with exposition of substance to open fire. We do not dwell here on a detailed discussion of multiple applications of pyroelectricity, since our interests at this stage are limited to electric charging of batteries through the use of shock waves or explosions. The corresponding chargers are not designed for repeated usage like the typical electric batteries and accumulators. They are designed to be used only once. This obvious drawback of explosive chargers has one important compensation: the process of charging is extremely quick. Sometimes, this feature can be of crucial importance. The analysis of such explosive chargers and, more generally, of the generators of electric power was pioneered more than a half-century ago in studies.^{4,5} A comprehensive review of more recent developments can be found in a monograph.⁶ A series of elegant experiments with shock wave-induced charging has been fulfilled at the US Army Combat Capabilities Development Command's (CCDC) Army Research Laboratory by Peter Bartkowski and Paul Berning.⁷ Very often pyroelectricity shows up in combination with other closely related phenomena (piezoelectricity, ferroelectricity, etc.). The corresponding theoretical literature counts hundreds of thousands of publications. Naturally, those theories face many difficulties, ranging from conceptual to rather superficial. As always, the fundamentals are poorly understood. Therefore, it does not make much sense to develop any comprehensive and therefore cumbersome theories, each chain of which is rather weak and questionable. To our understanding, CCDC Army Research Laboratory engineers prefer the standards of electrical engineering. That is what we try to suggest in this report. To do so, we decided not to touch the basics of ferroelectricity, which unavoidably would entail the discussions of phase transformations, thermodynamics, partial differential equations, and so forth.

2. What is a Pyroelectric Charger?

In fact, we intend to add to traditional electrical engineering only one element—the pyroelectric charger. The pyroelectric charger can be thought of as a capacitor with a special type of dielectric crystals inside it. The dielectric crystal inside the pyroelectric charger, however, differs quite essentially from the traditional capacitor of classical electrical engineering.⁸ In classical theory, the polarization inside the crystal vanishes when the exterior electric field vanishes. The essential peculiarity of the pyroelectric crystal is the presence of large enough macroscopic polarization $\mathbf{P}$ even in the absence of any exterior electric stimulus. Thus, nature itself supplied us with the free storage of a significant amount of electrostatic energy, associated with any pyroelectric crystal.

The central question appears: How is it possible to make the electrostatic energy of pyroelectric crystals usable in traditional devices?

To address this question, consider a uniform unbounded pyroelectric plate, as shown in Fig. 1. Its uniformity includes, among the traditional features, the uniform distribution of the natural polarization.

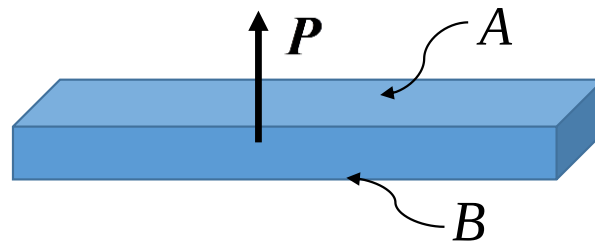


Fig. 1 An unbounded pyroelectric plate

Classical electrostatics claim that because of the uniformity of the plate and the distribution of the polarization inside it, the electrostatic field $\mathbf{E}$ 1) vanishes outside the crystal, and 2) remains uniform and vertically directed inside the crystal. If so, there is a free voltage U between the boundary points A and B .

This voltage opens the door to charge any traditional electric capacitor C or generate a pulse of current through any traditional resistor R (thus, converting the electrostatic energy of the pyrocrystal into the Joule heat). There is one obstacle, however, caused by the almost complete absence of free movable charges inside the dielectrics. Thus, we have to combine the pyroelectric crystal with metallic parts, abundant with the freely moving electrons. Technically, we can cover the pyrocrystal with the conducting foils, producing a capacitor-like sandwich, as shown in Fig. 2. Then, the voltage between the foils will be the same as the voltage between two opposite sides of the crystal. The role of the foils is very simple: they

supply the system with a multitude of free electrons that are absent in the pyrocrystal itself. Now, if we connect two opposite foils with a capacitor, then, under the action of the electric field, the free electrons will begin moving from one foil into one plate of the sandwich and, at the same time, the free electrons will be leaving the opposite plate of the sandwich and move inside the opposite foil. The case of using the resistor is even more straightforward: the free electrons of one of the foils will begin moving through the resistor to another foil, producing the electric current in the resistor. These processes will proceed until the potentials of the two foils become equal.



Fig. 2 Electric circuit with a pyroelectric charger (sandwich), capacitor, and resistor

Summarizing, the pyroelectric sandwich or charger permits the conversion of the electrostatic energy of the distributed dipoles to the electrostatic energy of the capacitor, or into Joule heat.

These processes include various details and secondary phenomena. When a very detailed description is required and multiple effects are taken into account, it makes sense to suggest more or less sophisticated models of theoretical physics. Such models are based on the formulation and analysis of the boundary value problems for systems of *partial* differential equations. However, a huge fraction of practical problems do not require such sophisticated models. In particular, many problems can be analyzed in a framework of the technically simpler models, known as circuit theory. Circuit theory is based on usage of the Kirchhoff systems (a.k.a., Kirchhoff laws), which reduces to the initial value problems for systems of *ordinary* differential equations.

The models, based on circuit theory, cannot provide exhaustive detail. However, they should not be treated as intellectually superficial. Researchers often confuse complex theories, using sophisticated mathematics, with deep theories. They believe that sophisticated theories appear *after* the simple theories become clear and exhaust their potential. In fact, with very rare (though important) exceptions, the situation is the opposite. The so-called simple theories are conceptually much deeper than the technically complex ones. The fundamentals of electrical engineering remain complex and mysterious: their interrelations with classical

electromagnetism include many sophistications, and the analysis for the same problems in the framework of theoretical and mathematical physics does not make those problems more clear. Quite the contrary, it makes those fundamental problems far more obscure. With these methodological thoughts in mind, we discuss in the next section how to model the circuits with pyroelectric sandwiches, using the language of electrical engineering.

3. Traditional Elements of Modeling in Electrical Engineering

To make sure that we are on the same page with the readers, who may have quite different specializations, we present our vision of the key elements of electrical engineering.

The circuit models of electrical engineering include four basic types of the elements: electric batteries, capacitors, resistors, and inductors. Each of these elements is characterized by one or two parameters. For instance, a capacitor is characterized by one constructive parameter C , called its electric capacity, and one state parameter, called capacitor's charge Q_C . Other characteristics—the voltage U_C and the current through it I_C —are calculated via C and Q_C by means of the equations of electrical engineering, reminiscent but not coinciding with the elements of classical electrodynamics. Needless to say, real-life capacitors cannot be characterized by the single parameter C . In fact, from the more general viewpoint, the parameter C should be treated as a placeholder, which should be replaced with much more sophisticated models when necessary.

The element of electrical engineering called the resistor is characterized by one constructive parameter called resistance R . Other characteristics like voltage U_R and the current through it I_R are connected with R by means of an equation of electrical engineering known as Ohm's law.

The battery in circuit theory is characterized by two constructive parameters: the magnitude E of electromotive driving force (EMF) and the internal resistance R_{EMF} . Other characteristics like voltage U_{EMF} and the current I_{EMF} are connected with the EMF E and the internal resistance via the equations of electrical engineering.

These equations are closely related with the basic principles of classical electrostatics. However, the distinctions between classical electrostatics and electrical engineering are also quite significant and can easily become sources of confusion or misunderstanding. In particular, we will not be surprised if our understanding of the interrelations between classical electrostatics (which is our

specialty) and electrical engineering (which is not our specialty) will differ essentially from the understanding of our readers. Therefore, in order to avoid confusion, it is even more important to explain our understanding of circuit theory and electrical engineering.

Let us give one illustration. When dealing with capacitors, it should be remembered that, contrary to the spatial charge distributions, any capacitor always carries a total charge equal to zero (therefore, we do not feel the electric field of a capacitor in its exterior).

Compare the central notion of electric charge in classical electrostatics on the one hand and in electrical engineering on the other. In classical electrostatics, when talking about a charge Q we mean the charge of a certain sign, positive or negative; even when using a “homogenized” description, we are talking about domains with the dominating amounts of charges of a certain sign. But when talking about a charge of the capacitor Q_C in the framework of electrical engineering, we have in mind something completely different. Namely, a capacitor, as a whole, is always electrically *neutral* and it contains the same amounts of positive and negative charges. Because of this central distinction, certain alterations are required in electrical engineering for the concepts of electric capacity, electric current, and so on.

When calculating the electric charges Q_C of the capacitors by solving the equations of electrical engineering, we get positive or negative results. However, this does not mean that there is an abundance of positive or negative charges in the capacitors. The net charge of the capacitor is always zero. But the sign of the charge in electrical engineering still plays a key role. It is because when formulating a system of the equations of electrical engineering we make an a priori choice of the plates carrying positive and negative charges. When, a posteriori, we get the answer for the Q_C with a positive sign, it is just the indication that our a priori guess was correct; when we get the negative value of Q_C , this is the indication that our a priori choice of the positively and negatively charged plates should be changed to the opposite. This change in the assigned signs does not imply the necessity of reformulating the master system of circuit theory and solving the corrected system from scratch. The same is true about the a priori chosen direction of the electric currents.

The main objects requiring calculation are voltages U , electric currents I , and charges Q , describing each of the elements composing the electric circuit under study. Usually, they are all functions of the time t . In these cases, we are talking

about transient or nonstationary processes; otherwise, we deal with stationary processes.

Quantitative analysis of electric circuits is based on Kirchhoff rules and the main relationships of electrical engineering. For resistors, the voltage $U(t)$, the electric current $I(t)$, and the resistance R are interconnected by Ohm's law

$$U_R(t) = RI_R(t) , \quad (1)$$

which is an algebraic relationship.

The capacitor's voltage $U_C(t)$ and its charge $Q_C(t)$ are connected with the capacitor's capacitance C by the relationship

$$U_C = \frac{Q_C}{C} , \quad (2)$$

which is also an algebraic relationship.

Although electric charges cannot move through the capacitor, it is convenient to talk about electric current moving "through the capacitor". Many quite sophisticated mathematicians, involved in mathematical physics, believe that there is a real current moving through capacitors. Of course, this has nothing in common with reality—perhaps they simply confuse the concept of a capacitor's breakdown with the concept of *apparent* current through the capacitor. By definition, the apparent current through capacitor $I_C(t)$ is given by the relationship

$$I_C(t) = \frac{dQ_C(t)}{dt} , \quad (3)$$

which is an ordinary differential equation. This is an electrical engineering surrogate of the equation of charge continuity in classical electrostatics.

By the way, when dealing with a spatial distribution of charges $Q(x, y, z, t)$, the electric current becomes a vector with three components: $I_x(x, y, z, t)$, $I_y(x, y, z, t)$, and $I_z(x, y, z, t)$. For the temporal change of the charge, we get the following differential equation:

$$\frac{\partial Q}{\partial t} = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} . \quad (4)$$

Basically, Eq. 3 plays the same role as Eq. 4. There is one typical confusion though, caused by using the same notation Q for the charge density in the spatial distributions of charges, on the one hand, and for the charges in capacitors, on the other hand.

The third canonical element of electrical engineering is the inductor or inductance. The voltage $U_L(t)$ across the inductance is connected with the current $I_L(t)$ through it by means of the ordinary differential equation

$$U_L(t) = L \frac{dI_L(t)}{dt}, \quad (5)$$

where L is the positive constant, called the inductance (or the coil inductance).

At last, the fourth canonical element of electrical engineering is the electromotive driving force of EMF. The EMF E is connected with the interior resistance R_{EMF} , the current I_{EMF} , and the voltage U_{EMF} via the following relationship of electrical engineering:

$$E = U_{EMF} + I_{EMF}R_{IMF}. \quad (6)$$

4. Modeling the Pyroelectric Charger

In this report, we introduce one more element to the circuit modeling in the framework of electrical engineering, called the pyroelectric charger. Of course, the realistic pyroelectric conductor may require far more elaborate modeling and formal instruments than those accepted in electrical engineering.

For the pyroelectric charger, we use the logo shown in Fig. 3.

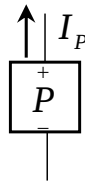


Fig. 3 The notation for the pyroelectric charger

By definition, the pyroelectric charger is characterized by the material parameter γ_p , called the capacitance of the pyroelectric charger. The state of the pyroelectric charger is characterized by two numbers: its polarization P and the accumulated charge Q_p . Like for any other element of electric circuit, we now have to formulate

how to calculate the current I_p through the pyroelectric charger and how to calculate the voltage U_p imposed on the pyroelectric charger.

For the current I_p , we postulate the same relationship between $I_p(t)$ and $Q_p(t)$ as for the traditional capacitor

$$I_p(t) = \frac{dQ_p(t)}{dt}, \quad (7)$$

In what concerns the voltage $U_p(t)$, we postulate the following relationship

$$U_p(t) = \frac{P + Q_p(t)}{\gamma_p}, \quad (8)$$

connecting the capacitance, the polarization, and the charge.

In the simplest situation, the polarization P and the capacitance γ_p are certain positive constants. However, the polarization P typically depends on the temperature, sometimes dramatically. In particular, it vanishes completely when the temperature exceeds the critical value T_c , called the Curie temperature. Also, under the action of a sufficiently intensive shock wave, the polarization can disappear and even change to the opposite orientation. This phenomenon was analyzed in several publications.^{4,5,7}

5. The Simplest Circuits with Pyroelectric Chargers

In the following sections, we illustrate how to analyze electric circuits with pyroelectric charges in the framework of generalized electrical engineering.

5.1 How to Charge the Pyroelectric Charger Itself?

Assume that the charger with the polarization P is originally disconnected from any elements and it carries the charge Q° . Let us check, using our model, what happens if we attach the plates of the charger to the exterior resistor R , as shown in Fig. 4.

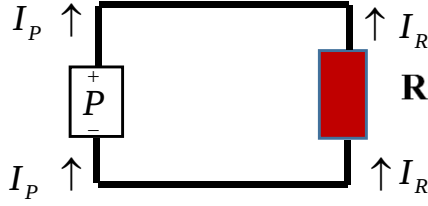


Fig. 4 A circuit for charging the pyroelectric charger

We choose a priori the directions of the currents through the charger I_P and the resistor I_R , as shown in Fig. 4. It is obvious that currents of the same magnitude and direction move through both elements. But the Kirchhoff methodology permits us to choose a priori those currents through the circuit's elements, with their directions and magnitudes, arbitrarily. This is very convenient since, more often than not, intuition is not helpful in making proper choices when dealing with complex circuits. Let us deliberately make the anti-intuitive choice, as shown in Fig. 4.

Now, the Kirchhoff methodology automatically leads us to the equations for 1) the currents

$$I_P + I_R = 0 \quad (9)$$

and 2) the voltages

$$U_P - U_R = 0 , \quad (10)$$

on the charger U_P and the resistor U_R .

The first of the Kirchhoff laws, Eq. 9, automatically shows us that the currents have opposite signs; thus, a posteriori, one of the chosen direction should be eventually changed to the opposite.

Ohm's law implies for the resistor

$$U_R = RI_R . \quad (11)$$

Combining the relationships between Eq. 7, Eq. 8, and Eqs. 9–11, we arrive at the following ordinary differential equation

$$\frac{dQ_P}{dt} + \frac{1}{R\gamma} Q_P = -\frac{1}{R\gamma} P , \quad (12)$$

for the charge of the pyroelectric charger.

The differential Eq. 12 should be amended with the so-called initial data

$$Q_p(0) = Q^\circ. \quad (13)$$

The ordinary differential Eq. 12 with the initial data (Eq. 13) has the following solution:

$$Q(t) = Q_0 e^{-\frac{1}{RC}t} - P \left(1 - e^{-\frac{1}{RC}t}\right). \quad (14)$$

At $t \rightarrow \infty$ the charge $Q_p(t)$ approaches its saturation value Q_p^∞ ,

$$Q_p^\infty = -P. \quad (15)$$

We see that the saturation value Q_p^∞ of the charger depends only on the polarization P and not on the initial charge Q° . In particular, the initial charge $|Q^\circ|$ can be either smaller or bigger than the saturation charge $|Q_p^\infty|$.

5.2 Charging an Exterior Capacitor

Let Q° be the initial charge of the pyroelectric charger with the polarization P . Let us use it for charging the initially uncharged capacitor C , as shown in Fig. 5. We choose the a priori currents' directions as shown in Fig. 5.

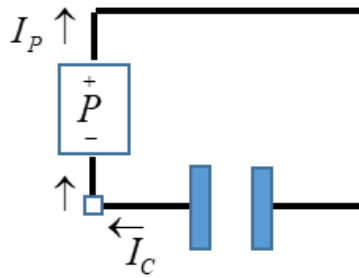


Fig. 5 A circuit for charging a capacitor with the pyroelectric charger

For the balance of the currents in the knot we get

$$I_p - I_C = 0. \quad (16)$$

The voltage balance implies

$$U_p + U_C = 0. \quad (17)$$

The charge conservation equations for the charger and the capacitor read

$$I_p = \frac{dQ_p}{dt} \quad (18)$$

and

$$I_c = \frac{dQ_c}{dt} , \quad (19)$$

respectively.

The “constitutive” equation for the charger reads

$$U_p = \frac{1}{\gamma} (P + Q_p) . \quad (20)$$

The “constitutive” equation for the capacitor reads

$$U_c = \frac{Q_c}{C} . \quad (21)$$

Using Eqs. 18 and 19, we eliminate the currents from the system, rewriting Eq. 16 as

$$\frac{dQ_p}{dt} - \frac{dQ_c}{dt} = 0 . \quad (22)$$

The “constitutive” Eqs. 20 and 21 allow us to eliminate the voltages U_p and U_c from Eq. 17

$$\frac{1}{\gamma} (P + Q_p) + \frac{Q_c}{C} = 0 , \quad (23)$$

which can be rewritten as

$$\frac{1}{\gamma} Q_p + \frac{1}{C} Q_c = -\frac{1}{\gamma} P . \quad (24)$$

The differential Eq. 22 implies

$$Q_p - Q_c = Q_{tot} , \quad (25)$$

where the constant Q_{tot} has the physical meaning of the total charge of the charger and the capacitors.

Solving the system of Eqs. 24 and 25, we get

$$Q_C = -\frac{C}{C+\gamma}(P+Q_{tot}), \quad Q_P = \frac{\gamma}{C+\gamma}Q_{tot} - \frac{C}{C+\gamma}P, \quad (26)$$

as implied by the following chain:

$$\begin{aligned} \frac{1}{\gamma}(Q_{tot} + Q_C) + \frac{1}{C}Q_C &= -\frac{1}{\gamma}P \\ \left(\frac{1}{C} + \frac{1}{\gamma}\right)Q_C &= -\frac{1}{\gamma}(P + Q_{tot}), \quad Q_C = -\frac{C}{C+\gamma}(P + Q_{tot}) \\ Q_P = Q_{tot} + Q_C &= Q_{tot} - \frac{C}{C+\gamma}(P + Q_{tot}) = Q_{tot} \frac{\gamma}{C+\gamma} - \frac{C}{C+\gamma}P \end{aligned}$$

Let us analyze the final result (Eq. 26) of our analysis. Consider a special case, when originally the charger is fully saturated and the exterior capacitor is originally free of charge, that is, when

$$Q_P(0) = -P, \quad Q_C(0) = 0. \quad (27)$$

Then, for the total charge Q_{tot} , we get

$$Q_{tot} = Q_P(0) + Q_C(0) = -P. \quad (28)$$

Inserting Eq. 28 in our final relationships, Eq. 26, we get

$$Q_C(t) = 0, \quad Q_P(t) = -P. \quad (29)$$

Per the relationships (Eq. 29), when the charger is fully saturated whereas the exterior capacitor lacks the original charge, the charger will not charge the originally empty capacitor—this situation, contradicting the analysis of the traditional capacitors, seems very natural when dealing with pyroelectric chargers; this fact confirms the consistence and usefulness of the suggested model.

With all other values of the initial charges of the charger and capacitor, per Eq. 26, there is a redistribution of charges between the charger and the exterior capacitor.

Based on Eq. 26, one can conclude that the final distribution of the charges does not depend upon the initial charges of the charger and the capacitor individually, but only upon the total charge Q_{tot} of both. Also, our model predicts only the final distribution of the charges but says nothing about the transient processes. These obvious shortcomings can be eliminated by taking into account the resistance and the inductance of the charging circuit, as shown in Section 6.

6. Explosive Pyroelectric Charger

Let us come back to the charging circuit shown in Fig. 5. Assume that the pyroelectric charger was initially fully charged so that

$$Q_p^\circ = Q_p^\infty = -P . \quad (30)$$

At $t = 0$, let the pyroelectric charger get exposed to an intensive shock load or an extensive heat wave. Assume that the pyroelectric charger loses its polarization P . Then, part of the charges will move from the pyroelectric charger to the capacitor C . The final charges Q_C and Q_p can still be calculated with the help of Eq. 26, in which we have to first put

$$P = 0, \quad Q^\circ = -P . \quad (31)$$

Then, we get

$$Q_C = \frac{CQ^\circ}{C + \gamma}, \quad Q_p = \frac{Q^\circ \gamma}{C + \gamma} . \quad (32)$$

The relationships (Eq. 32) are valid for any initial charge Q° of the pyroelectric charger.

If the charger is fully saturated, we have to put in Eq. 32 the condition of the saturation $Q^\circ = -P$. Then, we get the required relationships

$$Q_C = -\frac{CP}{C + \gamma}, \quad Q_p = -\frac{\gamma P}{C + \gamma} . \quad (33)$$

Since neither resistance R nor inductance L are taken into account, our analysis does not include any differential equations. To get a more detailed description, these elements should be taken into account. For instance, we can consider the chain, shown in Fig. 6.

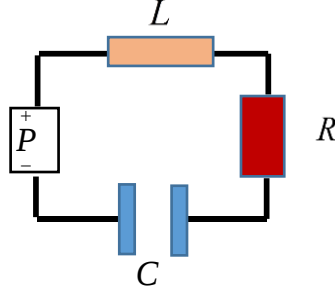


Fig. 6 The combined circuit for the pyroelectric charger with the inductance, resistor, and capacitor

The analysis of the circuit leads to the following second order ordinary differential equation for the charge $Q_C(t)$ of the capacitor

$$\frac{d^2 Q_C}{dt^2} + \frac{R}{L} \frac{dQ_C}{dt} + \frac{1}{L} \left(\frac{1}{C} + \frac{1}{\gamma} \right) Q_C = -\frac{P + Q_{tot}}{L\gamma}, \quad (34)$$

where $Q_{tot} = Q_p(0) + Q_C(0)$ is the total charge of the charger and the capacitor.

Equation 34 has a “particular” solution $Q_C(t) = Q_C^{part} = const$ such that

$$Q_C^{part} = -\frac{C}{C + \gamma} (P + Q_{tot}). \quad (35)$$

The solution carries a simple physical meaning. It gives the final value of the charge of the capacitor C at t approaching infinity.

In the regime of explosive charging, when $P = 0$ at $t > 0$, the final charge is equal to

$$Q_C^{part} = -\frac{C Q_{tot}}{C + \gamma}. \quad (36)$$

In particular, if in the initial moment the pyroelectric charger was fully charged, that is, $Q_p(0) = -P$, whereas the exterior capacitor was completely uncharged, that is, $Q_C(0) = 0$, we get $Q_{tot} = -P$.

Then, Eq. 36 implies

$$Q_C^{part} = \frac{CP}{C + \gamma}, \quad (37)$$

When we are interested in the transient process, accompanying the explosive charging, we have to use the general solution of the second order ordinary differential equation (Eq. 34). There are two different convenient forms of the general solution, depending on the validity of the inequalities

$$\text{a) } \frac{4L}{R^2} \left(\frac{1}{C} + \frac{1}{\gamma} \right) < 1 \quad \text{or} \quad \text{b) } \frac{4L}{R^2} \left(\frac{1}{C} + \frac{1}{\gamma} \right) > 1. \quad (38)$$

(The “convenience” here means that no complex numbers appear in the solution.)

In the case a), the convenient form of the general solution reads

$$Q_C^{gen}(t) = Q_C^{part} + A_+ e^{-\frac{R}{2L}(1-\Delta)t} + A_- e^{-\frac{R}{2L}(1+\Delta)t}, \quad (39)$$

where A_+ and A_- are the integration constants, and Δ is defined as

$$\Delta \equiv \sqrt{1 - \frac{4L}{R^2} \left(\frac{1}{C} + \frac{1}{\gamma} \right)}. \quad (40)$$

In the case b), the convenient form of the general solution reads

$$Q_C^{gen}(t) = Q_C^{part} + A_s e^{-\frac{R}{2L}t} \sin \frac{R\Delta^* t}{2L} + A_c e^{-\frac{R}{2L}t} \cos \frac{R\Delta^* t}{2L}, \quad (41)$$

where A_s and A_c are the arbitrary constants and Δ^* is defined as

$$\Delta^* \equiv \sqrt{\frac{4L}{R^2} \left(\frac{1}{C} + \frac{1}{\gamma} \right) - 1}. \quad (42)$$

Roughly speaking, the choice of the convenient form is essentially connected with the induction L : when the induction is very small, the form shown in Eq. 39 is more convenient; when the conductance is very large, the form shown in Eq. 41 is more convenient. We shall realize, though, that even a small induction plays a crucial role in the process of explosive charging. Formally, it is reflected in the fact that the inductance L appears in the denominators in Eq. 43 (or as a coefficient of the highest derivative if we multiply the terms by L); thus, the assumption $L = 0$ qualitatively changes the properties of this master equation.

Usually, the initial data are used to determine the arbitrary constants. Two initial items of data are required when dealing with a second order ordinary differential equation. When considering the shock-induced charging, it is natural to assume

that, at $t = 0$, the pyroelectric charger is saturated: $Q_p(0) = -P$, whereas the capacitor possesses no charge at all: $Q_C(0) = 0$. Then, the total charge in the system is equal to $Q_{tot} = Q_p(0) = -P$.

Since we are considering explosive charging, which eliminates the polarization P of the charger at $t > 0$, Eq. 34 should be replaced with the following one:

$$\frac{d^2 Q_C}{dt^2} + \frac{R}{L} \frac{dQ_C}{dt} + \frac{1}{L} \left(\frac{1}{C} + \frac{1}{\gamma} \right) Q_C = -\frac{Q_{tot}}{L\gamma}. \quad (43)$$

Now, in view of the initial data, we get $Q_{tot} = Q_p(0) = -P$, and the relationship (Eq. 43) should be replaced with the following one:

$$\frac{d^2 Q_C}{dt^2} + \frac{R}{L} \frac{dQ_C}{dt} + \frac{1}{L} \left(\frac{1}{C} + \frac{1}{\gamma} \right) Q_C = \frac{P}{L\gamma}. \quad (44)$$

We dwelled on the detailed derivation of Eq. 44 because it possesses an apparent inconsistency. Namely, it contains the polarization P on the right-hand side, although the polarization P vanishes at $t > 0$. Therefore, some concentration is necessary in order to avoid confusion.

The particular solution of Eq. 44 reads

$$Q_C^{part} = \frac{CP}{C + \gamma}. \quad (45)$$

Consider the case when the induction L is very small but does not vanish. Using the general solution in Eq. 39 and the relationship in Eq. 45, we get

$$Q_C^{gen}(t) = A_+ e^{\frac{R}{2L}(1-\Delta)t} + A_- e^{\frac{R}{2L}(1+\Delta)t} + \frac{CP}{C + \gamma}. \quad (46)$$

Further, assume that the initial charge of the capacitor C vanishes; the relationship $Q_C(0) = 0$ gives us the following relationship, connecting the arbitrary constants:

$$A_+ + A_- = -\frac{CP}{C + \gamma}. \quad (47)$$

Further, we assume that not only the charge $Q_C(t)$ but also the current $I_C(t)$ through the capacitor vanishes at $t = 0$. This assumption gives us the required additional relation for determination of the arbitrary constants; namely, we get

$$(1-\Delta)A_+ + (1+\Delta)A_- = 0 . \quad (48)$$

Solving the system (Eqs. 47 and 48), we get

$$A_+ = -\frac{1+\Delta}{2\Delta} \frac{CP}{C+\gamma}, \quad A_- = \frac{1-\Delta}{2\Delta} \frac{CP}{C+\gamma} . \quad (49)$$

Inserting Eq. 49 in Eq. 46, we arrive at the following solution of our problem

$$Q_C(t) = \frac{CP}{C+\gamma} \left(1 - \frac{1+\Delta}{2\Delta} e^{-\frac{R}{2L}(1-\Delta)t} + \frac{1-\Delta}{2\Delta} e^{-\frac{R}{2L}(1+\Delta)t} \right) . \quad (50)$$

Differentiating Eq. 50, we arrive at the following formula of the current $I_C(t)$:

$$I_C(t) = \frac{CP}{C+\gamma} \frac{R}{L} \frac{1-\Delta^2}{\Delta} e^{-\frac{R}{2L}t} \sinh \frac{R\Delta t}{2L} . \quad (51)$$

Using the relationship shown in Eq. 40, we can rewrite Eq. 51 as

$$I_C(t) = \frac{4}{R\gamma} \frac{P}{\Delta} e^{-\frac{R}{2L}t} \sinh \frac{R\Delta t}{2L} , \quad (52)$$

or else

$$I_C(t) = \frac{2P}{R\gamma} \frac{1}{\sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}} \left(e^{-\left(1 - \sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}\right) \frac{Rt}{2L}} - e^{-\left(1 + \sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}\right) \frac{Rt}{2L}} \right) . \quad (53)$$

The current $I_C(t)$ assumes its extremum at $t = t_{crit}$ such that

$$\tanh \frac{R\Delta t_{crit}}{2L} = \Delta . \quad (54)$$

After calculating the $t = t_{crit}$, the corresponding extremum current I_C^{crit} can be found from the formula

$$I_C^{crit}(t) = \frac{2P}{R\gamma} \frac{1}{\sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}} \left(e^{-\left(1 - \sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}\right) \frac{Rt_{crit}}{2L}} - e^{-\left(1 + \sqrt{1 - \frac{4L}{R^2} \frac{C+\gamma}{C\gamma}}\right) \frac{Rt_{crit}}{2L}} \right) . \quad (55)$$

For small inductance L , we can approximate the relationship in Eq. 53 by the following one:

$$I_C(t) = \frac{2P}{R\gamma} \left(e^{-\frac{C^{-1} + \gamma^{-1}}{R}t} - e^{-\frac{R}{L}t} \right). \quad (56)$$

In this approximation, the equation for the t_{crit} reads

$$e^{\frac{R^2 - C^{-1} - \gamma^{-1}}{RL}t_{crit}} = \frac{R^2}{(C^{-1} + \gamma^{-1})L}, \quad (57)$$

and it implies the following value of t_{crit}

$$t_{crit} = \frac{RL}{R^2 - C^{-1} - \gamma^{-1}} \ln \frac{R^2}{(C^{-1} + \gamma^{-1})L}. \quad (58)$$

Thus, for the extremum current I_C^{crit} we get the following expression:

$$I_C^{crit} = \frac{2P}{R\gamma} \left\{ \left[\frac{(C^{-1} + \gamma^{-1})L}{R^2} \right]^{\frac{L(C^{-1} + \gamma^{-1})}{R^2 - C^{-1} - \gamma^{-1}}} - \left[\frac{(C^{-1} + \gamma^{-1})L}{R^2} \right]^{\frac{R^2}{R^2 - C^{-1} - \gamma^{-1}}} \right\}, \quad (59)$$

as implied by the following chain:

$$\begin{aligned} I_C^{crit} &= \frac{2P}{R\gamma} \left(e^{-\frac{C^{-1} + \gamma^{-1}}{R} \frac{RL}{R^2 - C^{-1} - \gamma^{-1}} \ln \frac{R^2}{(C^{-1} + \gamma^{-1})L}} - e^{-\frac{R}{L} \frac{RL}{R^2 - C^{-1} - \gamma^{-1}} \ln \frac{R^2}{(C^{-1} + \gamma^{-1})L}} \right) = \\ &= \frac{2P}{R\gamma} \left(e^{-\frac{L(C^{-1} + \gamma^{-1})}{R^2 - C^{-1} - \gamma^{-1}} \ln \frac{R^2}{(C^{-1} + \gamma^{-1})L}} - e^{-\frac{R^2}{R^2 - C^{-1} - \gamma^{-1}} \ln \frac{R^2}{(C^{-1} + \gamma^{-1})L}} \right) = \\ &= \frac{2P}{R\gamma} \left\{ \left[\frac{(C^{-1} + \gamma^{-1})L}{R^2} \right]^{\frac{L(C^{-1} + \gamma^{-1})}{R^2 - C^{-1} - \gamma^{-1}}} - \left[\frac{(C^{-1} + \gamma^{-1})L}{R^2} \right]^{\frac{R^2}{R^2 - C^{-1} - \gamma^{-1}}} \right\} \end{aligned}$$

Per the explicit relationships in Eqs. 58 and 59, at the magnitude of the induction L , approaching zero, the critical time t_{crit} also approaches zero, whereas the magnitude of the critical current I_C^{crit} approaches the following value:

$$I_C^{crit} = \frac{2P}{R\gamma} . \quad (60)$$

Thus, in the approximation of small inductance, the explosive charging is accompanied by a single splash of the current, occurring at the moment t_{crit} , described by the transcendental relationship (Eq. 54), and assuming the value I_C^{crit} , described by the relationship in Eq. 55.

For a large enough induction L , technically, the analysis is the same as for small induction, but the general solution in Eq. 39 should be replaced with the general solution in Eq. 41 (if the reader is not experienced enough with using analytic continuation of real functions in complex domains). The process of charging in this case will include multiple oscillations.

7. Conclusion

Summarizing, we suggested a model of a pyroelectric charger and its analysis in the spirit of electrical engineering. The model of the pyroelectric charger is characterized by a positive parameter, called the capacity of the charger, and by the polarization P . We postulated the relationships in Eqs. 7 and 8, describing the voltage and the current through the charger. The model permits the analysis of the transient processes accompanying pyroelectric charging and, in particular, the explosive regime of charging.

We established Eq. 34, describing pyroelectric charging of a capacitor in the circuit, which includes the inductive element, as well as a resistor. It is a second-order linear ordinary differential equation that can be integrated explicitly in elementary (sometimes transcendental) functions. There are two convenient forms of the general solutions of Eq. 34: one of them (Eq. 39) is convenient in the case of a small magnitude of inductance; the other (Eq. 41) is convenient in the case of a large magnitude of inductance.

We illustrated the suggested engineering approach by analyzing the explosive charging of a capacitor by means of the pyroelectric charger. We limited ourselves with the case of small (but non-vanishing) inductance. The key relations describing this process are shown in Eqs. 53–60.

The suggested model is remarkably simple and does not require any sophisticated techniques going beyond standard college mathematics and physics. More sophisticated models of explosive charging would require much more elaborate modeling and will be discussed elsewhere.

8. References

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