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Sub-Hinze scale breakup model for high-fidelity simulation of bubbly flows

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Parviz Moin

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13. SUPPLEMENTARY NOTES

14. ABSTRACT

In this reporting period, algorithms to identify bubbles and to detect bubble breakup and coalescence events in a two-phase (air/water) simulation with knowledge of the volume fraction field (spatial distribution of the air and water phases) are detailed. These algorithms are then used to characterize the evolution of the bubble size distribution in an ensemble of simulations of periodic breaking waves, including the identification of breakup and coalescence events. Then, on the basis of trends observed in the evolution of the distribution and accompanying events, a pathway for the identification of potential mechanisms for bubble breakup and coalescence is outlined. The variation of the distribution of bubble sizes in a periodic breaking wave in time, which corresponds approximately to the variation of the distribution of bubble sizes with the distance downstream of wave-generating vessels, is important for the determination of the effects of bubble-mediated physical phenomena, such as the optical reflectance and acoustical properties of ship wakes, in particular their variation with distance downstream of the vessel.

15. SUBJECT TERMS
Bubble breakup, bubble coalescence, algorithms, large eddy simulation, multiphase flows

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**		16	Parviz Moin
					19b. TELEPHONE NUMBER *(Include area code)* (650) 723-9713

Final Technical Report

Sub-Hinze scale breakup model for high-fidelity simulation of bubbly flows
Grant N000014-15-1-2726

Prepared for

Office of Naval Research
75 N. Randolph Street
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For the Period

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Submitted by

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Principal Investigator: Parviz Moin, (650) 723-9713, moin@stanford.edu
Leland Stanford Junior University

Section I: Project Summary

1. Overview of Project

Abstract:
In this reporting period, algorithms to identify bubbles and to detect bubble breakup and coalescence events in a two-phase (air/water) simulation with knowledge of the volume fraction field (spatial distribution of the air and water phases) are detailed. These algorithms are then used to characterize the evolution of the bubble size distribution in an ensemble of simulations of periodic breaking waves, including the identification of breakup and coalescence events. Then, on the basis of trends observed in the evolution of the distribution and accompanying events, a pathway for the identification of potential mechanisms for bubble breakup and coalescence is outlined. The variation of the distribution of bubble sizes in a periodic breaking wave in time, which corresponds approximately to the variation of the distribution of bubble sizes with the distance downstream of wave-generating vessels, is important for the determination of the effects of bubble-mediated physical phenomena, such as the optical reflectance and acoustical properties of ship wakes, in particular their variation with distance downstream of the vessel.

Objective:
The cost of direct numerical simulation of turbulent breaking waves with realistic amplitudes and wavelengths is prohibitive due to the wide range of scales involved, necessitating the development of subgrid-scale (SGS) models to enable the large eddy simulation of these flows. A physics-based SGS model demands a fundamental understanding of the breakup and coalescence processes leading to the development and subsequent evolution of the bubble size distribution near and below the grid scale. With this goal in mind, an ensemble of simulations of breaking Stokes waves of a smaller amplitude and wavelength is employed to better understand these processes at the scales resolvable for these waves, in the hope that physical insights at the smaller length scales to be modeled may eventually be elucidated for more energetic waves.

Introduction:
Breaking waves in oceans generate bubbles of a wide range of scales, which in turn influence interfacial mass, momentum and energy transfer processes, surface reflectance, and the propagation of acoustic waves. Numerous experiments have been devised to measure bubble populations as a function of bubble size due to breaking waves generated under a wide variety of conditions. Of these, experiments by Deane & Stokes in 2002 and Blenkinsopp & Chaplin in 2010 were able to measure bubbles over a size range spanning more than two decades (between 100 μm and 10 mm) within a single experimental setup — one of the largest size ranges of simultaneously-measured bubbles to date. While there are differences in the bubble size distributions reported in both works due to the different wave parameters and measurement techniques used, as well as the stage of wave breaking during which the measurements were obtained, there seem to be two common

themes in the resulting measurements: first, the distribution of the larger bubbles appears to scale differently from the distribution of the smaller bubbles; second, these distributions evolve significantly in time as the wave breaks. Similar observations have been made in recent numerical simulations of breaking third-order Stokes waves by Wang *et al.* in 2016 and Deike *et al.* in 2016. It appears, then, that different physical mechanisms are at play at different length and time scales in the formation and dynamics of bubbles in these breaking waves. These mechanisms have not been straightforward to isolate and are a subject of active research, including the research undertaken in this project.

Background:

In this work, an unstructured node-centered geometric unsplit volume-of-fluid-based incompressible two-phase flow solver with consistent mass and momentum advection developed by the Center for Turbulence Research and Cascade Technologies was used to perform an ensemble of 30 simulations of breaking third-order Stokes water waves in air. The parameters were selected such that these simulations correspond to a 27-cm-long water wave at atmospheric conditions. The computational mesh consisted of about 4.2 million mesh nodes, the minimum grid spacing was 1/216 of the wavelength — equivalent to 1.25 mm at atmospheric conditions — and the length of the computational domain along each Cartesian axis was also the wavelength. Periodic boundary conditions were employed in the directions parallel to the interface, and slip boundary conditions were employed in the wave-normal direction.

In order to obtain the desired statistics, algorithms have been developed and refined to enable the computation of bubble size distributions, as well as the identification of bubble breakup and coalescence events via the tracing of bubble lineages. The computation of bubble size distributions involves the identification of the volumes of individual bubbles, which may be accomplished by the flood-fill algorithm. The flood-fill algorithm relies on a grouping criterion that determines whether pairs of computational cells containing some air should be included in a group (bubble). Several grouping criteria were explored in this work in order to reduce the influence of spurious numerical bubbles without incurring excessive volume errors. The identification of bubble breakup and coalescence events relies on the computation of bubble volumes and centroid locations, in tandem with the enforcement of the constraints that the bubbles satisfy the conservation of mass, and that the simulation satisfies the Courant-Friedrichs-Lewy (CFL) condition. With these constraints, as well as a good handle on the errors incurred by the bubble identification algorithm, lists of bubbles from consecutive snapshots may be compared and tested for these events.

2. Activities and Accomplishments

Using the refined flood-fill algorithm developed in this work, the bubble size distribution was computed for the ensemble of breaking wave simulations described in Section 1. The ensemble-averaged bubble size distribution is plotted after the wave has broken from a series of simulation snapshots regularly spaced in time in Figure 1. The time scale used to nondimensionalize time in this work is on the order of the integral time scale of the original wave, and the length scale is on the order of the wavelength of the fundamental harmonic of the original wave. The effective size of nonspherical bubbles is computed by measuring the size of a spherical bubble of equal volume. Some key observations are listed here. First, a $-10/3$ power law initially develops in the size distribution above the Hinze scale, and momentarily extends into the sub-Hinze-scale region. This scaling persists above the Hinze scale in the early wave-breaking stage. (The Hinze scale is the approximate scale below which bubbles no longer break under the action of turbulence due to the restoring action of surface tension. This is analogous to the Kolmogorov scale, which is the approximate scale below which eddies no longer transfer energy to smaller energies under the action of turbulence due to the dissipative action of viscosity.) These observations are consistent with the dominance of quasi-steady breakup by the action of turbulent eddies at scales both smaller and larger than the Hinze scale at an intermediate time, and at scales larger than the Hinze scale before and after that, keeping in mind that the Hinze scale is merely an order-of-magnitude estimate. At early times, the signature of the turbulent cascade in the bubble size distribution is present only at the large scales because air is being entrained and fragmented from the large scales; at late times, turbulent breakup persists at the large scales as the smallest eddies in the system have the fastest timescales and disappear most quickly. Second, a −3/2 power law eventually develops below the Hinze scale, and persists till late times. Note that this power-law scaling occurs fairly close to the grid scale. Third, a −4 power law is somewhat visible at intermediate sizes at late times as well. The two power-law fits that eventually emerge (−3/2 for the small scales and −10/3 for the large scales) corroborate the experimental observations of Deane & Stokes. A −4.36 power law was observed for what was postulated to be the super-Hinze-scale bubbles in a statistically-stationary set of measurements from air entrainment in a turbulent ship hull boundary layer by Masnadi *et al.* in 2018, lending credence to the steeper power-law fit observed at large scales and at later times in this work where the role of buoyancy is expected to be more pronounced (than at earlier times). The replacement of the −10/3 power-law scaling by other power-law scalings in the size distribution, first at the small scales and then at the large scales, appears to be concurrent with the decay of turbulence, which is also expected to occur first at the small scales and then at the large scales. The apparent emergence of these new scalings following the dissipation of turbulence may be caused by the eventual development of a cascading process from small scales to large scales. The flow of air from small scales to large is reminiscent of coalescence processes, and motivates a deeper analysis of the driving of these processes by the background flow field with and without turbulence.

Using a bubble tracking algorithm developed in this work, bubble breakup and coalescence events were identified in the simulation ensemble during the wave-breaking period. Figure 2 plots the ensemble-averaged and time-averaged breakup frequency for the first and second halves of the duration of interest. At early times, a −2/3 power law is clearly observed over a wide range of scales, suggesting the presence of a turbulent fragmentation cascade. This is consistent with the −10/3 power law observed in the ensemble-averaged bubble size distribution in Figure 1. The −2/3 power-law fit is less adequate at later times, suggesting the cascade has largely ceased by then. Note that the −2/3 power-law fit in Figure 2(a) does not extend completely into what was identified as the super-Hinze-scale region. There are a number of factors contributing to this peculiarity. First, super-Hinze-scale bubbles of different sizes are formed by the rupture of large air cavities, such as the cylindrical air cavity trapped when the wave overturns and breaks. This means that the turbulent fragmentation cascade does not uniformly begin at a single scale, since each of these super-Hinze-scale seed bubbles generates its own fragmentation cascade from a different originating scale. Second, the statistics in Figure 2 are time-averaged. The number of super-Hinze-scale seed bubbles varies in time, from a negligible number before the large cavities have fragmented, to a significant number after this fragmentation has taken place, and again to a negligible number once these seed bubbles have fragmented and no new seed bubbles are generated to take their place. This variation in the number of super-Hinze-scale seed bubbles may obfuscate the interpretation of the breakup frequency at these large scales. Ensemble averaging is the proper way to take statistics in a statistically-unsteady flow, and work is ongoing to collect more converged statistics after reducing the averaging time window for these events in an attempt to move towards ensemble averaging.

Third, the turbulence in the continuous phase may be sufficiently modulated at these large scales by the presence of the dispersed phase. Figure 3 plots the ensemble-averaged and time-averaged coalescence frequency for the first and second halves of the duration of interest. The coalescence frequency peaks close to the estimated Hinze scale. Coalescence is potentially inhibited at large scales where the effects of turbulence prevent imminent collisions from going to completion. In addition, the variation of the coalescence frequency with bubble size differs in both time intervals, suggesting that the nature of the background flow inducing the coalescence of bubbles is different in both time intervals.

Before analyzing the probability distributions of the relative sizes of the final (breakup) / initial (coalescence) bubbles involved in these breakup and coalescence events, it is instructive to divide the size range of observable bubbles into three distinct phenomenologies: resolved small bubbles (sub-Hinze-scale bubbles), large bubbles (super-Hinze-scale bubbles) and very large "bubbles" (super-Hinze-scale air cavities). This enables a more targeted analysis of the various breakup and coalescence events. In addition, the subsequent figure will analyze the occurrence of these events as a function of the volume ratio of the constituent bubbles. For breakup events, the volume of each of the two descendant (child) bubbles as a ratio of the sum of the volumes of these bubbles is of interest. For coalescence events, the volume of each of the two ancestor (parent) bubbles as a ratio of the sum of the volumes of these bubbles is of interest. Figure 4 plots the statistics of the breakup and coalescence events where the ancestor (breakup) / descendant (coalescence) bubble belongs to the category of large bubbles. The breakup distribution indicates that breakup generates bubble pairs of a broad range of size ratios. The coalescence distribution indicates that large-size-ratio coalescence events are favored. This supports a kinetic theory framework for bubble coalescence.

3. Findings and Conclusions

The bubble identification and tracking algorithms developed in this work are used to compute and analyze the bubble size distribution resulting from an ensemble of simulations of breaking Stokes waves. The distribution is compatible with a turbulent breakup mechanism at early times, and shifts, first at the small scales and then at the large scales, at later times. Breakup and coalescence frequencies and probabilities are computed directly from the simulation ensemble to provide insights into the population dynamics. The breakup frequencies support the turbulent breakup mechanism, and large-size-ratio coalescence events observed in the simulations support a kinetic-theory-based approach for describing and modeling coalescence. Based on the observations collected from these tools, a potential sequence of events governing the evolution of the bubbles is proposed as follows: as the wave breaks, turbulent fragmentation of large air cavities generates smaller bubbles of a wide range of sizes. A $-10/3$ power-law size distribution is generated by this turbulent fragmentation. Subsequently, after the turbulence has mostly dissipated and ceased to exist, a $-3/2$ power-law scaling develops in the size distribution for bubbles smaller than what was formerly the Hinze scale—the Hinze scale is no longer a relevant measure when the turbulence has mostly ceased to exist—and bubbles of intermediate size begin to conform to a -4 power-law size distribution. Because the power-law scaling at the small scales changes first, and then the scaling at the large scales, a coalescence cascade from small scales to large scales could be developing late in the wave-breaking process. A fundamental understanding of breakup and coalescence processes that govern the formation and evolution of bubbles in breaking waves may assist in developing SGS models for turbulent bubbly flows, and thereby predict the joint spatial and size distribution of bubbles in ship wakes. In light of the observations above, a preliminary SGS model for the largest sub-Hinze-scale bubbles may be conceived.

4. Plans and Upcoming Events

Based on the statistics of breakup and coalescence events detected in the work to date, efforts are ongoing to infer the physical mechanisms governing their occurrence. This demands deference to an overarching model that is general enough to describe the processes at various stages of the flow (viz., early wave-breaking before most of the wave energy has dissipated, and late wave-breaking after the surface amplitude has greatly decreased and most of the energy has dissipated). In the context of Naval applications, these correspond approximately to regions of varying distance from the vessel (viz., regions close to the ship where the bow and stern waves have just been generated, and regions further downstream where wave-breaking has largely gone to completion but the smaller bubbles still reside in the flow). A population dynamics model with cascade analogies is being developed to elucidate these mechanisms in three stages: first, the appropriate equations with suitable models for the constituent terms are being refined to ensure the equations reasonably capture the phenomena of interest. Second, the model will be simulated numerically in a sanitized test case to reduce the influence of inhomogeneity and determine the fundamental features of the model independent of the flow geometry. Third, scaling relations derived directly from the equations and implicitly via observations from the aforementioned numerical tests will be compared with the statistics compiled from the breaking wave simulation ensemble to determine the adequacy of the population dynamics model in oceanic flows. Conclusions from this endeavor will serve as the baseline model for subgrid-scale modeling of bubbles of sizes slightly smaller than the Hinze scale, subject to further refinements. Previous work on the subgrid-scale modeling of bubbles much smaller than the Hinze scale formed by breakup processes like film rupture and satellite formation that bypass a large range of scales, including the detection of collision events to determine when to activate the model, as well as the characterization of a microscale model problem that determines the final microbubble size distribution generated by each event, will also be continued to provide modeling support for a significant portion of the range of scales extending under the Hinze scale.

Recommendations for Future Work:

Work is ongoing to develop first iterations of subgrid-scale models (see above) for implementation in realistically-sized waves. Once development is complete, comparison with appropriate experiments will be sought and used as inspiration for further refinement. Interactions with Prof. Pablo Carrica (University of Iowa) may be initiated later this year in order to discuss transition opportunities for the proposed model (viz., implementation of the model in a range of codes), and also obtain insights from experience with existing RANS models for model refinement.

5. Transitions and Impacts

Not applicable or no information to report

6. Collaborations

Interactions with Prof. Kamran Mohseni (University of Florida) and his students were initiated during Q2/Q3 2018 to discuss inclusion of contact-line models in the large eddy simulation of oceanic flows around and near seafaring vessels for technological readiness (improved scale capabilities with increased accuracy).

Interactions with Prof. James Duncan (University of Maryland) and his students were initiated during Q3/Q4 2018 to learn more about experimental design for bubble measurements in breaking waves and to discuss potential collaboration opportunities in the future for validation of simulations and models.

7. Personnel

Principal investigator: Parviz Moin (4.0 calendar months) (National Academy Member)
Co-investigator or Co-PI: N/A
Business Contact: N/A
Team Members: Michael Dodd (12 calendar months)
Subs: N/A

8. Students

3 graduate students

9. Technology Transfer

Not applicable or no information to report

10. Products, Publications, Patents, License Agreements, etc.

Publications resulting from this project:

Archival Publications (publication reference information (article title, authors, journal, date, volume, issue) can be automatically entered using a DOI)

#1

a. Article Title: Formation and dynamics of bubbles in breaking waves: Part I. Algorithms for the identification of bubbles and breakup/coalescence events.
b. Journal: Center for Turbulence Research Annual Research Briefs
c. Authors: W. H. R. Chan, M. S. Dodd, P. L. Johnson, J. Urzay and P. Moin
d. Keywords: Bubble breakup, bubble coalescence, algorithms, large eddy simulation, multiphase flows
e. Distribution Statement: Approved for public release; distribution is unlimited.
f. Publication Status: Published
g. Publication Identifier Type: N/A
h. Publication Identifier: N/A
i. Publication Date: 10 January 2019
j. Volume: N/A
k. Issue: N/A
l. First Page Number: 3
m. Publication Location: Stanford, CA, United States
n. Acknowledgement of Federal Support? Yes
o. Peer Reviewed? Yes (internal)

#2

a. Article Title: Formation and dynamics of bubbles in breaking waves: Part II. The evolution of the bubble size distribution and breakup/coalescence statistics.

b. Journal: Center for Turbulence Research Annual Research Briefs
c. Authors: W. H. R. Chan, M. S. Dodd, P. L. Johnson, J. Urzay and P. Moin
d. Keywords: Bubble breakup, bubble coalescence, turbulent cascade, large eddy simulation, multiphase flows
e. Distribution Statement: Approved for public release; distribution is unlimited.
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g. Publication Identifier Type: N/A
h. Publication Identifier: N/A
i. Publication Date: 10 January 2019
j. Volume: N/A
k. Issue: N/A
l. First Page Number: 21
m. Publication Location: Stanford, CA, United States
n. Acknowledgement of Federal Support? Yes
o. Peer Reviewed? Yes (internal)

Conference Papers

#1

a. Title: Subgrid-scale modeling for microbubble generation amid colliding water surfaces
b. Authors: Wai Hong Ronald Chan, Javier Urzay, Parviz Moin
c. Conference Name: 32nd Symposium on Naval Hydrodynamics
d. Conference Date: 5 – 10 August 2018
e. Conference Location: Hamburg, Germany
f. Publication Status: Awaiting Publication
g. Publication Date: N/A
h. Publication Identifier Type: N/A
i. Publication Identifier: N/A
j. Acknowledgement of Federal Support? Yes

#2

a. Title: Formation and dynamics of bubbles generated by turbulent breaking waves
b. Authors: Wai Hong Ronald Chan, Michael Dodd, Perry L. Johnson, Javier Urzay, Parviz Moin
c. Conference Name: 71st Annual Meeting of the American Physical Society Division on Fluid Dynamics
d. Conference Date: 18 – 20 November 2018
e. Conference Location: Atlanta, GA, United States
f. Publication Status: Published
g. Publication Date: 18 November 2018
h. Publication Identifier Type: N/A
i. Publication Identifier: N/A
j. Acknowledgement of Federal Support? Yes

Books

Not applicable or no information to report

Book Chapter

Not applicable or no information to report

Theses
Not applicable or no information to report

Websites
Not applicable or no information to report

Patents
Not applicable or no information to report

Other Products: Identify any other significant products that were developed under this project. Describe the product and how it is being shared.

a. Description: Video submission to the Gallery of Fluid Motion at the 71st Annual Meeting of the American Physical Society Division on Fluid Dynamics describing a potential pathway for the formation of microbubbles in breaking waves (Link: https://gfm.aps.org/meetings/dfd-2018/5b986641b8ac31610362f3f1)

b. Product Type: Video

11. Point of Contact in Navy

Dr. Thomas Fu, Program Manager, (703) 696-7386, thomas.fu@navy.mil

12. Acknowledgement/Disclaimer

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Section II: Project Metrics

Grant or Contract Number: N00014-15-1-2726
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Project Title: Sub-Hinze scale breakup model for high-fidelity simulation of bubbly flows
Annual Summary Report: CY2018
PI: Parviz Moin, (650) 723-9713, moin@stanford.edu
Leland Stanford Junior University

Metrics

Number of faculty supported under this project during this reporting period: 1

Number of post-doctoral researchers supported under this project during this period: 1

Number of graduate students supported under this project during this reporting period: 3

Number of undergraduate students supported under this project during this period: 0

Number of scientists / engineers / technicians supported under this project during this reporting period: 1

Number of refereed publications during this reporting period for which at least 1/3 of the work was done under this effort: 0

Number of publications (all) during this reporting period: 2

Number of patents during this reporting period: 0

Number of M.S. students graduated during this reporting period: 0

Number of Ph.D. students graduated during this reporting period: 0

Awards received during this reporting period: 1

2018 American Physical Society Division of Fluid Dynamics Gallery of Fluid Motion Award Winner (Authors: Wai Hong Ronald Chan, Shahab Mirjalili, Suhas Jain Suresh, Javier Urzay, Ali Mani, Parviz Moin)

Invited talks given: 0

Conferences at which presentations were given (not including invited talks above): 2

1. Financial information

FY 2018	Total Budget	Obligated This Period	Obligated Cumulative	Expended This Period	Expended Cumulative	Grant/ Contract Period of Performance
6.1 (Basic Research Funding)	$691,868	$691,868		$691,852.77		05/28/2015 – 09/30/2018
6.2 (Applied Research Funding)						
Total (if both 6.1 and 6.2 funding was used)	$691,868	$691,868		$691,852.77		

2. Administrative notes and other items of interest

Interactions with Prof. Kamran Mohseni (University of Florida) and his students were initiated during Q2/Q3 2018 to discuss inclusion of contact-line models in the large eddy simulation of oceanic flows around and near seafaring vessels for technological readiness (improved scale capabilities with increased accuracy).

Interactions with Prof. James Duncan (University of Maryland) and his students were initiated during Q3/Q4 2018 to learn more about experimental design for bubble measurements in breaking waves and to discuss potential collaboration opportunities in the future for validation of simulations and models.

Appendix B (Data and Charts)

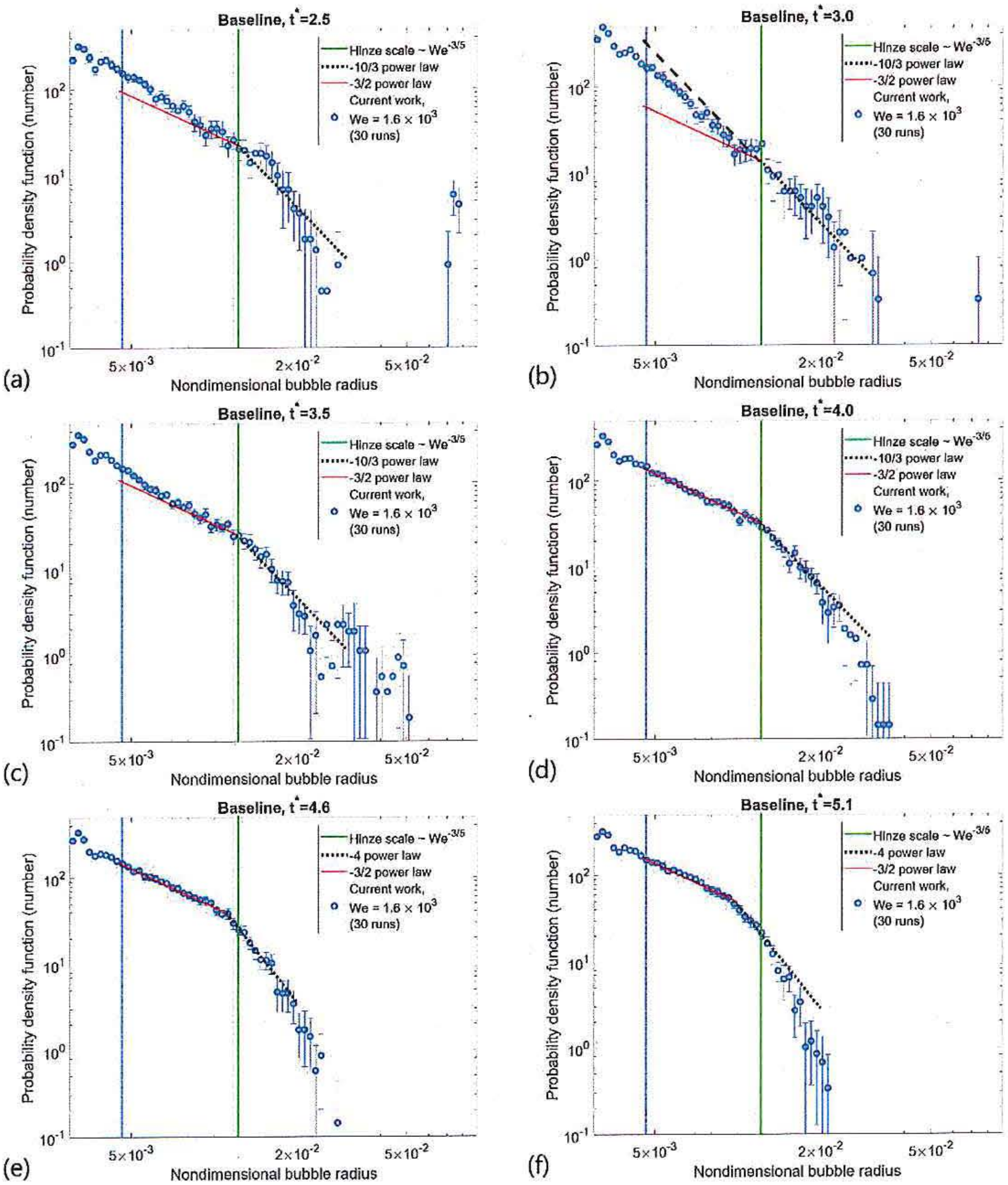

Figure 1. The ensemble-averaged bubble size distribution from a series of simulation snapshots that are equally spaced in time, some time after the wave has broken.

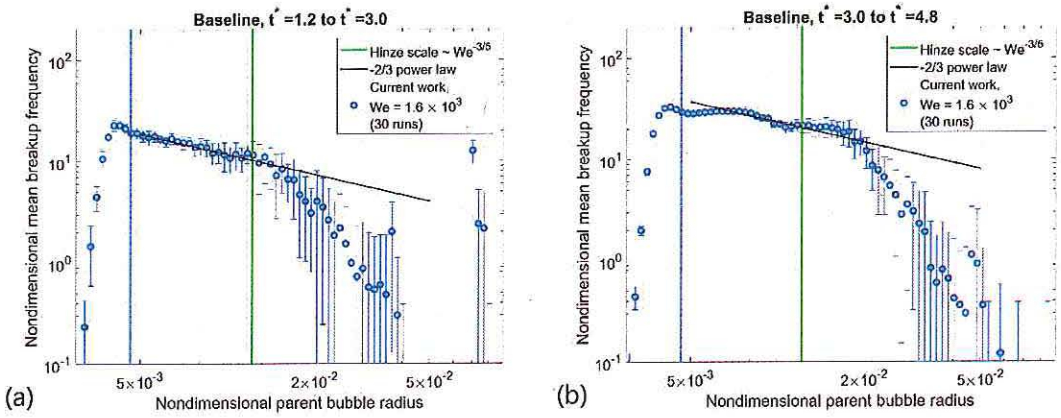

Figure 2. The ensemble-averaged and time-averaged breakup frequency (number of breakup events in each domain per unit time) from the two halves of the duration of interest.

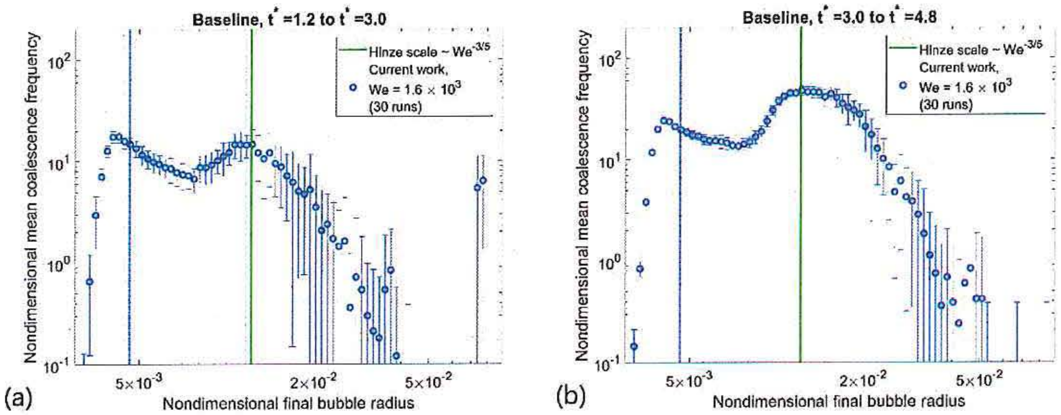

Figure 3. The ensemble-averaged and time-averaged coalescence frequency (number of coalescence events in each domain per unit time) from the two halves of the duration of interest.

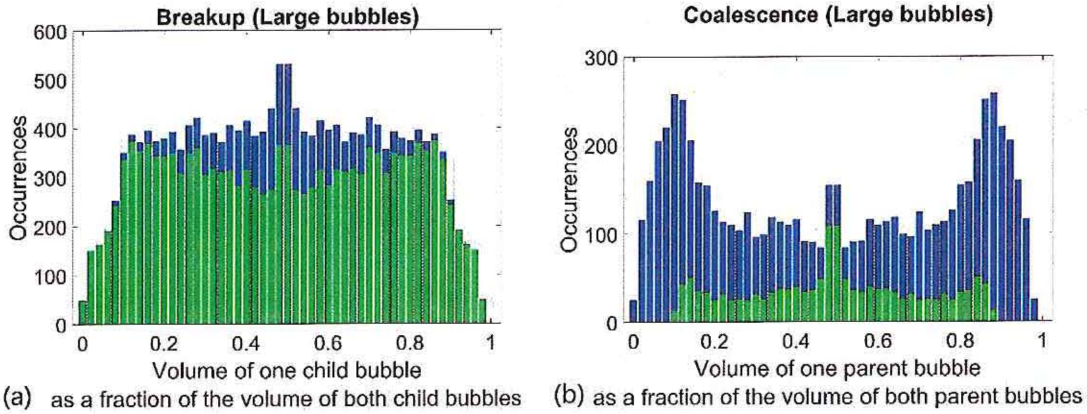

Figure 4. Conditional bubble volume ratio distributions due to some of the breakup (a) and coalescence (b) events in the simulation ensemble. For breakup, events where the ancestor bubble is a large bubble (super-Hinze-scale bubble) are included. The distribution is divided into two groups. The top portion of each bar (blue in color, darker in grayscale) denotes events where the smaller of the two descendant bubbles is a large bubble. The bottom portion (green in color, lighter in grayscale) denotes events where the smaller of the two descendant bubbles is a resolved small bubble (sub-Hinze-scale bubble). For coalescence, events where the descendant bubble is a large bubble are included. The top (blue/darker) portion of each bar denotes events where the larger of the two ancestor bubbles is a large bubble. The bottom (green/lighter) portion denotes events where the larger of the two ancestor bubbles is a resolved small bubble.