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Agency Code:

Proposal Number: 66137MA **Agreement Number: W911NF-15-1-0228**
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Report Date: 08-Nov-2018 Date Received: 11-Feb-2019
Final Report for Period Beginning 09-Jun-2015 and Ending 08-Aug-2018
Title: Novel Matrix-Based Methods for Fractional-Order Modeling
Begin Performance Period: 09-Jun-2015 **End Performance Period:** 08-Aug-2018
Report Term: 0-Other
Submitted By: DrSc. Igor PODLUBNY Email: igor.podlubny@tuke.sk
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STEM Degrees: **STEM Participants:** 1

Major Goals: Problem Area 1. Computational methods and numerical analysis for fractional differential equations. This is, in fact, the main goal of the proposed project. The work on various enhancements of the matrix approach, with the focus on using sparse matrices, parallel computations, grid computations, the “short- memory principle”, the “method of large steps”, etc.; all that including not only constant non-integer orders, but also variable and distributed orders.

Problem Area 2. Fractional-order modeling of real-world systems. The other direction of the proposed research is fitting experimental data with the help of the Mittag-Leffler function; this can be called the “Mittag-Leffler fitting” or the “self-tuning fitting”.

Problem Area 3. Experiments, demonstrations, software development.
We will work with laboratory objects (e.g., some types of electrical circuits) for studying fractional-order behavior “in situ” and demonstrating applicability of our tools developed for data fitting and for numerical solution of fractional differential equations.

Accomplishments: Problem Area 1. Computational methods and numerical analysis for fractional differential equations.

In the field of numerical analysis, we have obtained three important results regarding eigenvalues of the discretization matrix for symmetrical fractional-order derivatives (Riesz derivatives). Namely, we obtained the interval, in which all eigenvalues of the discretization matrices are contained for orders of fractional derivatives in the closed interval [1, 2]; we obtained the explicit formulas for the eigenvalues of the discretization matrices on N points for orders equal 1 and 2; and we proved that when the order approaches 1 or, respectively, 2, the eigenvalues of the corresponding discretization matrix of the fractional-order derivative approach the eigenvalues of the matrices for derivatives of order 1, or, respectively, 2. A paper on these results is in the process of preparation for publication.

In the field of computational methods and tools, we prepared an update of our MATLAB toolbox for solving

ordinary and partial fractional-order differential equations. The update includes the use of sparse matrices and the variable-order ordinary and partial differential equations.

An addition to our set of fractional-order control tools is represented by a toolbox for non-linear fractional-order PID controller.

While working on MATLAB toolboxes, we developed a totally new approach to designing stand-alone applications in MATLAB, which we named ReGUI (Responsive Graphical User Interface).

Another important theoretical result was the application of adjoint fractional-order operators in the study of regional controllability of the sub-diffusion process with the Caputo fractional derivative. In this direction, we are working towards discretization of adjoint fractional-order operators and their discretizations. This work is currently under development.

Problem Area 2. Fractional-order modeling of real-world systems.

In the field of fractional-order modeling, the most important result is the development of a totally new class of functions, which we provisionally call “P-functions”, which shows huge potential for modeling real-word materials and structures. We have developed first versions of MATLAB functions and routines for computation of “P-functions”, for plotting such functions, and made first steps towards their numerical differentiation and integration of arbitrary real order (integer and noninteger). A paper on “P-functions” is in the process of preparation for publication, as well as a toolbox with the corresponding MATLAB functions.

The Mittag-Leffler fitting method was used for creating a model of the combustion process in combustion engines. During this work, we developed better understanding of the roles of the parameters in the Mittag-Leffler function, and this understanding gives a new insight into fractional-order modeling of real dynamical processes and systems. Namely, now we interpret one of the parameters as the intrinsic time scale of the dynamic process, and the other as the shape parameters; furthermore, this allows proper consideration of the physical units in fractional-order modeling, which was until now considered as unclear.

We introduced a class of fractional-order extremal control that is based on sign changes of the derivative of the controlled value. For the first time, the control loop consists of a controlled process of the Wiener type described by a fractional-order linear model, and a term with extremal characteristic.

We also introduced a novel polarization index evaluation formula and investigated fractional-order dynamics in electric motor insulation resistance. On this way we used our tools for the Mittag-Leffler function and our methods for identification of parameters of fractional-order systems.

The developed methods, tools, and software were successfully used for modeling fractional-order systems and for simulations of fractional-order controllers.

Problem Area 3. Experiments, demonstrations, software development.

We prepared an update of our MATLAB toolbox for solving ordinary and partial fractional-order differential equations. The update includes the use of sparse matrices and the variable-order ordinary and partial differential equations.

One of the important experimental results is a new method for analog implementation of the fractional-order controller using an active element in form of the Operational Transconductance Amplifier has been developed. Transfer function of such con-troller is performed by the Inverse-Follow-the-Leader-Feedback topology. Presented implementation is resistorless, energy effective and suitable for realization in integrated form.

We also succeeded with modeling anomalous diffusion using a novel analog circuit, where for the first time instead of the classical capacitors in domino ladder structure, the ultracapacitors are used.

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Training Opportunities: We have presented our research findings and software tools at several conferences and workshops, which is a kind of “training” activities.

Since 2007, in collaboration with our international partners, we run a series of one-day workshops carried out in different countries and universities; we call these workshops "Fractional Calculus Day @ <university>", or "FC Day @ <university>". In line with that, we organized the following workshops:
(1) Fractional Calculus Day @ TUKE (May 12, 2017, Kosice, Slovakia);
(2) Fractional Calculus Day @ AGH Krakow (May 18, 2017, Krakow, Poland);
(3) Second Applied Fractional Calculus Day @ BJTU (July 1, 2017, Beijing, China)
(4) Third Applied Fractional Calculus Day @ BJTU (July 3, 2018, Beijing, China)

We have also developed and used MATLAB tools for teaching fractional-order modeling and control.

Results Dissemination: In the field of dissemination perhaps the most significant output is our investigation of the relationship between cybernetics and fractional calculus.

Cybernetics addresses complex problems in space and time, but, surprisingly, it does not often employ fractional calculus. We traced the history of cybernetics and fractional calculus to resolve this apparent paradox. Cybernetics played a key role in the birth of automatic control, artificial intelligence, and robotics in the mid-20th century, and its scope now includes cyberphysical systems. We provide arguments that fractional calculus can be applied more extensively in cybernetics as it continues to develop new methods to solve the complex, interconnected, and multiscale problems of the natural world.

The study of the history of the fractional calculus led us to exciting discovery of N. H. Abel's contribution to the birth of the fractional calculus. We show that in his first paper on the generalization of the tautochrone problem, that was published in 1823, Abel presented a complete framework for fractional-order calculus, and used the clear and appropriate notation for fractional-order integration and differentiation. We also examine his second paper, published in 1826, in which he solves the same problem by other method. Moreover, for the first time we provide the English translation to both Abel's obscure paper, and made all that material freely on the journal page and at Arxiv.org repository.

We also developed significant efforts in the field of dissemination through editing special issues of journals.

In order to attract attention of both general and professional public to fractional-order modeling, we prepared a bio sketch of Professor Richard Magin from the University of Illinois in Chicago, who is one of the most prominent researchers in the fractional-order modeling, on the occasion of his 70th birthday. It appeared in the dedicated journal “Fractional Calculus and Applied Analysis”.

We also prepared a bio sketch of Professor Virginia Kiryakova, DrSc., who is the Managing Editor of the journal Fractional Calculus and Applied Analysis (FCAA), and Prof. Igor Podlubny together with Prof. Yuri Luchko from Beuth Technical University in Berlin was the Guest Editor of a Special Issue of FCAA (namely, vol. 20, no. 5 2017).

Professor Igor Podlubny was the lead guest editor of the recent special issue of the ISA Transactions, focused on fractional-order signals, systems, and control.

Professor Ivo Petras was an editor of one of the volumes of the monumental multi-volume publication, namely of: Handbook of Fractional Calculus with Applications: Vol.6 Applications in Control, De Gruyter, 2019, ISBN 978-3-11-057174-5.

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Honors and Awards: (1) Prof. RNDr. Igor Podlubny, DrSc., elected (2017) a member of the Learned Society of Slovakia.
Comment: this is the highest level of scientific recognition in Slovakia; the Learned Society of Slovakia is roughly an analog of the Royal Society; currently, there are only 55 members (see: http://www.learned.sk/clenovia/riadni-clenovia/).

(2) Prof. Ing. Ivo Petras, DrSc., elected IEEE Senior Member (2016)
(3) Prof. RNDr. Igor Podlubny, DrSc., elected IEEE Senior Member (2017)
Comment: only less than 10% of IEEE members achieve this level.

(4) Prof. RNDr. Igor Podlubny, DrSc., received the Award of the Literary Fund of Slovak Republic (2016) for the citation impact of one work (for the paper: Li Y., Chen YQ, Podlubny I.: Mittag–Leffler stability of fractional order nonlinear dynamic systems. Automatica, vol. 45, no. 8, 2009, pp. 1965-1969).
(5) Prof. Ing. Ivo Petras, DrSc., received the Award of the Literary Fund of Slovak Republic (2017) for the citation impact of one work (for the book: Petras I., Fractional-Order Nonlinear Systems: Modeling, Analysis and Simulation, Springer, 2011, ISBN 978-3642181009.
(6) Prof. Ing. Ivo Petras, DrSc., received the Award of the Literary Fund of Slovak Republic (2017) for the 3-year total citation impact (First place).

(7) A very special recognition of the work of Professor Igor Podlubny is the fact that, according to MathSciNet (Mathematical Reviews), his book "Fractional Differential Equations" (Academic Press / Elsevier, 1999) is the most highly cited mathematical book during last six years, namely #1 in 2012, #1 in 2013, #1 in 2014, #4 in 2015, #2 in 2016, #1 in 2017 (go to MathSciNet - Citations - Top 10 lists).

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI
Participant: Igor Podlubny DrS
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Co PD/PI
Participant: Ivo Petras Dr
Person Months Worked: 12.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

BOOKS:

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Publisher: De Gruyter

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Book Title: Handbook of Fractional Calculus with Applications, Volume 6: Applications in Control
Authors: Ivo Petras (Ed.)
Editor:
Acknowledged Federal Support: N

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2016 17th International Carpathian Control Conference (ICCC)
Date Received: 07-Nov-2016 Conference Date: 28-May-2016 Date Published:
Conference Location: High Tatras, Slovakia
Paper Title: Fractional-order nonlinear controllers: Design and implementation notes
Authors: Ivo Petras
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2016 17th International Carpathian Control Conference (ICCC)
Date Received: 07-Nov-2016 Conference Date: 28-May-2016 Date Published:
Conference Location: High Tatras, Slovakia
Paper Title: An effective algorithm for implementation of non-linear fractional-order controller on PLC
Authors: Ivo Petras, Miroslav Kover Dorco
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2016 17th International Carpathian Control Conference (ICCC)
Date Received: 07-Nov-2016 Conference Date: 28-May-2016 Date Published:
Conference Location: High Tatras, Slovakia
Paper Title: Toolboxes and programs for fractional-order system identification, modeling, simulation, and control
Authors: Igor Podlubny, Ivo Petras,Tomas Skovranek, Jan Terpak
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2016 International Joint Conference on Neural Networks (IJCNN)
Date Received: 07-Nov-2016 Conference Date: 24-Jul-2016 Date Published: 03-Nov-2016
Conference Location: Vancouver, Canada
Paper Title: A Note on Fractional-Order Non-Linear Controller: Possible Neural Network Approach to Design
Authors: Ivo Petras
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: ?2016 UKSim-AMSS 18th International Conference on Computer Modelling and Simulation
Date Received: 07-Nov-2016 Conference Date: 06-Apr-2016 Date Published: 06-Apr-2016
Conference Location: Cambridge, UK
Paper Title: Computer-Generated Art: Madonna and Child - Infinity of Life
Authors: Igor Podlubny, Peter Kmetek
Acknowledged Federal Support: **Y**

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Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: TMSF 2017 - 8th International Conference on Transform Methods and Special Functions
Date Received: 03-Sep-2017 Conference Date: 26-Aug-2017 Date Published: 26-Aug-2017
Conference Location: Sofia, Bulgaria
Paper Title: Some Aspects of Modeling of Real Materials
Authors: Igor Podlubny
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2018 19th International Carpathian Control Conference (ICCC)
Date Received: 07-Feb-2019 Conference Date: 27-May-2018 Date Published:
Conference Location: Szilvasvarad, Hungary
Paper Title: An introduction to class of fractional-order extremal control: First results
Authors: Ivo Petras
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2018 19th International Carpathian Control Conference (ICCC)
Date Received: 07-Feb-2019 Conference Date: 27-May-2018 Date Published: 27-Jun-2018
Conference Location: Szilvasvarad, Hungary
Paper Title: Testing non reciprocal motion of a swimming flexible small robot with single actuation
Authors: Miguel López, Javier Prieto, José Traver, Inés Tejado, Blas Vinagre, Ivo Petras
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2017 IEEE Frontiers in Education Conference (FIE)
Date Received: 07-Feb-2019 Conference Date: 17-Oct-2017 Date Published: 13-Dec-2017
Conference Location: Indianapolis, IN
Paper Title: Design of a MATLAB-based teaching tool in introductory fractional-order systems and controls
Authors: Aleksei Tepljakov, Eduard Petlenkov, Emmanuel Gonzalez, Ivo Petras
Acknowledged Federal Support: **Y**

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: 2018 19th International Carpathian Control Conference (ICCC)
Date Received: 07-Feb-2019 Conference Date: 27-May-2018 Date Published: 27-Jun-2018
Conference Location: Szilvasvarad, Hungary
Paper Title: Responsive graphical user interface (ReGUI) and its implementation in MATLAB
Authors: Matej Mikulszky, Jana Pocsova, Andrea Mojzisova, Igor Podlubny
Acknowledged Federal Support: **Y**

Nothing to report in the uploaded pdf (see accomplishments).