

**ROUTING AND ACTION**

**MEMORANDUM**

---

ROUTING

---

TO:(1) Network Sciences Division (Zachery, Randy)

Report is available for review

(2) Proposal Files Report No.: -SR

Proposal Number: 63186-NS-SR.11

---

DESCRIPTION OF MATERIAL

---

CONTRACT OR GRANT NUMBER: W911NF-04-D-0001

INSTITUTION: Duke University

PRINCIPAL INVESTIGATOR: Bruce West

TYPE REPORT: Final Report

DATE RECEIVED: 8/3/16 10:15AM

PERIOD COVERED: 11/14/12 12:00AM through 11/13/15 12:00AM

TITLE: Final Report: Complex Networks: Structure and Function of Large-Scale Neural Networks

---

ACTION TAKEN BY DIVISION

---

(x) Report has been reviewed for technical sufficiency and IS  IS NOT  satisfactory.

(x) Material has been given an OPSEC review and it has been determined to be non sensitive and, except for manuscripts and progress reports, suitable for public release.

(x) Performance of the research effort was accomplished in a satisfactory manner and all other technical requirements have been fulfilled.

(x) Based upon my knowledge of the research project, I agree with the patent information disclosed.

Approved by SSL\PURUSH.S.IYER on 6/25/18 10:21AM

ARO FORM 36-E

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.  
PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE (DD-MM-YYYY) 03-08-2016	2. REPORT TYPE Final Report	3. DATES COVERED (From - To) 14-Nov-2012 - 13-Nov-2015
---	--------------------------------	---

4. TITLE AND SUBTITLE Final Report: Complex Networks: Structure and Function of Large-Scale Neural Networks	5a. CONTRACT NUMBER
	5b. GRANT NUMBER W911NF-04-D-0001
	5c. PROGRAM ELEMENT NUMBER 611102

6. AUTHORS M. Turalska, B.J. West	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES AND ADDRESSES Duke University C/O Office of Research Support 2200 W. Main St., Ste. 710 Durham, NC 27705 -4677	8. PERFORMING ORGANIZATION REPORT NUMBER
---	--

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211	10. SPONSOR/MONITOR'S ACRONYM(S) ARO
	11. SPONSOR/MONITOR'S REPORT NUMBER(S) 63186-NS-SR.11

12. DISTRIBUTION AVAILABILITY STATEMENT Approved for Public Release; Distribution Unlimited
--

13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.
---

14. ABSTRACT A two-state master equation (TSME), the decision making model(DMM), was developed and shown to generate phase transitions, to be topologically complex and to manifest temporal complexity through an inverse power-law (IPL) probability distribution function(PDF) in the switching times between the two critical states of consensus. These properties are entailed by the fundamental assumption that the network elements in the DMM imperfectly imitate one another. The process of subordination establishes that a single network element can be described by a <del>fractional master equation(FME), whose analytic solution</del>
--

15. SUBJECT TERMS network science, criticality, consensus, neuronal networks, fractional calculus, decision making model
---

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Bruce West
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 919-549-4257

## **Report Title**

Final Report: Complex Networks: Structure and Function of Large-Scale Neural Networks

### **ABSTRACT**

A two-state master equation (TSME), the decision making model(DMM), was developed and shown to generate phase transitions, to be topologically complex and to manifest temporal complexity through an inverse power-law (IPL) probability distribution function(PDF) in the switching times between the two critical states of consensus. These properties are entailed by the fundamental assumption that the network elements in the DMM imperfectly imitate one another. The process of subordination establishes that a single network element can be described by a fractional master equation(FME), whose analytic solution yields the observed IPL PDF, obtained by numerical integration of the TSME to a high degree of accuracy. The network dynamics describe the criticality of neuronal and genetic systems.

---

**Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:**

**(a) Papers published in peer-reviewed journals (N/A for none)**

<u>Received</u>	<u>Paper</u>
07/09/2013	2.00 P. Pramukul, A. Svenkeson, P. Grigolini, M. Bologna, B.J. West. Complexity and the Fractional Calculus, <i>Advances in Mathematical Physics</i> , (03 2013): 1. doi:
07/09/2013	1.00 B.J. West, P. Grigolini, M. Turalska. Role of committed minorities in times of crisis, <i>Scientific Reports</i> , (03 2013): 1. doi:
09/09/2014	4.00 M. turalska, B.J. West. Critical Social Networks, <i>Physica A: Statistical Mechanics and its Applications</i> , (06 2014): 466. doi:
09/09/2014	3.00 M. Turalska, B.J. West. Network of Echoes, <i>Chaos, Solitons and Fractals</i> , (10 2013): 109. doi:
09/09/2014	5.00 B.J. West, M. Turalska, P. Grigolini. Fractional calculus ties the microscopic and macroscopic scales of complex network dynamics, <i>New Journal of Physics</i> , (04 2014): 0. doi:
10/08/2015	6.00 Mauro Bologna, Adam Svenkeson, Bruce J. West, Paolo Grigolini. Diffusion in heterogeneous media: An iterative scheme for finding approximate solutions to fractional differential equations with time-dependent coefficients, <i>Journal of Computational Physics</i> , (07 2015): 297. doi: 10.1016/j.jcp.2014.08.027
10/08/2015	7.00 M. Turalska, B. J. West. Imitation versus payoff: Duality of the decision-making process demonstrates criticality and consensus formation, <i>Physical Review E</i> , (11 2014): 1. doi: 10.1103/PhysRevE.90.052815
10/08/2015	8.00 Bruce J. West. : Fractional calculus view of complexity: A tutorial, <i>Reviews of Modern Physics</i> , (10 2014): 1169. doi: 10.1103/RevModPhys.86.1169
10/08/2015	9.00 Bruce J. West. A mathematics for medicine: The Network Effect, <i>Frontiers in Physiology</i> , (12 2014): 1. doi: 10.3389/fphys.2014.00456
10/08/2015	10.00 M. Bologna, M. T. Beig, A. Svenkeson, P. Grigolini, B. J. West. Spectral Decomposition of a Fokker–Planck Equation at Criticality, <i>Journal of Statistical Physics</i> , (04 2015): 1. doi: 10.1007/s10955-015-1262-5
<b>TOTAL:</b>	<b>10</b>

Number of Papers published in peer-reviewed journals:

---

**(b) Papers published in non-peer-reviewed journals (N/A for none)**

Received      Paper

**TOTAL:**

Number of Papers published in non peer-reviewed journals:

---

**(c) Presentations**

Number of Presentations: 0.00

---

**Non Peer-Reviewed Conference Proceeding publications (other than abstracts):**

Received      Paper

**TOTAL:**

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

---

**Peer-Reviewed Conference Proceeding publications (other than abstracts):**

Received      Paper

**TOTAL:**

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

---

**(d) Manuscripts**

Received      Paper

**TOTAL:**

Number of Manuscripts:

---

**Books**

Received      Book

**TOTAL:**

Received      Book Chapter

**TOTAL:**

**Patents Submitted**

---

**Patents Awarded**

---

**Awards**

none

---

---

**Graduate Students**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

---

**Names of Post Doctorates**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
M. Turalska	1.00
<b>FTE Equivalent:</b>	<b>1.00</b>
<b>Total Number:</b>	<b>1</b>

---

**Names of Faculty Supported**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

---

**Names of Under Graduate students supported**

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

---

**Student Metrics**

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: ..... 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:..... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):..... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense ..... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:..... 0.00

---

**Names of Personnel receiving masters degrees**

<u>NAME</u>
<b>Total Number:</b>

---

## Names of personnel receiving PhDs

<u>NAME</u>
<b>Total Number:</b>

---

## Names of other research staff

<u>NAME</u>	<u>PERCENT SUPPORTED</u>
<b>FTE Equivalent:</b>	
<b>Total Number:</b>	

---

## Sub Contractors (DD882)

## Inventions (DD882)

## Scientific Progress

A two-state, master equation-based decision-making model has been shown to generate phase transitions, to be topologically complex, and to manifest temporal complexity through an inverse power-law probability distribution function in the switching times between the two critical states of consensus. These properties are entailed by the fundamental assumption that the network elements in the decision-making model imperfectly imitate one another. The process of subordination establishes that a single network element can be described by a fractional master equation whose analytic solution yields the observed inverse power-law probability distribution obtained by numerical integration of the two-state master equation to a high degree of accuracy.

A spectral decomposition method is used to obtain solutions to a class of nonlinear differential equations. We extended this approach to the analysis of the fractional form of these equations and demonstrate the method by applying it to the fractional Riccati equation, the fractional logistic equation and a fractional cubic equation. The solutions reduce to those of the ordinary nonlinear differential equations, when the order of the fractional derivative is  $\alpha = 1$ . The exact analytic solutions to the fractional nonlinear differential equations are not known, so we evaluate how well the derived solutions satisfy the corresponding fractional dynamic equations. In the three cases we find a small, apparently generic, systematic error that we are not able to fully interpret.

Many adaptive evolutionary systems display spatial and temporal features, such as long-range correlations, typically associated with the critical point of a phase transition in statistical physics. Empirical and theoretical studies suggest that operating near criticality enhances the functionality of biological networks, such as brain and gene networks, in terms for instance of information processing, robustness, and evolvability. While previous studies have explained criticality with specific system features, we still lack a general theory of critical behavior in biological systems. Here we look at this problem from the complex systems perspective, since in principle all critical biological circuits have in common the fact that their internal organization can be described as a complex network. An important question is how self-similar structure influences self-similar dynamics. Modularity and heterogeneity, for instance, affect the location of critical points and can be used to tune the system toward criticality. We review and discuss recent studies on the criticality of neuronal and genetic networks, and discuss the implications of network theory when assessing the evolutionary features of criticality.

## Technology Transfer

M. Turlaska became a post doctoral researcher at the Army Research Laboratory in order to transfer her skills in network science to the Army research community.