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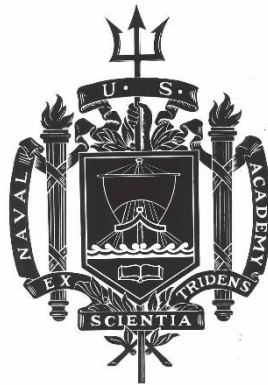
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**Asymmetrically Loaded Gridshells: A Parametric Study on Bracing Methods and Case  
Study for Use in Disaster Relief**

by

Midshipman 1/C Thomas B. Imhoff, USN

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## **Abstract**

From 1970 to 2000, 68 percent of United States naval operations were categorized as Humanitarian Assistance and Disaster Relief (HA/DR). These relief efforts identified a need for deployable, lightweight structures, capable of sheltering a large area. Large-span shelters are necessary in HA/DR because they provide space for community gathering, emergency services, educational facilities, and worship. Field work in Athens, Greece and input from external collaborators confirmed that this need is not only physical, but is both culturally and emotionally significant. Gridshells are a potential solution to this need because of their portability, sustainability, and efficiency. Formed from flat, linear members bent on-site into a curved surface, gridshells are relatively large structures that can be built without the use of heavy machinery. To broaden the use of gridshells, this paper quantifies the effect of asymmetric loading and bracing on their performance, as both are likely occurrences in HA/DR. The design requirements and constraints for the gridshell were established in Athens, Greece and with international building codes. The structural analysis included a parametric study that assessed the effect of the grid density, bracing patterns, and load cases (gravity, asymmetric, and point loads) on the failure load. These results, along with the design constraints, governed the design of the HA/DR gridshell. The results generated from this analysis will contribute to the overall design space of gridshells, specifically regarding the effects of bracing and the impact of asymmetric loads. This knowledge will broaden the design space by illustrating relationships between physical parameters and structural performance. The constructability was evaluated with small-scale models and a half-scale prototype. Having verified the structural performance, deployment mechanisms, storage schemes, and joint movements, gridshells were proven to be solutions that meet the distinct needs in HA/DR efforts.

**Keywords:** gridshells, humanitarian assistance, disaster relief, parametric studies, bracing, asymmetric loads

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## **1. Introduction**

This paper focuses on understanding the structural performance of gridshells under non-ideal load cases and their potential applications in Humanitarian Assistance and Disaster Relief (HA/DR) efforts. The following sections explain gridshell technologies in general, as well as current HA/DR operations as they pertain to both structures and the United States military.

### **1.1. Gridshells**

Gridshells are shell structures made up of linear members that are bent into a doubly curved surface. Gridshells are commonly used in architecture to enclose existing spaces, such as the Great Court at the British Museum in London (Figure 1a), or as permanent self-supporting structures, such as the Downland Gridshell at the Weald and Downland Museum in the United Kingdom (Figure 1b).

Gridshells structures are currently implemented in a variety of modes. The most common applications are museums, atriums, covered courtyards, public venues, and music halls. Gridshells are economically feasible and environmentally friendly structures that combine structural efficiency with the elegance and grace of free form architecture.



Figure 1: Examples of gridshells. (a) The Great Court at the British Museum in London [1]. (b) The Downland Gridshell at the Weald and Downland Museum in the United Kingdom [2].

Gridshells resist applied loads due to their inherent curvature. The specific curvature is determined by the natural catenary shape of a model. Engineers historically discovered this shape by hanging small chains onto a mock-up of the gridshell's footprint. Each chain represented a member of the gridshell and took on a catenary curvature due to the force of gravity (Figure 2). The resulting shape was the ideal form for that shell given the specific footprint and member lengths. This shape was interpreted using photographic and mathematical methods in order to model the organic curve as an analytical solution. This process, from hanging chain model to analytical solution, became known as "form-finding." Modern techniques for form-finding utilize computational methods including finite element analysis, transient stiffness methods, and dynamic relaxation techniques and have largely replaced hanging chain models.



Figure 2: Mannheim Gridshell in Germany. (a) Hanging chain model [3]. (b) Constructed gridshell [4].

Gridshell lattices can be made of nearly any material, but common options include wood, steel, concrete, and composites [5]. Lattices constructed with flexible materials such as wood or composites are initially assembled in a flat plane prior to being deformed on-site into a final curvature via tension cables, scaffolds, or gravity. Figure 3 shows the Downland Gridshell throughout the construction process [2]. Prior to deformation, the flat timber lattice was constructed above ground on scaffolding (Figure 3a). Utilizing gravity, the lattice was deformed as the scaffolds were slowly lowered and ultimately removed (Figure 3b). Once the shell was fully deformed, a tension tie was wrapped around the perimeter of the base to prevent the shell from expanding, and triangulation members were added to increase structural stiffness (Figure 3c).



Figure 3: Time-lapse of the construction of the Downland Gridshell [2]. (a) Prior to deformation. (b) Mid-deformation. (c) Fully deformed, complete gridshell structure.

The induced deformation of a gridshell introduces additional stiffness and stability to the structure due to the strain experienced by the members. These types of structures, those deformed on site, are the focus of this project. Lattices made of steel or other rigid materials are fabricated in their final geometry and assembled on-site, thus the stiffening effects due to deformation are less pronounced. After assembly, a gridshell can be clad in materials such as PVC, glass, or wood in order to better enclose the space [6, 7, 8, 9].

Timber gridshells made of deformed lattices rely on the lattice's lack of in-plane shear stiffness during the construction phase. This unique property allows the structure to deform into a doubly curved surface because the nodes are free to translate and rotate. However, once the final form is achieved, in-plane shear stiffness must be established in order to stiffen the structure. Current methods include tightening the joints to avoid sliding and adding additional diagonal bracing, though rules of thumb are yet to be established.

In-plane shear stiffness contributes to the stiffness, stability, and strength of the gridshell under load. Gridshells are designed to withstand symmetric loads because of their origins as catenary

structures formed by gravity. Global shell buckling caused by overload is the primary failure mode. Gridshells are designed to support symmetric loads rather than inherently destabilizing asymmetric loads.

### 1.2. HA/DR and the United States Military

Between 1970 and 2000, 68% of United States naval operations involved responding to a humanitarian crisis [10]. Since then, the U.S. and other world navies have continued to respond swiftly to HA/DR situations across the globe. Following a disaster, distressed regions require food, water, temporary shelter, and medical services. These services must be provided quickly to prevent additional loss of life and to help establish a continuum of care (Figure 4a). Following the immediate response to the emergency, efforts shift towards short term restoration operations and reconstruction. This process may take months but is ultimately aimed at empowering the regional government to rehabilitate and reconstruct its country in the long term. Military forces, especially expeditionary naval forces, are designed for quick reaction and have the material resources to provide support during the early stages of disaster response; as a result, military forces are often employed for HA/DR.

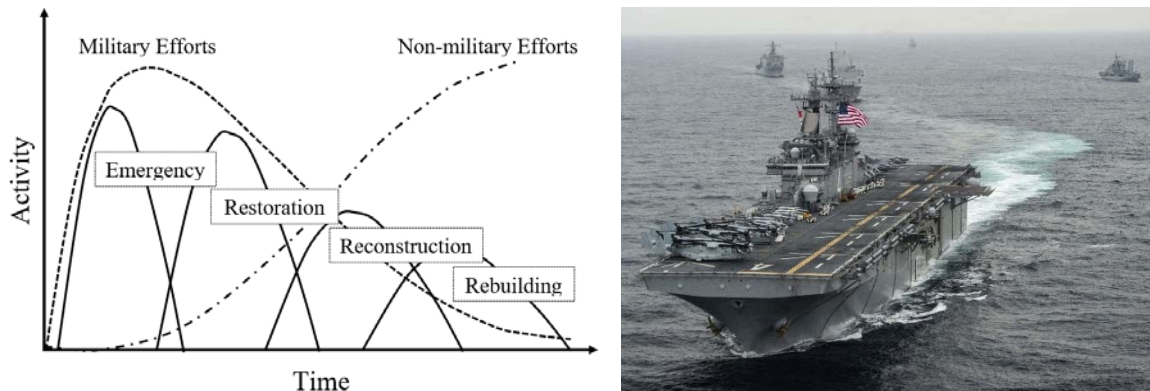


Figure 4: (a) HA/DR continuum of care. (b) U.S. Navy Amphibious Ready Group underway [11].

The U.S. Navy's premier platform for HA/DR is the Amphibious Ready Group, colloquially known as an "ARG" (Figure 4b). This mobile entity consists of an amphibious assault ship, an amphibious transport dock, a dock landing ship, a full complement of helicopters and tilt-rotor aircraft, air-cushion landing craft, traditional landing craft, and amphibious assault vehicles. Each is specially designed to quickly move cargo and personnel from "ship-to-shore," which greatly benefits HA/DR operations [10]. Among the supplies that an ARG can bring to a disaster zone are the means to build small shelters designed to hold four to eight people. The lack of heavy machinery and infrastructure in relief zones often prevents larger structures from being built, despite the need for large sheltered spaces for community gathering, dining, worship, education, and emergencies. This need is largely overlooked in HA/DR response. Gridshells could offer a solution because of their deployable construction methods, structural performance, and inherent beauty.

## **2. Literature Review**

The literature review is a cursory examination of relevant articles related to timber gridshells and other large span gridshell structures. The following sections examine bracing theory as it relates to gridshell structures in order to explain why gridshells must be braced. The report also discusses existing techniques used to brace gridshells and common loading cases for gridshells. Subsequently it discusses the cultural impacts of gridshells and methods for structural analysis.

### **2.1. Bracing Theory**

A shell resists applied loads due to its inherent shape [5]. Professor Frei Otto designed one of the first gridshell structures, the Mannheim Multihalle, using a hanging chain model. This model was based on the principal of catenary cables because it allowed gravity to pull small chains, which were attached to a scale footprint of the shell, into an ideal deformed shape resembling a bowl. When inverted, the bowl shape became a domed structure and represented the ideal catenary curvature of a shell given the same footprint and member lengths of the hanging chain model. During these tests, Otto noted that if the intersection point of the chains, representing the intersection nodes of the gridshell, could rotate, the resulting structure would have no in-plane shear stiffness [12]. While a continuous shell can resist in-plane shear stress, a gridshell lattice with rotating nodes cannot [13]. The lack of shear stiffness is a result of the lattice structure, which is fundamentally restricted to a single degree of freedom. Because lattice members are crisscrossed and held together on rotating joints, movement by one member will cause movement by all others. The single degree of freedom is necessary to form the doubly curved surface that defines a gridshell, yet after construction the shell must be stiffened in order to improve its overall stability. In order to create the required stiffness, Otto suggested tightening the joints or adding diagonal bracing, also known as a cross-tie. Joint tightening was effective for small deformations, while large deformations, like that seen in gridshell construction, required additional members [12, 13].

The lack of shear stiffness created by the lattice structure is necessary during the construction phase, yet once properly formed the gridshell must be braced in order to improve its stiffness. While tightening joints and adding diagonal members are viable solutions for stiffening gridshells, no past or ongoing research investigates the amount, frequency, and orientation of additional diagonal bracing.

### **2.2. Bracing Implementation**

Despite a lack of knowledge regarding the behavior of braced gridshells, structural engineers are aware of the importance of stiffening gridshells once they reach their final form. Timber gridshells are often braced by tightening the joints and adding cross-ties, while steel and other prefabricated gridshell rely more heavily on tension cables [12, 14].

Theory states that bracing a gridshell is necessary in order to establish resistance to in-plane shear stress. One common method to stiffen shells is to add cross-ties or diagonal bracing; however, the location, orientation, and frequency of these braces has historically been rather arbitrary and remains unsupported by engineering analysis. Designers often spend time determining an adequate cross-section size for additional bracing members, yet because the cross sectional area itself does not have a significant impact on in-plane shear stiffness, a bracing member's resistance to out-of-plane bending often dictates its size [12].

In some instances, such as the Mannheim Multihalle, the bracing was constructed using cables rather than wooden members [12]. The cables served as an effective means of bracing the structure using tension; however, once again little analysis supported the requirements and specifications of the bracing. The Palacio de Comunicaciones in Madrid, a steel gridshell, also utilized cables as a bracing mechanism, yet once again they were placed in an admittedly arbitrary manner [15].

A second type of bracing is accomplished using the joints themselves. The intersection nodes of timber gridshells are designed to rotate and slide during the construction phase, but once the final curvature is achieved they must be locked in place (Figure 5). One type of joint is the slot-screw joint where members overlap at the location of a bored slot and a loose bolt runs through the node. This setup allows the slotted members to slide and rotate freely during construction, but once the structure is assembled the bolt is tightened to prevent translation and rotation, thereby inducing stiffness. This solution is very easy to implement, though it lacks some versatility because some nodes will slide more than others, leading to non-standard slot lengths, and the slots themselves can create stress concentrations. A second type of joint is the screw-plate joint. These joints are made up of two plates, often metal, connected to one another in each corner by a long bolt. Initially the plates are loosely tightened at the intersection point to allow the members to slide during construction, but after the final form is achieved, the bolts are tightened to stiffen the structure and prevent movement. This method requires additional resources and is often heavier than its slotted counterpart, but is highly versatile and avoids stress concentrations. In both cases, the bolts can be made intentionally to protrude above the outer surface of the shell to support the attachment of bracing members after the initial construction phase. The Weald and Downland Museum used bolts protruding from the surface of the shell to add diagonal bracing ties and the Chiddingstone gridshell utilized the same long bolt protrusion method to support a frameless glass roof suspended above the grid [13].

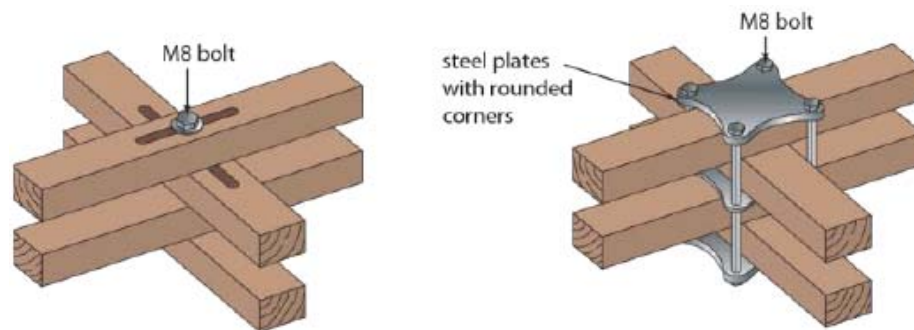


Figure 5: Timber gridshell joints. (a) Slot-screw joint. (b) Screw-plate joint [13].

A third method for bracing a gridshell structure is the addition of a retaining ring or tension tie to secure the outer footprint of the structure. The ring around the structure absorbs the horizontal thrust loads from the shell, allowing it to retain its shape. The ring itself allows the shell to transmit its load primarily in the vertical direction onto the surface or structure upon which it rests [16]. This technique is often used when bracing steel gridshell structures such as pavilion at the Palacio de Comunicaciones or the Dutch Maritime Museum, though a similar method could be adapted for timber shells [15, 17].

Regardless of the bracing material or bracing method utilized, implementing bracing requires further analysis and research. Little research has been dedicated to understanding how bracing affects structural performance or what parameters define an effective bracing scheme.

### 2.3. Load Cases

During any design process, the structure in question is subjected to various load cases in order to evaluate its performance [17]. The load cases that are presented during these analyses are often the worst case scenarios that the structure is likely to encounter over its lifetime based on the region, climate, and type of building [12].

Gridshell design requires the evaluation of two primary load cases. The first case encompasses the loads applied to the shell once it is fully constructed and the second case encompasses the loads that the shell experiences during the construction process itself [12].

After construction, the primary concern is a gridshell's ability to support its own self-weight [18]. This loading regime is based on a gravity load spread symmetrically over the entire shell. The self-weight is a distributed load that results from the weight of the physical building materials (members and joint hardware) that make up the shell. Ideally the hanging chain model, and similar computational simulations, will ensure that an unloaded gridshell is subject only to its self-weight. In reality, the induced loads are slightly greater and are not completely symmetrical, yet the assumption holds for most general analyses [12].

The self-weight of the structure is generally the benchmark performance indicator for a timber gridshell. The maximum load that a gridshell can support is often expressed as a multiple or factor of the self-weight of the structure, assuming that the new load applied is symmetric or close enough that the symmetric assumption is valid [12]. In addition to the self-weight of the structure, symmetric loads from precipitation and membranes are taken into account and referenced with the self-weight of structure as described above. The global geometry of the shell is the most fundamental indicator of stability, safety, and stiffness [19]. As a result, shell bending and unwanted deformation must be avoided at all costs, thus symmetrical loads are the primary loading cases that gridshells are designed to support because they do not inherently produce instabilities or deviations in global geometry.

Asymmetric loads are particularly difficult loading schemes for gridshells to handle because of their unbalanced nature and tendency to cause global deformation, leading to buckling and ultimate failure. Gridshells rely on their specific global curvature to support loads. This curvature is both the gridshell's greatest asset, allowing it to carry loads with extremely high efficiency, yet is also its greatest risk, as deviations from the curvature can lead to global buckling. Asymmetric loads are likely to cause deviations from the prescribed catenary curvature, thereby increasing the risk for failure. Some asymmetric load cases are understood and documented, though these are limited to point loads associated with maintenance personnel climbing atop large shells. In these cases the loads were not treated as a primary loading schemes because of the scale of the shell and the fact that the structure could withstand multiple times its self-weight distributed across its surface, making the additional load of a maintenance worker insignificant. In these instances, the impact of an individual person atop the structure was considered negligible [12].

Each of the above load cases discusses the loads that gridshells could encounter once their final curvature is achieved. In addition to the loads applied to the completed structure, the construction process itself must be analyzed to ensure that the members are not overloaded to the point of failure during the process. Once again, little research supports the analysis of the construction process, though most of the induced loads are a product of self-weight and bending stress [12].

#### 2.4. Timber Gridshells

Timber gridshells are a subset of gridshells structures that begin as flat structural lattices that are subsequently bent on-site into their final form. These structures use a “low tech” renewable resource to construct a “high tech” engineering solution [5]. Timber is especially suited for gridshell structures because it has a relatively low modulus of elasticity and can be easily bent into shape, but is subject to the effects of natural variation, differing moisture content, and grain direction. Wood is a simple and sustainable material which is advantageous for construction projects [12].

Timber gridshells range from small shells that are used as academic research projects to large scale structures commissioned by paying clients for commercial use. Small gridshells can cover areas of less than 50 m<sup>2</sup> (540 ft<sup>2</sup>) while large shells can cover areas in excess of 9,500 m<sup>2</sup> (102,205 ft<sup>2</sup>) [20]. The Mannheim Multihalle, Weald and Downland Museum, and Savill Gardens are examples of large timber gridshells, while the ZA Pavilion and the Toledo Shell are examples of small scale research projects [12, 20, 21]. Small shells often use more innovative techniques and test new methods so that these novel concepts can eventually be utilized in large scale gridshells [20].

Most trees from which the timber for gridshell beams is sourced are not long enough to cover the total span required by the design. To solve this problem, members are often spliced together using a running bond pattern. Each beam is fabricated as a “half cross-section.” These reduced cross-sections are layered atop one another in an alternating pattern to create a “full cross-section” member of the proper length [13]. The running bond pattern is illustrated in Figure 6.



Figure 6: Splicing pattern for beams made up of half cross-sections.

Most gridshell lattices are assembled on site as a large continuous grid. However, recent developments illustrate a new concept of combining multiple small lattices on-site to form the total structural lattice. Frei Otto conceived the idea of combining small prefabricated lattices into a large grid in order to cut down on transportation logistics and eliminate the need for advanced carpentry on-site. The Toledo Gridshell utilized the prefabrication method successfully [21].

The species of wood used to construct a gridshells differs based on the engineering requirements and material available. A combination of the material properties of the selected wood and the curvature required by the shell helps to determine an adequate cross section for the beams of the gridshell. Common woods used in gridshell construction for their relatively low modulus of elasticity and ability to bend naturally are hemlock, oak, and larch [13].

Timber gridshells are built using a variety of methods. The two most common methods are the “pull-up” and the “push-up” methods. The pull-up method uses cranes and scaffolds to pull the lattice from the ground into its final elevated form. The “push-up” method takes advantage of jacks and scaffolds beneath the lattice to push it into the final curvature. A less common method is the “let-down” method, which was used to construct the Weald and Downland Museum. In this process, the lattice begins lying flat on elevated jacks and scaffolds. These supports are slowly lowered, deforming the shell into its final form [20]. During any construction process the timber takes on high initial stress, but over time the loads relax and allow service loads to be applied without fear of failure [12].

One important aspect to note regarding the use of timber as a structural material is its inherent flammability. Building codes specify the extent to which timber must be fireproofed, if at all, but regardless, these fireproofing treatments often alter the material properties of the wood and should be taken into account during the analysis [5, 12].

## 2.5. Steel Structures

While fundamentally different from timber gridshell structures in the construction phases, steel gridshells are analyzed using similar methods. They are often the product of prefabricated nodes set to the specific form-found global geometry of the shell. Beams cut to specific lengths are affixed between the nodes, thereby forming the shell [18, 22]. Examples of famous steel gridshells are both the Great Court at the British Museum and the Dutch Maritime Museum, illustrated in Figure 7. Bracing is still required for steel gridshells and often comes in the form of steel retaining rings or tension cables. The Palacio de Comunicaciones uses both, though the cables are admittedly placed arbitrarily. The retaining ring serves to absorb the horizontal thrust loads rather than to establish in-plane shear stiffness, but still supports the shape of the shell [15]. The British Museum takes advantage of diagonal bracing at the corners of the shell to compensate for the bending stresses induced by the thrust loads from the shell itself [23].

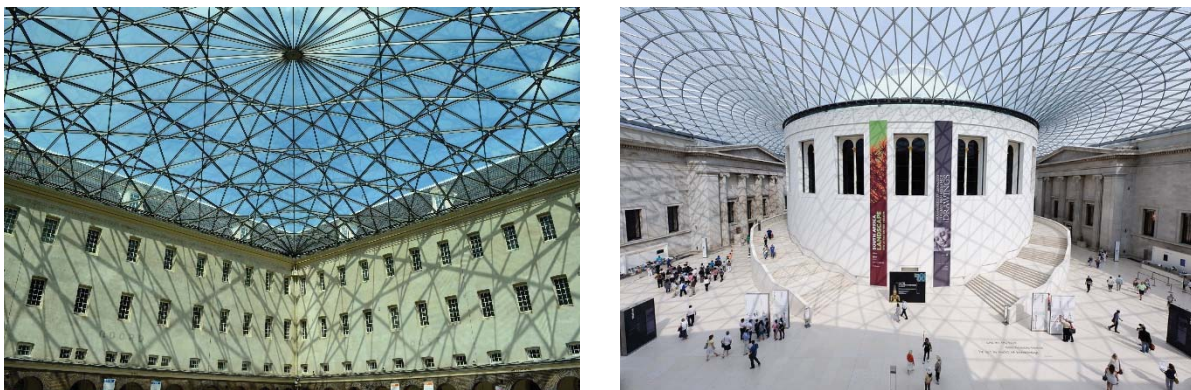


Figure 7: Steel gridshells. (a) Dutch Maritime Museum [24]. (b) The Great Court at the British Museum [25].

## 2.6. Cultural Impact

Gridshells design is driven by a combination of sculptural, geometrical, and physical considerations. The design process is a creative process and functions much like free-form architecture, giving designers the freedom to develop unique shapes and interesting forms [19]. Gridshells are innovative and fresh structures that cover large areas, can be assembled on-site, are

lightweight and sustainable, and are elegant solutions to structural problems. The Dutch Maritime Museum features a steel gridshell roof that replicates the patterns of a Dutch shipping map. This is just one example of how gridshell design can be adapted to meet very specific needs [17]. The structures can be complex during the design sequence, but gridshell technology ultimately exhibits untapped potential beyond purely architectural applications [20].

Gridshells are often used as functional structures within a community. The Mannheim Multihalle was purpose built to accommodate a biannual federal garden show in West Germany, while the Weald and Downland and Great Court gridshells were built as museums [12]. The Palacio de Comunicaciones features a shell made of steel and glass that covers a historic courtyard in Madrid, making it the premier location for high-level receptions [15]. Lesser known shells have been designed as music venues or works of art, yet each retain the overall purpose of serving as public gathering spaces [10]. As a result of their unique design, gridshells tend to draw the population, resulting in local pride, economic stimulus, and enhanced physical beauty [12].

## 2.7. Methods for Analysis

Gridshell design is based on a funicular method where beams are modeled as tiny chains pulled down by the weight of gravity. As long as the chains are free from in-plane shear stress, the resultant form of the hanging chain model is the most favorable shape of the shell for a given footprint. If the hanging geometry is set in place and inverted, forming a dome, the curvature of the resulting shell is ideally suited to carry its self-weight. In this form, a shell could theoretically be extremely thin, but in practice the thickness of the members is increased in order to withstand non-ideal loading and prevent global buckling [12]. The four main factors that dictate gridshell design are the existing environment where the shell is to be placed, the sculptural and aesthetic qualities of the shell, the geometric shape of the shell, and the structural efficiency of the shell. Often times these are approached as separate issues, yet the more that can be combined will result in a higher quality product [19].

Form-finding for gridshell like the Mannheim Multihalle and other early shells was done primarily through physical hanging chain models (Figure 2). These models were photographed and subsequently modelled as mathematical equations, thereby yielding the final curvature of the shell in an analytical form [12]. Modern gridshell form-finding techniques still utilize the same funicular method, yet most of the calculations and models are computational rather than physical. Often times the general curvature of the desired shell is drawn by a designer and computational tools are utilized to fit a shell to the prescribed shape [13]. The numerical methods applied were initially limited to a few shapes or types of geometry, thus many existing gridshells look similar to one another. As technology and computing continue to evolve, design is less limited by software and computing power [18].

In general, the numerical method for designing and analyzing a gridshell begins with forming a three dimensional model of the structure using gravity loads, thereby achieving the catenary shape previously mentioned. Once the model is finalized, it is transformed into a load bearing grid, or mesh, so that a finite element analysis (FEA) software can simulate the structural performance of the model [18]. Some steel gridshells require advanced mathematics during the form-finding phase to ensure that conditions such as planar grid spaces are maintained so that flat glass panes can be

inlaid once the gridshell is constructed [17]. For timber gridshells, planarity is generally not an issue because coverings are often made of plastic sheeting or textiles that do not need to remain flat in a single plane, and instead are quite pliable. Once the load bearing grid is modeled and supported, loads are applied to the structure and analyzed using FEA software [21].

During numerical analysis, the load cases applied are often the worst case scenarios. Global shell buckling is the primary mode of failure for these structures, thus it is the failure criteria that engineers focus on most heavily [19].

Gridshells are often constructed as single layer shells or double layer shells. These terms are slightly misleading, as single layer shells are actually made up of two layers, while double layer shells are made up of four layers. A single layer shell consists of one layer of beams arranged in a single direction, and a second layer of beams on top of the first layer, arranged in a near-perpendicular pattern. The result is one lattice, made of two layers of beams. A double layer gridshell consists of four layers of beams, and results in two single layer lattices stacked on top of one another. The beam directions alternate in a near-perpendicular pattern in order to yield a proper grind, but exact perpendicularity is not necessary, as the beams will shift during construction. Despite how gridshells are actually constructed, most basic numerical analysis engines model gridshells as a continuous lattice. In this analysis, there is no beam stacking, instead, the lattice is modelled within a single plane prior to deformation. The joints function as if the structure were a true single or double gridshell, but the structural analysis applies loads to the continuous lattice. During the design of the Schubert Club Band Shell in Minnesota the first analysis of the shell used the continuous lattice assumption, but further analysis utilized a physically accurate representation of the single layer shell by rendering both layers. The two analyses yielded results within fifteen percent of one another, well within a margin of error that would be covered by an appropriate factor of safety [14].

The mechanics of the calculations used to determine the structural performance of gridshells rely on the basics of FEA. Nodes and elements are discretized from the shell beams in order to create stiffness matrices that are solved to calculate the displacement, and thereby forces within the each member. This basic FEA technique holds up for small displacements but often fails when large displacements occur. As a result, many computer programs model the behavior of the gridshell with large deformation algorithms in mind. These algorithms can utilize linear or non-linear techniques to solve the model. Both are valid methods to numerically evaluate a gridshell structure [21].

## 2.8. Literature Review Summary

Gridshell structures offer a unique method of utilizing a sustainable material in a novel construction mode to produce elegant, yet structurally efficient large-span structures. The methods for analysis and construction of gridshells have been well researched, though the field is constantly developing. Areas where knowledge must be contributed to the field in order to continue its advancement revolve around bracing methods, asymmetric loading conditions, and adapting shells to broaden their use. Specifics of these lacking research areas include investigating the impact of bracing orientation, density, and type on the overall structural performance of a gridshell, as well as rules of thumb and overdesign factors for asymmetric loading cases. Research in these specific

areas would contribute to the advancing field of gridshell structures and has the possibility to open the design space for this novel construction method.

### **3. Problem Statement**

Most gridshells are designed to support symmetric distributed loads and are braced with additional material to increase the stiffness of the structure. As suggested by the literature review, one of the challenges associated with gridshell structures is that they function best when subjected to uniform loading [12]. Outside of architectural implementation, gridshells will likely be subjected to point loads and asymmetric loads, both of which are traditionally problematic for gridshell structures. Literature also suggests that gridshells have been braced in the past, and it is well known that bracing improves structural performance; however, the degree to which bracing affects the stiffness and strength of a gridshell has not been researched. In order to further advance gridshell technologies and adapt them for new applications such as HA/DR, additional research must investigate performance under non-traditional load cases and the measurable effects of bracing.

### **4. Project Methodology Overview**

In order to properly address both facets of the problem, namely understanding the structural effects of bracing and asymmetric load cases, as well as the design characteristics required to adapt the structure for use in HA/DR, the project was split into two distinct studies. The first was a computational parametric study on the stability of gridshells under various load cases and bracing layouts. The second study focused on the applications, constructability, and deployment mechanisms of gridshells as they relate to HA/DR. Combining both studies, the following actions were taken in order to achieve the ultimate objective of designing a gridshell for HA/DR:

- 1) Quantified the effects of bracing and asymmetric loads on the structural performance of gridshells through a computational parametric study.
- 2) Established constraints for an HA/DR gridshell using fieldwork in Athens, Greece and international standards.
- 3) Selected the most feasible design to carry forward based on the results of the parametric study and HA/DR considerations.
- 4) Built small scale models to test the constructability of the proposed design and used those lessons to construct a half-scale prototype HA/DR gridshell.

A flowchart representation of the research process is shown in Figure 8.

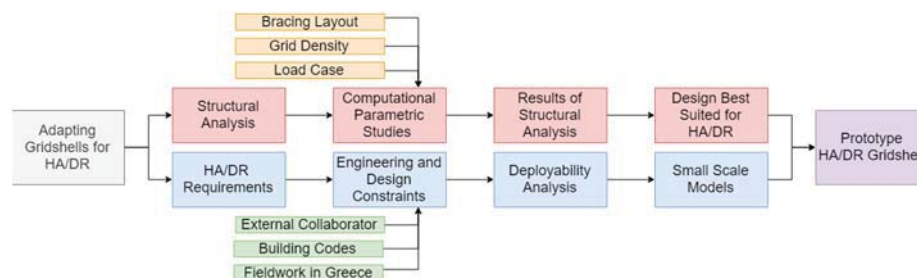


Figure 8: Research methodology flowchart.

## 5. Computational Parametric Studies and Structural Analysis

The first phase of this project focused on modelling the structure in the computational domain and subsequently analyzing its structural performance as various parameters were altered (grid density and bracing layout) and different loads (gravity, asymmetric, and point loads) were applied.

### 5.1. Structural Analysis Outline

The studies began by creating a candidate structure to serve as the subject of the parametric variation. The structure was influenced by the Essen Shell and the design of the ZA Pavilion in Cluj, Romania, one of the most successful small timber gridshells (Figure 9).

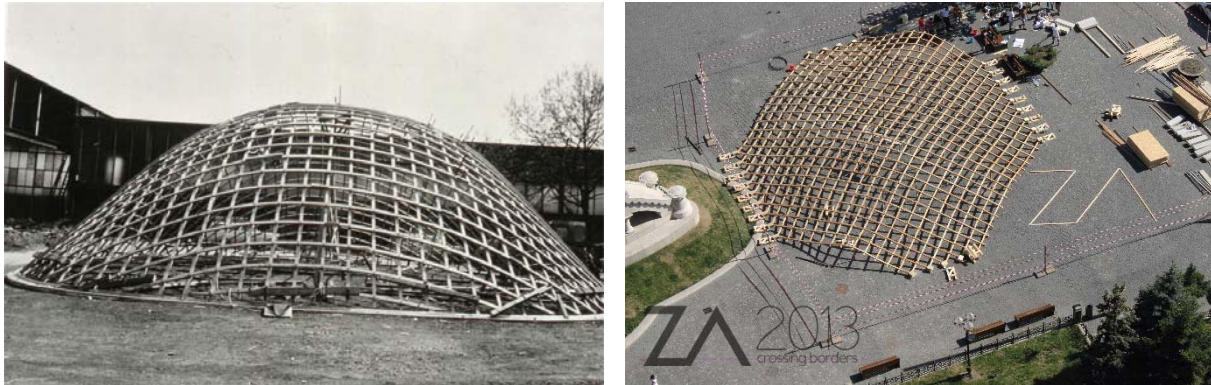


Figure 9: Gridshell influences. (a) The Essen Shell [26]. (b) The ZA Pavilion [27].

After some entry level tests and slight modifications, the parametric studies using the candidate gridshell began. The study focused on varying the grid density and bracing layout under various load cases. The cross-section of the members was held constant throughout. Data generated from these analyses was formatted to establish trends. These trends were used to map the structural performance of the gridshell as its geometric layout changed. At the conclusions of the parametric studies, the behavior of the shells under various conditions was documented and the structural design best suited for HA/DR was selected.

### 5.2. Candidate Structure Design

The candidate gridshell was designed to mimic the size and shape of what would become the HA/DR gridshell. The global geometry was based on the size of the ZA Pavilion (12 m by 18 m (40 ft. by 60 ft.)), the Essen Shell (15 m by 15 m (50 ft. by 50 ft.)), and other successful small timber gridshells. The candidate structure design was distilled during the validation period, before the parametric studies began, to ensure a quality design that would produce quality results. The selected candidate design was intentionally predictable and mimicked existing designs rather than introducing a novel global geometry so that the results could be applied to a broad range of structures.

#### 5.2.1. Global Geometry

The initial concept for the candidate structure was a 15.24 m (50 ft.) square structural lattice with four supports, one at each individual touchdown point in each corner. The center height was initially assumed to be 9.14 m (30 ft.) for each iteration of the structure. Figure 10 shows the initial representation of the gridshell.

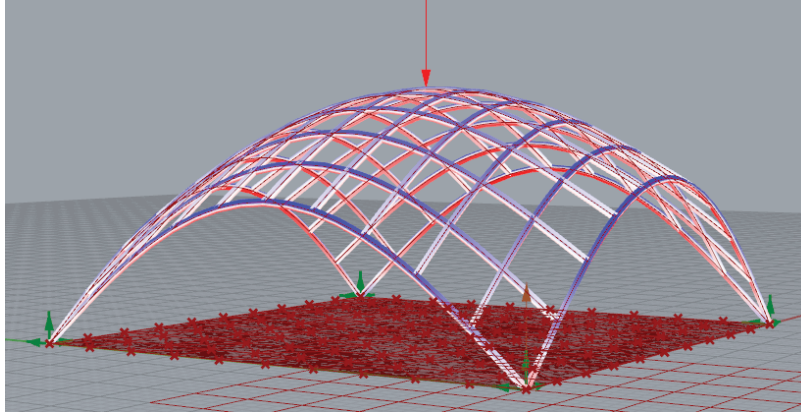


Figure 10: Initial concept gridshell formed by a corner-supported square lattice.

This initial global geometry was used to begin validating the FEA results by comparing Karamba's results with those of other FEA engines; however, over the course of these studies, two design flaws became apparent. First, the maximum height of the structure was much too tall to mimic existing gridshells and as a result was causing local failure in numerous members. The solution to this problem was to change the height constraint. Rather than setting the maximum height at an arbitrary threshold value of 9.14 m (30 ft.), the entrance height was henceforth set to a constant height, 2.13 m (7 ft.), to accommodate both ergonomic access and the egress constraints laid out in the International Fire Code [28]. This change decreased the tendency of the shell to experience local failure in individual members.

In addition to the height change, it became apparent that single touchdown points were structurally infeasible and impractical to construct. The initial design was supported by four single touchdown points, one in each corner. The result was that only eight individual members were holding the entire weight of the structure. Extremely high compressive forces in those members caused local failure in the beams before the final form could be achieved. In addition to the structural issues presented by individual touchdown points, building such a support system was deemed impractical and would require specialized hardware and load distribution schemes to prevent the supports from sinking into the ground. To rectify these issues, the supports at each corner were modified to encompass three distinct touchdown points. This method has been used in existing gridshells and solved the problem of overutilization and local failure in individual members. Figure 11 shows the original supports, the improved geometry, and an example of the support system used at the ZA Pavilion.

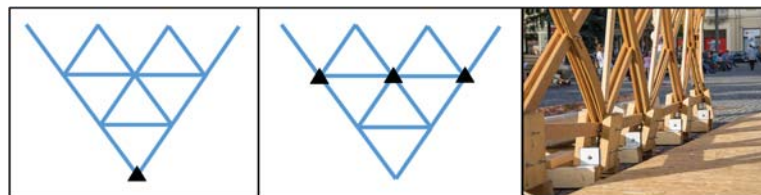


Figure 11: Support system. (a) Initial geometry. (b) Improved geometry. (c) Actual supports [29].

The improved candidate gridshell to be used in the subsequent parametric studies featured a 15.24 m (50 ft.) square grid with four corner supports, each encompassing three distinct touchdown points, and a fixed entrance arch height of 2.13 m (7 ft.). The computational model ultimately

assigned material properties to the structure so that it would mimic the behavior of wood (modulus of elasticity defined as 105 MPa (15 ksi), and yield strength defined as 0.13 MPa (0.019 ksi)). The initial member cross-section was 7 cm (2.76 in) wide by 3 cm (1.18 in) thick. The gridshell was modeled as a continuous lattice, though in reality, a single layer gridshell would be made up of two layers of stacked beams. A single layer shell and a true double layer shell exhibit the same overall trends and yield results within 15% of one another [13]. Figure 12 shows the final design of the candidate gridshell. The structure was form-found in Grasshopper, a graphical algorithm editor, using the method described in the following section.

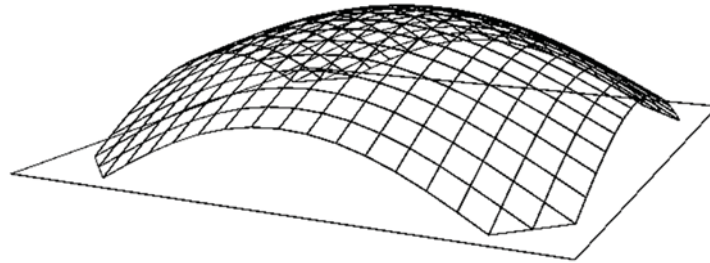


Figure 12: Improved candidate gridshell structure with improved supports and entrance height.

### 5.2.2. Computational Modelling

The software used to conduct the parametric studies was based in Rhinoceros, a 3D modelling software. Grasshopper, a graphical algorithm editor, is a plug-in that allows parameters that would normally be hard-coded into a Rhinoceros drawing to be altered in real time. This section explains how the candidate gridshell was modelled, varied, and evaluated in the computational realm.

First, the square border of the structural lattice was drawn in the XY plane. Additional lines were drawn in the X and Y directions at regular intervals to bisect the large square border into a grid. The number of intersecting lines was the same in the X and Y directions, but that number was variable in order to allow for the variation of the grid density. The intersection points of the grid lines became the points from which the lattice geometry was generated. These points were assigned local indices in a repeating pattern. By connecting various indices, the grid shells beams in the X and Y directions and the diagonal bracing members were generated. These lines were converted into structural beams by assigning each of them the cross-section and material properties specified above. Note that all members, including bracing members were modelled with the same cross-section and material properties. Figure 13 shows a visual representation of the assembly process.

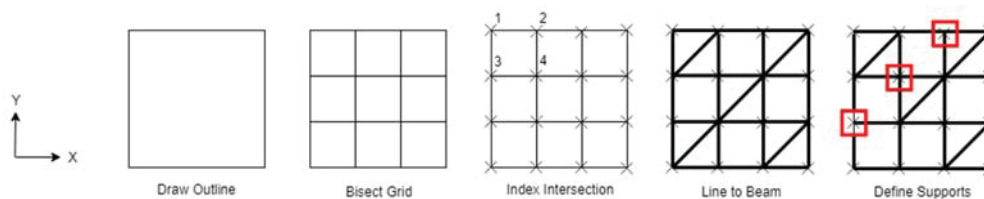


Figure 13: Progression of computational model generation.

Supports were added to the three touchdown points in each corner. All of the supports were fixed in the Z plane, but one support was also fixed in the X and Y plane for translation, as well as fixed in rotation in the Z direction. This support system prevented any rigid body modes during the analysis while ensuring that three of the four supports were still permitted to translate in the XY

plane as the structure changed form. Without translating three of the four supports, the members would be forced to stretch excessively into the final form rather than moving with the structure as it took shape. Stretching the members does not properly represent the physics of gridshells, namely inextensible deformation, but translating the supports mimics the behavior very closely.

The beams and supports were assembled into a functional model of the gridshell. An inverted gravity load was added to this structure to pull the lattice up into a catenary shape. The gravity factor, the percentage of gravity applied to the structure, was altered until the peak height of each entrance arch reached the desired 2.13 m (7 ft.). The deformation of the model during the form-finding process was analyzed using a large deformation algorithm. Traditional deformation analyses are based on small deformations and linear analysis, which was not feasible in this case. The large deformation algorithm took into account the global movement of the structure and iterated the solution with each change in geometry until the solution converged using an approach based on geometric nonlinearity. Material nonlinearities were not taken into account during this analysis. Once the solution was stable, the form of the gridshell was set.

The form-found geometry established in the previous analysis was converted into a new structural model for the parametric study. The points that defined the form-found model were extracted and new gridshell beams were constructed between these points. The new beams had the same cross section, material properties, and locations in space as the original structure. The structural model took on the exact same form as the form-found model, yet by disassembling and reassembling the model, all of the residual stress and strain from the large deformations experienced during the form-finding phase was eliminated. In theory, once a gridshell is constructed, the residual stress and strain in the beams that was generated during the construction process dissipates over time once the members are secured in their final form. To mimic this phenomenon, this simulation removed the residual stress and strain by reconstructing the model. Grasshopper calculated, displayed, and analyzed both the form-finding model and the structural model in real time as the various parameters were altered.

### 5.3 Simulation Validation

Prior to running a full battery of simulations, the model's FEA results were validated. The Karamba FEA engine itself was validated in the initial stages of the project by comparing the analytical solution for the deflection of a cantilever beam to the numerical solution generated by Karamba. The results matched exactly. Although this test validated Karamba's FEA engine, the overall gridshell simulation still required validation. Analytical solutions to solve for the buckling factors of gridshells do not exist and are impractical to derive, thus an independent analysis of a gridshell was completed using a separate FEA software, ADINA. To conduct this validation, the disassembled points from the form-found gridshell were exported and uploaded to ADINA. This was done to ensure that both FEA engines were analyzing the exact same structure. The supports, nodes, beams, cross-section, and material were also redefined in ADINA. A test load case was applied to both models to compare the reaction forces, buckling factors, and member utilization. The comparison yielded differences between 1 and 5 percent, well within the margins of error expected in gridshell analysis [14].

Potential explanations for these differences lie in differences native to the FEA software packages such as default mesh density and the behavior of intersection points. Despite the differences, the

results yielded a percent difference that is acceptable in the field and therefore served to validate the results of the Karamba simulation.

#### 5.4. Parametric Variation

During the parametric study, the grid density varied from 8 x 8 (coarse) to 30 x 30 (fine). The bracing schemes were defined as “half” or “full,” depending on whether all or only half of the grids were braced, and “continuous” or “discontinuous” depending on if the members braced single or multiple cells (Figure 14). Both the grid density and the bracing layout changed with each iteration of the study. The cross section, support conditions, and entrance height was held constant across all trials.

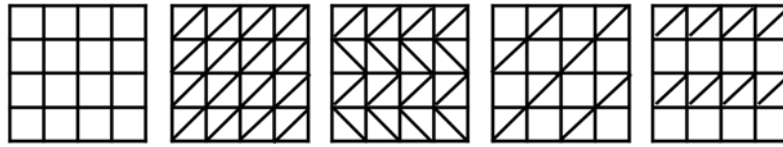


Figure 14: Variation of bracing layouts. (a) Unbraced. (b) Full continuous. (c) Full discontinuous. (d) Half continuous. (e) Half discontinuous.

Each iteration was subjected to the four load cases: a symmetric distributed load, an asymmetric distributed load, a centered point load, and an off-centered point load in a single quadrant of the shell. These loads are described in greater detail in subsequent sections. Table 1 details the range of each parameter. A total of 204 models were evaluated over the course of the study. The results of each test run are contained in Appendix A.

Table 1: List of varied parameters and their ranges.

| Bracing Scheme             | Grid Densities Tested                         | Load Cases Tested                                                                                                                                                                           | Total Models Tested |
|----------------------------|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| Unbraced                   | 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 | <ul style="list-style-type: none"> <li>• Symmetric distributed load</li> <li>• Asymmetric distributed load</li> <li>• Centralized point load</li> <li>• Decentralized point load</li> </ul> | 48                  |
| Half braced, continuous    | 10, 20, 30                                    | <ul style="list-style-type: none"> <li>• Symmetric distributed load</li> <li>• Asymmetric distributed load</li> <li>• Centralized point load</li> <li>• Decentralized point load</li> </ul> | 12                  |
| Half braced, discontinuous | 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 | <ul style="list-style-type: none"> <li>• Symmetric distributed load</li> <li>• Asymmetric distributed load</li> <li>• Centralized point load</li> <li>• Decentralized point load</li> </ul> | 48                  |
| Full braced, continuous    | 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 | <ul style="list-style-type: none"> <li>• Symmetric distributed load</li> <li>• Asymmetric distributed load</li> <li>• Centralized point load</li> <li>• Decentralized point load</li> </ul> | 48                  |
| Full braced, discontinuous | 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30 | <ul style="list-style-type: none"> <li>• Symmetric distributed load</li> <li>• Asymmetric distributed load</li> <li>• Centralized point load</li> <li>• Decentralized point load</li> </ul> | 48                  |

## **6. Results of Structural Analysis**

The results of the parametric study are presented below in terms of the applied load case. Though the grid density ranged from 8 x 8 to 30 x 30, all of the shells with a grid density less than 16 x 16 experienced local member failure in an unbraced condition prior to achieving the form required to test the global buckling of the shell. Global buckling is the primary failure mode for gridshell structures, as local failure indicates that the structure could not possibly achieve the prescribed catenary curvature, and would never exist as a gridshell. Even if adding bracing to a gridshell with a grid density below 16 alleviated local failure, these shells were still kept out of the analysis. The reason for this omission relates to the construction process, which states that shells must be built in an unbraced condition to allow the proper curvature to develop, and bracing is added to strengthen the shell after it has achieved its final form. Because all of the unbraced gridshells in this region of grid density failed, no braced shells could physically be constructed, thus all of the results were left out. The plots and trends describe only the behavior of the functioning shells. The discussion for each load case will focus on the entire range of grid densities, and subsequent sections will focus on those best suited for HA/DR. The MATLAB code used to generate the plots and figures is contained in Appendix B. Please note that the legends in Figures 15, 16, 17, 19, and 20 indicate the type of bracing schemes applied to the structure.

### **6.1. Symmetric Distributed Loads**

When subjected to a symmetric distributed load, the buckling factor of the gridshell was referenced to the self-weight of the structure. It is important to note that as the structure's grid density increases and additional bracing is added, the self-weight increases, which affects the buckling factor as defined in Equation 1.

$$BF_1 = \frac{\text{Maximum Supported Load (kN)}}{\text{Weight of Structure (kN)}} \quad (1)$$

Figure 15a shows that as grid density increased from 16 to 30 and additional bracing was added to the structure, the global buckling factor decreased by an average factor of 1.75 (Figure 15a). Native to each applied bracing scheme, increasing the grid density from 16 to 30 increased the weight by an average factor of 1.78. Regardless of grid density, adding a half bracing scheme caused an average increase in maximum supported load by 27% for a 34% increase in weight. Adding bracing to the whole shell caused an average increase in maximum supported load by 53% for a 66% increase in weight from an unbraced state (Figure 15b). The buckling factor trends related to the symmetric distributed load were misleading due to the ever changing weight of the structure. The maximum supported load was not referenced to the weight of the structure. The considerable weight increase due to the increasing grid density and additional bracing resulted in a decreasing buckling factor, but upon further examination, the maximum supported load actually increased. This behavior shows that the structure is able to support more load as grid density and bracing members increase, yet the weight increase associated with those physical changes is greater than the strength increase, resulting in a decreasing buckling factor (Equation 1). Varying grid density did not have a significant impact on the maximum supported load but additional bracing accounted for significant jumps. These jumps indicate that when

subjected to a symmetric load, full bracing most improves a gridshell's maximum supported load, followed by half bracing, and subsequently an unbraced shell.

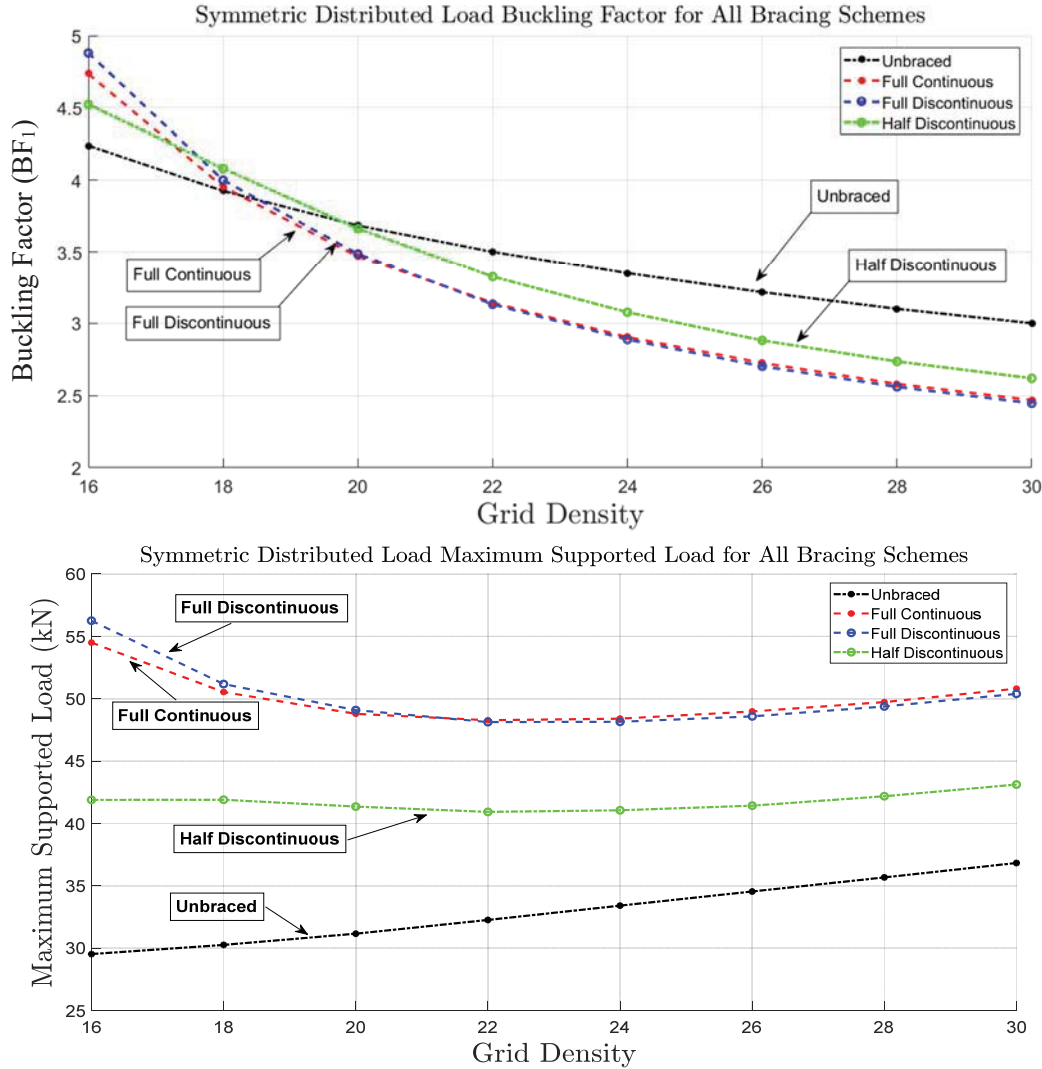


Figure 15: Symmetric distributed load. (a) Buckling factor. (b) Maximum supported load.

## 6.2. Asymmetric Distributed Loads

Applying asymmetric distributed loads to the gridshell presented significant problems (Figure 16). Due to software limitations, applying a true distributed load to only half of the structure was not possible. As a result, a load case was devised to mimic a distributed load. The particular load case utilized was defined as an array of point loads applied evenly to one half of the structure, thereby creating a load that was both asymmetric (covering half of the structure) and distributed (evenly spaced small point loads). This load case applied a point load to each joint on a single half of the structure and increased those loads in an iterative fashion until global buckling failure. The nominal point load added to each joint was 0.1 kN (22.5 lbf). The buckling factor was defined as the total maximum supported load divided by the reference load applied to the structure (Equation 2).

$$BF_2 = \frac{\text{Maximum Supported Load (kN)}}{\frac{(\text{Grid Density} + 1)^2}{2} \cdot 0.1 \text{ kN (kN)}} \quad (2)$$

Note that the reference load was 0.1 kN applied to half of the joints. The number of joints per each half of the structure was based on the grid density of the structure. In the event that the structure could not reach the 0.1 kN reference load per joint prior to failure, its buckling factor appeared to be less than 1, and the actual load applied at each joint can be thought of as the buckling factor multiplied by 0.1 kN. Shells with larger grid densities had more joints per half of the structure, thus the total load applied to each structure differed. The buckling factor initially decreased as grid density increased, but beyond a grid density of 22 the difference in structural performance was minimal, as each bracing scheme's buckling factor fell within  $\pm 0.11 BF_2$  of the group average. The maximum supported load shows that additional bracing improved the load carrying capacity of the shell in a manner similar to that of a gridshell subjected to a symmetric distributed load.

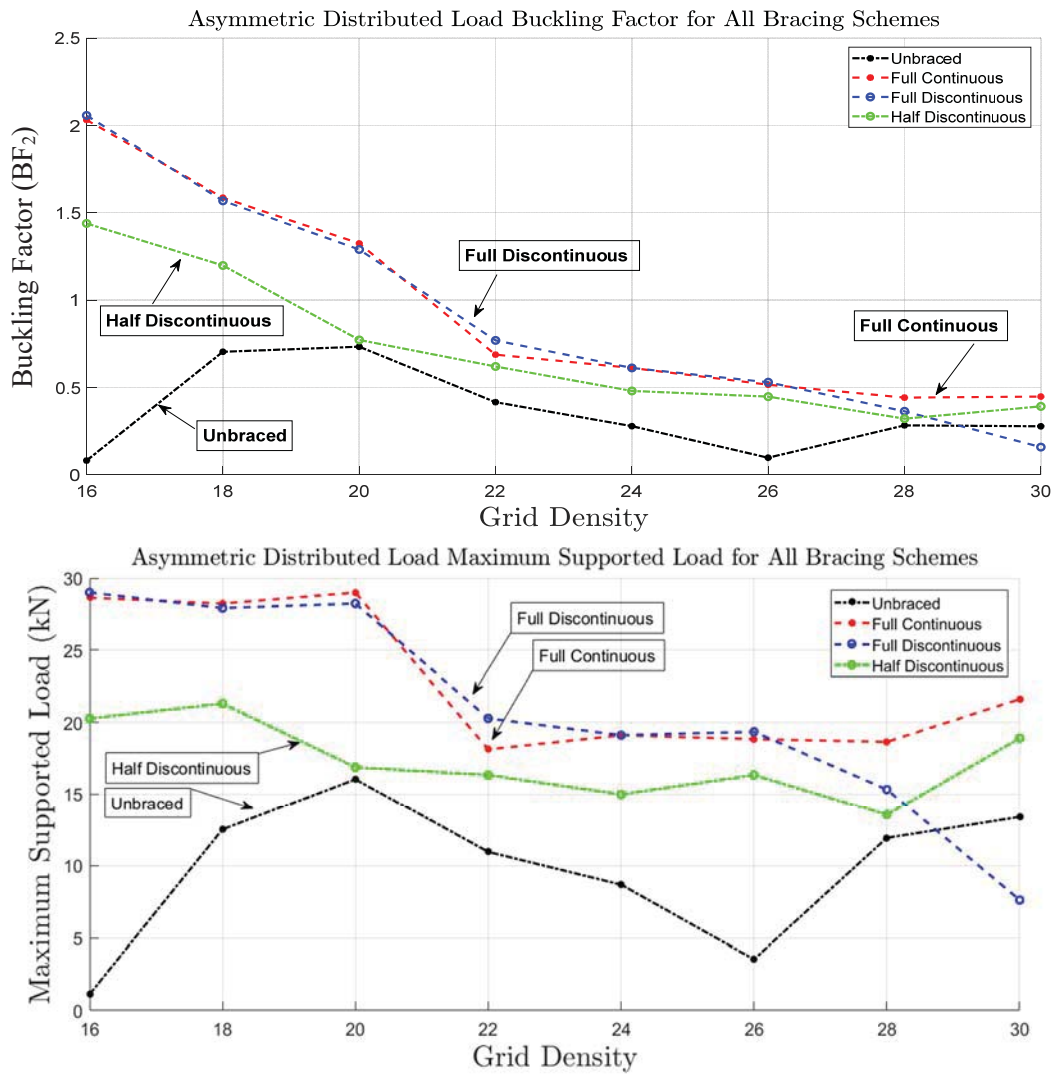


Figure 16: Asymmetric distributed load. (a). Buckling factor. (b) Maximum supported load.

Beyond a grid density of 22, the full bracing schemes supported an average of 11% more load than the half bracing scheme, or an additional 1.8 kN (405 lbf). While still an increase from the half bracing layout, it was less pronounced than the increase between half and full bracing schemes under the symmetric distributed load case.

### 6.3. Point Loads

Figure 17 illustrates the performance of gridshells subjected to point loads. The buckling factors for the centered point load and the off-center point load were calculated using Equation 3. Note that the buckling factor is a dimensionless version of the maximum supported load, thus the two can be used interchangeably.

$$BF_3 = \frac{\text{Maximum Supported Load (kN)}}{1 \text{ kN (kN)}} \quad (3)$$

Both point load buckling factors were referenced to 1 kN (225 lbf), indicating that a structure with a buckling factor of 1.0 could support exactly 1 kN. With respect to the centered point load, as grid density increased from 16 to 30 and additional bracing was added, the buckling factor increased by an average of 36%. The half bracing scheme showed the highest increase between the initial and final grid density (16 – 30) with 56%. Because the reference load was 1 kN, the maximum supported load plot showed identical behavior, albeit with different axes. The off-center point load was also referenced to 1 kN and showed that as grid density and bracing increased, both the buckling factor, and maximum supported load by association, increased. On average, the buckling factor of the half braced shell was +1.2  $BF_3$  (kN) and the buckling factor of the fully braced shell was +5.4  $BF_3$  (kN) compared to the unbraced shell. The oscillations are due to the fact that the load was not placed at the same location for each iteration due to layout restrictions.

(Continued on following page)

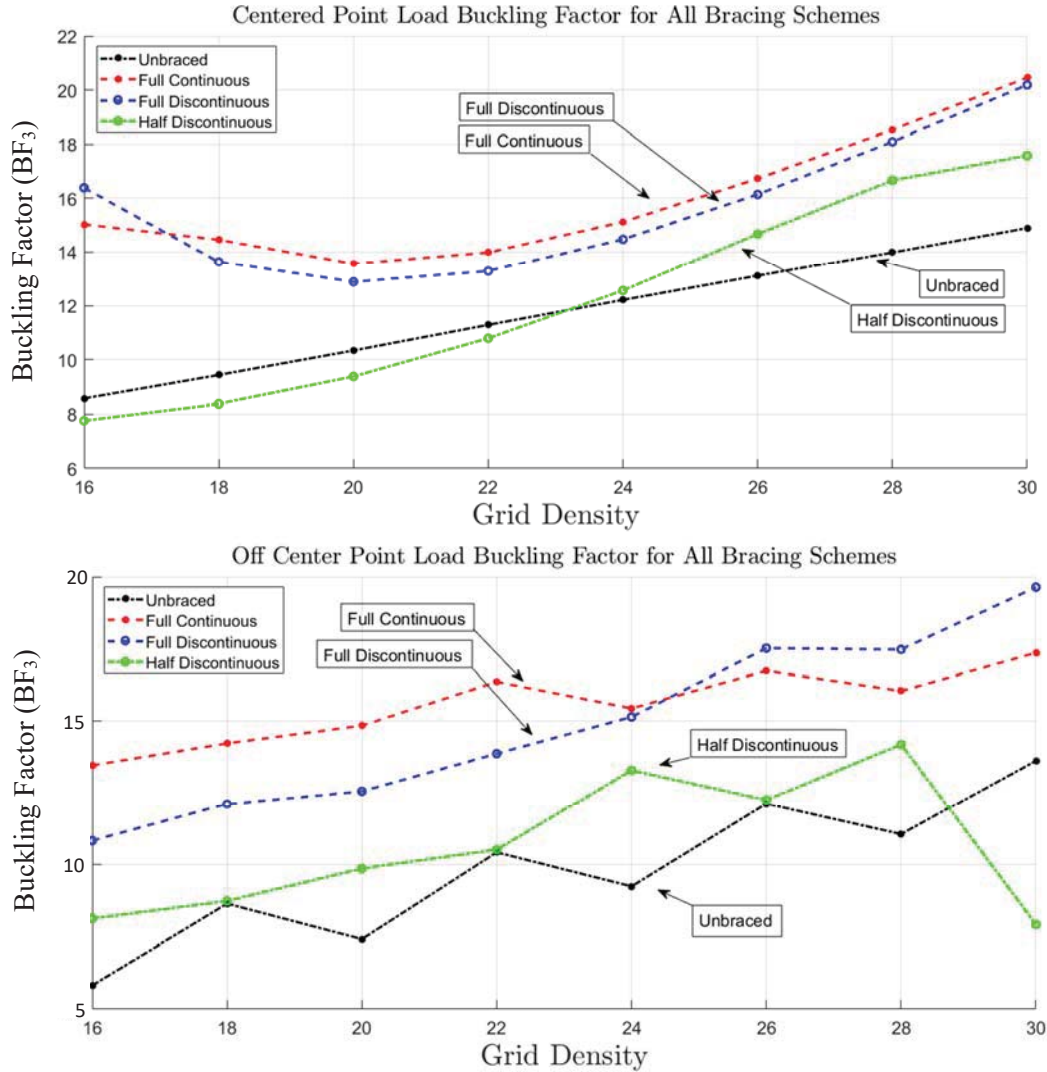


Figure 17: Point load buckling factor. (a) Centered. (b) Off-centered.

For the fully braced shell, the type of continuity of the bracing did not have a significant impact on its structural performance. Both full continuous bracing schemes and full discontinuous bracing schemes consistently produced results within 10% of one another. However, when testing select cases of half continuous bracing versus half discontinuous bracing at grid densities of 10, 20, and 30, the continuous bracing consistently exhibited higher performance, regardless of load case or grid density.

#### 6.4. Design Best Suited for HA/DR

A pavilion suited to hold between 150 and 200 people built using traditional means weighs between 53.4 and 66.7 kN (12,000 and 15,000 lbs) [30]. Concrete Canvas is a novel building solution for disaster relief and military forces, and features final forms similar to gridshells, but could weigh between 17.2 and 46.6 kN (3,900 and 10,500 lbs) based on the thickness of the concrete [31]. Additionally, constructing these structures requires an air blower and running water, both of which may not be available in disaster relief, and the structures are not reusable.

One of the lightest disaster relief solutions that can hold 150 to 200 people is the Modular General Purpose Tent (MGPT), which weighs 12.8 kN (2,900 lbs); however, it does not allow for wind to pass through the structure, as it is enclosed on all sides [32]. Figure 18 illustrates these existing structural solutions in use.



Figure 18: Pavilion structures. (a) Traditional pavilion [33]. (b) MGPT [34]. (c) Concrete Canvas [35].

Based on current solutions, the gridshell for use in HA/DR was designed to weigh approximately  $13.3 \pm 2.2$  kN ( $3,000 \pm 500$  lbs) in order to be competitive with existing solutions. Based on the results of the computational studies, this solution will fall between the grid densities of 18 and 22, where the average weight of the unbraced shell is 8.5 kN (1,900 lbs), a half-braced shell is 11.3 kN (2,500 lbs), and a fully braced shell is 14 kN (3,200 lbs). Each of these weights is an approximation based on the computational study and does not take into account the addition of hardware to the structure, which could add up to 0.7 kN (150 lbs) based on the type of hardware.

Within this grid density region, the results clearly indicated that with respect to symmetric distributed loads, adding additional bracing improved the maximum supported load, while increasing grid density showed little effect. Regarding asymmetric distributed loads, additional bracing again improved the maximum supported load, but also the buckling factor. Results indicated that generally speaking, lower grid densities in this region improved structural performance in both measures. The results of the centered and off-center point load indicate that additional bracing improved performance, as did increasing grid density. Figure 19 illustrates each of these trends. Given the trends, a region within each plot was selected as an “ideally suited region” for gridshells designed for HA/DR. Each region was selected based on load carrying capacity of that region and the potential mass that would be added to the structure. Most of these regions fell between the results of the half-discontinuous bracing scheme and the full-continuous/discontinuous bracing schemes. Figure 20 shows these regions.

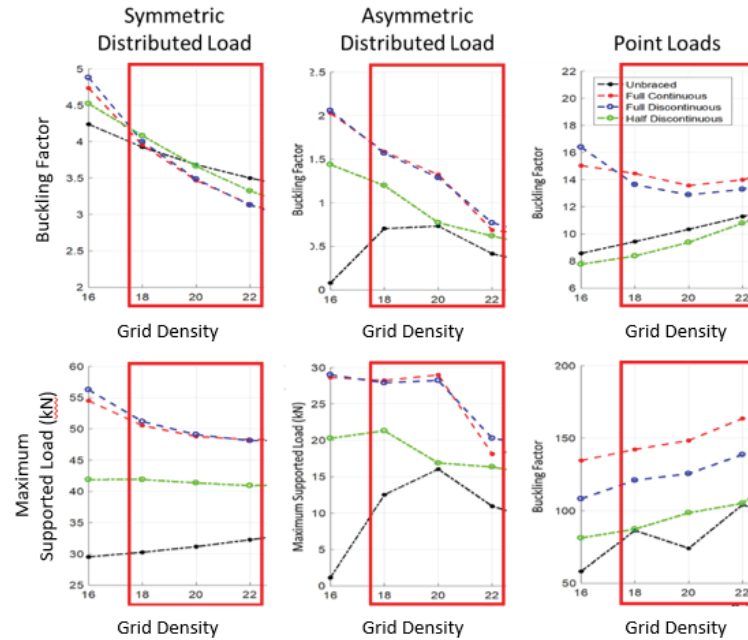


Figure 19: Structural performance of gridshell within grid density range of 18 to 22.

Half continuous bracing was difficult to simulate due to software indexing limitations, thus only select cases at grid densities of 10, 20, and 30 were tested. The results of these tests indicated that while the continuity of the bracing did not affect the full bracing schemes significantly, continuous bracing consistently outperformed discontinuous half bracing schemes. Figure 20 shows the results of the half continuous bracing applied to a gridshell with a grid density of 20.

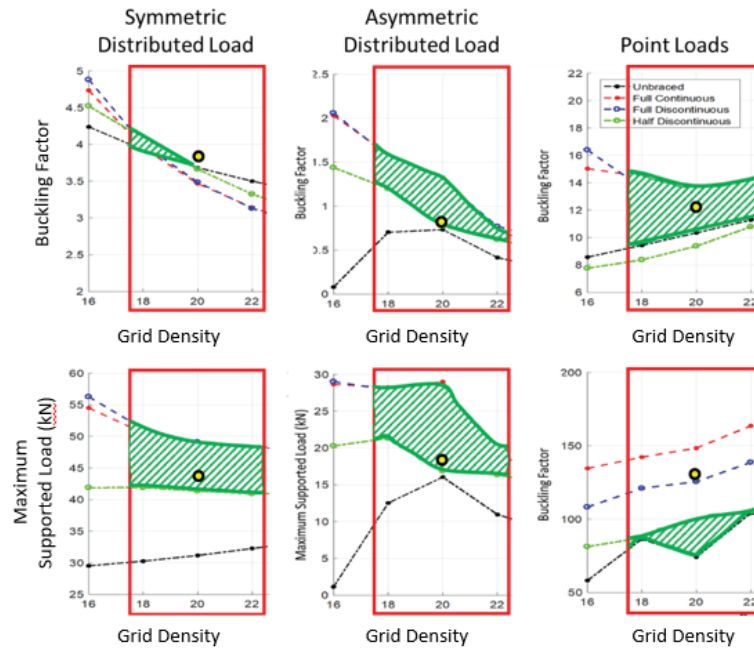


Figure 20: Ideal regions of structural performance within grid density range of 18 to 22 with added data point of “half braced, continuous” at a grid density of 20.

Similar trends were seen at grid densities of 10 and 30 and established that half continuous bracing outperforms half discontinuous bracing. The parametric studies indicated that bracing improved load carrying capacity; however, a full bracing scheme resulted in a 66% increase in mass, while half bracing schemes increased the mass by only 34%. Full bracing schemes consistently outperformed half bracing schemes, but relative to the additional mass, half bracing schemes proved to be more efficient. While the continuity of the bracing did not affect structural performance when fully braced, select studies indicated that when only half braced, continuous bracing consistently outperformed discontinuous bracing. Keeping in mind that the HA/DR gridshell must be mobile, as well as stable, a gridshell with continuous bracing over half of the structure was selected as best suited for HA/DR.

While results for the computational studies only gathered explicit data on a grid density of 20, the trend was extrapolated across all grid densities, therefore in order to further save weight for the final solution, the final design for HA/DR was selected with a grid density of 18.

## **7. Structural Design Constraints**

With the design best suited for HA/DR selected as a result of the parametric studies, specific constraints related to HA/DR were established. These constraints were established using pertinent civil and structural engineering codes including the Sphere Handbook, International Building Code (IBC), International Fire Code (IFC), and the American Society of Civil Engineers Code 7-16 (ASCE-7) [28, 36, 37, 38]. These codes established the worst case load scenarios and other structural requirements for the gridshell. International fieldwork sponsored by the USNA Summer International Service Leadership (SISL) Grant also contributed knowledge and firsthand experience of these constraints. Combined, the constraints helped to move the structure from the parametric modeling phase into a final proposed design for HA/DR.

### **7.1. Minimum Loading Constraints**

When designing a structure abroad, the most common governing codes are the IBC and the IFC. These codes could be supplemented by local codes; however, as a general rule of thumb, the requirements detailed in these codes serve as international standards. In addition to general structure guidelines, humanitarian structures fall under the Sphere Handbook, which establishes the minimum standards for HA/DR efforts.

#### **7.1.1. The Sphere Handbook [36]**

The Sphere Handbook is a set of minimum standards for HA/DR efforts. The Sphere Handbook states that all HA/DR efforts should provide, “safe, secure and equitable access to essential services. These include ... social facilities, places of worship, meeting points, recreational areas, including child-friendly spaces.” Gridshells can provide for each of these functions, and more, in an HA/DR setting. As a result, the Sphere Handbook lays out guidelines that are applicable to the construction of a gridshell in an HA/DR scenario such as a refugee camp or disaster relief zone.

The Sphere Handbook states that all temporary structures must remain safe and stable until more permanent solutions become available. A well designed gridshell will be able to support this criteria, as its intended use is a semi-permanent structure for shelter in temporary HA/DR relief

efforts. Next the Sphere Handbook states that efforts should be made to ensure that the temporary communal settlements do not become targets or create undue security risks for the local populations. In order to support this criteria, the shelter would be placed within the boundaries of the camp, near the center rather than the edges. The Sphere Handbook also states that structures must avoid placing unsustainable demands on natural environments. Gridshells are constructed from natural renewable resources so as long as local or foreign timber sources are reseeded and depletion to the point of extinction is avoided, this criteria will be met.

The Sphere Handbook states that it is not only permissible, but encouraged to utilize local volunteer and skilled labor in the construction and maintenance of the structure, as well as local supply chains during the logistics phase. This shell will be designed with a volunteer workforce in mind from the outset, and avenues to make the structure more sustainable through local sourcing have promise for future success.

The Sphere Handbook states that the structure should account for and design to wind, earthquake, landslides, volcanic activity, and other natural disasters in the region in order to prevent failure. In addition to structural considerations, the structure should not propagate disease by its design. Following sections of this document will explain and take into account various loading scenarios. By nature a gathering area such as a gridshell fosters a communal setting, but doesn't require occupants to touch any part of the structure. Periodic cleanings of the beams and potentially the membrane covering should keep the structure sufficiently clean to prevent the spread of disease.

With regards to permanence, the Sphere Handbook indicates that HA/DR structures should aim to be as temporary as possible while still maintaining safety standards. The sentiment behind this tenet is that HA/DR efforts are intended to be temporary relief efforts until domestic and international organizations begin rebuilding the actual institutions and infrastructure of the affected region. As a result, structures for use in HA/DR should be quickly deployable, and ideally reusable so that once a disaster is managed, the structure can be quickly removed and relocated to a new area of need.

Finally, in terms of fire prevention, the Sphere Handbook states that in an ideal situation, the separation distance between two structures should be twice the height of the taller structure when possible, though a strict minimum of 2 m (6.6 ft.) of spacing should always be enforced.

#### 7.1.2. International Building Code [37]

In addition to the Sphere Handbook, the IBC lays out general engineering restrictions and requirements for common structures. According to the IBC, gridshell structures fall under the category of tensile membrane structures or ground-level roofs. If they are erected for less than 180 days, their regulation officially falls under the IFC, but if erected for more than 180 days then that regulation falls under the IBC. The following are the IBC regulations for gridshells erected for more than 180 days.

Chapter 3 of the IBC lays out the official classifications of buildings. Gridshells fall under both the A-5 classification and the U classification. According to Section 303.6, A-5 structures are defined as, "Outdoor Assembly Area ... intended for participation in or viewing [of] outdoor activities including, but not limited to: amusement park structures, bleachers, grandstands,

stadiums.” Classification U structures are defined in Section 312.1 as, “buildings and structure of an accessory character and miscellaneous structures not classified in any specific occupancy.”

Chapter 10, Section 1006.2.1.1 defines minimum means of egress. By nature the gridshell will have four major egress points. With four egress points, the structure could hypothetically support an occupancy of over 1,000 individuals. However, due to its size of 232 m<sup>2</sup> (2,500 ft<sup>2</sup>) and an occupant load of 1.39 m<sup>2</sup> (15 ft<sup>2</sup>) per occupant for concentrated assembly areas per Section 1004, the gridshell can accommodate a maximum of 166 individuals.

Gridshells are constructed from timber members, and while fireproofing is available, it often alters the material properties of the wood. For the purpose of this analysis, the assumption will be made that the gridshell will be constructed with untreated timber. As a result, the timber does not pass any of the National Fire Protection Association (NFPA) fire safety regulations, yet is still a permissible building material. Section 602.5 states that, “Type V construction is that type of construction in which the structural elements, exterior walls and interior walls are of any materials permitted by this code.” Timber gridshells therefore fall under Type V construction.

Any structure greater than 11.1 m<sup>2</sup> (120 ft<sup>2</sup>) requires a local permit. Section 506 outlines the area restrictions for structures based on their classification and fire rating. The gridshell still falls under Type V construction, meaning that it has no fire retardant rating and no sprinkler system. A-5 structures of this type have unlimited area, while U structures of this type have a cap at 1,530 m<sup>2</sup> (16,500 ft<sup>2</sup>). Because the U classification is more restrictive, it will be utilized in further proceedings.

The IBC restricts a structure’s height based on the structure’s classification and construction type. As a Type V structure without sprinklers, both A-5 and U classification structures are limited to a height of 40 feet by Section 504.3.

Chapter 16 defines the loads that structures must withstand and the loading categories of each structure. A gridshell structure falls under Risk Category II (also known as Importance Factor 2) based on its occupancy. Because the structure is essentially a ground-supported roof, the IBC details live loads that it must be able to withstand. The live loads are defined as:

1. 1.33 kN (300 lbf) point load (support maintenance personnel climbing on the roof)
2. 0.239 kPa (5 psf) distributed load (accounts for any membrane or covering)

In addition to these live loads as defined by IBC, the chapter states that wind, snow, dead and seismic loads must be taken into account. Following sections of this report details how each additional load case was calculated.

### 7.1.3. International Fire Code [28]

The IFC defines the structural requirements for gridshell structures erected for less than 180 days. Under Section 105.6.45, gridshells still fall under the category of temporary membrane structures and tents and require a permit if the area is greater than 37.2 m<sup>2</sup> (400 ft<sup>2</sup>). The requirements for means of egress are explicitly written in the IFC and concur with the IBC means of egress clauses, indicating that with 4 egress points, the structure could potentially hold more than 1,000 people. However, based on the structure classification and the floor space that must be allotted to each

individual, the structure can hold a maximum of 166 people. Regarding structural engineering criteria, little information is provided, thus the IBC will be used to ensure safe and proper design.

#### 7.1.4. Wind Loads from ASCE 7 [38]

For the purpose of determining wind loads, the gridshell was modelled conservatively with a 15.24 m (50 ft.) square footprint, a maximum height of 4.57 m (15 ft.), a hemispherical curve on the upper half of the structure. Figure 21 shows a qualitative mock-up of the structure's elevation (side view). These dimensions are likely to change as the design improves, yet will ensure that a worst case load scenario is achieved.

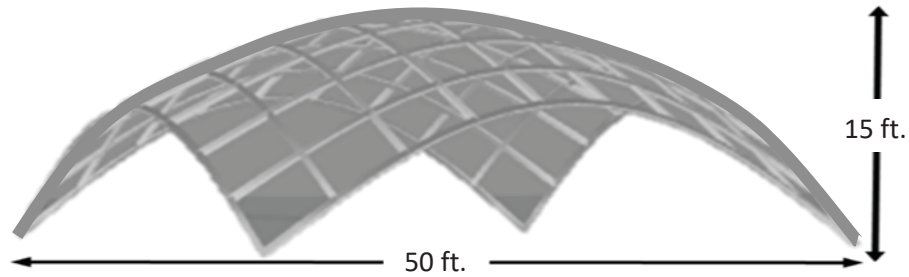


Figure 21: Mock-Up of gridshell elevation view.

Analysis of the wind conditions in Athens, Greece was conducted using data from *meteoblue*, a weather forecast and archiving system developed at the University of Basel in Switzerland based on the National Oceanic and Atmospheric Administration (NOAA) and National Centers for Environmental Prediction (NCEP) [39]. The climate wind rose shows that the historical maximum wind speed velocity is 38 mph (61 kph) from the north (Figure 22).

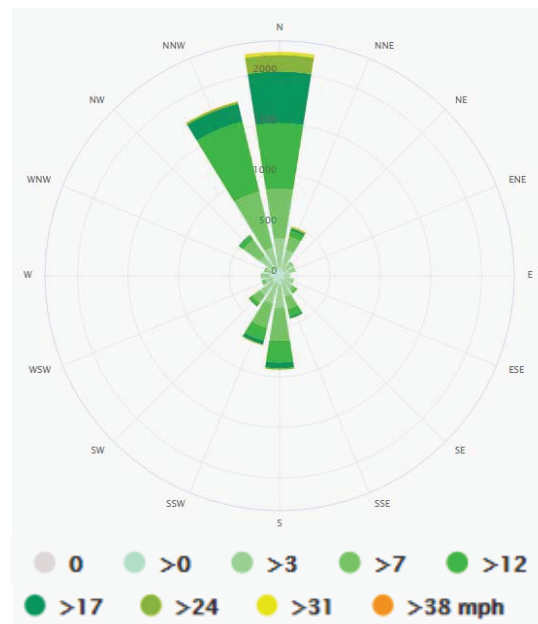


Figure 22: Wind rose for Athens, Greece [39].

According to Section 6.5.13 of ASCE 7, the design wind loads for open buildings and other structures are defined by Equation 4. ASCE 7 defines is equations using the Imperial System, therefore the calculations were carried out in that manner and converted upon completion.

$$P_{wind} = q_z G C_f (psf) \quad (4)$$

In Equation 4:

$P_{wind}$  = wind pressure (psf)

$q_z$  = velocity pressure evaluated a height  $z$  of the centroid area of the object (psf)

$G$  = gust effect factor from Section 6.5.8 of ASCE 7

$C_f$  = net force coefficients from Figure 6-18 of ASCE 7

To solve for the velocity pressure,  $q_z$ , Equation 5 was utilized.

$$q_z = 0.00256 K_z K_{zt} K_d V^2 I (psf) \quad (5)$$

In Equation 5:

$K_z$  = velocity pressure exposure coefficient defined in Section 6.5.6.6 of ASCE 7

$K_{zt}$  = topographic factor defined in Section 6.5.7.2 of ASCE 7

$K_d$  = wind directionality factor defined in Section 6.5.4.4 of ASCE 7

$V$  = wind velocity (ft./s)

$I$  = importance factor

The velocity pressure exposure coefficient,  $K_z$ , was defined by Table 2. Note that the Exposure Case defined in Section 6.5.6.3 of ASCE 7 indicated that gridshells are Exposure B, Case 1.

Table 2: Velocity Pressure Exposure Coefficient [38].

| Height above<br>ground level, $z$ |         | Exposure (Note 1) |        |             |             |
|-----------------------------------|---------|-------------------|--------|-------------|-------------|
|                                   |         | B                 |        | C           | D           |
| ft                                | (m)     | Case 1            | Case 2 | Cases 1 & 2 | Cases 1 & 2 |
| 0-15                              | (0-4.6) | 0.70              | 0.57   | 0.85        | 1.03        |
| 20                                | (6.1)   | 0.70              | 0.62   | 0.90        | 1.08        |
| 25                                | (7.6)   | 0.70              | 0.66   | 0.94        | 1.12        |
| 30                                | (9.1)   | 0.70              | 0.70   | 0.98        | 1.16        |
| 40                                | (12.2)  | 0.76              | 0.76   | 1.04        | 1.22        |
| 50                                | (15.2)  | 0.81              | 0.81   | 1.09        | 1.27        |
| 60                                | (18)    | 0.85              | 0.85   | 1.13        | 1.31        |
| 70                                | (21.3)  | 0.89              | 0.89   | 1.17        | 1.34        |
| 80                                | (24.4)  | 0.93              | 0.93   | 1.21        | 1.38        |
| 90                                | (27.4)  | 0.96              | 0.96   | 1.24        | 1.40        |
| 100                               | (30.5)  | 0.99              | 0.99   | 1.26        | 1.43        |
| 120                               | (36.6)  | 1.04              | 1.04   | 1.31        | 1.48        |
| 140                               | (42.7)  | 1.09              | 1.09   | 1.36        | 1.52        |
| 160                               | (48.8)  | 1.13              | 1.13   | 1.39        | 1.55        |
| 180                               | (54.9)  | 1.17              | 1.17   | 1.43        | 1.58        |
| 200                               | (61.0)  | 1.20              | 1.20   | 1.46        | 1.61        |
| 250                               | (76.2)  | 1.28              | 1.28   | 1.53        | 1.68        |
| 300                               | (91.4)  | 1.35              | 1.35   | 1.59        | 1.73        |
| 350                               | (106.7) | 1.41              | 1.41   | 1.64        | 1.78        |
| 400                               | (121.9) | 1.47              | 1.47   | 1.69        | 1.82        |
| 450                               | (137.2) | 1.52              | 1.52   | 1.73        | 1.86        |
| 500                               | (152.4) | 1.56              | 1.56   | 1.77        | 1.89        |

By modelling the structure as 15 feet tall,  $K_z$  was equal to 0.57.

The wind directionality factor,  $K_d$ , was referenced using Table 3. Note that the structure was modelled as a standard building in this simplified approach, therefore  $K_d$  was equal to 0.85.

Table 3: Wind Directionality Factor [38].

| Structure Type                                                                  | Directionality Factor $K_d^*$ |
|---------------------------------------------------------------------------------|-------------------------------|
| <b>Buildings</b><br>Main Wind Force Resisting System<br>Components and Cladding | 0.85<br>0.85                  |
| <b>Arched Roofs</b>                                                             | 0.85                          |
| <b>Chimneys, Tanks, and Similar Structures</b><br>Square<br>Hexagonal<br>Round  | 0.90<br>0.95<br>0.95          |

For this analysis, the specific topographic conditions for the gridshell were largely unknown, therefore a worst case scenario was modelled. To solve for,  $K_{zt}$ , Equation 6 was utilized.

$$K_{zt} = (1 + K_1 K_2 K_3)^2 \quad (6)$$

In Equation 6:

$K_1$  = Factor to account for shape of topographic feature and maximum speed-up effect.

$K_2$  = Factor to account for reduction in speed-up with distance upwind or downwind.

$K_3$  = Factor to account for reduction in speed-up with height above local terrain.

The factors  $K_1$ ,  $K_2$ , and  $K_3$  were defined in Table 4, Equation 7, and Equation 8 respectively.

To solve for  $K_1$  Table 4 was utilized.

Table 4: Parameters for Speed-Up Over Hills and Escarpments [38].

| Hill Shape                                                                | $K_1/(H/L_h)$ |      |      | $\gamma$ | $\mu$              |                      |
|---------------------------------------------------------------------------|---------------|------|------|----------|--------------------|----------------------|
|                                                                           | Exposure      |      |      |          | Upwind<br>of Crest | Downwind<br>of Crest |
|                                                                           | B             | C    | D    |          |                    |                      |
| 2-dimensional ridges<br>(or valleys with negative<br>H in $K_1/(H/L_h)$ ) | 1.30          | 1.45 | 1.55 | 3        | 1.5                | 1.5                  |
| 2-dimensional escarpments                                                 | 0.75          | 0.85 | 0.95 | 2.5      | 1.5                | 4                    |
| 3-dimensional axisym. hill                                                | 0.95          | 1.05 | 1.15 | 4        | 1.5                | 1.5                  |

Because the geometry of the topography is unknown, the maximum value of  $K_1$  under Exposure B was used to establish the worst case scenario, thus  $K_1$  was equal to 1.30.

To solve for  $K_2$  Equation 7 was utilized.

$$K_2 = \left(1 - \frac{|x|}{\mu L_h}\right) \quad (7)$$

Mathematically, the highest value for  $K_2$  was 1.0, thus by analyzing the worst case scenario,  $K_2$  was equal to 1.0.

To solve for  $K_3$  Equation 8 was utilized.

$$K_3 = e^{\frac{-\gamma z}{L_h}} \quad (8)$$

Mathematically, the highest value for  $K_2$  was 1.0, thus by analyzing the worst case scenario,  $K_3$  was equal to 1.0.

Referring to Equation 6, the values for  $K_1$ ,  $K_2$ , and  $K_3$  from above yield:

$$K_{zt} = (1 + 1.3)^2$$

that the value for the topographic factor,  $K_{zt}$ , was equal to 5.29.

In this analysis, the maximum wind speed,  $V$ , was equal to 38 miles per hour and the importance factor,  $I$ , was defined as 2 due to provisions in the International Building Code. Once all of the values are substituted into Equation 4, the velocity pressure equaled 18.9 psf.

$$q_z = 0.00256 \cdot 0.57 \cdot 5.29 \cdot 0.85 \cdot 38^2 \cdot 2 \text{ (psf)} = 18.9 \text{ psf}$$

The gust effect factor,  $G$ , was defined by Equation 9.

$$G = \sqrt{\frac{1 + 1.7 g_Q I_z Q}{1 + 1.7 g_v I_z}} \quad (9)$$

In Equation 9:

$g_Q$  = peak factor for background response defined as 3.4 by 6.5.8.1 of ASCE 7

$g_v$  = peak factor for wind response defined as 3.4 by 6.5.8.1 of ASCE 7

$I_z$  = intensity of turbulence defined by Equation 10

$Q$  = background response defined by Equation 11

To solve for the intensity of turbulence,  $I_z$ , Equation 10 was utilized:

$$I_z = c \left( \frac{33}{\bar{z}} \right)^{\frac{1}{6}} \quad (10)$$

In Equation 10:

$c$  = turbulence intensity factor from Table 5

$\bar{z}$  = equivalent height of the structure defined as 60 percent of the nominal height

The turbulence intensity factor,  $c$ , was referenced from Table 5.

Table 5: Terrain Exposure Constants [38].

| Exposure | $\alpha$ | $z_g$ (ft) | $\hat{a}$ | $\hat{b}$ | $\bar{\alpha}$ | $\bar{b}$ | $c$  | $l$ (ft) | $\bar{\epsilon}$ | $z_{min}$ (ft)* |
|----------|----------|------------|-----------|-----------|----------------|-----------|------|----------|------------------|-----------------|
| B        | 7.0      | 1200       | 1/7       | 0.84      | 1/4.0          | 0.45      | 0.30 | 320      | 1/3.0            | 30              |
| C        | 9.5      | 900        | 1/9.5     | 1.00      | 1/6.5          | 0.65      | 0.20 | 500      | 1/5.0            | 15              |
| D        | 11.5     | 700        | 1/11.5    | 1.07      | 1/9.0          | 0.80      | 0.15 | 650      | 1/8.0            | 7               |

In this case,  $c$  was referenced with Exposure B, therefore  $c$  was equal to 0.30. The equivalent height,  $\bar{z}$ , was defined as 60% of the nominal height, therefore  $\bar{z}$  was equal to 9 feet.

Using the above values to solve Equation 10, the intensity of turbulence,  $I_{\bar{z}}$ , equaled 0.373.

$$I_{\bar{z}} = 0.30 \cdot \left(\frac{33}{9}\right)^{\frac{1}{6}} = 0.373$$

To solve for the background response,  $Q$ , Equation 11 was utilized:

$$Q = \sqrt{\frac{1}{1 + 0.63 \left(\frac{B + h}{L_z}\right)^{0.63}}} \quad (11)$$

In Equation 11:

$B$  = base width (ft.)

$h$  = nominal height (ft.)

$L_z$  = the integral length scale of turbulence defined by Equation 9 (ft.)

To solve for the integral length scale of turbulence,  $L_z$ , Equation 12 was used.

$$L_z = l \left(\frac{\bar{z}}{33}\right)^{\bar{\epsilon}} \quad (12)$$

In Equation 12:

$l$  = the integral length scale factor from Table 5

$\bar{z}$  = equivalent height of the structure defined as 60 percent of the nominal height

$\bar{\epsilon}$  = ratio of solid area to gross area from Table 5

In this case,  $l$  and  $\bar{\epsilon}$  were referenced with Exposure B, therefore  $l$  was equal to 320 and  $\bar{\epsilon}$  was equal to 0.333. The equivalent height,  $\bar{z}$ , was defined as 60% of the nominal height, therefore  $\bar{z}$  was equal to 9 feet. Evaluated, Equation 10 yielded that the integral length scale of turbulence,  $L_z$ , equaled 208.

$$L_z = 320 \cdot \left(\frac{9}{33}\right)^{0.333} = 208$$

Returning to Equation 9, where  $B$  equaled 50 feet,  $h$  equaled 15 feet, and  $L_z$  equaled 208, the background response,  $Q$ , equaled 0.876.

$$Q = \sqrt{\frac{1}{1 + 0.63 \left( \frac{50 + 15}{207.61} \right)^{0.63}}} = 0.876$$

Referring to Equation 7, with  $g_Q$  and  $g_v$  defined as 3.4,  $I_z$  equal to 0.373, and  $Q$  equal to 0.876, the gust effect factor,  $G$ , equaled 0.957.

$$G = \sqrt{\frac{1 + 1.7 \cdot 3.4 \cdot 0.373 \cdot 0.876}{1 + 1.7 \cdot 3.4 \cdot 0.373}} = 0.957$$

The net force coefficient,  $C_f$ , was defined in Table 6.

Table 6: Net Force Coefficient [38].

| Roof angle $\theta$<br>degrees | L/B  |      |      |      |      |      |      |
|--------------------------------|------|------|------|------|------|------|------|
|                                | 5    | 3    | 2    | 1    | 1/2  | 1/3  | 1/5  |
| 10                             | 0.2  | 0.25 | 0.3  | 0.45 | 0.55 | 0.7  | 0.75 |
| 15                             | 0.35 | 0.45 | 0.5  | 0.7  | 0.85 | 0.9  | 0.85 |
| 20                             | 0.5  | 0.6  | 0.75 | 0.9  | 1.0  | 0.95 | 0.9  |
| 25                             | 0.7  | 0.8  | 0.95 | 1.15 | 1.1  | 1.05 | 0.95 |
| 30                             | 0.9  | 1.0  | 1.2  | 1.3  | 1.2  | 1.1  | 1.0  |

In this case  $L/B$  was defined as 1.0 because the footprint was square, and the worst case angle scenario was selected, thus  $C_f$  was defined as 1.3.

Returning the Equation 4, where  $q_z$  equaled 18.9 psf,  $G$  equaled 0.957, and  $C_f$  equaled 1.3, the wind pressure on the shell was approximately 4.45 psf.

$$P_{wind} = 18.9 \cdot 0.959 \cdot 1.3 = 23.6 \text{ psf}$$

Note that wind pressure is a horizontally distributed load on one face of the gridshell. A worst case scenario condition would be experienced if there were no uplift forces on the opposing side of the shell, thus the uplift forces due to wind flowing over the surface of the shell were set to zero for this analysis.

#### 7.1.5. Snow Loads

The density of snow is defined as a percentage of the density of water. Most snow falls somewhere between 3% and 20% water content, indicating densities between 30 and 200 kg/m<sup>3</sup>. Athens rarely experiences snowfall, yet a conservative estimate of 1 inch of snow at 20% water content was used to determine potential snow loads. The density of snow at 20% water content is approximately 12.5 lbf/ft<sup>3</sup>. Equation 13 was used to calculate the distributed pressure of the snow load.

$$P_{Snow} = \rho_{snow} \cdot t_{layer} \quad (13)$$

In Equation 13:

$$P_{Snow} = \text{snow pressure (psf)}$$

$$\rho_{snow} = \text{density of snow (lbf/ft}^3\text{)}$$

$$t_{layer} = \text{layer thickness (ft.)}$$

Using the conditions outlined above to approximate the snow load, the pressure on the structure provided by the snow,  $P_{Snow}$ , equaled 1.04 psf.

$$P_{Snow} = 12.5 \cdot 0.0833 = 1.04 \text{ psf}$$

#### 7.1.6. Dead Loads

The dead loads of the gridshell are made up of the structure's self-weight. Self-weight is defined by the weights of each individual member and the hardware used at the junction points. In this analysis, the dead load was accounted for during the finite element analysis of the gridshell in the Rhino-Grasshopper-Karamba software suite.

#### 7.1.7. Seismic Loads

Athens, Greece sits adjacent to an active converging fault line. Though Greece is located in a seismically active region, the effects of seismic loading are outside of the scope of this analysis [40]. Due to their flexible nature, gridshells can absorb some shock loads due to their design; however, a full shock loading analysis on the final structure will be necessary in future work.

#### 7.1.8. Summary of Loads

Table 7 contains a summary of the minimum loads that the structure must be able to support based on the IBC, IFC, and ASCE-7. The values have been converted from the Imperial System of Units to the International System of Units. Note that the dead load was calculated based on the self-weight of the structure and was applied as a default load when using Karamba. Figure 23 visually identifies these loads.

Table 7: Summary of Design Loads.

| Name                              | Type                   | Magnitude           | Source |
|-----------------------------------|------------------------|---------------------|--------|
| Maintenance Load                  | Point                  | 1.33 kN (300 lbf)   | IBC    |
| Membrane Load                     | Vertical Distributed   | 0.24 kPa (5.0 psf)  | IBC    |
| Wind Load                         | Horizontal Distributed | 1.13 kPa (23.6 psf) | ASCE 7 |
| Snow Load                         | Vertical Distributed   | 0.047 kPa (1.0 psf) | ASCE 7 |
| Dead Load                         | Vertical Distributed   | TBD                 | IBC    |
| Combined Load Type                |                        | Magnitude           |        |
| Total Vertical Distributed Load   |                        | 0.29 kPa (6.0 psf)  |        |
| Total Horizontal Distributed Load |                        | 1.13 kPa (23.6 psf) |        |
| Total Point Load                  |                        | 1.33 kN (300 lbf)   |        |

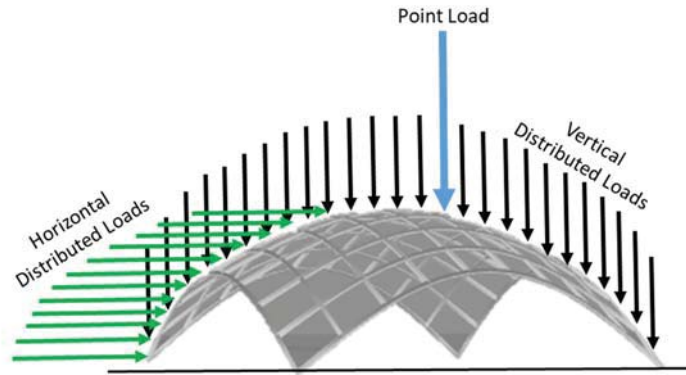


Figure 23: Diagram of design loads on mock-up gridshell.

## 7.2 Fieldwork in Athens Greece

The Summer International Service Leadership Grant sponsored by the U.S. Naval Academy is designed to offer select midshipmen the opportunity to travel abroad in order to garner enhanced knowledge and experience in a critical area. Athens, Greece was selected as the service location for this project in order to supplement written codes and standards through firsthand experience with refugees, migrants, and unaccompanied minors.

There is a lack of large sheltered spaces in refugee camps. Most relief efforts focus on constructing temporary housing for displaced people, ensuring access to sanitary water and food, and providing for the injured. The need for communal gathering spaces and pavilions is less immediate, yet vitally important beyond short-term survival. Relief efforts are semi-permanent by nature, but with the rise of refugee and migrant camps in the past five years, the lifespan of these efforts has greatly surpassed normal HA/DR expectations. Because camps are lasting longer, the structures to support displaced individuals in these situations must also evolve. The lack of heavy machinery and infrastructure in refugee camps can make constructing anything larger than tents increasingly difficult.

Refugee camps across the Middle East rely on canvas tents and container-like structures to house families. This solution is adequate for short term housing. However, most camps lack spaces to accommodate and shelter large gatherings of the population, functioning as a space for community gatherings, education, worship, and solidarity. Athens, Greece offered a window to understand the role of community gathering in refugee and migrant populations, to develop a better context for refugee camps, to learn about the climate and conditions in the Middle East and Mediterranean, and to determine what these affected populations want and need regarding a large shelter.

### 7.2.1. The Location: Athens, Greece

Athens is the largest city in Greece with 3.75 million people living in the metropolitan area [41]. Athens has long been a safe harbor for refugees and migrants, especially in the modern age. The majority of displaced populations within the city limits hail from Afghanistan, Pakistan, Kurdistan, and Syria. Prior to 2015, many refugees lived among the parks and squares in Athens proper. In August of 2015, the government of Greece built a refugee camp in the Eleonas neighborhood, a point far outside of the heart of Athens [42].

### 7.2.2. Key Takeaways

Below are the key takeaways from the SISL Project that contributed to the design of the HA/DR gridshell. Every structure serves a purpose and fits within a greater layout or city plan. In ancient Greece, the Agora was a central location that served as an athletic, political, commercial, and social center for the city by offering shelter and gather place for the population. In the case of an HA/DR gridshell, the purpose of the structure is similar to that of the Agora, as it would be designed to shelter large groups of people from the harsh desert environment and to provide a physical focal point for the community to gather. There are an unlimited number of potential solutions to fulfill that purpose, but given background conditions and constraints, certain structures and methods are more feasible and better able to accomplish the purpose of the structure than others. These takeaways will help to establish the boundary conditions and constraints associated with an HA/DR gridshell, particularly those that will impact the design and construction of the structure.

A full design team for any structure consists of more than a single engineer. A true design team features architects, structural engineers, electrical, mechanical, and plumbing experts who design ventilation and cooling systems, building physicists who analyze heating and cooling patterns, and not to mention the entire construction team required to make the structure. This research will focus specifically on the structural aspects of the gridshell, but when permissible will comment on additional considerations in order to develop a robust concept for an HA/DR gridshell.

### 7.2.3. Climate:

The climate in Athens is Mediterranean. These climates feature hot and dry summers, with short, mild, and rainy winters [43]. Farther east, where some of the largest refugee camps in the world are located, the climate is classified as arid. Arid climates experience large temperature variations during individual days and in between seasons. The summers are extremely hot and dry, with temperatures between 90 F° and 110 F° in the day, and between 60 F° and 75 F° at night. It is not uncommon during arid summers to experience 4 consecutive months without any precipitation. The winter months can bring temperatures around 40 degrees Fahrenheit and increased rainfall, and in some cases temperatures can dip below freezing [43]. Camps in regions similar to Bagdad, Iraq or Damascus, Syria may experience only a couple inches of precipitation annually.

For both Mediterranean and arid climates, shade is invaluable from both a physical and emotional standpoint. Of the engineering characteristics desired by refugees and migrants from a structure, shade is perhaps the easiest to provide. In addition to shade considerations, the effects of dry periods on the timber used to construct the structures must be taken into account during material selection.

Athens is located in a valley and is a subject of urban sprawl, thus breeze does not generally reach the inside of the city. However, on the outskirts of the city winds are more common and even in desert regions such as Bagdad and Damascus, winds average at 10 mph and can reach speeds of up to 24 mph. Consistent desert breezes are key to human comfort and should be treated as an important variable in the analysis, both in terms of maximizing the extent that breezes can flow through the structure, but also in terms of the effects of wind loading on the structure itself.

#### 7.2.4. Need for Community:

Based on conversations from external collaborators and interactions in Athens, it became clear that community was extremely important from a cultural standpoint. From daily greetings between friends, a preferential option to spend time together, and cultural customs emphasizing community, large gathering spaces were essential to the lives of the refugees and migrants. A structure designed to foster this type of community must have the capacity to shelter large populations, allow for the easy entry and exit of individuals and groups, and remain adaptable and configurable for multiple types of events.

#### 7.2.5. Desire for Beauty:

It is human nature to desire beauty and seek that which is pleasing to the eye. In Athens the refugees and migrants had the opportunity to learn woodworking and tailoring. Through these creative trades it was easy to see that each developed a pride in their work and sought beauty in the finished product. This need and desire for beauty extends beyond certain groups, rather it applies to the entirety of humankind. Architects and engineers design structures to suit the needs of those who will utilize them, and often seek to make structures as comfortable and appealing as possible. As a result, a structure designed for large group gatherings should seek to satisfy not only engineering requirements and a practical purpose, but beauty whenever permissible.

#### 7.2.6. Practicality and Overdesign:

Refugee camps and disaster relief efforts are similar in that all ages are likely to be present throughout the recovery period. One concern raised in Athens was that young children often identified pictures of gridshells as “playgrounds.” This type of point loading should be taken into account during the structural analysis.

Additionally, it is important to remember that prior to disaster, populations of affected individuals led normal lives. The Sphere Handbook urges relief personnel to employ willing and able volunteers from the affected regions to empower the local population in the recovery process. A gridshell can be constructed using manual labor alone, thus the design should take into account that volunteers may be from the local region.

#### 7.2.7. Membrane:

Given the importance of shade, and the fact that gridshells are rather porous, a membrane will have to be developed to cover the structures. A membrane for a desert climate must block or reflect the sun’s rays, remain breathable, and should be waterproof in the event of rain. From a structural standpoint, the membrane should be as light as possible to decrease the load on the gridshell itself. Considerations for the material for the membrane must be entertained, though they will not be the focus of the research. Experts in textiles and lightweight materials will be invaluable in determining a good fit for this aspect of the design.

#### 7.2.8. Packaging and Transportation:

The U.S. Navy’s ARG, and other commercial and government sea shipping entities, use standard shipping containers to transport goods across the ocean. Once these containers reach the port nearest to the disaster response, local couriers are employed to move aid packages inland. Packages that do not fit on common infrastructure platforms can become stuck in a port where they remain

unused. Special considerations for international shipping, local shipping, and construction of the structure will be taken into account pertaining to the size, shape, and packaging of the gridshell.

## **8. Applications, Constructability, and Deployment Study**

To translate the results of the parametric studies and design constraints into viable structural solutions, two small-scale models were used to evaluate the deployment and constructability of the gridshells. The insights generated from the small scale models were used to improve the fabrication and construction of the prototype.

The small-scale models did not mimic the *exact* design of the gridshell selected for HA/DR; rather, they were designed to evaluate fabrication methods and building techniques. The small-scale shells were approximately 1:25 scale with respect to the proposed HA/DR gridshell.

### **8.1. Small-Scale Models**

The beams for the first model, Model A (Figure 24b), were hand cut from 3.175 mm (0.125 in) thick high density fiberboard (HDF) into 6.35 mm (0.25 in) wide strips. HDF, a mixture of wood fibers and glue fused together under heat and pressure, was selected because of its isotropic material properties and relative pliability. The model was constrained by a rigid frame to keep the shell in place. The shell was assembled as a flat geometric lattice prior to being bent into its final form; however, certain holes did not match up exactly. It was subsequently discovered that the hole measurements for Model A failed to take into account the sliding of the beams that occurs during assembly. As a result, the model was constructed piecewise as each member was added individually, but even so some of the members experienced fracture at the hole locations due to bending stress in the small cross-section.

Shown in Figure 24a and 24c, Model B solved the issue of stress concentrations by utilizing wider members and stronger material. Model B was constructed using the same global dimensions as the previous shell, but now with 1.524 mm (0.06 in) thick acrylic plastic cut into 10 mm (0.39 in) strips and a finer grid density. Acrylic plastic was an improvement from HDF because it was still isotropic but was stronger and more flexible than HDF. Additionally, Model B was fabricated by a computer numerical control (CNC) laser cutter. To alleviate the issue of misaligned holes, the CNC laser cut slots approximately three times the diameter of the original hole. The slots allowed the lattice to lay flat prior to deformation while still permitting the joints to slide into their final location during assembly.

The laser cutter fabricated each member to exact, unique dimensions; however the use of slots later illustrated that the dimensions were similar enough to support using a common member size in future iterations. An advantage of gridshell structures is the ease in which they can be constructed, and by creating common member sizes that advantage can become even more pronounced.

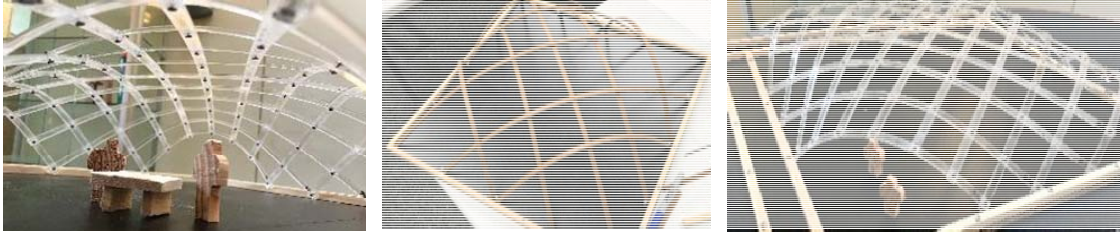


Figure 24: Scale models. (a) Inside of Model B. (b) Top view of Model A. (c) Top view of Model B.

## 8.2. Half-Scale Prototype

Improving on Model A and B, a half-scale prototype was built using 9 mm (0.35 in) thick sheets of marine grade okoume plywood. Okoume is often used to manufacture racing sailboats and was selected for its relatively low density and high flexibility. Okoume plywood was ultimately selected because it was readily available and met the criteria required to construct a gridshell, namely a low modulus of elasticity and a low density. This wood in particular was used in previous work at the U.S. Naval Academy to make a small-scale gridshell. Constructing a full-scale model or a gridshell intended for deployment in HA/DR would require a more robust material than okoume plywood, preferably a natural wood instead of plywood and a species such as oak, hemlock, or larch. The prototype was built to establish realistic manufacturing, transportation, assembly, and forming methods for gridshells designed for HA/DR. The model was not intended to be load tested, merely to prove the concept of implementing deployable gridshell structures in HA/DR.

The plywood sheets used to construct the prototype were purchased in 2.43 m (8 ft.) by 1.22 m (4 ft.) sheets. The sheets were cut into 2.43 m (8 ft.) long strips, each with a width of 38.1 mm (1.5 in). The shell was constructed using three common member sizes, a short, medium, and long member. Each of the three members were built with the same slot locations to ease the fabrication process. Each member (long, medium, or short) was made up of two to three individual segments, each approximately 2.43 m (8 ft.) long. The segments were joined together using small gusset plates.

Common member sizing was achieved by recording the unique member sizes from the Rhinoceros model of the prototype. These unique dimensions showed clearly that three general sizes of member lengths existed. Each similar member was virtually “stacked” on top of one another, with the center hole location of each member set as the origin point from which all of the other holes were referenced. Upon examining the “stack” of unique members, the slots at each location were set such that all of the stacked hole locations would fall within its bounds. The members, which spanned in excess of 7 m (21 ft.), were then cut into two or three common segments that did not interfere with the slotted hole locations. Figure 25 shows how the virtual “stack” yield common slot locations. While longer slots increased the stress concentrations in the members, the trade-off of interchangeability and decreased fabrication complexity was deemed to be of greater importance for this application.

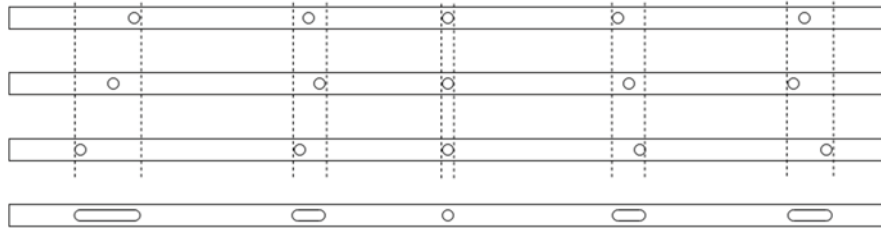


Figure 25: Virtual “stack” to yield common slot locations and lengths.

The implementation of common member sizes was a major improvement from previous small-scale models, and was a novel concept for gridshell construction as a whole. Timber gridshells often rely on unique members with slots drilled in specific locations to account for the sliding that occurs during the form finding process. Unique members must be fabricated individually, which is a time consuming process. Figure 26 show an example of the three common member sizes, long, medium, and short.



Figure 26: Common member sizes used in shell construction.

The gusset plates used to connect the members were made of 3 mm thick okoume plywood cut to 38.1 mm (1.5 in) wide and 101.6 mm (4 in) long plates. Four bolts were used to secure the plate on both sides of the connected segments. Figure 27 shows a section cut of the connection.

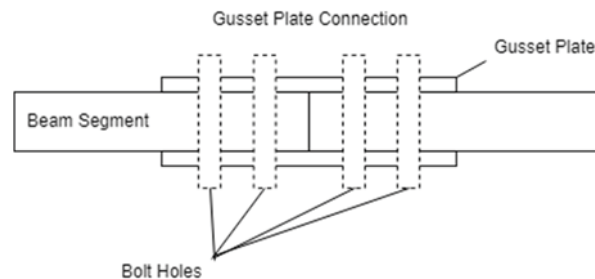


Figure 27: Gusset plate connection section cut.

The medium and long members were made up of three conjoined segments, while the short member required only two conjoined strips. The result was eight common segments that combined to make up three common large member sizes. To construct the shell in its entirety, four short members, four medium members, and eighteen long members were required. This tally does not include additional bracing members. The fabrication process itself involved marking the various hole locations on each strip, boring the holes using a drill press, routing the slots between the holes, and boring the gusset plate joining holes at the end of each member. Figure 28 shows a progression of the segment fabrication process.

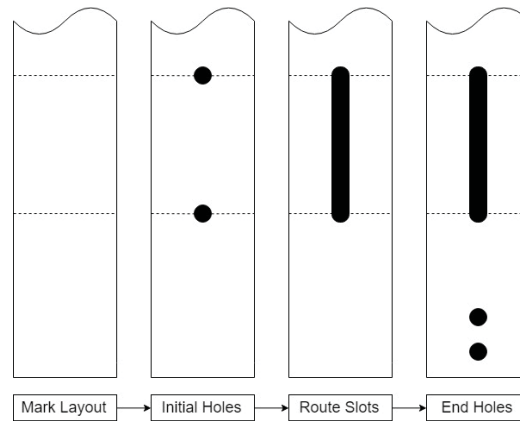


Figure 28: Progression of segment fabrication process.

Due to the common member sizes, much of the shell was interchangeable. Interchangeability improved not only the construction process, but also the reusability and replacement of the structure. Broken segments could be easily replaced and parts from one shell could potentially be used to construct other shells if needed.

Additional strips of material were fabricated to make up the bracing members. They were fabricated from the same strips of wood as the structure itself and featured the same cross-section. The gusset plate connections were utilized to make the members long enough, but slots were not drilled in the members. Instead, the bracing was designed to fit the precise form of the final shell, meaning that the holes were to be drilled on site, in piecewise fashion, as the bracing was pulled over the shell in real time. The bracing was designed to be applied in a continuous fashion over select locations on the shell. The prototype's joints were designed with bolts that protruded above the shell to allow additional bracing material to be added with ease. The small-scale models indicated that installing continuous bracing using such a method was easier than discontinuous bracing. Continuous slotted bracing was applied to the shell by securing one end of the bracing members to one side of the structure using the protruding screws mentioned earlier. That bracing member was bent over the structure, using its own weight to aid in the process, and secured at each subsequent intersection point through a hole drilled in the member on site. Figure 29 details a section cut of the installed bracing.

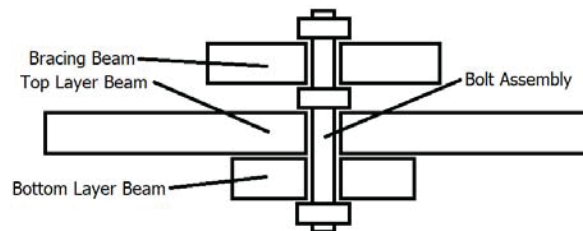


Figure 29: Section cut of prototype joint design with upper bolt protrusion for bracing installation.

In total, it took 35 work-hours to fabricate the shell in the workshop. All of the holes and slots in the segments and gusset plates were designed to have a diameter of 6.35 mm (0.25 in) so that drill bits and router bits did not have to be exchanged, and machine settings were largely

constant throughout the manufacturing process. Fabricating additional shells would take less time to manufacture with the machine settings, measurements, and process established. To manufacture the full scale shell in industry a large CNC wood-router could be utilized to translate the electronic blueprints directly to the work piece. This would greatly decrease the time and effort required to manufacture the shells.

Once the members were fabricated in the woodshop, the ends were marked with paint based on the type of segment. This was done to make the assembly process more convenient (Figure 30). With color coding properly established, a subject matter expert would not necessarily be required to assemble the shell, rather someone educated on the basic set-up instructions could coordinate its assembly. The common gusset plates ensured that the segments could be easily connected to one another on-site once the shell was ready for assembly.

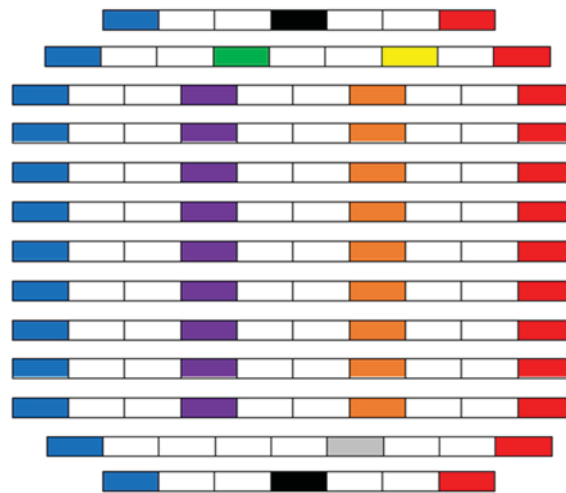


Figure 30: An example of the color code scheme for a single layer of the gridshell.

Based on the results of the parametric studies, constraints, and small-scale models, the prototype was built to prove the concept of a mobile and easy to assemble gridshell structure for HA/DR. The segments of each beam were purposefully designed to be interchangeable to ensure that assembling the shell was convenient and could be built with a team of laborers along with a few trained individuals. A computer rendering of the deployment of the shell shows that when subjected to an inverse gravity load, the four egress arches of the shell “pop-up” first, followed by the center of the shell (Figure 31). Many small timber gridshells can be assembled in this fashion without the use of heavy machinery if each of the egress arches are initially lifted off the ground before the center is pushed upwards.

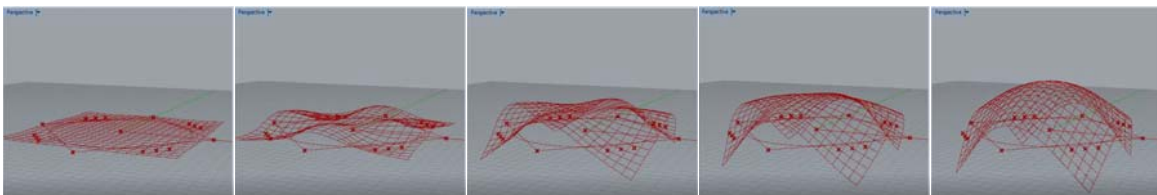


Figure 31: Time lapse of gridshell assembly when subject to an inverse gravity load.

The prototype was successfully assembled and set in its final form using the aforementioned method. After assembling the structural lattice, volunteers were able to elevate the structure into its final form (Figure 32). The touchdown points were supported with weights and tensioned ratchet straps to prevent the outward motion of the supports.



Figure 32: Construction process for prototype. (a) Lattice assembly. (b) Arch support. (c) Mid-deformation. (d) Fully deformed, weight supports in corner. (e) Tension straps added. (f) Final form achieved.

The final form of the shell was successful, but slightly less curved than the final form predicted by the computational model. Possible explanations include the material selection and support conditions. The computational rendering of the prototype shell was established using the material properties of wood as specified by Karamba's FEA solver. The material properties were also defined as isotropic throughout the member. In reality, plywood is not isotropic due to its many laminate layers and the material properties of okoume were not the same as those specified in the solver, resulting in a flatter shell than anticipated. Furthermore, the structural lattice was rather wet during the time of construction due to rainstorms throughout the day when the structural lattice was assembled. The result was less member stiffness, which also contributed to the sag in the shell. Additionally the supports were centralized at a single point at each touchdown location. As a result, the touchdown points on either side of the central touchdown location exhibited the tendency to push outward, contributing to a flatter shell with less curvature. Improvement to this design would look to improve the computational modelling of the material selected to construct the shell, as well as to lengthen the support abutments to support each of the three touchdown points at each support location.

Overall, the prototype was successful in demonstrating effective manufacturing, transportation, assembly, and forming methods for HA/DR gridshells. The manufacturing process was made more convenient by designing common member sizes, common hardware sizes, and interchangeable members. In the future, CNC manufacturing techniques could greatly improve the fabrication process. The transportation of the prototype was largely accomplished by human labor, demonstrating portability in potential disaster regions lacking proper infrastructure. However, if needed, the shell could be collapsed into segments spanning no more than 2.43 m (8

ft.) and placed inside of cargo van or pick-up truck. For the full scale gridshell, segmented common members would also be utilized and built such that the segments could fit inside of a standard international shipping container and flatbed truck so that the structures could reach regions of need. The assembly process was greatly improved with color-coded members and allowed for unskilled and unfamiliar laborers to assist in the construction process. Furthermore, the common member sizes made it easy to replace broken segments, thereby enhancing the speed of the assembly process. The forming method followed a method very similar to that predicted by the dynamic simulation and proved that a small number of volunteers can effectively assemble the structure. Ultimately, the prototype served its purpose a proof-of-concept for the idea that gridshells are viable structural solutions in HA/DR efforts. Compared to previous work, the shell's area coverage relative to the material utilized was much higher [44].

### 8.3. Full-Scale Design

The prototype demonstrated the feasibility and practical application of a gridshell structure in HA/DR. The full-scale shelter would have 15.24 m (50 ft.) square footprint, a grid density of 18 x 18, a half continuous bracing scheme, a center height of 5.3 m (17.4 ft.), a weight of approximately 13.5 kN (3,000 lbs), and a uniform cross-section of 7 cm (2.76 in) wide by 4 cm (1.57 in). The cross-section was altered from the initial parametric studies to better adapt the shell for practical use. The structural analysis concluded that the full-scale HA/DR gridshell could withstand 0.352 kPa (7.35 psf) of downward pressure (22% more than required), 2.2 kPa (45.9 psf) of horizontal pressure, 1.9 times higher than required, and a 21 kN (4,720 lbf) point load, 16 times higher than required. Without bracing, the structure would fail. As a result, the final shell will not only withstand the most strenuous load cases presented by HA/DR, but will do so in a mobile and easy to assemble package (Figure 33).

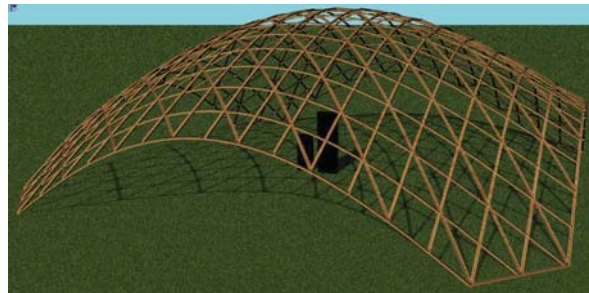


Figure 33: Computer rendering of final proposed HA/DR gridshell.

## 9. Conclusion

The role of the U.S. Navy is vital in HA/DR response because of its quick reaction capabilities and expeditionary design. The U.S. Navy's involvement in HA/DR has only grow over the past few decades, resulting a a renewed need for new technology to assist with these operations. One distinct and unfulfilled need in HA/DR response is a portable, temporary structure capable of serving as a large community gathering space. Traditional building methods are inadequate for this application because of limited infrastructure and a lack of heavy machinery available in disaster relief zones. Gridshell structures are capable of spanning large areas using a lattice that can be easily assembled

on-site and constructed using only hand tools. When properly braced, gridshell structures are highly effective in carrying load and can withstand the non-ideal loading conditions presented in HA/DR environments.

When designed with HA/DR and mobility in mind, these shells can be made using standardized slotted members that are easily packaged into generic shipping containers. By bracing the structure to support the non-ideal load cases presented by HA/DR and fabricating the members such that the shelter is portable and convenient to assemble, gridshells demonstrated their effectiveness as disaster relief infrastructure. In addition to their applications in HA/DR, this study demonstrated that gridshells can be adapted to serve in a broad array of capacities outside of an architectural context. This project demonstrated that given design loads, engineering constraints, assembly requirements, and a specific context, gridshell structures have the potential to serve in many capacities beyond architecture or even HA/DR.

### 9.1 Timeline

Compared to the proposed timeline, the project was completed as scheduled (Table 8). Some proficiencies and benchmark studies lasted longer than expected, but the fabrication and construction phases ended earlier than scheduled. This research has been accepted for presentation at the International Association of Shell and Spatial Structures Annual Symposium in October of 2019 in Barcelona, Spain.

Table 8: Proposed and Actual Project Timeline.

| Event or Task                                   | Duration | Proposed   |            | Actual     |            |
|-------------------------------------------------|----------|------------|------------|------------|------------|
|                                                 |          | Start      | Finish     | Start      | Finish     |
| Software Proficiency                            | 3mons    | 5/25/2018  | 8/16/2018  | 5/25/2018  | 1/11/2019  |
| Internship with NGO <i>Faros</i>                | 1mon     | 7/23/2018  | 8/17/2018  | 7/8/2018   | 8/1/2018   |
| SISL Report                                     | 2mons    | 7/23/2018  | 9/14/2018  | 7/8/2018   | 9/14/2018  |
| Literature Review                               | 1mon     | Unplanned  | Unplanned  | 9/10/2018  | 9/28/2018  |
| Constraints Finalized                           | 2wks     | 8/16/2018  | 8/29/2018  | 9/10/2018  | 10/5/2018  |
| Simple Parametric Model                         | 1mon     | 8/16/2018  | 9/12/2018  | 9/28/2018  | 10/18/2018 |
| Complete Parametric Model                       | 3mons    | 8/16/2018  | 11/7/2018  | 9/28/2018  | 11/12/2018 |
| Parametric Studies                              | 5mons    | 8/16/2018  | 1/2/2019   | 11/16/2018 | 2/1/2019   |
| Best Structure Criteria                         | 1wk      | 10/15/2018 | 10/19/2018 | 11/12/2018 | 2/1/2019   |
| Initial Findings Report                         | 2mons    | 11/26/2018 | 1/18/2019  | 10/5/2018  | 11/30/2018 |
| Simple Kinematic Model                          | 2mons    | 12/3/2018  | 1/25/2019  | 2/25/2019  | 2/28/2019  |
| Select Best Structure                           | 2wks     | 12/10/2018 | 12/21/2018 | 3/28/2019  | 3/31/2019  |
| Advanced Kinematic Model                        | 2mons    | 1/7/2019   | 3/1/2019   | 2/25/2019  | 3/4/2019   |
| Desktop Models                                  | 4mons    | 1/7/2019   | 4/26/2019  | 1/4/2019   | 3/7/2019   |
| IASS Abstract Due                               | 1 mon    | Unplanned  | Unplanned  | 1/18/2019  | 2/1/2019   |
| Joint Study                                     | 1mon     | 2/25/2019  | 3/22/2019  | 2/12/2019  | 2/21/2019  |
| IASS Conference Paper Due                       | 1 mon    | Unplanned  | Unplanned  | 3/6/2019   | 3/31/2019  |
| Final Report                                    | 2mons    | 3/5/2019   | 4/19/2019  | 3/10/2019  | 4/19/2019  |
| Practical Small-Scale Modelling with Deployment | 2mons    | 3/18/2019  | 5/10/2019  | 3/18/2019  | 4/16/2019  |
| Trident Presentation Day                        | 1day     | 4/26/2019  | 4/26/2019  | 4/26/2019  | 4/26/2019  |
| HumTech Boston                                  | 1wk      | 6/3/2019   | 6/7/2019   | Cancelled  | Cancelled  |
| IASS Symposium                                  | 1wk      | 10/7/2019  | 10/11/2019 | Confirmed  | Confirmed  |

### 9.2. Future Work

This research established the general behavior trends of braced gridshell subjected to symmetric, asymmetric, and point loads and proposed a viable proof-of-concept design for an HA/DR gridshell. Future work in computations could examine the effects of varying bracing cross-section and shock loading. Future work in construction could work on fabricating a full-scale shell in such a manner that it could be packaged in standard shipping containers.

### 9.3. Acknowledgements

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## Appendix A: Results of computational parametric studies.

|                                |          |          |          |          |          |          |          |          |          |          |          |          |
|--------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| UNBRACED                       | 8        | 10       | 12       | 14       | 16       | 18       | 20       | 22       | 24       | 26       | 28       | 30       |
| Number of Joints               | 69       | 109      | 157      | 213      | 277      | 349      | 429      | 517      | 613      | 717      | 829      | 949      |
| Maximum Height                 | 7.27     | 6.47     | 5.61     | 5.136    | 4.827    | 4.615    | 4.469    | 4.323    | 4.27     | 4.166    | 4.115    | 4.073    |
| Mass (kg)                      | 434.9    | 502.925  | 565.909  | 636.25   | 710      | 785.901  | 862.2    | 939.51   | 1017.1   | 1094.91  | 1172.9   | 1251.3   |
| Gravity Buckling Factor        | 7.908    | 6.4802   | 5.429036 | 4.708847 | 4.2387   | 3.924755 | 3.6836   | 3.5      | 3.348    | 3.2157   | 3.1      | 3        |
| Local Failure?                 | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 33.73845 | 31.97133 | 30.13966 | 29.3908  | 29.52297 | 30.25864 | 31.15656 | 32.25808 | 33.40551 | 34.54005 | 35.66906 | 36.82576 |
| Point Buckling Factor          | 10.512   | 8.7426   | 7.758133 | 7.936645 | 8.5697   | 9.436103 | 10.33899 | 11.287   | 12.21    | 13.11    | 13.988   | 14.8969  |
| Local Failure?                 | yes      | yes      | yes      | yes      | yes      | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 10.512   | 8.7426   | 7.758133 | 7.936645 | 8.5697   | 9.436103 | 10.33899 | 11.287   | 12.21    | 13.11    | 13.988   | 14.8969  |
| Half Gravity Buckling Factor   | 5.756    | 3.409618 | 2.190216 | 1.4762   | 0.080449 | 0.703372 | 0.732497 | 0.414876 | 0.27749  | 0.096618 | 0.282013 | 0.276644 |
| Local Failure?                 | no       | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      |
| Applied Load (kN)              | 3.3      | 5.4      | 7.9      | 10.8     | 14.1     | 17.8     | 21.9     | 26.4     | 31.3     | 36.6     | 42.3     | 48.4     |
| Max Supported Load (kN)        | 18.9948  | 18.41194 | 17.30271 | 15.94296 | 1.134331 | 12.52002 | 16.04168 | 10.95273 | 8.685437 | 3.536219 | 11.92915 | 13.38957 |
| Quadrant Point Buckling Factor | 45.348   | 55.89179 | 46.71191 | 70.12012 | 58.23    | 86.395   | 74.107   | 104.1453 | 92.348   | 121.06   | 110.49   | 136.21   |
| Local Failure?                 | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 4.5348   | 5.589179 | 4.671191 | 7.012012 | 5.823    | 8.6395   | 7.4107   | 10.41453 | 9.2348   | 12.106   | 11.049   | 13.621   |
| HALF BRACED - ALTERNATING      | 8        | 10       | 12       | 14       | 16       | 18       | 20       | 22       | 24       | 26       | 28       | 30       |
| Number of Joints               | 69       | 109      | 157      | 213      | 277      | 349      | 429      | 517      | 613      | 717      | 829      | 949      |
| Maximum Height                 | 8.412    | 6.375    | 5.533    | 5.054    | 4.785    | 4.57     | 4.44     | 4.32     | 4.24     | 4.15     | 4.1      | 4.07     |
| Mass (kg)                      | 475.76   | 606.1    | 746.9    | 842.95   | 944.27   | 1046.66  | 1150.87  | 1255.086 | 1360.38  | 1465.57  | 1571.68  | 1678     |
| Gravity Buckling Factor        | 3        | 5.6464   | 5.095358 | 4.8296   | 4.521477 | 4.080349 | 3.662435 | 3.323712 | 3.076057 | 2.881035 | 2.735499 | 2.6192   |
| Local Failure?                 | yes      | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 14.00162 | 33.5726  | 37.33414 | 39.9376  | 41.88375 | 41.89594 | 41.34902 | 40.92285 | 41.05099 | 41.42134 | 42.17642 | 43.11512 |
| Point Buckling Factor          | 8.0325   | 9.7737   | 8.4097   | 7.6905   | 7.762405 | 8.375987 | 9.378284 | 10.7897  | 12.56089 | 14.67632 | 16.667   | 17.5844  |
| Local Failure?                 | no       | yes      | yes      | yes      | yes      | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 8.0325   | 9.7737   | 8.4097   | 7.6905   | 7.762405 | 8.375987 | 9.378284 | 10.7897  | 12.56089 | 14.67632 | 16.667   | 17.5844  |
| Half Gravity Buckling Factor   | 2.331228 | 3.1385   | 2.135186 | 1.69246  | 1.438187 | 1.197727 | 0.771149 | 0.619092 | 0.479189 | 0.446578 | 0.320271 | 0.390651 |
| Local Failure?                 | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      |
| Applied Load (kN)              | 3.3      | 5.4      | 7.9      | 10.8     | 14.1     | 17.8     | 21.9     | 26.4     | 31.3     | 36.6     | 42.3     | 48.4     |
| Max Supported Load (kN)        | 7.693052 | 16.9479  | 16.86797 | 18.27857 | 20.27844 | 21.31954 | 16.88816 | 16.34403 | 14.99862 | 16.34475 | 13.54746 | 18.90751 |
| Quadrant Point Buckling Factor | 66.8869  | 49.56    | 46.03948 | 76.52886 | 81.256   | 87.30746 | 98.556   | 105.08   | 132.8529 | 122.499  | 141.8839 | 79.1362  |
| Local Failure?                 | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 6.68869  | 4.956    | 4.603948 | 7.652886 | 8.1256   | 8.730746 | 9.8556   | 10.508   | 13.28529 | 12.2499  | 14.18839 | 7.91362  |
|                                |          |          |          |          | 0.244559 | 0.245935 | 0.247204 | 0.248377 | 0.24943  | 0.250449 | 0.250916 | 0.251323 |
| FULL BRACING- ALTERNATING      | 8        | 10       | 12       | 14       | 16       | 18       | 20       | 22       | 24       | 26       | 28       | 30       |
| Number of Joints               | 69       | 109      | 157      | 213      | 277      | 349      | 429      | 517      | 613      | 717      | 829      | 949      |
| Maximum Height                 | 8.158    | 6.222    | 5.398    | 4.948    | 4.668    | 4.452    | 4.347    | 4.194    | 4.128    | 4.075    | 4.001    | 3.958    |
| Mass (kg)                      | 736.69   | 813.7    | 925      | 1047.534 | 1175.2   | 1304.07  | 1435.37  | 1566.82  | 1699.7   | 1832.62  | 1966.04  | 2099.72  |
| Gravity Buckling Factor        | 3.381913 | 15.60735 | 9.359494 | 6.393032 | 4.879197 | 4        | 3.485755 | 3.13088  | 2.887531 | 2.702226 | 2.560009 | 2.446204 |
| Local Failure?                 | yes      | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 24.44084 | 124.5841 | 84.93039 | 65.69675 | 56.25086 | 51.17171 | 49.08285 | 48.1232  | 48.14686 | 48.58062 | 49.37452 | 50.38753 |
| Point Buckling Factor          | 20.56803 | 16.0317  | 15.1939  | 15.81856 | 16.39433 | 13.63354 | 12.8824  | 13.28361 | 14.47809 | 16.14672 | 18.09392 | 20.19    |
| Local Failure?                 | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 20.56803 | 16.0317  | 15.1939  | 15.81856 | 16.39433 | 13.63354 | 12.8824  | 13.28361 | 14.47809 | 16.14672 | 18.09392 | 20.19    |
| Half Gravity Buckling Factor   | 1.687081 | 5.603    | 4.55863  | 2.9277   | 2.05692  | 1.568185 | 1.28928  | 0.768393 | 0.610822 | 0.528904 | 0.362486 | 0.157491 |
| Local Failure?                 | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      | yes      |
| Applied Load (kN)              | 3.3      | 5.4      | 7.9      | 10.8     | 14.1     | 17.8     | 21.9     | 26.4     | 31.3     | 36.6     | 42.3     | 48.4     |
| Max Supported Load (kN)        | 5.567367 | 30.2562  | 36.01318 | 31.61916 | 29.00257 | 27.91369 | 28.23523 | 20.28558 | 19.11873 | 19.35789 | 15.33316 | 7.622564 |
| Quadrant Point Buckling Factor | 70.10183 | 119.5677 | 116.435  | 138.99   | 108.23   | 121.0295 | 125.56   | 138.7    | 151.45   | 175.2155 | 174.77   | 196.38   |
| Local Failure?                 | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       | no       |
| Max Supported Load (kN)        | 7.010183 | 11.95677 | 11.6435  | 13.899   | 10.823   | 12.10295 | 12.556   | 13.87    | 15.145   | 17.52155 | 17.477   | 19.638   |



## **Appendix B: MATLAB code for processing results of computational parametric studies.**

```
% UNBRACED DATA
% _u means UNBRACED

space_u = xlsread('Trident Data Master Final.xlsx','Graphs','F1:M1');
joints_u = xlsread('Trident Data Master Final.xlsx','Graphs','F2:M2');
mass_u = xlsread('Trident Data Master Final.xlsx','Graphs','F4:M4');
grav_buck_u = xlsread('Trident Data Master Final.xlsx','Graphs','F5:M5');
point_buck_u = xlsread('Trident Data Master Final.xlsx','Graphs','F8:M8');
half_buck_u = xlsread('Trident Data Master Final.xlsx','Graphs','F11:M11');
quad_buck_u = xlsread('Trident Data Master Final.xlsx','Graphs','F15:M15');
grav_load_u = xlsread('Trident Data Master Final.xlsx','Graphs','F7:M7');
point_load_u = xlsread('Trident Data Master Final.xlsx','Graphs','F10:M10');
half_load_u = xlsread('Trident Data Master Final.xlsx','Graphs','F14:M14');
quad_load_u = xlsread('Trident Data Master Final.xlsx','Graphs','F17:M17');

% HALF DISCONTINUOUS DATA
% _hd means UNBRACED

space_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F19:M19');
joints_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F20:M20');
mass_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F22:M22');
grav_buck_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F23:M23');
point_buck_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F26:M26');
half_buck_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F29:M29');
quad_buck_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F33:M33');
grav_load_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F25:M25');
point_load_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F28:M28');
half_load_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F32:M32');
quad_load_hd = xlsread('Trident Data Master Final.xlsx','Graphs','F35:M35');

% FULL DISCONTINUOUS DATA
% _fd means UNBRACED

space_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F37:M37');
joints_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F38:M38');
mass_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F40:M40');
grav_buck_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F41:M41');
point_buck_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F44:M44');
half_buck_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F47:M47');
quad_buck_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F51:M51');
grav_load_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F43:M43');
point_load_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F46:M46');
half_load_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F50:M50');
quad_load_fd = xlsread('Trident Data Master Final.xlsx','Graphs','F53:M53');

% FULL CONTINUOUS DATA
% _fc means UNBRACED

space_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F55:M55');
joints_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F56:M56');
mass_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F58:M58');
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grav_buck_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F59:M59');
point_buck_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F62:M62');
half_buck_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F65:M65');
quad_buck_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F69:M69');
grav_load_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F61:M61');
point_load_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F64:M64');
half_load_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F68:M68');
quad_load_fc = xlsread('Trident Data Master Final.xlsx','Graphs','F71:M71');

% Half CONTINUOUS DATA
% _hc means UNBRACED

space_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F73:D73');
joints_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F74:D74');
mass_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F76:D76');
grav_buck_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F77:D77');
point_buck_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F80:B80');
half_buck_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F83:D83');
quad_buck_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F87:D87');
grav_load_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F79:D79');
point_load_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F82:D82');
half_load_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F86:D86');
quad_load_hc = xlsread('Trident Data Master Final.xlsx','Graphs','F89:D89');

% Data
mass_diff_half = 100*mean((mass_hd - mass_u)./mass_u)
mass_diff_full = 100*mean((mass_fd - mass_u)./mass_u)

grav_buck_diff_hd = 100*mean((grav_buck_hd - grav_buck_u)./grav_buck_u)
grav_buck_diff_fd = 100*mean((grav_buck_fd - grav_buck_u)./grav_buck_u)
grav_buck_diff_fc = 100*mean((grav_buck_fc - grav_buck_u)./grav_buck_u)

point_buck_diff_hd = 100*mean((point_buck_hd - point_buck_u)./point_buck_u)
point_buck_diff_fd = 100*mean((point_buck_fd - point_buck_u)./point_buck_u)
point_buck_diff_fc = 100*mean((point_buck_fc - point_buck_u)./point_buck_u)

half_buck_diff_hd = 100*mean((half_buck_hd - half_buck_u)./half_buck_u)
half_buck_diff_fd = 100*mean((half_buck_fd - half_buck_u)./half_buck_u)
half_buck_diff_fc = 100*mean((half_buck_fc - half_buck_u)./half_buck_u)

quad_buck_diff_hd = 100*mean((quad_buck_hd - quad_buck_u)./quad_buck_u)
quad_buck_diff_fd = 100*mean((quad_buck_fd - quad_buck_u)./quad_buck_u)
quad_buck_diff_fc = 100*mean((quad_buck_fc - quad_buck_u)./quad_buck_u)

per_grav_load_diff_hd = 100*mean((grav_load_hd - grav_load_u)./grav_load_u)
per_grav_load_diff_fd = 100*mean((grav_load_fd - grav_load_u)./grav_load_u)
per_grav_load_diff_fc = 100*mean((grav_load_fc - grav_load_u)./grav_load_u)

per_point_load_diff_hd = 100*mean((point_load_hd - point_load_u)./point_load_u)
per_point_load_diff_fd = 100*mean((point_load_fd - point_load_u)./point_load_u)
per_point_load_diff_fc = 100*mean((point_load_fc - point_load_u)./point_load_u)

per_half_load_diff_hd = 100*mean((half_load_hd - half_load_u)./half_load_u)
per_half_load_diff_fd = 100*mean((half_load_fd - half_load_u)./half_load_u)

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per_half_load_diff_fc = 100*mean((half_load_fc - half_load_u)./half_load_u)

per_quad_load_diff_hd = 100*mean((quad_load_hd - quad_load_u)./quad_load_u)
per_quad_load_diff_fd = 100*mean((quad_load_fd - quad_load_u)./quad_load_u)
per_quad_load_diff_fc = 100*mean((quad_load_fc - quad_load_u)./quad_load_u)

grav_load_diff_hd = mean((grav_load_hd - grav_load_u))
grav_load_diff_fd = mean((grav_load_fd - grav_load_u))
grav_load_diff_fc = mean((grav_load_fc - grav_load_u))

point_load_diff_hd = mean((point_load_hd - point_load_u))
point_load_diff_fd = mean((point_load_fd - point_load_u))
point_load_diff_fc = mean((point_load_fc - point_load_u))

half_load_diff_hd = mean((half_load_hd - half_load_u))
half_load_diff_fd = mean((half_load_fd - half_load_u))
half_load_diff_fc = mean((half_load_fc - half_load_u))

quad_load_diff_hd = mean((quad_load_hd - quad_load_u))
quad_load_diff_fd = mean((quad_load_fd - quad_load_u))
quad_load_diff_fc = mean((quad_load_fc - quad_load_u))

figure(1)
clf
hold on
grid on
plot(space_u,grav_buck_u,'-.* k','LineWidth',2)
plot(space_fc,grav_buck_fc,'--* r','LineWidth',2)
plot(space_fd,grav_buck_fd,'--o b','LineWidth',2)
plot(space_hd,grav_buck_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Density','FontSize',25,'Interpreter','latex')
ylabel('Buckling Factor','FontSize',25,'Interpreter','latex')
title('Symmetric Distributed Load Buckling Factor for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous');

figure(2)
clf
hold on
grid on
plot(space_u,grav_load_u,'-.* k','LineWidth',2)
plot(space_fc,grav_load_fc,'--* r','LineWidth',2)
plot(space_fd,grav_load_fd,'--o b','LineWidth',2)
plot(space_hd,grav_load_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Denstiy','FontSize',25,'Interpreter','latex')
ylabel('Maximum Supported Load (kN)','FontSize',25,'Interpreter','latex')
title('Symmetric Distributed Load Maximum Supported Load for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous');

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```

figure(3)
clf
hold on
grid on
plot(space_u,half_buck_u,'-.* k','LineWidth',2)
plot(space_fc,half_buck_fc,'--* r','LineWidth',2)
plot(space_fd,half_buck_fd,'--o b','LineWidth',2)
plot(space_hd,half_buck_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Density','FontSize',25,'Interpreter','latex')
ylabel('Buckling Factor','FontSize',25,'Interpreter','latex')
title('Asymmetric Distributed Load Buckling Factor for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous');

figure(6)
clf
hold on
grid on
plot(space_u,half_load_u,'-.* k','LineWidth',2)
plot(space_fc,half_load_fc,'--* r','LineWidth',2)
plot(space_fd,half_load_fd,'--o b','LineWidth',2)
plot(space_hd,half_load_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Density','FontSize',25,'Interpreter','latex')
ylabel('Maximum Supported Load (kN)','FontSize',25,'Interpreter','latex')
title('Asymmetric Distributed Load Maximum Supported Load for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous');
%
figure(7)
clf
hold on
grid on
plot(space_u,point_buck_u,'-.* k','LineWidth',2)
plot(space_fc,point_buck_fc,'--* r','LineWidth',2)
plot(space_fd,point_buck_fd,'--o b','LineWidth',2)
plot(space_hd,point_buck_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Density','FontSize',25,'Interpreter','latex')
ylabel('Buckling Factor','FontSize',25,'Interpreter','latex')
title('Centered Point Load Buckling Factor for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous');

figure(9)
clf
hold on
grid on
plot(space_u,quad_buck_u,'-.* k','LineWidth',2)

```

```

plot(space_fc,quad_buck_fc,'--* r','LineWidth',2)
plot(space_fd,quad_buck_fd,'--o b','LineWidth',2)
plot(space_hd,quad_buck_hd,'-.o g','LineWidth',2)

set(gca,'FontSize',15)
xlabel('Grid Density','FontSize',25,'Interpreter','latex')
ylabel('Buckling Factor','FontSize',25,'Interpreter','latex')
title('Off Center Point Load Buckling Factor for All Bracing
Schemes','FontSize',20,'Interpreter','latex')
h_legend=legend('Unbraced','Full Continuous','Full Discontinuous','Half Discontinuous')

```

|                                 |                                  |                                  |                                |
|---------------------------------|----------------------------------|----------------------------------|--------------------------------|
| mass_diff_half =<br>33.6187     | half_buck_diff_fc<br>= 420.9326  | per_point_load_diff_fc = 38.1416 | point_load_diff_hd<br>= 0.4948 |
| mass_diff_full =<br>66.8270     | quad_buck_diff_hd<br>= 13.2563   | per_half_load_diff_hd = 287.7697 | point_load_diff_fd<br>= 3.9082 |
| grav_buck_diff_hd<br>= -4.7446  | quad_buck_diff_fd<br>= 54.9563   | per_half_load_diff_fd = 411.7475 | point_load_diff_fc<br>= 4.2573 |
| grav_buck_diff_fd<br>= -8.0612  | quad_buck_diff_fc<br>= 66.4159   | per_half_load_diff_fc = 420.9326 | half_load_diff_hd<br>= 7.5549  |
| grav_buck_diff_fc<br>= -8.3361  | per_grav_load_diff_hd = 27.2543  | per_quad_load_diff_hd = 13.2563  | half_load_diff_fd<br>= 11.0850 |
| point_buck_diff_hd<br>= 2.2076  | per_grav_load_diff_fd = 53.2993  | per_quad_load_diff_fd = 54.9563  | half_load_diff_fc<br>= 13.0056 |
| point_buck_diff_fd<br>= 35.5877 | per_grav_load_diff_fc = 52.7292  | per_quad_load_diff_fc = 66.4159  | quad_load_diff_hd<br>= 0.8198  |
| point_buck_diff_fc<br>= 38.1416 | per_point_load_diff_hd = 2.2076  | grav_load_diff_hd = 8.7723       | quad_load_diff_fd<br>= 5.1044  |
| half_buck_diff_hd<br>= 287.7697 | per_point_load_diff_fd = 35.5877 | grav_load_diff_fd = 17.1852      | quad_load_diff_fc<br>= 5.77    |
| half_buck_diff_fd<br>= 411.7475 |                                  | grav_load_diff_fc = 17.0432      |                                |