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RPPR Final Report
as of 06-Feb-2019

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Final Report for Period Beginning 06-Apr-2017 and Ending 31-Aug-2018

Title: Rapid penetration of rigid bodies into elastic media: the temperature, boundary and nonuniform speed effects

Begin Performance Period: 06-Apr-2017

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STEM Degrees: 0

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Major Goals: The overall goal of the project was to advance constructive analytical methods for a class of mixed boundary value problems in elastodynamics and dynamic thermoelasticity. This project focused on modeling of rapid nonuniform penetration of thin two-dimensional and axisymmetric rigid bodies into a medium that can be categorized as an elastic or thermoelastic, infinite or semi-infinite, two- or three-dimensional body. Specifically, the PI aimed to study

(1) non-uniform speed effects on rapid penetration of a thin finite two-dimensional rigid body in an infinite elastic plane with finite fore and semi-infinite trailing crack-like cavities;

(2) temperature effects by analyzing a dynamic coupled thermoelastic model for a thin rigid two-dimensional body moving in an elastic medium when the conditions of Coulomb friction hold on the body-medium interface, and a semi-infinite crack-like cavity trails the body;

(3) boundary effects by examining penetration of a finite rigid body in a half-plane in the direction orthogonal to the boundary;

(4) an axisymmetric model on penetration of a cylindrical semi-infinite rigid body into a three-dimensional elastic medium.

Accomplishments: 1. Major activities. I was working on steady-state and transient two-dimensional problems on motion of a thin rigid body in an unbounded elastic medium, dynamic problems of coupled thermoelasticity, nonuniform speed motion, motion of a rigid body in an elastic medium with a boundary and motion of axis-symmetric bodies. I attended the Penetration Mechanics Short Course organized by SWRI at San Antonio (March 2018) to establish research links with scientists from SWRI working in penetration mechanics. I have written four papers on the results related to the project (three of them have been published and the fourth one is to be submitted) and made four presentations.

2. Specific objectives. Upon completion of this 15-month-research-work it was expected to obtain the following results:

(i) An analytically-numerical solution of a dynamic plane problem on a thin finite rigid body advancing at nonuniform speed. The model assumes that there are fore and trailing crack-like cavities and the body contacts with the surrounding elastic medium according to the Coulomb friction law. It was expected that this solution would

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help to understand how nonuniform speed and cavities affect the stress distribution around the body and design a criterion for determination of the terminal point of the body path.

(ii) A numerically-analytical solution of the steady-state plane problem of dynamic coupled thermoelasticity on transonic motion of a thin rigid body.

(iii) Derivation and analysis of a system of two two-dimensional singular integral equations governing the penetration process of a thin rigid body into an elastic half-plane at nonuniform speed in the direction orthogonal to the boundary of the half-plane.

(iv) Reduction of the dynamic axisymmetric problem on a semi-infinite cylindrical rigid body moving at constant speed in a three-dimensional elastic medium to four one-dimensional singular integral equations. It was aimed to use this model problem as a simplified model for penetration of a finite cylindrical rigid body in an elastic medium.

3. Significant results.

(i) To analyze the thermoeffects, in collaboration with B.T. Dickinson (Eglin AFB) we considered (Math. Mech. Solids, (2018) 23, 1274-1290) a model dynamic problem of coupled thermoelasticity for a two-dimensional elastic structure. We derived the governing integro-differential equations using the variational approach and solved them by applying integral transforms and the method of successive approximations for the resulting infinite system of linear algebraic equations for the Laplace transforms of the vibration modes.

(ii) I analyzed (Quart. J. Mech. Appl. Math. (2018) 71, 221-243) two steady-state models of penetration of a thin finite rigid body in an infinite elastic medium. Both models assume that the body and the medium contact according to the Coulomb friction law, and the body leaves a semi-infinite crack-like cavity behind. The difference between the two models is the presence of a fore finite crack-like cavity in the first model. Both problems were reduced to two exactly solved Riemann-Hilbert problems. I proposed a criterion for determination of the aft and fore separation points and a new integral formula for the resistance force which takes into account not only the tangential stress but also the other components of the stress tensor.

(iii) To analyze the transient effects, I considered the transient problem (the results are to be submitted) on motion of a thin finite rigid body at sub-Rayleigh speed in an elastic medium. When the body moves, it leaves a semi-infinite crack-like cavity behind, and a finite crack-like cavity is formed ahead of the body. By means of the Laplace and Fourier transforms the problem maps into a Riemann-Hilbert problem for three pairs of piecewise analytic functions with a triangular matrix coefficient. A new approximate scheme for its solution was proposed. The problem was also reformulated as a system of two singular integral equations on a finite and a semi-infinite intervals and solved by the method of orthogonal polynomials by reducing it to a second kind infinite system of linear algebraic equations. Numerical results demonstrate an efficiency of the method. Currently, based on this solution I am developing a numerical algorithm to compute the resistant force and stresses around the cavities.

(iv) To develop an efficient numerical procedure for the transient problem on motion of a body in an elastic medium with a semi-infinite trailing cavity and a finite fore cavity, in collaboration with S.M. Mkhitarian we studied (Quart. Appl. Math. (2018) 76, 739-766) the semi-infinite Hilbert transform, derived some new integral relations for orthogonal polynomials, and obtained new quadrature formulas for the Cauchy singular integral efficient for numerical purposes.

(v) Using the motion equations in cylindrical coordinates I derived a system of four integral equations for the three-dimensional axisymmetric model on rapid motion of a rigid semi-infinite cylinder in an elastic medium. Also a method of integral transformations for realistic ogive-nose rigid bodies was developed. This work is still in progress.

4) Key outcomes and discussion of stated goals not met.

(i) The numerical tests revealed that unless the coupling parameter is small the coupling of the heat and bending equations effects are essential on the deflection and the temperature distribution. If the coupling parameter is large, then the generally accepted practice of decoupling the heat and elasticity equations may lead to a significant error of computations.

(ii) We discovered that the resistance force formula based on the tangential traction only and widely employed in the literature underestimates its values and leads to the unrealistic zero value as speed reaches the Rayleigh speed. Numerical results show that for ogive-nose thin two-dimensional penetrators with a semi-infinite trailing cavity the resistance force varies significantly with motion speed. If the fore crack length is fixed and assumed to be independent of speed, then similarly to the variation of the Mode-I SIF with sub-Rayleigh speed in dynamic fracture the resistance force is a decreasing function of speed. If the fore crack length in Model 1 is recovered from the Griffith criterion as the root of the associated transcendental equation, then the crack length is an increasing function of speed, while the resistance force depends on speed not monotonically. It attains its maximum at a certain speed and then decreases as speed approaches the Rayleigh speed.

(iii) One of the objectives of the project was to work out a method for the model problem on motion of finite rigid bodies at nonuniform speed in an elastic medium. It was discovered that the Freund's procedure (L.B. Freund, Dynamic Fracture Mechanics) proposed for nonuniform motion of a semi-infinite crack is not applicable for rigid

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bodies having a nonzero thickness. The key step in the Freund's method is the solution of the problem on a suddenly stopping crack. The requirement that the stress intensity factor vanishes at the crack tip is sufficient for the crack to stop, while in the problem on a suddenly stopping finite rigid body that condition is not sufficient. In future we are planning to develop another method based on the full system of elasticity equations and the boundary layer theory.

(iv) It was also planned to analyze the boundary effect of rigid body motion in order to model the penetration of a rigid body into an elastic medium when it hits the medium surface. We managed to derive integral equations for the case of antiplane deformation. It turns out that even in this case the resulting integral equations are of a new type and have not been studied in the mathematical literature. In future we continue our efforts on the analysis of these integral equations and developing efficient numerical methods.

Training Opportunities: I did not manage to recruit a graduate student with one-semester support from the grant during the period April 2017- April 2018. However, during this period I interviewed several first-year graduate students and let them know the aims of my ARO proposal and how applied mathematics may help to solve hard problems of penetration mechanics. Several students whom I interviewed enrolled in my Fall 2018 graduate class Complex Analysis, and one of them became interested in working with me next semester on problems related to the grant supported research.

Results Dissemination: The results of the research work supported by ARO were reported at

the Canonical Scattering Workshop organized by the Department of Mathematics, University of Manchester, U.K. (November 2017). The title of presentation was "Nonuniform motion of cracks and thin rigid bodies in elastic media",

the Mathematics Department Colloquium in University of Memphis (March 2017). The title of presentation was "Penetration of thin rigid bodies into elastic media, the Riemann-Hilbert problem, and the semi-infinite Hilbert transform",

Annual SIAM Meeting in Portland (July 2018). The title of presentation was "Hilbert Transform, Quadrature Formulas for the Cauchy Integral in a Semi-axis, and Singular Integral Equations",

SIAM TX-LA Sectional Meeting in Baton Rouge (October 2018). The title of presentation was "Subsonic penetration of a thin rigid body into an elastic medium with crack-like cavities ahead and behind the body".

Also, during the Penetration Mechanics Course in South-West Research Institute at San Antonio in March 2018 I discussed the results obtained with attendees of the course and SWRI researchers.

Honors and Awards: Air Force Summer Faculty Fellowship Award 2017 (Eglin AFB)
Air Force Summer Faculty Fellowship Award 2018 (Eglin AFB)

Protocol Activity Status:

Technology Transfer: During the periods May - June 2017 and June - August 2018 I was collaborating with Dr Benjamin Dickinson (AFRL: Munitions Directorate, Eglin Air Force Base) on studying model problems of dynamic coupled thermoelasticity related to the second part of the current project supported by ARO.

PARTICIPANTS:

Participant Type: PD/PI

Participant: Yuri Antipov

Person Months Worked: 15.00

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Funding Support:

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