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**New GSD Modeling for Air Blast Wave
Supported by Non-Uniform Flow:
I. Modeling**

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14. ABSTRACT The goal of this research is to find a way to model complex shock interactions using Geo-metrical Shock Dynamics (GSD) as a more efficient alternative to computationally expensive hydrocodes, such as the CTH. But "classical" GSD, as initially described by Whitham [11], describes shock propagations in which the ow immediately behind the shock front is uniform. Many other shock dynamics phenomena in air, such as the Taylor point blast wave, are not characterized by a uniform flow behind the shock, so Whitham's GSD (WGSD) theory cannot provide an accurate shock simulation. We thus have developed a new expanded GSD (EGSD) model that can portray shock propagation with a non-uniform ow state behind the shock, which can arise from Taylor point blast; from a finite-sized, condensed explosive detonation; etc. We show the accuracy and efficiency of the new model in this paper by showing excellent agreement in the relation between Mach number and shock radius among hydrocode simulations, GSD simulations and theory.					
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1 Introduction

Air shock propagation problems generally can be solved accurately by hydrodynamic simulations using a full time-dependent gas dynamics solver; an Euler system of equations. However this can be extremely expensive computationally when the numerical resolution is high and the computational domain is large. A well-known approach to reduce the computational cost is to use the geometrical shock dynamics (GSD) theory, which can be solved using a system of two or three ordinary differential equations [11]. Whitham’s classical GSD theory (WGSD) assumes that the flow immediately behind the shock front is uniform, and WGSD has proven to be accurate for shock tube problems. The theory can be easily extended to solve more general problems in two or three dimensional space, including shock diffraction and shock-shock [11, 3]. The standard GSD implementation has been further developed by many researchers including Schedmann [4, 9].

When the flow state behind the shock front is not uniform, the WGSD model does not track the shock front accurately. Such cases include a point blast wave, as described by J. Taylor [10] and refined in an approximation by Bach-Lee [1], and a blast wave formed by a finite-size explosive detonation, as demonstrated in this paper. Therefore some effort has been made to include the effect of non-uniformity of flow behind a shock front by researchers such as J. P. Best [4], who introduced an additional term Q_m to the first order term in WGSD. While that extension can improve the unsteady shock propagation in high diffraction along a corner of high curvature, the improvement of GSD is still based on the assumption of initially uniform flow behind the shock. The algorithm is still far from being an accurate tracking method when the flow behind the shock front is highly nonuniform.

In this paper, we first give a quick review of WGSD theory and Bach and Lee’s approximation for point blast [1], then show how it can be integrated into the framework of GSD. We call the new GSD method for the point blast wave PGSD. We then show that the relation between Mach number M and shock radius R (M - R), derived from the PGSD method is identical to those computed from the analytic solutions of the Bach and Lee theory and from hydrocode simulations, while that from WGSD is far different.

We then consider a GSD model for an air blast from a finite, spherical explosive detonation. The Mach number of the air blast wave at a specified distance from the explosive depends on the explosive properties, which are reflected in the equations of state (EOS) and reactive flow model used in hydrocode simulations. The geometrical shock dynamics for a spherical explosive charge (hereafter, EGSD) described in this paper requires experimental data or a hydrocode simulation to establish the M - R relation. If no experimental data are available, this can be as simple as a 1-D simulation of a small charge, since we have utilized a scaling factor, which is a well-known property of expanding spheres, to extend the M - R relation to other charge sizes. The scaling laws are well summarized by J. M. Dewey in section 13.1.3 of Handbook of Shock Waves [3]. Using such an M - R relation from a simulation (or experiment) removes the necessity to develop a new analytic solution (as, e.g., the Bach-Lee theory), even though such a solution may offer a deeper insight to the underlying mechanism. We emphasize that our method of GSD described here is not sufficient for simulating shock interaction mechanisms; that is beyond of the scope of this paper and should be developed separately by following existing theories on shock waves ([2], [3], [11]). There are significant advantages to using the EGSD model instead of hydrocode simulations. Our

CTH hydrocode simulations require high resolution; on the order of 80 cells per millimeter. We found considerable numerical oscillation around the material interface of explosive products and air at lower resolutions. This means 160,000 cells for a one-dimension(1-D), 2 meter domain in spherical coordinates. Even for this simple case, a simulation of a 5 cm TNT spherical charge took several hours on a workstation with 32 processors. By contrast, the corresponding GSD simulation takes only a few minutes with a single CPU. The compu-

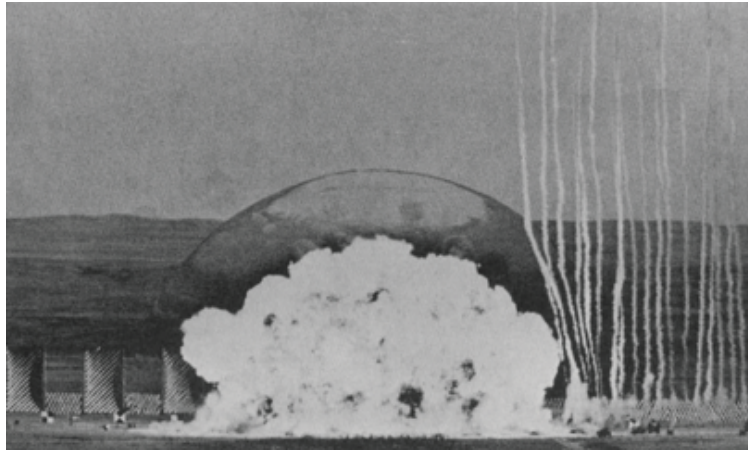


Figure 1. Surface detonation of 200 tons of TNT (Chapter 13, Expanding Spherical Shocks, by J. M. Dewey, Handbook of Shock Waves [3], Academic Press, 2001 by Ben-Dor *et al.*)

tational overhead highlights the impracticality of using hydrocodes for air shock simulations over a large domain in three-dimensional (3-D) space, and clearly validates the use of the EGSD method. Such computationally efficient simulation can be significant; e.g. in large scale military situations, as shown in Figure 1. As described by John M. Dewey [3, Chapter 13], the figure shows a large spherical ripple formed by the increase of the density at the front, which results an intense gradient of the refractive index of the ambient air to produce a distortion of the observed background behind the shock.

This paper describes only the new models for the GSD motion rule. The numerical algorithm for shock propagation will be described in a companion paper [13], since the technique also requires a lengthy description. Together, the GSD method can be a practical method to simulate shock propagation over a large size domain, when followed by non-uniform flow state, where "*the effect of the explosion is to force most of the air within the shock front into a thin shell just inside that front*" as described by G. Taylor [10]. A typical situation is seen in Figure 1.

2 New GSD formulation for a shock followed by non-uniform flow

Shock propagation through a quiescent air is defined as a system of ordinary differential equations in GSD as follows:

$$\begin{cases} \frac{dM}{dt} = -\frac{a_0 M}{\Phi(M)} \kappa \\ \frac{d\mathbf{x}}{dt} = a_0 M \mathbf{n}, \end{cases} \quad (1)$$

where M is Mach number; κ curvature; \mathbf{x} a point on the surface and \mathbf{n} the normal vector at point \mathbf{x} ; a_0 is the sound speed of air (nominally 340 m/sec); and t is time. The function Φ in equation (1) is a function of Mach number M . The specific formula is given according to the flow properties behind the shock. The primary work described in this paper is to give the formula Φ for three different cases: WGSD, PGSD and EGSD, and to show how the corresponding GSD can be a good alternative to hydrocode simulation in all these flow situations.

In the subsequent two sections, we describe the theory of point air blast waves developed by G. Taylor, Bach and Lee. While complete details are given in their papers ([10] and [1]), we provide some essential equations from their theories, which we used for developing the PGSD model and for running hydrodynamic simulations for comparison. The EGSD model for expanding spherical shock is described after these two sections. While relevant physics are well described by John Dewey [3, Chapter 13], the model we describe in the section is a simple method in the framework of GSD.

2.1 Whitham's theory (WGSD)

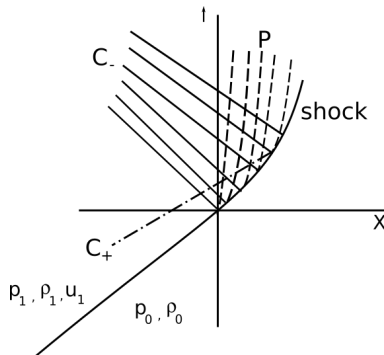


Figure 2. The (x, t) diagram for a shock entering a nonuniform region ([13], Fig. 8.1)

Whitham described a characteristic method to derive a motion rule for the propagation of a shock surface in a tube with a nonuniform cross-sectional area $A(x)$. The $x - t$ diagram for the problem is shown in Figure 2, with $t = 0$ at the point when the incident shock arrives at $x = 0$, where $A(x)$ changes. For $t \leq 0$, the flow is uniform with a discontinuity at the shock, and the pressure, density and particle velocity are represented by (p_1, ρ_1, u_1) behind

the shock and by $(p_0, \rho_0, 0)$ ahead of the shock. Disturbances to this state propagate on the particle paths P and the negative characteristics C_- . The slope of C_- depends upon whether the area is increasing or decreasing; the figure shows a decreasing area. The C_+ characteristic lines are attached to the shock front and any information on the shock front originates from the information on the initial uniform flow state along the C_+ line. This fact simplifies the shock speed relation dramatically, and Whitham's A-M relation for a finite area change ([11, equation (8.22)]) is

$$\frac{1}{A} \frac{dA}{dM} = -G(M), \quad (2)$$

where

$$G(M) = \frac{M}{M^2 - 1} \left(1 + \frac{2}{\gamma + 1} \frac{1 - \mu^2}{\mu} \right) \left(1 + 2\mu + \frac{1}{M^2} \right), \quad \mu^2 = \frac{(\gamma - 1)M^2 + 2}{2\gamma M^2 - (\gamma - 1)} \quad (3)$$

and A is the local area of the shock surface. Whitham assumed that A varies only by a spatial distance x in a 1-D shock tube, and that assumption can be interpreted as a 1-D variation of A in the shock normal direction. Therefore equation (2) can be converted to equation (1) with $G = \Phi$, and

$$\frac{A'}{A} = \kappa, \quad (4)$$

where $A' = dA/dx$. With a shock surface in 2-D parameterized as $\mathbf{x}(s(t), t)$ and $\frac{1}{A} \frac{dA}{dt} = a_0 M A' / A$ by the chain rule of differentiation, Best [4, equation (176)] derived the following relation:

$$\frac{1}{A} \frac{dA}{dt} = a_0 M(s(t), t) \frac{\partial \mathbf{x}(s(t), t)}{\partial s(t)} \cdot \frac{\partial \mathbf{n}(s(t), t)}{\partial s(t)}, \quad (5)$$

where \mathbf{n} is a normal vector to the surface. From the Frenet formula [7], we also have

$$\frac{\partial \mathbf{x}(s(t), t)}{\partial s(t)} \cdot \frac{\partial \mathbf{n}(s(t), t)}{\partial s(t)} = \mathbf{t} \cdot \mathbf{n}' = \kappa, \quad (6)$$

where \mathbf{t} is the tangent vector and \mathbf{n}' is its derivative with respect to the surface parameterization variable. These derivations prove equation (4) for 2-D curved shocks; extending the relation for 3-D surfaces, where κ is total curvature, is straightforward.

More GSD theory for wave propagation through quiescent gases and moving uniform and non-uniform gaseous fields ahead of a shock can be found in Chapter 3.7 of Shock Wave Handbook by Ahao-Yuan Han and Xie-Zhen Yin [3]. The primary reason to introduce curvature in equation (1) is so that we can apply it to an arbitrary convex shock surface. Then if the surface can be represented using a smooth spline, the curvature is a well defined geometrical quantity.

2.2 Point blast wave theory (PGSD)

As discussed previously, WGSD theory assumes that the flow immediately behind a shock front is uniform. By contrast, the analytic solution from the point blast theory developed by

G. I. Taylor [10], features sharp attenuation in the flow state behind the shock. The theory was further developed by other researchers ([1], [4]). In particular, Bach and Lee extended the theory to cover a wider range of Mach numbers, and generally match the Euler solutions over a wide range of time. The WGSD theory cannot accurately portray the Euler solution, because the flow profile behind the air shock does not satisfy the initial conditions. Figure 3 compares the M - R relation from WGSD with the Bach and Lee point blast calculation (Qiu [8]), and highlights the differences between the two. The new PGSD theory should be modified to include the conditions of point blast wave simulations. The simplest theory of point blast waves, as mentioned above, was developed by G. I. Taylor. This model works well for very strong, high Mach shocks, but breaks down at lower Mach numbers. We review the theory as a starting point for considering point blast.

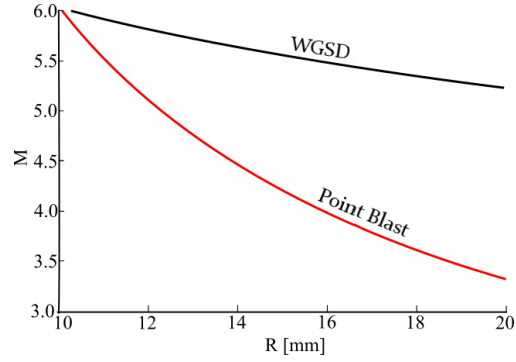


Figure 3. Comparison of Mach number vs Radius from WGSD and the point blast solution of Bach and Lee [1]. Figure from [8, USCD Ph.D. Thesis]

2.2.1 Taylor air blast theory for strong shock

The point blast wave theory was originally introduced by G. Taylor [10]. Let d ($= 2$ or 3) be the spatial dimension (*i.e.* 2D or 3D) and $j = d - 1$. G. Taylor introduced the necessary condition for a similarity solution as follows:

$$\frac{dR}{dt} = AR^{-\frac{j+1}{2}}, \quad (7)$$

where A is a constant which is related to the initial amount of energy to start the blast wave and R the radius of the expanding spherical shock front. The point blast wave for $0 < \eta (= r/R) \leq 1$ can be solved by solving the following ODE (see equations 7a-11a and 14 [10] and equations 16-18[6]) for the variables (f, ϕ, ψ) :

$$f' = \frac{d\eta(\eta - \phi) - (d - 2)\eta\phi\gamma/2 + (d - 1)\gamma\phi^2}{f - (\eta - \phi)^2\psi} \frac{\psi f}{\eta}, \phi' = \frac{f' - \frac{3}{5-d}\gamma\phi\psi}{\gamma\psi(\eta - \phi)} \text{ and } \psi' = \frac{\psi\{\eta\phi' + (d - 1)\phi\}}{(\eta - \phi)\eta}, \quad (8)$$

where the dependent variables are non-dimensionalized pressure, velocity and density so that $f = p/p_0 R^d a_0^2 / A^2$, $\phi = uR^{d/2} / A$ and $\psi = \rho/\rho_0$. The variables ρ, u, p are density, velocity and

pressure with $\rho_0 u_0, p_0$ are their ambient values. The initial conditions for the system of ODE (8) of (f, ϕ, ψ) are given at the shock ($r = R$ or $\eta = 1$) as follows:

$$(f(1), \phi(1), \psi(1)) = \left(\frac{2\gamma}{\gamma+1}, \frac{2}{\gamma+1}, \frac{\gamma+1}{\gamma-1} \right), \quad (9)$$

where γ is the ratio of specific heats for air ($\gamma = 1.4$ in this paper).

Any hydrocode that uses the solutions of the ODE system (8 and 9) as initial conditions should generate the same shock propagation as that defined by equation (7). So, using Φ derived from equation (7) in the GSD equation (1) for point blast wave propagation will result in an M - R relation that also matches such hydrocode simulations.

2.2.2 Bach and Lee approximation

Bach and Lee [1] proposed an analytical method to extend the point blast method from high Mach to the very low shock Mach number regions of a blast wave. In this model, the flow behind a shock front of radius R_s is given as follows: For $0 \leq \xi \leq R_s(t)$,

$$\begin{cases} \psi(\xi, \eta) = \psi(1, \eta)\xi^{q(\eta)}, & q(\eta) = (j+1)(\psi(1, \eta) - 1), \\ \phi(\xi, \eta) = \phi(1, \eta)\xi(1 - \Theta \ln \xi), \\ f(\xi, \eta) = f(1, \eta) + f_2 B_2 + f_3 B_3 + f_4 B_4, \end{cases} \quad (10)$$

where

$$\begin{aligned} f_2 &= \frac{\psi(1, \eta)}{(q+2)} \left[(1 - \Theta)\{\phi(1, \eta) - \phi^2(1, \eta)\} - \theta \left\{ \phi(1, \eta) - 2\eta \frac{d\phi(1, \eta)}{d\eta} \right\} \right] \\ f_3 &= \frac{\psi(1, \eta)}{(q+2)^2} \left(\theta \left\{ \Theta\phi(1, \eta) - 2\eta \frac{d}{d\eta}[\Theta\phi(1, \eta)] \right\} - \Theta\phi(1, \eta) - \Theta^2\phi^2(1, \eta) + 2\Theta\phi^2(1, \eta) \right) \\ f_4 &= \Theta^2\phi^2(1, \eta)\psi(1, \eta)/(q+2)^3, \\ B_2 &= \xi^{q+2} - 1, \\ B_3 &= \xi^{q+2}((q+2)\ln\xi - 1) + 1, \\ B_4 &= 2 - \xi^{q+2}[(q+2)^2(\ln\xi)^2 - 2(q+2)\ln\xi + 2]. \end{aligned}$$

The boundary values are

$$\phi(1, \eta) = \frac{2}{\gamma+1}(1 - \eta), \quad \psi(1, \eta) = \frac{\gamma+1}{\gamma-1+2\eta}, \quad f(1, \eta) = \frac{2}{\gamma+1} - \frac{\gamma-1}{\gamma(\gamma+1)}\eta.$$

The function Θ is the function that relates the flow profile to the Mach-Radius relation as follows:

$$\Theta = \frac{-2\theta\eta}{\phi(1, \eta)\psi(1, \eta)} \frac{d\psi(1, \eta)}{d\eta},$$

where $\eta = 1/M_s^2$ and

$$\theta = R_s \ddot{R}_s / \dot{R}_s^2. \quad (11)$$

Now note that the system equations (10) define a flow field from the center to the surface of a sphere of radius R_s if the variable θ in equation (11) is known as a function of η .

Using the energy integral for the energy conservation equation, Bach and Lee modeled the functional relation between η and θ by solving a system of two ordinary differential equations.

$$\frac{d\theta}{d\eta} = \mathcal{L}(\theta, \eta, y), \quad \frac{dy}{d\eta} = -\frac{(j+1)y}{2\theta\eta}, \quad (12)$$

where

$$\mathcal{L} = -\frac{1}{2\eta}A + \frac{D_1 + 4\eta}{8\eta^2(\gamma + 1)}B + \frac{2\theta[2 + (\gamma - 1)(j + 1)]}{D_1 + 4\eta}$$

and

$$\begin{aligned} A &= \theta + 1 - 2\phi(1, \eta) - \frac{D_1 + 4\eta}{\gamma + 1} - (\gamma - 1)(j + 1) \left[\phi(1, \eta) - \frac{(D_1 + 4\eta)^2}{4\theta y(\gamma + 1)} \right], \\ B &= \frac{(D_1 + 4\eta)\phi(1, \eta)}{\theta} - \frac{\phi(1, \eta)(\gamma + 1)}{\theta\psi(1, \eta)} + 2(\eta + 1) + (\gamma - 1)(j + 1)\frac{\gamma + 1}{2\theta}\phi^2(1, \eta), \\ D_1 &= \gamma(j + 3) + (j - 1). \end{aligned}$$

With the variable $\eta = 1/M_s^2$, the solution

$$y(\eta) = \left(\frac{R}{R_0} \right)^{j+1} \quad (13)$$

represents the relation of Mach number and shock radius R , scaled to a characteristic radius

$$R_0 = \left(\frac{E_0}{\rho_0 a_0^2 2j\pi} \right)^{\frac{1}{j+1}}, \quad (14)$$

where E_0 is the energy of the explosive. The characteristic radius is related to the energy density of the explosive.

We solved the system equations (12) and the results are shown in Figure 4. The function θ is used to model the shock acceleration behavior in this paper as described below. While Bach and Lee's system of ODE for (θ, y) is described in their paper[1] in detail, we noticed that the suggested initial values away from $\eta = 0$ did not solve the system reliably due to large variation of y in the vicinity of $\eta = 0$. For $\eta \ll 1$, careful attention should be paid when choosing the initial value of y , since the derivative of θ is very sensitive to the value of y due to the term A in \mathcal{L} . In the paper by Bach and Lee [1], the initial condition for $\theta(\eta) = \theta_0 + \theta_1\eta + \dots$ was OK as shown in the paper [1], but the estimate for $y(\eta) = y_1\eta + \dots$ in the paper did not give the correct solution due to high slope in the small η regime. We modified the initial value for y very carefully to avoid a large variation of \dot{y} at small η .

Now we describe the GSD model for point blast wave (PGSD) based on the Bach-Lee solution (θ, y) . Since the time variation of radius R in an expanding sphere and its Mach number M are related as $\dot{R} = a_0 M$, from the definition of θ in equation (11), the time derivative of Mach number is given as follows:

$$\dot{M} = -\frac{a_0 M}{\Phi(M)}\kappa, \quad \Phi(M) = -\frac{j}{M\theta}. \quad (15)$$

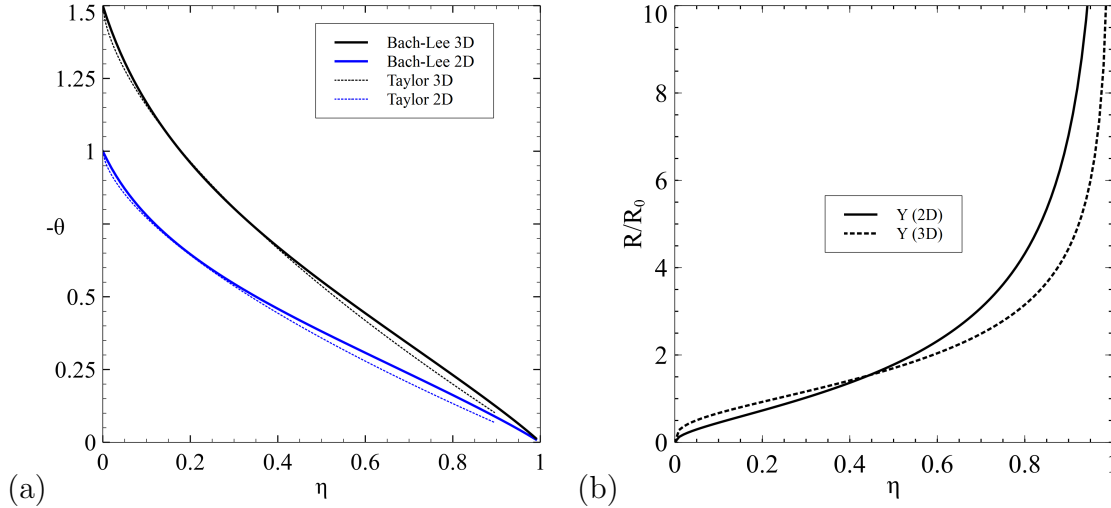


Figure 4. Solutions for (a) θ of the ODE (12) by Taylor (dotted) and Bach and Lee(solid) and (b) for $y^{1/j+1} = R/R_0$ from Bach-Lee solutions. A graph for θ represents a relation between shock acceleration of blast wave and Mach number M , where $\eta = 1/M^2$.

This equation (15) is the GSD formulation (1) for point blast waves, which have strong attenuation behind the shock. The initial condition of the PGSD system is determined as follows: First, the characteristic radius R_0 is calculated from the energy E_0 using equation (14). Then, for an initial PGSD radius r_0 , we can compute the initial Mach number M_0 from the equation (13). Interestingly then, given any two of R_0 , r_0 , and M_0 , we can compute the third. So, if we are given r_0 and M_0 , we can compute R_0 (and E_0 , the energy). We can also regard the equation as a relation of Mach and curvature M - κ as follows:

$$M(\kappa) = \frac{1}{\sqrt{y^{-1}[(R_0\kappa/j)^{-(j+1)}]}}, \quad (16)$$

where $y^{-1}[\cdot]$ is the inverse function of $y(\eta)$.

This equation (16) is the universal M - κ relation for PGSD, giving a relation of (M, κ, R_0) to define the motion rule of shock propagation for all energy sizes at the center. This is the scaling law embedded in equation (15), so the system can be applicable for any size of initial energy and even on an arbitrary convex surface shape. Figure 5 (a) shows the relations for both 2D and 3D cases.

As you see, the universal M - κ curve in 2-D is linear over a large curvature regime away from zero, while in 3-D, it is curved for small curvature. This fact implies that the character of shock propagation in 2D is quite different from that in 3D. We also see that higher curvature relates to higher Mach numbers, and as curvature approaches zero the Mach approaches one. Figure 5(b) shows a comparison between the Bach-Lee analytic solution, the Euler solution and the new PGSD solution. The figure shows excellent agreement between PGSD and Bach-Lee, but both differ very slightly from the Euler solutions at larger radii. Our new PGSD model corrects the deficiency shown in Figure 3. Note that the initial condition for the hydrocode simulation is defined by equation (10). The M - R relation is derived from the

time of arrival generated by the simulation. Therefore there can be a minor error associated with post-processing.

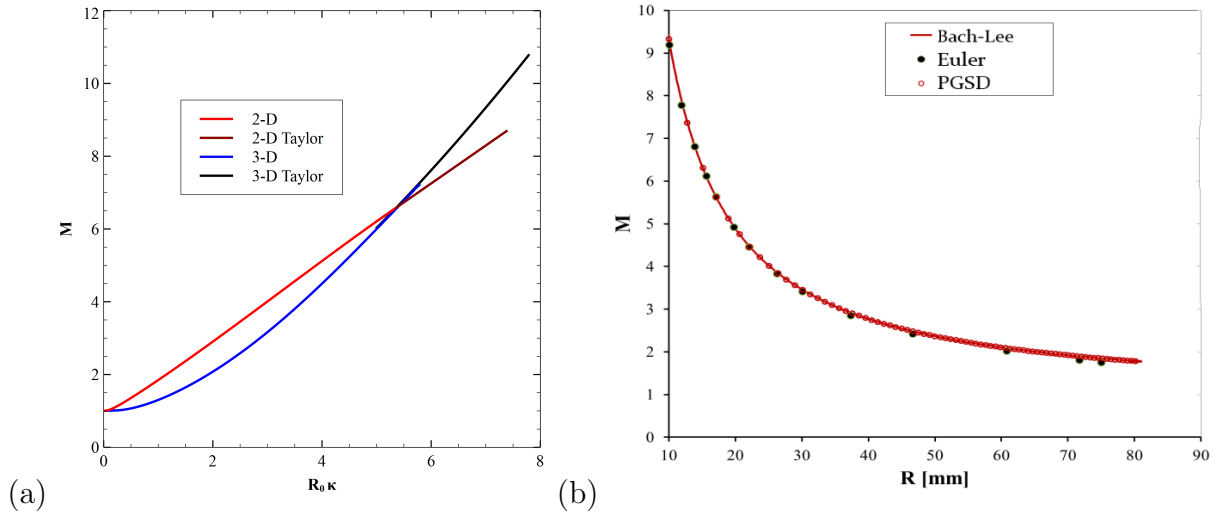


Figure 5. (a) The universal M - κ curves for 2-D and 3-D spaces from Bach-Lee theory and G. Taylor theory. (b) Comparison between the Bach-Lee analytical solution [4], the PGSD simulation and the Euler equations for a spherical shock wave expansion, with 5000 J initial energy (Qiu, [8] Figure 3.7).

Remark 1. Energy in a circle (2D, $j = 1$) or sphere (3D, $j = 2$) of radius R

The point blast theory "dumps" the energy, corresponding to an explosive charge, into a single point. To use the theory, an initial energy is specified and in turn, the constant A in equation (7) is related to the energy E as follows:

$$E = k_j A^2 \left[\frac{1}{2} \rho_0 \int_0^1 \psi \phi^2 \eta^2 d\eta + \left(\frac{p_0}{a_0^2 (\gamma - 1)} \right) \int_0^1 f \eta^2 d\eta \right], \quad (17)$$

where $k_j = 2\pi, 4\pi$ for $j = 1, 2$ and a_0 is the sound speed in the air as expressed $a_0^2 = \gamma p_0 / \rho_0$.

Remark 2. As shown above, for a strong air blast wave ($1/M^2 \rightarrow 0$), Geoffrey Taylor [10] and Shao-Chi Lin[6] showed that the shock front propagates by $\dot{R} = AR^{-(j+1)/2}$. Therefore the equation for y in equation (13) is approximated by a simple linear relation $y = L\eta$, $L = A^2/a_0^2 R_0^{-(j+1)/2}$ and so the $M - \kappa$ relation for such a strong shock is given by

$$M = L_1 \kappa^{\frac{j+1}{2}}, \quad L_1 = (LR_0/j)^{(j+1)/2}. \quad (18)$$

This M - κ relation (18) is shown in Figure 5 (a), which shows the curves overlapped over curves from Bach-Lee theory on the large regime of M and κ .

Now the expression, $\theta = R\ddot{R}/\dot{R}^2$ can be obtained from Taylor as follows:

$$\tilde{\theta}(\eta) = -\frac{j+1}{2}(1 - \eta^{1/2}). \quad (19)$$

According to our numerical inspection, this function of $\tilde{\theta}$ by Taylor deviates significantly from the θ by Bach-Lee theory as shown in Figure 4. Therefore the formulations of M from Taylor theory can not be used for the PGSD model on the intermediate and low Mach domain as we stated when we described the Taylor theory above.

2.3 GSD modeling for spherical condensed explosives (EGSD)

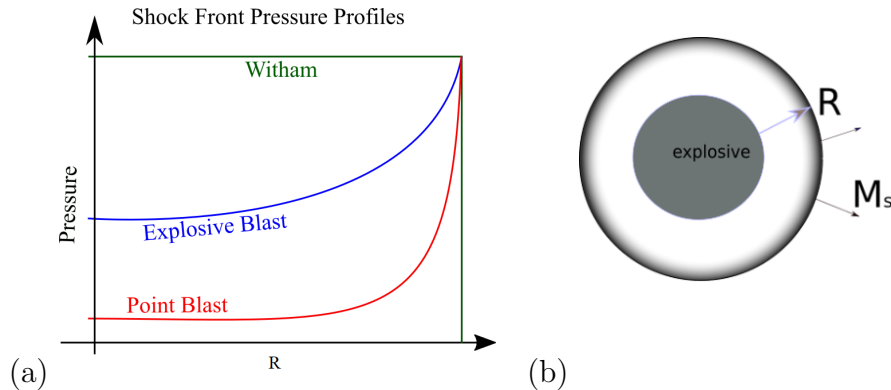


Figure 6. (a) Schematic of pressure profiles of the flow conditions behind a shock for the WGSD, point blast, and a finite-diameter explosive charge (b) Schematic of a spherical shock of radius R expanding at Mach M_s

WGSD and PGSD are extremes in the sense that the first represents an ideally uniform flow normal to a shock within a virtual shock tube, and the second represents a blast wave propagating from a strong blast from an arbitrarily small source. The flow profile behind the shock is attenuated sharply for the latter case. The actual blast wave propagation from a finite-diameter sphere is somewhere in between these two extremes, as shown in Figure 6 (a), and will be "bracketed" by these two cases, which are well described by the theories of Whitham, Taylor, and Lee&Bach *et. al.* briefly described previously. The theory of a blast wave from a spherical charge (BWSC) of finite size is not well developed to the authors' knowledge. Since the flow behind the wave front is very complicated due to the mixing of explosive product and air at the interface, penetrating particles, and so on [3], it is very difficult to derive any analytic solution as done for PGSD. The flow behind BWSC is not uniform, but is partially supported by the explosive products through detonation mechanisms as illustrated in Figure 6(b), unlike the sharp attenuation behind point blast waves. The initial Mach number of BWSC at a designated distance in the radial direction depends on the explosive properties and the charge radius, which increases the complexity in including BWSC in the EGSD model.

In the absence of an extensive database of relevant experimental data, we turn to hydrodynamic (CTH hydrocode) simulations of finite spherical charges to develop our general EGSD model. The detonation properties can be represented by various equations of state (EOS) and reactive burn models, independent of GSD modeling. We conducted hydrodynamic simulations of several spherical explosive charges of different diameters, and measured the speed of the shock as it expanded from the explosive charge into the surrounding air.

From these simulations we determined the initial Mach number M_0 , shock radius r_0 and the M - R relation $M(R; r_0, M_0)$. However, it would be highly expensive to use hydrodynamic computations to populate a database with a sufficiently large range of charge diameters to derive an EGSD model.

Note that equation (16) in the previous section represents an M - R relation for all energy values, and is scaled in terms of R_0 , which is a function of the explosive energy. In other words, this form expresses a relation between the velocity of blast wave propagation and the amount of energy available to support the shock strength, and is a material property. As such, the relation is not particular to the point blast wave phenomena, but also applies to an air blast wave caused by the explosion of a spherical charge, (the reader can find more discussion by John Dewey [3]). Therefore it is natural to assume that the M - R relation for a spherical charge air blast wave scales as

$$M_r(R) = M_{r_0}(r_0/rR), \quad (20)$$

where r is the radius of the explosive charge and r_0 is a reference radius. In other words, the M - R relation for a BWSC of radius r can be scaled from a BWSC of the same explosive with charge radius r_0 by the factor r_0/r . Considering the shock Hugoniot relation, the shock pressure must also satisfy the same scaling law for pressures P_r and P_{r_0} :

$$P_r(R) = P_{r_0}(r_0/rR). \quad (21)$$

To verify the scaling assumption, we ran 1-D simulations of spherical charges in CTH, in which a steady state detonation is formed before the material interface with air. We simulated two explosives, PBX9501 (density ≈ 1.8 g/cc) and TNT (density ≈ 1.6 g/cc). The former is considered an ideal explosive with a high detonation velocity, while the latter is a non-ideal explosive with a much lower detonation velocity. The results are shown in Figure 7, and show the excellent match using the 5 cm radius as a reference value for scaling charges, in this case of radii 20 cm and 80 cm.

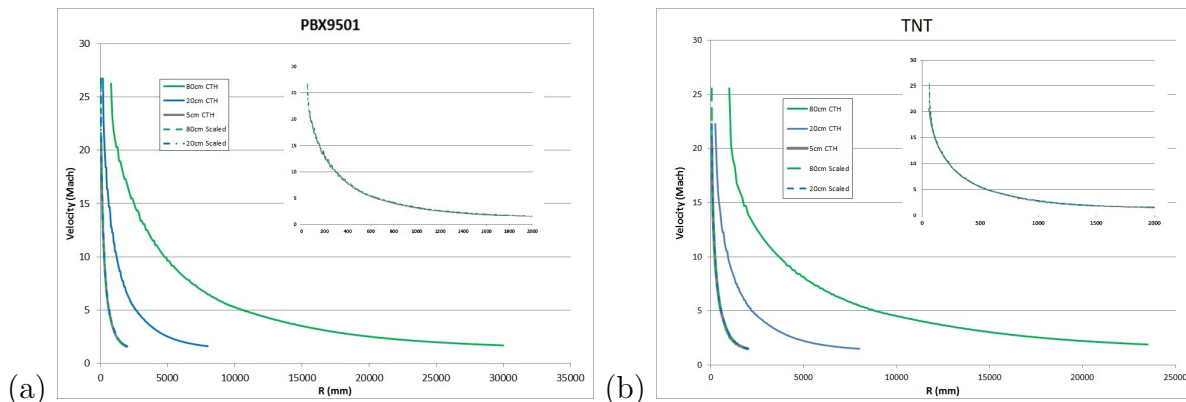


Figure 7. Verification of M - R scaling in CTH simulations for 5, 20, and 80 cm spherical charges of (a)PBX9501 and (b)TNT. The inset in each case shows the agreement with the curves scaled to the reference radius of 5cm.

In the previous section, we modeled the PGSD motion rule using the theory developed by Bach-Lee as defined by equation (15). There is no comparable analytic solution available for

the blast wave emanating from spherical explosives. However we need only one hydrodynamic simulation for a small explosive charge to get an M - R relation, and then apply the scaling law to model the GSD motion rule. Let us define the relation as a function $M = \tilde{F}(R)$, where R is the radius of a spherical shock surface. Figure 8 shows representative M-R and P-R relations for the two explosives, TNT (solid line) and PBX9501 (dotted line) for a reference charge radius $r_0 = 5$ cm. While PBX9501 produces a stronger air shock over the entire domain, the two curves approach each other asymptotically as the radius goes to infinity.

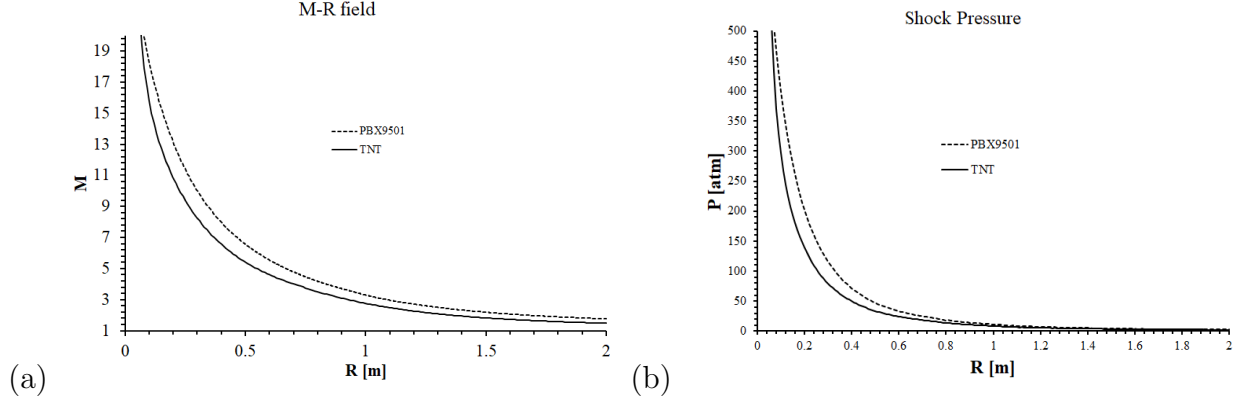


Figure 8. Reference (a) M - R and (b) P - R relations from hydrodynamic simulations of spherical charges of TNT (solid) and PBX9501 (dashed)

We can use the M-R relation from the hydrodynamic simulation, and express the relation in terms of curvature κ since the radius R is equal to $2/\kappa$ in three dimensional space. Then, using the function $M = F(\kappa)(\equiv \tilde{F}(2/\kappa))$ and its inverse function $\kappa = F^{-1}(M)$, the function Φ needed for equation (1), for a EGSD is:

$$\Phi(M) = \frac{j}{F^{-1}(M) \frac{dF(\kappa)}{d\kappa} \Big|_{\kappa=F^{-1}(M)}} \quad (22)$$

If the function $M = F(R)$ is given, it can be expressed by

$$\Phi(M) = \frac{-j}{F^{-1}(M) \frac{dF}{dR} \Big|_{R=F^{-1}(M)}} \quad (23)$$

Note that equations (22 and 23) are the expressions of function F only, without the scaling variable. This means that the time variation of Mach \dot{M} does not depend on the underlying scaling law, while the M - R (or κ) relation is. This is true for PGSD as well (Compare equation (22) with equation (15)). What this essentially means is that for air blast, the shock dynamics are completely determined by the initial shock strength and geometry. The GSD modeling can be dramatically simplified, since the initial M - κ relation can be obtained from that of a reference explosive charge through the scaling law, which relates the charge size, the expansion radius R and the Mach number M :

$$r(M, R) = r_0 \frac{R}{F^{-1}(M)}. \quad (24)$$

This function defines the radius r of a spherical explosive charge which can generate sufficient energy to produce an air blast wave of Mach number M (or pressure P) at a radius R . This relation would be useful for military or industrial applications requiring a prescribed shock strength at a specified distance from the charge.

Figure 9 shows a comparison of the CTH simulations and the corresponding GSD calculations. The initial radius and Mach number (r_0, M_0) for EGSD simulation are chosen from relation (24). The interface between explosive products and air is a complex mixing zone, which accounts for the departure from the smooth M - R curve in the low R regime of the figure (more evident in 20 cm curves). The air blast expands faster than this interface, so the match between CTH and EGSD improves. From the observation of flow profile from the CTH simulations, we can expect that eventually the flow structure behind the air blast becomes similar to a point blast wave.

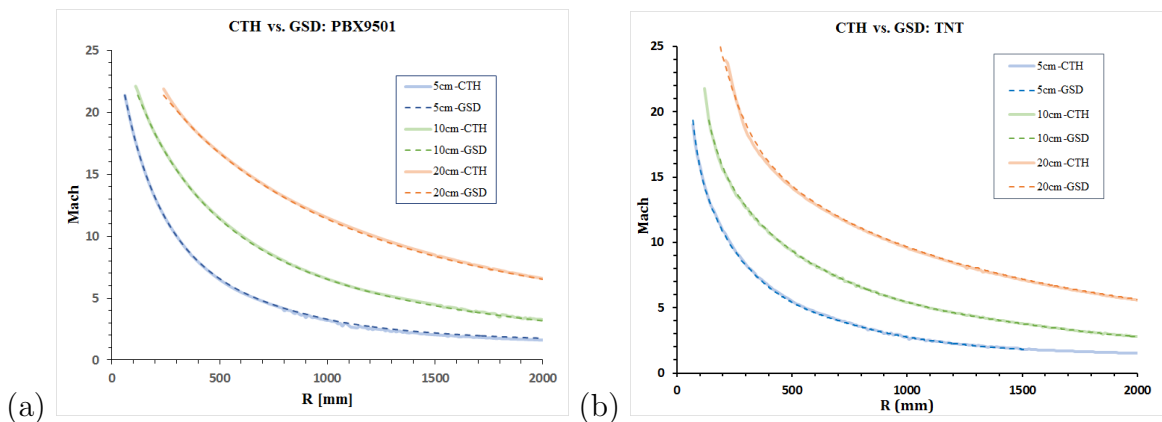


Figure 9. GSD fits to CTH simulation results, for spherical charges of (a)PBX9501 and (b)TNT.

Figure 10 shows the M - R relations (described by equation (24)) for TNT (solid contours) and PBX9501 (dotted contours), for a variety of initial charge radii. This function and figure provide us some useful information. For example, we can easily derive the fact that about 30 tons of TNT are required to have a spherical shock of Mach 3 at 30 m from the location of the explosion, while 20 tons of PBX9501 are required for the same effect. The corresponding radii of the explosives are 1.63 m and 1.38 m respectively. The values can also be estimated intuitively using Figure 10, in which a red circle marks the location (30 m, Mach 3), located slightly above the contour of TNT 1.6 m and below the contour of PBX9501 1.6 m.

A 30 m expansion radius for a charge radius of 1.6 m corresponds to an expansion radius of 92 cm for the reference radius of 5 cm. Referring to Figure 8 (b), the pressure of TNT at this radius is about 15 atm. In consideration of a historical event, the bomb dropped on Hiroshima in 1945 had a TNT equivalent of about 18 thousand tons (which would have a radius of about 13.3 m), and exploded at about 600 m above the ground. This is equivalent to an expansion radius of about 226 cm for the reference charge radius, and a Mach number of about 1.4, which is quite similar to estimates for conditions at Hiroshima. For more on this story, refer to Wikipedia, the free encyclopedia [12]: *"In Hiroshima almost everything within 1.6 kilometres (1.0 mi) of the point directly under the explosion was completely destroyed, except for about 50 heavily reinforced, earthquake-resistant concrete buildings, only the shells*

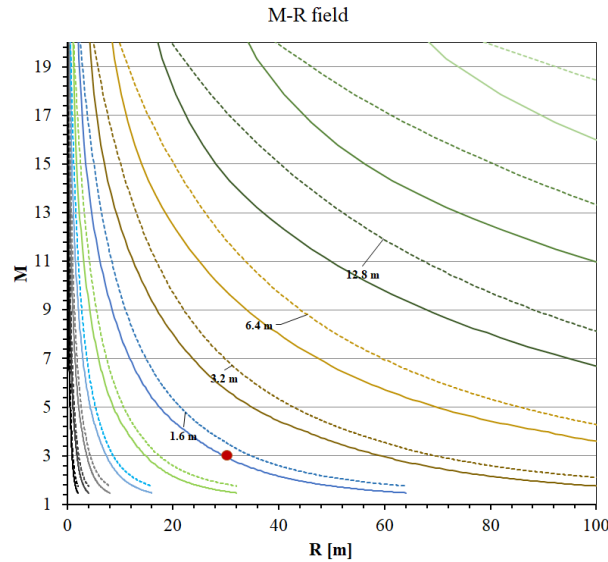


Figure 10. Countour plot of M - R field from hydrodynamic simulation: TNT (solid) and PBX9501 (dotted). The data labels on the curves are the charge radii. The charge sizes are doubled between adjacent countours.

of which remained standing. Most were completely gutted, with their windows, doors, sashes, and frames ripped out. The perimeter of severe blast damage approximately followed the 5 psi (34 kPa) contour at 1.8 kilometres (1.1 mi).” According to the extrapolation of CTH simulation data scaled to the equivalent size of TNT, the shock pressure was about 6 psi at a radius of 1.8 km from the point directly under the explosion. Therefore our model seems to match this real situation!

In summary, according to the scaling laws for pressure and Mach number, one hydrodynamic simulation at a selected charge size enables us to model the EGSD motion rules for a given explosive, so that the shock propagation from the blast of any size of explosive is completely determined and the shock state is also completely determined from the shock Hugoniot relation at any point on surface.

3 Conclusions

In summary, this report describes two new GSD models for the propagation of an air blast wave that is supported by non-uniform flow states. The PGSD model is a model for point blast waves, for which the flow character behind the expanding spherical shock is theoretically well characterized by G. Taylor and Bach-Lee. We showed that the PGSD motion rule produced an M - R relation identical to the Bach-Lee point blast theory. The relation also matched the results of hydrocode simulations. However, no such theory exists for the blast wave initiated from the detonation of a finite sized spherical explosive charge. The flow character behind the expanding shock is different from flows modeled by WGSD and PGSD, so we developed the EGSD model, which is for shock followed by flow partially supported by explosive product. This model requires a reference M - κ relation, which can be achieved

by a single 1-D, high resolution hydrodynamic simulation.

We presented typical examples for two explosives, TNT and PBX9501. The scaling law and M - R relations were confirmed for several charge sizes. We also showed that the underlying scaling law can be used to estimate the required size of charge to produce a desired wave strength at a prescribed distance. This estimation does not even require any simulation.

Simulating the propagation of a non-spherical surface, e.g. from a cylindrical charge or a more complex shape, is very important for many applications. Our EGSD model is general enough to give a motion rule for an arbitrary convex surface shape, since it substitutes κ (curvature) for the area relation A'/A in Whitham's original GSD theory. We have not tested this however; that will require a large 3-D hydrocode simulation and complex data analysis, and is left for future work.

Note that this report describes only the GSD motion rule for shock propagation, and does not include the effects of shock interactions with either a material surface or another shock. This will require combining the motion rule with an accurate and stable model for surface representation, and will be the focus of a companion paper [13]. The paper will contain discussions including shape-preserving surface representations using two-variable parameterization, mesh regularization, techniques for merging two surfaces, and a model for Mach stem formation which arises from the regular shock reflection at surface merging. The model requires very accurate techniques for extrapolating from the interior of each expanding surface to their intersection (a point in 2-D and a curve in 3-D) and experimental shock interaction data [5].

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References

- [1] G. G. Bach and J. H. S. Lee. An analytical solution for blast waves. *AIAA Journal*, 8(2):271–275, 1970.
- [2] G. Ben-Dor. *Shock Wave Reflection Phenomena, 2nd edition*. Springer, 2007.
- [3] Gabi Ben-Dor, Ozer Igra, and Tov. Elperin. *Handbook of Shock Waves*. Academic Press, 2001.
- [4] J. P. Best. A generalisation of the theory of geometrical shock dynamics. *Shock Waves*, 1:251–273, 1991.
- [5] Michael J. Hargather. 3D BOS imaging of shock wave interactions from multiple explosions. *Technical report, New Mexico Tech.*, 2018.

- [6] Shao-Chi Lin. Cylindrical shock waves produced by instantaneous energy release. *Journal of Applied Physics*, 25(1):54, 1954.
- [7] Barrett O' Neill. *Elementary Differential Geometry*. Academic Press, 1966.
- [8] Shi Qiu. *Numerical Study of Focusing Effects Generated by the Coalescence of Multiple Shock Waves*. PhD thesis, University of Southern California Los Angeles, California, 2017.
- [9] D. W. Schwedeman. Numerical shock propagation in non-uniform media. *Journal of Fluid Mechanics*, 188:383–410, 1988.
- [10] F. R. S Sir Gerfferey Taylor. The formation of a blast wave by a very intense explosion I. theoretical discussion. *Proc. Roy. Soc. (London)*, A201:159, 1950.
- [11] G. B. Whitham. *Linear and Nonlinear Waves*. A Wiley-Interscience Series, 1974.
- [12] Wikipedia. Atomic bombings of Hiroshima and Nagasaki. https://en.wikipedia.org/wiki/tomic_bombings_of_Hiroshima_and_Nagasaki or https://en.wikipedia.org/wiki/Little_Boy. [Online; accessed 28-May-2019].
- [13] Sunhee Yoo and G. Butler. New GSD modeling for air blast wave supported by non-uniform flow: II. GSD propagation. *in preparation*, 2019.

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