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Passive Space-Debris Trajectories: Description and Uncertainty
Propagation for Collision Prediction and Impact-Point Analysis

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14. ABSTRACT The final report presents the PhD research plan and progress for the first three years of the graduate program. The study investigates the propagation of Earth-orbiting objects to improve predictions, such as conjunction analysis and re-entry predictions. The main focus of the third year was on research related to the GOCE benchmarking re-entry (BMR) predictions. Three articles from refereed journals and published during the project are included in the report.						
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EOARD Progress and Planning

Passive Space-Debris Trajectories: Description and Uncertainty Propagation for Collision Prediction and Impact-Point Analysis

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Year 3 (complete, 15/07/2016–15/07/2017)
Year 4 (six months, 15/07/2017–15/01/2018)

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1

Introduction

This document presents the progress and planning of the PhD program with a focus on the research project. The research project is supported by the European Office for Aerospace Research and Development (EOARD), grant FA9550-14-1-0344.

The EOARD duration is three years. The PhD program has a four year duration and is conducted at the Delft University of Technology (TU Delft), Delft, The Netherlands. The three year EOARD research project is distributed equally over the entire period and executed alongside other TU Delft PhD requirements. The entire project duration is from July 15, 2014 to July 14, 2018.

The PhD program at the TU Delft consists of three major components, which are exercised in parallel over the four year period. First, the research project resulting in scientific output and a doctoral degree. Second, a doctoral education, consisting of 20% of the entire program duration, provided by the TU Delft graduate school, which aims to support the development of scholarly knowledge, personal generic competences and prospective professional development for future academic and non-academic career paths. Third, education support by PhD candidates for 10 to 15% of the program duration on teaching and other educational activities.

This document presents the PhD research plan and progress. The purpose of this document is provide an overview of the results achieved in the first three and a half years (contrasted with the second-year report) and a vision for the final six months. The reflection is presented in chapter 3, followed by the outlook in chapter 4.

2

Background

The study investigates the (uncertainty) propagation of Earth-orbiting objects (e.g., satellites, mission-related objects, fragments) to improve predictions, such as conjunction analysis and re-entry predictions.

The quality of conjunction/impact predictions is driven by model, state, and input uncertainties. By improving these predictions, the number of false positives can be reduced and allows for predicting further ahead in time. The orbit propagation problem is at the core of these predictions. This is highly dependent on the quality of the initial state and propagation models. Neither of these are perfect and subject to uncertainties, which grow (exponentially) over time. The quality of these predictions can therefore be significantly improved by decreasing the uncertainty in the initial state, improving the dynamics modelling, and so increasing the accuracy of generated ephemeris.

The following three research goals are identified:

1. increasing the quality of orbit description and propagation;
2. improving uncertainty modelling, estimation, and propagation;
3. improving conjunction and re-entry predictions.

These three goals are treated sequentially and constitute the main parts of the research.

3

Reflection

The reflection reviews on the first three and a half years of the PhD, with the primary focus on the third year and the first six months of the fourth year. For more details on the first two years, the reader is referred to the first and second yearly progress reports.

3.1. Research

The main focus of the third year was on research related to the GOCE benchmarking re-entry (BMR) predictions project by ESA. The participation in this project was initially not foreseen and therefore not taken into account in the initial PhD and the go/no-go planning. At the two-year review the discrepancy with the original planning was noted. The participation in the ESA study resulted in rescheduling of work packages, delay of work, and introduction of new packages.

The rescheduling concerns primarily work packages dealing with re-entry predictions. These activities were originally planned for the final year of the PhD and moved ahead.

Overall most work packages addressed in the GOCE BMR were carried out in more detail than initially scheduled, resulting in an accumulation of delay. Moreover, new work packages were introduced, such as the initial translational uncertainty modelling and estimation of the two-line elements (TLEs). Although this work is related to the propagation and state description initially scheduled, the work was carried out in more detail than initially planned and took additional time.

The total delay was estimated to be four months. By reducing the work on and expected output of the work packages concerning the space surveillance network (SSN) and novel observation techniques, the delay could be reduced by about two months.

The original planning accounted for a contingency of two months at the end of the PhD project. This contingency can be used to further address the delay in the project. Figure 3.1 gives an overview of the planning for the third and fourth year of the project at the two-year review, with the current completion of the tasks indicated.

This results in a very tight scheduling for the final six months. The updated scheduling with the remaining tasks rearranged is presented in Section 4.1.

3.2. Publications

An overview of the completed scientific output (SO) of the first three and a half years is given below. The outputs are numbered chronologically when a first version was completed and indicated in Figure 3.1. The dates directly after the output number indicates the final date of completion.

— previously discussed —

SO1. (2015-08) conference (paper+oral) on regularised propagation at the AAS/AIAA Astrodynamics Specialist Conference in Vail (completed).

Geul, J., Mooij, E., & Noomen, R. (2015). Regularised Methods for High-Efficiency Propagation. In Proceedings of the AAS/AIAA Astrodynamics Specialist Conference 156 (Vol. 31, pp. 4105–4124).

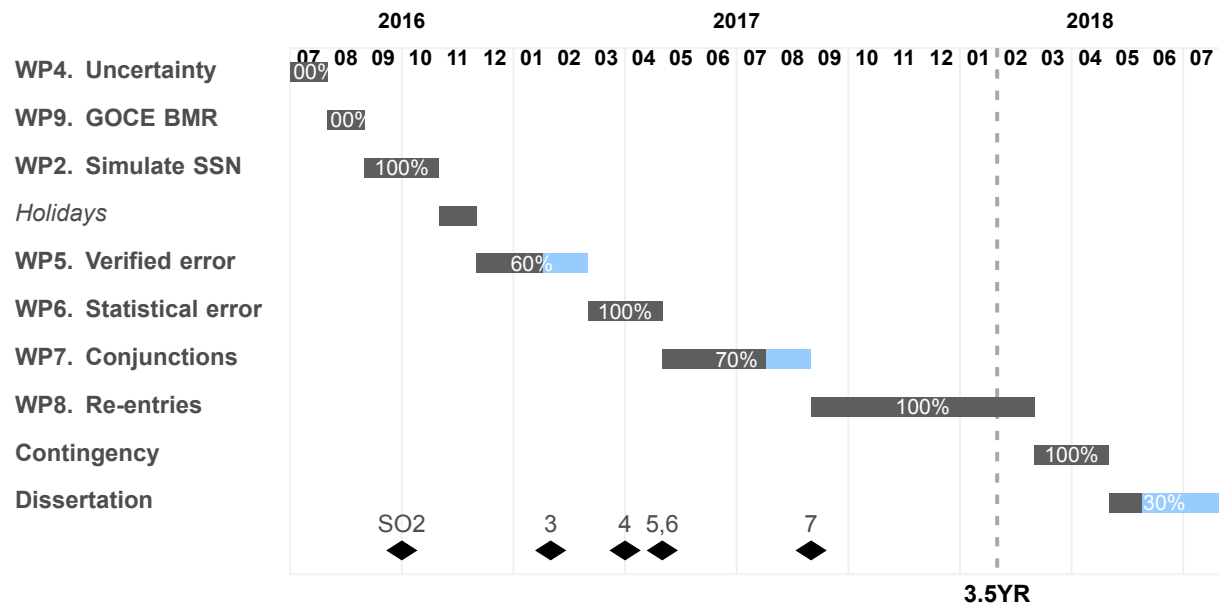


Figure 3.1: Gantt chart of planned tasks for years three and four.

— new scientific output —

SO2. (2017-03) journal (paper) on uncertainty modelling and estimation of TLEs.

Geul, J., Mooij, E., & Noomen, R. (2017). TLE Uncertainty Estimation using Robust Weighted Differencing. In *Advances in Space Research*, 59(10), 2522–2535. <http://doi.org/10.1016/j.asr.2017.02.038>

SO4. (2017-03) project (technical report) on benchmarking re-entry prediction uncertainties for the ESA contract on the re-entry study of GOCE.

SO5. (2017-04) conference (paper+poster) on simulating and assessing the current and future SSN at the European Conference on Space Debris at ESOC/ESA in Darmstadt.

Geul, J., Mooij, E., & Noomen, R. (2017). Modelling and Assessment of the Current and Future Space Surveillance Network. In *European Conference on Space Debris*.

SO6. (2017-04) conference (paper+poster) on performing statistical re-entry predictions of GOCE at the European Conference on Space Debris at ESOC/ESA in Darmstadt.

Geul, J., Mooij, E., & Noomen, R. (2017). GOCE Statistical Re-entry Predictions. In *European Conference on Space Debris*.

SO3. (2018-01) journal (paper, 3rd author) on uncertainty propagation methods for statistical re-entry predictions.

R. Hoogendoorn, E. Mooij and J. Geul (2018). Uncertainty Propagation for Statistical Impact Prediction of Space Debris. In *Advances in Space Research*, 61(1), 167–181. <https://doi.org/10.1016/j.asr.2017.10.009>

SO7. (2018-01) journal (paper, published online, in press) on the statistical re-entry predictions, benchmarked using GOCE.

Geul, J., Mooij, E., & Noomen, R. (2018). Analysis of Uncertainties and Modeling in Short-Term Re-entry Predictions. In *Journal of Guidance, Control, and Dynamics*. <http://arc.aiaa.org/doi/abs/10.2514/1.G003258>

Outputs SO1 and SO2 were previously discussed in the first and second year progress reports, respectively. SO2 presents the first journal publication of the PhD and was submitted in September 2016 and accepted for publication in January 2017.

The work of SO4 and SO6 has lead to the preparations for the second journal publication (SO7), which was submitted in August 2017 and accepted for publication. Outputs SO3, SO4, SO6, and SO7 concern packages WP6 and WP8.

Work done on the dissertation is conservatively estimated to be 30%. This concerns the literature review performed in the first year and outputs SO2, SO5, SO6, and SO7.

4

Outlook

The planning for the research and publications is presented in the following sections.

4.1. Research

The GOCE BMR study has lead to a significant deviation (part delay and part rescheduling) of the original planning. To successfully complete the PhD project an updated planning is given in Figure 4.1 and discussed next. In the description of the work packages the link is made to the outputs (SO8 and SO9) as described in Section 4.2, where applicable.

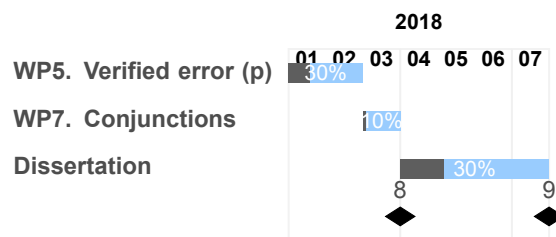


Figure 4.1: Gantt chart of planned tasks for the fourth year.

The first year focused on performing a literature study and researching the propagation of satellite orbits. Dromo, a computationally lean and numerically stable formulation of the equations of motion, was proposed (see SO1 in Section 3.2). Subsequently, a Taylor-series formulation of Dromo (Dromo+TSI) was developed. The Taylor-series formulation of Dromo presents rules for the automatic differentiation of the Dromo equations of motion. By itself the Dromo+TSI formulation was found to yield no additional computational benefits. However, it is expected to have an application in conjunction analysis.

In error propagation a distinction can be made between statistical and verified methods. The former deals with the propagation of uncertainty distributions and includes methods like Monte Carlo analysis, and was extensively addressed in outputs SO4, SO6, and SO7. Verified propagation methods, on the other hand, propagate the uncertainty bounds. These methods mathematically disprove events (e.g., conjunctions) and are generally very fast compared to statistical methods. Because of their powerful proof they are very suitable as a filtering technique. A promising class of verified methods includes the verified Taylor-series methods. These require a special formulation of the equations of motion, which was already derived for Dromo.

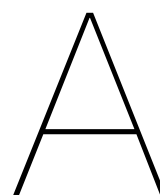
One of the primary challenges in applying verified methods to satellites orbits lies in the numerical stability and growth of the interval. A major contribution to this growth is the wrapping effect and can be attributed to the imperfect state representation of the interval. Dromo+TSI, due to its superior numerical stability and linearised motion, is an excellent candidate for verified error propagation. This work is novel and important for the application of interval methods to conjunction analysis. The aim is to publish the work in a journal (see SO8), completing WP5 and WP7.

4.2. Publications

The research and output for the fourth and final year was previously described. A summary of the output is given below. In total three journal publications (2 completed, 1 in progress) and a dissertation are planned for the successful completion of the PhD project.

SO8. (2018-03) journal (paper) on verified conjunction predictions using regularised methods.

SO9. (2018-07) dissertation integration of all previous research.



Paper S02

TLE uncertainty estimation using robust weighted differencing

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Abstract

Accurate knowledge of satellite orbit errors is essential for many types of analyses. Unfortunately, for two-line elements (TLEs) this is not available. This paper presents a weighted differencing method using robust least-squares regression for estimating many important error characteristics. The method is applied to both classic and enhanced TLEs, compared to previous implementations, and validated using Global Positioning System (GPS) solutions for the GOCE satellite in Low-Earth Orbit (LEO), prior to its re-entry. The method is found to be more accurate than previous TLE differencing efforts in estimating initial uncertainty, as well as error growth. The method also proves more reliable and requires no data filtering (such as outlier removal). Sensitivity analysis shows a strong relationship between argument of latitude and covariance (standard deviations and correlations), which the method is able to approximate. Overall, the method proves accurate, computationally fast, and robust, and is applicable to any object in the satellite catalogue (SATCAT).

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Keywords: Space debris; Error analysis; SGP4 accuracy; Space Situational Awareness (SSA); Conjunction analysis; Pairwise differencing

1. Introduction

The covariance describing the accuracy of a satellite state is an important input for exercises, such as conjunction analysis and re-entry predictions, which are increasingly important for operating in today's space environment. Through error propagation the probability of potential collisions can be calculated and a spread of impact locations and times anticipated. These efforts help significantly in managing and mitigating the problem of space debris. As the population of objects in space increases, so does the need to improve the knowledge of their states and associated uncertainty.

Two-line elements (TLEs) present the most comprehensive and up-to-date source of Earth-orbiting objects and are key in many monitoring and analysis activities. Despite

their importance, they suffer from major drawbacks: they have limited accuracy, are mis-tagged, miss manoeuvres, and perhaps most importantly, lack covariance information. Although the TLEs are publicly available, their input observations and/or fit residuals are not.

TLE differencing, amongst other methods, aims to estimate the missing covariance information by propagating TLEs (using the Simplified General Perturbations 4 (SGP4) model) to a common time and comparing them. This method presents a simple, robust and fast way of obtaining an uncertainty estimate. Although this method has been described in literature (Peterson et al., 2001; Deguine et al., 2002; Osweiler, 2006; Legendre et al., 2006), its implementation differs greatly, and, moreover, there is a lack of validation for important orbital regimes and little investigation into the sensitivity to the parameters involved.

This paper aims to present an extension of the general differencing method, offering improvements to increase its accuracy and robustness. Moreover, the method is demon-

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strated to work for both classic and enhanced TLEs and is validated for the GOCE satellite using Global Positioning System (GPS) data final phase (no orbit control) before re-entry. The demanding environment presented by the very low-Earth orbit (LEO), extensive knowledge about GOCE and an increased availability of TLEs make GOCE very interesting for this study.

The paper is structured as follows. First, the current state of differencing techniques is presented in Section 2, followed by the proposed methodology in Section 3. Next, Section 4 details the analysis. In Section 5, the results and validation of the method are presented and discussed. Lastly, the conclusions are given in Section 6.

2. Background

The lack of uncertainty information of TLEs has initiated numerous studies into the accuracy of TLEs, methods for estimating their covariance, and improvements to their accuracy (Vallado and Cefola, 2012). There is a wide range of methods that allow the uncertainty information to be estimated. These approaches differ greatly in complexity, accuracy and applicability. However, two classes can be distinguished: methods using only TLEs and methods relying on additional data. Methods relying on external data, such as observations or precise ephemeris (e.g., derived from satellite laser ranging (SLR) or GPS), are not further considered. Unfortunately, such data are not available for the far majority of objects. Moreover, uncertainties derived for a few objects are hard to extrapolate across the population or time, due to their dependency on, amongst others, satellite properties (shape, size, etc.), orbit type (semi-major axis, eccentricity, inclination, etc.), variability of the environment (solar, magnetic flux, etc.), configuration of the Space Surveillance Network (SSN), and the models and determination routines of TLEs.

Changes to SGP4 and how TLEs are estimated have not been made public since the official release of the “Space-track Report #3” in 1980, leaving users guessing to the current implementation of SGP4. Since 2013 the way TLEs are generated has changed significantly for many objects. TLEs are now fit to pseudo-observations generated from extrapolated higher-order orbital-theory solutions (Hejduk et al., 2013), as opposed to observations from the SSN directly. These new TLEs (referred to as enhanced TLEs) are found to differ significantly in terms of errors and behaviour compared to old TLEs (from hereon classic TLEs), as will be demonstrated in Section 5. A more obvious distinction between these two types is the argument of latitude (AOL) associated with their state at the TLE epoch. The AOL of classic TLEs is around zero degrees (i.e., at the ascending node) and around 90 degrees for enhanced TLEs.

TLE differencing and TLE fitting only use the TLE data and are thus applicable to any object for any epoch. The former technique propagates TLEs to a common epoch and compares them, while the latter technique treats TLEs

as pseudo-observations from which an orbit is estimated, usually using a higher-fidelity special-perturbation (SP) model (Levit and Marshall, 2011; Flohrer et al., 2008; Flohrer et al., 2009; Vallado et al., 2013). The benefits of TLE fitting are that an (improved) initial estimate as well as uncertainty information is obtained, which is convenient when further propagation is done using other models than SGP4. Furthermore, it has been demonstrated that both the initial error and error growth of TLE-derived states is significantly reduced compared to original TLE states (Levit and Marshall, 2011).

TLE differencing methods, on the other hand, are much simpler and faster: they only rely on SGP4 and require no external models and orbit-determination routines. Moreover, with the improved accuracy and propagation stability of enhanced TLEs the need for different propagation methods is reduced. Peterson et al. (2001) recommend a covariance generation (COVGEN) approach by differencing multiple preceding TLEs with the most forward one. Deguine et al. (2002) include backward propagation and note a temporal, and error bias and asymmetry.

The temporal bias is the shift of the epoch of minimum error, while temporal asymmetry refers to a difference in error growth between forward and backward propagation.

This bias was also demonstrated by Doornbos (2011) (for the CHAMP and GRACE satellites using GPS solutions), showing a clear dependency on the solar cycle and orbital altitude. Osweiler (2006) presents a detailed analysis of pairwise differencing and autocorrelation, while Mason (2009) compares the results against GPS solutions for the Stella satellite (near-circular 800 km orbit). Kelso (2007) assesses the COVGEN approach using precise ephemerides of GPS satellites and shows a good match in overall characteristics, but notes the importance of reflecting both forward and backward error growth. Legendre et al. (2006), Legendre et al. (2008) called on the work of Deguine et al. (2002), approximating the distribution as a Gaussian mixture. They mention the critical nature of outlier removal for data generation and sensitivity of the uncertainty to the AOL. Wang et al. (2009) present an analysis for many orbit regimes and identify that especially near-circular very low orbits ($h_p = 200 - 300$ km, $e = 0.00 - 0.02$) pose a major challenge, due to the importance of drag. Table 1 presents a small subset of the results found during that study, which demonstrates the difficulty of the aforementioned regime.

Table 1

Position errors in kilometres for one-day predictions as function of perigee height (h_p) and eccentricity (e) identified by Wang et al. (2009).

h_p [km]	e [–]		
	0 – 0.02	0.02 – 0.2	0.2 – 0.7
200 – 300	30.0	5.4	6.0
300 – 400	3.5	2.8	4.8
400 – 500	2.4	1.2	4.3
500 – 600	1.0	0.6	3.6

Unfortunately, this regime is critical for (human) space-flight and re-entering objects.

Despite almost two decades of investigation into TLE-differencing methods, several issues remain. A major problem is that the exact implementation of the differencing algorithms differs greatly, often without proper justification and comparison against previous implementations. The choice of analysis epoch(s) and the analysis/propagation window are often not properly motivated. Moreover, the implementations have been documented using varying detail, making it hard to reproduce results in many cases. Another major shortcoming is the scarcity of validation studies. Especially, the absence of validation for LEO is worrisome, considering its previously mentioned importance and difficulty. Moreover, most implementations only consider forward propagation to a single TLE epoch and do not consider the temporal bias. Furthermore, most studies consider a single TLE (at zero epoch) as reference, but especially the presence of a temporal bias sheds doubt on this practice. Lastly, it has not been investigated how this method is impacted by enhanced TLEs.

3. Methodology

Differencing methods generally propagate states to a common epoch and compare them to derive error samples. These methods are unique, as they only use TLE data and the SGP4 model. By solely relying on publicly available data this method is applicable to any (non-classified) object in the satellite catalogue (SATCAT).

Fig. 1 shows a schematic overview of the differencing process. Commonly a certain number of TLEs within a certain time period are considered at one time (TLEs at t_2, t_4 and t_7 in the example), which is determined by the propagation window (dashed line). The choice of propagation window is discussed in Section 3.3. Within this window the TLEs are propagated to a common differencing epoch (usually another TLE). In pairwise differencing, error samples are then obtained by creating pairs of TLEs at the differencing epoch. The following illustrates this:

$$\epsilon = x_n - x_m \quad (1)$$

where ϵ is the error sample created by differencing TLEs n and m , and x is an arbitrary state variable or vector of variables (discussed in Section 3.1). Note that n could be any TLE, but often represents the TLE at epoch (not propagated). If n is the TLE at epoch and paired with many m , n is considered the reference state. This assumption will be discussed in more detail in Section 3.2. Similarly, m usually represents any of the propagated TLEs.

TLEs can be propagated both forward and backward in time (see Section 3.4). In the example the propagation window is symmetric about the differencing epoch (e.g., equal maximum forward and backward propagation time).

To create many error samples, multiple differencing epochs are considered, for instance epochs $t_0, t_1, \dots, t_N$ as depicted in Fig. 1. As mentioned before, a common choice is to have the differencing epochs coincide with the TLE epochs. However, all possibilities will be explored in Section 3.6.

Next to the value of the ϵ itself, the propagation time is another important property of the error to consider. Throughout the paper this is referred to as the propagation time or age of the sample.

The process is illustrated in Fig. 1 and will be explained in further detail in the coming sections.

3.1. Reference frame

The reference frame for comparison is the RSW frame, composed of the radial, along-track and cross-track directions. The RSW frame is a local frame and intuitive for astrodynamics applications. In this frame, the difference in position and velocity are computed.

It is important to consider that RSW is a local frame, as the transformation between the Earth-centred inertial and RSW-frame depends on the inertial state itself. Eq. (1) can be rewritten to

$$\epsilon_{RSW} = \mathbb{T}(x_t)x_n - \mathbb{T}(x_t)x_m \quad (2)$$

where $\mathbb{T}(x_t)$ is the transformation matrix between the inertial and RSW frame, which is a function of the inertial state x_t . x_t can be any state (either n, m , or some other state), but it needs to be the same for both matrices in Eq. (2). It is

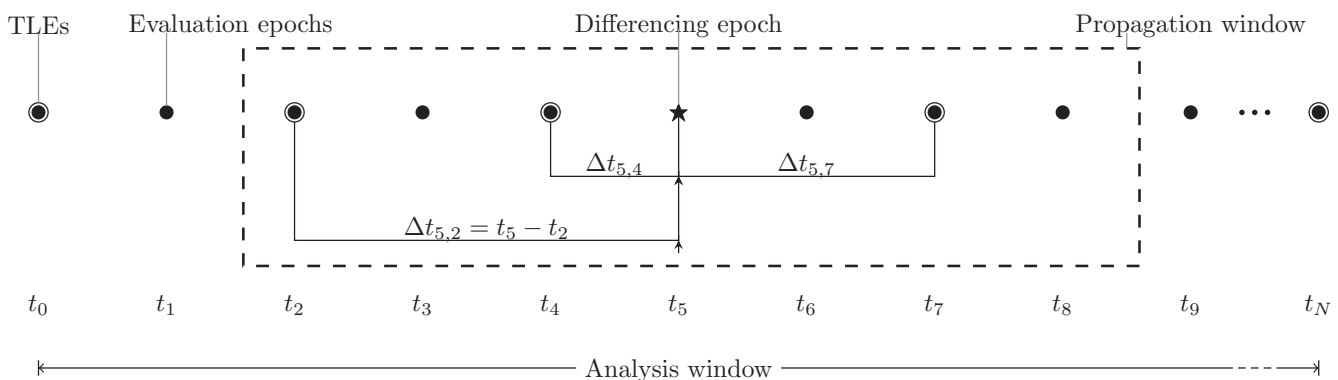


Fig. 1. Schematic overview of TLE differencing method.

intuitive to choose the TLE with highest (expected) accuracy, but, considering the temporal bias, it might not be as straightforward as choosing the nearest TLE.

3.2. Minimum error and reference

The consideration of the reference state in differencing is difficult, but important. The choice of the reference state influences the results of the method: directly (the state x_n) and indirectly (through the transformation matrix x_t). Previous studies consider the TLE at epoch to be the reference. This can be generalised to: the TLE that is closest to the epoch of interest is considered as reference. Although this is perhaps an intuitive strategy, it is not always the best. Deguine et al. (2002) already demonstrate that the epoch of minimum error does not coincide with the TLE epoch (i.e., there is a temporal bias Δt^*).

Typical values for Δt^* are in the order of 4–5 h for orbital altitudes of 200 – 300 km (as will be demonstrated) and 1.5–2.5 days for 400 – 500 km (Doornbos, 2011). Considering the similar or higher update frequency of TLEs, this can have a significant effect on any type of analysis.

A proper consideration of a reference epoch must thus account for this effect. It is proposed that the reference state is derived as the weighted mean of multiple states simultaneously. This method is named weighted differencing (WD) as opposed to pairwise differencing (PD). Further, an inversely-quadratic relation of the weights with the time since the epoch of minimum error is proposed.

$$w_i = \frac{1}{(t_i^* - t_{0_i} - t)^2} \quad (3)$$

$$\bar{w}_i = \frac{w_i}{\sum_{i \in P} w_i} \quad (4)$$

where $\bar{w}_i$ is the normalised weight of TLE_{*i*}, t_i^* is the epoch of minimum error (which is unknown), t_{0_i} the epoch of the *i*th TLE, and t the propagation time since t_{0_i} . The weights are normalised, such that the sum of all weights equals one for evaluation at each single epoch. The set P contains all TLEs under consideration, represented by the propagation window in Fig. 1.

As the epoch of minimum error of each individual TLE is unknown, it is assumed that all TLEs share the same offset (or shift), such that:

$$\Delta t_i^* = t_i^* - t_{0_i} \approx \Delta t^* \forall i \quad (5)$$

Due to this simplification Eq. (3) becomes:

$$w_i = \frac{1}{(\Delta t_i^* - t)^2} \quad (6)$$

The value of Δt^* is dependent mostly on the orbit determination process (fit length, etc.), which differs across orbital regimes and space-weather conditions (Doornbos, 2011). It is expected that the assumption is valid within the analysis window.

The reference state and Δt^* are mutually dependent: Δt^* determines the weights and in turn the reference state, from which error samples are computed, which determine Δt^* . This problem has to be solved iteratively. The initial guess $\Delta t^* = 0$ is proposed as a starting point. The level of convergence for Δt^* determines the accuracy of the method. A converging solution was obtained in all cases. Moreover, solutions were found to converge within three iterations to below a relative tolerance of $\epsilon < 0.01$, which is the criterion used throughout this study.

The weighted reference state can be found using:

$$x_{ref} = \sum_{i \in P} \bar{w}_i x_i \quad (7)$$

where P includes *all* states within the current propagation window. The weighted reference state is then used as reference in the generation of both the samples and transformation matrices ($x_t = x_n = x_{ref}$), such that Eq. (2) becomes:

$$\epsilon_{RSW} = \mathbb{T}(x_{ref})x_{ref} - \mathbb{T}(x_{ref})x_m \quad (8)$$

3.3. Analysis and propagation window

The analysis window, as depicted in Fig. 1, is the period from which the statistical properties are determined. The size of the window presents a trade-off. On one hand, the window should be large enough to allow for enough data for statistical significance, while on the other hand, it should be small enough to minimise the influence of longer-term effects, such as changes due to space weather (solar cycle) and orbit degradation.

The propagation window is a moving window within the analysis window. The differencing epoch resides within it. All TLEs that lie within the propagation window are propagated to the differencing epoch and compared. The size of the propagation window again presents a trade-off, depending on the application and object under consideration. For instance, for LEO objects a much shorter propagation window should be considered than for spacecraft at higher altitudes. Also the size of the propagation window can be influenced by manoeuvres. In previous studies windows up to 40 days have been considered. For very low-Earth orbits that are heavily influenced by atmospheric drag, the window could be as short as 4 days.

3.4. Forward/backward propagation

If the differencing epoch coincides with the upper bound of the propagation window, all considered TLEs precede the epoch, resulting in only forward propagation.

In general, both forward and backward propagation should be used. Forward propagation paints only half of the uncertainty picture. Kelso (2007) rightfully notes this in his assessment of the COVGEN method. Nonetheless, many previous studies have not looked at backward propagation. Not including both directions wrongly assumes a temporal symmetry (lack of bias and equal growth) of

the error. This is false, as the epoch of minimum error does not coincide with the TLE epoch and the rate of error growth can differ significantly between backward and forward direction. It is therefore recommended to choose the differencing epoch at the centre of the propagation window.

3.5. Modelling error growth

Regression, rather than data binning, is proposed for the estimation of the error growth from the samples. In previous studies binning was used. In binning, samples are collected based on their propagation time. The mean of the bin is then computed. The size of the bins is dependent on the number of samples and propagation-window size. Binning assumes no a priori shape of the error-growth pattern. Data binning, however, is not recommended for continuous problems, as it removes information from the problem. Moreover, the method is sensitive to the bin size and outliers. As will be shown, the shape of the error growth can be easily modelled, so non-parametric methods are not necessary at this point.

The most common fitting technique is least-squares regression, of which many variants exist. Essentially, a model is fit through the data and the sum of squared residuals is minimised. Unfortunately, least squares also suffer from outliers. Outlier removal can be employed to edit the data for fitting or binning. A common test uses the (dimensionless) studentized residual:

$$r_i = \frac{|\epsilon_i - \mu_\epsilon|}{\sigma_\epsilon} \quad (9)$$

where μ_ϵ and σ_ϵ are the mean and standard deviation of the error samples, respectively. A sample ϵ_i is discarded if the studentized residual is larger than a certain value r_{max} (discussed below). The mean μ_ϵ is included to account for any bias present. Note that μ_ϵ and σ_ϵ were determined using all data including the outliers. After eliminating the outliers, these properties need to be re-evaluated and the data retested, until no further outliers are detected.

A typical value of r_{max} is between 3 and 6 for normal-distributed data. Moreover, the recommended value of r_{max} depends on how far away outliers are expected to lie from the valid data. Ideally, the value of r_{max} should become smaller for each subsequent outlier-removal iteration, as the furthest outliers are removed first. Overall, the determination of outliers is troublesome and is sensitive to the choice of r_{max} .

Instead a robust least squares (RLSQ) method is proposed that is able to handle data with outliers directly. RLSQ is a form of weighted least-squares (specifically: iteratively reweighted least squares), where the weights of the data depend on their residuals. The residuals are obtained from the fit and are not affected by a bias. Also this method is iterative, as the residuals determine the weight, the weight determines the fit, and the residuals result from

the fit. The weighting scheme used is the bi-square weight function (Rousseeuw and Leroy, 1987):

$$v_i = \begin{cases} \left[1 - \left(\frac{\epsilon_i}{k}\right)^2\right]^2 & \text{for } |\epsilon_i| \leq k \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where v_i and ϵ_i are the weight and residual of sample i , respectively. The parameter k is commonly a function of the standard deviation of the data σ . For normal distributed errors $k = 4.685\sigma$ is recommended (Rousseeuw and Leroy, 1987).

3.6. Differencing epoch

The choice of differencing epoch has a big influence on the resulting estimation of the uncertainty. Previously, mostly TLE epochs have been considered. This is an obvious choice, especially when considering the TLE state at epoch as reference. However, this strategy results in only a limited number of differencing epochs (as the TLEs within the analysis window are usually few). Especially, if the propagation window is also short, the number of TLEs for comparison will be small. Moreover, TLEs are usually periodic, meaning that there will be gaps in the error samples. Lastly, the choice of the TLE epoch as differencing epoch seems very arbitrary when considering the temporal bias.

Therefore, intermediate differencing epochs are generated, in the case of Fig. 1 epochs $t_1, t_3, t_5, t_6, t_7, t_9$. The inclusion of intermediate differencing epochs will result in more samples and a more evenly spaced distribution of errors.

The choice and spacing of differencing epochs, however, should be made with care. Legendre et al. (2008) note a dependence of the error on the AOL u . As TLE states commonly share a similar AOL u_c , additional differencing epochs are conservatively chosen at this same AOL too, such that:

$$u_k = u_c + 2\pi k \quad (11)$$

where k is such that $t_0 < t_k < t_f$ and u_c is the common AOL of the TLEs within the analysis window $[t_0, t_f]$.

The effect of the choice of AOL will be demonstrated in Section 5.6. However, for now it is assumed that matching of the AOL to the (majority) of TLEs is the proper choice.

The differencing epoch t_k can be derived from the AOL u_k . However, here another issue arises with respect to the consideration of reference epoch and state. As the AOL is dependent on the state itself, many states propagated to the same epoch will slightly disagree on the exact AOL. Similarly, all TLEs within the propagation window propagated to have the exact same AOL, will slightly disagree about the current epoch t_i . Therefore, the true differencing epoch t_k is obtained as a weighted mean of all epochs t_i at the AOL u_k :

$$t_k = \sum_{i \in P} \bar{w}_i t_i \quad (12)$$

Practically, this results in the AOL becoming the independent variable for this exercise and time dependent. However, as the SGP4 propagator takes time steps as an argument, a simple bisection rootfinding scheme is employed to find $t_k = f(u_k)$.

The result of this is a series of differencing epochs, for a given analysis period and desired AOL. For a typical LEO satellite, this results in about 16 differencing epochs each day, as opposed to only 1–2.

4. Experimental set-up

In this section the details of the analysis are discussed. First, the proposed object of interest is introduced, followed by the periods considered and windows chosen. Next, statistical techniques and metrics are discussed. Finally, the analysis cases are introduced.

4.1. Object

For the analysis ESA's Gravity field and steady-state Ocean Circulation Explorer (GOCE) is considered. GOCE was a geodynamics and geodetic mission launched on 17 March 2009 and re-entered on 11 November 2013.¹ This object is interesting for many reasons.

GOCE's orbit is unique: it was in a very low near-circular orbit of only $h_p = 227$ km and $e = 9 \times 10^{-4}$. Therefore, the orbit was heavily influenced by atmospheric and gravitational disturbances. Even within the LEO regime, this orbit can be considered an extreme case, as illustrated by Table 1.

Second, precise orbits derived from GPS observations are widely available throughout the mission, against which the method can be validated.

Third, because of GOCE's high interest, it was tracked frequently during its mission. During the re-entry phase, the release frequency of TLEs even doubled from ~ 2 to ~ 4 per day.

4.2. Analysis window

GOCE low altitude orbit was maintained using a continuous-thrust ion-propulsion system, enabling a “drag-free” operational mode. The decay of the satellite started when the Xenon fuel was depleted early in the morning of October, 21, 2013.² The TLEs of the decay phase to re-entry are analysed: from October, 21, 2013 to November, 11, 2013. The analysis window is thus 21 days.

¹ http://www.esa.int/Our_Activities/Observing_the_Earth/GOCE/Facts_and_figures, accessed: August, 17, 2016.

² <https://directory.eoportal.org/web/eoportal/satellite-missions/g/goce>, accessed: August, 17, 2016.

A moving propagation window of four days ($\pm 2d$) is assumed.

TLEs are propagated to the argument of latitude (AOL) corresponding to the TLE set, such that samples are created every revolution.

4.3. Statistical techniques

Different estimation techniques for the error growth are compared. The traditional binning technique is compared to regression techniques. Several regression models and techniques are used. Unfiltered and filtered data (with outliers removed) are used to highlight differences between regular and robust methods.

The binning method is used to group together error samples of a similar propagation time. The propagation window is subdivided into 15 bins of 6.4 hours (or roughly 4 orbits). Polynomials are used as models of the errors and uncertainty:

$$p(\epsilon) = \sum_{n=0}^k c_n \epsilon^n \quad (13)$$

where k is the degree of the polynomial.

The errors in the RSW frame are modelled as a multi-variate (correlated) normal distribution (MVN). Moreover, even through propagation the errors are assumed to remain MVN within the propagation window considered. Such a distribution is fully defined by the covariance matrix. For very high levels of uncertainty (e.g., resulting from long propagation times), the errors tend to exhibit a banana shape distribution instead of a spheroid. It is therefore important to properly evaluate the validity of the MVN assumption.

Correlation matrices are reported instead of covariance matrices. Although covariances are often reported in literature and are a common format for analyses, it can be difficult to infer the strength of a dependency when the magnitudes and units of variances of the respective variables differ.

The joint distributions are also graphed, to fully assess the assumption of linearly-correlated multi-variate normal distributions. Margins will be plotted using Kernel density estimation, providing a continuous, but non-parametric model for distributions.

4.4. Cases

The analyses is divided into a number of test cases, to demonstrate/benchmark (various) aspects of the methods, provide validation, and analyse the sensitivity of the method. Each test case is represented by a separate section in Section 5. A brief overview of the different cases with a short description and rationale is given below.

1. Overview of data: general overview of the TLE data. Analysing frequency, count, and epochs of TLEs for the analysis window and dates prior. Providing insight into relationship TLEs and AOLs.
2. Error growth and minimum error: demonstrating temporal bias/asymmetry of TLE error growth using the absolute position error. The error data are constructed from comparison with GPS states. These data are estimated, to benchmark different statistical techniques. Robust regression is compared to binning and traditional regression methods.
3. Error growth in RSW components: robust regression is applied to each of position and velocity components, to demonstrate a good fit for all components and different model orders necessary.
4. Weighted differencing: weighted differencing is compared against pairwise differencing in terms of accuracy using GPS data for both enhanced and classic TLEs. Several (sub) tests are performed. First, the initial uncertainty is estimated. Second, the error and uncertainty growth are estimated. Third, the temporal bias is estimated.
5. Correlations: next to the uncertainty of various components, the correlations between them are important. The correlations are estimated using the robust weighted differencing technique and compared with GPS. Together with Case 3, the covariance matrix is now fully defined.
6. Covariance sensitivity: the sensitivity of the covariance (standard deviations and correlations) to the AOL is investigated to gain insight into TLE uncertainty and the importance of the choice of differencing epoch.

5. Results and discussion

The results of the analysis for each of the different cases are presented in the following sections.

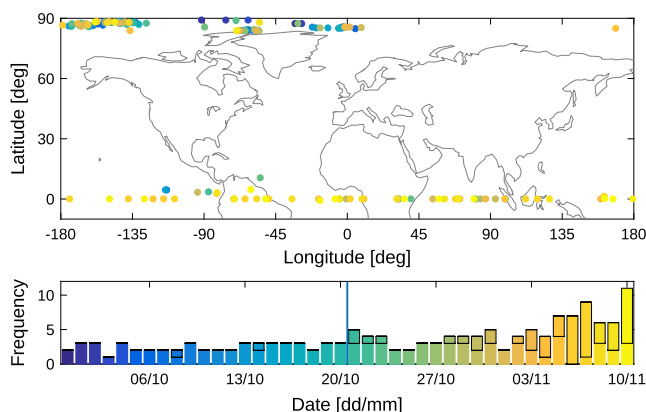


Fig. 2. (top) Overview of TLEs as a function of longitude and latitude and frequency of TLEs per day (bottom), where cTLEs can be differentiated by the outline of (part of the) bars.

5.1. Overview of data

The map in Fig. 2 gives an overview of the TLEs as a function of latitude and longitude at the TLE epoch for a period of 42 days: 21 days for analysis and 21 days prior. The colour of each marker corresponds to the date.

The bar plot show the number of TLEs for each day, the bars simultaneously function as a colourbar, relating the colour of the markers to the date. The begin of the analysis window is marked by the vertical blue line. Bars that are (partly) outlined in black indicate the portion of TLEs associated with the ascending node (latitude ~0°).

The first thing to note is that the analysis window contains almost twice the number of TLEs (101 versus 52). Second, it can be seen that almost all TLEs prior to October, 21, 2013 have a high latitude (~90°), after which there is an equal mix of TLEs associated with $u \approx 0^\circ$ and $u \approx 90^\circ$ (50 versus 51, respectively).

As the number of TLEs is nearly doubled and mixed in latitude, three assumptions are made. First, the additional TLEs are a result of additional tracking by Joint Space Operation Center (JSpOC) of GOCE during the re-entry phase. Second, these trackings result in TLEs with a distinct AOL. Third, the additional TLEs are of the classic type.

The latter suggests a mix of 50 classic TLEs and 51 enhanced TLEs for this period. This presents a unique opportunity to analyse these two types simultaneously for the same object and period.

5.2. Error growth and minimum error

The analysis of error growth provides a number of interesting insights: it can be used to demonstrate the presence of temporal bias and asymmetry, but also practically indicate when the TLE is on average the most accurate and, lastly, provide insight into the orbit determination process.

Fig. 3 shows the position errors of propagated TLEs with respect to GPS solutions as a function of the propagation time as scattered data (blue markers). For this result only the enhanced TLEs (eTLEs) are shown. It is remark-

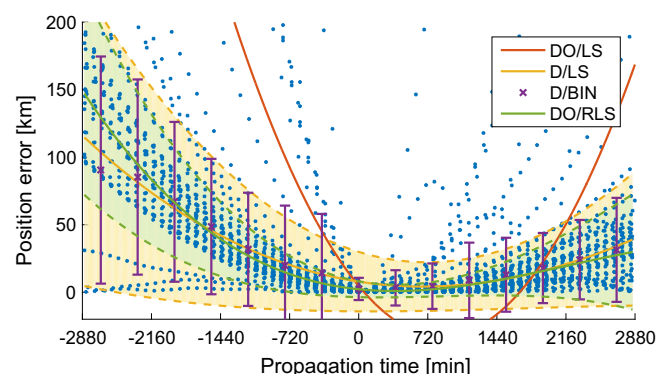


Fig. 3. Benchmark of different methods for estimating the error and (3 σ) uncertainty of the TLE propagation errors of Period 1a.

able that the minimum error is shifted ahead of the TLE epoch. The same analysis for classic TLEs (cTLEs) shows an opposite backward shift (as will be shown in Section 5.4), which is consistent with results from previous studies. This further confirms the difference between the two types.

The forward shift of the minimum (temporal bias) can be explained by the orbit-determination process of eTLEs, where future pseudo-observations are generated from using SP orbit determination and propagation.

It can further be seen that the forward error grows much more gradual than backwards growth. This peculiar shape can also be explained by the fitting process. Like for cTLEs, a quadratic error growth is expected. However, (part of) the pseudo-observations are derived from propagated states, which themselves are subject to error growth (due to limited predictive abilities). As a result the point of minimum error is actually shifted backwards (while remaining positive) from where one would expect it. This, perhaps counter-intuitively, results in a more gradual forward slope, as the error growth is a superposition of the uncertainty resulting from two separate sources.

The different estimation models are plotted as solid lines. A data/method naming scheme is used to differentiate the combinations. The data is split into data with outliers removed (D) (criterion $r_{max} = 3$) and data with outliers (DO). Therefore, methods starting with DO use unedited data. The methods used are (robust) least-squares regression (R) LS and data binning (BIN). The uncertainty bounds are also estimated using the different techniques and shown as the solid filled areas enclosed by dashed lines of the same colour.

The DO/LS serves as a benchmark for other methods and clearly performs worst. Only the error fit is shown, the resulting uncertainty was not plotted as it would fill the entire window. Data binning without outliers (not shown), similarly performs badly. This confirms the importance of data filtering for this type of analysis, as noted by Legendre et al. (2006).

Next, the two methods using filtered data D/LS and D/BIN provide comparable results as well. D/LS produces a more smooth result and shows better behaviour at the boundaries. Not only does it produce a continuous solution, but the solution is also more smooth, whereas D/BIN commonly suffers from jumps between adjacent bins.

The effectiveness of the robust variant is demonstrated when comparing D/LS and DO/RLS. It is expected that with careful tuning the D/LS (D/BIN) results can be improved. However, this is undesirable as it is very sensitive and dependent on the data.

Note that the orbit under consideration is very demanding, especially as it is decaying. Moreover, data outside the main band clearly forms streaks, which is due to a few bad TLE solutions. Another approach would therefore be to first identify and remove outlying TLEs. Doing so however, requires more statistical tests and tuning of parameters.

Overall, a number of observations stand out:

- a temporal bias shifts the epoch of maximum accuracy either forward or backward from the TLE epoch;
- the temporal asymmetry is much more pronounced for the enhanced TLEs, reducing the backward propagation stability significantly;
- the above effects are present in both classic and enhanced TLEs, but opposite/different in nature;
- robust least-squares regression using the polynomial models is able to capture the growth well, requiring no outlier removal of error samples and/or TLEs.

5.3. Error growth of RSW components

In the previous section the modelling of the absolute position error was investigated. Commonly, both the position and velocity error are analysed in their radial, along-track and cross-track components. The absolute position and velocity errors are dominated by the along-track position and radial velocity error, respectively. Due to the strong linear relationship between these two components, the errors in these components and the absolute errors will be very comparable. Therefore, a good fit is expected for these components. However, the goodness of fit of the remaining components remains to be demonstrated.

Figs. 4 and 5 show the results of the regression fit using the TLE errors with respect to the GPS reference states in all components.

It can be seen that the behaviour is significantly different per component. The top-left plots show the absolute position and velocity, respectively.

The contributions by the along-track position (bottom left) and radial velocity (top right) are clear. Moreover, the similarity between these two and the absolute errors are evident. These components also show the strongest growth in error. Therefore, these component should receive the highest priority in assessing the performance of the method. There is great similarity between these two (their correlation is discussed later) and the total position error, so previous conclusions apply here too.

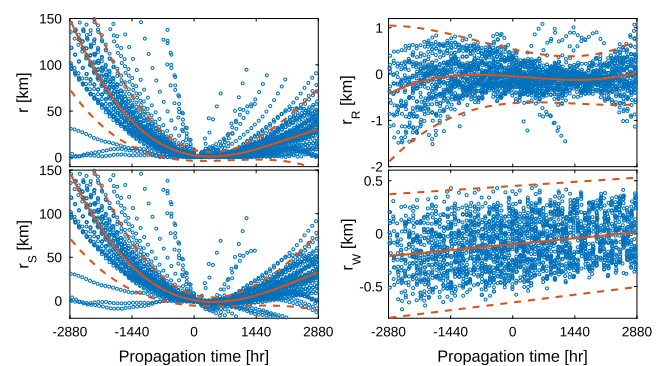


Fig. 4. Robust regression of the position errors and (3σ) uncertainty bounds decomposed in the RSW frame.

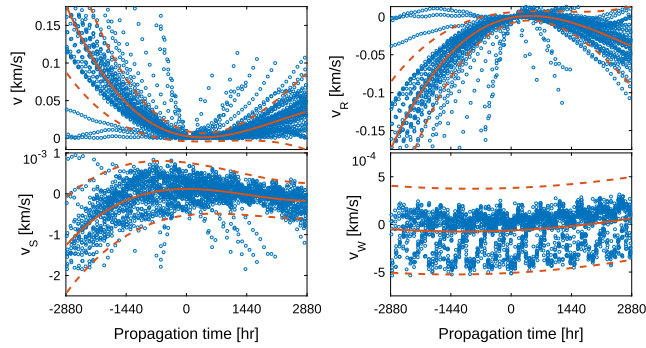


Fig. 5. Robust regression of the velocity errors and (3σ) uncertainty bounds decomposed in the RSW frame.

The radial position and along-track velocity errors are also comparable and show a complex curvature and a minimum uncertainty beyond the minimum total error. Although error growth is visible, it is not nearly as strong as for the previously discussed components.

The cross-track error of both position and velocity demonstrates a near-constant uncertainty and follow a simple mean trend (linear and quadratic, respectively). There is hardly any growth on this time scale.

It was assumed that the uncertainty bounds of the error are symmetric about the mean (e.g., equal distance of the lower and upper bound to the mean). It can be seen that this assumption is reasonable, except for the cross-track velocity error. The errors are clearly more concentrated above the mean. The exception is ignored due to the resulting overestimation of the upperbound and minor influence of this component on the overall error.

From these results it is obvious that the degree of the regression model of each component has to be carefully selected. As a general rule-of-thumb, the error uncertainty is modelled as an order less as the error trend. Table 2 gives an overview of the degree of the regression models used.

Note that a third-degree fit is enough to model all errors. It was found that for classic TLEs for some components the degree could be reduced further. However, this minimum degree was not further looked into.

Overall, the method is also able to accurately estimate the error and uncertainty growth of the individual RSW components. The following additional conclusions can be drawn:

- the error trend and growth are completely dominated by the along-track position and radial velocity components;
- varying orders of polynomial regression models are able to estimate error trends and growth for all components.

Next, the ability of the method to estimate this from differenced TLE data is investigated.

5.4. Weighted differencing

Table 3 shows the standard deviation of the error at the TLE epoch. It can be seen that both methods are fairly close to the truth. The largest differences are in the uncertainty in the cross-track state. This confirms findings by Kelso (2007) in estimating the error (growth) in this component. Fortunately, the error is smallest and most stable in this direction. Differences in r_W and v_W will therefore not be discussed further.

For the remaining components, the next largest difference is in the radial position and along-track velocity for eTLEs. These two components are underestimated by about 50%. In both cases WD performs about 25% better. Still the difference with the GPS results is not small. Nonetheless, the error in these components are quite small compared to the absolute error (about 8%). For cTLEs the results are closer and more favourable. The radial position the uncertainty is 12 – 15% overestimated, while the along-track velocity is underestimated by 10% only by PD.

The two major components (along-track position and radial velocity) show better results. Especially for WD, which conservatively overestimates each component for each type of TLE. PD on the other hand underestimates the errors for eTLEs by 25%, while overestimating (more than WD) for cTLEs. Overall, the results are can be considered acceptable, but not exact, depending on the analysis. It has to be restated that the data is very noisy. Better results are expected for different orbital regimes.

Figs. 6 and 7 show the error growth of the along-track position, estimated using the previously selected robust regression method for GPS and the standard (PD) and improved (WD) estimation methods for enhanced and classic TLEs, respectively. The error is plotted as the solid line, while the uncertainty (lower and upper) is shown using

Table 2
Overview of error regression modelling degree.

	Degree			Degree	
	Error	Residual		Error	Residual
r	3	2	v	3	2
r_R	3	2	v_R	3	2
r_S	3	2	v_S	3	2
r_W	1	1	v_W	2	1

Table 3
Initial standard deviation of errors at TLE epoch, derived from GPS (G), pairwise (P), and weighted (W) differencing.

	σ_{r_R} [km]	σ_{r_S} [km]	σ_{r_W} [km]	σ_{v_R} [m s ⁻¹]	σ_{v_S} [m s ⁻¹]	σ_{v_W} [m s ⁻¹]
<i>Enhanced</i>						
G	0.19	2.1	0.18	2.4	0.21	0.15
P	0.068	1.6	0.028	1.9	0.08	0.036
W	0.089	2.4	0.033	2.9	0.11	0.018
<i>Classic</i>						
G	0.39	4.4	0.19	5.3	0.44	0.25
P	0.44	6.6	0.22	8.0	0.39	0.20
W	0.46	6.2	0.14	7.6	0.46	0.13

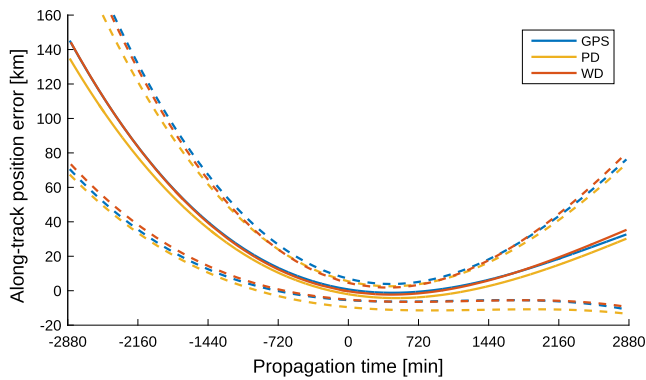


Fig. 6. Comparison of fit of the along-track position error and uncertainty for enhanced TLEs.

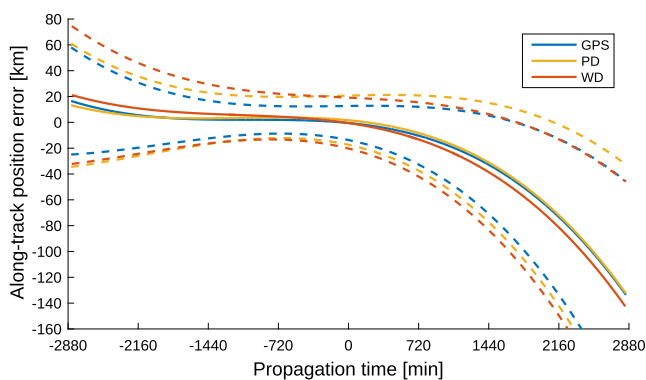


Fig. 7. Comparison of fit of the along-track position error and uncertainty for classic TLEs.

dashed lines of the same colour. PD, here, should not be confused with previous implementations (which used binning), as regression was applied to all data sets in this comparison. The difference between binning and regression was previously demonstrated. Moreover, recall that the absolute position, absolute velocity, along-track position, and radial velocity produce nearly identical results.

The figure highlights the similarity between PD and WD, and the good match with GPS data. This further demonstrates the capabilities of the RLS regression method. For enhanced TLEs, the error and bounds of GPS and WD match almost exactly. For classic TLEs the bounds of WD lie outside the GPS bounds, which is of course favourable.

Moreover, it can be seen that both methods are able to estimate the temporal bias well. Table 4 gives an overview of the exact figures of the temporal bias as obtained from the minimum absolute position error. As RLS provides a

continuous parametric solution, the minimum point is easily found through differentiation. The magnitudes of the bias are more than 6 and 4 h for eTLEs and cTLEs, respectively. Considering the update frequency of several times per day this is quite significant. Especially in the case of eTLEs it is thus common for older TLEs to be more accurate now when propagated than more recently released TLEs. It has to be noted that for the majority of objects and also the period preceding re-entry, the update frequency is lower (e.g., twice per day).

Both WD and PD are able to estimate the sign and magnitude satisfactory. Especially WD is almost spot on for eTLEs and off by about 3% for cTLEs.

Comparing enhanced and classical TLEs, it can be seen (Table 3) that overall the enhanced TLEs are much more accurate, having an uncertainty of roughly half across all components. Moreover, the uncertainty growth in forward propagation is reduced significantly. This clearly demonstrates the improved process of generation of enhanced TLEs. Their reduced accuracy and sharp growth in backwards propagation should be minded for applications that rely on this, such as splicing, where a (near) continuous state solution is created by combining the results from a series of TLEs.

This shows that:

- both methods are able to estimate the error uncertainty at TLE epoch reasonably well;
- WD perform better and tends to overestimate the uncertainty for the most important components;
- the error growth is much better approximated than the instantaneous uncertainty;
- the temporal bias is of the same order as the mean update interval;
- WD is able to very accurately estimate the temporal bias.

Next, the correlations between the components are analysed.

5.5. Correlations

Thus far only the errors and uncertainty have been analysed for all position and velocity components. However, these components are highly related. Such a strong correlation was demonstrated (in Figs. 4 and 5) between the along-track position and radial velocity, and radial position and along-track velocity. To complete the uncertainty model, the correlations have to be taken into account. For analyses, like error propagation and sampling, these relations can have a significant influence on the outcome.

Figs. 8 and 9 show joint and marginal distributions of the MVN distribution for enhanced and classic TLEs respectively. The marginal distributions are shown on the diagonal and are visualised using kernel density estimation. The joint distributions are shown as scattered data at the non-diagonal positions. The GPS and WD results are

Table 4
Temporal bias estimates using different methods.

	GPS [min]	PD [min]	WD [min]
Enhanced	375.3	314.2	372.1
Classic	−258.8	−292.2	−251.3

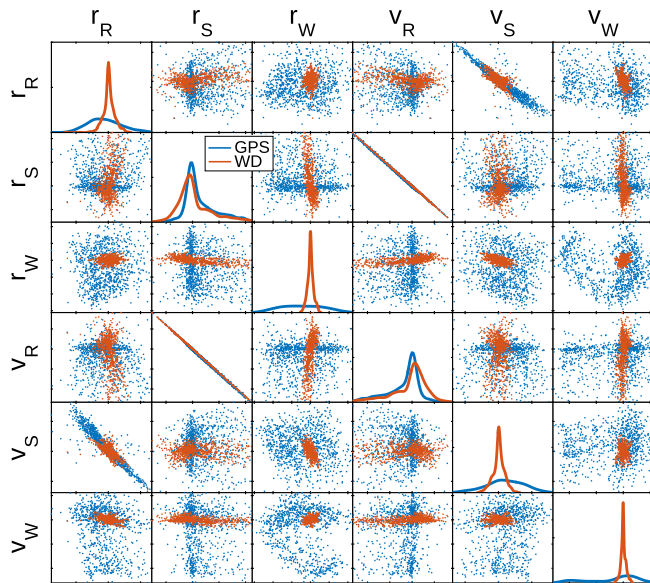


Fig. 8. Correlation estimation using weighted differences compared to GPS of enhanced TLEs.

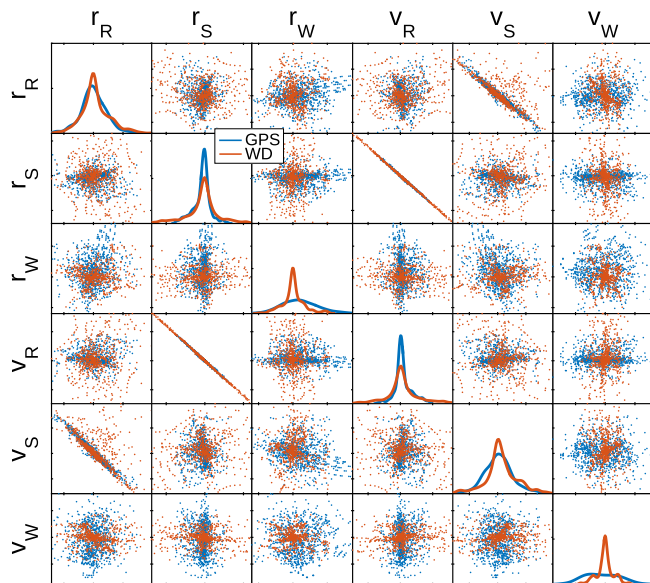


Fig. 9. Correlation estimation using weighted differences compared to GPS of classic TLEs.

shown in blue and red, respectively. Note that each joint distribution is shown twice, as correlations are commutative and mirrored across the diagonal. Thus only the joint distributions on one side of the diagonal need to be considered.

The first thing to note is that WD is able to estimate correlations for both TLE types. Visually it can be argued that for classic TLEs the best overlap with the GPS data is obtained. However, this does not necessarily hold true, as will be demonstrated shortly. For enhanced TLEs the WD results show narrower bands overlapping the GPS results for most combinations. This, however, does not affect the

correlations. The cause is attributed to narrow margins, which indicate underestimation of the radial position, along-track velocity and both cross-track components (diagonal positions 1, 3, 5 and 6). Moreover, the margins of the along-track position and radial velocity (diagonal positions 2 and 4) are properly estimated. This again confirms the results of the standard deviations as presented in Table 3.

The strong correlations between the along-track position and radial velocity, and radial position and along-track velocity, are approximated well for both TLE types. They present the only strong correlations present in both GPS- and weighted-differenced data.

Lastly, the WD distributions tend to have stronger central peaks in the marginals, but nonetheless the distributions appear symmetric and approximately Gaussian.

The actual values of the estimated and true correlation matrices are shown in Tables 5 and 6. The top half presents the correlation matrix obtained using GPS differenced TLEs, while the bottom half presents the correlations found through weighted TLE differencing.

For enhanced TLEs, looking at Table 5, it can be seen that the variables that are only very weakly correlated (values below ± 0.10), the correlations tend to be overestimated on average by a factor 3–4. At the same time the relationship between the radial position and along-track velocity is underestimated by 30%. The inverse relationship between the along-track position and radial velocity, however, is spot on. Lastly, although the magnitudes could be estimated better, it is important to note that the sign of the correlation is always estimated correctly (with the exception of ρ_{r_W, v_W}). This seems to indicate that correlation information could be potentially recovered from TLE data alone.

For classic TLEs, looking at Table 6, similar conclusions can be drawn. The magnitudes of (weak) correlations are overestimated, while the very large ρ_{r_R, v_S} is underesti-

Table 5

GPS (top) and weighted (bottom) differenced derived correlation matrices for enhanced TLEs.

	r_R	r_S	r_W	v_R	v_S	v_W
GPS						
r_R	1.00	0.08	0.06	−0.11	−0.93	−0.16
r_S	0.08	1.00	−0.09	−1.00	0.05	−0.08
r_W	0.06	−0.09	1.00	0.08	−0.11	−0.17
v_R	−0.11	−1.00	0.08	1.00	−0.03	0.10
v_S	−0.93	0.05	−0.11	−0.03	1.00	0.14
v_W	−0.16	−0.08	−0.17	0.10	0.14	1.00
WD						
r_R	1.00	0.33	0.05	−0.34	−0.67	−0.49
r_S	0.33	1.00	−0.56	−1.00	0.01	−0.34
r_W	0.05	−0.56	1.00	0.56	−0.58	0.27
v_R	−0.34	−1.00	0.56	1.00	−0.01	0.35
v_S	−0.67	0.01	−0.58	−0.01	1.00	0.07
v_W	−0.49	−0.34	0.27	0.35	0.07	1.00

Table 6

GPS (top) and weighted (bottom) differenced derived correlation matrices for classic TLEs.

	r_R	r_S	r_W	v_R	v_S	v_W
GPS						
r_R	1.00	0.04	0.25	−0.02	−0.98	0.06
r_S	0.04	1.00	0.04	−1.00	−0.09	−0.11
r_W	0.25	0.04	1.00	−0.04	−0.30	0.00
v_R	−0.02	−1.00	−0.04	1.00	0.07	0.12
v_S	−0.98	−0.09	−0.30	0.07	1.00	−0.03
v_W	0.06	−0.11	0.00	0.12	−0.03	1.00
WD						
r_R	1.00	−0.06	−0.13	0.06	−0.63	−0.22
r_S	−0.06	1.00	0.12	−1.00	0.01	−0.12
r_W	−0.13	0.12	1.00	−0.13	0.13	−0.07
v_R	0.06	−1.00	−0.13	1.00	−0.01	0.13
v_S	−0.63	0.01	0.13	−0.01	1.00	0.16
v_W	−0.22	−0.12	−0.07	0.13	0.16	1.00

mated significantly. Unlike previously, the signs of the weaker correlations are not even estimated correctly.

From this it has to be concluded that:

- correlations are difficult to estimate, where eTLEs show better behaviour than cTLEs;
- no very strong (false) correlations were introduced nor correlations present ignored by the method.

Combining Tables 3, 5 and 6, the covariance matrix is now fully defined. This information is important input to many types of error analyses, such as collision avoidance (COLA) and entry-point predictions (Ronse and Mooij, 2014). Note that also for the uncertainty the estimation at epoch proved slightly worse than the estimation of the growth. This clearly indicates a sensitivity at the TLE epoch, where values are small and switch sign. Therefore, the sensitivity of the correlations is investigated further in the next section.

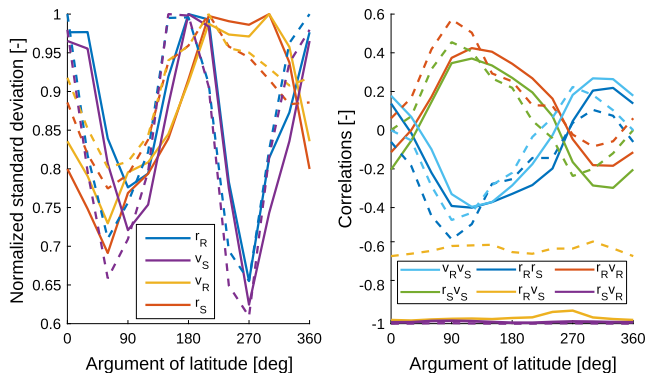


Fig. 10. Uncertainty and correlation of classic TLEs as a function of AOL for GPS (solid lines) and weighted differenced (dashed lines) results. Legend entries are sorted to corresponding value at AOL = 0°.

5.6. Covariance sensitivity

Intermediate propagation epochs were introduced to create a more evenly distributed set of error samples. It was chosen to generate samples at fixed positions in the orbit, rather than a fixed time interval, to eliminate any effect this might have on the error. In this section the sensitivity of the error as a function of AOL is investigated.

Fig. 10 shows the standard deviation of the position and velocity samples in the RSW components and their correlation coefficients. This analysis has been performed for both types and results were found to be similar. Previously, the classic TLEs showed worst results for the estimation of the correlation coefficients. Hence the results for this type are shown. The standard deviations are normalised with their maximum magnitude for each component. The plot shows both GPS (solid lines) and weighted TLE differenced (dashed lines) solutions. The cross-track components are not shown. As for previous exercises, TLE differencing could not be used to obtain satisfactory approximations of these components.

First, the amount of variation of both correlation and uncertainty is quite striking. A variation in uncertainty of more than 35% can be observed in the radial position and along-track velocity. Similarly, the along-track position and radial velocity vary significantly in uncertainty throughout the orbit.

For the correlation the case is perhaps even more variable. Correlations between most variables change from moderately correlated to uncorrelated and even switch sign, depending on the position in the orbit. These (estimated) correlations should be used with caution.

Secondly, these trends and extrema are captured relatively well using WD. The most notable deviation is the correlation between radial position and along-track velocity. The true correlation (almost 1) is underestimated by almost 35%, as already demonstrated by Table 6. Although the match between GPS and WD seemed very poor judging from this table, the right plot of Fig. 10 tells a different story.

A clear distinction between weakly and strongly correlated variables is present. The first class seems to vary significantly, change sign and average out over one orbit. The strongly correlated variables are almost invariant to the AOL.

Lastly, it is interesting to note that the enhanced TLEs are commonly reported around 90° from the ascending node. The uncertainty in the components is much lower there. Detailed analysis between enhanced and classic TLEs should take this into account.

Overall, the following can be concluded:

- uncertainty and correlations vary greatly throughout an orbital revolution;
- the trend of the correlations is captured rather well, despite the poor instantaneous estimation;

- the correlation between radial position and along-track velocity is underestimated, regardless of argument of latitude.

6. Conclusions

Enhanced TLEs offer improved uncertainty and error-growth properties. Even after the switch to enhanced TLEs the classic type remain in use, likely for additional tracking purposes. It is, however, simple to distinguish between the two in the presence of a mix by looking at the AOL.

The presence of a temporal bias was demonstrated and found to be of the order of the TLE update interval. Moreover, an asymmetry between forward and backward propagation was demonstrated. Both effects can have great influence on the propagation errors. This holds true especially considering their complete opposite nature in classic and enhanced TLEs. It is paramount to take these effects into account for any application relying on propagation of TLEs using SGP4, such as stitching TLEs for creating a continuous state-time solution.

Even for applications that employ the state at TLE epoch, it can be beneficial to consider the temporal bias and evaluate the state at the epoch of minimum mean error instead.

Robust least-squares regression was proposed as an alternative to data binning. This method has been shown to provide a continuous solution. This allows easy evaluation of the temporal bias. Moreover, it handles unedited data very well, reducing the need for TLE and data pruning and tuning of parameters. Furthermore, component-error analysis has shown that this approach extends well for all position and velocity components.

TLE differencing can accurately estimate important error characteristics, like the temporal bias, error uncertainty and growth. Even correlations can be estimated to some degree. The correlations seem very sensitive to variations in argument of latitude. This is likely true for other parameters. Nonetheless, there is a clear distinction between strong and weakly correlated variables. The former vary and average out over one orbit, while the latter remain invariant.

Weighted differencing was compared against pairwise differencing. WD uses temporal bias information to determine the reference state. It has been shown to perform better than PD on all exercises. Especially the consistent (marginal) overestimation of major error components makes the technique favourable.

In conclusion, weighted differencing method using robust least-squares regression was proposed to improve previous TLE differencing methods. The proposed method was shown to be superior on all tests, consistently providing better approximations of the GPS differenced solutions.

The method is validated for the GOCE satellite during the period of investigation. Due to the low orbital altitude and increased TLE-update frequency further investigation

is necessary in order to extend these conclusions to LEO in general. However, even for lower TLE-update rates, the method is expected to still perform satisfactorily, due to the intermediate evaluation epochs and the resulting near-continuous error data in time. Also, considering that the orbit considered presents a limit case in many respects, it is expected that the method will perform well in orbits in which errors are less.

The method is useful for the error analysis of a set of TLEs itself and in providing uncertainty input for other types of analysis. The method is computationally fast, can be applied to any object in the SATCAT, and deals well with noisy data. It is thus very suitable for demanding applications, such as conjunction analysis, that require frequent and fast processing for a large number of objects.

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References

- Deguine, B., Foliard, J., Alby, F., Donath, T., Bouchard, J., Quillien, C., 2002. Covariance modelling in satellite collision risk activities. In: *AIAA/AAS Astrodynamics Specialist Conference*.
- Doornbos, E.N., 2011. Thermospheric Density and Wind Determination from Satellite Dynamics PhD Thesis. Delft University of Technology.
- Flohrer, T., Krag, H., Klinkrad, H., 2008. Assessment and categorization of TLE orbit errors for the US SSN catalogue. In: *Advanced Maui Optical and Space Surveillance Technology Conference*.
- Flohrer, T., Krag, H., Klinkrad, H., 2009. ESAs process for the identification and assessment of high-risk conjunction events. *Adv. Space Res.* 44 (3), 355–363.
- Hejduk, M.D., Casali, S.J., Cappellucci, D.A., Ericson, N.L., Snow, D.E., 2013. A catalogue-wide implementation of general perturbations orbit determination extrapolated from higher order orbital theory solutions. *Adv. Astronaut. Sci.* 148, pp. AAS 13-240.
- Kelso, T.S., 2007. Validation of SGP4 and IS-GPS-200D against GPS precision ephemerides. In: *AAS/AIAA Space Flight Mechanics Meeting*.
- Legendre, P., Deguine, B., Garmier, R., Revelin, B., 2006. Two line element accuracy assessment based on a mixture of Gaussian laws. In: *AIAA/AAS Astrodynamics Specialist Conference*.
- Legendre, P., Garmier, R., Revelin, B., Delevault, S., 2008. Improvement of the TLE accuracy model based on a Gaussian mixture depending on the propagation duration. In: *AIAA/AAS Astrodynamics Specialist Conference*.
- Levit, C., Marshall, W., 2011. Improved orbit predictions using two-line elements. *Adv. Space Res.* 47 (7), 1107–1115.
- Mason, J., 2009. Development of a MATLAB/STK TLE accuracy assessment tool. Support of the NASA Ames Space Traffic Management Project, International Space University, MSc Thesis.

- Oswiler, V.P., 2006. Covariance Estimation and Autocorrelation of NORAD Two-Line Element Sets MSc Thesis. Air Force Institute of Technology.
- Peterson, G.E., Gist, R.G., Oltrogge, D.L., 2001. Covariance generation for space objects using public data. AAS/AIAA Space Flight Mechanics Meeting, p. 108.
- Ronse, A.L.A.B., Mooij, E., 2014. Statistical impact prediction of decaying objects. *J. Spacecraft Rockets* 51 (6), 1–14.
- Rousseeuw, P.J., Leroy, A.M., 1987. Robust Regression and Outlier Detection. John Wiley & Sons.
- Vallado, D.A., Cefola, P.J., 2012. Two-Line Element Sets – Practice and Use. In: *International Astronautical Congress*.
- Vallado, D.A., Virgili, B.B., Flohrer, T., 2013. Improved SSA through orbit determination of two-line element sets. In: *European Conference on Space Debris*.
- Wang, R., Liu, J., Zhang, Q.M., 2009. Propagation errors analysis of TLE data. *Adv. Space Res.* 43 (7), 1065–1069.

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Paper S03

Uncertainty propagation for statistical impact prediction of space debris

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Abstract

Predictions of the impact time and location of space debris in a decaying trajectory are highly influenced by uncertainties. The traditional Monte Carlo (MC) method can be used to perform accurate statistical impact predictions, but requires a large computational effort. A method is investigated that directly propagates a Probability Density Function (PDF) in time, which has the potential to obtain more accurate results with less computational effort. The decaying trajectory of Delta-K rocket stages was used to test the methods using a six degrees-of-freedom state model. The PDF of the state of the body was propagated in time to obtain impact-time distributions. This Direct PDF Propagation (DPP) method results in a multi-dimensional scattered dataset of the PDF of the state, which is highly challenging to process. No accurate results could be obtained, because of the structure of the DPP data and the high dimensionality. Therefore, the DPP method is less suitable for practical uncontrolled entry problems and the traditional MC method remains superior. Additionally, the MC method was used with two improved uncertainty models to obtain impact-time distributions, which were validated using observations of true impacts. For one of the two uncertainty models, statistically more valid impact-time distributions were obtained than in previous research.

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Keywords: Uncertainty propagation; Statistical impact prediction; Space debris; Liouville equation

1. Introduction

Space debris presents the largest group of observable objects in Earth orbit since the in-orbit explosion of the US Transit-4A satellite in 1961. These objects have frequently entered the Earth's atmosphere of which some impacted the Earth's surface (Klinkrad, 2006). Considering the kinetic energy with which these bodies impact, it is

valuable to have accurate predictions of the impact location and time.

Objects in orbital decay enter the Earth's atmosphere at the end of their life. During atmospheric entry, the body converts a large amount of kinetic energy into heat, which may result in breakup of the body. Impact prediction for these cases is performed using advanced tools that include breakup and survivability analysis (Portelli et al., 2004; Rochelle et al., 2004). Predictions of a decaying trajectory require a different approach.

The Space Surveillance Network (SSN) tracks a large number of objects orbiting the Earth. Orbit estimates resulting from these observations are published as Two-Line Elements (TLEs) in the Satellite Catalog (SATCAT). TLEs present a full description of the translational state of an object at the TLE epoch, as well as an estimate of its ballistic parameter. TLEs are updated in the SATCAT at

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regular intervals, resulting in a history of TLEs for each object. The orbit can be reconstructed by propagating the TLEs using the Simplified General Perturbations-4 (SGP4) simulator (Vallado et al., 2006). The SSN observations are also used to perform Tracking and Impact Predictions (TIPs)⁴ for bodies that have a higher than 5% risk of reaching the surface without breaking up. These TIPs provide an estimated decay time and window, but do not provide any additional statistical information. Predictions of the decaying trajectory of an object are highly influenced by uncertainties in the initial state of the object and the models that are used for propagation. Therefore, statistical methods are required to perform impact predictions.

In the research of Ronse and Mooij (2014) a method is proposed to obtain statistical impact predictions using the Monte Carlo (MC) method. A 6 Degrees-of-Freedom (DoF) simulator was used to perform a large number of simulations with a randomly varying initial state and atmospheric density. The data were used to obtain a distribution of the impact time, which can be used to derive the distribution of the impact location using the ground track of the orbit. Accurate statistical predictions were obtained, which were validated with re-entry observations. A property of the Monte Carlo method is that the solution approaches the exact solution for an increasing number of simulations. However, a large number of simulations is required to obtain accurate results, which results in a large computational effort.

In recent research, methods are proposed that have the potential to provide more accurate results with the same or even less computational effort than MC. These methods directly propagate uncertainty, expressed by probability distributions, in time. In the research of Halder and Bhattacharya (2011), a method is presented that models the uncertainty in the state of an entry vehicle by a Probability Density Function (PDF) and propagates this PDF in time. The method uses a direct solution of the stochastic Liouville equation, which is a quasi-linear Partial Differential Equation (PDE) that encompasses the conservation of probability. Results are obtained for an entry vehicle and compared with MC, and it is argued that the method provides more accurate results with less computational effort. This method will be referred to as the Direct PDF Propagation (DPP) method.

In addition to the direct method, several approximate methods can be used to propagate uncertainty in time (Pantano and Shotorban, 2007; Prabhakar et al., 2010; Attar and Vedula, 2008; Terejanu et al., 2008). These methods, propagate a parametric model of the PDF of the state. Promising results were obtained with these methods and some are successfully applied to spaceflight related problems (Xu, 2009; Xu, 2011). Although these methods can

provide significant reductions in computational effort, the DPP method potentially provides a higher accuracy.

In this paper, the DPP method is applied to statistical impact prediction, because of its promising accuracy. Statistical impact predictions are performed with the models developed by Ronse and Mooij (2014) of the Delta-K rocket body. Because 6-DoF trajectory simulations are performed, the dimension of this problem is considerably larger than the examples shown in Halder and Bhattacharya (2011). The goal is to investigate the limitations, accuracy and computational performance, and to compare the method to the MC method. Additionally, an improved uncertainty model is presented and used to obtain statistical impact predictions, which are validated and compared to the results of Ronse and Mooij (2014).

The paper starts with a discussion on the theory of the DPP method in Section 2. This theory is applied to develop simulators for statistical impact prediction, discussed in Section 3. An important part of the prediction method is the uncertainty model presented in Section 4, which determines the outcome of the predictions. The results of the research on the DPP method for statistical impact prediction are given in Section 5. The results of the statistical impact predictions with the improved uncertainty model are presented in Section 6. Finally, Section 7 describes the conclusions that are drawn from the research.

2. Probability density propagation

The trajectory of an object can be described using equations of motion that are defined as an initial-value problem of the form:

$$\dot{X} = F(X(t), p, t) \quad \text{with} \quad X(0) = X_0 \quad (1)$$

where X_0 is the initial state of the object. Solving these equations results in a unique solution of the trajectory. If uncertainty is present in the initial state, the trajectory cannot be defined in a deterministic way. The uncertainty in the initial state can be modeled by defining the initial state using a PDF, assigning a probability value to the possible initial states.

Instead of propagating a single trajectory using Eq. (1), the PDF of the state can be propagated in time. Hereby, all possible trajectories are propagated at once. The evolution of the PDF of the state, or the stochastic evolution of the state, is governed by the *Liouville Equation*⁵ (Halder and Bhattacharya, 2011):

$$\frac{\partial \varphi(X(t), t)}{\partial t} + \sum_{i=1}^{N_s} \frac{\partial}{\partial X_i} [\varphi(X(t), t) F_i(X(t), t)] = 0 \quad (2)$$

where $\varphi(X(t), t)$ is the PDF of the state and N_s is the number of states in the state-derivative model, Eq. (1). This equation is a mathematical description of the conservation

⁴ The TLEs and TIPs are obtained from a catalog available at <https://www.space-track.org/>.

⁵ The Liouville Equation is a special case of the Fokker-Planck Equation, which also includes a diffusion term (Gardiner, 2004).

of probability, because it forces that the integral of the PDF remains one when evolving in time. A similar equation originates from the conservation of mass.

In the research of [Halder and Bhattacharya \(2011\)](#), a solution of this equation is presented using the method of characteristics. A characteristic analysis results in the following equations that define the characteristic curves:

$$\begin{aligned} \frac{dt}{1} &= \frac{dX_1}{F_1} = \frac{dX_2}{F_2} = \frac{dX_3}{F_3} = \dots = \frac{dX_{N_S}}{F_{N_S}} \\ &= \frac{d\varphi(\mathbf{X}(t), t)}{-\varphi(\mathbf{X}(t), t)\Psi(\mathbf{X}(t))} \end{aligned} \quad (3a)$$

$$\Psi(\mathbf{X}(t)) = \sum_{i=1}^{N_S} \frac{\partial F_i}{\partial X_i} \quad (3b)$$

The fact that these equations define the characteristic curves shows that these are the trajectories of the state itself. These equations can be used to derive the solution, which is:

$$\varphi(\mathbf{X}(t), t) = \varphi_0(\mathbf{X}_0) \exp\left(-\int_0^t \Psi(\mathbf{X}(\tau)) d\tau\right) \quad (4)$$

The PDF of the state can thus be propagated in time by simulating trajectories using Eq. (1) and inserting the results in Eq. (4).

The integral of Ψ present in Eq. (4) can be solved by numerical quadrature after the state history is obtained from a numerical integrator. It was observed that, if a variable stepsize numerical integrator is used to solve the state history, the steps can be too large to solve the integral accurately using numerical quadrature. Therefore, a different method is employed to solve this integral, which involves introducing an extra state variable Ω :

$$\Omega(t) = \int_0^t \Psi(\mathbf{X}(\tau)) d\tau \quad (5)$$

This state variable can be solved along Eq. (1) using numerical integration, by finding an ODE that governs the evolution of Ω . This results in the following equations that are used to propagate the PDF:

$$\varphi(\mathbf{X}(t), t) = \varphi_0(\mathbf{X}_0) \exp(-\Omega(t)) \quad (6a)$$

$$\begin{pmatrix} \dot{\mathbf{X}} \\ \dot{\Omega} \end{pmatrix} = \begin{pmatrix} \mathbf{F}(\mathbf{X}(t), t) \\ \Psi(\mathbf{X}(t)) \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \mathbf{X}(0) \\ \Omega(0) \end{pmatrix} = \begin{pmatrix} \mathbf{X}_0 \\ 0 \end{pmatrix} \quad (6b)$$

Uncertainty in other parameters than the state variables can be included by converting them to state variables. The method is limited by the fact that all state variables should be included in the PDF, except for special cases where variables can be separated.

The only known in Eq. (6), is the uncertainty model of the initial state, so the definition of the PDF of the initial state $\varphi(\mathbf{X}_0)$. The process of propagating the PDF of the state in time is illustrated in Fig. 1. The figure shows that a sample of the initial PDF is propagated in time to obtain

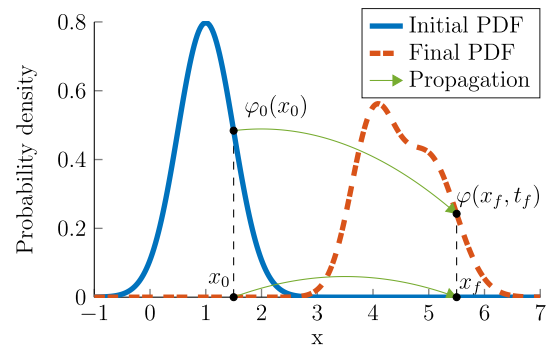


Fig. 1. Hypothetical example of propagating a PDF forward in time.

a sample of the PDF at the final time. The PDF can be propagated using two different methods:

- The initial state $\mathbf{X}_0$ can be sampled and forward propagated in time to $\mathbf{X}(t)$, and $\Omega(t)$ can be solved simultaneously. This is a shooting-like method that results in scattered datapoints of $\mathbf{X}(t)$ and the corresponding value of the PDF, $\varphi(\mathbf{X}(t), t)$.
- The state $\mathbf{X}(t)$ can be sampled and backward propagated in time to $\mathbf{X}_0$, while simultaneously solving $\Omega(t)$. This results in samples of the value of $\varphi(\mathbf{X}(t), t)$ at the chosen values of $\mathbf{X}(t)$.

Propagating the state forward in time has the benefit that the location x_f of the PDF $\varphi(x_f)$ does not have to be known. However, it results in a scattered dataset of the PDF at the final time, which may be difficult to process. Furthermore, the datapoints are concentrated near areas of higher probability density, which means that less information is obtained about the areas of lower probability density. Backward time propagation has the benefit that the location of the samples of the final PDF can be chosen. Unfortunately, for some state-derivative models it is challenging to accurately propagate the state backwards. Both methods are referred to as the DPP method.

It was found that the solution becomes unstable when propagating the state backwards in time from the Earth's surface. The reason is that the body approaches the terminal velocity and vertical flight at the end of the trajectory, which is an unstable process in the reverse direction. Therefore, only forward propagation is used in the research. One dimensional examples of propagating a PDF are described in [Halder and Bhattacharya \(2011\)](#). These examples were reconstructed during this research to verify the solution method.

3. Methodology

This section describes the models and methods to perform trajectory simulations, to propagate PDFs of the state of the body, and to process the PDF propagation data.

3.1. Reference vehicle and trajectory

The Delta-K rocket body is used for the analysis, because it has been shown that sufficient test cases are available for validation and the trajectories could be reconstructed. In the research of Ronse and Mooij (2014), an inertia matrix and aerodynamic model are obtained, which are used for this research. The aerodynamic model uses an aerodynamic-coefficient database to generate the forces and moments. This database contains coefficients for all possible angles of attack α , angles of sideslip β , and for Mach numbers M ranging from 10 to 48 generated using a panel method. Coefficients were generated for the rarefied-flow regime and the continuum-flow regime; a bridging function is used for the transition-flow regime.

The initial state of the Delta-K rocket body is obtained from TLEs using the SGP4 simulator. The simulations of the trajectory of the body are started in the decay phase with a time to impact ranging from 2 to 44 h.

3.2. Trajectory simulation

The statistical impact prediction methods require a trajectory simulator that simulates the motion of the Delta-K rocket body. This trajectory simulator is used with three different state models to investigate the DPP method.

3.2.1. State Model A

State Model A (SM-A) is a model to simulate two-dimensional atmospheric entry trajectories. The following state derivative model was used to perform these simulations (Mooij, 1994):

$$\dot{R} = V \sin \gamma \quad (7a)$$

$$\dot{V} = -\frac{\rho V^2 S C_D}{2m} - g \sin \gamma \quad (7b)$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{g}{V} \cos \gamma \left(1 - \frac{V^2}{gR}\right) \quad (7c)$$

where R is the radius from the center of the Earth to the spacecraft, V is the velocity of the spacecraft and γ is the flight-path angle. The atmospheric density ρ is determined using an exponential atmosphere model:

$$\rho = \rho_0 \cdot \exp(-\beta(R - R_E)) \quad (8)$$

where R_E is the radius of the Earth.

3.2.2. State Model B

For State Model B (SM-B), the trajectory simulator solves the evolution of the translational and rotational state in time using a numerical integrator. The state vector $\mathbf{X}$ is defined as:

$$\mathbf{X} = (R \quad \tau \quad \delta \quad V_N \quad V_E \quad V_D \quad \mathbf{q}^T \quad \boldsymbol{\omega}^T \quad X_\rho)^T \quad (9)$$

where R is the radial distance from the center of the Earth, τ is the longitude and δ is the latitude, and V_D , V_E and V_N are the velocity components in the down, east and north direction. The attitude of the rocket body is modeled using quaternion $\mathbf{q}$, which represents the attitude of the body with respect to the Earth pseudo-inertial reference frame. The rotational velocity $\boldsymbol{\omega}$ of the body is expressed in the body-fixed frame.

The NRLMSISE-00 atmosphere model of Picone et al. (2002) is used to provide the atmospheric properties to the trajectory simulator. The uncertainty in the atmospheric density is modeled by introducing an atmospheric density factor X_ρ :

$$\rho = X_\rho \rho_M \quad (10)$$

where ρ is the atmospheric density that is used in the simulations and ρ_M is the atmospheric density predicted by the NRLMSISE-00 model, which depends on longitude, latitude, altitude and time. This atmospheric density factor is only added as a state variable to include atmospheric density uncertainty for the DPP method, but remains constant:

$$\dot{X}_\rho = 0 \quad (11)$$

The following state-derivative model is used for the translational state (Mooij, 1994):

$$\frac{dR}{dt} = -V_D \quad (12a)$$

$$\frac{d\tau}{dt} = \frac{V_E}{R \cos \delta} \quad (12b)$$

$$\frac{d\delta}{dt} = \frac{V_N}{R} \quad (12c)$$

$$\frac{dV_N}{dt} = \frac{F_N}{m} - 2\omega_E V_E \sin \delta - \omega_E^2 R \sin \delta \cos \delta - \frac{V_E^2 \tan \delta - V_N V_D}{R} \quad (12d)$$

$$\frac{dV_E}{dt} = \frac{F_E}{m} + 2\omega_E (V_D \cos \delta + V_N \sin \delta) + \frac{V_E}{R} (V_N \tan \delta + V_D) \quad (12e)$$

$$\frac{dV_D}{dt} = \frac{F_D}{m} - 2\omega_E V_E \cos \delta - \omega_E^2 R \cos^2 \delta - \frac{V_E^2 + V_N^2}{R} \quad (12f)$$

The following equations are used for the rotational dynamics (Mooij, 1994):

$$\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{pmatrix} = \frac{1}{2} \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \quad (13a)$$

$$\dot{\boldsymbol{\omega}} = \mathbf{I}^{-1}(\mathbf{M}_B - \boldsymbol{\omega} \times \mathbf{I} \boldsymbol{\omega}) \quad (13b)$$

where $\mathbf{M}_B$ only includes the aerodynamic moment. The external forces in these equations are described by a force vector $\mathbf{F}_V$ expressed in the V -frame. This force vector is

composed of an aerodynamic force $\mathbf{F}_{A,V}$ and a gravitational force $\mathbf{F}_{G,V}$:

$$\mathbf{F}_V = \begin{pmatrix} F_N \\ F_E \\ F_D \end{pmatrix} = \mathbf{F}_{G,V} + \mathbf{F}_{A,V} \quad (14)$$

The gravitational force $\mathbf{F}_{G,V}$ is modeled using the central gravity model with J_2 perturbation (Mooij, 1994):

$$\mathbf{a}_{G,V} = \frac{\mu}{R^2} \begin{pmatrix} -3J_2 \left(\frac{R_e}{R}\right)^2 \sin \delta \cos \delta \\ 0 \\ 1 - \frac{3}{2}J_2 \left(\frac{R_e}{R}\right)^2 (3 \sin^2 \delta - 1) \end{pmatrix} \quad (15)$$

with the gravitational constant of the Earth $\mu = 398600.4418 \text{ km}^3/\text{s}^2$ and $J_2 = 1082.629821313305 \cdot 10^{-6}$ and the shape of the Earth is modeled as an ellipsoid (WGS, 2000).

The aerodynamic acceleration and moment are calculated using the aerodynamic coefficients obtained from the database described in Section 3.1, the atmospheric density ρ from Eq. (10) and the velocity obtained from the state vector. Linear interpolation is used to provide the aerodynamic coefficients, which are dependent on angle of attack α , angle of sideslip β and Mach number M . These variables are obtained from the state vector using a series of transformations.

3.2.3. State Model C

State Model C (SM-C) is identical to SM-B except for the model of the translational state. The state vector $\mathbf{X}$ is defined as:

$$\mathbf{X} = (\mathbf{r}_R^T \quad \mathbf{V}_B^T \quad \mathbf{q}^T \quad \boldsymbol{\omega}^T \quad X_\rho)^T \quad (16)$$

The translational state is now modeled using Cartesian position coordinates, $\mathbf{r}_R$, in the Earth-fixed reference frame and the velocity $\mathbf{V}_B = [u \ v \ w]^T$ is described by a ground-velocity vector expressed in the body-fixed reference frame. These state variables are chosen, because there is no dependence on the flight-path angle or the heading angle, which may cause singularities for vertical flight. The state derivative model of the position vector is defined as:

$$\dot{\mathbf{r}}_R = \mathbf{C}_{R,I}(\theta_g) \mathbf{C}_{I,B}(\mathbf{q}) \mathbf{V}_B \quad (17)$$

where θ_g is the angle between the Earth fixed frame and the inertial frame. The state derivative of the velocity vector can be calculated using the following equation:

$$\dot{\mathbf{V}}_B = \mathbf{V}_B \times \boldsymbol{\omega}_{B/R}^B + \frac{\mathbf{F}_B}{m} - 2\boldsymbol{\omega}_{R/I}^B \times \mathbf{V}_B - \boldsymbol{\omega}_{R/I}^B \times (\boldsymbol{\omega}_{R/I}^B \times \mathbf{r}_B) \quad (18)$$

where $\mathbf{r}_B$ is the Cartesian position vector $\mathbf{r}_R$ transformed to the body frame and $\boldsymbol{\omega}_{B/R}^B$ is the rotational velocity vector of the body with respect to the Earth expressed in the body frame:

$$\boldsymbol{\omega}_{B/R}^B = \boldsymbol{\omega}_{B/I}^B - \boldsymbol{\omega}_{R/I}^B \quad (19)$$

where $\boldsymbol{\omega}_{B/I}^B$ is angular rate of the body and $\boldsymbol{\omega}_{R/I}^B$ is the rotational velocity of the Earth expressed in the body frame:

$$\boldsymbol{\omega}_{R/I}^B = \mathbf{C}_{B,I}(\mathbf{q}) \begin{pmatrix} 0 \\ 0 \\ \omega_E \end{pmatrix} \quad (20)$$

3.3. PDF Propagation

The uncertainty is modeled using a PDF of the initial state (Section 4), which is propagated in time to obtain information about the uncertainty in the future. As explained in Section 2 and illustrated in Fig. 2, the initial PDF is sampled, and the state and probability density are propagated for each sample. The initial state is sampled using a Sobol sampler and propagated forward in time using the trajectory simulator. Simultaneously, Ω is solved and used to solve the probability density using Eq. (6).

For SM-A, the state-derivative model of Ω is defined as:

$$\dot{\Omega} = \frac{\partial \dot{\mathbf{R}}}{\partial \mathbf{R}} + \frac{\partial \dot{\mathbf{V}}}{\partial \mathbf{V}} + \frac{\partial \dot{\gamma}}{\partial \gamma} = -\frac{\rho V S C_D}{m} + \frac{g}{V} \sin \gamma \left(1 - \frac{V^2}{gR}\right) \quad (21)$$

Examining this equation, it can be observed that the first term on the right-hand side is driven by the aerodynamic drag. This term causes Ω to decrease, which means that the probability density increases (Eq. (6)). The second term originates from the centripetal acceleration and also becomes largely negative for $\gamma \rightarrow -90^\circ$ and $V \rightarrow 0$.

For SM-B, the state-derivative model of Ω is defined as:

$$\dot{\Omega} = \frac{1}{m} \frac{\partial F_D}{\partial V_D} + \frac{1}{m} \frac{\partial F_E}{\partial V_E} + \frac{1}{R} (V_\delta \tan \delta + V_D) + \frac{1}{m} \frac{\partial F_N}{\partial V_N} + \frac{V_D}{R} \quad (22)$$

where the aerodynamic acceleration $\mathbf{a}_{A,V}$ is the only component that contributes to this model:

$$\mathbf{a}_{A,V} = \mathbf{C}_{V,AG}(\gamma(u, v, w), \chi(u, v, w), \sigma(u, v, w)) \mathbf{a}_{A,AG}(u, v, w) \quad (23)$$

For SM-C, the state derivative is defined as:

$$\dot{\Omega} = \sum_{i=1}^{N_s} \frac{\partial F_i}{\partial X_i} = \frac{\partial a_{x,B}}{\partial u} + \frac{\partial a_{y,B}}{\partial v} + \frac{\partial a_{z,B}}{\partial w} \quad (24)$$

where the aerodynamic acceleration $\mathbf{a}_{A,B}$ is the only component that contributes to this model:

$$\mathbf{a}_{A,B} = \mathbf{C}_{B,AG}(\alpha(u, w), \beta(u, v, w)) \mathbf{a}_{A,AG}(u, v, w) \quad (25)$$

The above mentioned partial derivatives contain derivatives of the aerodynamic coefficients to α , β and M . These are determined by numerical differentiation using linear interpolation in the aerodynamic database. The step size is chosen by finding an optimum between the round-off error and the truncation error.

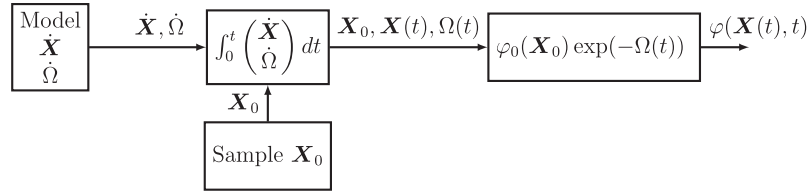


Fig. 2. Schematic presentation of the DPP method.

3.4. PDF data processing

The DPP method results in a scattered dataset of the 13-dimensional joint PDF of the state. To derive statistical information of a single state or a selection of states, marginal or conditional distributions need to be derived. Formally, these distributions are derived by integrating the joint PDF. A marginal distribution of dimension M can be derived using the following equation (Montgomery and Runger, 2014):

$$f_{X_1 \dots X_M}(x_1, \dots, x_M) = \int_{-\infty}^{\infty} f_{X_1 \dots X_N}(x_1, \dots, x_N) dx_{M+1} \dots dx_N \quad (26)$$

For impact prediction, a conditional distribution needs to be derived using the condition that the altitude has a certain value. From this conditional distribution, marginal distributions can be derived of the impact time and impact location.

Integrating the joint PDF obtained from the DPP method can be performed using two approaches. The first approach is to fit a curve through the scattered dataset to obtain a functional form of the joint PDF, which can be integrated analytically or numerically. The second approach is to use the data directly to derive the marginal distributions, without explicitly integrating.

3.4.1. PDF fitting

Accurate curve-fitting methods are available for a low number of dimensions, such as fitting splines. Unfortunately, fitting curves through higher dimensional scattered data is challenging. An attempt was made to fit a weighted summation of Radial Basis Functions (RBFs) to the 3-dimensional PDF data for SM-A. These RBFs can be integrated analytically, which can be used to generate marginal distributions.

No accurate results could be obtained through interpolation or Least-Squares regression with constraints. In Lazzaro and Montefusco (2002), it is acknowledged that instabilities occur with curve fitting of large datasets using RBFs. The use of compactly supported RBFs, which are only defined within a certain region, could partially overcome this problem. However, no significant improvement was obtained.

The reason that no accurate result could be obtained is that the propagation process causes a certain structure in the scattering of the data, which is difficult to model.

Because the full problem is a 13-dimensional problem, it is expected that no accurate approximation of the PDF can be obtained using this method. This is confirmed by the research of Allasia et al. (2014), which shows that fitting a large dataset of multi-dimensional data is challenging. Therefore, it is expected that obtaining accurate marginal distributions using a data fit is either highly challenging or infeasible.

3.4.2. PDF binning

In the research of Halder and Bhattacharya (2011), an approximate method is used to generate marginal distributions. This method produces a histogram-like PDF that is obtained by dividing the required marginal distribution in bins. The probability density of each bin is determined by taking the mean value of the PDF data in each bin. Subsequently, the PDF is normalized. This method is equivalent to MC integration if the following assumptions hold:

- The PDF data is uniformly distributed in each bin in all dimensions of the PDF.
- The enclosed volume of the scattered data in each bin is equal.

A Monte-Carlo integration process is performed by multiplying an estimate of the mean $\hat{\mu}$ of $f(\mathbf{x})$ with the enclosed volume V_x of the data samples of $\mathbf{x}$:

$$I = \int_V f(\mathbf{x}) d\mathbf{x} \approx \hat{\mu} \cdot V_x \quad (27)$$

which can be derived from the central limit theorem. If the above assumptions hold, the standard deviation of the random error is then (Dekking et al., 2005):

$$\sigma(I) = \sigma(\hat{\mu}) \cdot V_x \approx \frac{\sigma_f}{\sqrt{N}} \cdot V_x \quad (28)$$

which means that a large number of samples N is required within each bin to obtain an accurate estimate of the PDF.

The simulation process influences the distribution of the samples at the final time. Therefore, even if the initial distribution of the samples is uniform, the final distribution is not necessarily uniform and the enclosed volume of the data in each bin is not necessarily equal.

4. Uncertainty model

Performing statistical impact predictions requires an accurate model of the uncertainties that influence the

motion of the rocket body. Uncertainties are considered in the initial translational and rotational state, and two different uncertainty models were used for the atmospheric density. It is assumed that the uncertainty in the initial translational state is uncorrelated with the initial rotational state and the atmospheric density.

4.1. Translational state

The initial translational state is generated from TLEs, which can have a position inaccuracy in the order of several km (Kahr et al., 2013). This inaccuracy acts as an uncertainty in the simulation of the trajectory of the body. Therefore, it is important that an accurate statistical model of this uncertainty is used in the analysis.

These uncertainties are mostly included by assuming a (Gaussian) distribution based on a covariance matrix (Ronse and Mooij, 2014; Yim and Chung, 2012; Osweiler, 2006). However, no information on the distribution of these uncertainties is available. Therefore, the uncertainty model is derived from historic TLE data.

4.1.1. Pair-wise differencing

The SGP4 model is used to propagate several TLEs to a certain epoch at which the difference are analyzed to determine the uncertainty in the TLEs (Geul et al., 2017). A schematic of this TLE pair-wise differencing method is shown in Fig. 3.

As shown in the figure, a certain evaluation interval is chosen in which the error in the TLEs is determined. Several evaluation epochs are defined on this interval at which the TLEs are compared. A moving window is defined, which is centered on the current epoch. All the TLEs that are available within this window are propagated towards the current epoch and the results are pair-wise subtracted to determine the error. This process is performed for all the evaluation epochs within the evaluation interval to obtain samples of the error in the TLEs (Geul et al., 2017).

4.1.2. Considerations

One must realize that propagating TLEs in time, also propagates the uncertainty in the state, in a similar way to the methods discussed in Section 2. Because one is

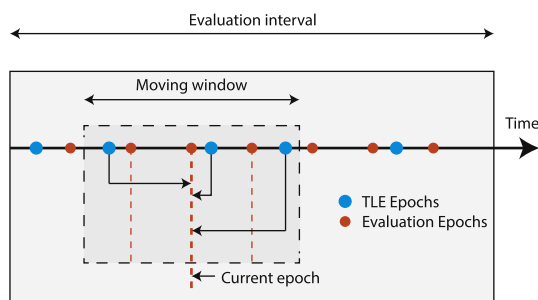


Fig. 3. Visualization of TLE propagation for the TLE pair-wise differencing method.

interested in the uncertainty of the initial state, the change in the uncertainty in state needs to be minimized. Therefore, the moving window width is chosen as small as possible with the constraint that enough TLEs are available to obtain an accurate model.

In the research of Geul et al. 2017, it was found that the inaccuracies in the TLEs vary with the position in its orbit. Therefore, the evaluation points are selected at the ascending node of each orbit, because most published TLEs states are around this region. It was further found that the errors produced by backward propagation are different than with forward propagation, resulting in a bias in the errors. This bias was mitigated by adding both differences of each pair.

A comparison was performed on the results obtained with the TLE pair-wise differencing method and errors in the TLEs obtained using GPS data. It was found that the magnitude of the errors grow with the propagation time, so no accurate measure of the magnitude of the uncertainty can be obtained. Nevertheless, it was observed that the correlation structure obtained using the TLE pair-wise differencing method is correct (Geul et al., 2017).

Therefore, the TLE pair-wise differencing method was used to obtain the datapoints and the dispersion is scaled by the standard deviation obtained from comparing GPS data of GOCE with TLEs. In the research of Yim and Chung (2012) it was shown that the standard deviations for a rocket body and a satellite are similar in magnitude. Therefore, it is expected that even though the GOCE satellite has a different shape than the Delta-K rocket body, this method will result in a more accurate uncertainty model than using TLE pair-wise differencing alone.

4.1.3. Uncertainty model

The data obtained using the TLE pair-wise differencing method is shown in Fig. 4 for one of the cases. It is presented in the orbital frame with R as the radial direction,

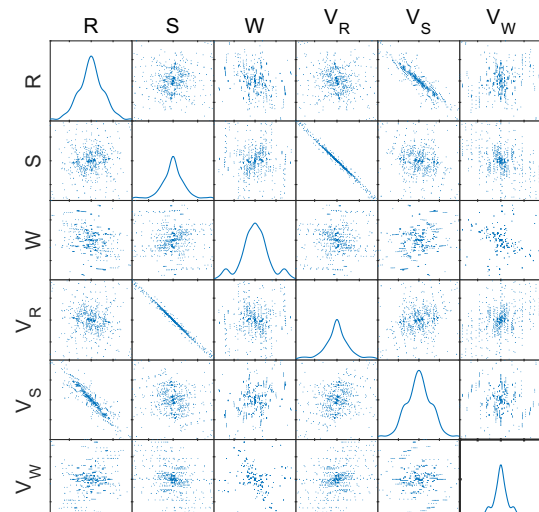


Fig. 4. Data samples obtained for Norad ID 28130 with a moving window size of 0.6 days.

S the along-track direction and W the cross-track direction. The figure shows several scatter plots that show the correlation between the state variables. From these scatter plots it can be observed that the radial velocity V_R and along-track position S error, and the along-track velocity V_S and the radial position R error are highly correlated. Both correlations are expected from orbital dynamics. On the diagonal a Kernel Density Estimate of the PDF of each state is shown. These plots indicate that the error has an approximate Gaussian distribution. The corresponding correlation matrix is shown in Table 1 and the standard deviation of the state variables are displayed in Eq. (29).

$$\begin{pmatrix} \sigma_R \\ \sigma_\tau \\ \sigma_\delta \\ \sigma_{V_D} \\ \sigma_{V_E} \\ \sigma_{V_N} \end{pmatrix} = \begin{pmatrix} 103.5 \text{ m} \\ 3.27 \cdot 10^{-4} \text{ rad} \\ 5.67 \cdot 10^{-4} \text{ rad} \\ 0.27 \text{ m/s} \\ 7.59 \cdot 10^{-3} \text{ m/s} \\ 3.05 \cdot 10^{-4} \text{ m/s} \end{pmatrix} \quad (29)$$

This data is used to fit an uncertainty model using Kernel Density Estimation, which results in a 6-dimensional PDF of the form (Bowman and Azzalini, 1997):

$$f_{X_1 \dots X_6}(x_1, \dots, x_6) = \frac{1}{N} \sum_{i=1}^N K_{i, X_1 \dots X_6}(x_1, \dots, x_6, \mathbf{h}) \quad (30)$$

where $\mathbf{h}$ is a vector containing the bandwidth of the kernel for each dimension and N is the number of samples. Gaussian kernels are used in the model and the bandwidth of the kernel is chosen using cross-validation to prevent overfitting. If one assumes that the distribution is Gaussian, the following optimal bandwidth can be derived for a kernel with dimension d (Bowman and Azzalini, 1997):

$$h_i = \left(\frac{4}{(d+2)N} \right)^{1/(d+4)} \sigma_i \quad (31)$$

where σ_i is the standard deviation of the data samples for dimension i .

Cross-validation is performed by obtaining two PDFs using half of the data and comparing the results. A result is shown in Fig. 5, where the bandwidth is chosen to be 1.5 times the optimal bandwidth. The figure shows that the two are almost equivalent, so this bandwidth is chosen for the uncertainty model. Oversmoothing is preferred to overfitting, because oversmoothing results in a wider PDF, so a more conservative uncertainty model.

Table 1
Correlation matrix of errors in orbital frame for one of the cases.

	R	S	W	V_R	V_S	V_W
R	1.000	0.165	−0.319	−0.167	−0.984	0.036
S		1.000	−0.156	−0.997	−0.318	0.296
W			1.000	0.158	0.305	−0.330
V_R				1.000	0.321	−0.313
V_S					1.000	−0.095
V_W						1.000

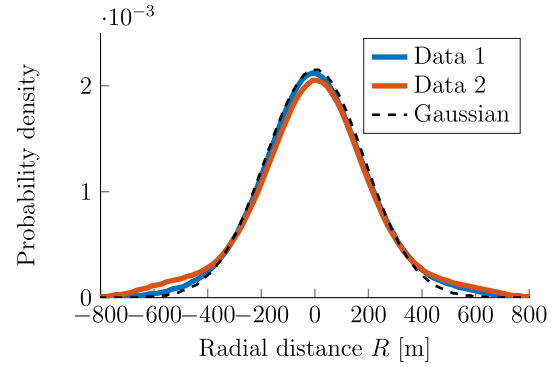


Fig. 5. Uncertainty model of the radial distance using two different datasets with $h = 1.5h_{opt}$.

4.2. Rotational state

The uncertainty in the initial attitude and initial rotational velocity are modeled using PDFs. Because no information of the initial attitude is available, a uniform random attitude is assumed. In Kirk (1992), a method is provided to sample quaternions such that a random uniform attitude is obtained. Three uniformly distributed random samples u_1, u_2, u_3 are generated and used to generate quaternions:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} \sqrt{1-u_1} \sin(2\pi u_2) \\ \sqrt{1-u_1} \cos(2\pi u_2) \\ \sqrt{u_1} \sin(2\pi u_3) \\ \sqrt{u_1} \cos(2\pi u_3) \end{pmatrix} \quad (32)$$

The outcome was analyzed and it was confirmed that this method results in a random uniform attitude. It was further found that the sampling method results in quaternions q_1, q_2 and q_3 that are uniformly distributed within a unit sphere. Therefore, the PDF of the quaternions is modeled using the following equation:

$$f_{q_1 q_2 q_3}(q_1, q_2, q_3) = \begin{cases} \frac{3}{4\pi} & q_1^2 + q_2^2 + q_3^2 \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

where q_4 follows from the quaternion unity constraint.

Like the initial attitude, the initial rotational velocity is also unknown. Ronse and Mooij (2014) found that the initial rotational velocity vector can be assumed to have a random orientation and a uniformly distributed length. Using Eq. (32) and a random sample of the length, samples can be generated. These samples will lie within a three dimensional sphere. If one draws several spheres with a radius r within this sphere, each sphere will contain the same number of samples on its surface, because the length is uniformly distributed. This means that the probability density of the samples is $\propto 1/r^2$. Using this result, the following PDF can be derived using spherical coordinates:

$$f_{r, \theta, \phi}(r, \theta, \phi) = \begin{cases} \frac{1}{4\pi r^2} & 0 < r \leq R \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

which has a total integral of 1 over its volume. The marginal PDF of the radius r is equal to $1/R$ with $r \in (0, R]$, which confirms that the radius is uniformly distributed. The PDF of the rotational velocity can be derived using Eq. (34):

$$f_{\omega}(\omega) = \begin{cases} \frac{1}{4\pi\omega_{max}(\omega_1^2 + \omega_2^2 + \omega_3^2)} & 0 < \omega_1^2 + \omega_2^2 + \omega_3^2 \leq \omega_{max}^2 \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

with $\omega_{max} = 1$ rotation per minute, which is based on data examined by Ronse and Mooij (2014).

4.3. Atmospheric density

In the research of Picone et al. (2002), Bezdek (2007), and Prag (1983) it is indicated that the natural logarithm of the observed density to the predicted density by the MSIS-86 has a normal distribution. Because the NRLMSISE-00 model is based on this atmosphere model, such a distribution is assumed for the uncertainty in the atmospheric density. Therefore, the atmospheric density factor X_{ρ} , Eq. (10), has a lognormal distribution (Montgomery and Runger, 2014):

$$f_{X_{\rho}}(x_{\rho}) = \frac{1}{x_{\rho}\omega\sqrt{2\pi}} \exp\left(-\frac{(\ln x_{\rho} - \theta)^2}{2\omega^2}\right) \quad (36)$$

where the location parameter θ and the scale parameter ω are defined by:

$$\theta = \ln\left(\frac{\mu_{x_{\rho}}}{\sqrt{\frac{\sigma_{x_{\rho}}^2}{\mu_{x_{\rho}}^2} + 1}}\right) \quad (37)$$

$$\omega^2 = \ln\left(\frac{\sigma_{x_{\rho}}^2}{\mu_{x_{\rho}}^2} + 1\right) \quad (38)$$

The simulations were performed with two different uncertainty models: Uncertainty Model A (UM-A) and B (UM-B). The properties of these uncertainty models are defined in Table 2. Uncertainty Model A is based on the mean and standard deviations specified by Ronse and Mooij (2014) for the altitude region of 120–200 km, which used a uniform distribution. Uncertainty Model B is based on the parameters specified by Picone et al. (2002) for the altitude region of 200–400 km. As shown in the table, UM-A is unbiased and has a smaller standard deviation than UM-B, which also contains bias.

Table 2

Parameters of the lognormal distribution describing the uncertainty in the atmospheric density.

	UM-A	UM-B
$\mu_{X_{\rho}}$	1	0.93
$\sigma_{X_{\rho}}$	0.12	0.17

5. PDF propagation results

Simulations were performed with the DPP simulator and the three different state models, with the goal of comparing the computational performance. The results of these simulations are discussed in this section.

5.1. Case A: Two-dimensional test

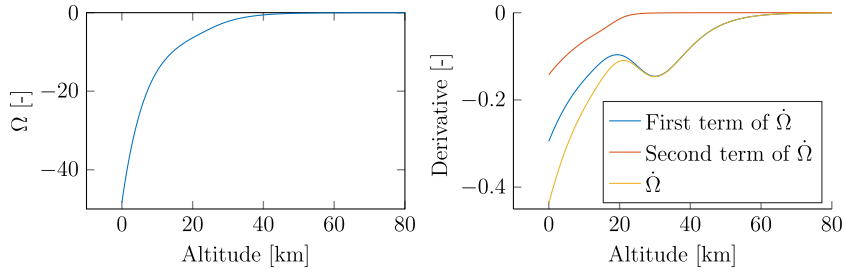
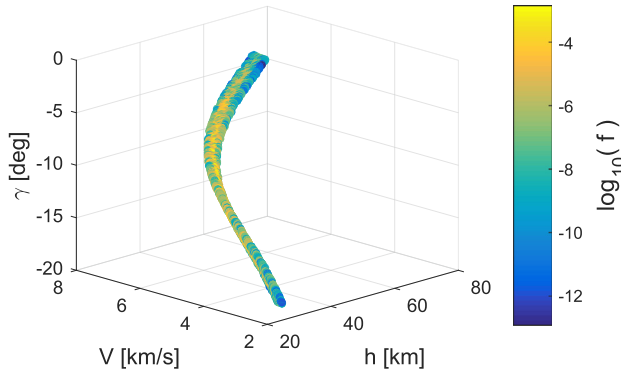
The evolution of Ω is studied first to investigate how the PDF will deform in time. The evolution of Ω is shown in Fig. 6 for a ballistic trajectory with an initial altitude of 80 km, an initial flight-path angle of -10° and an initial velocity of 7.2 km/s. The figure also shows $\dot{\Omega}$ and the two individual components present in Eq. (21). These results are independent of the initial PDF (Uncertainty Model) and only indicate how the PDF of the state deforms in time. The first term of Eq. (21) is the most dominant, so the evolution of Ω or probability density is mostly caused by the aerodynamic drag. The centripetal acceleration (second term) only has a significant effect in the final part of the trajectory. The value of Ω decreases to a large negative value, which means that the probability density increases. This corresponds to a convergence of the multi-dimensional PDF, so converging marginal distributions or increasing correlations, and a decrease in uncertainty. This is an expected result, because the velocity converges towards the terminal velocity and the dispersion in altitude also gets smaller in the final part of the trajectory. This behavior is a consequence of the use of the Liouville equation, because it does not include noise. Therefore, the trajectory of the body is fully deterministic and the PDF of the state converges in the multi-dimensional space.

After examining the evolution of Ω , a DPP simulation was performed to investigate the evolution of the three-dimensional PDF of the state. A total of 2000 trajectory simulations were performed with a Gaussian distribution of the initial state with the following properties:

$$\begin{pmatrix} \sigma_R \\ \sigma_V \\ \sigma_\gamma \end{pmatrix} = \begin{pmatrix} 40.0 \text{ m} \\ 50.0 \text{ m/s} \\ \sqrt{10^{-3}} \text{ rad} \end{pmatrix} \quad (39)$$

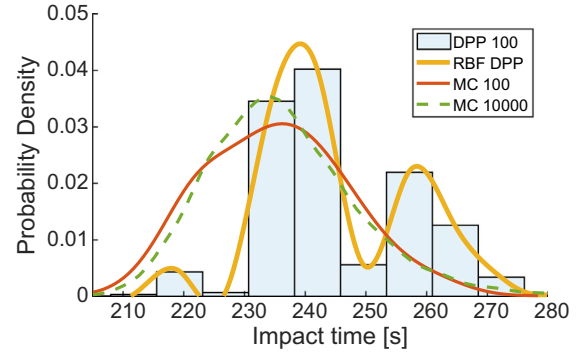
The initial state was uniformly sampled from a region bounded by 4σ . The results are shown in Fig. 7, which is a result that is similar to the results obtained in Halder and Bhattacharya (2011). The figure shows a scatter-plot of the PDF values after 30 s flight time. From the plot it can be observed that the values of the PDF vary largely and the location of the datapoints are highly correlated, which makes it difficult to process (Section 3.4.1).

The binning method described in Section 3.4.2 was used to obtain marginal distributions of the altitude, which are shown in Fig. 8. The barplot in the figure is a direct result of this method and an RBF interpolation is added to compare the shape with the reference solution. The reference

Fig. 6. The evolution of Ω and $\dot{\Omega}$ as a function of altitude (SM-A).Fig. 7. 2000 datapoints of the PDF of the state (SM-A) at $t = 30$ s generated using the DPP method. The color denotes the $\log_{10}$ of the probability density f . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

PDF is obtained using kernel density estimation using 10^4 datapoints obtained from an MC simulation.

The figure shows that the binning method approximates the shape of the PDF and that the accuracy is dependent on the number of bins. However, no convergence to the MC solution is observed. In Fig. 9 a comparison is shown of impact-time distributions that are obtained using the DPP method with 100 samples and MC with the same number of samples. The figure shows that the binning method produces inaccurate results, while the MC solution is fairly accurate. This shows that the assumptions associated with the binning method do not hold or the number of datapoints is insufficient to obtain accurate results. Considering that the accuracy of the PDF value in each bin is of the order $N^{-1/2}$ (Eq. (28)), the number of datapoints in each bin is insufficient. This shows that MC provides a

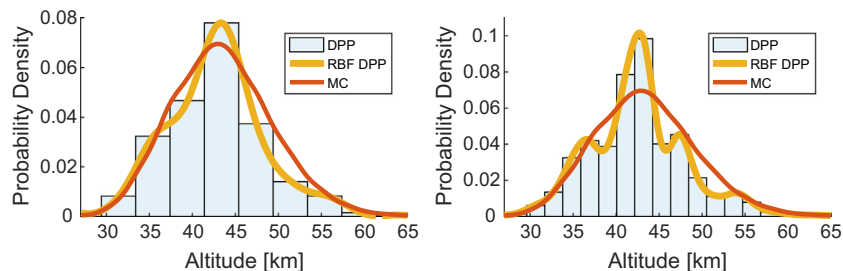
Fig. 9. Impact time distributions obtained from 100 DPP samples and 10^4 MC samples (SM-A).

more accurate result than the DPP method combined with the binning method.

5.2. Case B: Full uncertainty analysis

Several simulations were performed with SM-B of the Delta-K rocket body as described in Section 3.2.2. The evolution of Ω for a full simulation with initial conditions obtained from one of the TLEs is shown in Fig. 10.

The figure shows that Ω rapidly decreases in the lower altitude regions, which is the same behavior observed in case A (Section 5.1). This indicates that the PDF converges and the uncertainty gets smaller. Because uncertainty in the atmospheric density is added, the velocity of the Delta-K rocket body will not converge to the terminal velocity. Therefore, it is not really intuitive that the uncertainty decreases. This can be explained by the correlation between the velocity (and other states) and atmospheric density,

Fig. 8. Marginal distributions of the altitude at $t = 30$ for 10 and 20 bins obtained with 2000 DPP samples and 10^4 MC samples (SM-A).

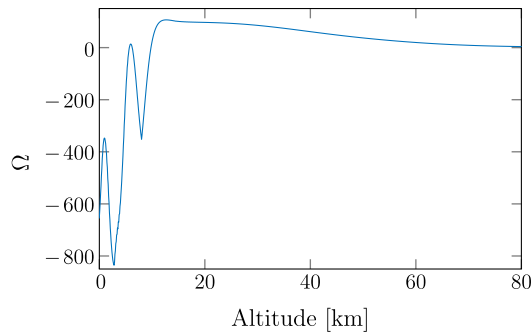


Fig. 10. Evolution of Ω as a function of altitude and flight-path angle (SM-B).

which largely increases during the final part of the trajectory, resulting in a decrease in uncertainty.

The behavior of Ω causes problems when propagating the PDF of the state to lower altitude regions, because inserting a large negative value of Ω in Eq. (6) can result in numbers that are too large to process. Fig. 10 also shows discontinuities in the value of Ω . These originate from singularities in the model of Ω that are a consequence of the choice of state variables. The use of the velocity components in the local-vertical-local-horizontal frame causes problems, because the heading angle becomes undefined when approaching vertical flight.

For a statistical analysis on a decaying trajectory, it is sufficient to derive a (decay) time distribution at a higher altitude for which the time to impact is small. Therefore, the behavior of Ω at lower altitudes does not affect the solution. In Fig. 11 a time distribution is shown for 70 km altitude. An outlier-removal method is applied to the DPP data before constructing the distribution, because the dispersion in Ω was too large to obtain accurate results. The outlier removal method removed any samples that were located outside the 5σ range.

A correction was made to the binning method such that the necessity of the assumption of an equal enclosed volume in each bin is removed. The mean value in each bin is multiplied with a measure of the enclosed volume of the data in each bin. This measure is obtained by multiply-

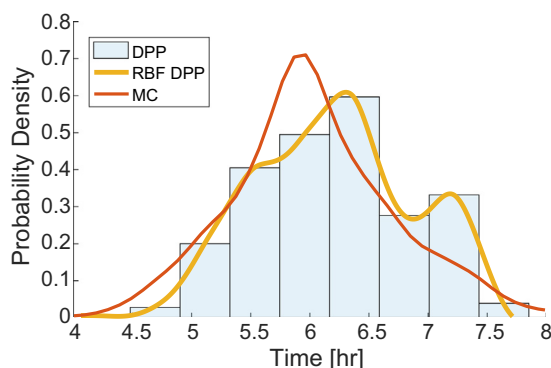


Fig. 11. Marginal PDFs of the time at an altitude of 70 km obtained using the binning method with 2298 MC data samples (SM-B).

ing the standard deviations of the principle components of the data. Although this correction highly improved the results, the accuracy of this method is insufficient as shown in Fig. 11. This shows that the DPP method fails to produce accurate results due to the lack of an accurate method to post-process the data. Because post-processing the multi-dimensional data of the DPP method becomes increasingly challenging with increasing dimension, the method suffers from a curse of dimensionality.

5.3. Case C: Different state variables

An additional test is performed with a different set of state variables to verify that the rapid decrease of Ω is not caused by a wrong choice in state variables and to show that a solution can be obtained without singularity problems. A DPP simulation is performed with state model C (Section 3.2.3), which only differs in its translational-state model.

In Fig. 12, the evolution of Ω is shown versus altitude. The figure shows that Ω decreases during the final part of the trajectory and that there are no singularities that cause problems. This verifies that the conclusions drawn from the previous case are correct and that the numerical problems associated with a large negative Ω are inherent to this type of entry problems.

As indicated earlier, it is expected that Ω decreases rapidly, because the PDF of the state converges rapidly in the final part of the trajectory. This is an inherent property of the uncontrolled atmospheric entry problem, because simulating a trajectory is fully deterministic and different trajectories converge to a similar end state. Including uncertainty in the atmospheric density only adds a state variable, but does not add noise to the system of equations. Therefore, the multi-dimensional PDF will converge and Ω will become largely negative.

6. Statistical impact predictions

The two uncertainty models described in Section 4 are used to perform statistical impact predictions of the Delta-K rocket body. The MC Method is used to predict the PDFs of the impact time for 21 different entry events.

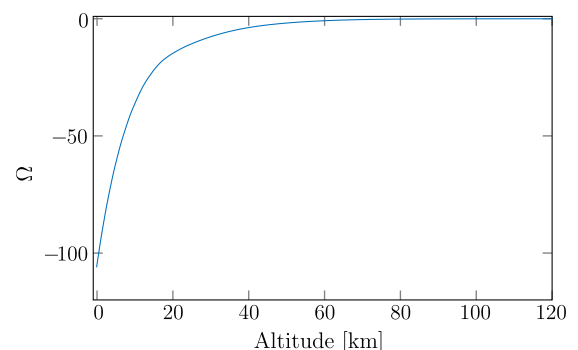


Fig. 12. Omega as a function of the altitude (SM-C).

For each PDF, 2000 Sobol samples are generated and used to obtain samples of the initial state and atmospheric density factor with distributions defined by the uncertainty model. Kernel density estimation is used to obtain the impact time PDFs, which are validated using data of the true impact points.

A selection of predicted impact-time distributions is shown in Fig. 13. The PDF of the impact time is shown, its median and a corresponding 99.73% probability window associated with a 99.73% confidence level. A bootstrapping method is applied that uses random sampling to derive the probability window with a certain confidence level (Montgomery and Runger, 2014). For comparison the figure includes the TIP window and epoch. The oscillations in the PDFs are related to the orbital period of the rocket body.

Data of the true impact time are used to validate the predicted impact time PDFs using the following property (Thomopoulos, 2013):

$$U = F_x(X) \quad (40)$$

where X is the random impact time, F_x is the Cumulative Distribution Function (CDF) of the impact time and U is a random variable uniformly distributed on $[0, 1]$. This means that the outcome should be uniformly distributed if the true impact time is inserted in an accurate prediction of the CDF of the impact time.

6.1. Uncertainty Model A

A total of 119 valid TLEs (cases) could be used to generate impact-time distributions, which were compared with the true impact time.

In Fig. 14, the number of cases is shown where the true impact time falls within a certain probability band centered around the median. In an ideal case, the probability band contains a percentage of the true impact points that is equal to that probability band. The figure shows that the number of cases is less than expected in each probability band. This indicates that the prediction of the probability band is too small, so the uncertainty is larger than predicted.

For each case, the true impact time is inserted in the predicted CDF of the impact time. According to the property described by Eq. (40), the distribution of these cumulative probabilities should be uniform if the predicted CDF is accurate. A histogram of these cumulative probabilities is shown in Fig. 15. The figure shows that the distribution of these values is not uniform, which indicates that the predicted impact time PDFs are inaccurate. The mean cumulative probability of the impact points is equal to 0.73, which means that the derived distributions largely underpredict the impact time.

An analysis performed by Picone et al. (2002) showed that the NRLMSISE-00 model overestimates the atmospheric

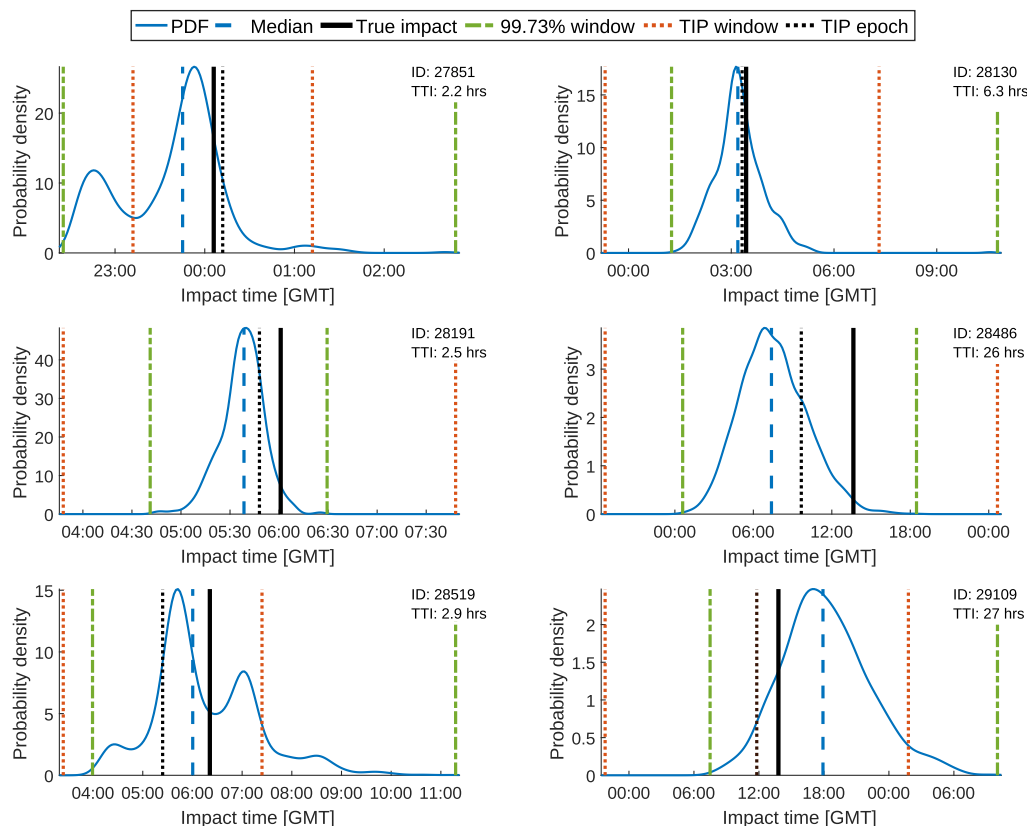


Fig. 13. Selection of impact-time distributions derived using UM-A (TTI: Time To Impact, ID: Norad ID).

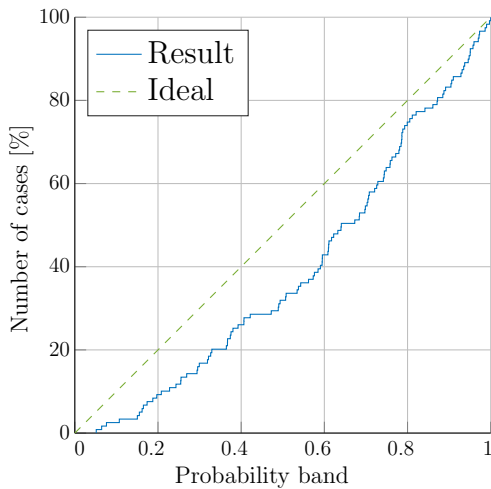


Fig. 14. Number of simulations versus the probability band of the true impact point centered around median for 119 Monte Carlo simulations (UM-A).

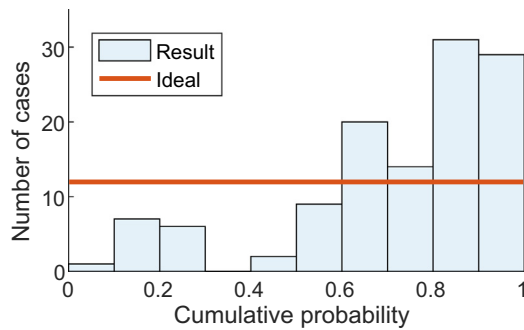


Fig. 15. Histogram of the cumulative probability that is obtained from inserting the true impact point in the generated CDF (UM-A).

density for lower altitude regions, which can explain the observed bias in the impact time. Therefore, additional impact predictions were performed using uncertainty model B, which is based on the standard deviation and mean obtained by Picone et al. (2002).

6.2. Uncertainty Model B

For Uncertainty Model B, a total of 31 TLEs (cases) impact-time distributions were generated and compared to the true impact time.

Because less impact predictions are performed, bootstrapping was performed to obtain a statistical bandwidth for these figures. A bound (99.73% confidence level) of the cumulative probability at the true impact time is obtained and the result for the lower and upperbound are shown in Fig. 16. The figure shows that the number of cases in each probability band is higher than ideal, which means that impact windows generated from the impact-time distributions are overestimated.

The overestimation is a direct consequence of the change in the uncertainty model of the atmospheric density

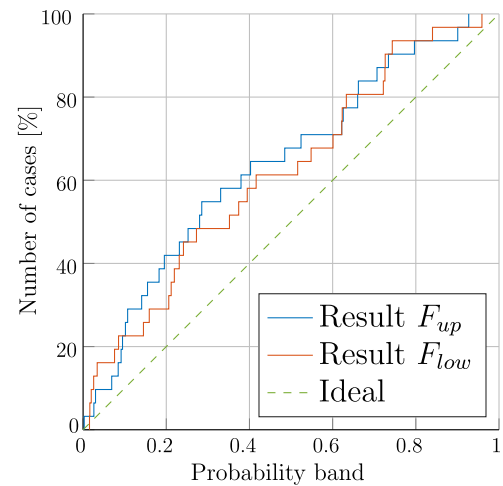


Fig. 16. Number of simulations versus the probability band of the true impact point centered around median for 31 Monte Carlo simulations (UM-B).

and shows the large influence of the atmospheric density in the predictions. This result is expected because the standard deviation of UM-B is 0.17 compared to 0.12 for UM-A.

A PDF of the cumulative probability of the impact points is shown in Fig. 17. The figure shows that the bias in the predictions is almost removed compared to Fig. 15. This is confirmed by the mean cumulative probability of the true impact time, which is bounded by [0.5, 0.56] with 99.73% confidence.

From Fig. 17 it cannot be concluded that the shape of the distribution is correct, because the distribution of the cumulative probabilities is not uniform. It confirms however that the higher probability bands are overestimated, which is preferred over underestimation.

Important to note is that the improved performance of the simulation results for UM-B does not mean that UM-A is less accurate. The better performance could also be attributed to the fact that uncertainty is present in other parameters that is now accounted for by making the uncertainty in the atmospheric density larger. Because the atmospheric density only influences the aerodynamic forces and it has a large effect on the impact time dispersion, the uncertainty in the aerodynamic model could be a source of large influence.

6.3. Comparison

The statistical predictions can be compared to TIPs for 31 cases (TLEs) for which the TIPs were published within 10% of the time to impact to the TLE. The window size of the distributions obtained with UM-A are in 21 of the 31 cases smaller than the TIP windows. On average the obtained windows are 7% smaller than the TIP windows. For the results with UM-B only 6 of the 31 impact windows are smaller than the TIP window and the impact windows are on average 29% larger than the TIP window.

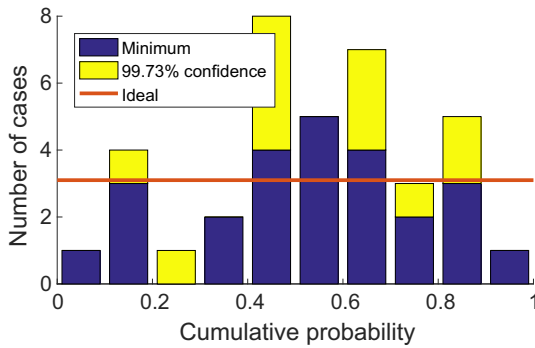


Fig. 17. Histogram of the cumulative probability that is obtained from inserting the true impact point in the generated CDF (UM-B).

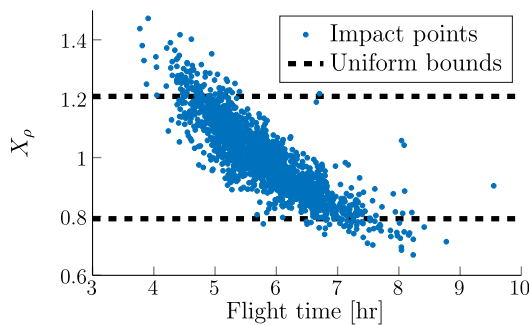


Fig. 18. Scatter plot ($N = 1940$) of the atmospheric density factor X_ρ versus the flight time until impact of the Delta-K rocket body for UM-A and the bounds of a uniform distribution with $\sigma = 0.12$.

No consistent conclusions can be drawn from these results, because the probability and confidence level associated with the TIP windows are unknown.

Assuming that the TIP window corresponds to a uniform distribution with the TIP window width corresponding to a 3σ value of the distribution, a mean cumulative probability of 0.57 is calculated. Assuming a Gaussian distribution, the mean cumulative probability is equal to 0.62, which means that the TIPs also generally underestimate the impact time.

It was observed that for one of the 31 cases, the true impact time lies outside the TIP window. Because the probability distribution associated with the TIP window is not known and the derived distributions are not ideal, it cannot be concluded that the predictions from the MC method are more accurate. However, it can be concluded that this method provides significantly more information on the probability distribution of the impact time.

In the research of Ronse and Mooij (2014), a plot was generated to validate the obtained results that is similar to Figs. 14 and 16. In this plot it is shown that the lower probability bands are overestimated and the higher probability bands are underestimated. This means that the higher probability bands that are derived are too small, thus unconservative. This is a consequence of using a uniform distribution for the atmospheric density, because a uniform distribution is more flat and less wide than a lognormal distribution.

The effect of using a uniform distribution for the atmospheric density instead of a lognormal distribution is shown in Fig. 18. The figure shows samples of the flight time to impact versus the atmospheric density factor, and the bounds of a uniform distribution with the same standard deviation. The figure shows that these parameters are highly correlated ($\rho = -0.85$), which confirms the sensitivity of the flight time to atmospheric density uncertainty. Considering the correlation between these parameters, the figure shows that using a uniform distribution instead of a lognormal distribution results in underestimated impact time windows.

7. Conclusions

The DPP method is investigated for the use in a 13-dimensional uncontrolled entry problem and higher dimensional problems in general. It was found that the numerical integral that needs to be solved for each trajectory, $\Omega = \int \Psi$ in Eq. (4), should be converted to an ODE, which can be solved using the numerical integrator. Hereby, the numerical integrator can be used to control the accuracy. It was further observed that the total uncertainty decreases during the trajectory due to a converging end state or increasing correlations. This can cause numerical problems for applications that require the method to propagate to lower altitudes, due to a large negative value of Ω .

The DPP method results in a scattered dataset of the PDF of the state, which is challenging to post-process. No accurate curve fit could be obtained using this data due to the structure of the data and high dimensionality of the data. The binning method, which was used in previous research, was employed to derive time distributions. The results showed that the binning method leads to inaccuracies that are too large to obtain accurate time distributions. Post-processing the DPP data becomes increasingly challenging with increasing dimension, which indicates that the DPP method suffers from a curse of dimensionality. It is observed that the MC method is capable of producing accurate results for higher dimensions. Therefore, the conventional MC method is still recommended over the DPP method.

An improved uncertainty model is developed and validated by comparing impact predictions with true impact data. From the MC simulations, it was found that the shape of the PDF for the uncertainty in the atmospheric density is of large influence for the impact time and that a bias in the atmospheric density cannot be neglected. Using a uniform distribution for the atmospheric density leads to unconservative predictions of impact time windows with high probability. A lognormal distribution for the atmospheric density provides more accurate results. The improvements to the uncertainty model resulted in more conservative, thus statically more valid, results. Considering the evacuation of an area, it is important that conservative estimates of probability bands are used to minimize the risk for casualty.

References

- Allasia, G., Cavoretto, R., De Rossi, A., 2014. Multidimensional Lobachevsky spline integration on scattered data. *Appl. Math. Inf. Sci.* 8 (1), 145–151.
- Attar, P.J., Vedula, P., 2008. Direct quadrature method of moments solution of the FokkerPlanck equation. *J. Sound Vib.* 317, 265–272.
- Bezdek, A., 2007. Lognormal distribution of the observed and modelled neutral thermospheric densities. *Stud. Geophys. Geodaet.* 51 (3), 461–468.
- Bowman, A.W., Azzalini, A., 1997. *Applied Smoothing Techniques for Data Analysis*. Oxford University Press, Inc., New York.
- Dekking, F.M., Kraaikamp, C., Lopuhaa, H.P., Meester, L.E., 2005. *A Modern Introduction to Probability and Statistics*. Springer.
- Gardiner, C.W., 2004. *Handbook of Stochastic Methods*. Springer.
- Geul, J., Mooij, E., Noomen, R., 2017. TLE uncertainty estimation using robust weighted differencing. *Adv. Space Res.* 59 (10), 2522–2535. <https://doi.org/10.1016/j.asr.2017.02.038>.
- Halder, A., Bhattacharya, R., 2011. Dispersion analysis in hypersonic flight during planetary entry using stochastic Liouville equation. *J. Guid., Control Dynam.* 34 (2), 459–474.
- Kahr, E., Montenbruck, O., O’Keefe, K.P.G., 2013. Estimation and analysis of two-line elements for small satellites. *J. Spacecr. Rock.* 50 (2), 433–439.
- Kirk, D. (Ed.), 1992. *Graphics Gems III - A Collection of Practical Techniques for the Computer Graphics Programmer*. AP Professional.
- Klinkrad, H., 2006. *Space Debris Models and Risk Analysis*. Springer.
- Lazzaro, D., Montefusco, L.B., 2002. Radial basis functions for the multivariate interpolation of large scattered data sets. *J. Comput. Appl. Math.* 140 (1–2), 521–536.
- Montgomery, D.C., Runger, G.C., 2014. *Applied Statistics and Probability for engineers*, sixth ed. Wiley.
- Mooij, E., 1994. *The Motion of a Vehicle in a Planetary Atmosphere*. Delft University Press.
- Oswiler, V.P., 2006. *Covariance Estimation and Autocorrelation of Norad Two-line Element Sets* M.Phil. thesis. Air Force Institute of Technology (March).
- Pantano, C., Shotorban, B., 2007. Least-squares dynamic approximation method for evolution of uncertainty in initial conditions of dynamical systems. *Phys. Rev. E* 76 (6), 066705.
- Picone, J.M., Hedin, A.E., Drob, D.P., 2002. NRLMSISE-00 empirical model of the atmosphere: statistical comparisons and scientific issues. *J. Geophys. Res.* 107 (A12), SIA 15-1–SIA 15-16.
- Portelli, C., Salotti, L., Anselmo, L., Lips, T., Tramutola, A., 2004. BeppoSAX equatorial uncontrolled re-entry. *Adv. Space Res.* 34 (5), 1029–1037.
- Prabhakar, A., Fisher, J., Bhattacharya, R., 2010. Polynomial chaos-based analysis of probabilistic uncertainty in hypersonic flight dynamics. *J. Guid., Control Dynam.* 33 (1), 222–234.
- Prag, A.B., 1983. *A Comparison of the MSIS and Jacchia-70 Models with Measured Atmospheric Density Data in the 120–200 km Altitude Range*, Technical Report A500821. The Aerospace Corporation El Segundo, CA 90245.
- Rochelle, W.C., Marichalar, J.J., Johnson, N.L., 2004. Analysis of reentry survivability of UARS spacecraft. *Adv. Space Res.* 34 (5), 1049–1054.
- Ronse, A.L.A.B., Mooij, E., 2014. Statistical impact prediction of decaying objects. *J. Spacecr. Rock.* 51 (6), 1797–1810.
- Terejanu, G., Singla, P., Singh, T., Scott, P.D., 2008. Uncertainty propagation for nonlinear dynamic systems using gaussian mixture models. *J. Guid., Control Dynam.* 31 (6), 1623–1633.
- Thomopoulos, N.T., 2013. *Essentials of Monte Carlo Simulation*. Springer.
- Vallado, D., Crawford, P., Hujsak, R., Kelso, T.S., 2006. *Revisiting Spacetrack Report # 3*. AIAA/AAS Astrodynamics Specialist Conference and Exhibit. Keystone, Colorado.
- WGS, 2000. *Department of Defense World Geodetic System 1984*, Tech. rept. Department of Defense.
- Xu, Y., 2009. Nonlinear robust stochastic control for unmanned aerial vehicles. *J. Guid., Control Dynam.* 32 (4), 1308–1319.
- Xu, Y., 2011. Nonlinear stochastic control for space launch vehicles. *IEEE Trans. Aerosp. Electron. Syst.* 47 (1), 98–108.
- Yim, H., Chung, D., 2012. Validation on residual variation and covariance matrix of USSTRATCOM two line element. *J. Astron. Space Sci.* 29 (3), 287–293.

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Paper S05

MODELLING AND ASSESSMENT OF THE CURRENT AND FUTURE SPACE SURVEILLANCE NETWORK

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ABSTRACT

Two-Line Elements (TLEs) and Tracking and Impact Predictions (TIPs) resulting from the Space Surveillance Network (SSN) are important for many Space Situational Awareness (SSA) activities. The network consists of many different radar and optical stations, contributing in either a dedicated, collateral or contributing fashion. For the majority of objects in low-Earth orbit (LEO) phased-array radar (PAR) stations are essential for maintaining the satellite catalog (SATCAT). A survey of the current state of the SSN and a methodology for simulating the network is presented. The current state is compared to a future scenario with the new Space Fence System (SFS) included. The SSN is simulated as a collection of current and hypothetical dedicated and collateral PARs. The location and coverage of each sensor is investigated and modelled. Observations are generated for a test rocket body in LEO. TLEs are estimated from the simulated measurements using the Simplified General Perturbations (SGP4) model. The sensitivity of the fit residuals and propagated accuracy with respect to eccentricity, inclination, and number of observations and sensors is analysed. The orbit-determination solution is found to be most sensitive to the eccentricity, and number of observations and sensors involved. The new SFS improves the solution, especially for lower inclinations.

Key words: Space Debris; Space Surveillance; Phased-Array Radar; Satellite Catalog (SATCAT); Orbit Determination; Two-Line Elements (TLEs).

1. INTRODUCTION

The Space Surveillance Network (SSN) is responsible for tracking over 18 000 objects in the satellite catalog (SATCAT)¹. Its measurements are used for generating Two-Line Elements (TLEs) and Tracking and Impact Predictions (TIPs). These products are often the only available source for many Space Situational Awareness (SSA) ac-

tivities, such as conjunction analyses and re-entry predictions. The SSN is the largest network of its kind and consists of optical and radar sensors. The sensors contribute in various degrees, and are classified as either dedicated, collateral or contributing. Most sensors are located on the north-western hemisphere. The new Space Fence System (SFS), with two sites in the Pacific Ocean and Australia, promises improved network geometry and detection capabilities.

To properly assess the current and future configurations of the SSN, the technical capabilities, coverage, roles, and location of individual sensors must be taken into account. Previous surveys of the SSN are a good starting point, but are either lack completeness in these aspects or are outdated [1, 2, 3, 4, 5].

This paper presents a survey of the most important sensors, focusing on sensor type, role, coverage, and capabilities. The current network geometry is simulated. A methodology for generating and editing observations will be presented.

For this study the tracking of rocket bodies in low-Earth orbit (LEO) is considered. The focus is on phased-array radars (PARs), as they are primarily used for this purpose. TLEs are estimated from these simulated observations for a test rocket body. The sensitivity of the solutions to several orbital and orbit determination (OD) parameters is investigated. Finally, through the addition of two new sensors, the future network geometry is assessed.

The paper is outlined as follows. First, the SSN is described in Sec. 2. Section 3 presents an overview of the simulation set-up. The results are presented and discussed in Sec. 4. Finally, conclusions and recommendations are given in Sec. 5.

2. SPACE SURVEILLANCE NETWORK

The United States Air Force Space Command Space Surveillance Network (US AFSPC SSN) is the largest governmental network. The network was initially developed as a missile-warning network [4]. The network consists of plain radar, optical, and PAR sensors, and can be

¹<https://www.celestrak.com/satcat/boxscore.asp>, retrieved: April 3, 2017

Table 1. Overview of PARs sensors in the SSN. The sensor role (R) is either dedicated (D) or collateral (C). The azimuth and elevation of the boresight of the radar face is indicated in degrees.

Name / location	Model	R	Lat [°]	Lon [°]	Alt [m]	Boresight [°]
Eglin, FL, US	AN/FPS-85	D	30.572	−86.215	36	180x45
Clear ^a , AK, US	AN/FPS-132	C	64.300	−149.191	175	6,244x20
Fylingdales, UK	AN/FPS-132	C	54.362	−0.670	258	5,125,245x20
Thule, GL	AN/FPS-132	C	76.569	−68.299	424	117,357x20
Beale, CA, US	AN/FPS-132	C	39.136	−121.351	117	186,246x20
Cape Cod ^a , MA, US	AN/FPS-132	C	41.752	−70.538	67	47,167x20
Cobra Dane, Shemya, AK, US	AN/FPS-108	C	52.737	174.092	68	319x20
PARCS, Cavalier ND, US	AN/FPQ-16	C	48.725	−97.900	344	8x25
SFS1 ^b , Kwajalein Atol, US	AN/FSY-3	D	8.723	167.719	7	0x90
SFS2 ^b , Exmouth, AU	AN/FSY-3	D	−21.816	114.165	11	0x90

^a The upgrades of the Clear and Cape Cod started in 2012 and 2013 and are to be fully certified in 2017 and 2018, respectively [6].

^b SFS1 will become operation in 2018, while SFS2 is expected to become operational in 2021 [7].

grouped into dedicated, collateral, and contributing sensors. Dedicated sensors have space surveillance as a primary mission. Collateral sensors are connected directly to the Space Surveillance Center (SSC). These sensors still have primary (or even secondary) missions other than space surveillance. In the case of PARs, usually a dedicated sub-array of elements (also fence) is operated for the detection and tracking of objects. This fence covers only a portion of the total coverage and duty cycle, in parallel to the primary mission. Finally, contributing sensors provide space surveillance only upon request from the Joint Space Operations Center (JSpOC). The following examples give a rough idea of how much these sensors contribute [3]: the Millstone radar contributes 80 hours per week and the Haystack radar about one week every six. All four mechanical radars at Kwajalein contributed a total of 138 hours in 2014. Some sensors are only used exclusively for certain purposes, such as to track launches.

The two radar types currently used in the SSN are mechanical and PARs. Previously, the SSN also included a multi-static radar system (more details below). PARs consist of flat faces containing several thousands of antenna elements. Compared to mechanical radars, PARs use electronic steering by shifting the phase of incoming signals. Due to this, PARs can track numerous targets simultaneously and are suitable for detection [3]. PARs have a limited coverage (if less than three faces are used), narrowing beamwidth away from the face normal, lower operating frequencies, and higher cost.

In the past Several authors have consolidated space-surveillance sensors [1, 3, 4, 5]. Although they provide valuable information, none prove to be a complete and sufficient source for simulating the SSN. The SSN is rapidly changing, so even information a few years old can already be outdated. For example, the Air Force Space Surveillance System (AFSSS) and the Moron Optical

Space Surveillance (MOSS) telescope were both closed in 2013, the Antigua radar was relocated to Australia in 2015, and the Ascension radar has become a dedicated sensor. Moreover, a categorisation into dedicated, collateral, and contributing sensors is important. Sensor limitations, such as coverage and range, have a large impact on the simulated observations. Lastly, instrument accuracy, observation rate, tasking, and scheduling should ideally all be taken into account.

The AFSSS formed a major asset for the radar surveillance of LEO satellites. Also known as the space fence, the AFSSS consisted of three continuous transmitter and six receiver stations at 33° latitude, spanning approximately 70° in longitude across the US. The fence was closed in August 2013. Construction of the AN/FSY-3 Space Fence System (SFS) began in 2014. SFS will consist of two sites and use S-band radar (2 GHz to 4 GHz). Traditional PARs operate at around 400 MHz, with the exception of Cobra Dane which has a nominal frequency of 1275 MHz [8]. The first and largest site is located on Kwajalein Atoll in the Pacific Ocean and is expected to be operational in 2018. The second and smaller site is located in Western Australia and is expected to be operational in 2021 [7]. The new fence will have a higher accuracy and detection capabilities than current sensors, potentially increasing the number of tracked objects to more than 100 000 [9].

An updated inventory of all SSN sensors is made, including all mechanical radars and optical stations. The closing, relocation, and upgrades of sensors are taken into account. The current role of each sensor is investigated, as well as a preliminary investigation into sensor accuracy. Tab. 1 shows a selection of this survey for PARs.

The table reports the azimuth and elevation of the normal of the radar faces (i.e., boresight). The horizontal field of view (FOV) is generally reported to be 120° (thus

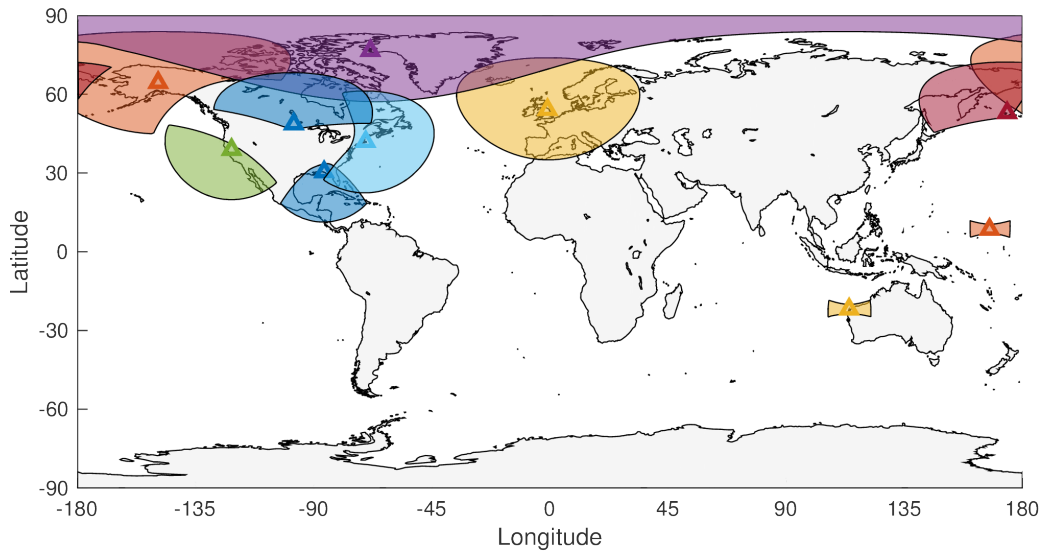


Figure 1. Space Surveillance network coverage for an object in a circular 600 km orbit.

a full 360° for three faces). For faces with a tilt of 20° , the resulting maximum elevation differs amongst sources, most report either 80° or 85° of maximum elevation. The latter value suggests a vertical FOV of 130° . However, it is assumed that the FOV of all PAR systems is 120° in horizontal and vertical direction per face, as they are roughly square. An exception is SFS, which receiving antenna array is more rectangular and spread in longitudinal direction, for this sensor a FOV of 40° is assumed for the latitudinal direction. The location of SFS1 was estimated from satellite images of the construction site. The location of SFS2 is unknown, but is assumed to be at the Naval Communication Station Harold E. Holt in Exmouth, Western Australia, although this is not confirmed.

The contract for the construction of a new AN/FPS-132 early-warning radar in Qatar was awarded to Raytheon and is expected to be completed in 2021 [10]. Its exact location, number of faces, and possible role in the SSN is not known.

3. SIMULATION SET-UP

The SSN is simulated for LEO. First, the configuration of the network is explained. Second, the test object is introduced, including a sampling strategy to randomise the orbits. Third, the set up of the orbit propagator is given. Fourth, the process of simulating observations is detailed. Finally, the orbit determination is explained.

3.1. Network Configuration

The coverage is dependent on the orbit of an object. Due to the FOV, the coverage grows with the range, resulting in longer observation arcs. A coverage map is generated

for a test object in a 600 km orbit. The azimuth, elevation, and range is calculated of the test object with respect to each station using the WGS84 reference ellipsoid² and coordinates from Tab. 1. The observation vectors are then transformed to the local observation plane, as determined by the boresight of each face. Only observations within the FOV are kept.

Fig. 1 shows the location and coverage of the dedicated and collateral PARs for a test object in a circular orbit of 600 km. The elevation and azimuth limitations of each sensor have been taken into account. Single-, double-, and triple-face PARs are easily distinguished. The primary mission of many of these sensors is providing ballistic missile warnings, which is obvious from the outward facing coverage of the stations along the US border. The footprint of the two SFS sensors seems small in comparison to the other sensors. This is due to projecting the (mostly) vertical coverage on the horizontal plane, reduced latitudinal coverage, and choice of map projection (i.e., a tendency to exaggerate sizes at higher latitudes).

The figure also shows how most PARs are located in North America and northern Europe. In this configuration, the southern and eastern hemisphere are underrepresented. The two SFS sensors will improve geographic coverage.

3.2. Test Object

The Delta-K rocket body is selected as the test object. The object has frequently re-entered and fragments have been previously recovered due to its high-temperature resistant components. The Delta-K was also subject for the 6 degree-of-freedom (6DOF) re-entry simulator in [11].

²<http://earth-info.nga.mil/GandG/wgs84/index.html>, accessed: April, 7, 2017

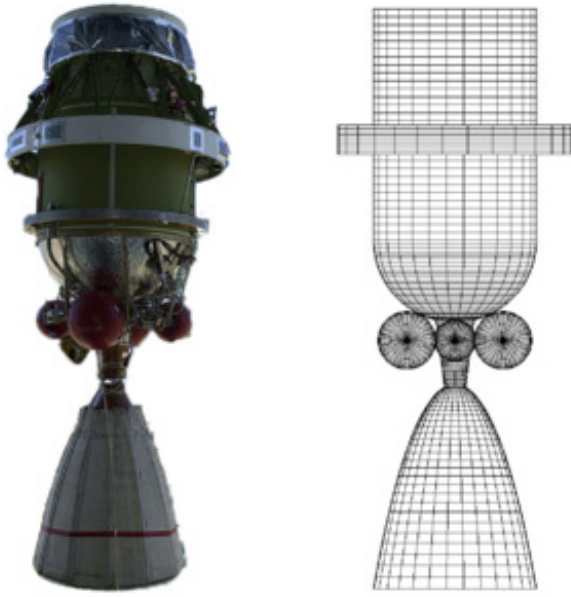


Figure 2. True and modelled Delta-K rocket body [11].

As the aerodynamic model and databases for the Delta-K were already derived, a 6DOF propagation of the test object is performed. Fig. 2 shows the true and modelled Delta-K rocket body.

A nominal state has been derived from 33 re-entered Delta-Ks in the period of 2004 to 2017. For each object the orbital state 30 days prior to re-entry is taken. The variations in semi-major axis, eccentricity, and inclination are shown in Fig. 3. A strong correlation between semi-major axis and eccentricity can be observed, related to the circularisation of the orbit due to the aerodynamic drag. Moreover, the objects have either a low or high inclination, depending on the target orbits in which these rockets were launched (i.e., GTO or LEO, respectively). The groups are indicated by the different markers, where the legend indicates the mean and standard deviation of the inclinations found in each group.

Random initial states are sampled from each group with an equal ratio. The eccentricity is sampled uniformly within the observed bounds. The semi-major axis is obtained through a linear correlation with the eccentricity. Finally, the inclination is sampled from a normal distribution. The right-ascension of the ascending node and argument of perigee are assumed to be uniformly distributed between 0 and 360°, and the initial true anomaly is zero. The initial aerodynamic angles are assumed to be zero, while the rotational rate is taken as one revolution per minute about the pitch axis. Finally, variations in the atmospheric density are modelled using a log-normal distribution with parameters $\mu^* = 1.0$ and $\sigma^* = 0.12$ [11].

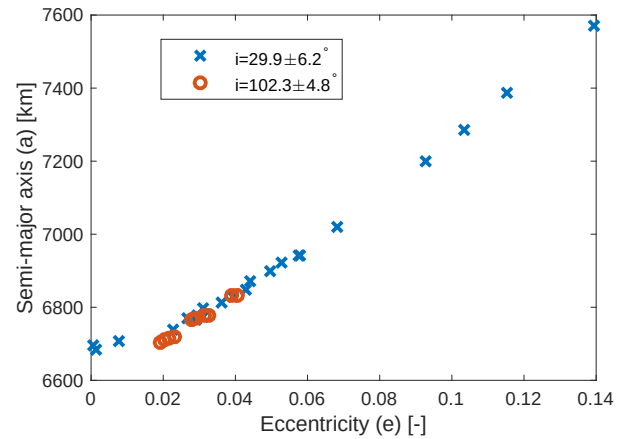


Figure 3. Semi-major axis versus eccentricity grouped by inclination of 33 Delta-K rocket bodies.

3.3. Orbit propagation

Observations are generated from numerically propagated orbits for the varying initial conditions. As mentioned previously, both the translational state and rotational state of the object is propagated. The equations of motion are integrated using a Runge-Kutta-Fehlberg 4(5) variable step-size integrator. The tolerance is set to $\epsilon_{rel} = \epsilon_{abs} = 1 \times 10^{-8}$. The step-size is allowed to vary between a minimum and maximum value of 1×10^{-4} and 1×10^3 s, respectively. The EGM2008 gravity field model is used³. The atmospheric density is modelled using the NRLMSISE-00 model [12]. These settings provided a good trade-off between accuracy and computational efficiency.

3.4. Observations

Realistically, observations are limited by a number of technical and physical constraints, and are subject to numerous error sources. Not properly taking these into account would lead to too many and too idealistic observations. Accurate modelling of individual sensor capabilities, and scheduling is difficult based on the available information, and therefore considered beyond the scope of the current work. The main focus is on modelling the network geometry and coverage.

The observation conditions are taken into account for each sensor, as previously explained in Sec. 3.1. An elevation cut-off of 5° is used for all sensors. A measurement rate of 0.1 Hz has been chosen. The measurements are further edited to account for sensor availability, viewing conditions, and other limitations. The process uses an observation probability factor to discard measurements. An individual observation has a 20% chance of being re-

³http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html, retrieved: April 7, 2017.

jected, while the probability for discarding an entire pass is 25% for collateral sensors only.

Measurement noise is simulated to account for sensor capabilities and atmospheric disturbances. The noise is assumed to be unbiased Gaussian noise, with a standard deviation of 1 km in radial direction and 0.1° (1 km at $r = 600$ km) in azimuth and elevation.

3.5. Orbit Determination

The observations are used to fit TLEs using the Simplified General Perturbations (SGP4) model [13]. Range, azimuth and elevation measurements are generated for each MC sample for three consecutive days. The fit span consists of the first two days of observations. These are in the form of range, azimuth, and elevation, which are edited and used to for orbit fitting. The test span consists of the third and final day and provides unedited observations in inertial Cartesian coordinates, which are used for studying the propagation accuracy of the solution.

4. RESULTS

The orbit determination is performed for 1000 randomly initialised orbits. The mean, median, and standard deviation of the fit and test span are collected for each sample.

On average, the object is observed by 4.8 stations over two days, resulting in 1560 observations. The mean fit residual is 1.7 km, while the residuals for test period are on average 5.8 km. The large error growth in the test period is due to the low orbit and resulting atmospheric drag, and in line with actual results for one-day predictions of TLEs [14].

4.1. Sensitivity

The rank correlation between the median residual and a number of parameters is computed to obtain the sensitivity of the solution. Fig. 4 shows the result for the fit and test span for 5 parameters: eccentricity, inclination, density, number of stations, and number of observations, respectively. The results between the fit and test span are very similar. The correlations are slightly larger for the test span, which are enhanced through propagation and the resulting error growth.

The residuals are negatively correlated with the eccentricity and number of observations. Near-circular orbits are more difficult to estimate due to sensitivity of the argument of perigee near its singularity. The reduction of the residuals with the number of observations is obvious. However, there is a strong positive correlation with the number of stations. It is found that the spread of solutions increases significantly with multiple stations. Some

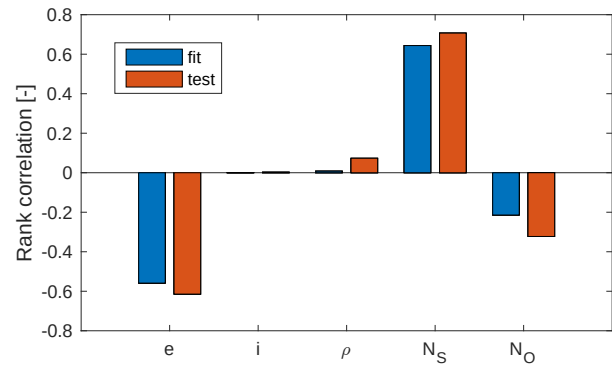


Figure 4. Sensitivity of orbit fit and propagation to the eccentricity e , inclination i , density ρ , number of stations N_s , and number of observations N_o .

solutions are improved, while other are worsened. Improved performance can be achieved by only including observations from sensors, if the fit is improved.

Inside each inclination group the variation has little effect the solution quality. However, there is a small difference between the two groups, which will be discussed next.

4.2. Space Fence

Fig. 5 shows the residuals of the test span for the two inclination groups (L and H, indicating the lower and higher inclinations, respectively) and two observation scenarios (C and F indicating the current and future scenarios, respectively).

It can be seen that the lower-inclined group has a slightly higher accuracy, but larger spread. The difference is attributed to the larger spread in eccentricity for the lower-inclined group. The additional sensors have a positive effect on the orbit solutions. The mean of the lower-inclined objects is reduced with 650 m, while the higher-inclined mean is reduced by 257 m. The larger difference for the low-inclined group is due to the location of the new sensors closer to the equator.

More detailed modelling of these sensors will likely improve these results further, as the SFS is expected to be more accurate than other PARs [7]. The spread of the solutions is higher, especially for the higher-inclined objects, which is due to the small number of observations resulting from short passes registered by the SFS sensors. The passes are shorter than for traditional PARs due to the vertical boresight and narrow latitudinal width of the fence. For a near-polar orbit the pass duration is only several minutes.

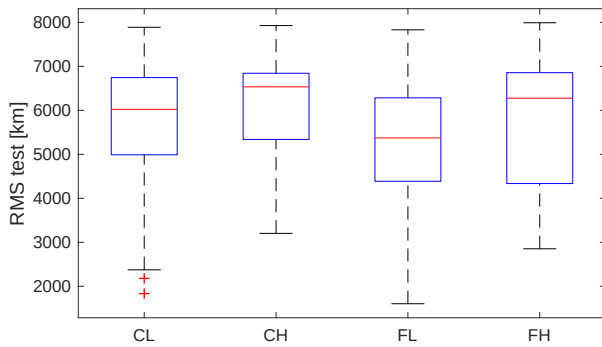


Figure 5. Boxplot of the propagated residuals for one day for the current and future network configuration (C and F, respectively) and the low- and high-inclined orbits (L and H, respectively).

5. CONCLUSIONS

The SSN is continuously changing. New sensors are added, as existing ones are relocated, upgraded, or closed. Moreover, the participation of sensors can change over time. An overview of the current configuration was presented for PARs, including the planned SFS. The coverage of each sensor was investigated. Observations for a Delta-K rocket-body reference object were generated for a randomly initialised orbit with varying eccentricity, inclination, and atmospheric density. From the observations the TLEs were estimated.

The residuals of both the fit and test span are highly sensitive to the eccentricity, number of observations, and number of sensors, while the inclination has minor influence.

The two additional SFS sensors provide a reduction of the median error in the solutions, especially for low-inclined orbit. However, due to the shorter observation arcs, the spread in solution quality is increased.

The study only took the network geometry into account. Further improvements can be made by simulating the accuracy of individual sensors to a higher degree. The capabilities are found to differ greatly across sensors. Specifically, the SFS promise a significant improvement in performance over traditional PARs. So the observed benefits are likely very conservative and much higher in reality. However, more details on the sensors are required, and these are often not available.

ACKNOWLEDGEMENT

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REFERENCES

1. Johnson, N. L., "U.S. Space Surveillance," *Advances in Space Research*, Vol. 13, No. 8, 1993, pp. 5–20.
2. Klinkrad, H., Fritsche, B., and Koppenwallner, G., "Re-Entry Prediction and On-Ground Risk Estimation," *Space Debris - Models and Risk Analysis*, edited by H. Klinkrad, chap. 9, Springer Verlag, Berlin.
3. Chatters, E. and Crothers, B. J., "Space Surveillance Network," *Space Primer AU-18*, Vol. 18, 2nd ed., 2009, pp. 249–258.
4. Vallado, D. A. and Griesbach, J. D., "Simulating Space Surveillance Networks," *AAS/AIAA Astrodynamic Specialist Conference*, 2011.
5. Flohrer, T., *Optical Survey Strategies and their Application to Space Surveillance*, Schweizerischen Geodätischen Kommission, Zurich, 2012.
6. Missile Defense Agency, "Upgraded Early Warning Radars, AN/FPS-132," Tech. Rep. Fact Sheet 16-MDA-8777, 2016.
7. Hack, P. J., Carbaugh, K., and Simon, K. J., "Automated Space Surveillance Using the AN/FSY-3 Space Fence System," *Advanced Maui Optical and Space Surveillance Technologies*, 2016.
8. Allen, R. S., Donatelli, D. E., and Picardi, M. C., "Correction for Ionospheric Refraction for COBRA DANE," *Air Force Surveys in Geophysics*, , No. 376, 1977.
9. Pechkis, D. L., Pacheco, N. S., and Botting, T. W., "Statistical Approach to the Operational Testing of Space Fence," *IEEE Aerospace and Electronic Systems Magazine*, Vol. 31, No. 11, 2016, pp. 30–39.
10. Binnie, J., "Qatari long-range radar procurement moves forward," *IHS Jane's Defence Weekly*, dec 2016.
11. Ronse, A. and Mooij, E., "Statistical Impact Prediction of Decaying Objects," *Journal of Spacecraft and Rockets*, Vol. 51, No. 6, 2014, pp. 1797–1810.
12. Picone, J. M., Hedin, A. E., Drob, D. P., and Aikin, A. C., "NRLMSISE-00 Empirical Model of the Atmosphere: Statistical Comparisons and Scientific Issues," *Journal of Geophysical Research*, Vol. 107, No. A12, 2002, pp. 1468–1483, doi: 10.1029/2002JA009430.
13. Vallado, D. A. and Crawford, P., "SGP4 Orbit Determination," *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, 2008, pp. 18–21.
14. Wang, R., Liu, J., and Zhang, Q., "Propagation errors analysis of TLE data," *Advances in Space Research*, Vol. 43, No. 7, 2009, pp. 1065–1069, doi: 10.1016/j.asr.2008.11.017.

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Paper S06

GOCE STATISTICAL RE-ENTRY PREDICTIONS

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ABSTRACT

Satellite re-entry predictions are used to determine the time and location of impacts of decaying objects. Large uncertainties result from unknowns in the initial state and environment models. The complex evolution of the attitude further complicates these predictions, especially for slender bodies. Typically, predictions only propagate a point-mass and assume a static error window. Full six degrees-of-freedom (6DOF) statistical re-entry predictions of ESA's Gravity field and steady-state Ocean Circulation Explorer (GOCE) are proposed. Improved error models for the initial state and atmospheric density are introduced. The uncertainty parameters are estimated using Global Positioning System orbit solutions. The predictions are compared against Tracking and Impact Predictions (TIPs) and predictions by the ESA/ESOC Space Debris Office. The 6DOF predictions are consistently closer to the true decay epoch for several starting epochs, while providing narrower windows than TIPs.

Key words: Space Debris; Re-entry Predictions; Error Propagation; Rigid-Body Dynamics.

1. INTRODUCTION

Re-entry predictions are important for determining the impact locations and times of decaying objects. For large spacecraft, with high-temperature resistant components, fragments often survive re-entry and impact Earth. Re-entry predictions can be distinguished by their forecasting period into long- to medium-term, short-term, and break-up and survival predictions. For each type the tools, underlying methods, and analyses are distinct [1].

Short-term predictions range from several days to hours prior to re-entry and focus on predicting the final decay epoch, corresponding to around 80 km altitude, when break up commonly occurs. The associated uncertainty is often also reported, and is roughly 40% of the remaining orbital lifetime. About 100 Tracking and Impact Predictions (TIPs) are released each year by the Joint Space Operations Center (JSpOC)¹. Similar predictions are reg-

ularly performed by other agencies, institutes, and corporations [2, 3, 4, 5].

In these predictions, the object is modelled as point-mass under the influence of aerodynamic drag acceleration alone [1]. The drag coefficient is known to be dependent on the flight environment and rotational state [6, 7]. Changes with respect to the environment are either not included [2, 4] or through simplified analytic relations [3]. Changes in attitude are even more difficult to accurately model. Especially slender bodies can have multiple stable and tumbling modes, resulting in different drag (and lift) scenarios [8].

All these effects can be accurately modelled with full six degrees-of-freedom (6DOF) simulations, improving the fidelity of the simulations and reducing the number of assumptions on the evolution of the satellite's attitude. 6DOF simulations are already employed in spacecraft-oriented break-up and survival analyses, such as Spacecraft Atmospheric Re-Entry and Aerothermal Breakup (SCARAB) tool by the European Space Agency (ESA) [9], and dispersion analyses of planetary entry vehicles [10]. 6DOF simulations have been previously applied to re-entry predictions of Delta-K rocket bodies [8].

The modelling and quantification of the uncertainties present in the state and models, and subsequent propagation of these uncertainties, can further improve the accuracy of predictions. Most tools assume a static symmetric uncertainty of $\pm 20\%$ of the orbital lifetime on the predicted decay epoch [3, 11]. However, the true boundaries and asymmetry are very sensitive to the atmospheric density and the initial (rotational) state [8], which can vary significantly per re-entry. Statistical methods provide more accurate windows and insight into the shape of the distribution [8, 11].

The paper focuses on improving re-entry predictions of ESA's Gravity field and steady-state Ocean Circulation Explorer (GOCE) satellite, which re-entered on 11 November, 2013². GOCE's re-entry took significantly longer than initially predicted, due to its aerodynamic shape and controlled attitude, and much lower atmo-

April, 7, 2017

²http://www.esa.int/Our_Activities/Observing_the_Earth/GOCE/Facts_and_figures, accessed: April, 7, 2017

¹<https://www.space-track.org/#/decay>, accessed:

spheric densities than predicted. The re-entry of GOCE was previously studied by the ESA/ESOC Space Debris Office (SDO) [12] and the Japan Aerospace Exploration Agency (JAXA) [5].

For the re-entry of GOCE, SDO simulated a transition from controlled to uncontrolled motion, by changing the drag coefficient after the attitude controller was assumed to be saturated, effectively using two values for the drag coefficient. For controlled and stable motion the drag coefficient was estimated from GPS and ground station tracking data, while the drag coefficient for uncontrolled motion was obtained as the mean value from 6DOF simulations using SCARAB [12]. The latter drag coefficient was held constant, so is not clear whether the systematic errors in the atmospheric density at the time of re-entry and the dependency on the flight regime were properly accounted for. The atmosphere models are generally not unbiased for the lower regions [13, 14], and especially not during the GOCE re-entry period (see Sec. 3.2).

This paper performs statistical re-entry predictions of GOCE using the 6DOF re-entry simulator as described in [8]. The simulator is extended by modelling GOCE's attitude controller. Further improvements are made to uncertainty models and estimation techniques for the initial translational state and atmospheric density. The parameters are derived from Global Positioning System (GPS) orbit solutions. Finally, the predictions are compared against TIPs and SDO predictions.

The paper is structured as follows. First, the trajectory simulator, including propagation, spacecraft, and environment models, is detailed in Sec. 2. The error propagation and the process of uncertainty modelling and estimation for both GPS- and TLE-derived states is presented in Sec. 3. In Sec. 4, the re-entry predictions are presented, compared, and discussed. Lastly, the conclusions are given in Sec. 5.

2. TRAJECTORY SIMULATION

The explanation of the complete re-entry-prediction simulator is split up in the (deterministic) propagation of a single trajectory (this section), and the modelling, estimation, and propagation of errors (Sec. 3).

GOCE is has a slender aerodynamic shape, which is modelled as a rigid body with attitude control. The drag coefficient for $\alpha = \beta = 0^\circ$ is 3.15, compared to $C_D = 13.24$ for $\alpha = 90^\circ$. For a constantly tumbling motion around the pitch axis the effective mean drag coefficient would be 9.41, almost three times higher than in a stable configuration. Not accounting for the controlled attitude would result in predictions that significantly underestimate the re-entry epoch.

2.1. Propagation

The re-entry simulator consists of a 6DOF trajectory propagator. The complete state is represented by 13 state variables: position and velocity in inertial Cartesian coordinates, attitude quaternion, and the rotational rate in the body-frame. The equations of motion are integrated using a Runge-Kutta-Fehlberg 4(5) variable step-size integrator. The tolerance is set to $\epsilon_{rel} = \epsilon_{abs} = 1 \times 10^{-8}$. The step-size is allowed to vary between a minimum and maximum value of 1×10^{-4} and 1×10^3 s, respectively. The EGM2008 gravity field model is used up to order and degree 5³. These settings were determined to provide a good trade-off between accuracy and computational efficiency. The atmospheric density is modelled using the NRLMSISE-00 model [14]. The aerodynamic model is explained in further detail below.

2.2. Aerodynamic Model

GOCE is modelled as a combination of 44 flat panels. The mass, centre of mass, and inertia properties of GOCE are provided by ESA for the entire mission duration⁴. For the period after October 28, 2013 the values are:

$$\mathbf{x}_{com} = [2.536, 0.004, 0.001] \text{ m} \quad (1)$$

$$m = 1002.152 \text{ kg} \quad (2)$$

$$\mathbf{I} = \begin{pmatrix} 173.0 & 20.2 & -1.7 \\ 20.2 & 2750.5 & -1.1 \\ -1.7 & -1.1 & 2723.2 \end{pmatrix} \text{ kg m}^2 \quad (3)$$

where $\mathbf{x}_{com}$ is the position of the center of mass and $\mathbf{I}$ the inertia tensor with respect to the body-fixed reference frame.

Three separate flow regimes are considered, namely the free-molecular, transitional, and hypersonic continuum regime. Aerodynamic coefficients are derived for the free-molecular and continuum flow. Shadowing is not taken into account, as its effects are considered negligible for the GOCE panel model. Coefficients for the transitional regime are found using the following bridging function [15]:

$$C_X = F(Kn)(C_{X_{FM}} - C_{X_{cont}}) + C_{X_{cont}} \quad (4)$$

$$F(Kn) = \sin^2 \left(\pi \frac{3 + \log_{10} Kn}{8} \right) \quad (5)$$

³http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html, retrieved: March 22, 2017.

⁴<https://earth.esa.int/web/guest/-/goce-mass-property-file-8276>, retrieved: March 22, 2017.

where $F(Kn)$ is the bridging function, and has a value of 0 at $Kn = 0.001$, 1 at $Kn = 10$, and in between zero and one for $Kn \in (0.001, 10)$. The bridging parameters used were derived for the Shuttle, but are also in line with other (blunt) bodies [16].

2.3. Controller Model

GOCE was equipped with magnetorquers to stabilise its attitude. These torquers continued to function throughout the re-entry. For a slender body like GOCE, this has significant impact on the re-entry predictions.

A linear state-feedback controller is implemented to simulate the torque control of GOCE [17]. A control frequency of 10 Hz is chosen. The values of the input and output weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ were obtained from maximum control torques observed during normal operational modes for GOCE and drag-free mode specifications [18]. The gains are solved using optimal control theory. Tab. 1 list the gains of the X , Y , and Z control torques with respect to the aerodynamic angles α , β , and σ , and attitude rates p , q , and r . The gains were solved along a reference trajectory and found to remain constant for altitudes >60 km.

3. ERROR PROPAGATION

Error propagation is central to performing statistical re-entry predictions. Error propagation, much like regular propagation, solves an initial-value problem, represented by the non-linear transformation of an initial uncertainty under the influence of a stochastic environment over time.

For re-entry predictions the uncertainty in atmospheric density outweighs all other model contributions [19], it is therefore the only environment-model uncertainty considered.

The final decay-time distribution is the uncertainty of main interest. The uncertainty is propagated to obtain a final uncertainty using a Monte-Carlo (MC) simulator. The uncertainties are modelled as a deviation on top of the nominal state. The probability density function (PDF) of the decay-time is estimated using kernel-density estimation (KDE). For each simulation 1000 samples are used, which was determined to provide sufficient convergence of the final distribution.

3.1. Initial State Uncertainty

The translational uncertainty is modelled as a six-dimensional multi-variate normal (MVN) distribution. The translation uncertainty is expressed in the orbital frame as the radial, along-track, and cross-track position and velocity, in which the errors behave quasi-normal.

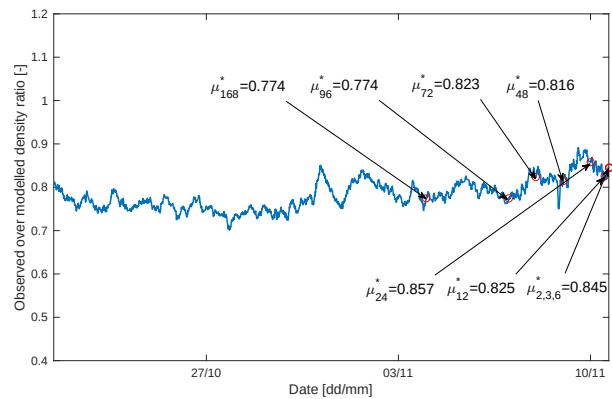


Figure 1. Estimated mean density error μ^* for GPS-based density estimates.

Tabs. 2 and 3 give the estimates of the initial translational uncertainty obtained from the orbit determination residuals of November 9, 2013.

Uncertainty in the initial rotational state is assumed to be distributed uniformly within the maximum values for the drag-free mode [18]. The values are given in Tab. 4.

3.2. Atmospheric Density Uncertainty

The atmospheric density has a log-normal uncertainty. The mean and variance mainly depend on the altitude and space-weather conditions. The uncertainty for the 120 – 200 km altitude range was derived in [8] and found to be $\mu^* = 0.98$ and $\sigma^* = 1.13$. The uncertainty for the 200 – 400 km altitude range is $\mu^* = 0.93$ and $\sigma^* = 1.17$ [14]. Note that the mean here is an average mean and can vary significantly depending on space-weather conditions. In [13] several atmospheric models are compared for altitudes ranging from 150-240 km. It is found that the mean is typically given within $\pm 10\%$ and a standard deviation around 15%.

The parameters μ^* and σ^* can be estimated in several ways. In [8], values from literature were used, which presents a conservative approach. Estimating the parameters for a specific re-entry will provide more narrow and tailored uncertainty distributions. For this, however, additional data is required.

GPS-derived non-gravitational accelerations (as obtained by [20]) are used to estimate uncertainty parameters. The ratio of observed and modelled accelerations is computed. Fig. 1 shows the 10-minute averaged density ratios. The estimate of σ^* is obtained by subtracting the 10-minute averaged signal from the 10 Hz signal. The value for the entire analysis period is used and estimated to be 1.082.

For the estimates of μ^* the average over one orbital revolution is taken at several epochs prior to re-entry: 168, 96, 72, 48, 24, 12, 6, 3, and 2 hours. These values are

Table 1. Gains for the LQR controller.

	α	β	σ	p	q	r
K_X	0	3.92×10^{-3}	-1.60×10^{-2}	1.00×10^1	0	0
K_Y	2.67×10^{-2}	0	0	0	1.33×10^2	0
K_Z	0	-3.17×10^{-1}	3.92×10^{-2}	0	0	1.00×10^2

Table 2. Initial standard deviation of errors of GPS-derived state.

σ_{r_R} cm	σ_{r_S} cm	σ_{r_W} cm	σ_{v_R} mm s^{-1}	σ_{v_S} mm s^{-1}	σ_{v_W} mm s^{-1}
1.03	12.7	1.31	0.15	0.009	0.016

Table 3. Correlation matrix for initial state obtained through POD from GPS.

	r_R	r_S	r_W	v_R	v_S	v_W
r_R	1.00	0.80	-0.03	-0.80	-0.93	-0.06
r_S	0.80	1.00	-0.00	-1.00	-0.54	-0.05
r_W	-0.03	-0.00	1.00	0.00	0.04	0.02
v_R	-0.80	-1.00	0.00	1.00	0.55	0.05
v_S	-0.93	-0.54	0.04	0.55	1.00	0.06
v_W	-0.06	-0.05	0.02	0.05	0.06	1.00

also indicated in the figure.

4. RESULTS

Apart from the final decay message, seven TIP messages were released for GOCE at various epochs prior to re-entry. SDO has similarly performed several predictions prior to re-entry. SDO predictions started on October 23, 2013, with the assumption that the attitude controller would fail for a 20 mN drag threshold, resulting in tumbling motion. These early predictions placed the re-entry at around November 6. From November 1, 2013, the threshold was raised to 50 mN. Later, the saturation of the attitude controllers was expected to occur late into the re-entry [12]. Only these last predictions are used for the comparison.

For the 6DOF predictions, it is chosen to start the re-entry predictions at common TIP moments prior to the true re-entry epoch; these are 2h, 3h, 6h, 12h, 1d, 2d, 3d, 4d, and 7d prior to the true decay epoch.

Fig. 2 shows the results for this analysis. The horizon-

tal axis corresponds to date of prediction, and the vertical axis to the prediction error as a percentage of time to decay (TTD). For each prediction the confidence interval (CI) and median is indicated. As a general trend the relative prediction window (as a percentage of the TTD) remains roughly constant. The 6DOF, TIP, and SDO intervals are on average 46%, 65%, and 43% wide, respectively. All are thus wider than the 40% rule of thumb [21, 22].

The CIs of the TIPs were 41% wider than for 6DOF. The smaller widths for 6DOF can be attributed to the attitude controller, which reduces the rotational uncertainty, leaving only the atmospheric uncertainty as the main driver of the width. A true comparison with the TIP window, of course, depends on the probability of the CI, for which only empirical values are available from old sources [21]. As the TIP window for GOCE is significantly wider than the 40% earlier mentioned, it could be the TIP window and process have changed.

For all predictions the median of the 6DOF is found to be closer to the true re-entry epoch, compared to the window mid-points of the closest TIP and SDO's medians. The 6DOF distributions are slightly asymmetric, where the median is on average located at 7.3% (in terms of the total distribution width) from the midpoint towards the lower bound. This is characteristic for decay-time distributions, where long re-entries are more likely than short re-entries.

Except for the final three predictions (6, 3, and 2h), respectively, the median overshoots the true decay epoch. A maximum of 10% (about 5 hours) is observed for the prediction made at 48 hours prior. The median of the SDO predictions is in all cases earlier (13% on average) than the true re-entry epoch. The difference grows for the last 2 prediction epochs, hinting perhaps at an issue with the later stages of the prediction. Two possible explanations were already discussed: not taking the low densities into account in the SCARAB-derived drag coefficient and/or keeping the drag-coefficient invariant of the flight environment.

5. CONCLUSIONS

The decay-time distribution accuracy for GOCE is dominated primarily by systematic errors in atmospheric density and attitude control, while its precision is dominated by the atmospheric uncertainty. This is followed by the initial translational uncertainty.

Table 4. Bounds on uniform rotational uncertainty [18].

		Roll	Pitch	Yaw
Attitude	rad	± 0.15	± 0.06	± 0.15
Rates	mrads^{-1}	± 0.20	± 0.03	± 0.20

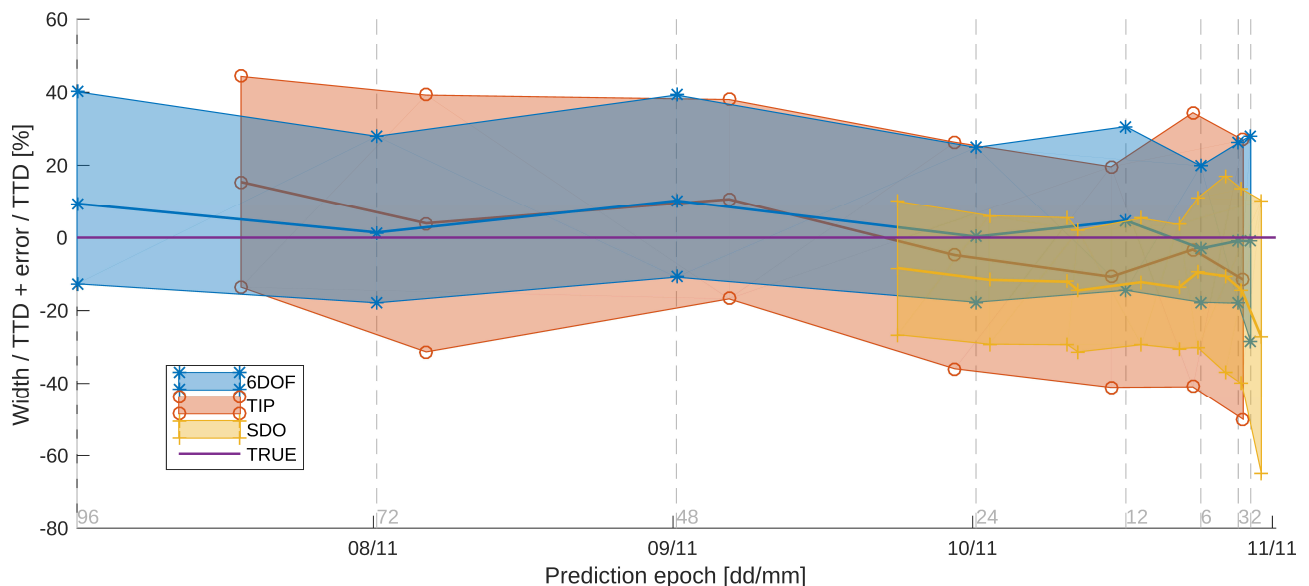


Figure 2. Comparison of 6DOF with TIPs and SDO re-entry predictions for several prediction epochs.

By analysing the density-modelling error from GPS-derived non-gravitational accelerations, a better estimate of the density uncertainty could be obtained. The attitude control is found to prolong the re-entry significantly. By modelling the attitude control and motion in a 6DOF simulator, it is not necessary to assume a constant drag coefficient and introduce any discontinuities to this value.

The median of the 6DOF re-entry predictions was found consistently closer to the true decay epoch than neighbouring TIPs and predictions by SDO. Moreover, the TIP window was found to be on average 41% wider than for 6DOF. The 6DOF and SDO predictions are found to be of comparable relative width. The SDO predictions already employed 6DOF techniques to determine the final drag coefficients for the 3DOF propagation. Two potential implementation issues with this approach were identified.

It is recommended to perform full 6DOF re-entry predictions for objects for which aerodynamic models are available or can be easily constructed. For instance, for rocket bodies a single model can be used for many re-entry predictions.

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REFERENCES

1. Klinkrad, H., Fritsche, B., and Koppenwallner, G., "Re-Entry Prediction and On-Ground Risk Estimation," *Space Debris - Models and Risk Analysis*, edited by H. Klinkrad, chap. 9, Springer Verlag, Berlin.
2. Strizzi, J. D., *An Improved Algorithm for Satellite Orbit Decay and Re-Entry Predictions*, MSc thesis, Massachusetts Institute of Technology, 1993.
3. Klinkrad, H., "Methods and Procedures of Re-Entry Predictions at ESA," *European Conference on Space Debris*, Vol. 6, 2013.
4. Anselmo, L. and Pardini, C., "Satellite reentry predictions for the Italian civil protection authorities," *Acta Astronautica*, Vol. 87, 2013, pp. 163–181, doi: 10.1016/j.actaastro.2013.02.004.
5. Ikeda, S., Tajima, T., Abe, J., and Matsuda, I., "Improved Re-Entry Prediction Method," *SpaceOps Conference*, No. AIAA 2014-1836, 2014.
6. Crowther, R., "Re-entry Aerodynamics Derived from Space Debris Trajectory Analysis," *Planetary and Space Science*, Vol. 40, No. 5, 1992, pp. 641–646.
7. Bowman, B. R. and Moe, K., "Drag coefficient variability at 100-300 km from the orbit decay analyses of rocket bodies," *AIAA/AAS Astrodynamics Specialist Conference*, , No. AAS 05-256, 2005.
8. Ronse, A. and Mooij, E., "Statistical Impact Prediction of Decaying Objects," *Journal of Spacecraft and Rockets*, Vol. 51, No. 6, 2014, pp. 1797–1810.
9. Koppenwallner, G., Fritsche, B., Lips, T., and Klinkrad, H., "SCARAB - A Multi-Disciplinary Code for Destruction Analysis of Spacecraft During Re-Entry," *European Symposium on Aerothermodynamics for Space Vehicles*, 2005.

10. Desai, P. N., Schoenenberger, M., and Cheatwood, F. M., "Mars Exploration Rover Six-Degree-of-Freedom," *Journal of Spacecraft and Rockets*, Vol. 43, No. 5, 2006, pp. 1019–1025, doi: 10.2514/1.6008.
11. Neuenfeldt, B., *A survey of uncontrolled satellite reentry and impact prediction*, MSc thesis, Naval Postgraduate School, 1993.
12. Virgili, B. B., Flohrer, T., Lemmens, S., and Krag, H., "GOCE Re-entry Campaign," *5th International GOCE User Workshop*, No. ESA SP-728, 2014.
13. Marcos, F. A., "Accuracy of atmospheric drag models at low satellite altitudes," *Advances in Space Research*, Vol. 10, No. 3-4, 1990, pp. 417–422, doi: 10.1016/0273-1177(90)90381-9.
14. Picone, J. M., Hedin, A. E., Drob, D. P., and Aikin, A. C., "NRLMSISE-00 Empirical Model of the Atmosphere: Statistical Comparisons and Scientific Issues," *Journal of Geophysical Research*, Vol. 107, No. A12, 2002, pp. 1468–1483, doi: 10.1029/2002JA009430.
15. Blanchard, R. C. and Buck, G. M., "Rarefied-flow Aerodynamics and Thermosphere Structure from Shuttle Flight Measurements," *Journal of Spacecraft and Rockets*, Vol. 23, No. 1, 1986, pp. 18–24.
16. Wilmoth, R. G., Blanchard, R. C., and Moss, J. N., "Rarefied Transitional Bridging of Blunt Body Aerodynamics," *Symposium on Rarefied Gas Dynamics*, 1998.
17. Mooij, E., *The Motion of a Vehicle in a Planetary Atmosphere*, Series 08: Astrodynamics and Satellite Systems 01, Delft University Press, 1997.
18. Sechi, G., André, G., Andreis, D., and Saponara, M., "Magnetic attitude control of the GOCE satellite," *Guidance, navigation and control systems*, , No. 606, 2006.
19. Klinkrad, H., "On the Use of Atmosphere Models in Re-Entry Predictions," *Environment Modelling for Space-based Applications*, No. SP-392, 1996, pp. 287–298.
20. Visser, P. N. A. M. and van den IJssel, J. A. A., "Orbit determination and estimation of non-gravitational accelerations for the GOCE reentry phase," *Advances in Space Research*, Vol. 58, No. 9, 2016, pp. 1840–1853, doi: 10.1016/j.asr.2016.07.013.
21. Lefebvre, S., *An Analysis of Tracking and Impact Predictions*, MSc thesis, Air Force Institute of Technology, 1991, No. AFIT/GSO/ENG/91D-11.
22. Kronebusch, R. M., "USAF's Role in Surveillance," *The Space Congress*, No. 16, 1979.

E

Paper S07



Analysis of Uncertainties and Modeling in Short-Term Reentry Predictions

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Satellite reentry predictions are used to determine the time and location of impacts of decaying objects. These predictions are complicated by uncertainties in the initial state and environment models, and the complex evolution of the attitude. Typically, the aerodynamic and error propagation are done in a simplistic fashion. Full six-degrees-of-freedom modeling and attitude control is proposed for studying the historic reentry case of the Gravity Field and Steady-State Ocean Circulation Explorer satellite. Improved error modeling and estimation of the initial state and atmospheric density are introduced for both Global Positioning System and two-line elements states. A sensitivity analysis is performed to identify the driving parameters for several models and epochs. The predictions are compared against Tracking And Impact Predictions, and predictions by the European Space Agency Space Debris Office. The performed predictions are consistently closer to the true decay epoch for several starting epochs, while providing narrower windows than other predictions with higher confidence.

Nomenclature

a	=	acceleration, m/s ²
C_D, C_S, C_L	=	aerodynamic drag, side, and lift force coefficient
$D_{\max}$	=	Kolmogorov–Smirnov test statistic
d	=	dummy sensitivity parameter
d_p	=	particle collision diameter, m
$\mathcal{E}_p$	=	specific orbital energy, m ² /s ²
F	=	cumulative distribution function
F_K	=	ballistic coefficient scaling factor
$\hat{F}$	=	sample cumulative distribution function
f_C	=	aerodynamic bridging function
I	=	inertia tensor, kg · m ²
i	=	orbital inclination, rad
K	=	control gain matrix
K	=	ballistic coefficient, m ² /kg
Kn	=	Knudsen number, kg · m ²
k	=	number of uncertainty parameters
L	=	characteristic flowfield dimension, m
m	=	mass, kg
N	=	total number of samples
N_R	=	number of subsampling repeats
N_S	=	number of search curve samples
N_C	=	sample size of subset
n	=	mean motion, rad/s
P	=	specific work, J/(kg · s)
p, q, r	=	roll, pitch, and yaw rates, rad/s
Q	=	state weighting matrix, various
R	=	control weighting matrix, 1/(N · m ²)
r	=	position, m
S_i	=	first-order sensitivity index
S_{ref}	=	reference area, m ²
S_{Ti}	=	total-order sensitivity index
T	=	control torque, N · m

u	=	control vector, N · m
v	=	velocity, m/s
x_{com}	=	location of the center of mass, m
x_0	=	nominal state vector
α_a	=	angle of attack, rad
β_a	=	side-slip angle, rad
β^*	=	TLE BSTAR drag term
ϵ	=	tolerance
θ	=	Earth's rotational rate, rad/s
λ	=	mean molecular free path length, m
μ	=	normal mean
μ^*	=	log-normal mean
ρ	=	atmospheric density, kg/m ³
σ_a	=	bank angle, rad
σ	=	normal standard deviation
σ^*	=	log-normal standard deviation

I. Introduction

RENTRY predictions are important for determining the impact locations and times of decaying objects. Fragments often survive reentry and impact Earth, especially in case of large spacecraft with high-temperature-resistant components. These predictions can be distinguished by their forecasting period into long-term to medium-term, short-term, and break-up and survival predictions. For each type the tools, underlying methods, and analyses are distinct [1].

Short-term predictions range from several days to hours before reentry, starting from around 200 until 80 km altitude, when break-up commonly occurs. Tracking and Impact Predictions (TIPs), publicly released by the Joint Space Operations Center (JSpOC),[§] are the most common type, but short-term predictions are regularly performed by various actors [2–5]. Short-term predictions commonly report the expected decay epoch and the associated uncertainty as a percentage of the remaining orbital lifetime (for TIPs generally $\pm 20\%$ or more). The uncertainty in remaining time can be used to compute the potential footprint along the satellite ground-track. Short-term reentry predictions are invaluable for many activities, such as tracking, informing relevant authorities, avoiding false detection by missile-warning systems, and attributing liability resulting from impact [6].

Despite their importance, there is little research into these types of predictions, despite many promising developments in related fields. Areas of potential improvement are identified as spacecraft and aerodynamic modeling, statistical uncertainty propagation, and atmospheric modeling. The former two areas are the main focus of

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<https://www.space-track.org/#decay> Retrieved 8 August 2017].

this paper and explained in more detail below. The influence of atmosphere modeling is highlighted in the paper, and uncertainty estimation techniques for the atmospheric density are investigated. Improvements in thermospheric density and wind modeling [7–10] and new and better forecasting of space-weather proxies [11] can benefit reentry predictions significantly, but are not further treated.

Improving the spacecraft and aerodynamic modeling can provide more accurate results, while reducing the number of assumptions. Commonly, the object is modeled as point-mass under the influence of aerodynamic drag deceleration, neglecting lift and side-force contributions [1]. Performing these predictions is far from straightforward. The drag coefficient should account for changes in the flight environment [12,13], which is either not included [2,4] or through simplified analytic relations [3]. Likewise, changes in attitude orientation and motion should be accounted for, but it is difficult to predict all possible modes. This especially holds for slender bodies, where stable and tumbling modes create a myriad of different drag (and lift) scenarios. Finally, it is important to account for the relative likelihood of the possible modes to fully assess the final probability distribution.

All these effects can be accurately modeled in full six-degrees-of-freedom (6DOF) simulations, improving the fidelity of the simulations, while reducing the number of assumptions on the evolution of the satellite's attitude. 6DOF simulations are already employed in spacecraft-oriented break-up and survival analysis, such as Spacecraft Atmospheric Re-Entry and Aerothermal Breakup (SCARAB) by the European Space Agency (ESA) [14], and dispersion analysis of planetary entry vehicles [15]. Statistical 6DOF reentry simulations have been previously applied to reentry predictions of Delta-K rocket bodies [16]. Compared with point-mass predictions, the 6DOF simulations are more computationally expensive and require the object's shape, mass, and inertia properties to be known in greater detail.

The modeling and quantification of the uncertainties present in the state and models, and subsequent propagation of these uncertainties, can further improve the accuracy of predictions. Many methods assume a static symmetric uncertainty of $\pm 20\%$ on the predicted decay epoch [3,17]. Little research has been performed into the validity of these assumptions for a statistically significant number of objects over a long period. One study found that the true empirical confidence of the TIP window was dependent on the time to decay (TTD) and varied from 93 to 98% for 180 objects between 1987 and 1990 [6]. A study of reentry campaigns for 15 objects found that 5% of the predictions fell outside the $\pm 20\%$ window used by ESA [3]. The true dispersion boundaries and asymmetry are most sensitive to the atmospheric density and the initial rotational state [16], which can vary significantly per reentry. Statistical methods, such as Monte-Carlo (MC) propagation, transform the initial uncertainty under the influence of stochastic models to obtain a final desired distribution, providing more accurate intervals and insight into the shape of the distribution [16,17]. The drawback is that performing these statistical predictions is considerably more computationally expensive. Several promising techniques offer more efficient error propagation [18–20] and sampling strategies [21], but are not further investigated at this stage.

This paper aims to investigate the effect of uncertainties in short-term reentry predictions by studying the past reentry of ESA's Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) satellite using statistical error propagation and improved spacecraft and uncertainty modeling. GOCE was a geodynamics and geodetic mission launched on March 17, 2009, and re-entered on November 11, 2013.[†] The attitude of GOCE was controlled with magnetorquers. However, the reentry of GOCE is not considered a controlled de-orbit, which involves targeting low-risk impact zones. GOCE's reentry took significantly longer than initially predicted, due to its aerodynamic shape and controlled attitude, and much lower atmospheric densities than predicted. The reentry was previously studied by the ESA/ESOC Space Debris Office (SDO)

[22] and Japan Aerospace Exploration Agency (JAXA) [5]. Furthermore, the reentry of GOCE is particularly interesting, because of the availability of many different measurements, such as TLEs, additional tracking by JSpOC, dedicated radar trackings, GPS solutions, and accurate solutions for the attitude. All these data allow for benchmarking the models and approaches.

For the reentry of GOCE, SDO implemented a number of improvements to their prediction method, including an asymmetric uncertainty window and by accounting for the transition from controlled to uncontrolled attitude motion. The latter was simulated by changing the drag coefficient after the attitude controller was assumed to be saturated, effectively using two values of the drag coefficient. For controlled and stable motion the drag coefficient was estimated from GPS and ground station tracking data, while the drag coefficient for uncontrolled motion was obtained as the mean value from 6DOF simulations using SCARAB [22]. The latter drag coefficient was held constant. It is not clear whether the systematic errors in the atmospheric density at the time of reentry and the dependency on the flight regime were properly accounted for. The atmosphere models are generally not unbiased for the lower regions [23,24] and especially not during the period of GOCE's reentry.

This paper proposes statistical reentry predictions of GOCE using an improved version of the 6DOF reentry simulator as described in [16]. The simulator is extended by modeling the attitude controller. Further improvements are made to uncertainty models and estimation techniques for the initial translational state and atmospheric density. Initial conditions (both state and uncertainty parameters) are derived from both Global Positioning System (GPS) and two-line elements (TLEs). A sensitivity analysis is performed to identify the driving uncertainty parameters and their time dependency. Finally, the predictions are benchmarked against TIPs and SDO predictions.

The paper is structured as follows. First, the trajectory simulator, including propagation, spacecraft, and environment models, is detailed in Sec. II. The error propagation and the process of uncertainty modeling and estimation for both GPS-derived and TLE-derived states is presented in Sec. III. In Sec. IV, the results of several reentry prediction cases and sensitivity analysis are presented and discussed. Lastly, the conclusions are given in Sec. V.

II. Spacecraft Propagation and Modeling

The explanation of the complete reentry prediction simulator is split up into the (deterministic) propagation of a single trajectory (this section), and the modeling, estimation, and propagation of errors (Sec. III).

A. Equations of Motion

The reentry simulator consists of a 6DOF trajectory propagator. The translational and rotational state are propagated using a numerical integrator. The complete state is represented by 13 state variables: the position and velocity in inertial Cartesian coordinates, the attitude quaternion, and the rotational rate in the body frame.

The equations of motion are integrated using a Runge-Kutta-Fehlberg 4(5) variable step-size integrator. The tolerance is set to $\epsilon_{\text{rel}} = \epsilon_{\text{abs}} = 1 \times 10^{-8}$. The step-size is allowed to vary between a minimum and maximum value of 1×10^{-4} and 1×10^3 s, respectively. The EGM2008 gravity field model is used up to order and degree 5.^{**} These settings were determined to provide a good trade-off between accuracy and computational efficiency. Finally, the aerodynamic accelerations are modeled using

$$\mathbf{a}_A = -\frac{1}{2} \begin{pmatrix} C_D \\ C_S \\ C_L \end{pmatrix} \frac{S_{\text{ref}}}{m} \rho v_A^2 \quad (1)$$

where C_D , C_S , and C_L are the aerodynamic force coefficients, which will be defined in Sec. II.C. S_{ref} is the reference area of the satellite,

[†]http://www.esa.int/Our_Activities/Observing_the_Earth/GOCE/Facts_and_figures [retrieved 8 August 2017].

^{**}http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html [retrieved 8 August 2017].

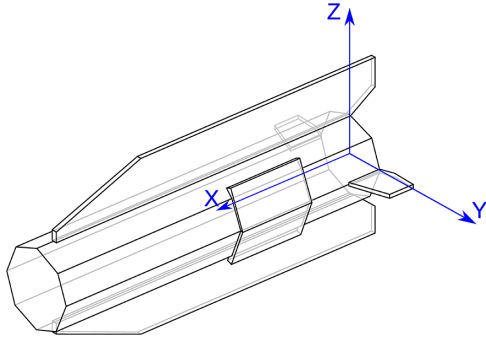


Fig. 1 Schematic of the 44-panel GOCE aerodynamic model, which includes the origin and principal axes of the satellite physical coordinate frame (SPCF).

normalized as a unit area of 1 m^2 in both the generation of the coefficients and Eq. (1). The spacecraft mass m is given in Sec. II.B. Lastly, v_A is the velocity with respect to the atmosphere and ρ the atmospheric density, modeled using the NRLMSISE-00 model [24].

B. Spacecraft Model

GOCE is modeled as a combination of 44 flat panels, as illustrated in Fig. 1. The mass, center of mass, and inertia properties of GOCE are provided by ESA for the entire mission duration.^{††} These properties remained constant after October 28, 2013, and are used for the simulation. The position of the center of mass $\mathbf{x}_{\text{com}}$, the mass m , and the inertia tensor $\mathbf{I}$ in the satellite physical coordinate frame (SPCF) are shown in Table 1.

C. Aerodynamic Model

Throughout the decay and reentry the aerodynamic forces and moments need to be accurately modeled. To this end an aerodynamic database is constructed that relates aerodynamic force and moment coefficients to the spacecraft attitude (in terms of angle of attack α_a and side-slip angle β_a) and the Mach number, M .

However, as the spacecraft re-enters Earth's atmosphere, the atmospheric properties (reflected by the Knudsen number), and therefore the aerodynamic behavior, change substantially. In space ($>100 \text{ km}$) the atmospheric density is low and interaction between the gas molecules is rare, while at lower altitudes intermolecular collisions start playing an important role. Different aerodynamic theories are thus required to model the aerodynamics. To accommodate this, three separate flow regimes are considered. The transitional regime provides a bridging between the free-molecular and continuum regimes.

These three regimes are distinguished by the Knudsen number:

$$Kn = \frac{\lambda}{L} = \frac{m_p}{\sqrt{2\pi d_p^2 \rho L}} = \frac{M}{N_A \sqrt{2\pi d_p^2 \rho L}} \quad (2)$$

$$Kn \approx \frac{\bar{M}}{N_A \sqrt{2\pi d_p^2 \rho L}} \quad (3)$$

where λ is the mean molecular free path length and L the characteristic flowfield dimension. λ can be expressed as a function of the density ρ , particle collision diameter d_p , and particle mass m_p . In turn, m_p can be expressed as the ratio of the molar mass M and Avogadro's number N_A . The formula is simplified by assuming the mean values $\bar{M} = 28.9 \text{ kg} \cdot \text{kmol}^{-1}$ and $\bar{d}_p = 367 \text{ pm}$ [16].

The aerodynamic coefficients can then be generated and used depending on the regime. This approach is summarized in Table 2. For the free-molecular flow, a diffuse reflection model is used to describe the molecule-surface interaction [25]. The continuum flow

Table 1 GOCE satellite properties as of October 28, 2013

Symbol	Value	Unit
$\mathbf{x}_{\text{com}}$	[2.536, 0.004, 0.001]	m
m	1002.152	kg
$\mathbf{I}$	$\begin{bmatrix} 173.0 & 20.2 & -1.7 \\ 20.2 & 2750.5 & -1.1 \\ -1.7 & -1.1 & 2723.2 \end{bmatrix}$	$\text{kg} \cdot \text{m}^2$

Table 2 Summary of aerodynamic regimes and flow theories

Regime	Knudsen	Theory
Free molecular	$Kn \geq 10$	Diffuse reflection
Transitional	$0.001 \leq Kn < 10$	Bridged
Continuum	$Kn < 0.001$	Mod. Newtonian

is modeled using the modified-Newtonian theory, a local-inclination method suitable for hypersonic aerodynamics [16,26]. A routine is implemented to derive coefficients for both free-molecular and continuum regimes. A coefficients database for the free-molecular flow was previously derived for GOCE and used to verify the implementation [27]. Note that shadowing is not taken into account, as its effects are considered negligible for the GOCE panel model. A selection of the coefficients is shown in Fig. 2.

The drag and lift force, and pitch-moment coefficients (Figs. 2a–2b, 2c–2d, and 2e–2f, respectively) are shown as a function of the angles of attack and sideslip (Figs. 2a, 2c, 2e, and 2b, 2d, 2f, respectively). FM_V indicates the free-molecular solution obtained from an external source. The figure highlights the differences between the free-molecular and continuum-flow regimes. It can be seen that the drag coefficient is almost half for the continuum flow regime, whereas the lift coefficient is substantially higher. The two free-molecular solutions (FM and FM_V in the figure) show good agreement, demonstrating correct implementation of the coefficient database generator.

The intermediate transitional regime is modeled using a bridging function between the coefficients of the other regimes. Any force or moment coefficient C_X can be found using the following relation [28]:

$$C_X = f_C(Kn)(C_{X_{\text{FM}}} - C_{X_{\text{cont}}}) + C_{X_{\text{cont}}} \quad (4)$$

$$f_C(Kn) = \sin^2\left(\pi \frac{3 + \log_{10} Kn}{8}\right) \quad (5)$$

where $f_C(Kn)$ is the bridging function, and has a value of 0 at $Kn = 0.001$, 1 at $Kn = 10$, and in between 0 and 1 for $Kn \in (0.001, 10)$.

This particular bridging function is often referred to as the “Shuttle” bridging function and was deduced from Space Shuttle data [28]. A “universal” bridging function does not exist and differs per object. The sine-squared form of Eq. (5) is a common choice, as it provides a smooth transition between the free-molecular and continuum-flow regimes. Moreover, it is preferred over the erf-log form, which approaches the bridging coefficients of 0 and 1 asymptotically, making the sine-squared form more computationally efficient. The function is typically tuned by its two parameters, in this case $a_1 = 3/8$ and $a_2 = 1/8$, which are typical values found for the Shuttle, but also in line with other (blunt) bodies [29].

Figure 3 shows the value of the bridging function throughout a typical reentry for GOCE. It can be seen that between 160 and 80 km the spacecraft is in the transitional regime. The altitude is not strictly decreasing, due to the oblate surface model. The decay epoch is achieved at 80 km, so the trajectory is not simulated into the continuum regime. Nevertheless, the continuum coefficients are

^{††}<https://earth.esa.int/web/guest/-/goce-mass-property-file-8276> [retrieved 8 August 2017].

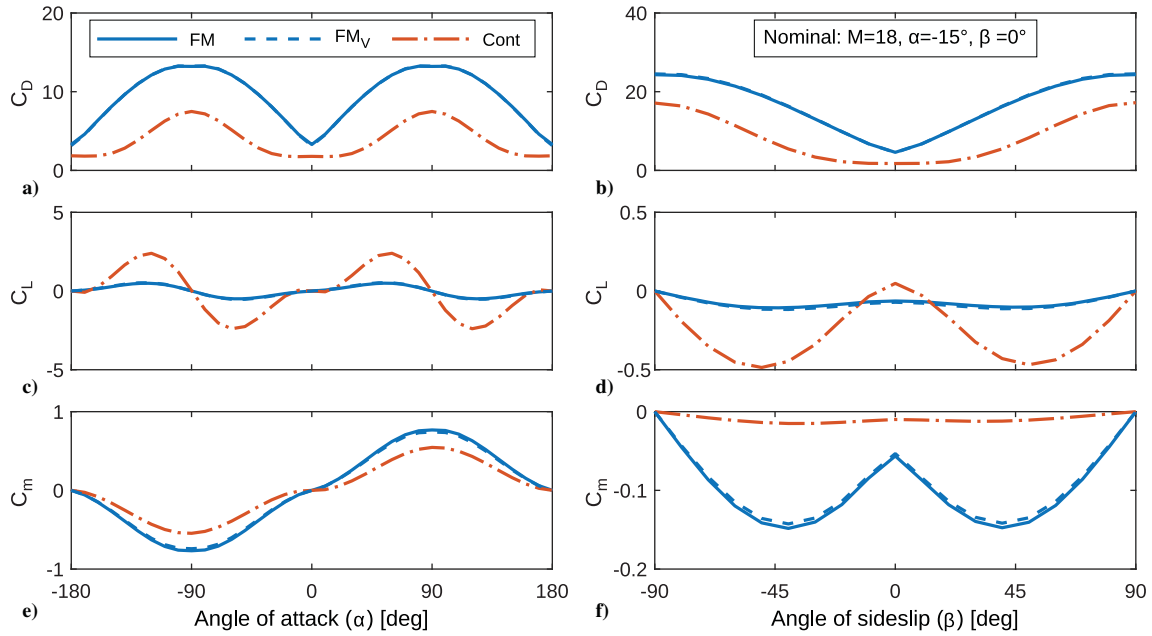


Fig. 2 Selection of aerodynamic force and moment coefficient for GOCE for the free-molecular (FM) regime and continuum (Cont) regime.

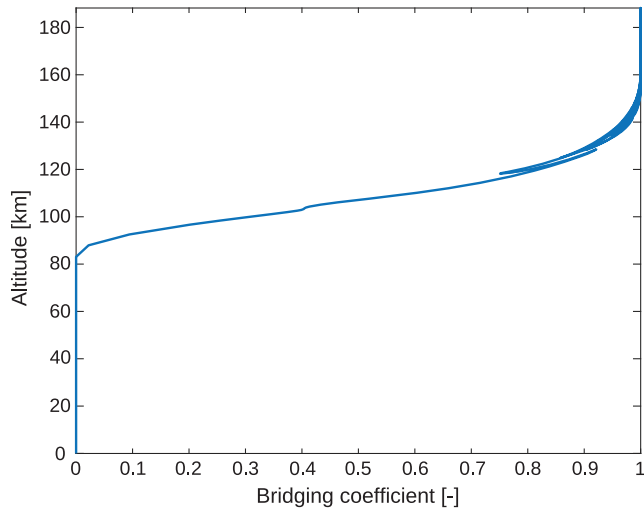


Fig. 3 Example of the bridging function value for a typical reentry trajectory of GOCE.

D. Controller

GOCE was equipped with magnetorquers to stabilize its attitude. These torquers continued to function throughout the reentry. For a slender body like GOCE, a stable attitude has significant impact on the reentry predictions.

In Fig. 2 it can be seen that the drag coefficient for $\alpha_a = \beta_a = 0^\circ$ is 3.15, compared with $C_D = 13.24$ for $\alpha_a = 90^\circ$. For a constantly tumbling motion around the pitch axis the effective mean drag coefficient would be 9.41, almost three times higher than in a stable configuration. Not accounting for the controlled attitude would result in predictions that significantly underestimate the reentry epoch.

The maximum step size of the integrator needs to be limited to the control frequency to ensure proper response. The total number of integration steps is therefore increased, causing additional round-off errors and computational cost. Nevertheless, a control frequency of 10 Hz is chosen. The frequency was empirically found to provide a reasonable trade-off between control stability and computational cost.

A simple linear-quadratic regulator is implemented to simulate the torque control of GOCE. The input to the controller is the deviation from the nominal rotational state $\Delta \mathbf{x}$.

$$\Delta \mathbf{x} = [\Delta p, \Delta q, \Delta r, \Delta \alpha_a, \Delta \beta_a, \Delta \sigma_a]^T \quad (6)$$

representing full-state feedback. Note that the attitude is represented here by the aerodynamic angles α_a , β_a , and σ_a . For propagation, however, quaternions are used instead.

The nominal state at epoch is assumed as follows:

$$\mathbf{x}_0 = [0, -n, 0, 0, 0, 0] \quad (7)$$

where the only nonzero value is the pitch rate with respect to the inertial frame, assumed to be equal to the mean motion, n . The control outputs are torques around the body-frame axes. The control response $\mathbf{u}$ is defined as a function of the deviation multiplied by the control gains:

$$\Delta \mathbf{u} = [T_x, T_y, T_z] = -\mathbf{K} \Delta \mathbf{x} \quad (8)$$

where $\mathbf{K}$ is the control gain matrix, which needs to be determined.

The sensitivity of the controller is determined by input and output weighting matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively. Only the diagonals of the matrices are nonzero, the values are derived from maximum values observed during normal operational modes for GOCE, and drag-free mode specifications are given in Table 3 [30].

Linearized equations of motion are used, and longitudinal and lateral control are decoupled [31]. The control gain matrix $\mathbf{K}$ is obtained by numerically solving the matrix Riccati equation along a nominal 3DOF reentry trajectory.

The optimal gains remain nearly constant throughout the trajectory. Only for lower altitudes (<60 km) there is some variation in the optimal gains. As the reentry simulations are stopped at 80 km, the gains are assumed to stay constant throughout the trajectory. Table 4 lists the control gains obtained.

Table 3 Diagonal elements of the controller input and output weighting matrices $\mathbf{Q}$ and $\mathbf{R}$

Elements	Value	Unit
$[q_{11}, q_{22}, q_{33}]$	$[0.2^{-2}, 0.03^{-2}, 0.02^{-2}]$	$\text{nrad}^{-2} \cdot \text{s}^2$
$[q_{44}, q_{55}, q_{66}]$	$[0.15^{-2}, 0.06^{-2}, 0.15^{-2}]$	rad^{-2}
$[r_{11}, r_{22}, r_{33}]$	$[20^{-2}, 40^{-2}, 20^{-2}]$	$\text{nN}^{-2} \cdot \text{m}^{-2}$

Table 4 Gains for the LQR controller

Axis	p	q	r	α_a	β_a	σ_a
x	1.00×10^1	0	0	0	3.92×10^{-3}	-1.60×10^{-2}
y	0	1.33×10^2	0	2.67×10^{-2}	0	0
z	0	0	1.00×10^2	0	-3.17×10^{-1}	3.92×10^{-2}

III. Error Modeling, Estimation, and Propagation

Error propagation is central to performing statistical reentry predictions. Error propagation, much like regular propagation, solves an initial-value problem, represented by the nonlinear transformation of an initial uncertainty under the influence of a stochastic environment over time.

There are different error sources. A distinction can be made between state and model uncertainties. The initial state is uncertain due to uncertainties in the original observations, the imperfect models that relate observations to state variables, and approximations made in the orbit determination process. The environment and acceleration models are inherently deficient, and are further affected by errors in model inputs. For reentry predictions the uncertainty in atmospheric density outweighs all other model contributions [32]. It is therefore the only model uncertainty considered.

A. Error Propagation

The final decay-time distribution is the uncertainty of main interest. The initial uncertainty is propagated to obtain a final uncertainty using MC. The uncertainties are modeled as a deviation on top of the nominal state and sampled using a pseudo random-number generator (PRNG). The final probability density function (PDF) is estimated using kernel-density estimation (KDE) to obtain a nonparametric solution of the decay-time distribution.

The convergence of the final distribution is investigated. A simulation with 10,000 samples is performed starting at 48 h before the reentry. Six different subsample sizes N_C are defined as test cases, where $N_C = 100, 200, 400, 1000, 2000$, and 4000 samples. For each value of N_C , a random subsampling of the total number is done to obtain an estimate of the final PDF using KDE. Depending on the subsample size N_C , the subsampling is repeated N_r times: $N_r = 3200, 1600, 800, 320, 160$, and 96, respectively. These values were empirically derived and found to provide a sufficient convergence on the test statistic.

To investigate the convergence of the simulation as a function of the number of samples, a two-sample Kolmogorov–Smirnov (KS) test has been performed. The KS test is a nonparametric statistical test to assess whether two sample sets are of the same continuous distribution, by comparing the cumulative distribution function (CDF) of each set. The KS test statistic is computed as the maximum absolute difference between the CDFs:

$$D_{\max} = \max_x \left(|\hat{F}(x) - F(x)| \right) \quad (9)$$

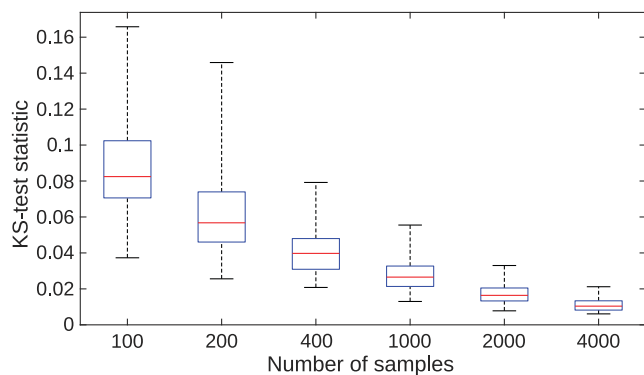


Fig. 4 Range of Kolmogorov–Smirnov test values for MC sample size

where $D_{\max}$ is the test statistic, and $\hat{F}$ and F present the CDF of the subsample and full set, respectively.

The KS test statistic $D_{\max}$ is a measure of how well a subset approaches the full set. For each subsampling a different $\hat{F}$ is obtained. Therefore, for each subsample size N_C the test can be performed N_r times, providing an estimate of the $D_{\max}$ value. Figure 4 shows boxplots of $D_{\max}$ as a function of N_C . As expected, the variability and mean go down with increasing number of samples. These values asymptotically approach zero, as the number of samples goes to infinity. The trend suggests that an exponentially increasing number of samples is required for a linear increase in accuracy.

A 5% error in the CDF would result in a maximum error of 45 min or 1.7% of TTD near the median. Such an error is considered acceptable. $N_C = 1000$ exceeds the maximum error exactly once ($D_{\max} = 5.5\%$ in all 320 comparisons). The median error for 1000 samples is merely 2.6% (or 24 min in median shift, less than 0.9% of TTD).

B. Uncertainty in the Translational State

The translational uncertainty is modeled as a six-dimensional multivariate normal (MVN) distribution, expressed in the orbital frame as the radial, along-track, and cross-track position and velocity. The orbital frame is a local frame, in which the errors behave quasi-normal. Standard deviations and means are considered for each dimension, as well as correlations between parameters. The MVN is defined by the mean vector and covariance matrix. These values and how they are estimated is different for GPS-derived and TLE-derived initial states and treated separately next.

1. GPS-Derived Initial Uncertainty

For the GPS-derived state, an estimate of the uncertainty is obtained from the orbit determination residuals. The orbit determination was performed in [33]. The corresponding residuals of November 9, 2013, were taken to obtain the standard deviations and correlations as reported in Tables 5 and 6, respectively.

2. TLE-Derived Initial Uncertainty

The process of estimating uncertainty from TLEs is different. For TLEs, observations or residual information is not publicly available. Fortunately, there are several methods to estimate this uncertainty [34]. Methods that rely on external data (e.g., raw observations or GPS/SLR solutions) are not considered, as these data are not available for many objects.

For this study a newly developed robust weighted differencing (WD) method is used [35]. It differs from traditional differencing methods in that the TLE at epoch is not considered as truth. Instead, a weighted truth is estimated using multiple TLEs. The method is also robust, removing the need for outlier removal and filter tuning. Moreover, the method requires only TLEs and simplified general perturbations (SGP4) model for propagation, so it is generally applicable to any object in the satellite catalog (SATCAT).

Figure 5a gives an overview of the TLEs as a function of latitude and longitude at the TLE epoch for a period of 42 days before reentry [35]. The color of each marker corresponds to the bars in Fig. 5b.

The bar plot in Fig. 5b shows the number of TLEs for each day. Bars that are (partly) outlined indicate the portion of TLEs associated with the ascending node (latitude $\sim 0^\circ$).

The period after October 21, 2013, is considered as the reentry period. It has almost twice the number of TLEs (101 vs 52) and an equal mix of TLEs associated with a latitude of $\approx 0^\circ$ and $\approx 90^\circ$ (50 vs 51, respectively). The additional TLEs are assumed to result from additional tracking by JspOC. It is shown that these are, in fact,

Table 5 Initial standard deviation of errors of GPS-derived state

σ_{r_R}	σ_{r_S}	σ_{r_W}	σ_{v_R}	σ_{v_S}	σ_{v_W}
cm	cm	cm	mm · s ⁻¹	mm · s ⁻¹	mm · s ⁻¹
1.03	12.7	1.31	0.15	0.009	0.016

Table 6 Correlation matrix for initial state obtained through POD from GPS

Component	r_R	r_S	r_W	v_R	v_S	v_W
r_R	1.00	0.80	-0.03	-0.80	-0.93	-0.06
r_S	0.80	1.00	-0.00	-1.00	-0.54	-0.05
r_W	-0.03	-0.00	1.00	0.00	0.04	0.02
v_R	-0.80	-1.00	0.00	1.00	0.55	0.05
v_S	-0.93	-0.54	0.04	0.55	1.00	0.06
v_W	-0.06	-0.05	0.02	0.05	0.06	1.00

classic TLEs and enhanced TLEs, respectively, which exhibit significantly different error characteristics [35]. Therefore, they will be treated separately.

Table 7 shows the standard deviation of the error at the TLE epoch as obtained by the WD method. Comparing the two TLE types, it can be seen that the uncertainty in eTLEs is roughly half that of cTLEs.

C. Uncertainty in the Rotational State

The uncertainty in the rotational state is not estimated separately for GPS and TLEs. Rather, the rotational state is assumed to be distributed uniformly within the maximum values for the drag-free mode and given in Table 8 [30].

As the angles are very small it is acceptable to sample uniformly directly within the reported bounds. However, for larger bounds direct uniform sampling of the angles becomes invalid. As sampling in spherical coordinates causes roughly an equal number of samples per elevation, the samples will tend to get more concentrated toward the poles. In such a case the samples should be drawn uniformly over a sphere, as described in [16].

D. Uncertainty in the Atmospheric Density

The atmospheric density is modeled using the NRLMSISE-00 model. The atmospheric density has a log-normal uncertainty. The mean and variance depend on the altitude and space-weather conditions. The uncertainty for the 120–200 km altitude range was derived in [16] and found to be $\mu^* = 0.98$ and $\sigma^* = 1.13$. The uncertainty for the 200–400 km altitude range is $\mu^* = 0.93$ and $\sigma^* = 1.17$ [24].

Note that the mean here is an average mean and can vary significantly depending on space-weather conditions. In [23] several atmospheric models are compared for altitudes ranging from 150 to

Table 7 Initial standard deviation of errors at TLE epoch, derived using weighted differencing [35]

Types	σ_{r_R} , km	σ_{r_S} , km	σ_{r_W} , km	σ_{v_R} , m · s ⁻¹	σ_{v_S} , m · s ⁻¹	σ_{v_W} , m · s ⁻¹
eTLE	0.089	2.4	0.033	2.9	0.11	0.018
cTLE	0.46	6.2	0.14	7.6	0.46	0.13

Table 8 Bounds on uniform rotational uncertainty [30]

Type	Unit	Roll	Pitch	Yaw
Attitude	rad	±0.15	±0.06	±0.15
Rates	mrad · s ⁻¹	±0.20	±0.03	±0.20

240 km. It is found that the mean is typically given within ±10% and a standard deviation around 15%.

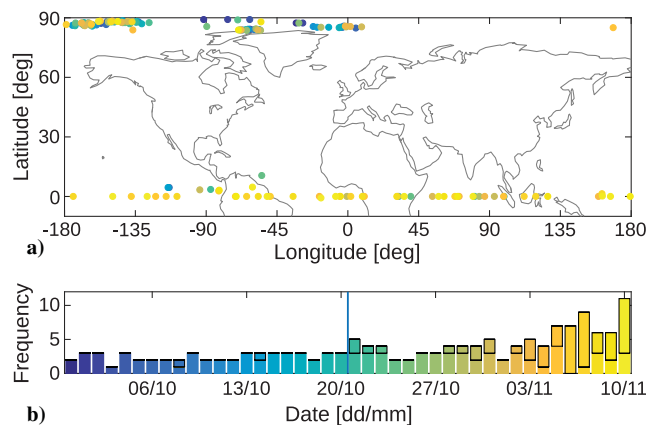
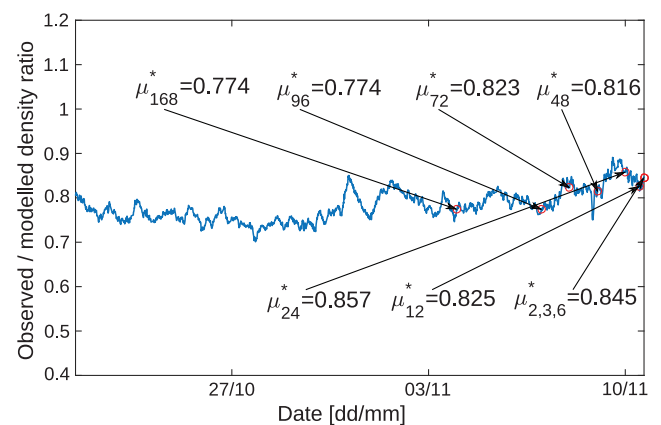
To sample the uncertainty in the atmospheric density, the values of μ^* and σ^* need to be determined. As with the initial translational state uncertainty, there are several ways of determining these values. In [16], values from literature were used. However, these values are conservative, as they are derived for many different objects and conditions simultaneously. Determining the true parameters on a per-object, or per-reentry basis, will provide more narrow and tailored uncertainty distributions. For this, object-specific data are required.

The mean density error μ^* is estimated for each starting epoch and each input set separately. The atmospheric bias is sampled at the start of the simulation and remains fixed throughout a single simulation. The atmospheric uncertainty σ^* is assumed to be 1.082 for GPS and derived from analyzing the GPS-derived accelerations in the next section. Because of the limited temporal resolution of TLE-derived density ratios (see Sec. III.D.2), σ^* is assumed to be 1.13, in accordance with the uncertainty found for several test objects [16].

1. GPS-Derived Density Estimates

To estimate the atmospheric density uncertainty parameters, GPS-derived nongravitational accelerations are used, as obtained in [33]. These accelerations are assumed to primarily consist of the aerodynamic contribution. Modeled aerodynamic accelerations are computed using attitude solutions, the NRLMSISE-00 atmospheric model, and the aerodynamic database. The differences between the modeled and observed accelerations are explained by (i) random errors in the density modeling, (ii) systematic errors in density models, and (iii) systematic errors in the aerodynamic models. The contributions of (ii) and (iii) are coupled through Eq. (1) and estimated jointly as the mean density error μ^* . The contribution of (i) is estimated as σ^* .

Figure 6 shows the 10-min averaged density ratios. The estimate of σ^* is obtained by subtracting the 10-min averaged signal from the

**Fig. 5** Overview of TLEs as a function of longitude and latitude (a) and frequency of TLEs per day (b), where the portion of TLEs are outlined.**Fig. 6** Estimated mean density error μ^* for GPS-based density estimates.

10 Hz signal. The value for the entire analysis period is used and estimated to be 1.082, as mentioned earlier. It is assumed that it is representative for the reentry period as well.

For the estimates of μ^* the average over one orbital revolution was taken at several epochs before reentry: 168, 96, 72, 48, 24, 12, 6, 3, and 2 h. These values are indicated in Fig. 6. The method allows for very accurate predictions of the atmospheric uncertainty.

2. TLE-Derived Density Estimates

The drawback of the GPS-derived density estimates is that it relies on accurate and frequent GPS solutions. GPS data are generally not available or existent for most objects in the SATCAT.

For TLE states the ratio of the observed and modeled ballistic coefficients is derived instead, which is similar to the ratio of observed and modeled aerodynamic accelerations, and can be used to estimate the mean density error μ^* . The density variability σ^* , however, cannot be estimated using this method, due to the low frequency of the TLEs (in the order of two per day). This value needs to be taken from literature instead. Moreover, the mean density-error obtained using TLEs is considered less accurate, as it presents the average over a longer period.

The ballistic coefficient is already reported as β^* as part of the TLEs. During orbit determination, the estimate of β^* includes the difference between the true and predicted atmospheric density by the simple exponential density model of SGP4, as well as other modeling deficiencies. Estimates of β^* often have large spikes and negative values, and are therefore not used in this study.

Instead the ballistic coefficient is obtained through retrofitting. A history of the semimajor axis is used to determine the rate of change of the orbital energy. The energy and semimajor axis are related through the following equation for an unperturbed two-body problem:

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \quad (10)$$

where $\mathcal{E}$ is the energy per unit mass. The work done by the atmosphere can be approximated by

$$P_A = \mathbf{v} \cdot \mathbf{a}_A = -\frac{1}{2} K \rho n^3 r^3 \left(1 - 2 \frac{\dot{\theta}}{n} \cos i \right) \quad (11)$$

where K is the ballistic coefficient, P the work, a the acceleration, $\dot{\theta}$ the Earth's rotational rate, i the inclination, and n the mean motion.

The energy at each state is computed and numerically differentiated to compute the rate of change of the energy, $\dot{\mathcal{E}}$. By assuming that the atmospheric drag dominates other nonconservative perturbations, Eq. (11) can be equated to $\dot{\mathcal{E}}$, from which an estimate of K can be obtained.

Figure 7 shows the result of the ballistic coefficient retrofitting for both TLE types (solid lines) as well as the ballistic coefficients reported by the TLEs (dashed lines). The β^* from the TLEs was related to the ballistic coefficient K using the following formula^{††}:

$$\beta^* = \frac{\rho_0 K}{2} \quad (12)$$

Discrepancies between ballistic coefficients obtained using retrofitting and those reported by the TLEs originate from errors absorbed by β^* during the orbit determination, and differences between NRLMSISE-00 and the atmospheric model of SGP4. Overall, eTLE solutions appear to be much more consistent and stable. For cTLEs, a similar trend (with a clear bias) between $c\beta^*$ and cRF can be observed until November 5, 2013. For the remaining period this difference diverges, due to increasing differences between the two atmosphere models. For the same (final) period the retrofit is

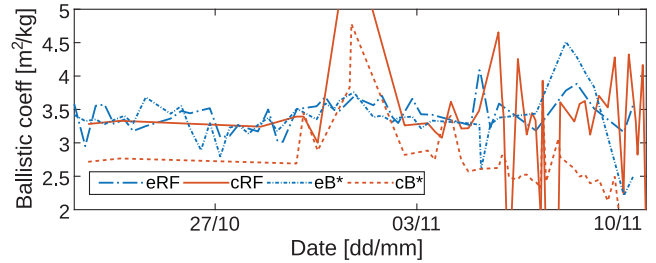


Fig. 7 Ballistic coefficients for both TLE types as estimated using retrofitting and as reported in the TLEs.

seen to fluctuate greatly, due to increasing variations in TLE accuracy. The mean, however, remains stable. Especially for cTLEs, it is recommended to use smoothing to obtain a better estimate of the ballistic coefficient at the cost of further reducing the temporal resolution.

The modeled ballistic coefficients are obtained from the aerodynamic model for the same period, and evaluated at the TLE epochs. A constant attitude $\alpha_a = \beta_a = 0^\circ$ is assumed, whereas the speed ratio is derived from the TLE states. The ratio of the retrofit and modeled ballistic coefficients is used to estimate μ^* . The value has to be obtained by smoothing the results over several TLEs, resulting in $\mu^* = 0.810$ for the entire reentry period under consideration.

IV. Results

First, the effect of the controller and atmospheric modeling is investigated in Sec. IV.A. Second, the differences between the GPS-based and TLE-based reentry predictions are explored in Sec. IV.B. Third, the statistical uncertainty propagation is compared against simplified uncertainty propagation methods in Sec. IV.C. For these first three studies, GPS-based predictions starting at 48 h before reentry are used as a baseline with 1000 samples. Fourth, the sensitivity of the predictions to the uncertainty parameters for various models and starting epoch is investigated in Sec. IV.D. Fifth, GPS-based predictions are made at several epochs before the true decay epoch. These predictions are compared with the multiple historical TIP messages and SDO predictions in Sec. IV.E.

A. Controller and Atmospheric Modeling

To investigate the effect that the modeling has on the simulation results, several cases are ran and compared. These cases focus on the comparison of modeled, ideal, and no control (cases A–C), and the effect of the atmospheric uncertainty (cases A, D, and E). Table 9 gives an overview of the different settings for each case.

Note that case A presents the baseline, with all models implemented, and translational and rotational motion simulated. The logarithmic standard deviation of atmospheric uncertainty σ^* as reported in literature is used, to highlight the proportionality of this uncertainty in general, compared with other error sources.

The results are shown in Fig. 8. The decay-time distributions are shown as the probability density as a function of the simulation time. Note that the simulation start epoch was exactly 48 h before the decay epoch.

Case A overestimates the true decay time, having a mean of 53 h (10% more). The width of the distribution is rather large, that is, 40 h (3σ bounds), covering over 26 orbits.

Table 9 Overview of simulation cases for different modeling scenarios

Case	DOF	Attitude control	μ^*	σ^*
A	6	Yes	0.816	1.13
B	3	Ideal	0.816	1.13
C	6	No	0.816	1.13
D	6	Yes	0.980	1.13
E	6	Yes	0.816	1.06

^{††}FAQs: Two-line Element Set Format, <http://celestrak.com/columns/v04n03/> [retrieved 8 August 2017].

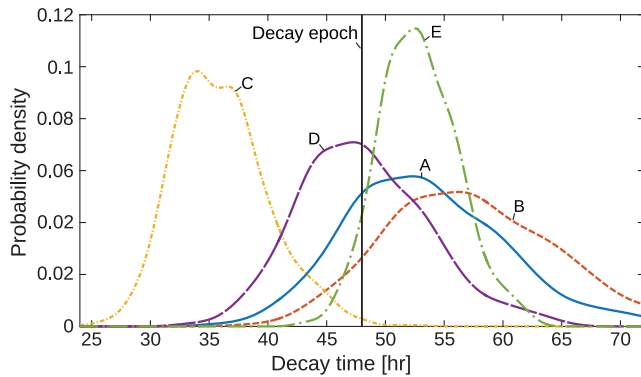


Fig. 8 Decay-time distributions for different modeling scenarios, compared with the true reentry epoch.

Case B presents several simplifications: only translational motion (3DOF) and the influence of the drag is simulated, reducing the state vector from 13 to 6 variables. The major advantage of 3DOF is that the computational efficiency is much higher, as the variable step-size integrator is not limited by the control frequency. Because the aerodynamic angles of $\alpha_a = \beta_a = 0^\circ$ have been fixed, the drag coefficient only varies with Mach number. The same aerodynamic database of the 6DOF simulation is used for the computation of C_D for the 3DOF simulation. As can be seen, the resulting decay-time distribution for this case is shifted toward the right by 4.3 h, overshooting the true decay time by almost 10 h. This case represents an overly idealized attitude-control scenario. Small aerodynamic angles of the order of the rotational uncertainty during controlled motion, the aerodynamic lift contribution, and uncontrolled motion after controller saturation are not accounted for. The width of the distribution is about 10% wider than for case A, which is caused by the shift in median.

Case C illustrates the effect of an uncontrolled attitude. The controller, together with the aerodynamic shape of GOCE, makes the reentry unique and difficult to predict. Compared with stable motion the drag of tumbling motion is nearly 3 times higher, shortening the reentry significantly. The results show that the inclusion of a controller is crucial for the proper reentry simulation of GOCE and by extension other controlled satellites. The median is situated at 36 h. The spread is similar to before at 25 h, or 70%.

Cases B and C present limit cases for the controlled reentry. Underestimation of the maximum control torques will result in impact predictions between cases C and A, whereas overestimation of the maximum torques will result in predictions within cases A and B.

Case D illustrates the effect of not accounting for the bias in the atmospheric density. Such a case presents itself when the mean density error cannot be estimated due to insufficient knowledge of the attitude and/or spacecraft mass and geometry. In this case values from literature have to be used, as was done in [16]. The result shows that for GOCE the density would be overestimated, causing a faster reentry than in reality. Together with case C, the atmospheric-density bias presents the major driver in prediction accuracy. Clearly, estimating the bias improves the results significantly and should be applied whenever possible.

Case E presents what would happen if the atmospheric uncertainty were halved. Ongoing research into atmospheric models shows great potential for reducing the atmospheric uncertainty. The PDF in this case is much narrower, reducing the width of the PDF to 25 h (at 3σ bounds). The large difference highlights the importance of reducing

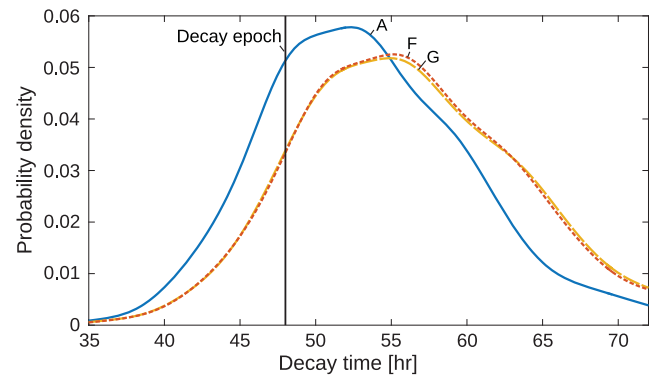


Fig. 9 Decay-time distributions for TLE-derived states and uncertainty parameters, compared with GPS baseline prediction and the true reentry epoch.

the atmospheric uncertainty. Moreover, the averaged density bias used for case D, as derived in [16], is likely too close to one and should in reality be smaller. Especially, considering the mean bias of $\mu^* = 0.93$ obtained using the Jacchia dataset for the 200–400 km range, as presented in [24]. Unfortunately, data to accurately assess the uncertainty of atmosphere models in the 120–200 km range are extremely limited.

B. GPS-Based Versus TLE-Based Predictions

Different uncertainty estimation methods were derived and presented in Secs. III.B and III.D. Based on the availability of GPS solutions, a more accurate initial state and better atmospheric uncertainty parameters can be obtained. The GPS-based reentry predictions are compared against the more general TLE-based predictions. For TLEs a distinction is made between enhanced and classic TLEs.

Figure 9 shows the result of cases F and G compared with case A. Cases F and G show the results for the enhanced and classic TLEs, respectively. The enhanced TLEs were shown to have better accuracy and reduced error growth for forward propagation. The improved propagation stability of enhanced TLEs is limited to SGP4 and does not necessarily improve the prediction ability. The improved accuracy at epoch, however, causes a small, but positive difference.

Compared with case A both decay-time distributions are wider, which is to be expected due to increased uncertainty in the initial state. The 3σ width is 54 and 56 h for enhanced and classic TLEs, respectively. TLE-based predictions are found to be about 35% wider than the GPS-based predictions. Moreover, a shift toward the right can be observed for both TLE predictions, due to the slightly lower estimated atmospheric-density bias as a result of the smoothing process.

However, considering that the positional accuracy is on the order of decimeters for GPS and kilometers for TLEs, the difference in reentry prediction uncertainty is not that large. In contrast to the atmospheric-density bias, the positional uncertainty has only a moderate effect on the final decay-time distributions. By extension, the differences between cases F and G are even less. Enhanced TLEs were found to be twice as accurate, but this only results in a 4% reduction of the final uncertainty window. Table 10 gives an overview of the methods used to obtain the error model parameters.

C. Simplified Error Propagation

Commonly, simplified strategies are employed to propagate the error in time. Benefits of such strategies are low computational cost and a reduced (uncertainty) modeling effort. Two common approaches are investigated further and compared with statistical error propagation. In both approaches, only the nominal state of the spacecraft is propagated to obtain the median of the final decay-time distribution. The uncertainty window is then derived, either by placing an empirical uncertainty on the nominal state or by performing two additional propagations to obtain the bounds.

In the case of empirical uncertainty bounds, generally the uncertainty is assumed to be uniform over the remaining orbital lifetime

Table 10 Overview of simulation cases for different modeling scenarios

Case	Source	State	Atmosphere
A	GPS	OD residuals	Accelerations
F	eTLE	Differencing	RF, literature
G	cTLE	Differencing	RF, literature

on top of the nominal decay time. These bounds are derived by studying historic re-entries. Lefebvre [6] shows that confidence intervals of TIPs range from 93 to 98%, for orbital lifetimes ranging from 7 days to 3 h, respectively. The assumption that the uncertainty window width is always $\pm 20\%$ provides a good rule of thumb for estimating 2σ bounds. This assumption, however, is population based and does not necessarily extend well to individual (future) cases. In the case of propagating additional samples to obtain the bounds, commonly the ballistic coefficient is varied $\pm 20\%$ to obtain samples for the lower and upper limit of the uncertainty.

Both approaches assume that the nominal state represents the uncertainty median. Furthermore, the first approach assumes that the uncertainty is symmetrical about the assumed median, whereas the second approach assumes a symmetrical uncertainty in the product of the atmospheric and aerodynamic uncertainty, yielding an asymmetric distribution in time.

To investigate these two approaches further, their results are compared against the statistical reentry predictor. The starting epoch is chosen to be 48 h before the actual decay. At this epoch both 6DOF and TIP predictions were found to be close to each other, but overestimating the true decay time significantly.

Figure 10 shows the reentry predictions. The true impact is indicated using the solid black vertical line at 48 h. For the 6DOF statistical reentry predictions, the obtained uncertainty distribution is visualized using a histogram (using 1 h bins) and KDE. Moreover, the 2.5σ (98.758%) confidence interval is indicated using the dashed lines. This value represents the upper limit of the TIP confidence. The actual TIP results are plotted as the three dash-dotted lines, to demonstrate the empirical uncertainty bounds. Lastly, the approach using varying values of the ballistic coefficient is shown using the long-dashed lines. The different variations are indicated by their F_K value, where $F_K = 1 \pm 0.20$ represents the $\pm 20\%$ case described above. In total nine different values for F_K were chosen (including the nominal case $F_K = 1$), to illustrate the effect of using different values.

The median of the statistical error propagation is comparable to the TIP prediction for this epoch. The nominal single propagation corresponding to $F_K = 1$ also lies very close to the median of the statistical propagation. Such agreement only occurs when the uncertainties of the translational and rotational state the distributions are relatively symmetrical and unbiased. For this case a good estimate of the median of the decay-time uncertainty can be obtained using a single propagation of the nominal state. Note that this holds only for a nominal 6DOF propagation and not a single 3DOF propagation. It

was previously demonstrated that the nominal 3DOF propagation would overshoot the true decay time even further.

As can be seen, the uncertainty distribution is asymmetrical. The upper limit is further away from the median than the lower limit, which is a characteristic of reentry distributions. Both the TIP window (TW) and the 2.5σ window of the KDE (KW) are of comparable width: 1440 and 1463 min, respectively. However, by default the uncertainty of the TIP is symmetrical in time. The lower bound of the TIP overestimates the uncertainty, whereas the upper bound fails to capture the full width of the uncertainty.

Looking at the four pairs of equal variation to the ballistic coefficient, $F_K = 1 \pm (0.10, 0.12, 0.15, 0.20)$, it can be seen that resulting windows are indeed asymmetric in time. These windows are, however, much more asymmetric than the uncertainty distribution obtained using the statistical approach. The lower bound of $F_K = 0.88$ shows good agreement with the statistical window, but its corresponding upper bound of $F_K = 1.12$ overshoots by more than an hour. The propagation $F_K = 0.88, 1.00, 1.11$ would likely result in the best agreement with the statistical method in this case. The earlier described $F_K = 1 \pm 0.20$ results in too conservative estimates. It is apparent that the bounds are very sensitive to the choice of F_K . Considering the results of this study, the choice of F_K depends highly on the satellite under consideration, the remaining orbital lifetime, and the propagation setup.

D. Sensitivity Analysis

1. Introduction

The full uncertainty model is 13-dimensional. To investigate which parameters drive the uncertainty, a sensitivity analysis is performed. The sensitivities of the reentry simulation with respect to the uncertainty parameters are computed for various degrees of modeling and for several starting epochs.

For the sensitivity analysis the extended Fourier amplitude sensitivity test (eFAST) method is used [36,37]. eFAST is suitable for analyzing the sensitivities of nonlinear systems. The method varies the input parameters along a sinusoidal search curve of a particular frequency, causing a variation in simulation output. eFAST decomposes the variance of the output to the input parameters by Fourier analysis. The search curve has to be sampled with enough fidelity to recover the search-curve frequencies in the Fourier analysis. The minimum recommended number of samples, N_S , is 65. Moreover, the curves are re-sampled N_R times. The total number of samples is thus $N = N_S N_R k$, where k is the number of uncertainty parameters.

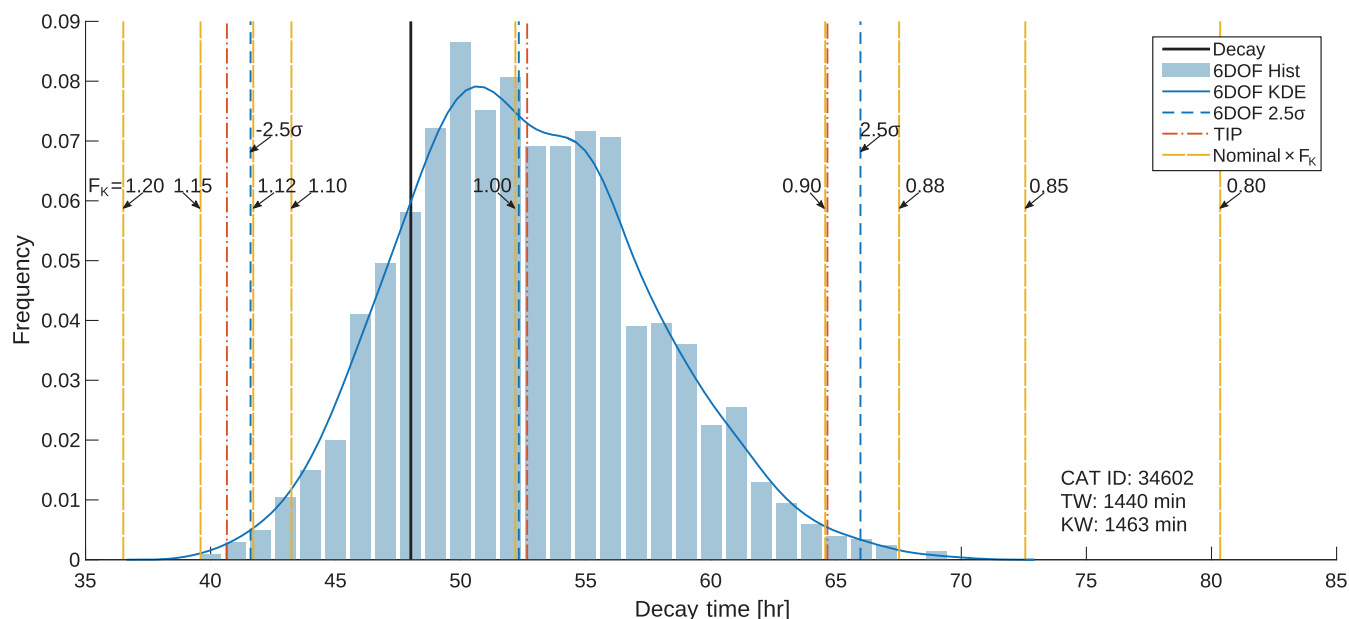


Fig. 10 Statistical error propagation compared with simplified error propagation strategies for performing reentry predictions of GOCE 48 h before reentry.

The primary outputs of the eFAST method for the analysis are the first-order and total-order sensitivity indices. Both indices assume values between 0 and 1, where 0 indicates no influence and 1 indicates complete dependence. The first-order sensitivity index is the variance of the output at the parameter's particular frequency divided by the total output variance. The total-order sensitivity index is calculated as the remaining variance after the contribution of all other parameters is removed, and includes nonlinear and correlated effects.

An additional uncertainty parameter is introduced, which is not used in the reentry simulator. This parameter is referred to as the dummy parameter and has no influence on the output. Ideally, the sensitivity indices of the dummy parameter should be zero. However, due to the stochastic nature the indices will be nonzero. The sensitivity to the dummy is used to test for significance of the other sensitivities. Parameters with a sensitivity index of less than or equal to the dummy sensitivity should be considered not significantly different from zero.

The sensitivity analysis is performed for three different simulation setups: 3DOF, controlled 6DOF, and uncontrolled 6DOF. Moreover, for most time-dependent models the sensitivity of the output to its parameters is also time dependent. To investigate this dependency further the sensitivity indices are computed for simulations starting at several values of the remaining orbital lifetime.

The number of search curves per parameter is chosen to be 5 with 260 samples per curve. Previous trials were conducted with $N_S = 65$ and $N_S = 130$, but resulted in high-sensitivity indices for the dummy parameter. As the atmospheric uncertainty is the primary contributor to the total uncertainty, the other parameters are relatively small in comparison. Including the dummy parameter and with the chosen settings for the search curves, this results in $N = 18200$ samples for 6DOF and $N = 10400$ for 3DOF per analysis.

The sensitivity indices were found to be severely affected by outliers in the data. A routine was therefore implemented to check for outliers in the results. The studentized residuals of the samples of each search curve were calculated separately. Samples with a residual of higher than 4 were labeled as outliers. Figure 11 shows an example of the simulation output along a single search curve with outliers. Two types of outliers are present in the figure: a null result (caused by a crash in the simulation) and an unrealistic reentry time. These spikes cause interference in the Fourier analysis and can throw off the resulting indices significantly. The values of the outliers are replaced by a linearly interpolation of neighboring samples. The occurrence of outliers was found to be well below 1%.

2. Model Sensitivity

Figure 12 shows the result of the eFAST analysis for all three simulation setups at two different starting epochs. The starting

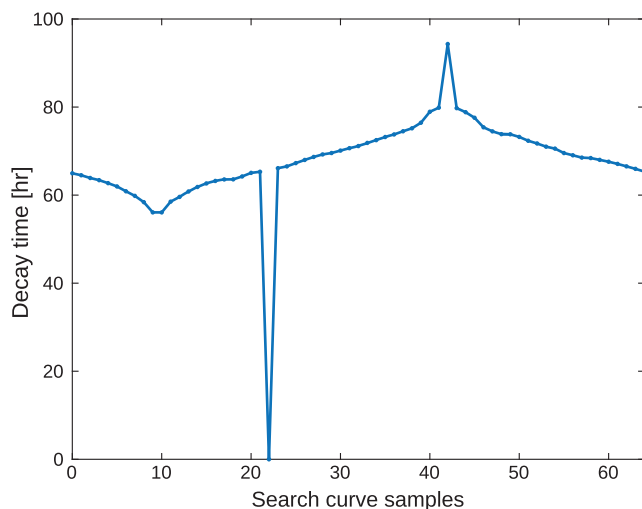


Fig. 11 Example of two types of outliers in the simulation output results along a single search curve for the radial velocity.

epochs chosen for this visualization are 12 and 2 h of remaining orbital lifetime. The epochs were selected close to reentry, yet significantly far apart in time. The model was found to be more sensitive to other uncertainty parameters close to reentry, except the atmospheric uncertainty, resulting in higher values for the sensitivities in general and values further outside the dummy's range. The time dependency and sensitivities at earlier epochs will be analyzed in more detail in the next section.

Figure 12 shows the first-order and total-order sensitivity indices for each parameter and each setup. The different setups are distinct by the different colors, where the first-order sensitivity index is hatched. Moreover, by default the total-order sensitivity index is always equal or greater than the first-order sensitivity.

It is immediately clear from Fig. 12 that for any setup at any starting epoch, the simulation is most sensitive to the atmospheric density, ρ . For this reason the bars of the remaining parameters are shown on a scale that is four times smaller. Two factors diminish the sensitivity of the predictions to the atmospheric density: the remaining orbital lifetime (less time is spent in the atmosphere) and the growing influence of other parameters. The atmospheric density sensitivity is almost completely made up of the first-order sensitivity index, confirming that the atmospheric density has a very direct contribution to the reentry predictions.

The sensitivity of the dummy parameter is indicated as d in the figure. Using the increased number of samples, false first-order sensitivities were reduced to negligible values. Increasing the number of samples does not significantly reduce the total-order sensitivity, but primarily affects its precision. At the selected number of samples the sensitivity indices of the dummy parameter were consistently $S_d < 0.0005$ and $S_{T_d} < 0.01$. Convergence of these values was further confirmed by performing t -tests on the indices. Moreover, it was found that values of dummy sensitivities tend to decrease with the introduction of additional parameters and shorter propagation times. This can be seen in the figure, where the S_{T_d} for the uncontrolled 6DOF set up at 2 h is smallest.

Among the translational parameters, the along-track velocity component is consistently found to be the primary first-order driver of the final uncertainty. The radial-position component is the second most influential translational parameter. Although these two parameters are known to be highly correlated, their sensitivities differ due to the magnitude and bias of their uncertainty. The sensitivity indices of both parameters also increase closer to reentry and will be further investigated in the next section.

For the rotational parameters of the controlled reentry, the first-order sensitivity indices are negligible and the total-order indices are at the threshold of the dummy parameter's level. Therefore, these sensitivities should be considered nonsignificant from zero. The insignificance of the rotational parameter is caused by their small initial uncertainties and the rotational control. Regarding the other parameters, the sensitivities of the controlled 6DOF and 3DOF setups are very similar.

For the uncontrolled 6DOF setup the aerodynamic angles α_a and β_a were found to be the most influential parameters. The lesser significance of σ_a can be explained by the high-degree of rotational symmetry GOCE around the X axis. The larger influence of β_a is due to the slightly larger projected area in the XZ plane compared with the XY plane. The minor first-order sensitivity to the initial pitch and yaw rates (q and r , respectively) can be attributed to their small initial values and aerodynamic stability of GOCE. Conversely, the roll has a significant first-order sensitivity and the highest total-order of the rotational rates.

From the uncontrolled case it can also be seen that introduction of additional significant parameters yields an increase in the total-order sensitivity of parameters with weak first-order sensitivities. It is difficult to assign the true significance to such parameters and this primarily demonstrates the interdependency of translational and rotational state.

3. Time Dependency

Previously, it was already shown that the sensitivity indices at 12 and 2 h of remaining orbital lifetime are slightly different. To

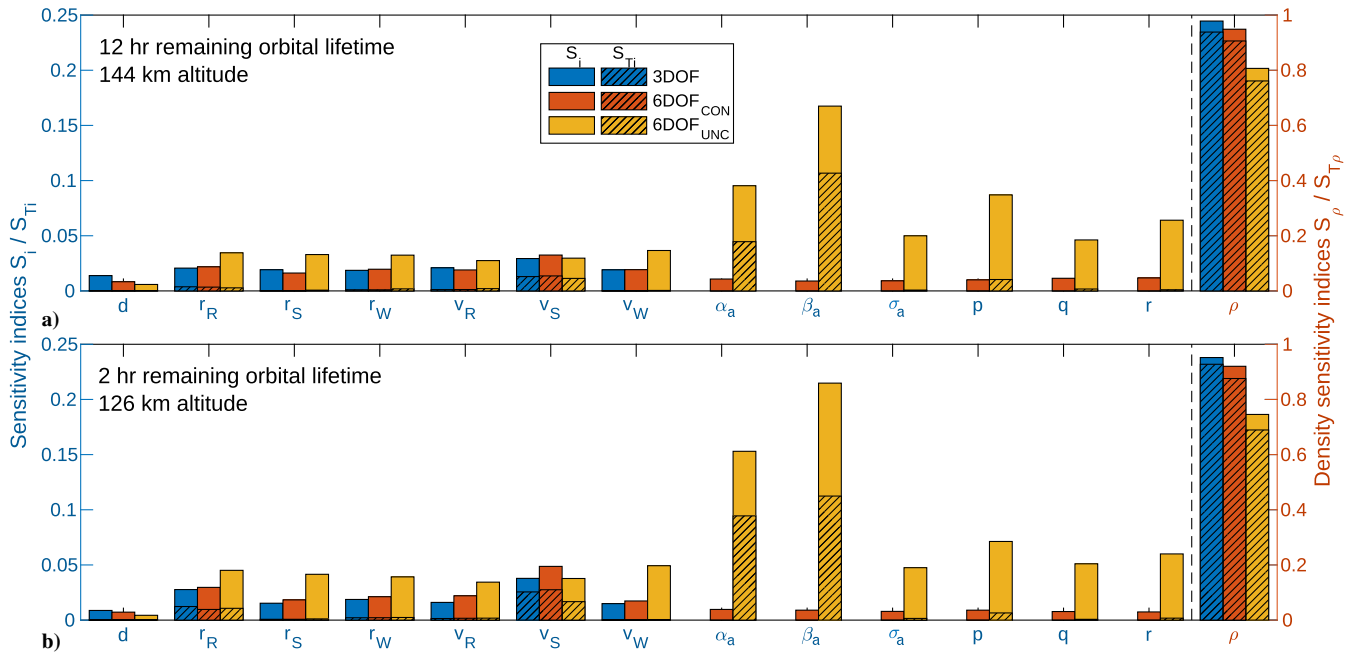


Fig. 12 Sensitivity indices of the uncertainty parameters for various simulation setups at two different starting epochs: a) 12 h and b) 2 h remaining orbital lifetime.

investigate the time dependency of these parameters in more detail the analysis is repeated for other starting epochs, namely, at 168, 96, 72, 48, 24, 12, 6, 3, and 2 h of remaining orbital lifetime. Figure 13 shows the time evolution of the sensitivity indices. Only the parameters with nonsignificant sensitivities are shown. The sensitivity indices of the atmospheric density are not shown in the figure. Its sensitivity indices slowly decrease as the others increase, starting at $S_\rho = 0.9563$ and $S_{T_\rho} = 0.9967$ at 168 h.

Figure 13 confirms the importance of the along-track velocity component, followed by the radial position. The radial position, however, converges to the other uncertainties (the cross-track position and radial velocity) for increased orbital lifetime, whereas the along-track velocity remains clearly the dominant parameter in this model at all epochs.

Overall, the sensitivity (both first and total order) of the predictions grows sharply as the orbital lifetime decreases. In practice, this effect is expected to be only larger, as the magnitude of the uncertainty in these parameters also increases close to reentry, due to the growing influence of disturbances (such as the atmospheric density) on the orbit determination.

E. Prediction Comparison

Apart from the final decay message, seven TIP messages were released for GOCE at various epochs before reentry. The SDO has

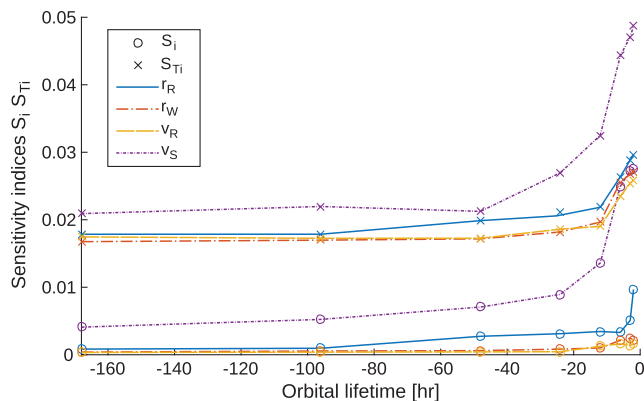


Fig. 13 Sensitivity of all nonsignificant translational parameters of the controlled 6DOF model with respect to time.

similarly performed several predictions before reentry. SDO predictions started on October 23, 2013, with the assumption that the attitude controller would fail for a 20 mN drag threshold, resulting in tumbling motion. These early predictions placed the reentry at around November 6. From November 1, 2013, the threshold was raised to 50 mN. Later, the case where the attitude controllers would fail only very late into the reentry was considered [22]. Only the predictions resulting from this latter case are compared.

For the 6DOF predictions, it is chosen to start the reentry predictions at common TIP moments before the true reentry epoch; these are 2 h, 3 h, 6 h, 12 h, 1 d, 2 d, 3 d, 4 d, and 7 d before the true decay epoch. The decay-time distribution was estimated using KDE. A 2.5σ confidence interval was used to estimate the bounds. This value was chosen for comparison against TIPs and SDO predictions. Lefebvre [6] shows that TIPs range from 93 to 98% confidence, depending on the orbital lifetime. Similarly, the SDO windows have 2σ confidence based on multiple predictions over only 15 reentry campaigns [3]. Using 2σ bounds would likely be too optimistic, whereas 3σ is not comparable to the confidence associated with the other predictions.

Figure 14 compares the resulting predictions. The horizontal axis corresponds to date of prediction, and the vertical axis to the prediction error as a percentage of TTD. For each prediction the confidence interval and median is indicated. As a general trend the relative prediction window (as a percentage of the TTD) remains roughly constant. The 6DOF, TIP, and SDO intervals are on average 46, 65, and 43% wide, respectively. All are thus wider than the 40% rule of thumb.

The windows for the TIPs were on average 41% wider than for 6DOF. A true comparison with the TIP window, of course, depends on the confidence level of the windows, for which only empirical values are available from the literature. As the width of the TIP is significantly more than the 40% described in those studies, it is assumed that the method for determining the window has changed. A true comparison is therefore very hard to make. Nonetheless, the confidence level chosen for the 6DOF windows is likely higher than TIPs and SDO predictions, while providing windows of the same order or much less. These relatively small widths, while providing higher confidence for statistical 6DOF, can be attributed to the attitude controller and improved atmospheric uncertainty modeling.

For all predictions the median of the 6DOF is found to be closer to the true reentry epoch, compared with the TIP midpoints and SDO's medians. For the SDO predictions the distributions are slightly

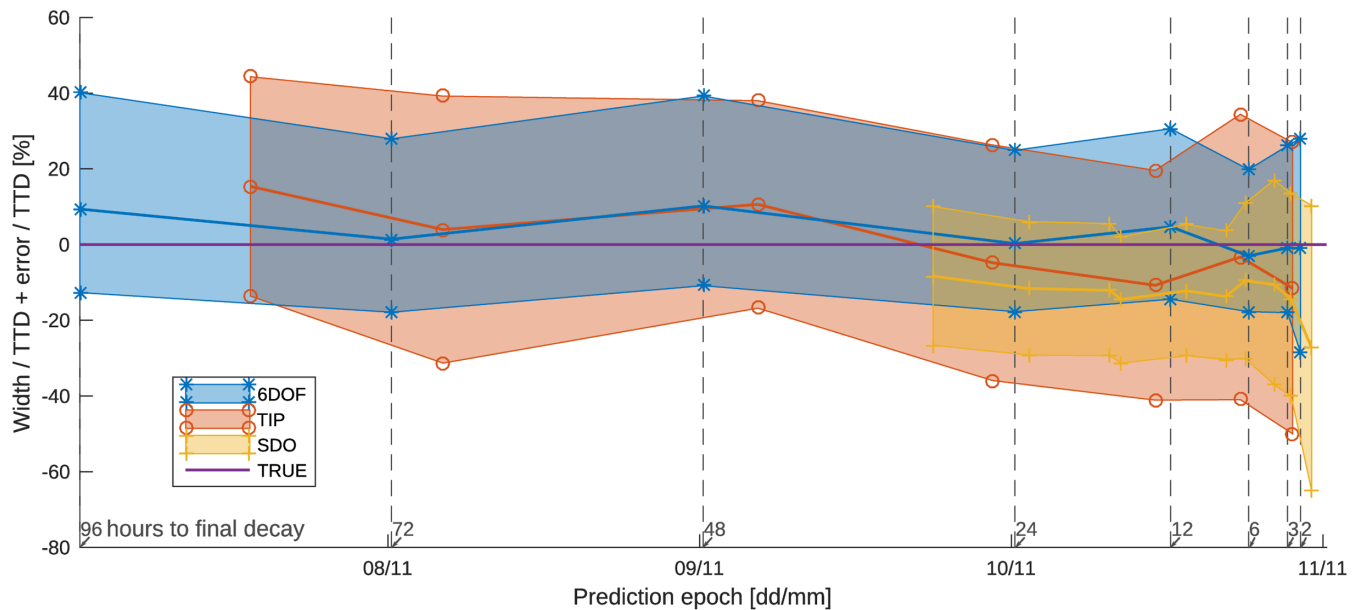


Fig. 14 Comparison of 6DOF with TIPS and SDO reentry predictions for several prediction epochs.

asymmetric, where the median is on average located at 7.3% (in terms of the total distribution width) from the midpoint toward the lower bound. The asymmetry is found to be characteristic for decay-time distributions, where long re-entries are statistically more likely than short re-entries.

Except for the final three predictions (starting at 6, 3, and 2h, respectively), the median overshoots the true decay epoch. A maximum of 10% (about 5 h) is observed for the prediction made at 48 h prior. This offset was already observed in the previous results presented. The median of all SDO predictions seems to be earlier than the true reentry epoch. The earlier medians can be explained by the modeling of the uncontrolled motion using the drag coefficient obtained from SCARAB, without accounting for the lower atmospheric densities.

V. Conclusions

The decay-time distribution accuracy for the GOCE satellite is dominated primarily by systematic errors in atmospheric density and attitude control, whereas its precision is dominated by the atmospheric uncertainty. To a lesser degree, the precision is also affected by the initial translational uncertainty.

The study proposed an uncertainty model for the atmospheric density to account for (systematic) modeling errors in the atmospheric and aerodynamic models. Subsequently, two different uncertainty estimation methods were presented for GPS- and TLE-derived states. The resulting approach produces accurate results for dealing with a biased atmosphere and/or aerodynamic model. By analyzing the density-modeling error from GPS-derived nongravitational accelerations, a better estimate of the density uncertainty could be obtained. For TLEs similar results could be obtained, although at a much coarser temporal resolution. Considering the importance of the atmospheric uncertainty, additional modeling (especially of its evolution) should improve results even further.

The attitude control is found to significantly affect the median of the reentry predictions. The predicted median for a remaining orbital lifetime of 48 h was found to lie anywhere between 17 and 55 h depending on when the controller was assumed to be saturated. Therefore, it is important to properly account for this control. By modeling the attitude control and motion in a 6DOF simulator, it is not necessary to assume a constant drag coefficient and introduce any discontinuities to this value. The assumptions on controller saturation could be reduced to maximum torques, which were sourced from the literature. Finally, systematic errors and uncertainties in the atmospheric density were included throughout the simulation.

The effect of atmospheric uncertainty on the reentry prediction uncertainty for GOCE is found to outweigh all other contributions in the uncertainty modeling effort. Halving the atmospheric uncertainty results in a halving of the final decay-time uncertainty. Moreover, the difference between TLE-derived and GPS-derived initial translational uncertainty results in a 35% improvement, despite GPS being up to four orders of magnitude more precise. In previous studies the initial rotational uncertainty was found to be another main driver for the precision, but the attitude control mostly negated this influence.

Two simplified error propagation strategies were examined. Propagating only the translational motion of a single nominal trajectory resulted in significant overshooting of the decay epoch, even for such a strongly attitude controlled satellite as GOCE. A single 6DOF propagated nominal state, however, showed good agreement with the median of the uncertainty distribution for GOCE. This agreement is due to the relatively unbiased and symmetrical initial uncertainty distributions. The empirical uncertainty window, as used by the TIPS, does not capture the asymmetry of the uncertainty distribution. Moreover, the window width is not a constant factor of the remaining orbital lifetime. Although these effects can be included in an empirical approach, it needs to be stressed that it remains based on populations of historic re-entries and does not necessarily correspond to individual future cases. Approaching the lower and upper limits of the uncertainty distribution by varying the ballistic coefficient produced better bounds. Varying the ballistic coefficient by $\pm 12\%$ produces limits with the best agreement to the statistical distributions. The case of $\pm 20\%$ produced far too conservative results. The approach showed clear asymmetry, although slightly too much, resulting in a too conservative upper bound. This suggests that perhaps different percentages should be used for the lower and upper bounds. The bounds were found to be very sensitive to these values. Selecting the best value(s) is unfortunately not straightforward and likely depends on the satellite, modeling, and also the orbital lifetime. The approach is only recommended when using a high-fidelity (6DOF) propagation model and when many samples are infeasible.

The sensitivity analysis further demonstrates that the atmospheric uncertainty is the primary driver of the decay-time distribution. For the controlled reentry the along-track velocity and radial position are of secondary importance. The first-order and total-order sensitivity of these parameters is on the order of 0.01–0.03 and 0.03–0.05, respectively. The significance of these parameters is nonnegligible for performing accurate reentry predictions. Moreover, the sensitivity analysis indicates that the atmospheric uncertainty is the primary driver of the remaining orbital

lifetime. In addition, the estimation error of these parameters also grows close to the re-entry epoch.

From the sensitivity analysis of the controlled 6DOF setup it can be concluded that the dimensions of the uncertainty model can be greatly reduced, at no significant loss of accuracy. The entire rotational uncertainty and several position and velocity components can be ignored. This should improve the performance of the reentry simulations significantly. It is, however, not possible to extend these conclusions for reentry simulations in general. Especially not on the importance of rotational uncertainty.

For the uncontrolled reentry setup it was shown that the rotational parameters were more influential than the translational parameters. In this study, the angles of attack and sideslip were found to be the dominant rotational parameters. The ranking of the rotational parameters and absolute values of the sensitivity indices is highly dependent on the initial rotational uncertainty and the aerodynamic characteristics of the satellite. The rotational uncertainty is expected to play an even larger role for other bodies, considering GOCE's unique aerodynamic shape and the very small initial rotational uncertainty. To improve the efficiency of the MC error propagation in general a more intelligent sampling strategy should be employed that takes into account the sensitivity of the uncertainty parameters.

For the 6DOF reentry predictions of GOCE, the median was found consistently closer to the true decay epoch than neighboring TIPs and predictions by the SDO of the ESA. Moreover, the statistically obtained windows are on average 41% narrower than the TIP windows. The 6DOF and SDO predictions are found to be of comparable relative width. A true comparison on the width is difficult as the associated confidence can only be empirically determined, for which not enough cases have been studied. Nonetheless, statistical 6DOF predictions provide smaller or equal windows with a high confidence for GOCE. For noncontrolled slender bodies, such as rocket bodies, the windows at the same confidence level will likely be much wider.

The results demonstrate that by modeling the uncertainties per object and performing proper error propagation, the resulting distributions are tailored specifically to the reentry under consideration. It is recommended to perform statistical 6DOF reentry predictions for objects for which aerodynamic models are available or can be easily constructed.

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References

- [1] Klinkrad, H., Fritsche, B., and Koppenwallner, G., "Re-Entry Prediction and On-Ground Risk Estimation," *Space Debris—Models and Risk Analysis*, edited by H. Klinkrad, Springer-Verlag, Berlin, Chap. 9.
- [2] Strizzi, J. D., "An Improved Algorithm for Satellite Orbit Decay and Re-Entry Predictions," M.Sc. Thesis, Massachusetts Inst. of Technology, Cambridge, MA, 1993.
- [3] Klinkrad, H., "Methods and Procedures for Re-Entry Predictions at ESA," *Proceedings of 6th European Conference on Space Debris*, edited by L. Ouwehand, Vol. 6, ESA Communications SP-723, Noordwijk, The Netherlands, April 2013.
- [4] Anselmo, L., and Pardini, C., "Satellite Reentry Predictions for the Italian Civil Protection Authorities," *Acta Astronautica*, Vol. 87, No. 1, 2013, pp. 163–181.
doi:10.1016/j.actaastro.2013.02.004
- [5] Ikeda, S., Tajima, T., Abe, J., and Matsuda, I., "Improved Re-Entry Prediction Method," *SpaceOps Conference*, AIAA Paper 2014-1836, 2014.
- [6] Lefebvre, S., "An Analysis of Tracking and Impact Predictions," M.Sc. Thesis, Air Force Inst. of Technology, AFIT/GSO/ENG/91D-11, Wright-Patterson AFB, OH, 1991.
- [7] Storz, M. F., Bowman, B. R., Branson, J. I., Casali, S. J., and Tobiska, W. K., "High Accuracy Satellite Drag Model (HASDM)," *Advances in Space Research*, Vol. 36, No. 12, 2005, pp. 2497–2505.
- [8] Doornbos, E., "Thermospheric Density and Wind Determination from Satellite Dynamics," Ph.D. Thesis, Delft Univ. of Technology, Delft, The Netherlands, 2011.
- [9] Visser, P., Doornbos, E., van den IJssel, J. A. A., and Da Encarnação, J. T., "Thermospheric Density and Wind Retrieval from Swarm Observations," *Earth, Planets and Space*, Vol. 65, No. 11, 2013, pp. 1319–1331.
doi:10.5047/eps.2013.08.003
- [10] Vallado, D. A., and Finkleman, D., "A Critical Assessment of Satellite Drag and Atmospheric Density Modelling," *Acta Astronautica*, Vol. 95, No. 1, 2014, pp. 141–165.
doi:10.1016/j.actaastro.2013.10.005
- [11] Wit, T. D. D., and Bruinsma, S., "The 30 cm Radio Flux as a Solar Proxy for Thermosphere Density Modelling," *Journal of Space Weather and Space Climate*, Vol. 7, No. A9, 2017, pp. A9-p1–A9-p11.
doi:10.1051/swsc/2017008
- [12] Crowther, R., "Re-Entry Aerodynamics Derived from Space Debris Trajectory Analysis," *Planetary and Space Science*, Vol. 40, No. 5, 1992, pp. 641–646.
doi:10.1016/0032-0633(92)90004-8
- [13] Bowman, B. R., and Hrcir, S., "Drag Coefficient Variability at 100-300 km From the Orbit Decay Analyses of Rocket Bodies," *Advances in the Astronautical Sciences*, Vol. 129, No. 1, American Astronautical Soc. Paper 07-262, San Diego, CA, 2008.
- [14] Koppenwallner, G., Fritsche, B., Lips, T., and Klinkrad, H., "SCARAB—A Multi-Disciplinary Code for Destruction Analysis of Spacecraft During Re-Entry," *Proceedings of the 5th European Symposium on Aerothermodynamics for Space Vehicles*, edited by D. Danesy, ESA Publications Div. SP-563, Noordwijk, The Netherlands, Nov. 2004.
- [15] Desai, P. N., Schoenenberger, M., and Cheatwood, F. M., "Mars Exploration Rover Six-Degree-of-Freedom," *Journal of Spacecraft and Rockets*, Vol. 43, No. 5, 2006, pp. 1019–1025.
doi:10.2514/1.6008
- [16] Ronse, A., and Mooij, E., "Statistical Impact Prediction of Decaying Objects," *Journal of Spacecraft and Rockets*, Vol. 51, No. 6, 2014, pp. 1797–1810.
doi:10.2514/1.A32832
- [17] Neuenfeldt, B., "A Survey of Uncontrolled Satellite Reentry and Impact Prediction," M.Sc. Thesis, Naval Postgraduate School, Monterey, CA, 1993.
- [18] Terejanu, G., Singla, P., Singh, T., and Scott, P. D., "Uncertainty Propagation for Nonlinear Dynamic Systems Using Gaussian Mixture Models," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 6, 2008, pp. 1623–1633.
doi:10.2514/1.36247
- [19] Prabhakar, A., Fisher, J., and Bhattacharya, R., "Polynomial Chaos-Based Analysis of Probabilistic Uncertainty in Hypersonic Flight Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 1, 2010, pp. 222–234.
doi:10.2514/1.41551
- [20] Halder, A., and Bhattacharya, R., "Dispersion Analysis in Hypersonic Flight During Planetary Entry Using Stochastic Liouville Equation," *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 2, 2011, pp. 459–474.
doi:10.2514/1.51196
- [21] Plazolles, B., Spel, M., Rivola, V., and El Baz, D., "Monte-Carlo Analysis of Object Reentry in Earth's Atmosphere Based on Taguchi Method," *Proceedings of the 8th European Symposium on Aerothermodynamics for Space Vehicles*, ESA Communications, Noordwijk, The Netherlands, 2015.
- [22] Virgili, B. B., Flohrer, T., Lemmens, S., and Krag, H., "GOCE Re-Entry Campaign," *Proceedings of the 5th International GOCE User Workshop*, edited by Ouwehand, L., ESA Communications SP-728, Noordwijk, The Netherlands, 2015.
- [23] Marcos, F. A., "Accuracy of Atmospheric Drag Models at Low Satellite Altitudes," *Advances in Space Research*, Vol. 10, Nos. 3–4, 1990, pp. 417–422.
doi:10.1016/0273-1177(90)90381-9
- [24] Picone, J. M., Hedin, A. E., Drob, D. P., and Aikin, A. C., "NRLMSISE-00 Empirical Model of the Atmosphere: Statistical Comparisons and Scientific Issues," *Journal of Geophysical Research*, Vol. 107, No. A12, 2002, pp. SIA 15-1–SIA 15-16.
doi:10.1029/2001JA001606

- [25] King-Hele, D., *Satellite Orbits in an Atmosphere: Theory and Applications*, Blackie and Son Ltd., Glasgow, U.K., 1987, pp. 22–23.
- [26] Anderson, J., *Hypersonic and High-Temperature Gas Dynamics*, AIAA, Reston, VA, 2006, pp. 61–63.
- [27] Dumontel, M., Bacchetta, A., Vinai, B., Allasio, A., and Berto, A., “GOCE Stand-Alone Aerodynamic Model How-to,” Thales Alenia Space, Tech. Rept. GO-TN-AI-0179, Turin, Italy, 2010.
- [28] Blanchard, R. C., and Buck, G. M., “Rarefied-Flow Aerodynamics and Thermosphere Structure from Shuttle Flight Measurements,” *Journal of Spacecraft and Rockets*, Vol. 23, No. 1, 1986, pp. 18–24. doi:10.2514/3.25078
- [29] Wilmoth, R. G., Blanchard, R. C., and Moss, J. N., “Rarefied Transitional Bridging of Blunt Body Aerodynamics,” *Proceedings of the 21st International Symposium on Rarefied Gas Dynamics*, edited by Brun, R., Cépadues Editions Rept. EOARD CSP 98-1024, Toulouse, 1999.
- [30] Sechi, G., André, G., Andreis, D., and Saponara, M., “Magnetic Attitude Control of the GOCE Satellite,” *6th International Conference on Guidance, Navigation & Control*, edited by D. Danesy, ESA Publ. Div. SP-606, Noordwijk, The Netherlands, 2006.
- [31] Mooij, E., *The Motion of a Vehicle in a Planetary Atmosphere*, Series 08: Astrodynamics and Satellite Systems 01, Delft Univ. Press, Delft, The Netherlands, 1997, pp. 71–90.
- [32] Klinkrad, H., “On the Use of Atmosphere Models in Re-Entry Predictions,” *Symposium Proceedings on Environment Modelling for Space-Based Applications*, edited by W. Burke, and T. D. Guyenne, ESA Publ. Div. SP-392, Noordwijk, The Netherlands, 1996, pp. 287–298.
- [33] Visser, P. N. A. M., and van den IJssel, J. A. A., “Orbit Determination and Estimation of Non-Gravitational Accelerations for the GOCE Reentry Phase,” *Advances in Space Research*, Vol. 58, No. 9, 2016, pp. 1840–1853. doi:10.1016/j.asr.2016.07.013
- [34] Vallado, D. A., and Cefola, P. J., “Two-Line Element Sets Practice and Use,” *63rd International Astronautical Congress*, International Astronautical Federation, Paris, France, 2012, pp. 5812–5826.
- [35] Geul, J., Mooij, E., and Noomen, R., “TLE Uncertainty Estimation Using Robust Weighted Differencing,” *Advances in Space Research*, Vol. 59, No. 10, 2017, pp. 2522–2535. doi:10.1016/j.asr.2017.02.038
- [36] Saltelli, A., Tarantola, S., Campolongo, F., and Ratto, M., *Sensitivity Analysis in Practice—A Guide to Assessing Scientific Models*, Wiley, Chichester, 2004, pp. 123–132.
- [37] Marino, S., Hogue, I. B., Ray, C. J., and Kirschner, D. E., “A Methodology for Performing Global Uncertainty and Sensitivity Analysis in Systems Biology,” *Journal of Theoretical Biology*, Vol. 254, No. 1, Sept. 2008, pp. 178–196. doi:10.1016/j.jtbi.2008.04.011