Modeling & Simulation, Test & Validation

GVSETS GROUND VEHICLE SYSTEMS ENGINEERING & TECHNOLOGY SYMPOSIUM
& ADVANCED PLANNING BRIEFING FOR INDUSTRY

NDIA Michigan

Amphibious Vehicle Water Egress Modeling and Simulation Using CFD and Wong's Methodology

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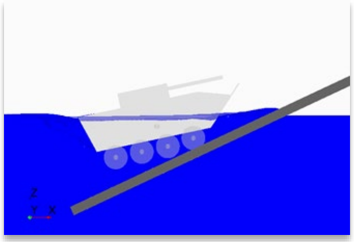
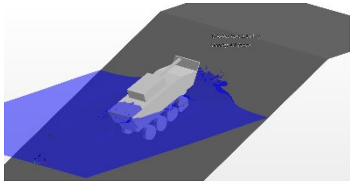
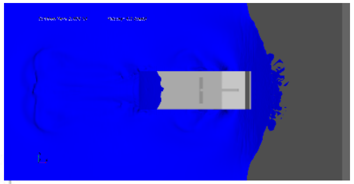
Introduction

- **Background / Impetus:**
  - A significant challenge for amphibious vehicles during swimming operations involves egress from a body of water onto the ramp of a vessel.
  - The vehicle generally needs to be able to swim to the ramp of a vessel, and then propel itself up the ramp (substrate) using water propellers and wheels simultaneously.
  - To accurately predict water egress, it is important to accurately model: (1) the interaction of the flows through the propellers, around the vehicle hull, and over the substrate; (2) the wheel / substrate interaction; (3) wheel and hull motions (both translation and rotation); (4) the suspension system spring, damping, and jounce- and rebound-limiting forces; (5) tire deformation and loading characteristics; and (6) the drivetrain power distribution to the wheels.
  - Detailed modeling and simulation of the associated physics and processes would be highly computationally expensive.
  - Therefore, to make the water egress problem more tractable to solve, various modeling simplifications need to be introduced to facilitate rapid simulation.

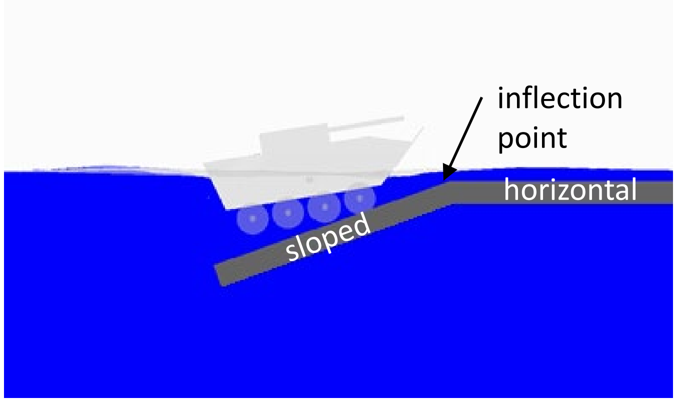


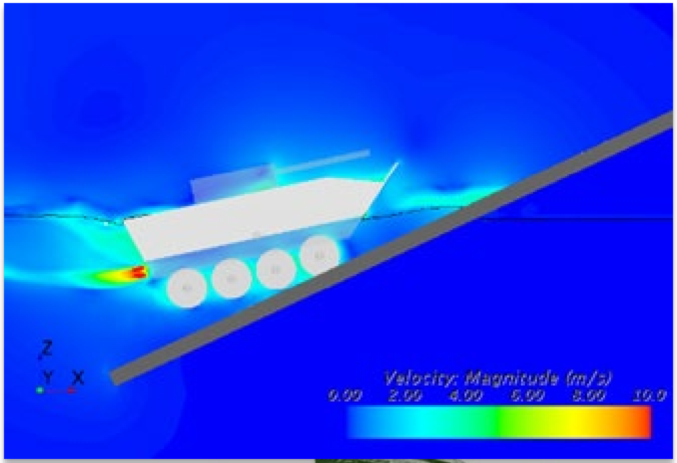




Introduction (cont.)

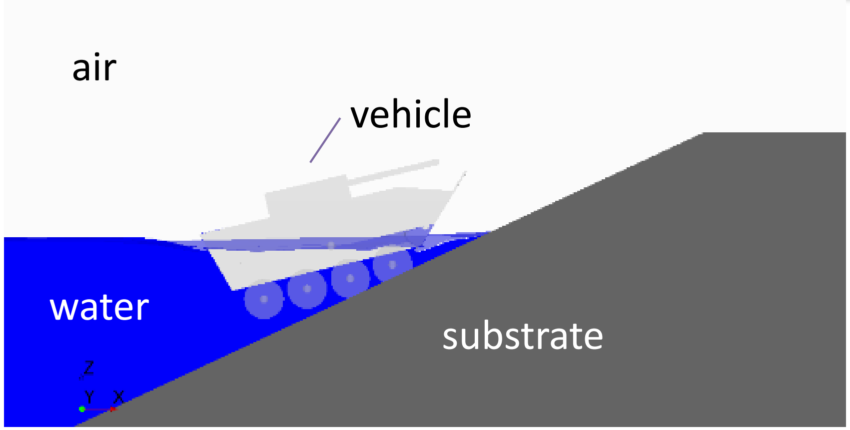
- **Purpose / Scope:**
  - A simplified, analytic wheel-substrate interaction modeling approach based upon Wong’s methodology [1] was developed, and simplified propeller, powertrain, suspension, tire, and wheel rotation modeling were incorporated.
  - The resulting simplified vehicle solver was integrated with a computational fluid dynamics (CFD) and six-degree-of freedom (6DOF) body dynamics solver (STAR-CCM+) [2], resulting in a comprehensive methodology for modeling and simulating amphibious vehicle egress from water to a ramp for various vehicle characteristics and operational conditions.
- **Organization**
  - **Modeling:** environment, vehicle (fictitious)
  - **Simulation:** using vehicle solver and flow / 6DOF solver
  - **Results:** via parametric studies

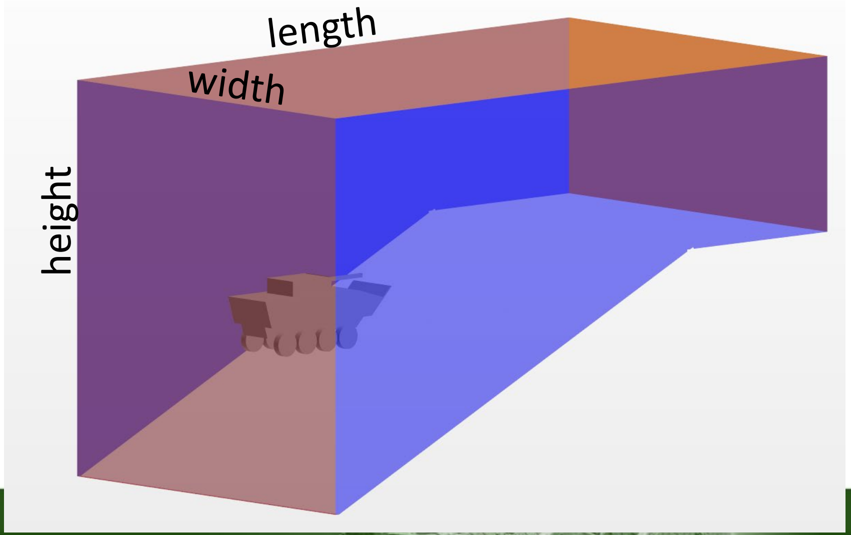




Modeling
Environment – General

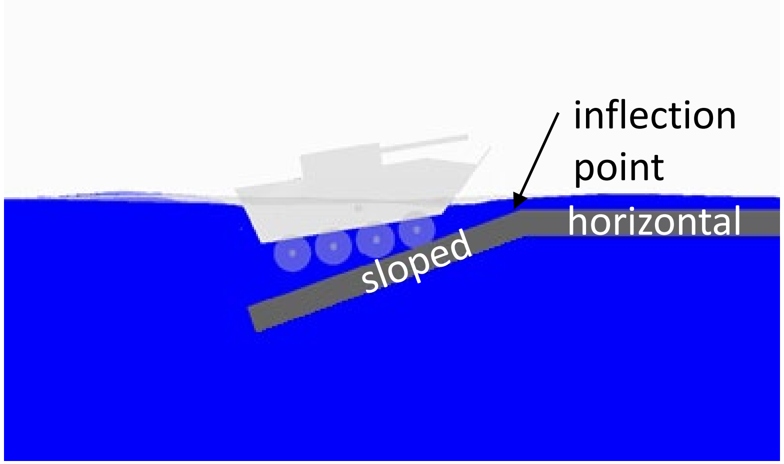
- **Components:**
  - Flow
    - Water (fresh)
    - Air
  - Substrate – ramp
- **Envelope:**
  - Width: 20 m
  - Length: 40 m
  - Height: 30 m

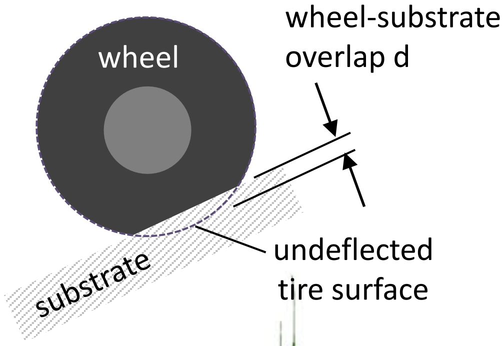




Modeling
Environment – Substrate

- **Shape:**
  - Sloped portion
  - Horizontal portion
- **Firmness:**
  "hard", with flow underneath substrate
- **Wheel Interaction Characteristics:**
  - **Wheel-substrate overlap:**
    The overlap direction is normal to the substrate surface and away from the vehicle.
  - **Tractive force limitation:**
    The coefficient of adhesion attribute, μ, is used as a constant of proportionality between the wheel-substrate normal and tractive forces. A limiting (or maximum) value, μ_L, can be used to appropriately limit tractive force for specific limiting-case scenarios (e.g., wet and slippery surfaces).
  - **Coefficient of adhesion:**
    - Peak coefficient of adhesion value, μ_p
    - Sliding coefficient of adhesion value, μ_s



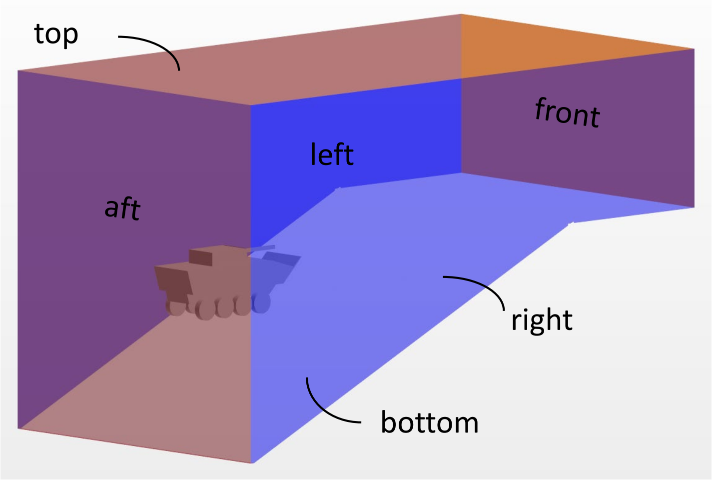


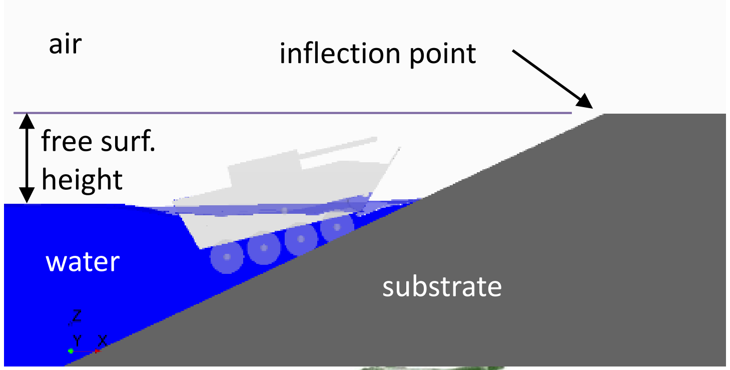
Modeling

Environment – Flow

- **Boundary constraints:**
  - Vehicle: moving walls (6DOF solving used)
  - Substrate: stationary wall
  - Aft, top, bottom flow: zero velocity inlet ("flat wave"), with turbulent intensity and viscosity ratio of 1% and 10, respectively
  - Front flow: ambient pressure outlet ("flat wave"), with turbulent intensity and viscosity ratio of 1% and 10, respectively
  - Side flow (right / left): symmetry
- **Interior constraints:**
  - Reynolds-averaged Navier-Stokes model – continuity and conservation of momentum
  - Two-equation turbulence model – Realizable k-epsilon
  - Multiphase model – Volume of Fluid (VOF)

Implemented via CFD solver – STAR-CCM+ [2]

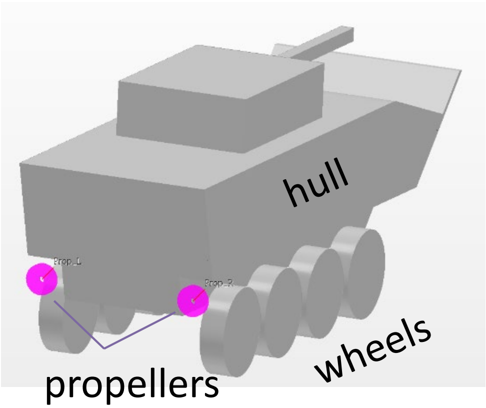




Modeling

Vehicle – General

- **Subsystems:**
  - **Propellers –** rigidly attached to hull, and used to provide water propulsion (thrust) through their interaction with the environmental flow (water)
  - **Hull –** the dominant inertial component of the vehicle which interacts with the other vehicle components and the environmental flow (via flow drag, buoyancy)
  - **Powertrain –** provides tractive power for the land propulsion running gear (wheels)
  - **Suspension –** transmits forces (and moments) between the hull and each wheel based upon the relative distances and motions
  - **Wheels –** attached to the suspension, and used to provide land propulsion through interaction with the environmental terrain (substrate)

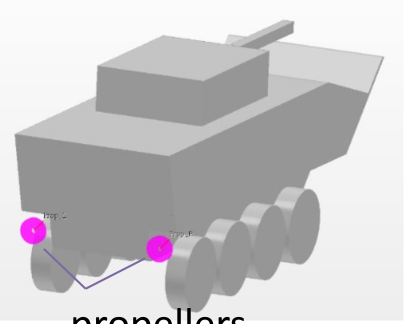


- **Motions / Forces:**
  - The force interactions among the vehicle subsystems and between the vehicle and its environment, along with the associated motions, are modeled using STAR-CCM+'s flow and 6DOF solving capability [2].
  - The hull and each of the wheels are modeled independently, allowing their translational and rotational motions to be affected by all of the forces to which each body is subjected to.

Modeling

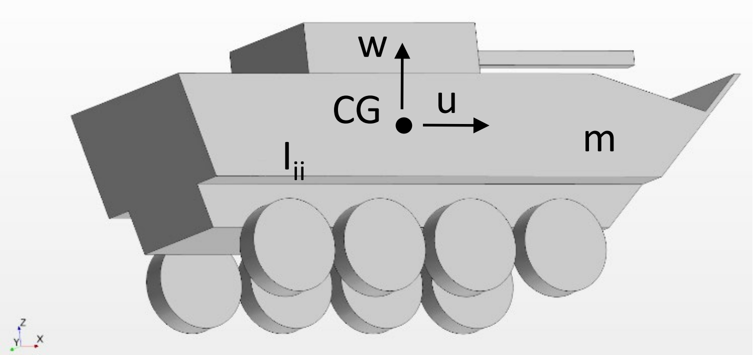
Vehicle – Propellers, Hull

- **Propellers**
  - **Hydrofoil:**
    The selection of the outer diameter, pitch, blade area ratio, and number of blades – together with the usage of the unducted Wageningen B series shaped propeller – results in specified inner diameter, axial length, and performance characteristics [3].
  - **Propulsion:**
    - Specified via "open water" performance data table [3]
    - Modeled using an actuator (virtual) disk method with the thrust and torque distributed in the radial direction based upon Goldstein's optimum [2]



propellers

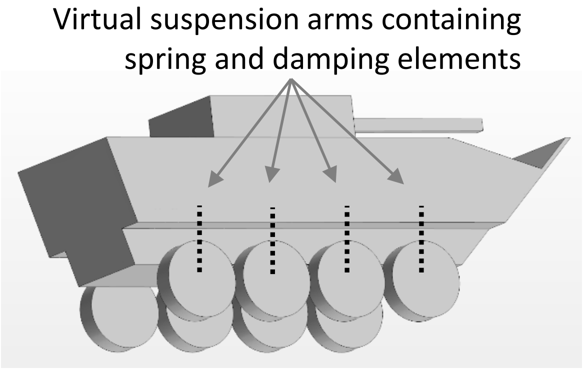
- **Hull**
  - **Directions:**
    - **Hull-longitudinal:** x-axis (from aftward to forward)
    - **Hull-transverse:** y-axis (from starboard / right to port / left)
    - **Hull-vertical:** z-axis (from bottom to top)
  - **Velocity:**
    Hull-longitudinal, -transverse, and -vertical components:
    u, v (presently assumed to be zero), and w, respectively
  - **Forces:**
    Gravitational, flow (buoyancy, pressure drag, and viscous drag), suspension

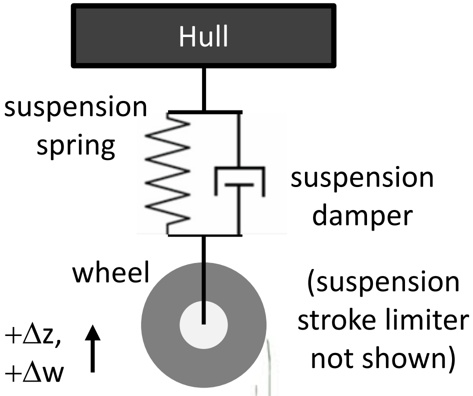


Modeling

Vehicle – Suspension

- The suspension system is primarily intended to manage the hull-vertical motion of the wheel assemblies relative to hull; however, limited hull-horizontal motion (forward-aftward) is also accommodated.
- The wheel-hull relative motion is affected by the suspension system through both damping and spring suspension forces. The damping and spring forces are applied to both the wheels and the hull in the appropriate directions, with the hull force direction opposite of that of the wheel – i.e., there is no directional change of the forces transmitted across the suspension system.



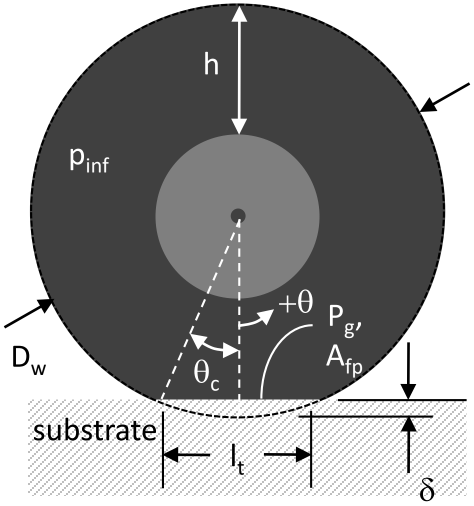


Modeling

Vehicle – Wheels

- **Assumptions / Methods:**
  - The substrate resistance and tractive forces on the tire are parallel to the substrate surface.
  - The substrate normal force on the tire is normal to the substrate surface.
  - The CG location of each wheel is tracked relative to the substrate inflection point, allowing the wheel-substrate normal and shear forces to be applied at the appropriate locations (on the tire deflection plane) and in the appropriate directions.

- **Parameters:**
  - Tire outer diameter, D_w , or radius, R_w
  - Tire section height, h
  - Tire tangential spring rate, k_t
  - Wheel mass, m_w
  - Wheel translational moments of inertia (for each of the cardinal vehicle directions, with products of inertia neglected), $I_{w,t,ii}$
  - Wheel rotational moment of inertia (about its intended spinning axis), $I_{w,r}$
  - Wheel CG location (relative to hull)



Modeling

Vehicle – Wheels (cont.)

- **Variables:**
  - Wheel-substrate distance, d, or tire deflection, δ
  - Tire inflation pressure, p_{inf}
  - Tire effective mass load, $m_L(\delta, p_{inf})$
  - Tire deflected contact length, $l_t(\delta)$
  - Tire width, $b_{ti}(\delta)$
  - Tire compression (deflection) half-angle, $\theta_c(\delta)$
  - Tire deformation motion resistance parameter, $\varepsilon(h)$
  - Tire footprint area, $A_{fp}(\delta)$
  - Tire ground pressure, $p_g(F_N, A_{fp})$
  - Tire normal spring rate, $k_N(p_{inf})$
  - Tire equilibrium deflection, δ_{eq}
  - Tire normal force, $F_N(m_L, \delta, k_N)$
  - Tire rolling resistance force, F_R:

$$F_R = 3.581 b_{ti} {D_w}^2 p_g \varepsilon \frac{(0.0349\theta_{c,\text{deg}} - \sin(2\theta_c))}{2\theta_{c,\text{deg}}(R_w - \delta)} + R_w b_{ti} \int_{\theta_c}^{\theta_1} p \sin\theta \, d\theta$$

  - Wheel slip, i:

$$i = 1 - \frac{u}{R_w \omega}$$

  - Wheel critical slip, i_c:

$$i_c = \frac{F_N}{k_t l_t^2}$$

  - Tire tractive force, F_T:

$$F_{T*} = \begin{cases} F_{T*} = 0.5 i k_t l_t^2 \text{ for } i \le i_c \\ F_N \left[\mu_p \left(1 - \frac{\mu_p F_N}{2 k_t l_t^2 i}\right) \frac{(1-i)}{(1-i_c)} + \mu_s \frac{(i-i_c)}{(1-i_c)}\right] \text{ for } i > i_c \end{cases}$$

  - Tire tractive torque, T_T
  - Wheel rotational speed, ω:

$$T_P + T_F - T_T = I_{w,r} \frac{d\omega}{dt}$$

where T_P is the applied powertrain torque

  - Wheel equivalent speed, v_{eq}:

$$v_{eq} = \omega(R_w - \delta_{eq})$$

Modeling

Vehicle – Powertrain

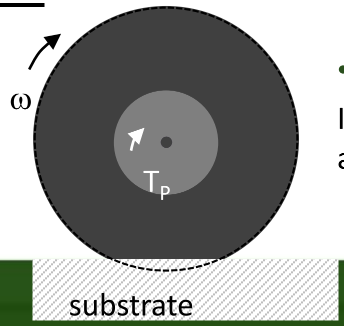
- **Available powertrain torque for each wheel, $T_{AP,i}$:**

$$T_{AP,i} = \sum_{j=0}^{3} \frac{c_{APF,j}}{8} v_{eq,i}^{j} \left(R_w - \delta_{eq}\right)$$

where: i identifies the wheel; v_{eq} is the wheel equivalent speed; $c_{APF,j}$ is a piecewise-continuous, four-by-one parameter matrix relating the maximum available vehicle powertrain force to v_{eq} based on vehicle tractive force and speed data involving specified vehicle mass, tire deflections, substrate grade, and optimum slip conditions; and the number eight appears because the total vehicle powertrain force is assumed to be distributed equally among the eight wheels.

- **Available powertrain torque for all wheels, T_{AP}:**

$$T_{AP} = \sum_{i=0}^{8} T_{AP,i}$$



- **Applied powertrain torque (for each wheel), T_P:**
  - **Distribution:**
    - The total maximum available powertrain torque T_{AP} is distributed among the wheels by assuming that all of the wheels are "locked" together such that they all share the same rotational speed.
    - Torque is applied positively (as available), or negatively (as required), in order to have all of the wheels spin with the same rotational speed – based on the assumption that powering / braking torque can be distributed appropriately.
  - **Limits:**

    Powertrain torque is not applied to a given wheel if …
    - That wheel's rotational speed has already surpassed an upper limit (ω_{lim})
    - That wheel's slip is less than a target slip value (i_{tar})

    In such cases, only the torque loads (tractive, flow) are applied to the wheel.

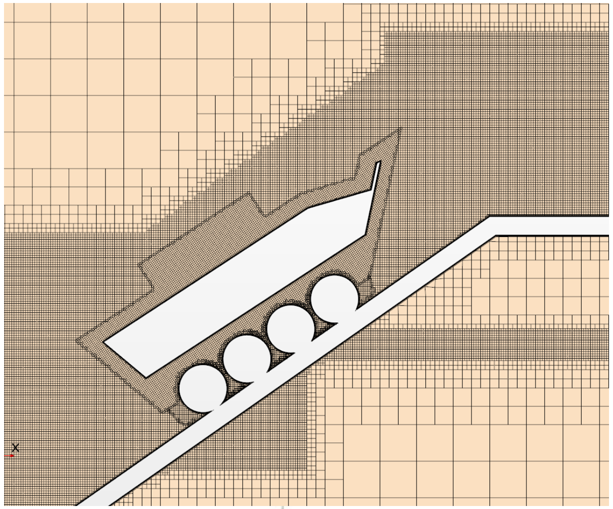
Simulation
Overview

- **Solver Utilization:**
  - **Flow / 6DOF Solver**
    - **Scope** – modeling of the …
      - Hull / wheel motions / forces and suspension spring coupling models (6DOF)
      - Flow dynamics, including the propeller-, hull-, and substrate-flow interactions (flow)
    - **Embodiment** –
      A commercial-off-the-shelf (COTS) CFD solver – STAR-CCM+ [2] – is used to simulate the flow and hull / wheel motions and forces based upon its unsteady Reynolds-Averaged Navier-Stokes (uRANS) equations, Volume of Fluid (VOF) multi-phase flow modeling method, six degree of freedom (6DOF) rigid body motion modeling, and overset meshing capabilities.
      - A “two-layer all y+ wall treatment” is used for the near-wall prism layer cells, and hull and wheel flow cells are allowed to be no larger than 50mm.
      - Overset meshing (or “Chimera” or overlapping meshing) is used to allow the hull and wheel meshes to move relative to one another as well as the environment mesh, and overlap each other in such a way that the governing physical equations can still be solved.
  - **Vehicle Solver**
    - **Scope** – modeling of the suspension, wheels, hull, and powertrain characteristics and interactions
    - **Embodiment** – within the STAR-CCM+ Java run script

$$\frac{\partial}{\partial t}(\rho\vec{v}) + \vec{\nabla}\cdot\rho\vec{v}\vec{v} = \rho\vec{g} + \vec{\nabla}\cdot\bar{\bar{\tau}}$$

where

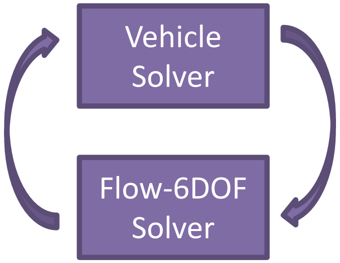
$$\bar{\bar{\tau}} = \mu\left[\left(\vec{\nabla}\vec{v}\right) + \left(\vec{\nabla}\vec{v}\right)_T\right] - \left(p + \frac{2}{3}\mu\vec{\nabla}\cdot\vec{v}\right)\bar{\bar{I}}$$



Simulation
Overview (cont.)

- **Solver Execution:**
  - A STAR-CCM+ Java run script is used to control both the flow / 6DOF solver (STAR-CCM+) as well as the vehicle solver – which is embedded within the run script – and facilitate the passing of data back and forth between the solvers.
  - Department of Defense Supercomputing Resource Center (DSRC) resources – namely, Thunder, Mustang, Centennial, Onyx, Topaz, Gaffney, and Koehr – are used to run the solvers.
- **Solver Data Passing:**

Flow-6DOF Solver Output, Vehicle Solver Input –
- Wheel and hull horizontal and vertical relative positions and speeds
- Wheel flow moments

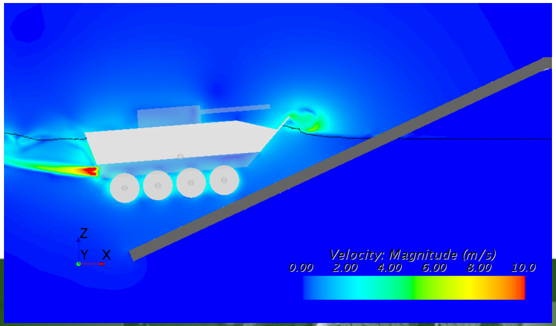


Vehicle Solver

Flow-6DOF Solver

Flow-6DOF Solver Input, Vehicle Solver Output –
- Locations, directions, and magnitudes of substrate tractive and normal forces on wheels and hull
- Suspension spring hull-side locations, spring rates, and damping forces on wheels and hull
- Tire tractive force and moment, substrate pressure, substrate normal force, substrate resistance force, drivetrain torque, and rotational speed

- **Solver Main Steps:**
  - **Initialization**
  - **Time-Marching** (for each wheel and time step)
    - Vehicle solver
    - Flow-6DOF solver



Results

General Conditions

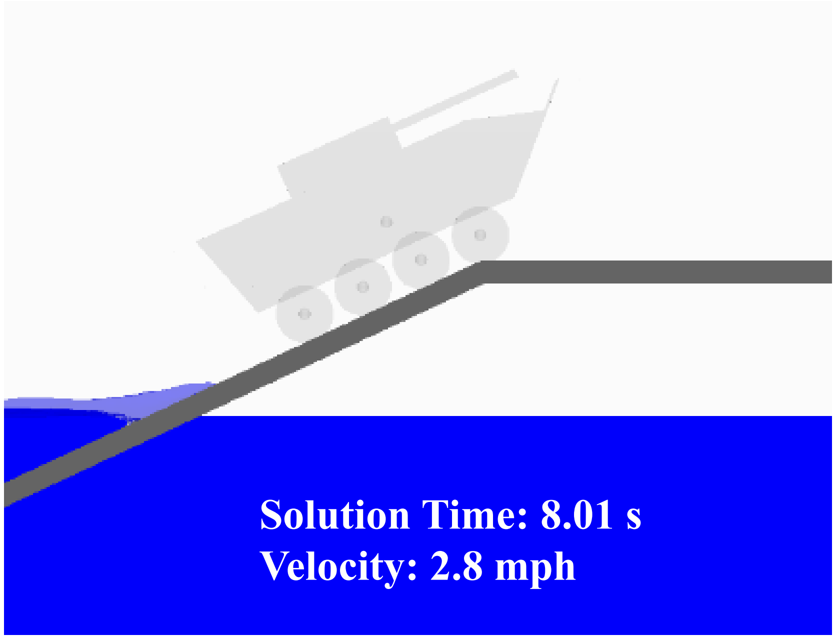
Class	Entity	Sub-Entity	Parameter				
			Name	Symbol	Comments	Setting / Value	Units
Modeling	Environment	Flow	Water free surface location	-			m
			Water density	-		999.98	kg/m^3
			Water viscosity	-		1.51E-03	kg/m-s
			Air density	-		1.18415	kg/m^3
			Air viscosity	-		1.86E-05	kg/m-s
			Water-air surface tension	-		0.072	N/m
		Substrate	Peak coefficient of adhesion	μ_p		0.6	-
			Sliding coefficient of adhesion	μ_s		0.4	-
			Limiting coefficient of adhesion	μ_L		0.8	-
			Substrate angle	–		25	degrees
	Vehicle	General	Objective speed in water	-		5	mph
			Objective speed on substrate	-		8	mph
			Datum longitudinal position	-	Rel. to center of front-axel axis	-1814.4	mm
			Datum transverse position	-		0	mm
			Datum vertical position	-		1104.4	mm
			Center of gravity longitudinal position	CG_x	Relative to vehicle datum	0	mm
			Center of gravity transverse position	CG_y		0	mm
			Center of gravity vertical position	CG_z		0	mm
			Mass	m		30000	kg
		Hull	Moment of inertia about longitudinal axis	I_{xx}	Rotational axis goes through CG	6.78E+04	kg-m^2
			Moment of inertia about transverse axis	I_{yy}		2.04E+05	kg-m^2
			Hull Moment of inertia about vertical axis	I_{zz}		2.07E+05	kg-m^2
			Center of gravity longitudinal position	-	Relative to vehicle datum	0	mm
			Center of gravity transverse position	-		0	mm
			Center of gravity vertical position	-		-7.36E+01	mm
		Suspension	Jounce stroke length	Δz_j		1.50E+02	mm
			Rebound stroke length	Δz_r		1.50E+02	mm
		Powertrain	Torque distribution ratio limit	-		0.375	-

Class	Entity	Sub-Entity	Parameter				
			Name	Symbol	Comments	Setting / Value	Units
Modeling	Vehicle	Propeller	Longitudinal position	-	Relative to vehicle datum	-3733.4	mm
			Transverse position	-		+/-1187.5	mm
			Vertical position	-		-450	mm
			Inner radius	-		50	mm
			Outer radius	-		225	mm
			Axial length	-		100	mm
			Rotational speed	-		2000	rpm
		Wheel	Tire tangential spring rate	k_t		4.00E+06	N/m^2
			Tire section height	h		0.33575	m
			Tire construction parameter	k_e		7	-
			Tire maximum deflection	d_{max}		140	mm
			Wheel radius	R_w		6.72E+02	mm
			Wheel target slip	i_{tar}		2.50E-01	-
			Wheel rotational moment of inertia	$I_{w,r}$		7.50E+01	kg-m^2
			Assembly moment of inertia about long. axis	$I_{w,xx}$	Not used	3.31E+01	kg-m^2
			Assembly moment of inertia about trans. axis	$I_{w,yy}$		9.97E+01	kg-m^2
			Assembly moment of inertia about vert. axis	$I_{w,zz}$		1.01E+02	kg-m^2
			Wheel rotational speed upper limit	ω_{lim}		5.00E+02	rpm
			Wheel assembly mass	m_w		2.34E+02	kg
			Wheel assembly missing volume	-		6.25E-04	m^3
Simulation	General		Time step	-		2	ms
	Flow-6DOF		Discretization order	-		1	-
	Vehicle		Initial speed	-		5	mph
			Initial suspension stroke distance	-		1.50E+02	mm

Results

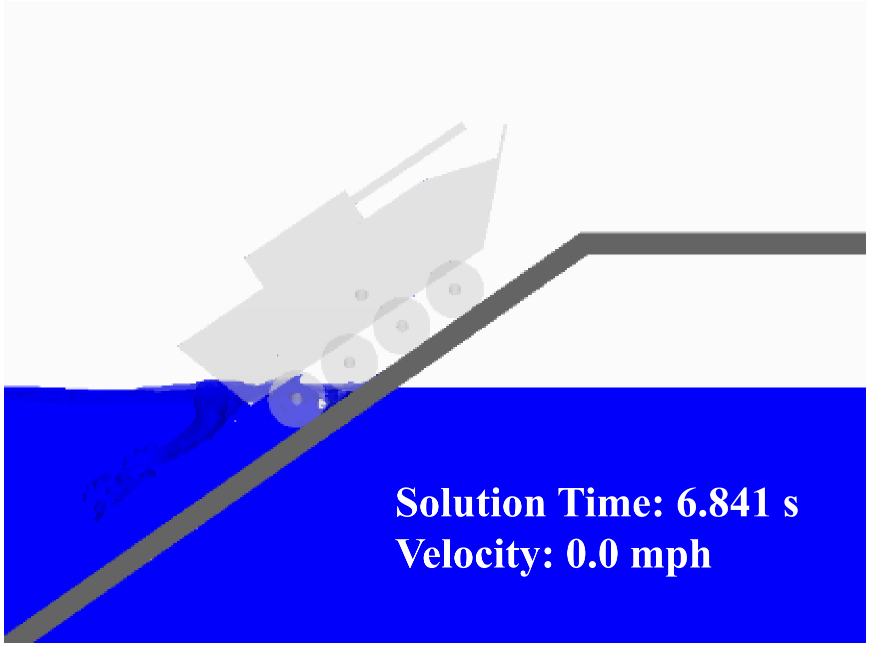
Specific Scenarios – Effect of Substrate Angle

25° ramp angle



Ramp successfully climbed

35° ramp angle

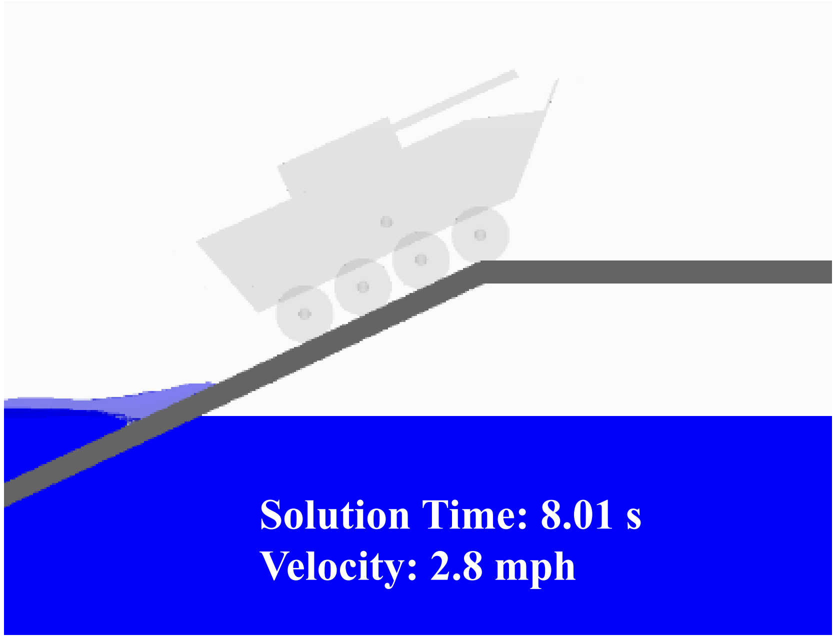


Ramp unsuccessfully climbed

Results

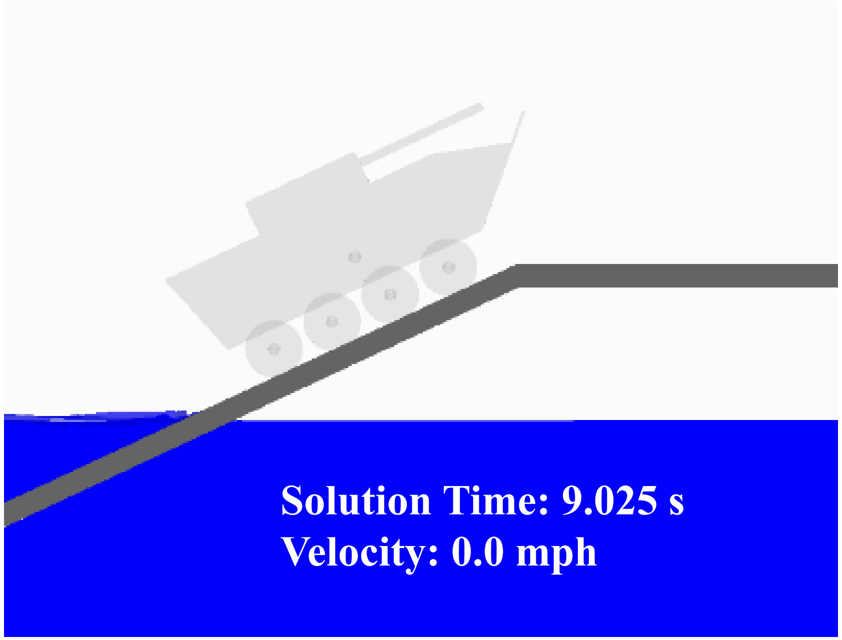
Specific Scenarios – Effect of <u>Reduced Powertrain Torque</u>

Full powertrain torque



Ramp <u>successfully</u> climbed

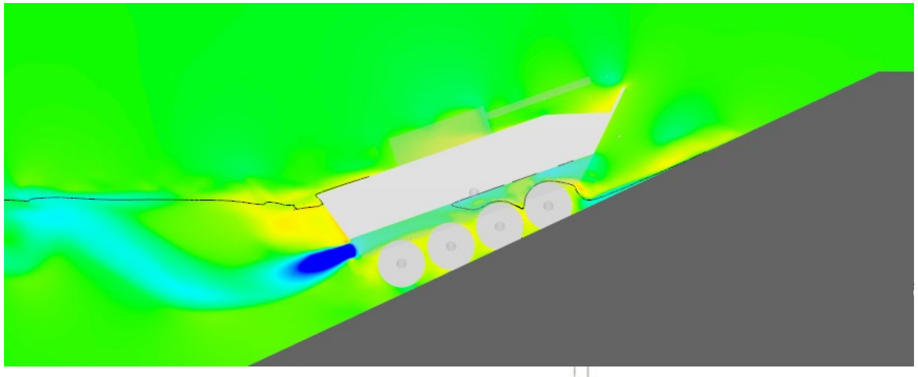
Half powertrain torque



Ramp <u>unsuccessfully</u> climbed

Conclusion

- **Summary**
  - A simplified wheel-substrate interaction modeling approach based upon Wong’s methodology was developed for hard substrates, involving the incorporation of simplified propeller, powertrain, suspension, tire, and wheel rotation modeling.
  - The resulting simplified vehicle solver was integrated with a flow-6DOF solver, resulting in a comprehensive methodology for modeling and simulating amphibious vehicle egress from water to a ramp for various vehicle characteristics and operational conditions.
  - Limited parametric studies demonstrated the method’s capability and reasonableness.
- **Path Forward**
  - Transverse current
  - Additional powertrain torque distribution methods, driver modeling
  - Soft substrate capability development:
    - Using analytical (Bekker-based) method
    - Using empirical (cone-index-based) method
  - Flow accuracy improvement
  - Validation



References

[1] J. Y. Wong, *Theory of Ground Vehicles*. Hoboken, NJ: John Wiley & Sons, Inc., 2008.

[2] Siemens Technical Staff, *STAR-CCM+ User Guide*, Siemens, 2019.

[3] M. W. C. Oosterveld and P. van Oossanen, “Further Computer-Analyzed Data of the Wageningen B-Screw Series”, *International Shipbuilding Progress*, vol. 22, no. 251, pp. 251-262, 1975.

Back-Up

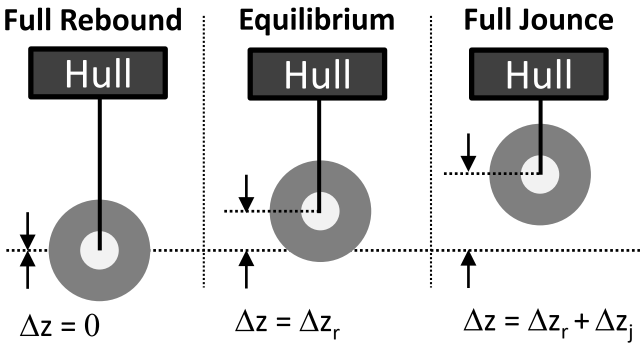
Modeling

Vehicle – Suspension (cont.)

- **Vertical Motion:**
  - **Stroke length:**
    - Rebound stroke length, Δz_r
    - Jounce stroke length, Δz_j
  - **Wheel-hull vertical distance Δz …**
    - At full rebound: $\Delta z = 0$
    - At equilibrium: $\Delta z = \Delta z_r$
    - At full jounce: $\Delta z = \Delta z_r + \Delta z_j$
  - **Spring forces acting on wheel:**
    - **Cylinder, $F_{S,spring,cyl}$:**

$$F_{S,spring,cyl} = \begin{cases} -c_{Fcyl,0} \text{ for } \Delta z \leq 0 \\ -\sum_{i=0}^{6} c_{Fcyl,i} \Delta z^i \text{ for } 0 < \Delta z \leq (\Delta z_r + \Delta z_j) \\ -\sum_{i=0}^{6} c_{Fcyl,i} (\Delta z_r + \Delta z_j)^i \text{ for } \Delta z > (\Delta z_r + \Delta z_j) \end{cases}$$

where $c_{Fcyl,i}$ is a piecewise-continuous, seven-by-one parameter matrix relating the cylinder spring forces, $F_{S,spring,cyl}$, to Δz.



- **Rebound limit, $F_{S,spring,rl}$:**

$$F_{S,spring,rl} = \begin{cases} -k_{rl} \Delta z \text{ for } \Delta z \leq 0 \\ 0 \text{ for } \Delta z > 0 \end{cases}$$

where k_{rl} is the rebound-limited spring rate constant.

- **Jounce limit, $F_{S,spring,jl}$:**

$$F_{S,spring,jl} = \begin{cases} -k_{jl}(\Delta z - \Delta z_r - \Delta z_j) \text{ for } \Delta z \geq (\Delta z_r + \Delta z_j) \\ 0 \text{ for } \Delta z < (\Delta z_r + \Delta z_j) \end{cases}$$

where k_{jl} is the jounce-limited (bumpstop) spring rate constant.

Modeling

Vehicle – Suspension (cont.)

- **Vertical Motion (cont.):**
  - **Damping Forces acting on wheel:**
    - **Cylinder, $F_{S,damping,cyl}$:**

$$F_{S,damping,cyl} = -\beta_{cyl}\Delta w$$

where Δw (first time-derivative of Δz) is the upward velocity of the wheel relative to the hull, and the cylinder damping rate $\beta_{cyl} = \sum_{i=0}^{3} c_{\beta cyl,i}\Delta w^i$ and $c_{\beta cyl,i}$ is a piecewise-continuous, four-by-one parameter matrix relating the suspension damping rate to Δw.

- **Rebound limit, $F_{S,dampring,rl}$:**

$$F_{S,damping,rl} = -\beta_{rl}\Delta w \text{ for} \Delta z \leq 0$$

where the return-limited damping rate coefficient

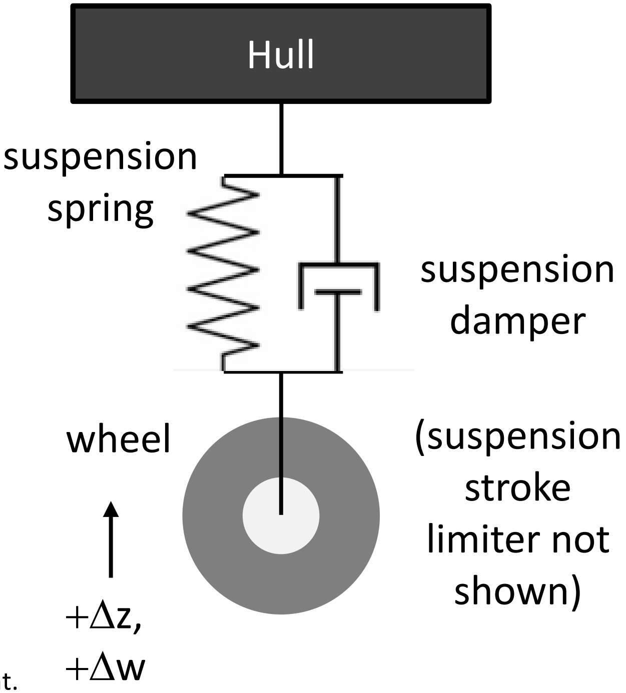
$$\beta_{rl} = \begin{cases} 0 \text{ for } \Delta w > 0 \\ \sqrt{2\zeta_{rl} m_w k_{rl}} \text{ for } \Delta w \leq 0 \end{cases}$$ and ζ_{rl} is the rebound-limited damping ratio constant.

- **Jounce limit, $F_{S,damping,jl}$:**

$$F_{S,damping,jl} = -\beta_{jl}\Delta w \text{ for } \Delta z \geq \left(\Delta z_r + \Delta z_j\right)$$

where the jounce-limited damping rate coefficient

$$\beta_{jl} = \begin{cases} 0 \text{ for } \Delta w < 0 \\ \sqrt{2\zeta_{jl} m_w k_{jl}} \text{ for } \Delta w \geq 0 \end{cases}$$ and ζ_{jl} is the jounce-limited damping ratio constant.



- **Composite Force:**

$$F_{SV} = F_{S,spring,cyl} + F_{S,spring,jl} + F_{S,spring,rl} + F_{S,damping,cyl} + F_{S,damping,jl} + F_{S,damping,rl}$$

Modeling

Vehicle – Suspension (cont.)

- **Horizontal Motion:**
  - **Spring Force:**

$$F_{S,spring,hor} = -k_{hor}\Delta x$$

where k_{hor} is the spring rate constant associated with horizontal wheel-hull relative motion and Δx is the horizontal distance by which the wheel moves forward of the wheel's hull equilibrium position.

  - **Damping Force:**

$$F_{S,damping,hor} = -\beta_{hor}\Delta u$$

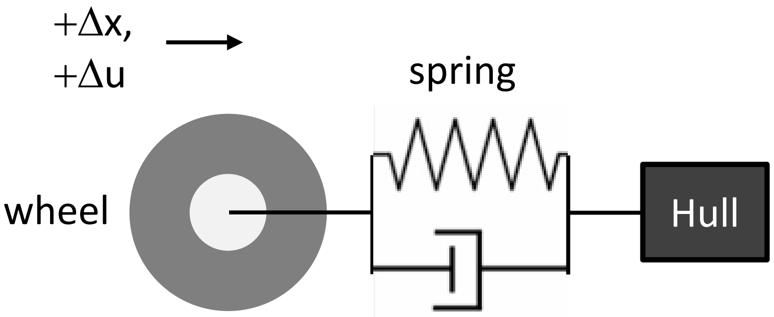
where Δu is the horizontal velocity of the wheel relative to that of the hull, the horizontal wheel-hull relative motion damping rate constant

$$\beta_{hor} = \sqrt{2\zeta_{hor} m_w k_{hor}},$$

and ζ_{hor} is the horizontal damping ratio constant.

  - **Composite Force:**

$$F_{SH} = F_{S,spring,hor} + F_{S,damping,hor}$$



(horizontal stroke limiter not shown)

Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Wheel-substrate distance, d:** outputted by the 6DOF solver
  - **Tire inflation pressure, p_{inf}:**

$$p_{inf} = \sum_{i=0}^{3} c_{pi,i} \left(\frac{m}{8}\right)_w^i$$

where $c_{pi,i}$ is a four-by-one matrix containing parameters relating the appropriate tire inflation pressure to equilibrium wheel load and m is the total vehicle mass.

  - **Tire deflection, δ:**

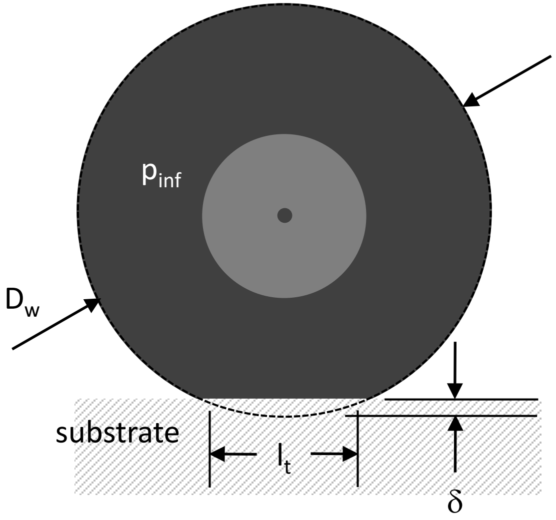
$$\delta = \begin{cases} d \text{ for } d \leq \delta_{max} \\ \delta_{max} \text{ for } d > \delta_{max} \end{cases}$$

where δ_{max} is the tire maximum deflection (based on the limits of the available test data).

  - **Tire effective mass load, m_L:**

$$m_L = \sum_{i=1}^{3} \sum_{j=1}^{3} 10^{c_{mL,ij} p_{inf}^j} \delta^i$$

where $c_{mL,ij}$ is a three-by-four parameter matrix relating tire mass load to deflection and inflation pressure.



Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Tire deflected contact length, l_t:**

$$l_t = 2\sqrt{D_w\delta - \delta^2}\sum_{i=0}^{2} c_{lt,i}\delta^i$$

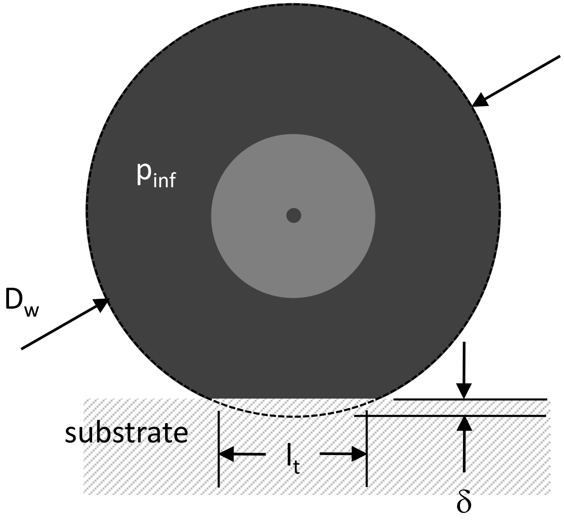
where $c_{lt,i}$ is a three-by-one matrix containing parameters relating tire deflected contact
length to tire deflection.

  - **Tire width, b_{ti}:**

$$b_{ti} = \sum_{i=0}^{2} c_{b,i}\delta^i$$

where $c_{b,i}$ is a three-by-one matrix containing parameters relating tire width to tire deflection.

  - **Tire contact patch minimum dimension, b:**
minimum value of l_t and b_{ti}



Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Tire compression (deflection) half-angle, θ_c:**

$$\theta_c = \cos^{-1}\left(1 - \frac{\delta}{R_w}\right)\sum_{i=0}^{2} c_{lt,i}\delta^i$$

  - **Tire deformation motion resistance parameter, ε:**

$$\varepsilon = 1 - e^{\frac{-k_e\delta}{h}}$$

where the tire construction parameter k_e is 7 for radial-ply tires and 15 for bias-ply tires [1].

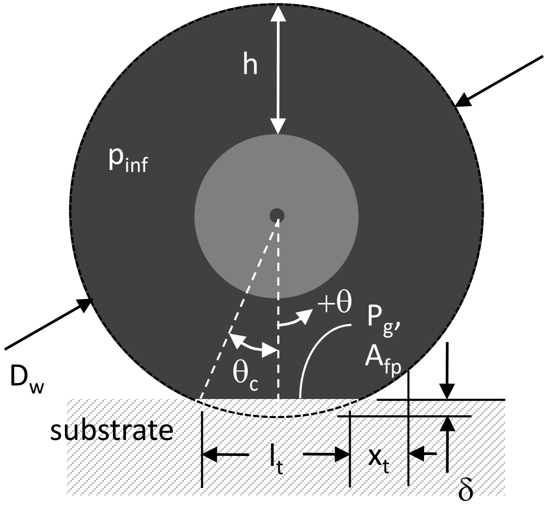
  - **Tire footprint area, A_{fp}:**

$$A_{fp} = c_{A0}\delta^{c_{A1}} = l_t b_{ti}$$

where c_{A0} and c_{A1} are parameters relating the tire footprint area to deflection.

  - **Tire ground pressure, p_g (over deflected portion):**

$$p_g = \frac{F_N}{A_{fp}}$$



Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Tire normal spring rate, k_N:**

$$k_N = \sum_{i=0}^{2} c_{kN,i}\, p_{\mathrm{inf}}$$

where $c_{kN,i}$ is a three-by-one parameter matrix relating the tire normal spring rate to inflation pressure.

  - **Tire equilibrium deflection, δ_{eq}:**

$$\delta_{eq} = \frac{mg}{8k_N}$$

where the number eight appears because the vehicle mass m is assumed to be equally distributed among the eight vehicle tires.

  - **Tire normal force, F_N:**

$$F_N = m_L g + k_N(d - \delta)$$

    - For tire deflections δ less than δ_{max}, the first term on the right-hand side fully accounts for all of the normal force, and the second term equals zero since δ should be the same value as d for hard substrates.
    - For δ greater than δ_{max}, the first term on the right-hand side does not fully account for all of the normal force (because δ, which is used to determine m_L, is limited at δ_{max}), and the second term then accounts for the remainder of the normal force associated with tire deflection beyond δ_{max}.

Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Tire rolling resistance force, F_R:**

$$F_R = 3.581 b_{ti} {D_w}^2 p_g \varepsilon \frac{\left(0.0349\theta_{c,\text{deg}} - \sin(2\theta_c)\right)}{2\theta_{c,\text{deg}}(R_w - \delta)}$$

where $\theta_{c,\text{deg}}$ and θ_c are measured in degree and radians, respectively.

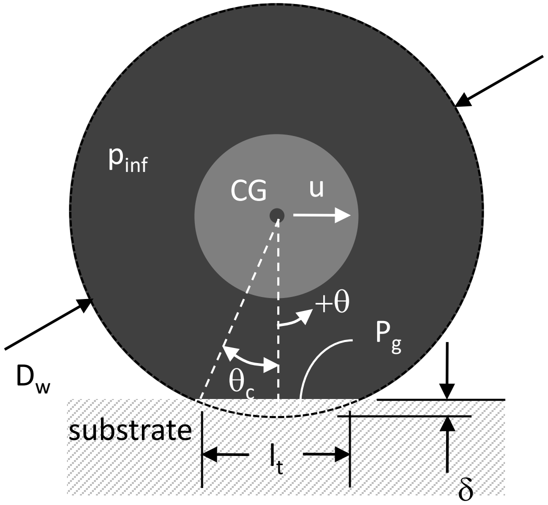
  - **Wheel slip, i:**

$$i = 1 - \frac{u}{R_w \omega}$$

where u is the horizontal velocity of the hull.

  - **Wheel critical slip for hard substrates, i_c [1]:**

$$i_c = \frac{F_N}{k_t l_t^2}$$



Modeling

Vehicle – Wheels (cont.)

- **Variables (cont.):**
  - **Tire tractive force, F_T:**

$$F_T = \begin{cases} F_{T*} \text{ if } F_{T*} \leq \mu_L F_N \\ \mu_L F_N \text{ if } F_{T*} > \mu_L F_N \end{cases}$$

where μ_L is a limiting adhesion coefficient value and

$$F_{T*} = \begin{cases} 0.5 k_t l_t^2 i \ \text{ for } \ i \leq i_c \\ F_N \left[\mu_p \left(1 - \frac{\mu_p F_N}{2 k_t l_t^2 i} \right) \frac{(1-i)}{(1-i_c)} + \mu_s \frac{(i-i_c)}{(1-i_c)} \right] \ \text{ for } \ i > i_c \end{cases}$$

which involving a customized transition from critical slip ($i = i_c$) to complete slip ($i = 1$) partially based upon Wong's methodology [1].

  - **Tire tractive torque, T_T:**

$$T_T = \begin{cases} T_{T*} \text{ if } F_{T*} \leq \mu_c F_N \\ T_{T*} \left(\frac{F_N \mu_L}{F_{T*}} \right) \text{ if } F_{T*} > \mu_L F_N \end{cases}$$

where $T_{T*} = (R_w - \delta) F_T$

  - **Wheel rotational speed, ω:**

$$T_P + T_F - T_T = I_{w,r} \frac{d\omega}{dt}$$

where ω is determined by integrating the above equation with respect to time and T_P and T_F are the torques on the wheel resulting from applied powertrain forces and flow forces as determined from the CFD solver, respectively.

  - **Wheel equivalent speed, v_{eq}:**

$$v_{eq} = \omega \left(R_w - \delta_{eq} \right)$$