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AN OPTIMIZATION-BASED APPROACH TO MEASURING ROBUSTNESS IN COMMAND AND CONTROL NETWORKS

by

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This thesis frames command and control (C2) design as a type of assignment problem involving tasks, missions, resources, and teams. We formulate and solve a sequence of four different optimization problems that assign resources to tasks, missions to teams, tasks to missions, and resources to teams. Our models are based on the assumption that any resource can execute any task; however, the cost for a resource to execute a task depends on the suitability of the resource for that task. An optimal assignment is one that leaves the fewest missions and tasks incomplete while also yielding the lowest execution cost. We characterize the type of work required by a task in terms of a functional requirement, and we characterize the capability of a resource in terms of its resource specialization (e.g., Military Occupational Specialty). We also include nonlinear effects based on the interactions within a team and across a mission. Our goal is to understand the performance effects of optimal teams on the organizational architecture of a C2 network. We find optimal teaming and C2 architecture for a given mission is dependent on problem-specific information such as task requirements, resource specializations, force organization, mission needs, allowed computational time, and data availability.

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AN OPTIMIZATION-BASED APPROACH TO MEASURING ROBUSTNESS IN COMMAND AND CONTROL NETWORKS

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This thesis frames command and control (C2) design as a type of assignment problem involving tasks, missions, resources, and teams. We formulate and solve a sequence of four different optimization problems that assign resources to tasks, missions to teams, tasks to missions, and resources to teams. Our models are based on the assumption that any resource can execute any task; however, the cost for a resource to execute a task depends on the suitability of the resource for that task. An optimal assignment is one that leaves the fewest missions and tasks incomplete while also yielding the lowest execution cost. We characterize the type of work required by a task in terms of a functional requirement, and we characterize the capability of a resource in terms of its resource specialization (e.g., Military Occupational Specialty). We also include nonlinear effects based on the interactions within a team and across a mission. Our goal is to understand the performance effects of optimal teams on the organizational architecture of a C2 network. We find optimal teaming and C2 architecture for a given mission is dependent on problem-specific information such as task requirements, resource specializations, force organization, mission needs, allowed computational time, and data availability.

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List of Acronyms and Abbreviations

C2	Command and Control
C4ISR	Command, Control, Communications, Computers and Intelligence, Surveillance, & Reconnaissance
CCRP	Command and Control Research Program
DoD	Department of Defense
JP	Joint Publication
JSOTF	Joint Special Operations Task Force
MAGTF	Marine Air Ground Task Force
MCDP	Marine Corps Doctrinal Publication
NCW	Network-Centric Warfare
NPS	Naval Postgraduate School
OR	Operations Research
USMC	U.S. Marine Corps

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Executive Summary

With the advent of information technology and asymmetric threats, military organizations are being tasked with increasingly diverse and complex operations beyond traditional warfare. A key aspect of leadership and organization in the context of these operations is command and control (C2). The fundamental C2 support structure in the U.S. military still relies on traditional architectures, characterized by rigid hierarchies that have been optimized for industrial-age conflicts. As a result, the C2 architecture does not perform optimally against today's dynamic, asymmetric threats.

One promising solution is the idea of a team-of-teams approach to restructuring an organization, designed to enable lower levels of a hierarchy additional information flow and decision rights to adequately respond to such threats. However, to date the research on team-of-teams and C2 is largely qualitative to describe expected benefits, rather than quantitative to offer C2 design guidance and recommend new organizational structures. More work is needed to measure the effectiveness of a given C2 architecture to perform military missions and offer recommendations on how to restructure organizations to have these architectures.

This thesis identifies optimal C2 architectures by framing C2 design as a type of assignment problem between *tasks* (i.e., units of work to be done) and *resources* (i.e., personnel who perform work). As the number of tasks and resources get large, we assume that tasks are organized into *missions* and that resources are organized into *teams*.

We formulate and solve a sequence of four different optimization problems that assign resources to tasks, missions to teams, tasks to missions, and resources to teams. Our models are based on the assumption that any resource can execute any task; however, the cost for a resource to execute a task depends on the suitability of the resource for that task. We characterize the type of work required by a task in terms of a functional requirement, and we characterize the capability of a resource in terms of its resource specialization (e.g., Military Occupational Specialty). We also include nonlinear effects based on the interactions within a team and across a mission. Our goal is to understand the performance effects of optimal teams on the organizational architecture of a C2 network.

We compare all four assignment models using large and realistic data sets that demonstrate their distinct effectiveness and recommendations. We show that assigning tasks to resources without considering military missions or teams can efficiently produce a globally optimal task-resource assignment. However, implementing this assignment in practice is difficult because it requires a flat organizational architecture with high management overhead. Instead, models with predefined missions and teams simplify the computational effort of finding a good assignment, but often at the expense of a solution that is globally sub optimal. Models that form adaptive teams and/or mission sets can achieve better assignment and lower execution costs than current C2 architectures, but at the expense of significant increases in computational time. Thus, each model provides a new perspective on what teaming means while setting baselines for computational requirements, optimality, robustness, adaptability, and flexibility. Our results demonstrate that optimal teaming is data-, context-, and mission-dependent.

We conclude that organizations with different C2 architectures should consider different models to guide task-resource assignment decisions. Extremely flat organizations can benefit from the globally optimal solution found with the simple Task-to-Resource assignment model. However, current military force organization is not flat enough to harness these benefits, and missions and teams must be considered in assignment. Models with predefined missions and teams should only be used when task-resource decisions are time constrained. Instead, when faced with changing and unforeseen mission environments, military organizations may want a team-of-teams approach and would benefit from the adaptability of an assignment model that includes team formation. Moreover, when organizations are rigid but tasks can be redistributed to synchronize missions, the optimal assignment is best provided by the mission assignment model.

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> "Without counsel plans fail, but with many advisers they succeed."
>
> — Proverbs 15:22 English Standard Version (ESV)

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CHAPTER 1:
Introduction

> *A fighting force with good individual training, a solid handbook, and a sound strategy can execute a plan efficiently, and as long as the environment remains fairly static, odds of success are high. But a team fused by trust and purpose is much more potent. Such a group can improvise a coordinated response to dynamic, real-time developments.*
>
> — General Stanley S. McChrystal (McChrystal et al. 2015, p. 98)

1.1 Overview of the Problem

With the advent of information technology and increasingly asymmetric threats, military organizations are being tasked with increasingly diverse and complex operations beyond traditional warfare. A key aspect of leadership and organization in the context of these operations is command and control (C2). Per Marine Corps Doctrinal Publication 6 (MCDP 6), *Command and Control*, "the basic elements of the C2 system are people, information, and the C2 support structure" (U.S. Marine Corps 1996, p. 47). C2 enables the military to carry out contingency operations in peace, in crisis, or in war.

With the information age and new technology, C2 in the U.S. military has expanded into a concept called C4ISR: Command, Control, Communications, Computers and Intelligence, Surveillance, & Reconnaissance. However, while this concept introduces new systems to address dynamic, asymmetric threats, the fundamental C2 support structure still relies on the traditional integration of communication. Traditional C2 architectures are characterized by rigid hierarchies that have been optimized for industrial-age conflicts; as a result, the C2 architecture does not perform optimally against today's dynamic, asymmetric threats as it does not enable the information flow and decision rights at various organizational levels required in modern warfare domains.

One promising solution is the idea of a *team-of-teams* approach to restructuring an organization, designed to enable lower levels of a hierarchy additional information flow and decision rights in order to adequately respond to dynamic, asymmetric threats. The purpose of this research is to gain insight into how a team-of-teams approach to a C2 architecture can improve flexibility over traditional C2 hierarchies. Operations Research (OR) is a highly sought-after discipline with tools that can be applied to investigate networks like C2 architectures. This research therefore uses techniques from the field of OR to investigate the following question: can an optimization-based team-of-teams approach be used to measure robustness in C2 Networks?

1.2 The Evolution of C2 to the 21st Century

In *Power to the Edge*, Alberts and Hayes (2003) identify that "two key force capabilities needed by Information Age militaries are interoperability and agility" (p. 56). Interoperability is the main issue with organizations transitioning to the information age: Industrial age hierarchies like traditional C2 are susceptible to stovepiping, which results in a lack of information sharing between individuals and organizational entities. Interoperability also affects the configuration and development of systems and processes within an organization to communicate with outside entities. Alberts (2011) defines agility as "the ability to successfully effect, cope with, and/or exploit changes in circumstances" (p. 190). From a military perspective, an agile and flexible force often gives up firepower and strength in favor of speed or mobility. Likewise, an agile and flexible force will also require an agile and flexible C2 capability. Alberts and Hayes therefore propose that "a 21st century force needs to be robustly networked with information management capabilities that enable widespread information sharing and support simultaneous collaborations" (Alberts and Hayes 2003, p. 102). How can the military and civilian organizations make information available in a secure, timely manner and in a usable form to everyone who needs it?

A real-world example of interoperability and agility is demonstrated in the book *Team of Teams: New Rules of Engagement for a Complex World*, which describes how retired Army General Stanley S. McChrystal "restructured our force from the ground up on principles of extremely transparent information sharing (what we call 'shared consciousness') and decentralized decision-making authority ('empowered execution')" (McChrystal et al. 2015, p. 20). General McChrystal committed a disciplined Joint Special Operations Task Force (JSOTF) organization to deliberate sharing of information which served as a catalyst for change.

The implementation of General McChrystal's team-of-teams approach is similar to how Alberts and Hayes describe the potential of Network-Centric Warfare (NCW) for shared situation awareness and self-synchronization in C2. NCW is a new theory of warfare first articulated in the book *Network Centric Warfare: Developing and Leveraging Information Superiority* (Alberts et al. 2000); Rubel (2001) defines NCW as "the style of warfare that is possible when individual combat units are robustly connected by information" (p. 65). However, both General McChrystal's team-of-teams approach and Alberts and Hayes' NCW offer a descriptive view of C2, while the purpose of this thesis is to address the advantages of teams through a prescriptive C2 architecture model.

Overall, the evolution of C2 to the 21st century involves the difficult task of changing an industrial mindset and transforming an organizational hierarchy to operate in today's information age.

1.3 Thesis Objectives

The objective of this research is to understand the performance effects of teams on the organizational architecture of a C2 network. Drawing upon previous studies and organizational structures, we develop the framework for the team-of-teams C2 network optimization model. The primary hypothesis and problem framing questions are as follows:

- Is there an optimal teaming of resources (i.e., people) for a given set of work (i.e., mission)?
- Will the performance of teams with a hierarchical set of work (i.e., structured) degrade when the team is given a diverse set of work (i.e., unstructured)?
- Will the performance of teams with a diverse set of work (i.e., unstructured) improve when the team is given a hierarchical set of work (i.e., structured)?

In answering these questions we seek to explain why some C2 organizational structures work better in some environments than others. The warfighting domains have changed due to the information age, and this change opens the door to a wide range of other C2 architectures that are now applicable to organizations, leadership, and hierarchies. In this age, there is a need to explore C2 organizational structure options with modern fields of research. This thesis presents a team-of-teams C2 optimization model that assesses whether robust advantages exist in a small- and large- network. This research can help military leaders seeking to understand the effects of teams within organizational architectures and how modifications in hierarchy can enhance communications, workflow, and mission accomplishment. The thesis provides insight into how teams within a C2 architecture can both enhance and diminish the effectiveness of old practices in C2 and translate into a new adaptable, flexible architecture for leading war fighters.

CHAPTER 2:
Literature Review

> *This is the difference between "education" and "training." ... Education requires fundamental understanding, which can be used to grasp and respond to a nearly infinite variety of threats; training involves singular actions, which are useful only against anticipated challenges. Education is resilient, training is robust.*
>
> — General Stanley S. McChrystal (McChrystal et al. 2015, p. 135)

Investigating the question of how the U.S. military might develop a more effective modern C2 structure requires drawing on a number of ideas related to control theory, organization theory, network science, and OR. This chapter begins by breaking down C2 through doctrine, which is necessary to define in military terms and establish a C2 foundation. Next, we provide a C2 discussion on the development of NCW theory and its utilization to transform C2 into the information age. Subsequently, we discuss the current issues and dilemmas in C2 research and implementation. Lastly, we provide an overview of current optimization-based OR methods for studying force structure and the agility of military teams. A particularly important point, put forth in *Understanding Command and Control* by Alberts and Hayes (2006) is that C2 is everyone's concern and should no longer be left to the "C2 Specialist." Understanding the elements and functions of C2 allows us to establish a foundation for our objectives and constraints in our model development.

2.1 Doctrine on C2

Creating an accurate model first requires an understanding of current definitions of C2, which, at the highest level of U.S. Military, is the Joint doctrine followed by the service-level Marine Corps doctrine.

2.1.1 Joint Doctrine

The DOD's definition of C2, put forth by the *Doctrine for the Armed Forces of the United States*, Joint Publication (JP) 1 (JP-1), is "the exercise of authority, responsibility, and direction by a commander over assigned and attached forces to accomplish the mission" (Joint Chiefs of Staff 2017, p. I-18). C2 is an art based on leadership. Mission accomplishment is achieved through leadership and the ability to "command" by "motivating and directing people and organizations into action" (p. I-18). Control is built into command and is "how" command manages and directs forces to execute "requirements, allocate means, and integrate efforts" (p. I-18).

Ultimately, from the joint perspective the preferred method of exercising C2 is mission command. Mission command is "the conduct of military operations through decentralized execution based upon mission-type orders" (Joint Chiefs of Staff 2017, p. V-15). A mission-type order is "(1) an order issued to a lower unit that includes the accomplishment of the total mission assigned to the higher headquarters, or (2) an order to a unit to perform a mission without specifying how it is to be accomplished" (Joint Chiefs of Staff 2019, p. 148). Mission command is successful when leaders issue orders on the purpose of the operation, which allows subordinate leaders the ability to exercise initiative and action independently rather than scripted details of how to perform assigned tasks. Per JP-1, mission command "empowers individuals to exercise judgment in how they carry out their assigned tasks and it exploits the human element in joint operations, emphasizing trust, force of will, initiative, judgment, and creativity " (Joint Chiefs of Staff 2017, p. V-15).

There is also a command and control system definition put forth by the *Joint Communications System*, JP 6-0, which is "the facilities, equipment, communications, procedures, and personnel essential for a commander to plan, direct, and control operations of assigned and attached forces pursuant to the missions assigned" (Joint Chiefs of Staff 2015, p. GL-4). Today's C2 system, with information-age technology, facilitates freedom of action that is flexible and responsive to command and that can also provide the opposite effect if used incorrectly.

2.1.2 Marine Corps Doctrine

As a service, the Marine Corps doctrine takes the C2 definition from the joint doctrine and describes how we execute C2 within the Marine Air Ground Task Force (MAGTF). According to MCDP 6, *Command and Control*, there are two fundamental approaches to the spectrum of C2:

1. Detailed C2: a "centralized command" and formal approach to execution.
2. Mission C2: a "decentralized control," informal, and flexible approach to execution. (U.S. Marine Corps 1996)

In order to be successful in future conflicts, Commanders and Marines must understand the limitations and strengths of both C2 approaches. The practical application is detailed C2 ("centralized command") and mission C2 ("decentralized control").

Detailed C2 applies a "centralized command" and formal approach to execution which involves explicit orders or plans requiring "strict adherence, effectively minimizing subordinate decisionmaking and initiative" (U.S. Marine Corps 1996, p. 78). A natural response to the unknown is by optimization towards a highly efficient C2 system that can minimize and reduce uncertainty on the battlefield. Detailed C2 derives from, "a belief that a powerful and highly efficient C2 system can impose order and certainty on the disorderly and uncertain operational environment" (U.S. Marine Corps 1996, p. 47). Detailed C2 utilizes a vertical approach where orders flow down the chain of command while information flows up. As a result, the C2 process is centralized which has a tendency to move more slowly and may not "react well to rapidly changing situations" (U.S. Marine Corps 1996, p. 78).

Mission C2 applies a "decentralized control," informal, and flexible approach to execution. Unlike the degree level of certainty needed with detailed C2, mission C2 focuses on an understanding of the commander's intent and delegation of authority which is distributed to lower-echelon units. Orders are brief and simple which allow subordinates flexibility in decision-making. As a result, the C2 process is decentralized which allows lower-echelon units to execute the intent of mission-type orders more freely in rapidly changing situations. The Marine Corps ability to "increase tempo" and optimize effective responses to "fluid and disorderly situations" is a product of the mission C2 approach (U.S. Marine Corps 1996, p. 79).

Per MCDP 6, "the basic elements of the C2 system are people, information, and the C2 support structure" (U.S. Marine Corps 1996, p. 47). The C2 system is fundamental to the Corps and essential to the application of maneuver warfare which is discussed in Marine Corps Doctrinal Publication (MCDP 1), *Warfighting* (U.S. Marine Corps 1997). In order for the C2 system to be effective in the coordination of supporting arms and aviation assets it requires two elements: (1) people that use explicit communication to control personnel and equipment and (2) technology to direct, monitor, and control operations constituted in facilities, equipment, and weapon systems (Joint Chiefs of Staff 2015, p. vii). Both C2 approaches have separate and distinct characteristics which complement the task organization of the MAGTF.

2.2 NCW and C2 in the Information Age

The US Department of Defense (DOD) Command and Control Research Program (CCRP) developed the NCW doctrine to transform C2 processes from independent platforms to networked systems with the adoption of information age technologies (Fewell and Hazen 2003). NCW is a guiding resource and tool in C2 theory that breaks down "the structure and function of sociotechnical systems for analysis and design ... emphasizes the relationship between successful C2 processes and a networked system structure" (Eisenberg et al. 2018, p. 68783). The NCW doctrine provides "a comprehensive theoretical overview of C2 systems, processes, and needs, while also identifying characteristics of successful C2 systems" (Eisenberg et al. 2018, p. 68783). C2 has benefited by NCW doctrine through the incorporation of information age technologies which through a network force, rapidly enables automated and distributed systems to work together.

In the book, *Understanding Command and Control*, Alberts and Hayes (2006) provide a deeper view and purpose for C2 in the information age: "The purpose of command and control is to bring all available information and all available assets to bear. Mission success may, in fact, not be achieved even if the best C2 Approach for the situation is employed and C2 is executed perfectly. In these cases, other factors dominate" (p. 67). Thus, according to Alberts and Hayes, mission accomplishment is one way to assess the quality of C2 yet does not present a full picture of the essential functions and characteristics associated with C2. The essential C2 functions and characteristics are: "Establishing intent," "Determining roles, responsibilities, and relationships," "Establishing rules and constraints," "Monitoring and assessing the situation and progress," "Inspiring, motivating, and engendering trust," "Training and education," and "Provisioning" (Alberts and Hayes 2006, p. 35-36). Next, the doctrine of NCW introduces domains of warfare in the context of C2 and its span within the information age.

2.2.1 NCW Domains of Warfare

The NCW doctrine introduces a set of domains which are different from the domains in U.S. military doctrine. The U.S. military doctrine recognizes five domains of warfare which are: Land, Sea, Air, Space, and Information which Fogleman (1995) discusses the newest information domain in the article, "Information Operations: The Fifth Dimension of Warfare." However, C2 spans an entirely different set of domains that in part describe human behavior and interactions with information and technology. In the book *Power to the Edge*, Alberts and Hayes (2003) delineate that C2 agility spans four interrelated NCW domains of warfare defined as physical, information, cognitive, and social. Following is a brief example of each domain according to, "The Implementation of Network-Centric Warfare," by Cebrowski (2005):

- Physical Domain: "the traditional domain of warfare where a force is moved through time and space" (p. 20).
- Information Domain: "the domain where information is created, manipulated, and shared" (p. 20).
- Cognitive Domain: "the mind of the warfighter" (p. 20).
- Social Domain: "describes the necessary elements of any human enterprise ... where humans interact, exchange information, form shared awareness and understandings, and make collaborative decisions" (p. 20).

The NCW value chain in Figure 2.1 exists within the domains of warfare. The tenets of NCW and the domains of warfare are represented in this value chain as follows: "(1) a robustly networked force improves information sharing, (2) information sharing and collaboration enhances the quality of information and shared situational awareness, (3) shared situational awareness enables collaboration and self synchronization, and enhances sustainability and speed of command, and (4) these in turn dramatically increase mission effectiveness" (Alberts et al. 2010, p. 27).

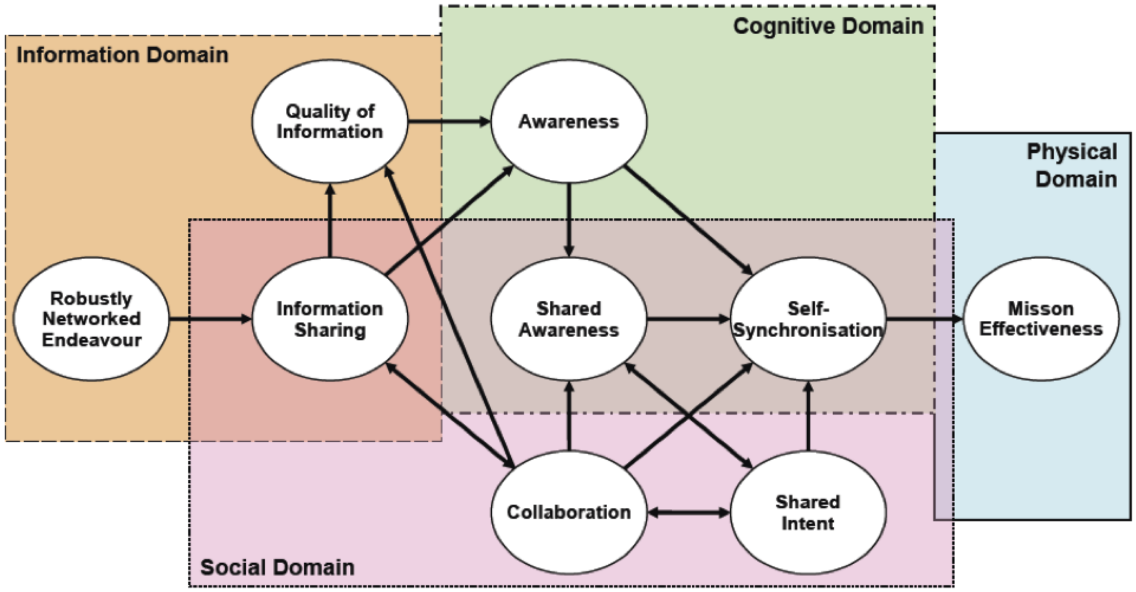

Figure 2.1. Network Centric Value Chain within the Domains of Warfare.
Source: Alberts et al. (2010).

C2 as a function of the domains of warfare is represented in Figure 2.2. Alberts and Hayes (2006) highlight that the function of command over time initiates and shapes the conditions for operations and sets the rules for control. Likewise, the function of control is to maintain specific elements of the operating environment efforts on track and make adjustments within guidelines (bounds/intent) established by the command.

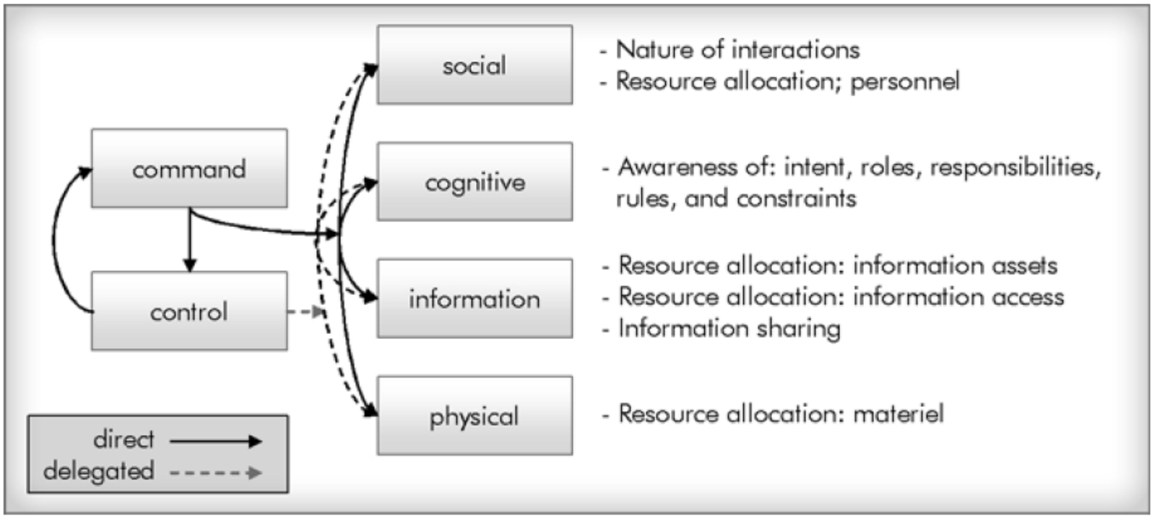

Figure 2.2. C2 as a Function of Domain.

Source: Alberts and Hayes (2006).

The NCW domains of warfare are also represented in system architecture which can be utilized to model structure and implement functions to establish constraints. Building on work originally developed by Alderson and Doyle (2010) on system architecture in complex systems, Eisenberg et al. (2018) define system architecture design constraints for NCW domains of warfare:

- Component Constraints: physical laws and requirements that dictate the capability of network nodes.
- Protocols: rules for the configuration and/or interaction of system components.
- System-level Constraints: higher-level functional purpose of a single network layer including objectives and design criteria the system is meant to serve.
- Emergent Constraints: the laws that dictate physical limitations of real systems often expressed as needs and interactions across systems. (p. 68785)

The NCW domains describe how C2 is related to warfare at the highest level, and can be further simplified to architecture dimensions which can be used to distinguish between different C2 approaches.

2.2.2 NCW C2 Architecture Dimensions

The NCW domains lead to C2 architecture dimensions which will provide a useful interpretation of the C2 approach space and how it can be applied to model development. The C2 architecture dimensions are best described in the book, *NATO NEC C2 Maturity Model*, by Alberts et al. (2010). NCW defines a three dimensional C2 approach space with axes that correspond to "(1) the ***allocation of decision rights*** to the collective, (2) the ***patterns of interactions*** that take place between and among entities, and (3) the ***distribution of information*** across entities" (Alberts et al. 2010, p. 47). Figure 2.3 provides a graphical representation of the mappings involved and will guide the development of the model in this research.

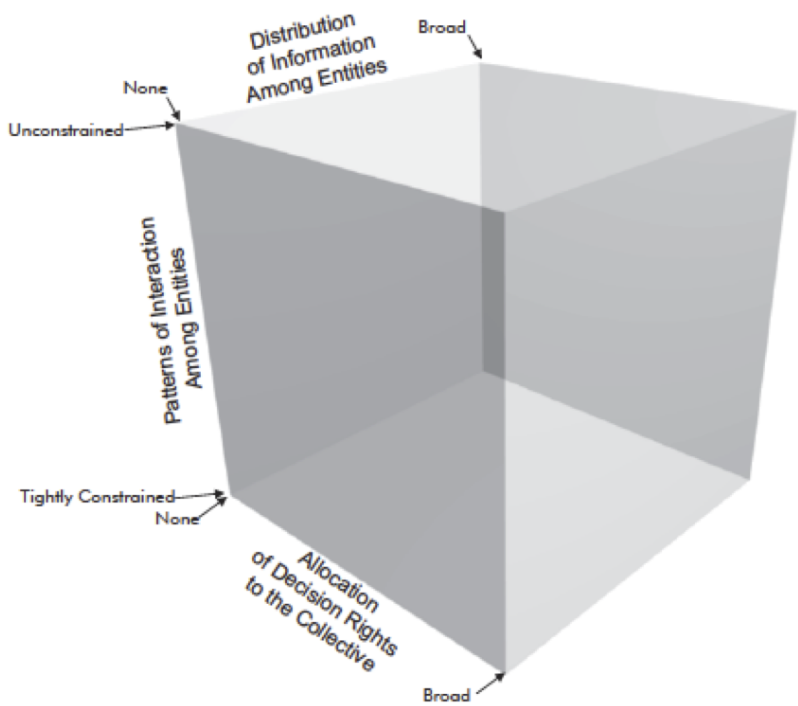

Figure 2.3. C2 Approach Space.

Source: Alberts et al. (2010).

Allocation of Decision Rights

Guiding model-development question: In a network, who "does what" based on the type of decisions?

In a collection of entities, Alberts et al. (2010) describe that the allocation of decision rights, "can likewise be explicit, implicit or emergent and refers to the degree to which individual entities have given up their respective rights for the benefit of the endeavour as a whole." Cohick (2018) phrases the key question for distribution of information as: "Who gets to make what type of decisions?"

Patterns of Interaction
Guiding model-development question: How does the network dictate who "collaborates," "communicates," and "shares"?

Alberts et al. (2010) describes the patterns of interaction among participating entities as, "a function of their respective abilities and willingness to interact as well as the opportunities they have as a result of the actual occurrence of interactions and collaborations." Cohick (2018) phrases the key question for patterns of interaction as, "Who communicates or interacts with whom?" and "What rules are in place that constrain or enhance collaboration?"

Distribution of Information
Guiding model-development question: How is information passed through the network from where it is "found" to where it is "needed"?

Alberts et al. (2010) describe the distribution of information across participating entities as, "the extent to which the information needed to accomplish required tasks is available to each participant." Cohick (2018) phrases the key question for distribution of information as, "Who knows what?" and "Who gets which resources?"

We have defined the C2 architecture dimensions in the terms of the C2 approach space, as well as provided the dimension guidance for consideration in model development. The C2 approach space presents architecture dimensions that are agile and useful in characterizing the differences in C2 approaches.

2.2.3 NCW Goals: C2 Agility and Shared Consciousness

NCW utilizes the architecture dimensions to distinguish and represent agility within different C2 approaches. Alberts and Hayes (2006), describes agility as a synergistic multidimensional concept with six key areas that interact with the operating environment for

C2: robustness, resilience, responsiveness, innovation, flexibility, and adaptation. Similarly to agility, General McChrystal introduces the idea of "shared consciousness" or information sharing within an organization which deals with transparency and trust among people within a hierarchy and results in collaboration. The modelling of agility and collaboration within the C2 community is not well understood and prescriptive research is necessary in understanding these operational concepts.

In Figure 2.4, Alberts and Hayes (2003) take the domains of warfare and incorporate the six key areas of agility. The elements of agility are interdependent and influence one another and represent a degree of self-synchronization. Therefore, when all of the elements of agility are present, the likelihood of mission accomplishment increases and the opposite holds true if one is missing achieving the other elements is more difficult (Alberts and Hayes 2003, p. 128). Alberts and Hayes (2006) provide an interconnected network statement of the six key areas of agility, "Hence, while it remains possible to specify definitions and metrics for the elements of agility, the concept must be considered holistically" (p. 189).

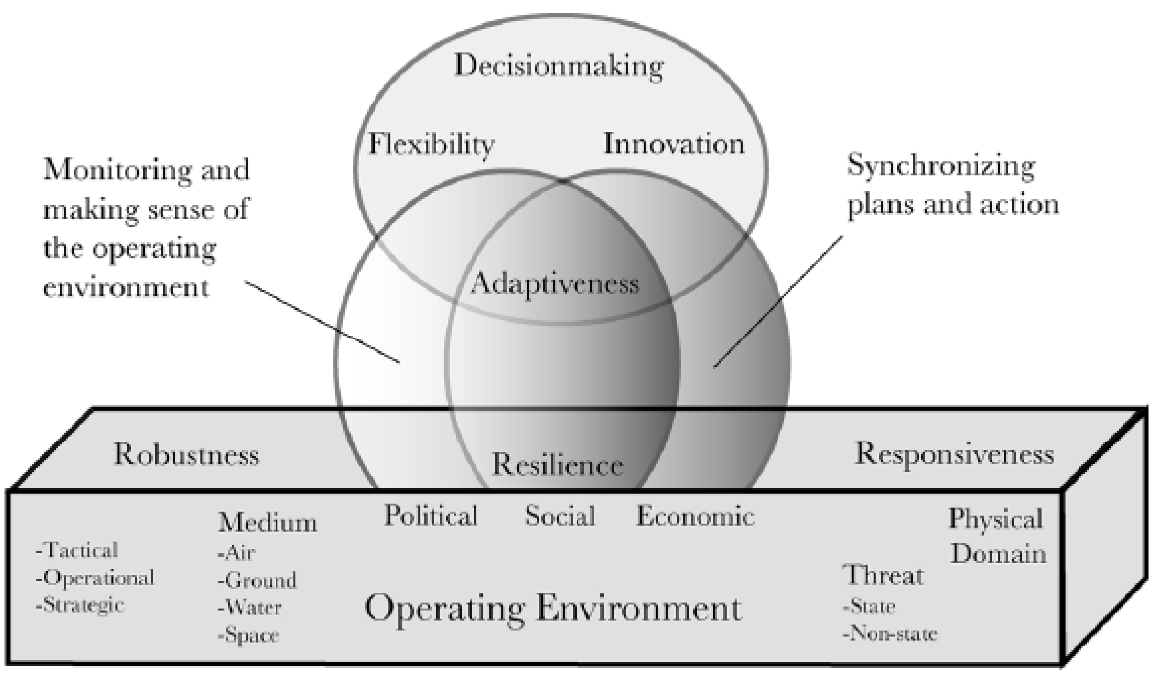

Figure 2.4. The Six Aspects of Agility in the Domains of Warfare.

Refer to *Power to the Edge*, for a complete discussion of Agility. Source: Alberts and Hayes (2003).

General McChrystal and Alberts both discuss agile C2 organization for decision-makers; following is a brief description of the attributes of agility as applied to decisions:

- Robustness: the degree to which a decision is effective across a range of situations.
- Resilience: the degree to which a decision remains applicable under degraded conditions or permits recovery from setbacks.
- Flexibility: the degree to which a decision allows force entities to maintain flexibility (i.e., incorporates multiple ways of succeeding).
- Adaptability: the degree to which a decision facilitates force entities' ability to alter or modify the decision or decision process. (Alberts and Hayes 2006, p. 148-149)

It is easier to optimize and make decisions in a traditional C2 model when the situation is known, understood, and predictable which represents a lower degree of agility. In the missions that General McChrystal's team-of-teams encountered, an agile C2 support structure empowered with these four attributes of robustness, resilience, flexibility, and adaptability is necessary to handle a less known, less understood, and a less predictable enemy. Agility with an emphasis on robustness is our goal in modelling a C2 network. Currently, to this point there are few research papers that can serve as a guide in the objective of this thesis. The next section of the literature review will focus on the research problem and issues with the current approaches to C2.

2.3 Current Issues in C2 Research and Implementation

The main issue with researching C2 is that it is embedded at the core of all disciplines within the military, governments, civilian workforce, and academia. So far, C2 research is often limited to a single discipline, which limits and isolates best practices to common problems that, as a whole, the community of interest faces daily. C2 does not have its own discipline and within the past two decades has emerged as a topic of interest in control theory, organization theory, and network science. However, the core C2 dilemma in the "Network Foundation for Command and Control (C2) Systems: Literature Review," by Eisenberg et al. (2018) notes that C2 "literature remains disorganized and isolated in publication" and states “one consequence is that the majority of research remains descriptive rather than prescriptive in experimental design, modeling, and analysis” (p. 68783). Eisenberg et al. (2018) describes descriptive and prescriptive C2 research as follows:

> *Descriptive* for which social and technological relationships will succeed in performing shared goals ... Descriptive research reveals underlying factors that enable successful completion of missions, often measured as the speed an organization can complete a task, the diversity of different tasks an organization can complete, the amount of shared information among organizational members, or a combination of all three. (p. 68782-68783)
>
> *Prescriptive* for how to design better C2 systems ... Prescriptive research then builds on these results to rearrange existing social relationships and/or introduce disruptive technologies to improve existing C2 processes in an organization. (p. 68782-68783)

Eisenberg et al. (2018) presents that the lack of organization, standard practices, and integration of knowledge across descriptive and prescriptive studies limits advances in C2 due to “a disconnected landscape of research, which inhibits the possibility of advancing research towards an applied practice” (p. 68783). Also important to C2 research is how prescriptive research can have a “broad impact on social systems of decision hierarchy, interpersonal interactions, knowledge sharing, training, and skills and technological systems” (p. 68783). A different approach to C2 is necessary as the evolution of work (i.e, mission) transitions from the industrial age to the information age, likewise, a C2 support structure must adapt to a change in work.

Today in the context of research, the direction and evolution of C2 requires intervention towards a unified front. OR is the ideal discipline to take a deep dive into the subject matter of C2 and determine a path to addressing this multiple-discipline and community-wide issue.

2.3.1 C2 System Complexity

Another issue to discuss is the complexity of C2 systems. In Alberts and Hayes (2006) book, *Understanding Command and Control*, complexity means braided, entwined together, inseparable, or interdependent and implies that "a complex system cannot be deconstructed into a series of manageable or predictable pieces" (p. 203). Gell-Mann (1995), a Nobel Physicist stated that, "effective complexity can be high only in a region intermediate between total order and complete disorder." Alberts and Hayes, further define total order as linear and predictable and complete disorder as randomness and impossible to predict. Per MCDP-6 (U.S. Marine Corps 1996), in Marine Corps terms a complex system is, "any system composed of multiple parts, each of which must act individually according to its own circumstances and which, by so acting, changes the circumstances affecting all the other parts" (p. 44). Complexity in the system emerges from part interactions that can be random or unpredictable and tend to exhibit a nonlinear behavior similar to open systems. Part interactions tend to be frequent and free in open systems which are capable of working with other systems as well as the external environment. MCDP-6 (U.S. Marine Corps 1996), provides several important features that define the reciprocal action and feedback view of C2 as a complex system, following:

1. "Effective C2 must be sensitive to changes in the situation" (p. 46).
2. "The action-feedback loop makes C2 a continuous, cyclic process and not a sequence of discrete actions" (p. 46).
3. "The action-feedback loop also makes C2 a dynamic, interactive process of cooperation" (p. 46).
4. "The commander is not above the system" and considered "an integral part of this complex web of reciprocal influence" (p. 47).

Ultimately, the C2 systems cooperation between action ("command") and feedback ("control") contributes to the organizations effectiveness and adaptability to changing situations. Overall MCDP-6, states that, "it is unreasonable to expect command and control to provide precise, predictable, mechanistic order to a complex undertaking of war" (U.S. Marine Corps 1996, p. 47).

2.3.2 C2 Networks and System Architecture

As a response to the complex systems, the field of network science has been applied to the study of large scale networks. In the book, *Networks: An Introduction*, Newman

(2010) defines Network Science (NS) as, "...concerned with understanding and modeling the behavior of real-world networked systems and observational data are the starting point for essentially all the developments of the field" (p. 22). Networks often model complex systems. In the article, "Network Foundation for Command and Control (C2) Systems: Literature Review," by Eisenberg et al. (2018), provide a comprehensive understanding of C2 research and identify network evaluation methods for complex systems which can benefit communities of interest. Eisenberg et al. (2018) states that, "network science methods are useful for studying the structure and function of C2 sub-systems by ranking nodes and links, where social and technological networks use the same methods to represent different constructs and dependencies," such that NS models and analysis methods can, "provide a consistent basis for comparing technical advances in the C2 literature" (p. 68784).

In the article, "Catching the Network Science Bug: Insight and Opportunity for the Operations Researcher," Alderson (2008), summarizes the advantages and disadvantages of network science and highlights how OR can provide a critical benefit to decision-makers:

> However, when it comes to the research agenda now popularized by network science, OR has been an underutilized resource, with the result that many decision-makers tasked with important problems are headed in a direction that does not benefit from this vast body of theory and experience of making decisions based on analysis. (p. 1063)

Network science takes a snapshot view of deriving details and information from an existing or a generated network. What OR can provide is the ability to build and test a network model and provide analysis based off of the findings for a decision-maker. The next section will introduce optimization and cover how agility is a potential course of action to navigate C2 system complexity.

2.3.3 Optimization and C2 Agility

C2 optimization presents a particular difficulty unlike other optimization problems. The book, *Optimization in Operations Research*, Rardin (1998) defines optimization as, "optimization models (also called mathematical programs) represent problem choices as decision variable and seek values that maximize or minimize objective functions of the decision variables subject to constraints on variable values expressing the limits on possible decision variables." Rardin also presents three fundamental dimensions of modeling problems as: "the *decisions* open to decision-makers," "the *constraints* limiting decision choices," and "the *objectives* making some decisions preferred to others" (p. 4). In *Power to the Edge*, Alberts and Hayes (2003) provide an analytic definition of optimization as, "a process that seeks to find a solution (a military option, organizational form, process, system design) that gives the best possible result, a global maximum" (p. 60). The decisions, constraints, and objectives of optimization inherently involves tradeoffs. Given a choice, options are based off the objectives such as the global optimum that yields the best result or another choice that is sub-optimal but maintains the objective over a larger range of conditions.

An underlying optimization problem for industrial age organizations is that centralize planning was created to increase efficiency throughout military and civilian organizations. Moreover, Alberts and Hayes (2003) suggest that there are problems with that approach in a modern context of the information age:

> For centralized planning to work, it must be possible for a relatively small group of people to do all of the following: make sense of the situation, maintain this understanding in the face of a dynamic environment, predict the future, develop an appropriate response strategy, decompose the response into a coherent set of executable tasks, allocate resources, task subordinates, monitor execution, and make adjustments as required, all in a timely manner. In fact, despite a belief in the power of reductionism and a strong desire to optimize, centralized planning has evolved into a set of processes that often prevent optimization. Ironically, centralized planning processes are designed to deconflict tasks and elements of the force so that they will not get in each other's way or do harm to one another. They prize deconfliction over synergy. This prevents simultaneity and the synergies necessary to perform anywhere near optimality. Centralized planning is antithetical to agility because it (1) is relatively slow to recognize and respond to changes in the situation, (2) results in ill-informed participants, and (3) places many constraints on behavior. (p. 63)

In the context of centralized planning there is a way to incorporate optimization and introduce agility, for example, a cornerstone of the USMC planning process is centralized command and decentralized control. This ideology is built into the fabric of the Marine Corps and enables leaders and subordinates at lower levels of the chain of command to exercise decision-making which fosters the agility to respond to "real-world" changes in a situation. Similar to how the USMC conducts operations, how can an optimization model represent this type of organizational structure?

In the book, *NATO NEC C2 Maturity Model*, Alberts et al. (2010) argues against an assumption "that one can optimize or employ a C2 approach that is optimal for a given situation" (p. 22):

> One cannot and should not think about *optimizing* command and control in the 21st century. There is no single approach, no best system design or configuration, no best process for all situations and circumstances. Uncertainty in the mission space and complexity in the environment, the effects space, and the complexity inherent in a collective dominate. (p. 22)

Alberts thus contends that C2 should focus on agility rather than optimization which leverages engineers and analysts towards an opportunity to develop a creative approach and solution. The past and future issues of C2 do not lead down a path of a rigid C2 system but more towards an agile C2 system which incorporates elements of a top down hierarchy and a flat organization with teams that can provide a dynamic response. Lastly, for the purpose of this thesis we discuss current C2 literature in OR and network science.

2.4 Optimization-based OR Methods for Studying C2 Force Structure and Agility of Military Teams

The literature on C2 organization with teams falls into two broad categories: descriptive and prescriptive. Recent work on the topic of C2 organization with teams is limited in the area of prescriptive research, with few sources offering recommendations for how to improve C2 processes in an organization.

Despite reservations by Alberts and Hayes (2006) that C2 agility should not be "optimized", several optimization-based OR methods exist that can support the descriptive and prescriptive study of C2 force structure and agility in military teams. In particular, optimization-based methods for well-studied OR problems are relevant for understanding how military team force structure influences their capability to complete diverse and rapidly changing tasks:

1. Stochastic Processes and Monte Carlo Estimation Problems
2. Scheduling Problems
3. Team Formation Problems

We consider each of these well-studied OR problems in turn via related research.

2.4.1 Stochastic Processes and Monte Carlo Estimation

Cohick (2018), in his NPS thesis titled, "A first-principles approach to measuring robustness in command and control systems" develops a computational framework in the form of a game to measure and compare the effectiveness of organizational architectures. Likewise, Cohick's thesis uses a stochastic Monte-Carlo simulation to explore the advantages of different architectures when an organization's objective and/or operating environment change. While Cohick's thesis presents a prescriptive approach to General McChrystal's team-of-teams concept using mathematical analysis and simulation, he provides options for future work with more sophisticated architectures and leaves open the question of whether network optimization can be utilized in the assignment of work to teams in a C2 organization.

2.4.2 Scheduling Problems

Whereas Cohick investigates a team-of-teams concept with respect to C2, Beavers (2019) offers a prescriptive treatment of C2 resilience in maritime operations centers with a focus on mission planning, task organization, and scheduling of resources. Beavers, in his NPS thesis titled, "A method for quantifying manpower, personnel, training, and education (MPT&E) impacts on Command and Control (C2) resilience in maritime operations centers," focuses on a project scheduling problem for considering staff activities and develops a simulation-based tool to quantify the risk to mission accomplishment when staff levels, training, and equipment are under-funded. Likewise, Beavers identifies the difficulty organizations encounter with task organization and the appropriate scheduling of tasks to resources. While Beaver's thesis presents a prescriptive approach to incorporating resilience in the C2 of maritime operations centers through resilience in scheduling, he states in future work to, “lean efforts towards better understand the organization’s structure and improvements to model utility,” which leaves open the question of how C2 optimization of teams can be applied to accomplishing work.

2.4.3 Team Formation Problems

An article by Salas et al. (2001), “Understanding Command and Control Teams Operating in Complex Environments,” provides a descriptive approach that focuses on the social aspect of how teams behave within organizations. The article offers a prescriptive approach based on a compilation of qualitative descriptive research and does provide a contribution via tables with the characteristics of C2 teams, as well as distributed C2 teams. Salas et al. (2001), identify guiding principles related to effective C2 team performance from literature and also present challenges related to the increased use of distributed C2 teams. However, the authors’ recommendations are limited to principles applicable to understanding existing teams; they do not make recommendations about how “best” to create C2 teams with theory or empirical evidence.

A combination of Salas et al.’s article and Cohick’s thesis, Farasat and Nikolaev (2016) extends the research on teams by utilizing social structure optimization and incorporates a combination of tools used in network science and OR. Their article, “Social structure optimization in team formation,” takes both a descriptive and prescriptive approach to teams and applies OR methods to incorporate a social component on the team formation problem. As a stand alone product using social network analysis theory on teams optimization this article is one of few seminal works that produces a “mathematical framework model for the Team Formation Problem explicitly incorporating Social Structure (TFP-SS)” (p. 27). Farasat and Nikolaev quantifies social structure through the local network of each individual on their work-related outcome and is described as, “Given a pool of individuals, the TFP-SS objective is to assign them to teams so as to achieve an optimal structure of individual attributes and social relations within the teams” (p. 127). That said, while Farasat and Nikolaev’s article presents a prescriptive approach to the team formation problem framework to generate a range of models based on social structures, he states in his conclusion that, “the question of selecting the best model for a given application deserves more attention,” which leaves open the question of whether there is an optimal assignment of work to teams that can be tailored to a C2 force structure (p. 140).

Our research question is about whether organizing around teams can give C2 more of the desirable qualities it needs such as agility, interoperability, robustness etc. to respond to the dynamism and asymmetry of present and future threats. In the context of well-studied OR problems, the related works provide depth into "real-world" C2 issues and problems that are and can be viewed through the lens of optimization-based methods in order to analysis the implementation of teams within a C2 support structure. The potential role of teams within a military's force structure and insight from optimization-based methods can provide C2 with a value added capability to complete diverse and rapidly changing tasks.

2.5 Contributions in Context

Successful organizations have to stay on pace or ahead of the game to maintain effectiveness among competitors and the best C2 practices can make a difference in an organization. The underlying core components of C2 are people, information, and the C2 support structure. Understanding the core construction of any C2 support structure will enable organizations the ability to make adjustments as necessary with changes in that specific environment. As called for in "Catching the Network Science Bug: Insight and Opportunity for the Operations Researcher," by Alderson (2008), this research takes a step in the right direction by shifting the emphasis from "network science research towards the development and validation of explanatory models ... and it is in this area that the OR community has an important role to play" (p. 1056). The literature review provides a snapshot of the problem set and how C2 is a subject that presents a dynamic realm of complexity which is difficult to generate models for optimization. Chapter III facilitates a first-principles approach towards the development of our C2 models.

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CHAPTER 3:
Models

This chapter introduces the basic terminology and notation used in the development of four optimization models for team formation and work (i.e., mission) assignment.

3.1 Lexicon

Fundamentally, this thesis is about the relationship between four types of entities: *tasks*, *resources*, *missions*, and *teams*.

Task. A *task* is an atomic unit of work. A task requires resources in order to be completed. We say that when a task that has sufficient resources assigned, it is *covered*; otherwise the task is *uncovered*. Mathematically, we let $k \in K$ index the set of tasks under consideration.

Resource. A *resource* is the basic unit that performs work. In general, we think of resources as people, although our definition is general enough to include different types of resources (e.g., equipment). Mathematically, we let $i \in R$ index the set of resources under consideration.

Mission. We refer to a group of tasks as a *mission*. We use the term *synchronization* to refer to the process of assigning tasks to missions. Mathematically, we let $n \in N$ index the set of missions under consideration.

Team. A *team* refers to a group of resources that work together. We say that resources on a team *coordinate* their work. Mathematically, we let $m \in M$ index the set of teams under consideration.

Ultimately, all work is performed by resources, but the way in which resources are matched to tasks depends on the way in which the respective missions and teams are formed. Accordingly, we are interested in four types of *assignment* problems:

1. *Task-to-Resource Matching:* How to assign individual resources to tasks (without regard to teams or missions) so that work gets done?

2. *Mission-to-Team Matching:* How to assign teams of resources to missions of tasks (and thereby assign resources to tasks) so that work gets done?
3. *Team-Formation:* Given a set of missions that need to be covered, how best to form teams?
4. *Mission Assignment:* Given a set of available teams, how best to group tasks into missions?

We consider each of these assignment problems in turn via an illustration and a description of the concept, which is used to formulate the associated assignment model.

3.2 Task-to-Resource Matching

The most basic assignment problem is task-to-resource matching, which requires knowledge of all the available tasks and resources. Figure 3.1 depicts the matching of resources to tasks. We use color to illustrate the potential fit between resources and tasks; the optimal assignment is the one that does the best overall job matching resources to tasks.

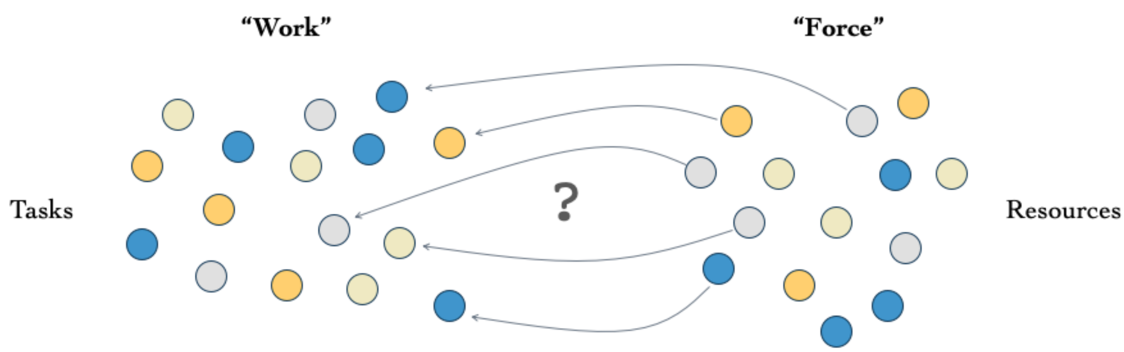

Figure 3.1. Task-to-Resource Matching.

The main question to consider here: "Which resources should execute which tasks?" The optimal matching for this assignment involves the task and resource entities. The assignment represents the optimal match for a selected resource to execute an appropriate specified task.

We formalize the task-to-resource matching problem according to the following assignment model.

3.2.1 Task Functional Requirements and Resource Specialization

We assume that any resource can execute any task, however that the cost for a resource to execute a task depends on the suitability of the resource for that task, which we represent as follows.

Task Functional Requirement. We use a *functional requirement* to characterize the type of work required by a task among a broad spectrum of work. Mathematically, we use an angle on the unit circle, $0 \le \phi_k \le 2\pi$, to denote the functional requirement for task k.

Resource Specialization. We use a *resource specialization* to characterize the type of skills (e.g., Military Occupational Specialty, or MOS) of a resource among a broad spectrum of resources. Mathematically, we use an angle on the unit circle, $0 \le \theta_i \le 2\pi$, to denote the specialization of a resource i.

We use the angular difference $\theta_i - \phi_k$ to represent the suitability of a resource for a task, and we define the function $f(\theta_i, \phi_k) = (1 - \cos(\theta_i - \phi_k))/2$ to quantify the cost for resource i to execute task k. The basic idea is that a resource whose specialization "matches" the functional requirement of a task will have a low execution cost, while a resource whose specialization "greatly differs" from the functional requirement can still execute the task but at a high execution cost. At the extremes, we have $f(0) = 0$ and $f(\pi) = 1$. Figure 3.2 illustrates this concept.

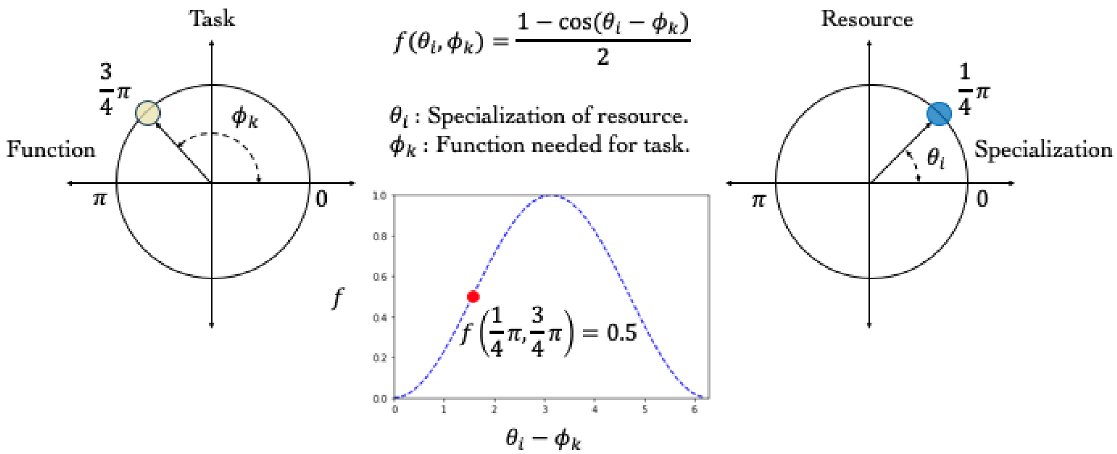

Figure 3.2. Task Functional Requirements and Resource Specialization.

Each task entity will be equipped with a task functional requirement and each resource will be equipped with a resource specialization. The angles will vary depending on the function and specialization of the entities. In this example, the angular difference $\theta_i - \phi_k = \pi/2$ and the suitability is 0.5.

Thus, the optimal solution to the Task-to-Resource model is the assignment of resources to tasks that covers all tasks while minimizing the aggregate differences between resource specialization and task functional requirements.

3.2.2 Task-to-Resource Model

The goal of the Task-to-Resource model is to produce the best assignment of tasks to resources in order to accomplish work. The Task-to-Resource model requires two given inputs: tasks and resources, and their associated data.

Indices and Sets

$i \in R$	resources (alias j)
$k \in K$	tasks

Data [units]

ϕ_k	functional requirement for task k [radians], $\phi_k \in [0, 2\pi]$
θ_i	specialization of resource i [radians], $\theta_i \in [0, 2\pi]$
p_k	per unit shortfall penalty for task k [cost units]

Calculated Data [units]

f_{ik}	cost for resource i to execute task k [cost units]
	$f_{ik} = (1 - \cos(\theta_i - \phi_k))/2$

Decision Variables [units]

S_k	1 if task k is not covered (shortfall), 0 otherwise [binary]
Y_{ik}	1 if resource i assigned to task k, 0 otherwise [binary]

Formulation

$$\min_{S,Y} \quad \sum_i \sum_k f_{ik} Y_{ik} + \sum_k p_k S_k \tag{A0}$$

$$\text{s.t.} \quad \sum_k Y_{ik} \leq 1 \qquad \forall i \in R \tag{A1}$$

$$S_k \geq 1 - \sum_i Y_{ik} \qquad \forall k \in K \tag{A2}$$

Discussion

This formulation implements assignment via the *Task-to-Resource model* — which matches a selected resource to execute an appropriate specified task based on a matching suitability function. The objective function (A0) combines the total task-to-resource execution cost and the total penalty cost for any coverage shortfall. Constraint (A1) limits each resource to be assigned to at most one task. Constraint (A2) calculates whether there is a shortfall for each task k.

Overall, the Task-to-Resource model is the simplest assignment model which mathematically captures how an organization can execute and accomplish work at the most basic level. The Task-to-Resource model will serve as the primary foundation for the models that follow.

3.2.3 Small Example: Task-to-Resource Model

We consider a small example problem with two tasks and two resources. Table 3.1 depicts the problem data, Figure 3.3 illustrates the optimal solution to the Task-to-Resource model, and Table 3.2 depicts the Task-to-Resource model report.

Tasks			**Resources**	
k	ϕ_k	p_k	**i**	θ_i
k1	$3\pi/4$	1	i1	$\pi/4$
k2	$-\pi/4$	1	i2	$-\pi/4$

Table 3.1. Task-to-Resource Data (Two Tasks, Two Resources).

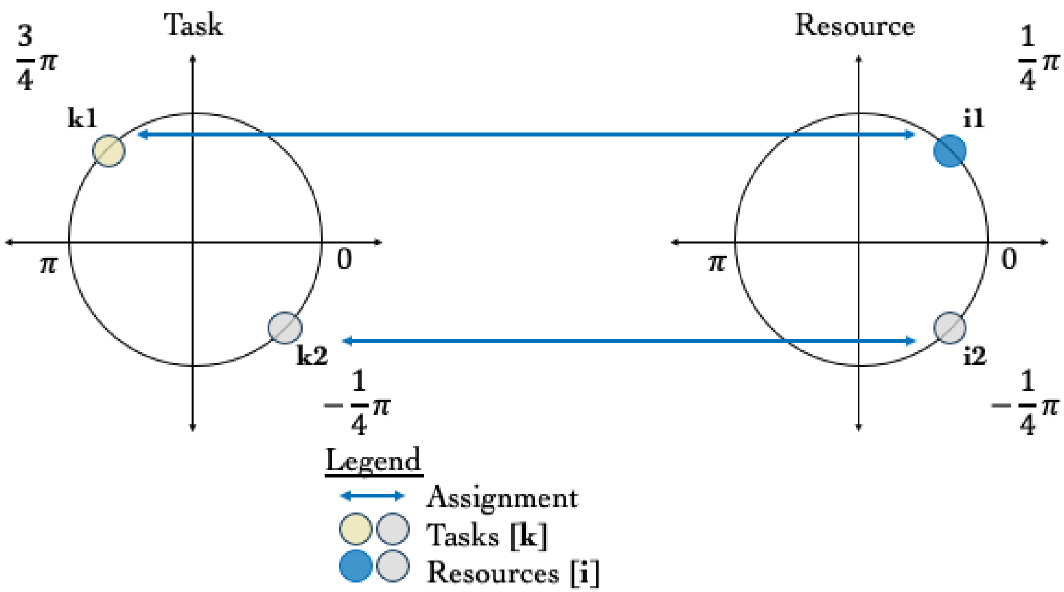

Figure 3.3. Direct Task-to-Resource Model (Two Tasks, Two Resources).

The optimal assignment for two tasks and two resources. The execution cost is minimized when resource $i1$ is assigned to task $k1$ and $i2$ is assigned to $k2$. Both tasks are covered, so there is no shortfall penalty cost.

Task-to-Resource Model Report

```
Objective:  0.7240
Assigned resources:
i1 -> k1, execution cost:  0.7240
i2 -> k2, execution cost:  0.0000
Shortfall penalties:
None.
```

Table 3.2. Task-to-Resource Report (Two Tasks, Two Resources).

3.3 Mission-to-Team Matching

Assigning resources to tasks is straightforward conceptually but can become daunting as the number of tasks and resources becomes large. Here, therefore, we consider a simplified (restricted) problem where the tasks have previously been grouped into missions and the resources have previously been formed into teams.

We consider the assignment problem of missions to teams, wherein we can then assign tasks to resources.

Figure 3.4 describes the relationships between the mission and team entities, and it illustrates the optimal matching of missions to teams with the emphasis of accomplishing work.

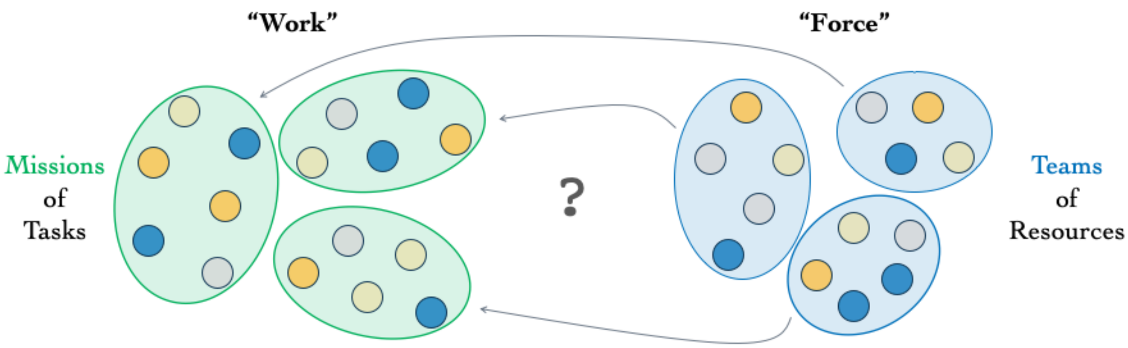

Figure 3.4. Mission-to-Team Matching.

The main question to consider here: "Which teams should execute which missions?" The optimal matching for this assignment involves the mission and team entities. The assignment represents the optimal match for a selected team to execute an appropriate specified mission.

We formalize the Mission-to-Team matching problem in the following assignment model.

3.3.1 Mission-to-Team Model

The Mission-to-Team model requires four given inputs: tasks, resources, predetermined missions and predetermined teams in order to choose the best assignment of missions to teams and allocate appropriate tasks to resources.

Indices and Sets

$i \in R$	resources (alias j)
$k \in K$	tasks
$m \in M$	teams
$n \in N$	missions

Data [units]

ϕ_k	functional requirement for task k [radians], $\phi_k \in [0, 2\pi]$
θ_i	specialization of resource i [radians], $\theta_i \in [0, 2\pi]$
p_k	per unit shortfall penalty for task k [cost units]
q_n	per unit shortfall penalty for mission n [cost units]
$\hat{W}_{kn}$	1 if task k assigned to group n, 0 otherwise [binary]
$\hat{X}_{im}$	1 if resource i assigned to team m, 0 otherwise [binary]
b_m	the maximum missions allowed for team m

Calculated Data [units]

f_{ik}	cost for resource i to execute task k [cost units]
	$f_{ik} = (1 - \cos(\theta_i - \phi_k))/2$

Decision Variables [units]

S_k	1 if task k is not covered (shortfall), 0 otherwise [binary]
U_n	1 if mission n is not assigned (shortfall), 0 otherwise [binary]
Y_{ikmn}	1 if resource i assigned to task k, 0 otherwise [binary]
Z_{mn}	1 if team m assigned to group n, 0 otherwise [binary]

Formulation

$$\min_{S,U,Y,Z} \quad \sum_i \sum_k \sum_m \sum_n f_{ik} Y_{ikmn} + \sum_k p_k S_k + \sum_n q_n U_n \tag{B0}$$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_k \sum_m \sum_n Y_{ikmn} \leq 1 \qquad \forall i \in R && \text{(B1)} \\
& S_k \geq 1 - \sum_i \sum_m \sum_n Y_{ikmn} \forall k \in K && \text{(B2)} \\
& \hat{W}_{kn} + S_k - U_n \leq 1 \qquad \forall k \in K, \forall n \in N && \text{(B3)} \\
& Y_{ikmn} \leq \hat{W}_{kn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N && \text{(B4)} \\
& Y_{ikmn} \leq \hat{X}_{im} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N && \text{(B5)} \\
& Y_{ikmn} \leq Z_{mn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N && \text{(B6)} \\
& \sum_n Z_{mn} \leq b_m \qquad \forall m \in M && \text{(B7)} \\
& \sum_m Z_{mn} = 1 \qquad \forall n \in N && \text{(B8)}
\end{aligned}
$$

Discussion

This formulation implements assignment via the *Mission-to-Team model* — that is, the assignment of missions to teams based on the given grouping of missions each with a predefined set of tasks and the given formation of teams each with a predefined set of resources. The Mission-to-Team model makes the problem simpler to match mission and team entities vice a large scale collection of task and resource entities, albeit at a potentially less optimal solution.

The objective function (B0) combines the total task-to-resource execution cost, the total task-to-resource coverage shortfall penalty cost, and the total mission-to-team coverage shortfall penalty cost. Constraints (B1) limit the assignment of one resource to at most one task. Constraints (B2) calculates whether there is a shortfall for each task k. Constraints (B3) calculate the mission shortfall decision variable. Constraints (B4), (B5), and (B6) enforce that a resource can be assigned to a task if and only if a resource on a team is assigned a mission that has the task. Constraints (B7) place an upper bound on the number of missions that each team is assigned. Constraints (B8) ensure that each mission is assigned exactly one team.

Overall, the Mission-to-Team model mathematically captures how an organization can manage, execute, and accomplish work within a tiered hierarchy (of predefined teams and predefined missions). However, this formulation is a restriction on the Task-to-Resource assignment problem. The use of predetermined missions and teams makes the assignment problem simpler (in the sense that there are fewer combinations to consider), but the ultimate solution of resources to tasks cannot be better (lower cost) than in the task-to-resource problem (and could be considerably worse).

3.3.2 Small Example: Mission-to-Team Model

We consider a small example problem with two missions and two teams. Table 3.3 depicts the problem data, Figure 3.5 illustrates the optimal solution to the Mission-to-Team model, and Table 3.4 depicts the Mission-to-Team model report.

Tasks			***Resources***		***Missions***			***Teams***		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$	**i**	**m**	$\hat{X}$
k1	$3\pi/4$	1	i1	$\pi/4$	k1	n1	1	i1	m1	1
k2	$-\pi/4$	1	i2	$-\pi/4$	k2	n2	1	i2	m2	1

Table 3.3. Mission-to-Team Data (Two Missions, Two Teams).

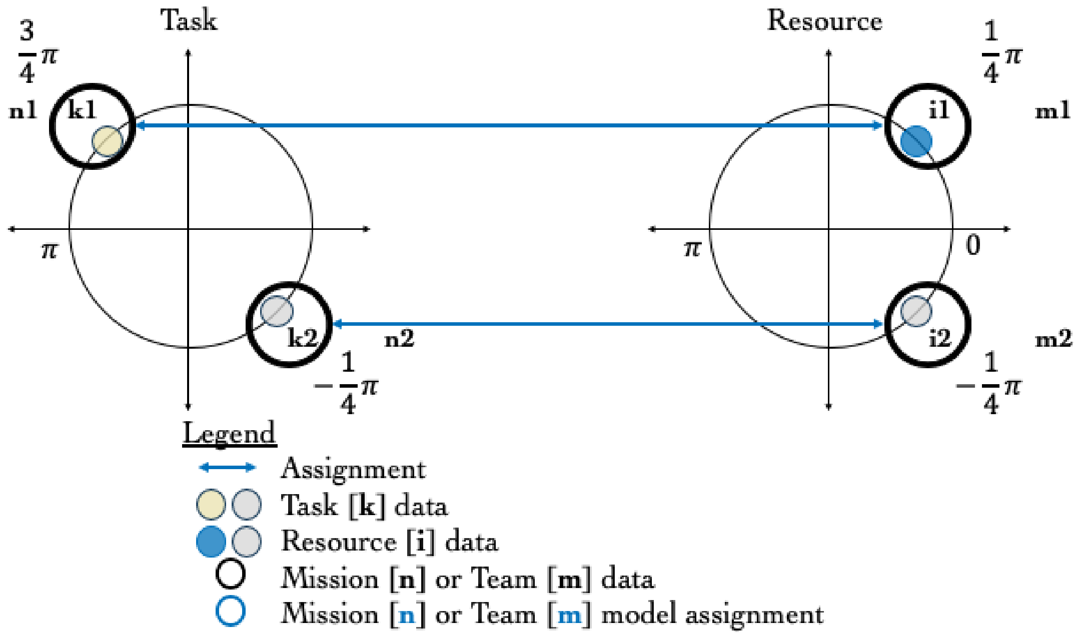

Figure 3.5. Direct Mission-to-Team Model (Two Missions, Two Teams).

The optimal assignment for two missions and two teams. Team $m1$ is assigned to mission $n1$ and $m2$ is assigned to $n2$. The execution cost is minimized when resource $i1$ is assigned to task $k1$ and $i2$ is assigned to $k2$. Both tasks are covered, so there is no shortfall penalty cost.

Mission-to-Team Model Report

```
Objective:  0.7240
Team-Mission Assignment:  Team membership:  Mission membership:
        m1:  n1                m1:  i1              n1:  k1
        m2:  n2                m2:  i2              n2:  k2
Assigned resources:
i1 ->k1, execution cost:  0.7240
i2 ->k2, execution cost:  0.0000
Shortfall penalties:
None.
```

Table 3.4. Mission-to-Team Report (Two Missions, Two Teams).

3.4 Team Formation

In a resource-constrained environment, assigning resources to teams can be a difficult process that requires a fine balance of resource specialization. Teams that are formed with balanced resource specialization may have the breadth and coverage necessary to accomplish a mission with varying task functions.

Here, therefore, we consider a relaxation of the mission-to-team problem where the tasks have been grouped into predetermined missions while the teams are yet undefined. In this Team-Formation model, the resources will be formed into the "best" possible teams to accomplish the designated missions. We then consider the assignment problem of resources into team-formation, wherein we can then assign mission-to-team and then task-to-resource.

Figure 3.6 describes the relationships between the mission and team entities, and it illustrates a potential assignment of resources to teams with the emphasis of generating a team to accomplishing a predefined mission. In this case tasks are already grouped into predetermined missions while teams are yet undefined. In team-formation the goal is to put resources into teams, and then assign mission-to-team, thereby assigning task-to-resource.

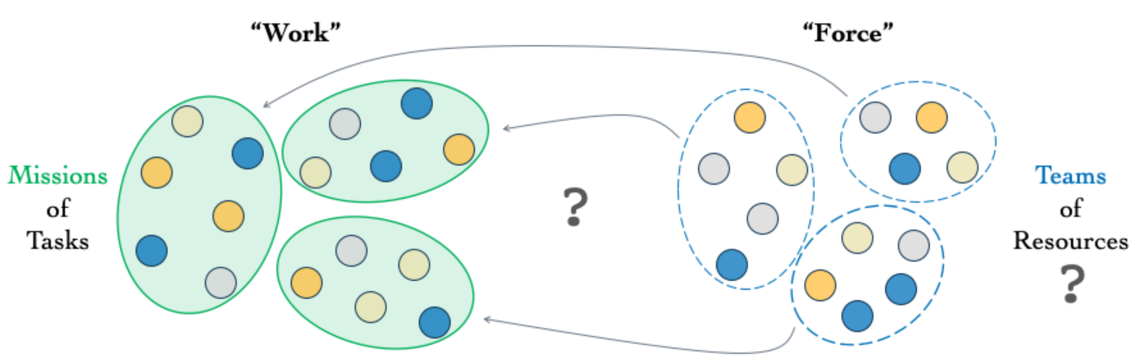

Figure 3.6. Team-Formation.

There are two questions to consider here: (1) "Which teams should execute which missions?" and (2) "What are the "best" teams for a given set of missions?" The optimal formation for this assignment involves the resource and team entities. The assignment represents the optimal formation of a selected resource to join a specified team in order to accomplishing a predefined mission.

We formalize the team-formation problem with the following assignment model.

3.4.1 Team-Formation Model

The Team-Formation model requires three given inputs: tasks, resources, and predetermined missions in order to choose the best assignment of resources into team-formation, then mission-to-team, assignments and thereby assigning tasks to resources.

Indices and Sets

$i \in R$	resources (alias j)
$k \in K$	tasks
$m \in M$	teams
$n \in N$	missions

Data [units]

ϕ_k	functional requirement for task k [radians], $\phi_k \in [0, 2\pi]$
θ_i	specialization of resource i [radians], $\theta_i \in [0, 2\pi]$
p_k	per unit shortfall penalty for task k [cost units]
q_n	per unit shortfall penalty for mission n [cost units]
$\hat{W}_{kn}$	given: 1 if task k assigned to group n, 0 otherwise [binary]
b_m	the maximum missions allowed for team m

Calculated Data [units]

f_{ik}	cost for resource i to execute task k [cost units]
	$f_{ik} = (1 - \cos(\theta_i - \phi_k))/2$
ρ_{ij}	balance "cost" for teaming resource i with resource j [cost units]
	$\rho_{ij} = \cos((\theta_i - \theta_j)/2)$

Decision Variables [units]

S_k	1 if task k is not covered (shortfall), 0 otherwise [binary]
U_n	1 if mission n is not assigned (shortfall), 0 otherwise [binary]
X_{im}	1 if resource i assigned to team m, 0 otherwise [binary]
Y_{ikmn}	1 if resource i assigned to task k, 0 otherwise [binary]
Z_{mn}	1 if team m assigned to group n, 0 otherwise [binary]

Formulation

$$\min_{S,U,X,Y,Z} \quad \sum_i \sum_k \sum_m \sum_n f_{ik} Y_{ikmn} + \sum_k p_k S_k + \sum_n q_n U_n + \sum_{m \in M} g(X_m) \tag{C0}$$

$$\text{s.t.}\quad g(X_m) = \sum_{i\in R} X_{im}\left(\sum_{j\in R; j\neq i} \rho_{ij} X_{jm}\right) \qquad \forall m \in M \tag{C1}$$

$$\sum_k \sum_m \sum_n Y_{ikmn} \leq 1 \qquad \forall i \in R \tag{C2}$$

$$S_k \geq 1 - \sum_i \sum_m \sum_n Y_{ikmn} \qquad \forall k \in K \tag{C3}$$

$$\hat{W}_{kn} + S_k - U_n \leq 1 \qquad \forall k \in K, \forall n \in N \tag{C4}$$

$$Y_{ikmn} \leq \hat{W}_{kn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \tag{C5}$$

$$Y_{ikmn} \leq X_{im} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \tag{C6}$$

$$Y_{ikmn} \leq Z_{mn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \tag{C7}$$

$$\sum_m X_{im} = 1 \qquad \forall i \in R \tag{C8}$$

$$\sum_n Z_{mn} \leq b_m \qquad \forall m \in M \tag{C9}$$

$$\sum_m Z_{mn} = 1 \qquad \forall n \in N \tag{C10}$$

Discussion

This formulation implements assignment via the *Team-Formation model* — that is, the assignment and composition of teams based on the given predetermined grouping of missions each with a predefined set of tasks. The Team-Formation model makes the problem simpler to match mission and team entities due the ease of forming teams with resources that can accomplish the mission, albeit potentially at a less optimal solution.

The objective function (C0) combines the total task-to-resource execution cost, the total task-to-resource coverage shortfall penalty cost, the total mission-to-team coverage shortfall penalty cost, and the "team effect" cost.

Figure 3.7 illustrates the team effect concept and Figure 3.8 is a plot representation of the team effect function. The "team effect" function (C1) calculates the balance of a given team, by calculating the "conflict" between each resource on the specified team. Here, we define "conflict" between resources to occur when resources with similar specialization are assigned to the same team. In practical terms, resources with the same specialization θ_i will prefer to do the same tasks in optimal task-to-resource assignment. When resources with the same specialization are on the same team, this creates conflict in optimal task-to-resource assignment as resources have a limited pool of potential task assignments due to mission grouping. In contrast, a "balanced" team is composed of resources where there is low conflict between resources for optimal task-resource assignment. Thus, it is better to form teams of resources with specializations that reduce resource-resource conflict. Equation (C1) captures this effect in the objective function by increasing the cost of the Team-Formation model when teams are not balanced.

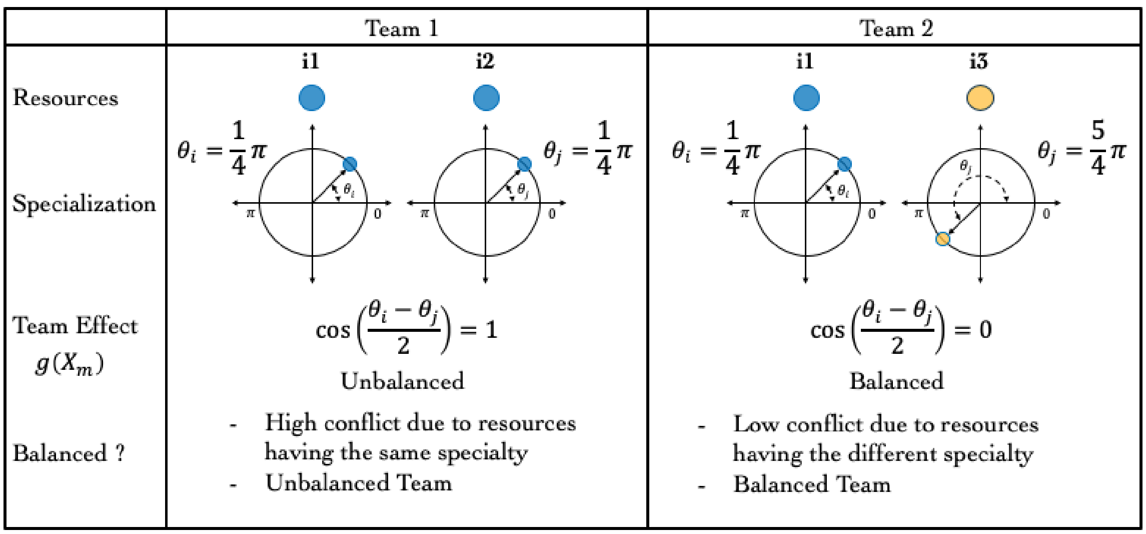

Figure 3.7. Team Effect.

The value in the team effect is that we are in search of creating a balanced team of resources with multiple specializations. In the case of similar resource-resource specialization this will produce an unbalanced team while a difference in resource-resource specialization will favor a more well balanced team with the capability of executing a more diverse set of missions with varying task functional requirements.

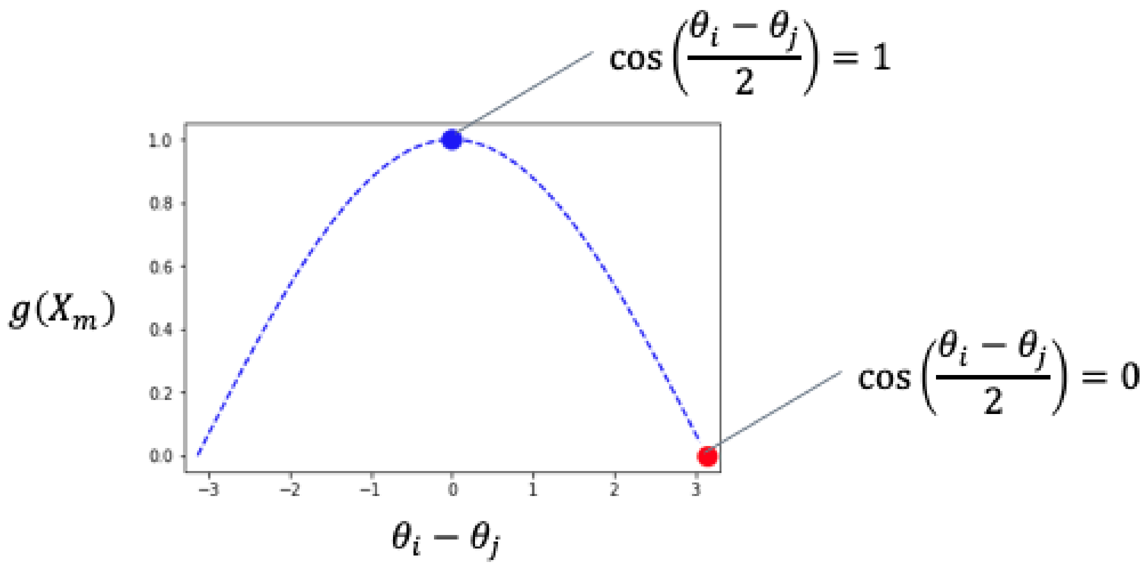

Figure 3.8. Team Effect Balance.

A plot of the team effect function $g(X_m)$ for values $-\pi \leq (\theta_i - \theta_j) \leq \pi$.

Constraints (C2) limit the assignment of one resource to at most one task. Constraints (C3) calculates whether there is a shortfall for each task k. Constraints (C4) calculate the mission shortfall decision variable. Constraints (C5), (C6), and (C7) enforce that a resource can be assigned to a task if and only if a resource on a team is assigned a mission that has the task. Constraints (C8) ensure that each resource is assigned to exactly one team. Constraints (C9) place an upper bound on the number of missions that each team is assigned. Constraints (C10) ensure that each mission is assigned exactly one team.

Overall, the Team-Formation model mathematically captures how an organization can manage and adjust its force structure through team-formation to execute designated missions within a tiered hierarchy. This model allows an organization to be versatile in the assignment of resources. The Team-Formation model also incorporates an added level of complexity in that the "team effect" function builds a well balanced team in terms of resource specialization. In this model the execution of work occurs at the team-formation, then assignment of mission-to-team, and then execution of work via task-to-resource.

3.4.2 Small Example: Team-Formation Model

We consider a small example problem with two predefined missions and undefined teams. Table 3.5 depicts the problem data, Figure 3.9 illustrates the optimal solution to the Team-Formation model, and Table 3.6 depicts the Team-Formation model report.

Tasks			*Resources*		*Missions*		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$
k1	$3\pi/4$	1	i1	$\pi/4$	k1	n1	1
k2	$-\pi/4$	1	i2	$-\pi/4$	k2	n2	1

Table 3.5. Team-Formation Data (Two Predefined Missions, Undefined Teams).

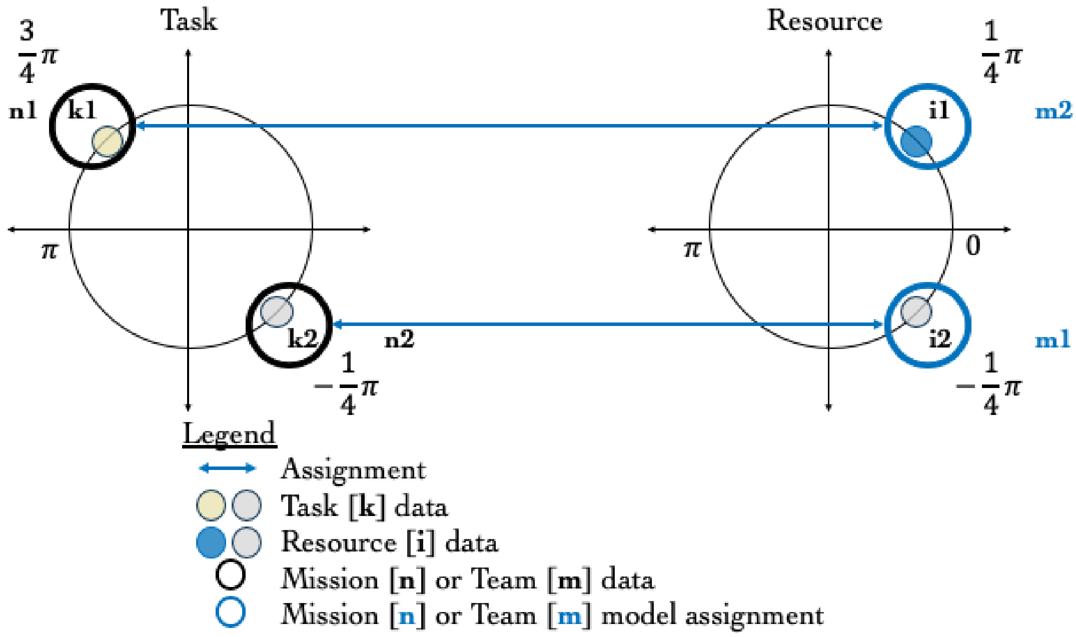

Figure 3.9. Direct Team-Formation Model (Two Predefined Missions, Undefined Teams).

The optimal assignment for two predefined missions and undefined teams. Resource $i2$ is formed into team $m1$ and is assigned to mission $n2$, and $i1$ is formed into $m2$ and is assigned to $n1$. The solver produces the same result as the mission-to-team but switches the teams. The execution cost is minimized when resource $i1$ is assigned to task $k1$ and $i2$ is assigned to $k2$. Both tasks are covered, so there is no shortfall penalty cost.

Team-Formation Model Report

```
Objective:  0.7240
Team-Mission Assignment:  Team membership:  Mission membership:
        m1:  n2               m1:  i2             n1:  k1
        m2:  n1               m2:  i1             n2:  k2
Assigned resources:
i1 -> k1, execution cost:  0.7240
i2 -> k2, execution cost:  0.0000
Shortfall penalties:
None.
```

Table 3.6. Team-Formation Report (Two Predefined Missions, Undefined Teams).

3.5 Mission Assignment

The fourth assignment problem involves mission assignment. In a mission-tailored environment, assigning tasks to missions is a difficult process that requires a fine balance of task functional requirements. Missions that are grouped with balanced task functions may require teams to provide coverage with varying resource specializations. Here, we consider a relaxation of the mission-to-team problem where the resources have been formed into predetermined teams while the missions are yet undefined.

Figure 3.10 describes the relationships between the mission and team entities, and it illustrates how tasks are grouped into the "best" possible missions to be accomplished by designated teams. We then consider the assignment problem of tasks into mission-assignment, wherein we can then assign mission-to-team and then task-to-resource. The illustration presents the optimal assignment of tasks to missions with the emphasis of generating a mission to be matched to a team. In this case resources are already formed into teams while missions are undefined. In mission assignment the goal is to put tasks into a mission, and then assign mission-to-team, thereby assigning task-to-resource.

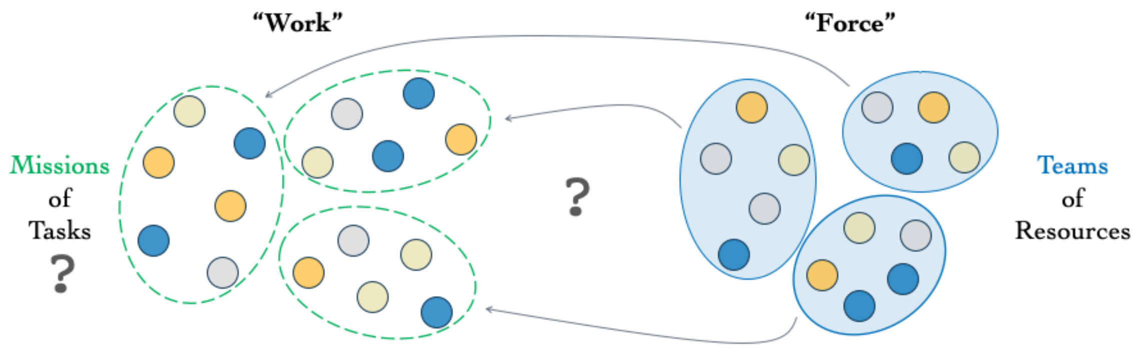

Figure 3.10. Mission Assignment.

There are two questions to consider here: (1) "Which teams should execute which missions?" and (2) "What are the "best" missions for a given set of teams?". The optimal assignment involves the task and mission entities. The assignment represents the optimal grouping of missions with selected tasks to be matched to a predefined team.

We formalize the mission assignment problem in the following assignment model.

3.5.1 Mission Assignment Model

The mission assignment model requires three given inputs: tasks, resources, and predetermined teams in order to choose the best assignment of tasks into mission-assignment, then mission-to-team, and thereby assigning task-to-resource.

Indices and Sets

$i \in R$	resources (alias j)
$k \in K$	tasks
$m \in M$	teams
$n \in N$	missions

Data [units]

ϕ_k	functional requirement for task k [radians], $\phi_k \in [0, 2\pi]$
θ_i	specialization of resource i [radians], $\theta_i \in [0, 2\pi]$
p_k	per unit shortfall penalty for task k [cost units]
q_n	per unit shortfall penalty for mission n [cost units]
$\hat{X}_{im}$	given: 1 if resource i assigned to team m, 0 otherwise [binary]
b_m	the maximum missions allowed for team m

Calculated Data [units]

f_{ik}	cost for resource i to execute task k [cost units] $f_{ik} = (1 - \cos(\theta_i - \phi_k))/2$
σ_{kl}	timing "cost" for synchronization between task k and task l [cost units] $\sigma_{kl} = \cos((\phi_k - \phi_l)/2$

Decision Variables [units]

S_k	1 if task k is not covered (shortfall), 0 otherwise [binary]
U_n	1 if mission n is not assigned (shortfall), 0 otherwise [binary]
W_{kn}	1 if task k assigned to group n, 0 otherwise [binary]
Y_{ikmn}	1 if resource i assigned to task k, 0 otherwise [binary]
Z_{mn}	1 if team m assigned to group n, 0 otherwise [binary]

Formulation

$$\min_{S,U,W,Y,Z} \quad \sum_i \sum_k \sum_m \sum_n f_{ik} Y_{ikmn} + \sum_k p_k S_k + \sum_n q_n U_n + \sum_{n \in N} h(W_n) \tag{D0}$$

$$\text{s.t.} \quad h(W_n) = \sum_{k \in K} W_{kn} \left(\sum_{l \in K; l \neq k} \sigma_{kl} W_{ln} \right) \qquad \forall n \in N \tag{D1}$$

$$\sum_k \sum_m \sum_n Y_{ikmn} \le 1 \qquad \forall i \in R \qquad \text{(D2)}$$

$$S_k \ge 1 - \sum_i \sum_m \sum_n Y_{ikmn} \qquad \forall k \in K \qquad \text{(D3)}$$

$$W_{kn} + S_k - U_n \le 1 \qquad \forall k \in K, \forall n \in N \qquad \text{(D4)}$$

$$Y_{ikmn} \le W_{kn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \qquad \text{(D5)}$$

$$Y_{ikmn} \le \hat{X}_{im} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \qquad \text{(D6)}$$

$$Y_{ikmn} \le Z_{mn} \qquad \forall i \in R, \forall k \in K, \forall m \in M, \forall n \in N \qquad \text{(D7)}$$

$$\sum_n Z_{mn} \le b_m \qquad \forall m \in M \qquad \text{(D8)}$$

$$\sum_m Z_{mn} = 1 \qquad \forall n \in N \qquad \text{(D9)}$$

$$\sum_n W_{kn} = 1 \qquad \forall n \in N \qquad \text{(D10)}$$

Discussion

This formulation implements assignment via the *Mission-Assignment model* — that is, the assignment and composition of missions based on the given composition of predetermined teams with a predefined set of resources. The Mission-Assignment model makes the problem simpler to match mission and team entities due the ease of forming a mission with tasks that can be accomplish by a team.

The objective function (D0) combines the total task-to-resource execution cost, the total task-to-resource coverage shortfall penalty cost, the total mission-to-team coverage shortfall penalty cost, and the "mission effect" cost.

Figure 3.11 illustrates the mission effect concept and Figure 3.12 is a plot representation of the mission effect function. The "mission effect" function (D1) calculates the synchronization among missions, by calculating the "conflict" between each task on a specified mission. Here, we define "conflict" between tasks to occur when tasks with similar execution are assigned to the same team. In practical terms, tasks that need to be executed with the same timing ϕ_k are more efficiently done by separate resources in optimal scheduling problems and, thus, by separate teams in mission-to-team assignment. When tasks with the same timing are on the same mission, this creates conflict in optimal task-to-resource assignment.

In contrast, “synchronized” missions assign tasks with similar timing to separate teams to optimize the overall mission. Thus, synchronized missions should be composed of tasks with low conflict. Equation (D1) captures this effect in the objective function by increasing the cost of the Mission-Assignment model when missions are not synchronized.

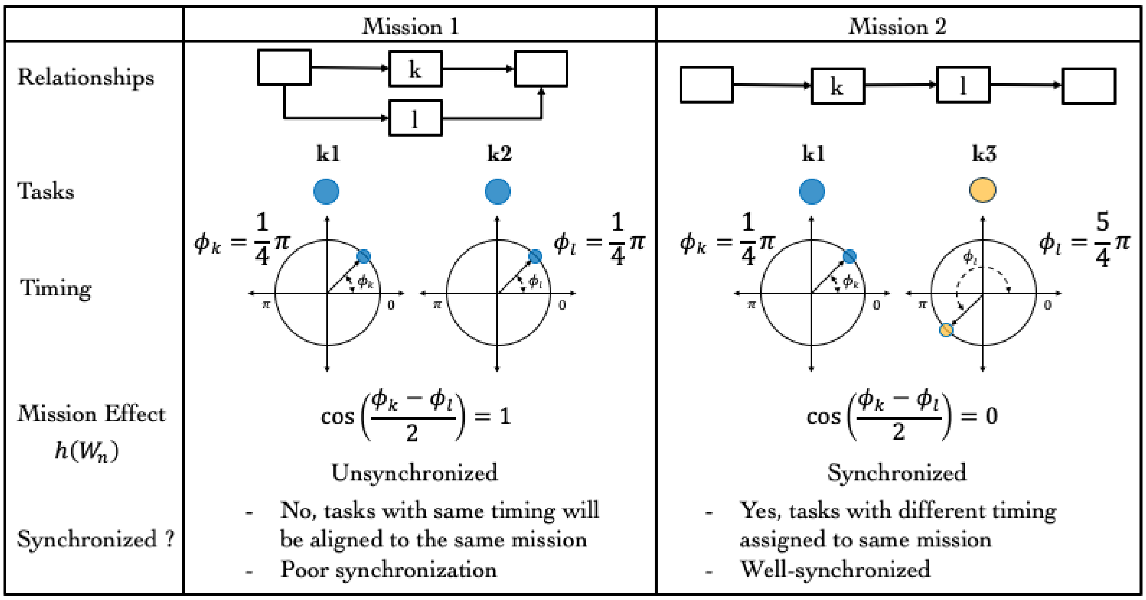

Figure 3.11. Mission Effect.

The value in the mission effect is that we are in search of creating a synchronized mission of tasks with multiple varying functional requirements. In the case of similar task-task timing this will produce a mission with poor synchronization while a difference in task-task timing will favor a well synchronized mission which can be assigned to a team with a varying set of resource specializations.

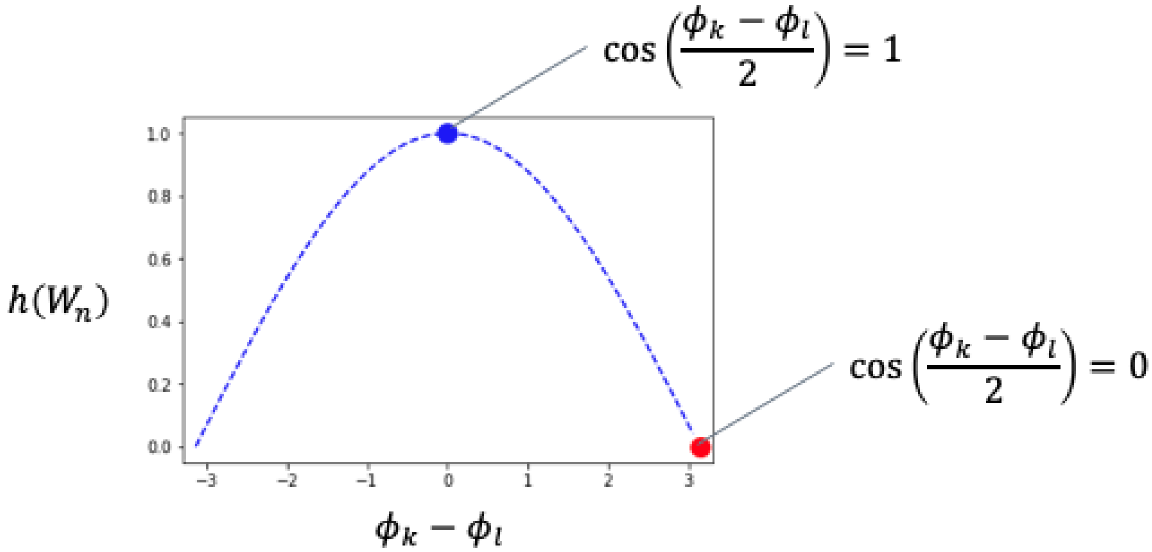

Figure 3.12. Mission Effect Synchronization.

A plot of the mission effect function $h(W_n)$ for values $-\pi \leq (\phi_k - \phi_l) \leq \pi$.

Constraints (D2) limit the assignment of one resource to at most one task. Constraints (D3) calculates whether there is a the shortfall for each task k. Constraints (D4) calculate the mission shortfall decision variable. Constraints (D5), (D6), and (D7) enforce that a resource can be assigned to a task if and only if a resource on a team is assigned a mission that has the task. Constraints (D8) place an upper bound on the number of missions that each team is assigned. Constraints (D9) ensure that each mission is assigned exactly one team. Constraints (D10) ensures that each task is assigned to exactly one mission.

Overall, the Mission-Assignment model mathematically captures how an organization can tailor and adjust mission-assignment to an established force structure within a tiered hierarchy. This model allows an organization to be versatile in the assignment of tasks. The Mission-Assignment model also incorporates an added level of complexity in that the "mission effect" function builds a well balanced mission in terms of task function. In this model the execution of work occurs at mission-assignment, then assignment of mission-to-team, and then execution of work via task-to-resource.

3.5.2 Small Example: Mission Assignment Model

We consider a small example problem with undefined missions and two predefined teams. Table 3.7 depicts the problem data, Figure 3.13 illustrates the optimal solution to the Mission-Assignment model, and Table 3.8 depicts the Mission-Assignment model report.

Tasks			***Resources***		***Teams***		
k	ϕ_k	p_k	**i**	θ_i	**i**	**m**	$\hat{X}$
k1	$3\pi/4$	1	i1	$\pi/4$	i1	m1	1
k2	$-\pi/4$	1	i2	$-\pi/4$	i2	m2	1

Table 3.7. Mission-Assignment Data (Undefined Missions, Two Predefined Teams).

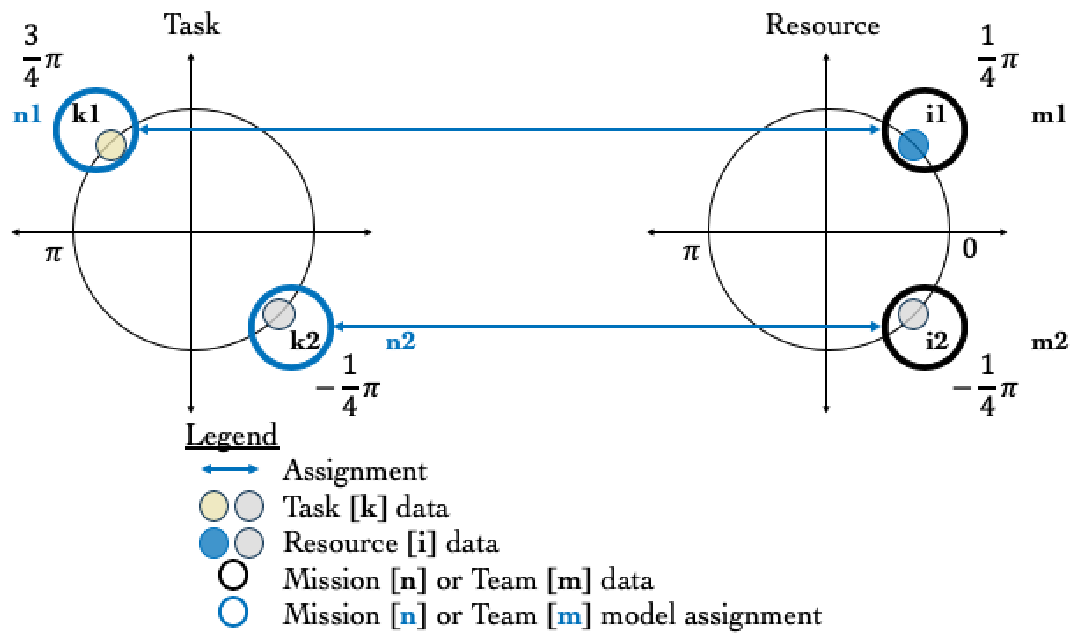

Figure 3.13. Direct Mission-Assignment Model (Undefined Missions, Two Predefined Teams).

The optimal assignment for undefined missions and two predefined teams. Task $k1$ is grouped into mission $n1$ and is assigned to team $m1$ and $k2$ is grouped into $n2$ and is assigned to $m2$. The solver produces the same result as the Mission-to-Team model. The execution cost is minimized when resource $i1$ is assigned to task $k1$ and $i2$ is assigned to $k2$. Both tasks are covered, so there is no shortfall penalty cost.

Mission-Assignment Model Report

```
Objective:  0.7240
Team-Mission Assignment:  Team membership:  Mission membership:
        m1:  n1               m1:  i1             n1:  k1
        m2:  n2               m2:  i2             n2:  k2
Assigned resources:
i1 -> k1, execution cost:  0.7240
i2 -> k2, execution cost:  0.0000
Shortfall penalties:
None.
```

Table 3.8. Mission-Assignment Report (Undefined Missions, Two Predefined Teams).

3.6 Summary

This chapter has introduced four assignment models that consider the allocation of resources to teams, tasks to missions, and teams to missions, in various combinations. The small examples demonstrate the use of these models and the tensions between objectives and constraints, while also providing some validation of their effectiveness. In the next chapter, we consider larger problem instances to show how an optimization-based team-of-teams approach can be used to measure robustness in C2 Networks.

CHAPTER 4:
Analysis

This chapter validates the four assignment models using larger problem data sets and provides an analysis on how an optimization-based team-of-teams approach can be used to measure robustness in C2 networks.

4.1 Von Mises Distribution Function

In order to analyze the differences between each model formulation and optimal assignment, we use a continuous probability distribution function to generate larger numbers of tasks and resources. We consider a circular normal distribution known as the *von Mises distribution*. "The von Mises distribution M(μ,κ) has a probability density function:

$$g(\theta|\mu,\kappa) = \frac{e^{\kappa\cos(\theta-\mu)}}{2\pi I_0(\kappa)}, \tag{4.1}$$

where $I_0(\kappa)$ denotes the modified Bessel function of the first kind and order 0" (Mardia and Jupp 2009, p. 36). The parameter μ is the mean direction, which is an angular equivalent to the mean of a normal distribution on the unit circle. The parameter κ is known as the concentration, and its inverse $1/\kappa$ (called dispersion) is equivalent to the variance of a normal distribution on a unit circle.

We use the von Mises distribution to generate tasks with an associated functional requirement and resources with an associated specialization. Specifically, when generating a distribution of tasks or resources, we set μ as the target requirement or specialization and adjust κ to control the variation in need for similar tasks or the similarity of resources with the same specialization. Using the larger data sets generated via the von Mises distribution, we consider each of the assignment models for mathematical validation and further analysis.

4.2 Large Assignment Problems: von Mises Distributions

4.2.1 Task-to-Resource Model: von Mises

We consider a large task-to-resource assignment that includes 200 tasks to be completed and 200 resources capable of completing each task. We use the von Mises distribution to generate the functional requirements for the tasks, half of which are centered with mean direction $\mu = 3\pi/4$ and the other half of which are centered with mean direction $\mu = -\pi/4$. We also use the von Mises distribution to generate the task specialization of resources, half with mean direction $\mu = \pi/4$ and the other half with mean direction $\mu = -\pi/4$. Table 4.1 presents the parameters used in the von Mises distribution to generate the task and resource data.

Tasks			***Resources***	
k	ϕ_k	p_k	**i**	θ_i
k1-k100	μ=3π/4, κ=50	1	i1-i100	μ=π/4, κ=50
k101-k200	μ=-π/4, κ=50	1	i101-i200	μ=-π/4, κ=50

Table 4.1. Task-to-Resource Data: von Mises (200 Tasks, 200 Resources).

We solve the Task-to-Resource model from Section 3.2.2 to optimality. Figure 4.1 illustrates the results, and Table 4.2 presents the corresponding model report. Figure 4.1 illustrates the optimal solution to the Task-to-Resource model, and Table 4.2 presents the Task-to-Resource model report.

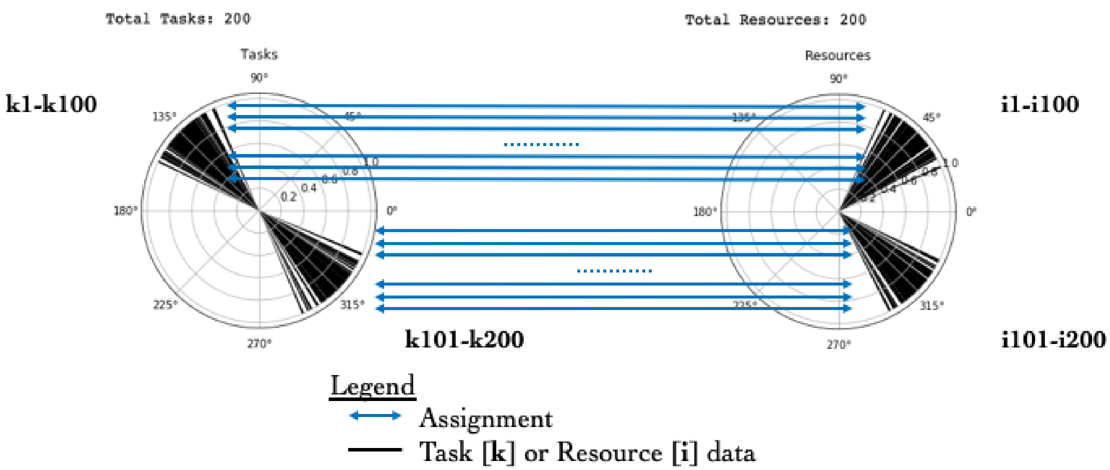

Figure 4.1. Task-to-Resource Model: von Mises (200 Tasks, 200 Resources).

The optimal assignment for 200 tasks and 200 resources. The execution cost is minimized when resources $i1 - i100$ is assigned to task $k1 - k100$ and $i101 - i200$ is assigned to $k101 - k200$. All tasks are covered, so there is no shortfall penalty cost.

Task-to-Resource Model Report

```
Objective:  50.3169
Assigned resources:
i1 -> k40, execution cost:  0.4403
i10 -> k51, execution cost:  0.7355
i100 -> k2, execution cost:  0.4406
                  ...
i97 -> k56, execution cost:  0.6523
i98 -> k76, execution cost:  0.4836
i99 -> k89, execution cost:  0.6428
Shortfall penalties:
None.
```

Table 4.2. Task-to-Resource Report: von Mises (200 Tasks, 200 Resources).

This large assignment problem demonstrates both the effectiveness of the Task-to-Resource model for optimal assignment and the difficulty of using its results for practical applications.

The optimal assignment for the 200 task and 200 resource case is qualitatively similar to the small example presented in Section 3.2. This is because the von Mises distributions are chosen to have μ match the task requirements and resource specializations used in the small two task and two resource example (see Table 3.1). However, because the tasks and resources generated in this example form a normal distribution centered on μ, there is a possibility for reductions in overall assignment costs by matching tasks and resources that are close together. Thus, the objective value is much less than the optimal assignment if all tasks and resources had precise requirements and specializations.

Still, the one-to-one assignment required to achieve these reductions in costs is difficult to implement in practice. For smaller numbers of tasks and resources, it is easy to optimally arrange people to work on tasks that they are best suited for. When the pool of tasks and resources is large, it becomes more difficult to implement the optimal assignment due to high management costs and other contextual factors. Instead, additional organization of tasks into missions and resources into teams is required to simplify the implementation of an optimal task-resource assignment.

4.2.2 Mission-to-Team Model: von Mises

We implement a similar large example of 200 tasks and 200 resources organized into two missions and two teams to test optimal task-resource assignment for more manageable situations. Table 4.3 presents the parameters used in the von Mises distribution to produce problem data and the organization of tasks into missions and resources into teams, Figure 4.2 illustrates the optimal solution to the Mission-to-Team model, and Table 4.4 presents the Mission-to-Team model report.

Tasks			*Resources*		*Missions*			*Teams*		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$	**i**	**m**	$\hat{X}$
k1	μ=3π/4, κ=50	1	i1	μ=π/4, κ=50	k1	n1	1	i1	m1	1
...	...	...	...	...	...	...	...	...	...	...
k100	...	...	i100	...	k100	...	...	i200	...	...
k101	μ=-π/4, κ=50	1	i101	μ=-π/4, κ=50	k101	n2	1	i101	m2	1
...	...	...	...	...	...	...	...	...	...	...
k200	...	...	i200	...	k200	...	...	i200	...	...

Table 4.3. Mission-to-Team Data: von Mises (Two Sets of Missions, Two Sets of Teams).

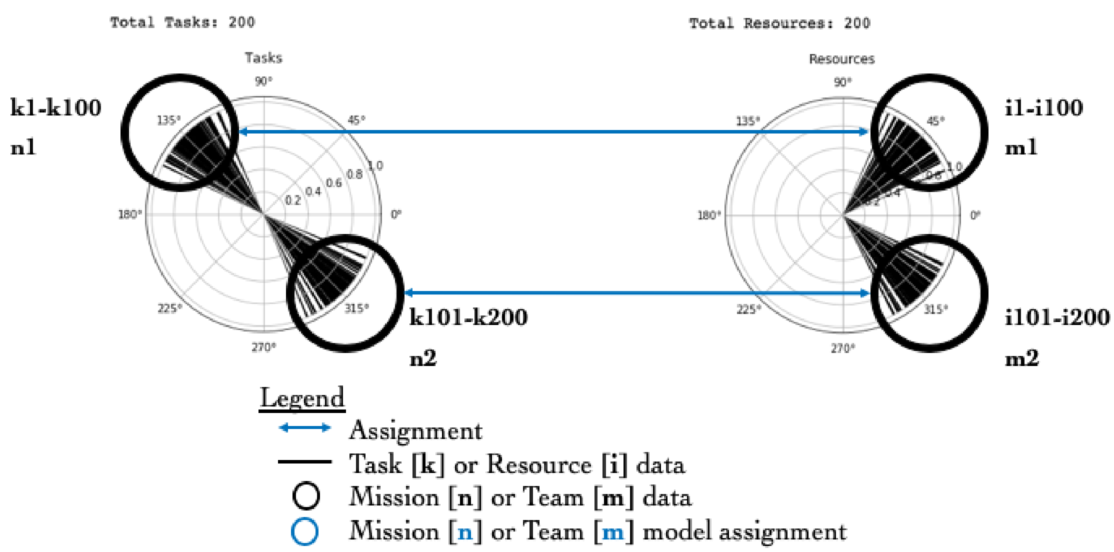

Figure 4.2. Mission-to-Team Model: von Mises (Two Sets of Missions, Two Sets of Teams).

The optimal assignment for two sets of missions and two sets of teams. Team $m1$ is assigned to mission $n1$ and $m2$ is assigned to $n2$. The execution cost is minimized when resource $i1-i100$ is assigned to task $k1-k100$ and $i101-200$ is assigned to $k101-200$. All tasks are covered, so there is no shortfall penalty cost.

Mission-to-Team Model Report

```
Objective:  50.3169
Team-Mission Assignment:
m1:  n1
m2:  n2
Team membership:
m1:  i1 i2 i3 ...  i98 i99 i100
m2:  i101 i102 i103 ...  i198 i199 i200
Mission membership:
n1:  k1 k2 k3 ...  k98 k99 k100
n2:  k101 k102 k103 ...  k198 k199 k200
Assigned resources:
i1 ->k22, execution cost:  0.4413
i10 ->k51, execution cost:  0.7355
i100 ->k2, execution cost:  0.4406
                   ...
i97 ->k56, execution cost:  0.6523
i98 ->k33, execution cost:  0.4846
i99 ->k89, execution cost:  0.6428
Shortfall penalties:
None.
```

Table 4.4. Mission-to-Team Report: von Mises (Two Sets of Missions, Two Sets of Teams).

This large assignment problem produces similar results to the Task-to-Resource model, but in a more practical way for real-world implementation. The optimal assignment for the 200 tasks and 200 resources using the Mission-to-Team model achieves the same objective value as the Task-to-Resource model. This occurs because the von Mises distribution parameters remain the same and the missions and teams organize tasks and resources in a manner that guarantees optimal assignment. In military C2, the results of this model are easier to implement as missions can be assigned to teams by commanders, then resources can be optimally assigned to tasks by team leaders and mission owners.

Despite the benefit provided by this approach for implementation, it requires even more knowledge about task functional requirements and resource specialization than the Task-to-Resource model. The missions and teams defined here require prior knowledge about task and resource parameters to ensure optimal assignment. For example, if tasks with diverse μ and κ values were assigned to the same mission with the teams held the same, there is a high possibility that the Mission-to-Team model would not achieve the global optimum for its assignment. This same is true for resources with diverse μ and κ values assigned to the same teams. In these situations, it may not be clear prior to analysis how to form teams of resources or missions of tasks. Instead, having the teams and missions as an output from analysis may be required to ensure optimal task-resource assignment with practical C2.

4.2.3 Team-Formation Model: von Mises

We test a realistic assignment problem with two predefined missions and with the optimal teams as the output of analysis. Due to the computational complexity of this task, this analysis is completed with a smaller problem of 20 tasks and 20 resources. To simulate the more complex situation of not knowing what resources should be on what teams, tasks are generated with von Mises distributions with high concentration, but resources are generated with von Mises distributions with low concentration ($\kappa < 1$). Table 4.5 presents the problem data, Figure 4.3 illustrates the optimal solution to the Team-Formation model, and Table 4.6 depicts the Team-Formation model report.

Tasks			***Resources***		***Missions***		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$
k1	μ=3π/4, κ=50	1	i1	μ=0, κ=0.5	k1	n1	1
...	...	...	...	...	...	...	...
k10	...	...	...	...	k10	...	...
k11	μ=-π/4, κ=50	1	...	...	k11	n2	1
...	...	...	...	...	...	...	...
k20	...	...	i20	...	k20	...	...

Table 4.5. Team-Formation Data: von Mises (Two Sets of Predefined Missions, 20 Resources with High Dispersion Not Assigned to Teams).

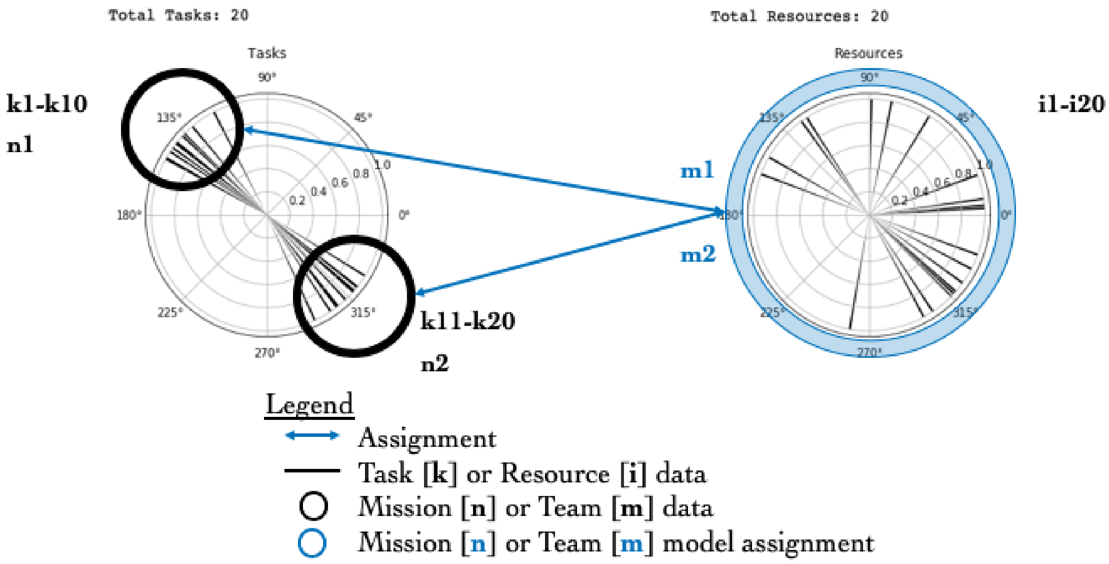

Figure 4.3. Team-Formation Model: von Mises (Two Sets of Predefined Missions, 20 Resources with High Dispersion Not Assigned to Teams).

The optimal assignment for two predefined missions and undefined teams with a dispersed range of resources. Resources identified in the report are formed into team $m1$ and $m2$ and are assigned to mission $n1$ and $n2$, respectively. All tasks are covered, so there is no shortfall penalty cost.

Team-Formation Model Report

```
Objective:  115.1271
Team-Mission Assignment:
m1:  n1
m2:  n2
Team membership:
m1:  i1 i2 i4 i6 i7 i8 i9 i14 i19 i20
m2:  i3 i5 i10 i11 i12 i13 i15 i16 i17 i18
Mission membership:
n1:  k1 k2 k3 k4 k5 k6 k7 k8 k9 k10
n2:  k11 k12 k13 k14 k15 k16 k17 k18 k19 k20
Assigned resources:
i1 ->k8, execution cost:  0.9231
i10 ->k12, execution cost:  0.9472
i11 ->k15, execution cost:  0.8550
                    ...
i7 ->k2, execution cost:  0.1567
i8 ->k5, execution cost:  0.9590
i9 ->k4, execution cost:  0.6648
Shortfall penalties:
None.
```

Table 4.6. Team-Formation Report: von Mises (Two Sets of Predefined Missions, 20 Resources with High Dispersion Not Assigned to Teams).

The results for the Team-Formation model demonstrate the complexity of optimal teaming and task-resource assignment in military C2. The results are qualitatively similar to the Mission-to-Team model, as two teams are formed to complete the two predetermined missions. However, even with a factor of 10 less tasks and resources to be assigned, the execution cost for these missions is more than double that from the previous two models. This is due to the lack of organization captured by the dispersed range of resources.

Importantly, the Team-Formation model shows how resource agility to perform unforeseen missions is required by military organizations. The Team-Formation model is capable of forming teams that would otherwise be very difficult to produce even with prior knowledge of resource specializations. Forming these teams requires agility as prior military force organization would likely not contain such diverse teams. Moreover, to avoid task shortfalls, some resources are assigned tasks that they are not suitable for. For example, task $k5$ is assigned to resource $i8$ incurring an execution cost of 0.959. In practice, knowing that resource $i8$ should be on team $m1$ and complete task $k5$ when this task requires a very different specialization is very difficult to know *a priori*. Without agile teams that can adapt to the given missions, this unsuitable, yet optimal assignment would not be possible.

4.2.4 Mission Assignment Model: von Mises

We further test a realistic assignment problem with undefined missions and two predefined teams. Due to the computational complexity of this task, this analysis is also completed with a smaller problem of 20 tasks and 20 resources. To simulate the more complex situation of not knowing what task should be on what missions, resources are generated with von Mises distributions with high concentration, but tasks are generated with von Mises distributions with low concentration ($\kappa < 1$). Table 4.7 presents the parameters to generate problem data and team assignment, Figure 4.4 illustrates the optimal solution to the Mission-Assignment model, and Table 4.8 depicts the Mission-Assignment model report.

Tasks			***Resources***		***Teams***		
k	ϕ_k	p_k	**i**	θ_i	**i**	**m**	$\hat{X}$
k1	μ=3π/4, κ=0.5	1	i1	μ=π/4, κ=50	i1	m1	1
...	...	...	...	...	...	...	...
...	...	...	i10	...	i10	...	...
...	...	...	i11	μ=-π/4, κ=50	i101	m2	1
...	...	...	...	...	...	...	...
k20	...	...	i20	...	i20	...	...

Table 4.7. Mission-Assignment Data: von Mises (20 Tasks with High Dispersion Not Assigned to Missions, Two Sets of Predefined Teams).

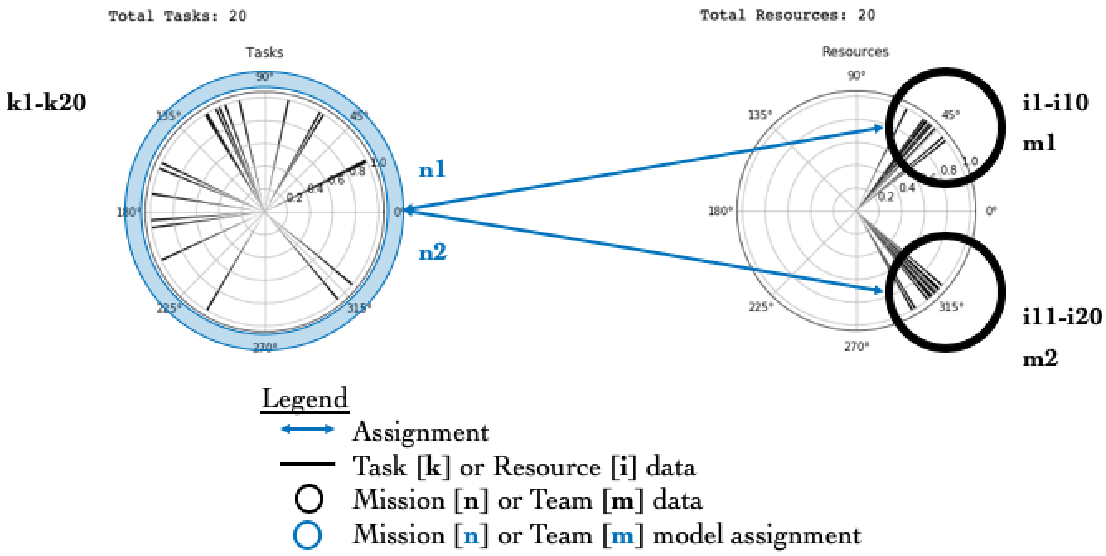

Figure 4.4. Mission-Assignment Model: von Mises (20 Tasks with High Dispersion Not Assigned to Missions, Two Sets of Predefined Teams).

The optimal assignment for undefined missions with a dispersed range of tasks and two predefined teams. Tasks identified in the report are grouped into mission $n1$ and $n2$ and are assigned to team $m1$ and $m2$, respectively. All tasks are covered, so there is no shortfall penalty cost.

Mission-Assignment Model Report

```
Objective:  63.3816
Team-Mission Assignment:
m1:  n1
m2:  n2
Mission membership:
n1:  k1 k2 k5 k7 k10 k11 k12 k16 k17 k18
n2:  k3 k4 k6 k8 k9 k13 k14 k15 k19 k20
Team membership:
m1:  i1 i2 i3 i4 i5 i6 i7 i8 i9 i10
m2:  i11 i12 i13 i14 i15 i16 i17 i18 i19 i20
Assigned resources:
i1 ->k10, execution cost:  0.8711
i10 ->k7, execution cost:  0.0259
i11 ->k20, execution cost:  0.3112
                     ...
i7 ->k17, execution cost:  0.2347
i8 ->k12, execution cost:  0.2482
i9 ->k18, execution cost:  0.0162
Shortfall penalties:
None.
```

Table 4.8. Mission-Assignment Report: von Mises (20 Tasks with High Dispersion Not Assigned to Missions, Two Sets of Predefined Teams).

The results for the Mission Assignment model further demonstrate the complexity of optimal teaming and task-resource assignment in military C2. The results are qualitatively similar to the Team-Formation model, as two missions are formed to have optimal task-resource assignment with static teams. Minor changes in the value of μ representing prior knowledge of mission requirements results in half the execution cost of the Team-Formation model. This is due to the fact that even novel missions are composed of tasks with some form of sequential organization (e.g., via precedence in task networks). However, the results further indicate a need for agile teams as the optimal task-to-mission assignment is very different from the Team-Formation model. The fact that task requirements may be more organized in practice than resource specializations implies that optimal missions will vary from team to team.

4.3 Implications of Diverse Task Requirements and Resource Specializations

We explore the implications of forming missions and teams across all four assignment models using a representative example with diverse tasks and resources. Unlike previous examples, where each mission or team was generated with a single von Mises distribution, here, distributions are organized into three predefined missions and three predefined teams. Moreover, tasks have varying shortfall penalties, missions have different numbers of tasks, and teams have varying numbers of resources. Thus, each mission and team is a mix of each distribution, representing the difficulty of real work and the breadth of skill-sets on real teams. Each assignment problem and sub-problems are defined with the following general characteristics: (P1) 15 tasks and 15 resources with no missions or teams, (P2) tasks and resources organized into three missions and three teams, (P3) tasks organized on three predefined missions with undefined teams, and (P4) resources organized on three predefined teams with undefined missions. Tables 4.9, 4.11, 4.13, 4.15 present the parameters used to generate problem data, Figures 4.5 and 4.6 provide representative illustrations of the optimal solution for the Task-to-Resource and Mission-to-Team assignment models, and Tables 4.10, 4.12, 4.14, and 4.15 present the optimal assignment model reports. *(Note: no visualizations are provided for team-formation and mission-assignment due to complicated output.)*

Tasks			*Resources*	
k	ϕ_k	p_k	**i**	θ_i
k1-k3	μ=3π/4, κ=50	1	i1-i3	μ=π/4, κ=50
k4-k6	μ=-π/4, κ=50	1	i4-i6	μ=-π/4, κ=50
k7-k9	μ=5π/4, κ=50	5	i7-i9	μ=3π/4, κ=50
k10-k12	μ=π/4, κ=50	5	i10-i12	μ=5π/4, κ=50
k13-k15	μ=π, κ=50	1	i13-i15	μ=π/2, κ=50

Table 4.9. Task-to-Resource Data: von Mises (15 Diverse Tasks, 15 Diverse Resources).

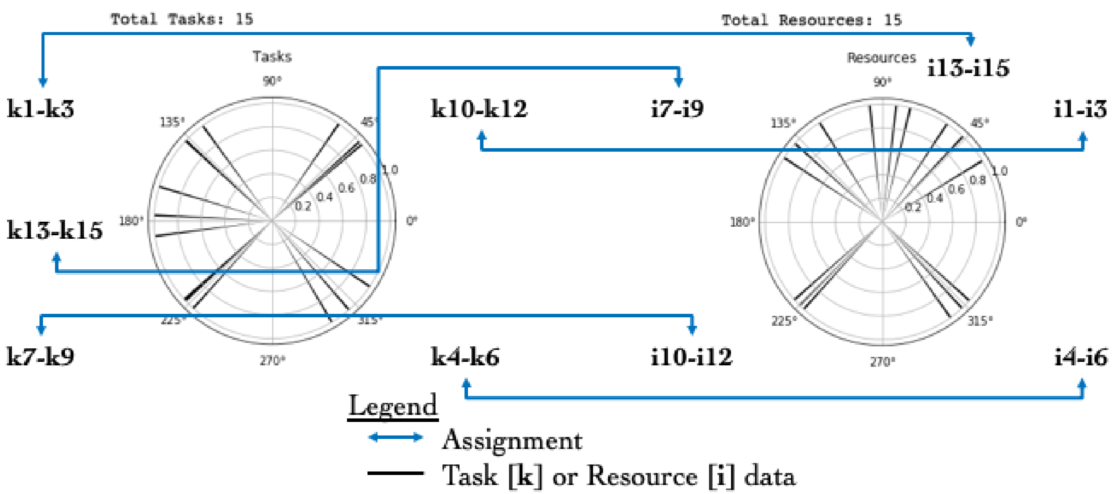

Figure 4.5. Task-to-Resource Model Implications: von Mises (15 Diverse Tasks, 15 Diverse Resources).

The optimal assignment for 15 tasks and 15 resources. The execution cost is minimized when resources $i1-i3$ are assigned to tasks $k10-k12$, $i4-i6$ are assigned to $k4-k6$, $i7-i9$ are assigned to $k13-k15$, $i10-i12$ are assigned to $k7-k9$, and $i13-i15$ are assigned to $k1-k3$. All tasks are covered, so there is no shortfall penalty cost.

Running each model with the same basic assignment problem affords direct comparison of results. As the Task-to-Resource model has neither missions nor teams, it achieves the global optimal value for the objective function. However, like the 200 task 200 resource example in Section 4.2.1, this assignment provides limited guidance in practice for C2.

First, it assumes perfect information regarding all tasks and resources, which may be unavailable to decision-makers. Moreover, it is difficult to implement in practice even if perfect information is available because resources are often not available to complete any given task.

Task-to-Resource Model Report

```
Objective:  0.8727
Assigned resources:
i1 -> k10, execution cost:  0.0062
i10 -> k7, execution cost:  0.0000
i11 -> k8, execution cost:  0.0000
                   ...
i7 -> k13, execution cost:  0.1210
i8 -> k15, execution cost:  0.1200
i9 -> k14, execution cost:  0.1119
Shortfall penalties:
None.
```

Table 4.10. Task-to-Resource Report: von Mises (15 Diverse Tasks, 15 Diverse Resources).

Tasks			*Resources*		*Missions*			*Teams*		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$	**i**	**m**	$\hat{X}$
k1	μ=3π/4, κ=50	1	i1	μ=π/4, κ=50	k1	n1	1	i1	m1	1
...	...	...	...	...	...	...	...	...	...	...
k3	...	...	i3	...	k3	...	...	i3	...	...
k4	μ=-π/4, κ=50	1	i4	μ=-π/4, κ=50	k4	n1	1	i4	m2	1
...	...	...	...	...	...	...	...	...	...	...
k6	...	...	i6	...	k6	...	...	i6	...	...
k7	μ=5π/4, κ=50	5	i7	μ=3π/4, κ=50	k7	n2	1	i7	m1	1
...	...	...	...	...	...	...	...	...	...	...
k9	...	...	i9	...	k9	...	...	i9	...	...
k10	μ=π/4, κ=50	5	i10	μ=5π/4, κ=50	k10	n2	1	i10	m2	1
...	...	...	...	...	...	...	...	...	...	...
k12	...	...	i12	...	k12	...	...	i12	...	...
k13	μ=π, κ=50	1	i13	μ=π/2, κ=50	k1	n3	1	i1	m3	1
...	...	...	...	...	...	...	...	...	...	...
k15	...	...	i15	...	k15	...	...	i15	...	...

Table 4.11. Mission-to-Team Model Data: von Mises (Three Predefined Missions, Three Predefined Teams).

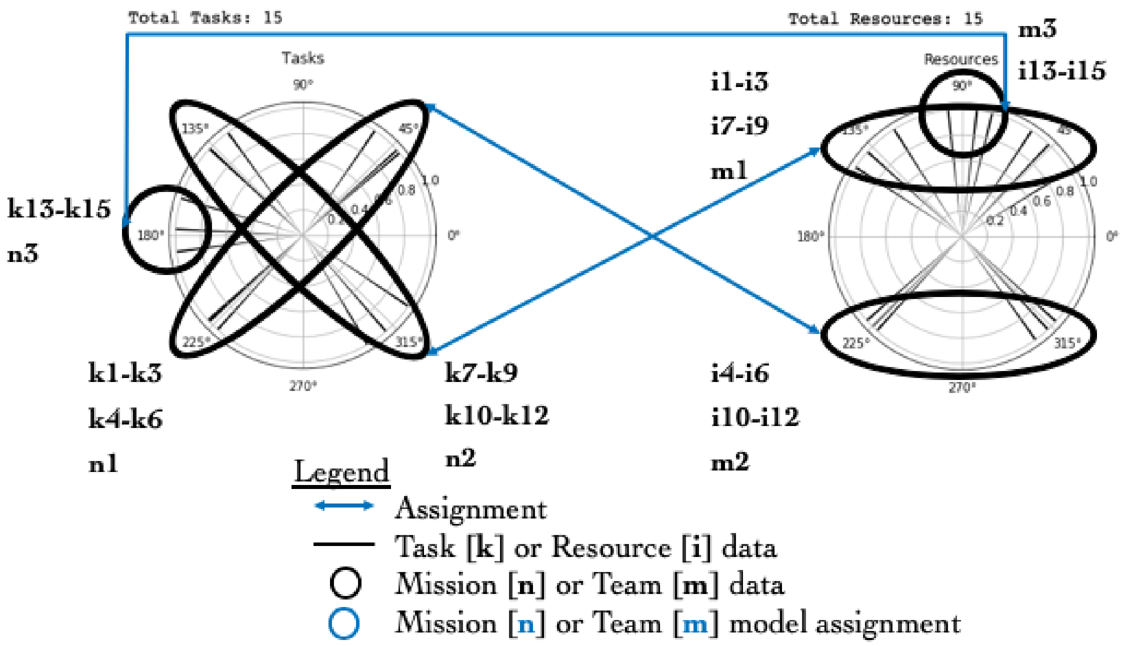

Figure 4.6. Mission-to-Team Model Implications: von Mises (Three Predefined Missions, Three Predefined Teams).

The optimal assignment for three predefined missions and three predefined teams. Team $m1$ is assigned to mission $n2$, $m2$ is assigned to $n1$, and $m3$ is assigned to $n3$. All tasks are covered, so there is no shortfall penalty cost.

In real-world practice, tasks are combined into larger missions that are assigned to teams that lack much flexibility to change membership, training, or otherwise adapt their function to changing environments. This more rigid view of military operations is captured by the Mission-to-Team model, where missions and teams are known prior to optimal assignment. However, the objective value for the optimal assignment found with the Mission-to-Team model is far from the global optimum found with the Task-to-Resource model. Moreover, this model is not as computationally efficient, and may simply lead to slower, less optimal decisions overall.

Mission-to-Team Model Report

```
Objective:  4.5280
Team-Mission Assignment:
m1:  n2
m2:  n1
m3:  n3
Team membership:
m1:  i1 i2 i3 i7 i8 i9
m2:  i4 i5 i6 i10 i11 i12
m3:  i13 i14 i15
Mission membership:
n1:  k1 k2 k3 k4 k5 k6
n2:  k7 k8 k9 k10 k11 k12
n3:  k13 k14 k15
Assigned resources:
i1 ->k10, execution cost:  0.0062
i10 ->k2, execution cost:  0.6029
i11 ->k1, execution cost:  0.4524
               ...
i7 ->k7, execution cost:  0.6353
i8 ->k9, execution cost:  0.3780
i9 ->k8, execution cost:  0.4454
Shortfall penalties:
None.
```

Table 4.12. Mission-to-Team Model Report: von Mises (Three Predefined Missions, Three Predefined Teams).

Tasks			*Resources*		*Missions*		
k	ϕ_k	p_k	**i**	θ_i	**k**	**n**	$\hat{W}$
k1	μ=3π/4, κ=50	1	i1	μ=π/4, κ=50	k1	n1	1
...	...	...	...	...	...	...	...
k3	...	...	i3	...	k3	...	...
k4	μ=-π/4, κ=50	1	i4	μ=-π/4, κ=50	k4	n1	1
...	...	...	...	...	...	...	...
k6	...	...	i6	...	k6	...	...
k7	μ=5π/4, κ=50	5	i7	μ=3π/4, κ=50	k7	n2	1
...	...	...	...	...	...	...	...
k9	...	...	i9	...	k9	...	...
k10	μ=π/4, κ=50	5	i10	μ=5π/4, κ=50	k10	n2	1
...	...	...	...	...	...	...	...
k12	...	...	i12	...	k12	...	...
k13	μ=π, κ=50	1	i13	μ=π/2, κ=50	k1	n3	1
...	...	...	...	...	...	...	...
k15	...	...	i15	...	k15	...	...

Table 4.13. Team-Formation Data: von Mises (Three Predefined Missions, Undefined Teams).

Team-Formation Model Report

```
Objective:  23.1510
Team-Mission Assignment:
m1:  n3
m2:  n1
m3:  n2
Team membership:
m1:  i1 i5 i13
m2:  i4 i6 i7 i9 i10 i15
m3:  i2 i3 i8 i11 i12 i14
Mission membership:
n1:  k1 k2 k3 k4 k5 k6
n2:  k7 k8 k9 k10 k11 k12
n3:  k13 k14 k15
Assigned resources:
i1 ->k14, execution cost:  0.9139
i10 ->k5, execution cost:  0.3508
i11 ->k9, execution cost:  0.002
                ...
i7 ->k1, execution cost:  0.0160
i8 ->k8, execution cost:  0.3703
i9 ->k3, execution cost:  0.0001
Shortfall penalties:
None.
```

Table 4.14. Team-Formation Report: von Mises (Three Predefined Missions, Undefined Teams).

In this work, the Team-Formation assignment model is the best representation of General McChrystal's team-of-teams approach to C2. General McChrystal describes team-of-teams as an approach for how C2 can benefit from building adaptable teams that can form in response to given mission sets (McChrystal et al. 2015, p. 91). For our example assignment problem, the Team-Formation model does not achieve the same objective value as the Task-to-Resource model but forms teams with a very different membership than those given in the Mission-to-Team model, which incorporates the team effect function. In particular, the computational efficiency of the Team-Formation model significantly decreases as the problem complexity increases, in large part due to the team effect function. Thus, C2 can benefit from the Team-Formation assignment model, which can achieve near optimal results in adapting to unpredictable missions but is not as computationally efficient as a completely flat organization.

Tasks			***Resources***		***Teams***		
k	ϕ_k	p_k	**i**	θ_i	**i**	**m**	$\hat{X}$
k1	μ=3π/4, κ=50	1	i1	μ=π/4, κ=50	i1	m1	1
...	...	...	...	...	...	...	...
k3	...	...	i3	...	i3	...	...
k4	μ=-π/4, κ=50	1	i4	μ=-π/4, κ=50	i4	m2	1
...	...	...	...	...	...	...	...
k6	...	...	i6	...	i6	...	...
k7	μ=5π/4, κ=50	5	i7	μ=3π/4, κ=50	i7	m1	1
...	...	...	...	...	...	...	...
k9	...	...	i9	...	i9	...	...
k10	μ=π/4, κ=50	5	i10	μ=5π/4, κ=50	i10	m2	1
...	...	...	...	...	...	...	...
k12	...	...	i12	...	i12	...	...
k13	μ=π, κ=50	1	i13	μ=π/2, κ=50	i1	m3	1
...	...	...	...	...	...	...	...
k15	...	...	i15	...	i15	...	...

Table 4.15. Mission-Assignment Data: von Mises (Undefined Missions, Three Predefined Teams).

Mission-Assignment Model Report

```
Objective:  10.9207
Team-Mission Assignment:
m1:  n1
m2:  n2
m3:  n3
Mission membership:
n1:  k1 k3 k8 k10 k11 k15
n2:  k4 k5 k6 k9 k13 k14
n3:  k2 k7 k12
Team membership:
m1:  i1 i2 i3 i7 i8 i9
m2:  i4 i5 i6 i10 i11 i12
m3:  i13 i14 i15
Assigned resources:
i1 ->k10, execution cost:  0.0062
i10 ->k9, execution cost:  0.0020
i11 ->k13, execution cost:  0.2375
                ...
i7 ->k3, execution cost:  0.0166
i8 ->k8, execution cost:  0.3703
i9 ->k15, execution cost:  0.1740
Shortfall penalties:
None.
```

Table 4.16. Mission-Assignment Report: von Mises (Undefined Missions, Three Predefined Teams).

Finally, the Mission-Assignment model represents situations where the teams are preset (e.g., when specialized training and/or circumstances make it difficult to adapt teams) but tasks can be reconfigured and synchronized across missions for better assignment to resources. Like the Team-Formation model, the adaptability provided by the Mission-Assignment model allows for the task-to-resource assignment to achieve a near optimal solution. The Mission-Assignment model also has similar computational efficiency to the Team-Formation model while the objective result is more favorable. However, unlike the Team-Formation model, three missions were formed that best suit each of the individual teams. This discrepancy between models results from the fact that conflict can arise when multiple teams are assigned the same mission set. To avoid shortfalls due to lack of coverage, from a teaming perspective, it may be better to have multiple missions on the same team, but from a missions perspective, it is better to have each team do exactly one mission.

4.4 Computational Results

All of the computational results on this thesis were obtained on a laptop computer with a 2.5 gigahertz (Ghz) Intel Core i5 MacBook Pro, 16 GB of RAM DDR3 memory, while running macOS Mojave Version 10.14.5. We implement the optimization models in the Python programming language (Python Software Foundation 2019) version 3.6.8 using the Pyomo optimization package (Hart et al. 2017) version 5.6.5 for constructing optimization models and the IBM CPLEX Optimizer (IBM 2019) version 12.9 for solving each optimization problem.

Table 4.17 presents the runtimes for all optimization models. In the small example problems with 2 tasks and 2 resources, the Task-to-Resource model has the longest runtime, yet all runtimes are similar and relatively fast due to the simplicity of the given assignment problem. The large example assignment problem with 200 tasks and 200 resources show that the Task-to-Resource model is the most efficient, incurring only a small increase in computational time (roughly 3x the small example model). The Mission-to-Team model also scales to produce efficient calculations even though it includes more complex constraints to handle additional assignment variables and parameters. In contrast, the team-formation and the mission-assignment incur a significant computational runtime due to the added complexity of nonlinear team effect and mission effect functions. The mission effect function has the greatest impact on computational time, requiring more than three times more runtime than the Team-Formation model. The diverse task requirements and resource specialization example further demonstrates these results across all four models with only 15 tasks and 15 resources.

Assignment Model Computation Runtime Report

Small Example Models
Task-to-Resource (2): Time - 0.8758 sec.
Mission-to-Team (2): Time - 0.0320 sec.
Team-Formation (2): Time - 0.0341 sec.
Mission-Assignment (2): Time - 0.0425 sec.
Large Example Models - von Mises.
Task-to-Resource (200): Time - 2.2573 sec.
Mission-to-Team (200): Time - 7.8011 sec.
Team-Formation (20): Time - 262.7214 sec.
Mission-Assignment (20): Time - 889.9943 sec.
Implications of Diverse Task Requirements and Resource Specializations Across All Four Models
Task-to-Resource (15): Time - 0.1547 sec.
Mission-to-Team (15): Time - 0.3273 sec.
Team-Formation (15): Time - 126.1497 sec.
Mission-Assignment (15): Time - 756.7638 sec.
Note: () indicates the number of tasks and resources

Table 4.17. Assignment Model Computation Runtime Report.

4.5 Summary

This chapter presents analysis and comparison of all four assignment models using large and realistic data sets generated with von Mises distributions. The large examples demonstrate the effectiveness of the Task-to-Resource and Mission-to-Team models and global optimal task-resource assignment. The smaller, more realistic examples show the benefits of adaptable teams via the Team-Formation and Mission-Assignment models. Although the optimal solution to each model is not necessarily the global optimal solution for the given problem, the different teaming and mission sets produced by each model suggest that there are multiple ways to produce an optimal team.

In particular, different contexts benefit from the use of each model. Extremely flat organizations can utilize the global optimal solution provided by the Task-to-Resource model. Real-world requirements for resource management and mission needs may be best met by optimal task-resource assignment with the Mission-to-Team model. When faced with changing and unforeseen mission environments, military organizations may want a team-of-teams approach and would benefit from the adaptability of the Team-Formation model. Finally, when organizations are rigid but tasks can be redistributed to synchronize missions, the optimal task-resource assignment is best provided by the Mission-Assignment model.

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CHAPTER 5: Conclusion

The objective of this research is to understand the performance effects of teams on the organizational architecture of a C2 network. We explore modern fields of research to explore C2 organizational structure to understand the effects of teams and how modifications in hierarchy can enhance communications, workflow, and mission accomplishment.

We adopt a first-principles approach to develop four optimization-based assignment models which allow us to measure the robustness of a C2 network with the emphasis on a team-of-teams approach and methodology. We introduce four assignment models that consider the allocation of resources to teams, tasks to missions, and teams to missions, in various combinations. The small and large examples demonstrate the use of these models and the tensions between objectives and constraints while also providing validation of their effectiveness through larger problem instances used to measure robustness in C2 networks.

We produce findings that are pertinent and applicable to C2 and organizational hierarchy. We test the primary hypothesis, that there is an optimal teaming of resources (i.e., people) for a given set of work (i.e., mission), with our core assignment models. We found that contrasting assignment-model performance results with various entity data reveals multiple force structures that may be optimal for different situations. Thus, results in Chapter 4 suggest that there are *multiple* ways to achieve an optimal teaming for a given set of tasks thereby motivating us to reject our initial hypothesis.

Hypothesis: *There is an optimal teaming of resources (i.e., people) for a given set of work (i.e., mission).*

Result: *It depends. There are different ways to achieve optimal teaming depending on data requirements, calculation time, military organizational structure, and mission needs.*

Task-to-Resource: Given only tasks and resources, the Task-to-Resource assignment model can offer an efficient calculation of task assignment but requires an extremely flat organization where complete information on all tasks and resources is known and any resource is allowed to complete any task. Furthermore, this model produces an optimal solution and at "best" a one-to-one matching. In a resource-constrained environment, the one-to-one matching will result in a task shortfall due to limited resources, where grouping resources into teams may alleviate these shortfalls. As a force structure grows beyond a certain threshold, the performance of the Task-to-Resource model is computationally possible but will pose a quite difficult challenge in human terms of practical performance and management.

Mission-to-Team: The Mission-to-Team assignment model is a restriction on the Task-to-Resource assignment model, meaning it can not obtain a better solution but is simpler to solve. Given tasks, resources, missions, and teams, an organization can utilize the Mission-to-Team model to produce at "best" a sub-optimal or local optimal teaming of resources to missions of tasks for a given set of work. This model is constrained with restrictions due to the inability of preexisting missions and teams to change. As work changes and teams need to adapt, the objective value and assignment produced from this model can serve as a baseline measure of robustness for an existing organization and highlight whether a predefined organizational structure can handle a diverse set of work. The Mission-to-Team model best fits military organizations with a tiered force structure where resources are formed into teams and tasks are grouped into missions and then assignment occurs between teams and missions. With larger architectures, this model can be used to maximize performance advantages of a centralized force structure that executes a strict hierarchy in the execution of work.

Team-Formation: Given tasks, resources, and missions, an organization can utilize the Team-Formation model and produce at "best" a near-optimal or local optimal teaming of resources. This model is a relaxation of the Mission-to-Team model and sets a baseline understanding of the adaptability and flexibility a force structure needs to handle a diverse set of work. Effective team formation requires a collaborative architecture, and the Team-Formation model determines in what way an organization that is amenable to changes in force structure can accomplish work. This model incorporates a team effect function that balances resource specialization, meaning it is inappropriate for a force with few different specializations. For example, an extremely specialized force of resources will be incapable of performing various tasks, which will always result in a large optimality gap when compared to more balanced organizations. Thus, the Team-Formation model maximizes the advantages of cooperation while limiting conflict between resource specializations to produce more effective teams capable of varying missions and tasks.

Mission-Assignment: Given tasks, resources, and teams, an organization can utilize the Mission-Assignment model and produce at "best" near-optimal or local optimal missions for a given force. This model is also a relaxation of the Mission-to-Team model and provides a different baseline measure of adaptability and flexibility required by a force structure to handle a diverse set of work. This is because the Mission-Assignment model instead incorporates a mission effect to produce a more synchronized task architecture. This ensures that even a rigid force structure is only assigned missions that are de-conflicted and easily completed in tandem. Thus, the Mission-Assignment model maximizes the advantages of synchronization while limiting connections between task functional requirements to produce more effective missions even when teams are unable to change.

Overall, an optimal C2 hierarchy depends on given computational requirements, known data, organizational structure, and mission requirements. Each optimization-based assignment model provides a new perspective on what teaming means while setting baselines for computational requirements, optimality, robustness, adaptability, and flexibility. Choice of optimal teaming is therefore data-, context-, and mission-dependent.

5.1 Future Work

Future work and options in this C2 research topic include the following.

Master Assignment Model: There is a fifth and final assignment model that has yet to be explored. Given simply tasks and resources, the model will take the workings of the previous four models and provide the "best" assignment of tasks to resources, missions to teams, team formation, and mission assignment.

Decentralized Assignment: A decomposition-based approach can be applied to a decentralized assignment of tasks, resources, missions, and teams. The decentralized execution can be used to represent how work is accomplished through an organizational hierarchy.

Machine Learning: A machine learning approach can be incorporated into the assignment models by leveraging previous solutions to determine the best allocation of tasks and resources into missions and teams. A team social network analysis may be applied to represent a learning, sensing, and adapting team.

Team of Teams: Lastly, taking this a step further would be connecting multiple teams to carry out a combination of missions and measuring the robustness of an interconnected team architecture with various objectives for accomplishing work.

These future architecture options will require additional tools and expertise within the field of OR or other disciplines to measure robustness in C2.

5.2 Final Thoughts

The descriptive work of General McChrystal's team of teams led us toward how Marines execute C2 through MCDP-6, *Command and Control*. In striking fashion, at the core of both texts, which the MCDP-6's commander's intent and delegation of authority which closely correlated to General McChrystal's team of teams representation of shared consciousness and empowered execution.

By examining C2 architectures, we have established an optimization-based approach with a prescriptive assignment framework that models processes of an organization. The four assignment models are supported by quantitative analysis that identifies the best composition of entities and measures the robustness of a C2 organizational architecture in accomplishing work.

In order to remain relevant in accomplishing varying degrees of work, it is paramount that organizations are capable and amenable to change. The depth and breadth of this research has led to key insights into how teams within a C2 architecture can both enhance and diminish the effectiveness of old practices in C2 and translate into a new adaptable, flexible architecture for leading war fighters.

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