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1. REPORT DATE (DD-MM-YYYY) 28-01-2019		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 21-Aug-2015 - 20-Aug-2018	
4. TITLE AND SUBTITLE Final Report: Three Dimensional Fracture Mechanics Capability for Structures operating in High Temperature Thermal Environments			5a. CONTRACT NUMBER W911NF-15-1-0456		
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7. PERFORMING ORGANIZATION NAMES AND ADDRESSES University of Texas at San Antonio One UTSA Circle San Antonio, TX 78249 -1644			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
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a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU	UU		Harry Millwater
					19b. TELEPHONE NUMBER 210-458-4481

RPPR Final Report
as of 30-Jan-2019

Agency Code:

Proposal Number: 67230EGREP

Agreement Number: W911NF-15-1-0456

INVESTIGATOR(S):

Name: Arturo Montoya
Email: arturo.montoya@utsa.edu
Phone Number: 2104587516
Principal: N

Name: Harry Millwater
Email: harry.millwater@utsa.edu
Phone Number: 2104584481
Principal: Y

Name: Justin Wilkerson
Email: justin.wilkerson@utsa.edu
Phone Number: 2104585521
Principal: N

Organization: **University of Texas at San Antonio**

Address: One UTSA Circle, San Antonio, TX 782491644

Country: USA

DUNS Number: 800189185

EIN: 741717115

Report Date: 20-Nov-2018

Date Received: 28-Jan-2019

Final Report for Period Beginning 21-Aug-2015 and Ending 20-Aug-2018

Title: Three Dimensional Fracture Mechanics Capability for Structures operating in High Temperature Thermal Environments

Begin Performance Period: 21-Aug-2015

End Performance Period: 20-Aug-2018

Report Term: 0-Other

Submitted By: Harry Millwater

Email: harry.millwater@utsa.edu

Phone: (210) 458-4481

Distribution Statement: 1-Approved for public release; distribution is unlimited.

STEM Degrees: 5

STEM Participants: 10

Major Goals: 1.1. Incorporation of a multicomplex thermal solver into ZFEM through a user element (UEL)

1.1.1. 2D with verification

1.1.2. 3D with verification

1.2. Integration of sequential thermal solver with solid mechanics solver

1.2.1. 2D with verification

1.2.2. 3D with verification

1.3. Development and validation of experimental design

2. Yr 2

2.1. Incorporation of thermally-dependent material properties into ZFEM through a user material (UMAT)

2.1.1. 2D with verification

2.1.2. 3D with verification

2.2. Analysis of 2D DoD-provided example problem

2.3. Validation experiments

2.3.1. C(T) sample with an imposed high-temperature field in front of the crack tip

3. Yr 3

3.1. Validation experiments

3.1.1. High-temperature modified Sandia Fracture Challenge specimen

3.2. Analysis of 3D DoD-provided example problem

Accomplishments: The goals of the project were completed. Additional capabilities were also accomplished, as shown below

1.1. Incorporation of a multicomplex thermal solver into ZFEM through a user element (UEL) - COMPLETE

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- 1.2. Integration of sequential thermal solver with solid mechanics solver - COMPLETE
 - 1.3. Development and validation of experimental design - COMPLETE
 - 2.1. Incorporation of thermally-dependent material properties into ZFEM through a user material (UMAT) - COMPLETE
 - 2.2. Analysis of 2D DoD-provided example problem - COMPLETE
 - 2.3. Validation experiments - PARTIALLY COMPLETE (tests conducted at AFRL. Still waiting on load frame fixes as UTSA).
 - 3.1. Validation experiments - COMPLETE
 - 3.1.1. High-temperature modified Sandia Fracture Challenge specimen
 - 3.2. Analysis of 3D DoD-provided example problem - COMPLETE
- Additional accomplishments:
- developed multicomplex and multidual numerical library
 - extended and verified ZFEM method to elasto-plastic fracture
 - extended and verified ZFEM method for interface and mixed mode cracks
 - developed a fast "local" ZFEM method for energy release rate calculations
 - developed a progressive thermally-driven crack growth code

Training Opportunities: 13 conference papers were presented in total - 6 were presented by PhD students.

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Results Dissemination: Peer-Reviewed Published Paper with acknowledgement to DoD support

Montoya, A., Ramirez Tamayo, D., Millwater, H. and Kirby, M. (2018). "A Complex Variable Virtual Crack Extension Finite Element Method for Elastic-Plastic Fracture Mechanics." *Engineering Fracture Mechanics*, 202: 242-258.

<https://doi.org/10.1016/j.engfracmech.2018.09.023>

Ramirez Tamayo, D., Montoya, A., and Millwater, H. (2018). "Application of the Complex Variable Finite Element Method to Mixed Mode Fracture and Interface Cracks." *AIAA Journal*, 56(11):4632-4637. <https://doi.org/10.2514/1.J057231>

Ramirez Tamayo, D., Montoya, A. and Millwater, H. (2018). "A Virtual Crack Extension Method for thermoelastic fracture using a complex-variable finite element method." *Engineering Fracture Mechanics*, 192:328-342. <https://doi.org/10.1016/j.engfracmech.2017.12.013>

Montoya, A. and Millwater, H. (2017). "Sensitivity Analysis in Thermoelastic Problems Using the Complex Finite Element Method." *Journal of Thermal Stresses*, 40(3): 302-321. <http://dx.doi.org/10.1080/01495739.2016.1264871>

Under Review Papers with acknowledgement to DoD support

Wagner, D., Millwater, H., Garcia, M., and Montoya, A. (2019). "A Finite Element-based Adaptive Energy Response Function Method for 2D Curvilinear Progressive Fracture." Submitted to *International Journal of Fatigue* on 11/15/2018.

Conference presentations:

H. Millwater, David Wagner, Daniel Ramirez-Tamayo, Andres Aguirre, Arturo Montoya, Manuel Garcia, Brendy Rincon-Troconis, "Two and Three Dimensional Progressive Fracture under Thermal Loading: Computational Modelling and Experimental Validation, International Conference on Fatigue Damage of Structural Materials XII. Hyannis, MA, 16-21, Sept. 2018

H. Millwater, J. Ocampo, and N. Crosby, "An Ultrafast Crack Growth Lifting Model for Efficient Prognosis, International Conference on Fatigue Damage of Structural Materials XII. Hyannis, MA, 16-21, Sept. 2018

M. Aristizabal, M. Garcia, and H. Millwater, "General Purpose Finite Element Library for Computation of High-Order Multivariable Sensitivities," 13th World Congress of Computational Mechanics, New York, NY July 2018

A. Aguirre, D. Ramirez-Tamayo, M. Garcia, and H. Millwater, "Computation of the Energy Release Rate using a Complex Stiffness Derivative Approach," 13th World Congress of Computational Mechanics, New York, NY July 2018

D. Ramirez-Tamayo, A. Montoya, H. Millwater, and D. Wagner, "A Multicomplex Finite Element Approach for Thermoelastic Curvilinear Progressive Fracture," 13th World Congress of Computational Mechanics, New York, NY July 2018

A. Montoya, H. Millwater, R. Fielder, & P. Golden, "A Complex Variable Finite Element Based Approach for Rapid Estimates of Residual Stress Variance," 13th World Congress of Computational Mechanics, New York, NY July 2018

M. Garcia, M. Aristizabal, A. Aguirre, H. Millwater, and A. Montoya, "An Overview of Hypercomplex Algebras for Sensitivity Analysis in the Finite Element Method," 13th World Congress of Computational Mechanics, New York, NY July 2018

A. Aguirre, M. Garcia, M. Aristizabal, and H. Millwater, "MCX: A Multicomplex Finite Element Library for High Order Derivatives," World Congress on Computational Mechanics, July 2016, Seoul, Korea

H.R. Millwater, "Overview of the ZFEM Multicomplex Finite Element Method", Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016

M. Garcia, D. Wagner, H.R. Millwater, "Two Dimensional Curvilinear Progressive Fracture using a Multicomplex Finite Element Method," Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016

A. Aguirre, M. Garcia, H.R. Millwater, "Multi-Complex and Multi-Dual Numbers," Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016

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Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Nam Phan, NavAir, MD, August 2018 - "High order sensitivity analysis using the hypercomplex finite element method"
Patrick Golden, AFRL/RX, May 2017 - "Overview of the Hypercomplex Finite Element Method and its application to Fracture Mechanics"
R. Bateman, K. Schweitzer ARL, Aug. 2017 - "Overview of the Hypercomplex Finite Element Method and its application to Fracture Mechanics"

PARTICIPANTS:

Participant Type: PD/PI

Participant: Harry Millwater

Person Months Worked: 12.00

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Funding Support:

Participant Type: Co PD/PI

Participant: Arturo Montoya

Person Months Worked: 12.00

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Funding Support:

Participant Type: Co PD/PI

Participant: Justin Wilkerson

Person Months Worked: 6.00

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Funding Support:

Participant Type: Co-Investigator

Participant: Brendy Rincon-Troconis

Person Months Worked: 12.00

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Funding Support:

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as of 30-Jan-2019

CONFERENCE PAPERS:

Publication Type: Conference Paper or Presentation **Publication Status:** 1-Published
Conference Name: AIAA SciTech
Date Received: 11-Jan-2019 Conference Date: 08-Jan-2018 Date Published: 08-Jan-2018
Conference Location: Kissimmee, FL
Paper Title: A New Complex-variable Thermal Fracture Finite Element Method for Evaluating the Structural Integrity of Aircraft Structures
Authors: Daniel Ramirez-Tamayo, Arturo Montoya, Harry Millwater
Acknowledged Federal Support: **Y**

DISSERTATIONS:

Publication Type: Thesis or Dissertation
Institution: University of Texas at San Antonio
Date Received: 11-Jan-2019 Completion Date: 5/2/18 1:23AM
Title: Full-Field Experimental Analysis of Ductile and Fatigue Fracture and the Accompanying Thermal Effects
Authors: Zachary Huber
Acknowledged Federal Support: **N**

Publication Type: Thesis or Dissertation
Institution: University of Texas at San Antonio
Date Received: 11-Jan-2019 Completion Date: 8/1/18 5:00AM
Title: A FINITE ELEMENT-BASED ADAPTIVE ENERGY RESPONSE FUNCTION METHOD FOR CURVILINEAR PROGRESSIVE FRACTURE
Authors: DAVID WAGNER
Acknowledged Federal Support: **Y**

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a. REPORT	b. ABSTRACT	c. THIS PAGE			19b. TELEPHONE NUMBER (Include area code)

1) Submissions or publications under ARO sponsorship during this reporting period.

(a) Papers published in peer-reviewed journals

4

(b) Papers published in non-peer-reviewed journals

0

(c) Presentations

i. Presentations at meetings, but not published in Conference Proceedings

None

ii. Non-Peer-Reviewed Conference Proceeding publications (other than abstracts)

13

iii. Peer-Reviewed Conference Proceeding publications (other than abstracts)

1

(d) Manuscripts

None

(e) Books

None

(f) Honor and Awards

1 – promotion of Co-PI Dr. Arturo Montoya to Associate Profesor

(g) Title of Patents Disclosed during the reporting period

None

(h) Patents Awarded during the reporting period

None

(2) Student/Supported Personnel Metrics for this Reporting Period (name, % supported, %Full Time Equivalent (FTE) support provided by this agreement, and total for each category):

(a) Graduate Students

Daniel Ramirez-Tamayo, (PhD ME, current): 100%, 50% FTE 3 years

Andres Aguirre-Mesa (PhD ME, current): 50%, 25% 2 years FTE

Zachary Huber (MS ME, graduated May 2018): 100%, 50% FTE (3 years)

Hamid Eslami (MS ME, graduate May 2015): 100%, 50% FTE (1 semester)

Mauricio Aristizabal (PhD ME Eafit Univ. Medellin Colombia, current) 50%, 25% FTE (1 year)

Total: 5 students, 4 FTE

(b) Post Doctorates

None

(c) Faculty

Harry Millwater (0.5 summer months, years 1-3)
Arturo Montoya (1 summer month year 1, 0.5 summer months, years 2-3)
Justin Wilkerson (0.5 summer months, years 1-2) – left UTSA after year 2
Brendy Rincon-Troconis (0.5 summer months, years 3)
Total: 4 summer months

(d) Undergraduate Students (FTE estimated as \$4800/year)

Kyle Fernandez (BS ME), 1/10 FTE 1 year
Jose Barajas (BS CE), $\frac{3}{4}$ FTE 1 year
Daniela Gomez (CE senior - current), $\frac{1}{4}$ FTE 1 year
Brandon Ard (ME senior - current), 1/6 FTE 1 year
Zach Roussel (BS ECE), $\frac{3}{4}$ FTE 1 year

(e) Graduating Undergraduate Metrics (funded by this agreement and graduating during this reporting period):

i. Number who graduated during this period

3

ii. Number who graduated during this period with a degree in science, mathematics, engineering, or technology fields

3

iii. Number who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields

0

iv. Number who achieved a 3.5 GPA to 4.0 (4.0 max scale)

2

v. Number funded by a DoD funded Center of Excellence grant for Education, Research and Engineering

None

Vi. Number who intend to work for the Department of Defense

None

vii. Number who will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields

None

(f) Masters Degrees Awarded (Name of each, Total #)

Zachary Huber (MS ME, graduated May 2018)

Hamid Eslami (MS ME, graduate Dec 2016)

Total 2

(g) Ph.D.s Awarded (Name of each, Total #)

None

(h) Other Research staff (Name of each, FTE % Supported for each, Total % Supported)

None

REPORT OF INVENTIONS AND SUBCONTRACTS
(Pursuant to "Patent Rights" Contract Clause) (See Instructions on back)

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b. ADDRESS (Include ZIP Code) 1 UTSA Circle San Antonio, TX 78249		d. AWARD DATE (YYYYMMDD)		b. ADDRESS (Include ZIP Code)		d. AWARD DATE (YYYYMMDD) 20150821		4. REPORTING PERIOD (YYYYMMDD)	
								a. FROM 20150821	
								b. TO 20180820	

SECTION I - SUBJECT INVENTIONS

5. "SUBJECT INVENTIONS" REQUIRED TO BE REPORTED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)

NAME(S) OF INVENTOR(S) <i>(Last, First, Middle Initial)</i>	TITLE OF INVENTION(S)	DISCLOSURE NUMBER, PATENT APPLICATION SERIAL NUMBER OR PATENT NUMBER	ELECTION TO FILE PATENT APPLICATIONS (X)				CONFIRMATORY INSTRUMENT OR ASSIGNMENT FORWARDED TO CONTRACTING OFFICER (X)	
			d. (1) UNITED STATES		(2) FOREIGN		e.	
			(a) YES	(b) NO	(a) YES	(b) NO	(a) YES	(b) NO
None								

f. EMPLOYER OF INVENTOR(S) NOT EMPLOYED BY CONTRACTOR/SUBCONTRACTOR		g. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED	
(1) (a) NAME OF INVENTOR (Last, First, Middle Initial)	(2) (a) NAME OF INVENTOR (Last, First, Middle Initial)	(1) TITLE OF INVENTION	(2) FOREIGN COUNTRIES OF PATENT APPLICATION
(b) NAME OF EMPLOYER	(b) NAME OF EMPLOYER		
(c) ADDRESS OF EMPLOYER (Include ZIP Code)	(c) ADDRESS OF EMPLOYER (Include ZIP Code)		

SECTION II - SUBCONTRACTS (Containing a "Patent Rights" clause)

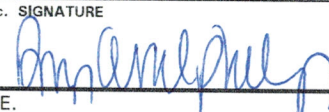
6. SUBCONTRACTS AWARDED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)

NAME OF SUBCONTRACTOR(S)	ADDRESS (Include ZIP Code)	SUBCONTRACT NUMBER(S)	FAR "PATENT RIGHTS"		DESCRIPTION OF WORK TO BE PERFORMED UNDER SUBCONTRACT(S)	SUBCONTRACT DATES (YYYYMMDD)	
			(1) CLAUSE NUMBER	(2) DATE (YYYYMM)		(1) AWARD	(2) ESTIMATED COMPLETION
None							

SECTION III - CERTIFICATION

7. CERTIFICATION OF REPORT BY CONTRACTOR/SUBCONTRACTOR (Not required if: (X as appropriate))

I certify that the reporting party has procedures for prompt identification and timely disclosure of "Subject Inventions," that such procedures have been followed and that all "Subject Inventions" have been reported.

SMALL BUSINESS or		NONPROFIT ORGANIZATION	
a. NAME OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL (Last, First, Middle Initial)	b. TITLE Director, RSC for SCI/ENG	c. SIGNATURE 	d. DATE SIGNED 20190124

**Project Summary - Grant # W911NF-15-1-0456
(Reporting Period: August 2015 – August 2018)**

**Three Dimensional Fracture Mechanics Capability for Structures operating in High
Temperature Thermal Environments**

Harry Millwater
Mechanical Engineering
The University of Texas at San Antonio, San Antonio, Texas, 78249

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Statement of the Problem Studied

The objective of this research is to provide DoD analysts a new high accuracy method for computing 2D and 3D fracture of structures and materials undergoing thermally driven crack growth. This research seeks to obviate the use of complicated energy conservation integrals, such as the J-integral and its extensions, to compute the energy release rate and to provide a more robust and accurate method. The proposed new method investigated here calculates the energy release rate through a virtual crack extension approach facilitated through the use of hypercomplex mathematics within the finite element formulation. Although this method has shown excellent results for isothermal problems, this project evaluated the validity of the method for high temperature problems with thermally driven crack growth and temperature dependent materials. This project will provide analysts an accurate straightforward method to assess the structural integrity of DoD assets. The results indicate that the hypercomplex finite element provides a highly accurate, robust, and general method for computing the energy release rate for linear and nonlinear material undergoing thermal loading.

Summary of the Most Important Results

A summary of the accomplishments are provided below. Multiple new developments were accomplished that were not listed in the original proposal. These accomplishments extend the method into new domains of importance to DoD and provide significant efficiency enhancements. The new capabilities are so marked in the summary below. The accomplishments for this grant include:

1. Enhanced ZFEM software with thermal capabilities
 - Developed, implemented and verified a thermal solver within ZFEM in order to compute the energy release rate for thermally and mechanically loaded structures.
 - Developed complex- and hypercomplex-valued 2D and 3D user elements in which temperature-dependent material behaviors can be specified, e.g. linear, exponential, etc.
2. Verified ZFEM Energy Release rate results against analytical solutions and J-integral results for thermally loaded structures
 - ZFEM energy release rates results were compared against analytical solutions and Abaqus J-integral results for: (a) a 2D cracked plate with fixed temperatures on its walls (Figure 1 and Table 1) [1, 2] (b) 3D cracked plates with fixed temperatures on the walls (Figure 3 and Table 2) [1, 2], (c) circumferential crack in a pipe under steady state heat transfer loading (Figure 4 and Table 3), and a (d) bimaterial edge cracked plate under thermal and mechanical loading [3] (Figure 5 and Table 4). ZFEM was found to be of the same accuracy as the J-integral formulation for all cases.
 - Performed convergence studies of the ERR with respect to mesh density for a (a) homogeneous (Figure 2) and (b) bimaterial cracked plate (Figure 6). For homogeneous materials, ZFEM was found to converge at the same rate as the J-integral. For nonhomogeneous materials, the J-integral loses its path independent property for nonhomogeneous materials and must be corrected in order to provide a consistent energy value for increasing contours [4–6]. ZFEM was found to con-

- verge more quickly and be more robust than the corrected J-integral. These results are relevant as fracture problems considering temperature-dependent materials under temperature gradients are analogous to the analysis of cracked structures in nonhomogeneous materials.
3. Implemented a thermal progressive fracture code that can take large curvilinear crack steps (see Figures 7 and 8). In Figure 8, three different curvilinear paths are obtained by varying the initial temperature and loading conditions. The results are compared against those provided by Prasad and Aliabadi [7]. The ability of ZFEM to compute high order derivatives was exploited to construct an efficient progressive fracture code. The code works as follows:
 - Build a Taylor series expansion (up to 4th order) of the strain energy with respect to the crack propagating in self-similar or perpendicular directions (see Figure 9).
 - Use a gradient descent algorithm to compute the curvilinear crack path using the maximum energy release method, see Figure 7 (other criterion can be implemented).
 - Use error-correction adaptive step size to determine the longest accurate step before remeshing.
 4. (NEW) Enhanced ZFEM to include multicomplex and multidual variables. During the course of the project, it was realized that ZFEM can be extended to hypercomplex numbers which include both multicomplex and multidual numbers as, in some cases, multidual numbers are more efficient.
 - Developed a Fortran hypercomplex library (MultiZ) to support hypercomplex variables and operations. MultiZ works by operator overloading.
 - MultiZ contains a number of features such as: hypercomplex variable types, addition/subtraction, multiplication/division, intrinsic functions (sine, cosine, exponential, log) and matrix operations (multiplication, transpose), see Table 5.
 5. (NEW) Extended ZFEM to elasto-plastic analysis. It was realized that many fracture problems of relevance to DoD involve plasticity. As such, ZFEM was extended to compute the energy release rate of structures undergoing plastic deformation. It was shown that ZFEM can accurately compute the energy release rate for loading and unloading whereas the J-integral can only compute an accurate energy release rate for loading. Figure 10 shows a comparison verifying that ZFEM agrees with the finite difference results. The J-integral values for the two different integration contours provide erroneous, non-converging approximations during unloading.
 6. (NEW) Verified the computation of the energy release rate for mixed mode and interface cracks. For example, Figure 11 shows a semi-infinite plate made of two different materials with a center crack that was used as a benchmark problem. The results were verified against the analytical solution provided by Rice and Sih [8] (see Table 6).
 7. (NEW) Enhanced ZFEM to use a stiffness derivative approach such that the energy release rate was computed with roughly a 2X reduction in compute time. This approach is similar to the paper by Parks [9] but far superior in numerical accuracy. In practice, the time to compute the ERR requires only an additional 1 or 2 percent of the solution time.
 8. Experimental fatigue calculations were conducted in order to predict the curvilinear crack growth from a structure with varying potential cracks paths. A new specimen was developed based on a compact tension specimen that was modified by the drilling of three holes in front of the crack. A dozen different crack paths were possible depending upon the location of the starter crack. Successful experiments completed at the Air Force Research Lab (AFRL); however, experiments conducted at UTSA have not been successful to date due to issues with the load frame.
 9. Designed experimental fracture mechanics capability for elevated temperatures.
 - Conducted fracture mechanics experiments at room temperature with 15-5PH Martensitic Stainless Steel. Samples were acquired, machined, and heat treated. Multiple dogbone and modified C(t) specimens were tested under different heat treatment conditions - H900, H1075, and the solution annealed Condition A. The results are presented in Figure 12. An efficient triggering system was utilized to provide a common clock for the thermal, load, and deformation data for the tests completed. This allowed for temperature data to be presented as a function of crack opening displacement (COD) and position. Figure 13 shows the temperature distribution along half of the gauge length of a dogbone sample loaded in uniaxial tension for various amounts of gauge length extension. Temperature distribution plots at different COD were created for the modified C(t) samples tested along the lines presented in Figure 14. Isotherm images were also created for different interesting events such as that seen in Figure 15. The raw experimental data has successfully

been turned into a rich data set for use in validation of computational algorithms. Results are being compiled in a manuscript to be submitted to The International Journal of Fracture.

- Built the required experimental capabilities by integrating an Instron loading frame, a OEM Model 4085 Infrared Spot Heater (capable of 2000 °F), a FLIR A6753sc MWIR thermal camera with 50 mm lens, and an Epsilon Technology crack opening displacement gauge. The experimental set-up is capable of providing experimental measurements of reaction forces, crack opening displacement, and full-field surface temperature measurements. Furthermore, we utilized the experimental set-up to provide full-field surface temperature measurements (see Figure 13) of the Sandia Fracture Challenge specimens loaded to failure [10]. We also demonstrated a capability to provide full-field surface measurements of the sum of in-plane stresses through thermomechanical stress analysis.
- Successfully laser machined dog-bone, compact tension fracture toughness, and Sandia Fracture Challenge samples within tight tolerances. Developed an in-house capability to heat treat 15-5PH martensitic stainless steel to the H1075 condition utilized in the Sandia Fracture Challenge specimens [10]. We confirmed our specimen preparation and experimental set-up (see Figure 14) provides consistent results to those obtained at Sandia National Laboratory as well as UT Austin [10].

Collaborations and Technology Transfer

- Presentation at Wright-Patterson AFB with Patrick Golden, AFRLRX and staff, May 2017
- Presentation at Naval Air Station with Nam Phan and staff, Aug. 2018

Resulting Journal Publications during Reporting Period

- Montoya, A., Ramirez Tamayo, D., Millwater, H. and Kirby, M. (2018). "A Complex Variable Virtual Crack Extension Finite Element Method for Elastic-Plastic Fracture Mechanics." *Engineering Fracture Mechanics*, 202:242-258. <https://doi.org/10.1016/j.engfracmech.2018.09.023>
- Ramirez-Tamayo, D., Montoya, A., and Millwater, H. (2018). "Application of the Complex Variable Finite Element Method to Mixed Mode Fracture and Interface Cracks." *AIAA Journal*, 56(11):4632-4637. <https://doi.org/10.2514/1.J057231>
- Ramirez Tamayo, D., Montoya, A. and Millwater, H. (2018). "A Virtual Crack Extension Method for thermoelastic fracture using a complex-variable finite element method." *Engineering Fracture Mechanics*, 192:328-342. <https://doi.org/10.1016/j.engfracmech.2017.12.013>
- Montoya, A. and H. Millwater. (2017) "Sensitivity Analysis in Thermoelastic Problems Using the Complex Finite Element Method". *Journal of Thermal Stresses*, 40(3): 302-321 <http://dx.doi.org/10.1080/01495739.2016.1264871>
- Huber, Z. and Wilkerson, J.W. "Revisiting the Sandia Fracture Challenge with full-field thermography." To be submitted to Int. J. Fracture. Status: 90% complete.

Peer-reviewed Conference Papers during Reporting Period

- D. Ramirez Tomayo*, A. Montoya, and H. Millwater, "A New Complex-variable Thermal Fracture Finite Element Method for Evaluating the Structural Integrity of Aircraft Structures," AIAA SciTech Conference, Kissimmee, FL, Jan 8-12, 2018, AIAA-0649, <https://doi.org/10.2514/6.2018-0649>

Non-peer-reviewed Conference Papers during Reporting Period

- H. Millwater, David Wagner, Daniel Ramirez-Tamayo, Andres Aguirre, Arturo Montoya, Manuel Garcia, Brendy Rincon-Troconis, "Two and Three Dimensional Progressive Fracture under Thermal Loading: Computational Modelling and Experimental Validation, International Conference on Fatigue Damage of Structural Materials XII. Hyannis, MA, 16-21, Sept. 2018
- D. Ramirez Tamayo, A. Montoya, H.R. Millwater, D. Wagner, "A Multicomplex Finite Element Approach for Thermoelastic Curvilinear Progressive Fracture", 2018 conference of Engineering Mechanics Institute, Boston, MA, May 29-June 1, 2018
- M. Aristizabal, M. Garcia, and H. Millwater, "General Purpose Finite Element Library for Computation of High-Order Multivariable Sensitivities," 13th World Congress of Computational Mechanics, New York, NY July 2018
- A. Aguirre, D. Ramirez-Tamayo, M. Garcia, and H. Millwater, "Computation of the Energy Release Rate using a Complex Stiffness Derivative Approach," 13th World Congress of Computational Mechanics, New York, NY July 2018
- D. Ramirez-Tamayo, A. Montoya, H. Millwater, and D. Wagner, "A Multicomplex Finite Element Approach

for Thermoelastic Curvilinear Progressive Fracture,” 13th World Congress of Computational Mechanics, New York, NY July 2018

- A. Montoya, H. Millwater, R. Fielder, & P. Golden, “A Complex Variable Finite Element Based Approach for Rapid Estimates of Residual Stress Variance,” 13th World Congress of Computational Mechanics, New York, NY July 2018
- M. Garcia, M. Aristizabal, A. Aguirre, H. Millwater, and A. Montoya, “An Overview of Hypercomplex Algebras for Sensitivity Analysis in the Finite Element Method,” 13th World Congress of Computational Mechanics, New York, NY July 2018
- H. Millwater, J. Ocampo, and N. Crosby, “An Ultrafast Crack Growth Lifting Algorithm for Probabilistic Damage Tolerance Analysis, Aircraft Airworthiness & Sustainment Conference, Jacksonville, FL, April 2018
- H. Millwater, R. Fielder, F. Garcia, and C. Labowsky, Probabilistic and Sensitivity Method Development and Application in Life Prediction of metallic Materials and Structures,” AFRL Minority Leaders Research Collaboration Program Review, Dayton, OH, Sept 20-22, 2016
- A. Aguirre, M. Garcia, M. Aristizabal, and H. Millwater, “MCX: A Multicomplex Finite Element Library for High Order Derivatives,” World Congress on Computational Mechanics, July 2016, Seoul, Korea
- H.R. Millwater, “Overview of the ZFEM Multicomplex Finite Element Method”, Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016
- M. Garcia, D. Wagner, H.R. Millwater, “Two Dimensional Curvilinear Progressive Fracture using a Multicomplex Finite Element Method,” Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016
- A. Aguirre, M. Garcia, H.R. Millwater, “Multi-Complex and Multi-Dual Numbers,” Parameterized Reduced Order Modeling Workshop Organized by Sandia National Laboratories in Albuquerque, New Mexico, June 1-2, 2016

Graduate Students Involved during Reporting Period

- Daniel Ramirez-Tamayo (PhD ME, current)
- Andres Aguirre-Mesa (PhD ME, current)
- Zachary Huber (MS ME, graduated May 2018)
- Hamid Eslami (MS ME, graduate Dec 2016)
- Mauricio Aristizabal (PhD ME Eafit Univ. Medellin Colombia, current)

Undergraduate Students Involved During Reporting Period

- Kyle Fernandez (BS ME)
- Jose Barajas (BS CE)
- Daniela Gomez (CE senior - current)
- Brandon Ard (ME senior - current)
- Zach Roussel (BS ECE)

Awards, Honors and Appointments

Dr. Arturo Montoya, Co-PI, was awarded tenure starting Fall 2018. The work conducted on this grant, as evidenced in the journal publications, was a significant contribution towards his promotion.

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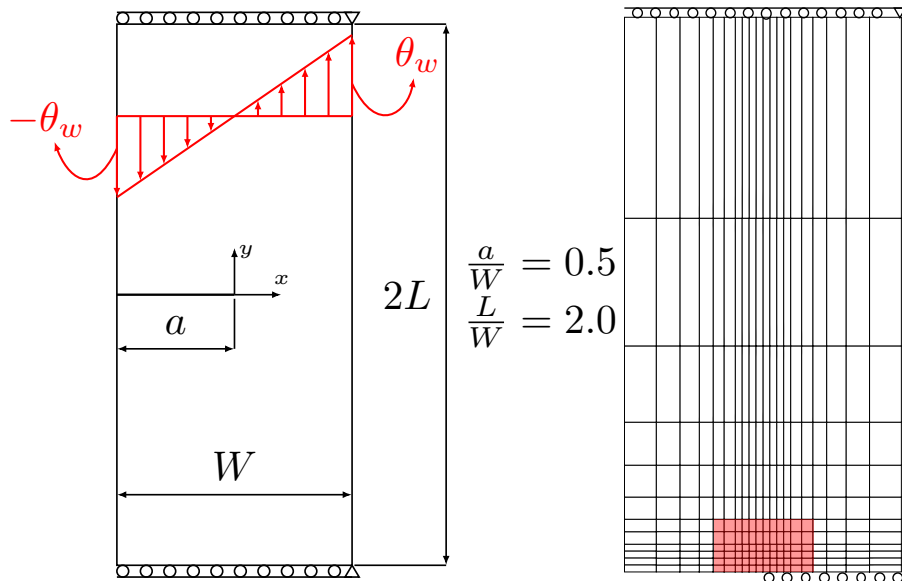


Figure 1: 2D cracked plate with fixed temperature on its walls (left) and corresponding finite element mesh (right). ZFEM perturbed region is indicated by the red-shadowed area.

Normalized Energy Release Rate $[G/(\sigma_{\theta}^2 a/E)]$					
ZFEM	Contour Based Approaches				
	Contour	Abaqus	Wilson and Yu [2]	Shih et al. [1]	Superposition [1]
0.7064	1	0.7072	0.7273	0.6366	0.6408
–	2	0.7066	0.7084	0.6618	0.6900
–	3	0.7064	0.7239	0.6712	0.6923
–	4	0.7063	0.7093	0.6759	0.6954
–	5	0.7063	0.7565	0.6800	0.7004
–	6	0.7063		0.6827	0.7033
–	7	0.7063		0.6848	0.7113
–	8	0.7063		0.6869	0.7253
–	9	0.7064		0.6899	0.7503

Table 1: Comparison of the Energy Release Rate results between ZFEM and Contour-based Approaches for the problem shown in Figure 1.

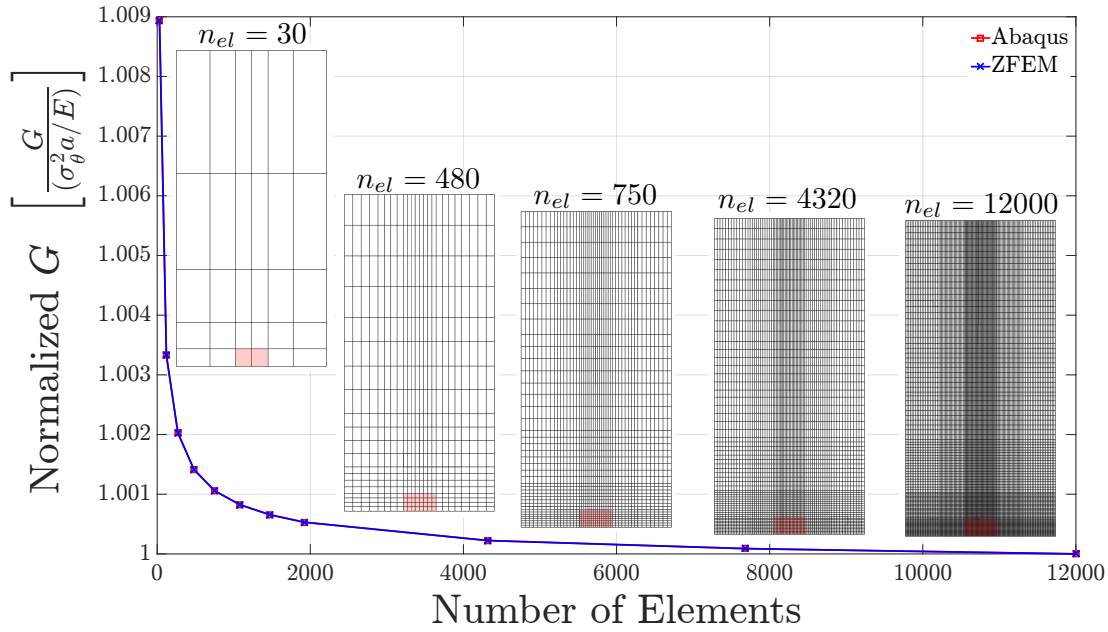


Figure 2: Mesh density convergence of energy release rate for homogeneous cracked plate.

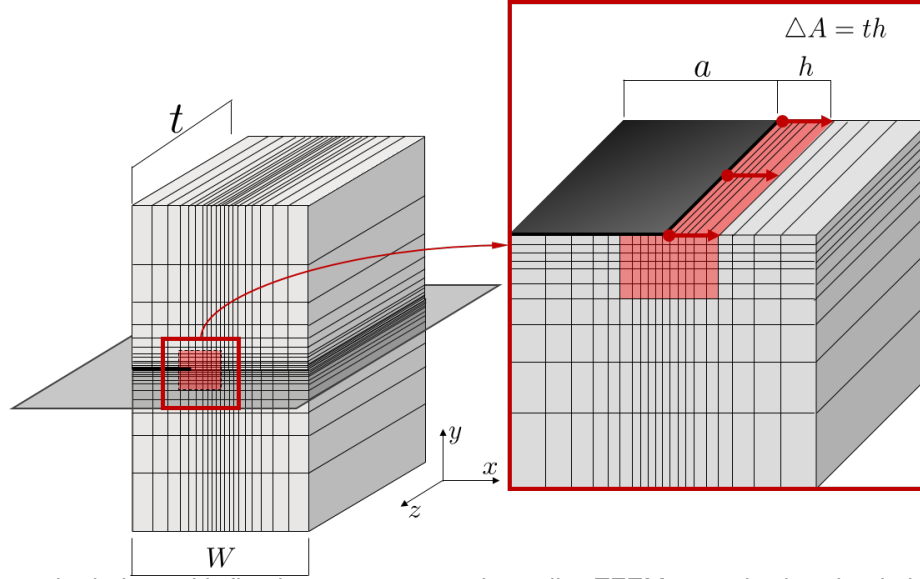


Figure 3: 3D cracked plate with fixed temperature on its walls. ZFEM perturbed region is indicated by the red-shadowed region.

Normalized G [$G/(\sigma_\theta^2 a/E)$]			
ZFEM 2D	ZFEM 3D	2D Abaqus 10 th Contour	3D Abaqus 10 th Contour
0.70641	0.70662	0.70640	0.70662

Table 2: Comparison of the Energy Release Rate results between 2D and 3D ZFEM and 10th contour of the 2D and 3D Abaqus J-integral for the problem shown in Figure 3.

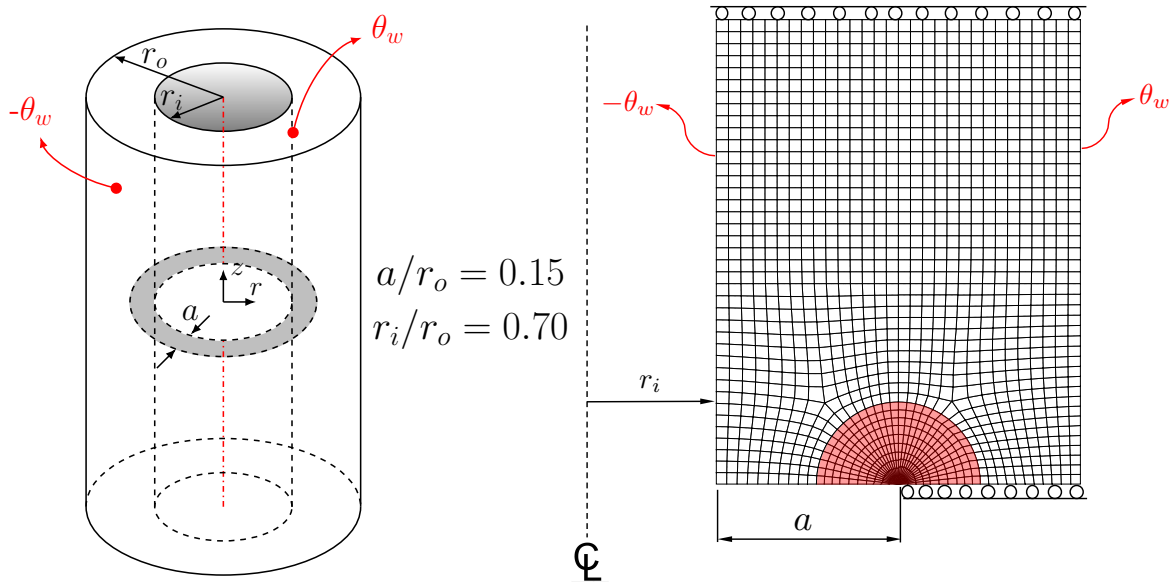


Figure 4: Circumferential crack in a pipe under steady heat transfer loading (left) and corresponding finite element mesh (right). ZFEM perturbed region is indicated by the red-shadowed area.

Normalized G [$G/(\sigma_\theta^2 a/E)$]	
ZFEM	Abaqus 10 th Contour
0.19359	0.19359

Table 3: Comparison of the Energy Release Rate results between ZFEM and 10th contour of the Abaqus J-integral for the problem shown in Figure 4.

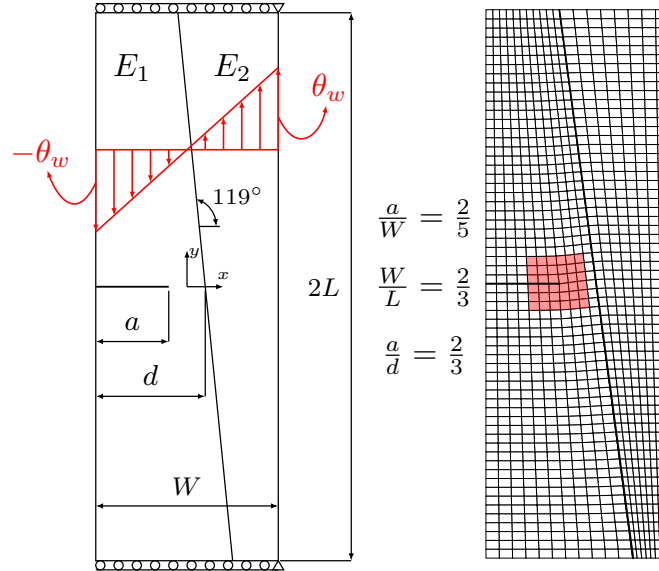


Figure 5: 2D bi-material cracked plate with fixed temperature on its walls (left) and corresponding finite element mesh (right). ZFEM perturbed region is indicated by the red-shadowed area.

Normalized G [$G/(\sigma_\theta^2 a/E)$]			
ZFEM	Contour Based Approaches		
	Contour	J	J_{corr}
11.82	1	11.82	11.82
-	2	11.85	11.85
-	3	11.84	11.84
-	4	11.84	11.84
-	5	5.940	11.54
-	6	5.790	11.54
-	7	5.700	11.53
-	8	5.740	11.63
-	9	5.700	11.63
-	10	5.650	11.63

Table 4: Comparison of the Energy Release Rate results between ZFEM and 10th contour of the Abaqus J-integral and corrected J-integral for the problem shown in Figure 5.

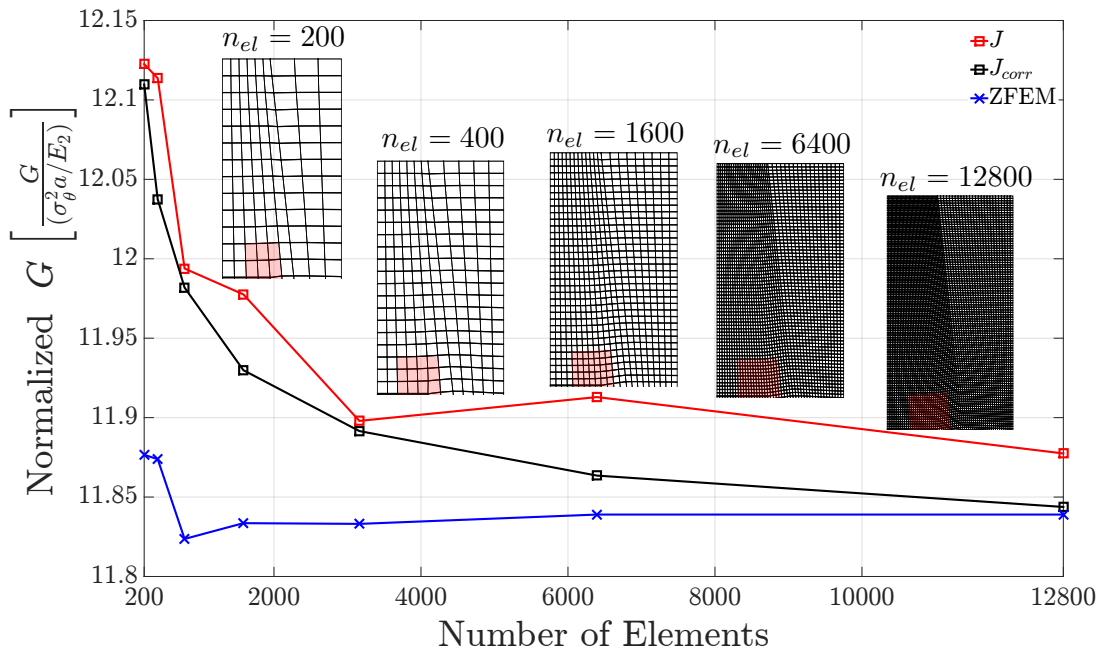


Figure 6: Mesh density convergence of energy release rate for a homogeneous cracked plate.

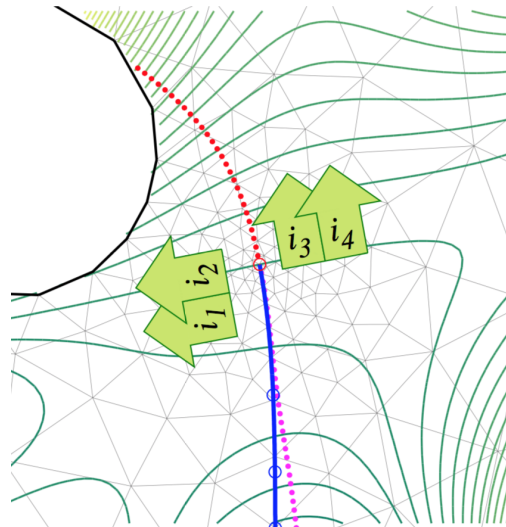


Figure 7: Example of a curvilinear crack path using high order derivatives and a Taylor series expansion.

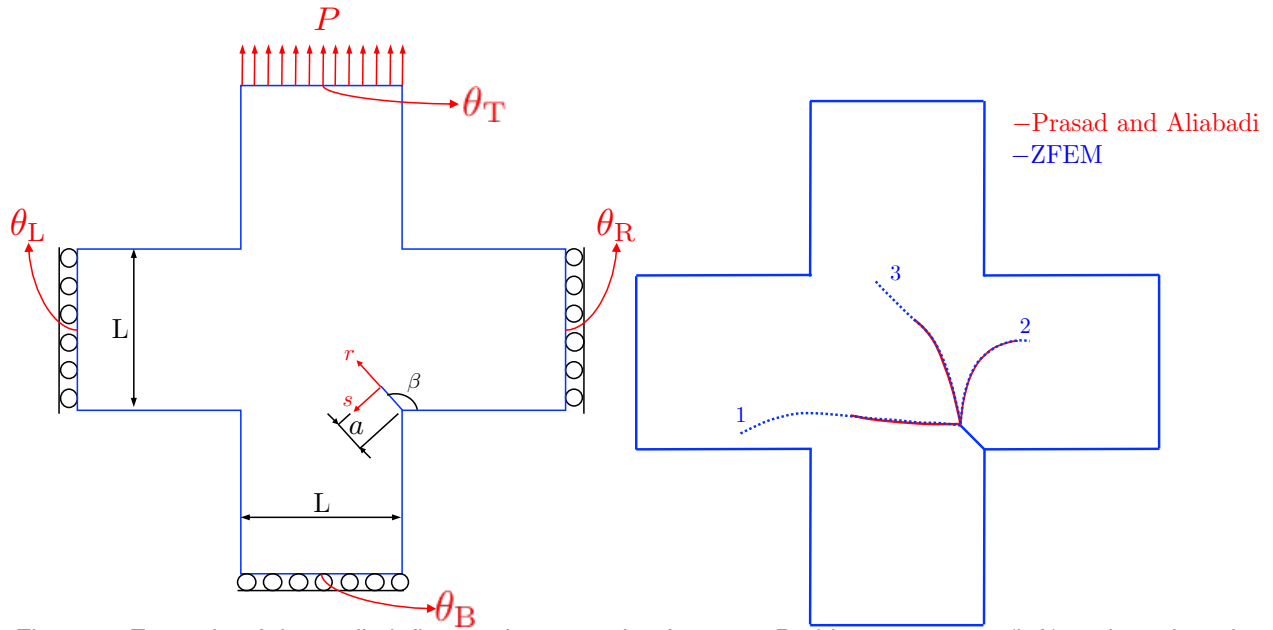


Figure 8: Example of thermally influenced progressive fracture. Problem statement (left) and crack paths comparison (right).

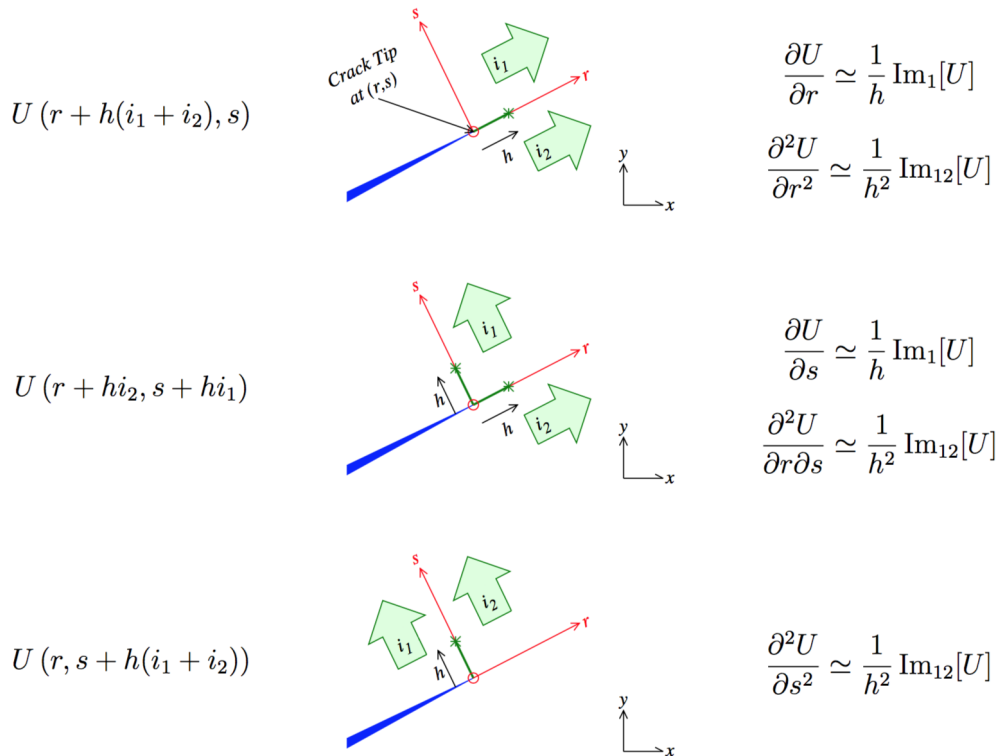


Figure 9: Example of perturbations required for a second order Taylor series expansion.

Name	Description	Name	Description
mcomplex, mdual	Create a multicomplex or multidual number by providing the coefficients	mallocate	Allocate space for arrays of hypercomplex numbers
im, eps	Create a unit value imaginary part for multicomplex or multidual numbers	mchange_order	Increase or decrease number of imaginary coefficients
real	Extract the real part of a number or array	mget	Extract a single hypercomplex number from an array element
aimag	Extract the imaginary part of a number or array	mset	Assign a hypercomplex number to an array element
conj_im	Compute the conjugate of a hypercomplex with respect to the specified imaginary unit	mget_slice	Extract a slice (subarray)
+- * / **	Mathematical operators for addition, subtraction, multiplication, division and power.	mset_slice	Assign values to a slice (subarray)
		mget_row	Extract a row from a matrix
sin, cos, exp, log, sqrt	Elementary functions	mget_col	Extract a column from a matrix
transpose	Transpose of a hypercomplex matrix	shape	Get shape of an array. Number of elements per dimension
matmul	Matrix-matrix and matrix-vector multiplication	size	Get total number of elements of the array
dot_nc	Dot product without conjugation	mto_cr	Convert hypercomplex data structures (numbers, vectors, matrices) to Cauchy-Riemann compatible arrays.
		mcr_to_mcomplex	Convert CR matrix to multicomplex number
		mcr_to_mdual	Convert CR matrix to multidual number

Name	Description
mcr_to_mcxvec	Convert CR compatible vector to multicomplex vector
mcr_to_mduevec	Convert CR compatible vector to multidual vector
mcr_to_mcxmat	Convert CR compatible matrix to multicomplex matrix
mcr_to_mdumat	Convert CR compatible matrix to multidual matrix
get_cr	Compute a CR matrix element at the specified position from a hypercomplex number

Table 5: Functions to handle hypercomplex numbers

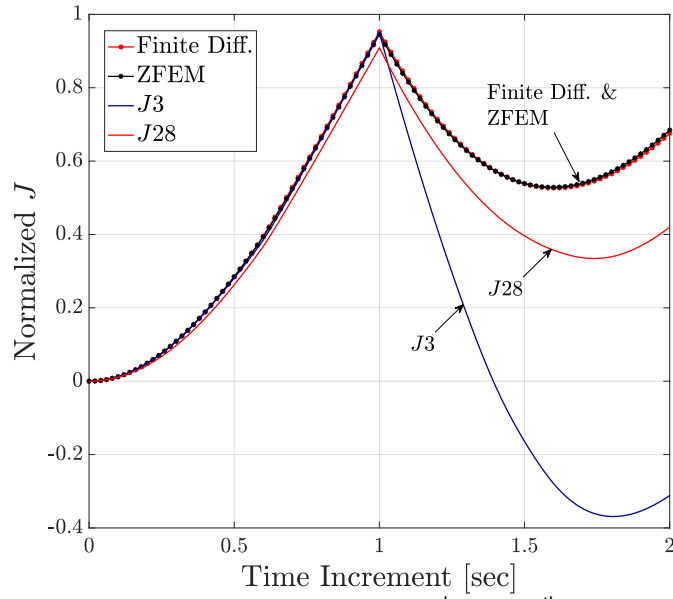


Figure 10: Comparison of ZFEM, Finite Difference and 3rd and 28th contours of Abaqus J-integral for a elasto-plastic analysis.

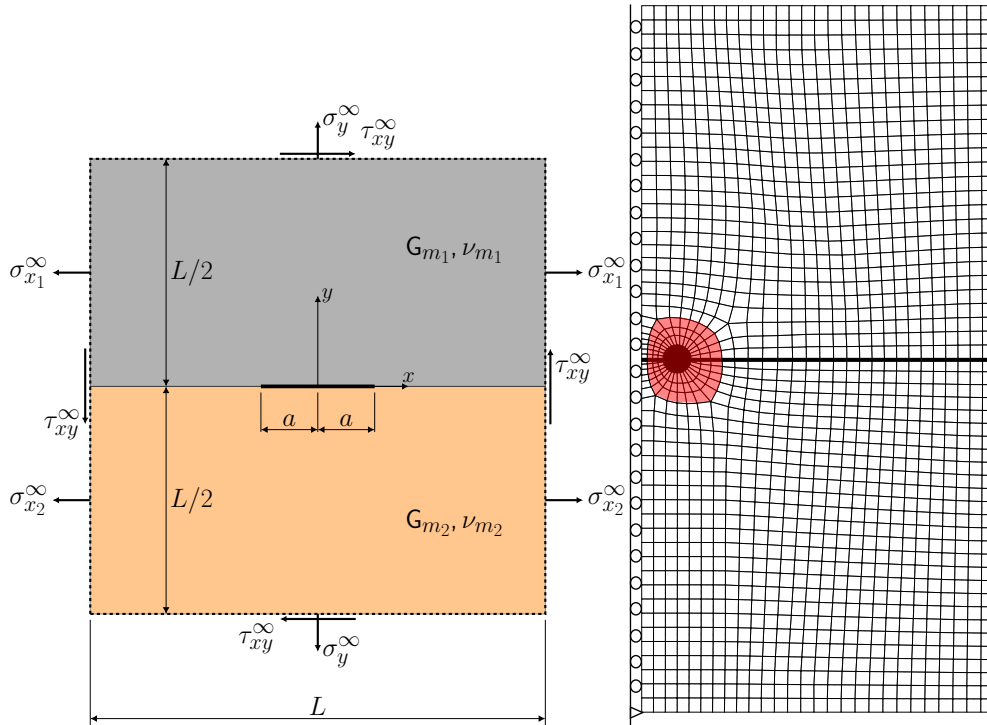


Figure 11: Semi-infinite bimaterial plate with a center crack subjected to remote stresses (left) and corresponding finite element mesh (right). ZFEM perturbed region is indicated by the red-shadowed area.

	ZFEM	Rice and Sih [8]
K_I [MPa \sqrt{m}]	1.795	1.790
K_{II} [MPa \sqrt{m}]	-0.219	-0.217

Table 6: Comparison of the Stress Intensity Factor results between ZFEM and the analytical solution for the problem shown in Figure 11.

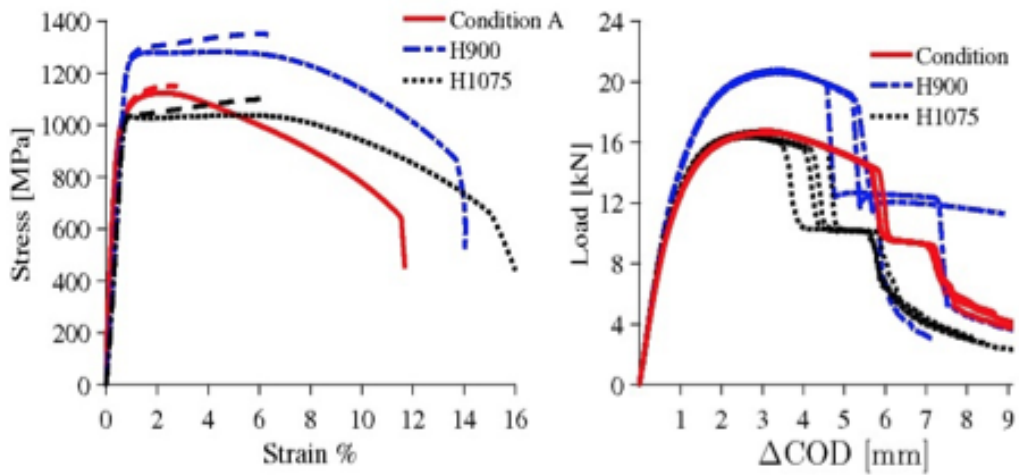


Figure 12: (Left) Stress vs. Strain response from uniaxial tension testing with true stress strain responses up to UTS. (Right) Load vs. COD plot for the modified C(t) samples.

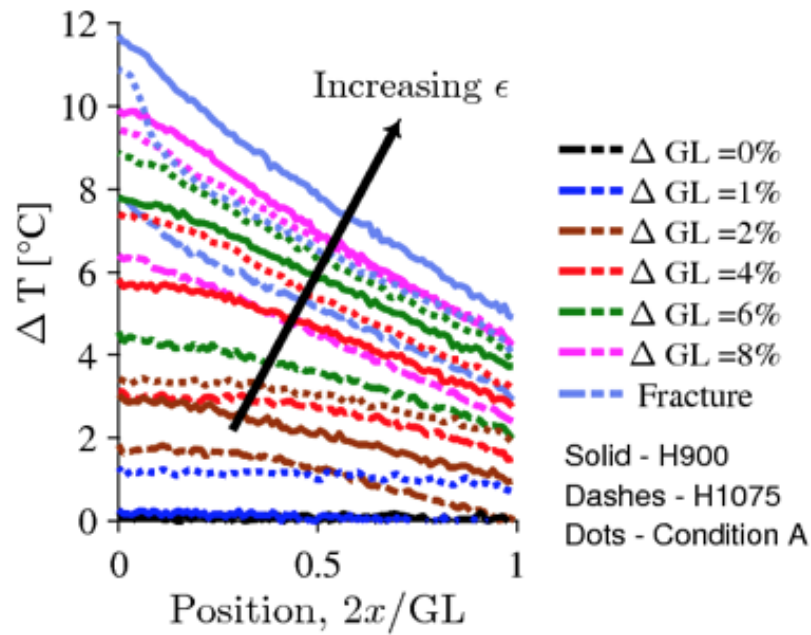


Figure 13: Temperature vs. position for different strains along the gauge length (GL). The GL is 1 inch, so the strain is equivalent to the change in GL. $x = 0$ is the center of the dogbone sample. The temperature is measured along the center of the width of the sample.

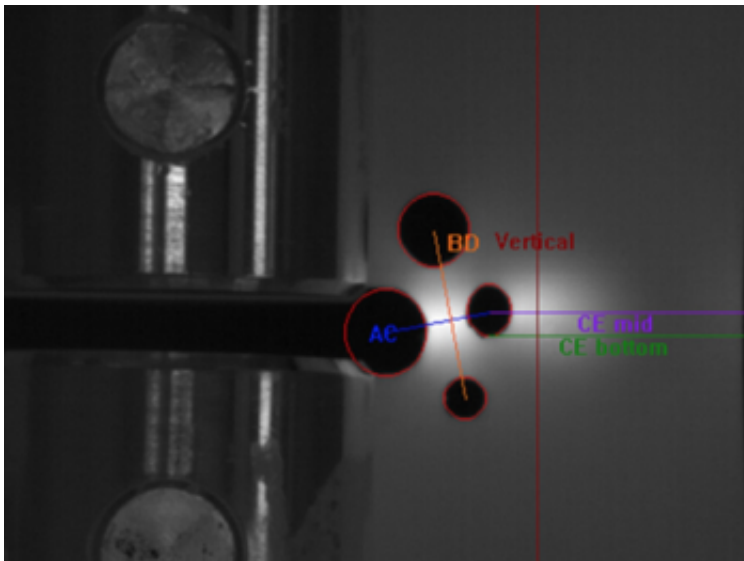


Figure 14: Spatial distribution of temperature is plotted for different COD along each of the shown lines.

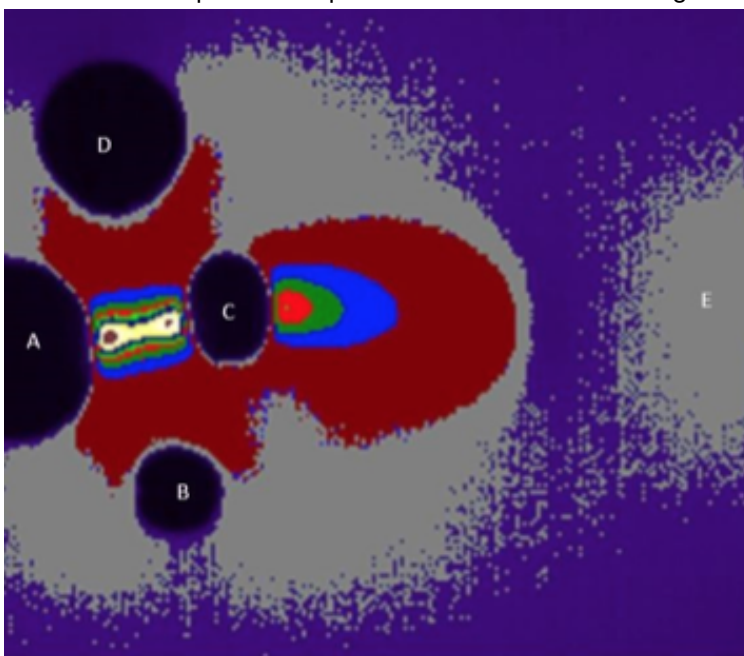


Figure 15: Isotherms for the first fracture event in a H900 modified C(t) sample. The colors represent the following temperature ranges in °C: Brown >80; Yellow-White 55-80; Dark Blue 50-55; Light Green 45-50; Red 40-45; Dark Green 35-40; Royal Blue 30-35; Maroon 25-30; Gray 23.9 – 25, Purple-Black <23.9.