

# REPORT DOCUMENTATION PAGE

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# RPPR Final Report

as of 11-Jan-2019

Agency Code:

Proposal Number: 67906MA

Agreement Number: W911NF-16-1-0041

**INVESTIGATOR(S):**

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**Principal:** Y

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**Report Date:** 03-Apr-2019

Date Received: 09-Jan-2019

**Final Report** for Period Beginning 04-Jan-2016 and Ending 03-Jan-2019

**Title:** Mathematics: Probability and Statistics: Computational Probability: Explorations of Hybrid Symbolic-Probabilistic Mathematics

**Begin Performance Period:** 04-Jan-2016

**End Performance Period:** 03-Jan-2019

**Report Term:** 0-Other

Submitted By: Andrew Glen

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**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

**STEM Degrees:** 7

**STEM Participants:**

**Major Goals:** The major goals, as outlined in the proposal fall into three categories. 1. Improve the computer algebra system called A Probability Programming Language (APPL) in various ways: make it completely open source and free by transferring it to Python (not Maple), beta test the new version in the class room and in research. 2. Use APPL to find new distributions and new properties of probabilistic models in a number of ways: explore the base proportion method, explore the fully enumerated bootstrap, explore ad hoc distributions that are a super set of the triangular distributions. 3. Propagate and advertise the use of APPL to the academic community through publications, presentations, and articles on how to use APPL and what are some of the results from APPL.

**Accomplishments:** In the past year of the project the following accomplishments have been achieved.

In category 1 of the goals:

- A Beta version of APPL has been further beta tested in 2 probability class. The improvements have been added to the Python website on github.
- This version was beta tested by 24 more students in the spring class of MATH313 Probability at The Colorado College. Results of the testing have resulted in updates to APPLPy.

In Category 2 of the goals:

- Research was accomplished by six undergrad students and myself over three summers. The students were research interns study the creation of new families of distributions based on the marginal distribution method and the maximums and minimums method. Also we looked at expansions of the triangular distribution into ad hoc distributions that improve modeling and statistical methods. Also included is work done on fully enumerated bootstrap distributions. This research greatly improves empirical modeling. These results include creating and cataloging over 1000 new families of probability distributions. Also included now is a draft paper on the Ratio of Life Distributions, another way of producing new distributions.
- Further research has been started on the ratios of independent random variables to include exploring their properties and animating their transitions in various video clips.
- Work has begun on a third volume to the Computational Probability series of research monographs. This monograph will be titled Computational Probability: Producing and Assessing New Families of Univariate Continuous Distributions. So far the draft is over 200 pages and reports on over 1000 new families of distributions, as well as an assessment of the utility of each distribution.

In Category 3 of the goals:

- A research paper entitled "SURVIVAL DISTRIBUTIONS BASED ON THE INCOMPLETE GAMMA FUNCTION RATIO" was submitted and accepted at the 2016 Winter Simulation Conference in December of 2016.



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as of 11-Jan-2019

Acknowledged Federal Support: Y

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Publication Location:  
Publisher: Springer  
**Book Title:** Computational Probability: Producing and Assessing New Families of Univariate Continuous Distributions  
**Authors:** Andrew Glen  
**Editor:**  
Acknowledged Federal Support: Y

**CONFERENCE PAPERS:**

**Publication Type:** Conference Paper or Presentation **Publication Status:** 2-Awaiting Publication  
**Conference Name:** Winter Simulation  
Date Received: Conference Date: 11-Dec-2016 Date Published: 11-Dec-2016  
Conference Location: Arlington, VA  
**Paper Title:** SURVIVAL DISTRIBUTIONS BASED ON THE INCOMPLETE GAMMA FUNCTION RATIO  
**Authors:** Andrew Glen, Larry Leemis, Daniel Lockett  
Acknowledged Federal Support: Y

**DISSERTATIONS:**

**Publication Type:** Thesis or Dissertation  
**Institution:** The Colorado College  
Date Received: 30-Aug-2018 Completion Date: 5/1/18 3:18PM  
**Title:** CONTINUOUS RANDOM VARIABLES WITH PIECEWISE, LINEAR DENSITY FUNCTIONS  
**Authors:** Lyujiangnan Ye, Andrew Glen  
Acknowledged Federal Support: Y

**Publication Type:** Thesis or Dissertation  
**Institution:** The Colorado College  
Date Received: 30-Aug-2018 Completion Date: 6/13/18 6:00AM  
**Title:** Fully Enumerated Bootstrap for Convolutions and Other Random Variable Algebraic Operations  
**Authors:** Kelli Sullivan, Pan Gu, Sohair Abdullah, Andrew Glen  
Acknowledged Federal Support: Y

**Publication Type:** Thesis or Dissertation  
**Institution:** The Colorado College  
Date Received: Completion Date: 4/16/19 3:10PM  
**Title:** Ratios of Life Distributions  
**Authors:** Kelli Sullivan, Andrew Glen  
Acknowledged Federal Support: Y

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as of 11-Jan-2019

Nothing to report in the uploaded pdf