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Major Goals: The climate is an extremely complex multi-scale turbulent dynamical system with an extremely large phase space with a large dimension of instabilities which strongly interact and influence the large scale state. Statistical uncertainty quantification (UQ) to the response to the change in forcing or uncertain initial data in such complex turbulent systems in a grand challenge with the fundamental difficulty that by necessity imperfect models (model error) are needed both for lack of physical understanding and the overwhelming computational demands of Monte Carlo simulation with a large phase space, “the curse of ensemble size” or “the curse of dimension”. One natural way to constrain the behavior of imperfect models is to use active data assimilation and data driven methods to constrain the dynamics but these approaches are also hampered by model error and the curse of ensemble size for complex turbulent dynamical systems. Three recently developed mathematical approaches are reported here for attacking these complex dynamical systems.

Accomplishments: I. Improving prediction skill of forced imperfect turbulent dynamical systems through statistical response theory and information theory;
II. Novel algorithms for multi-scale data assimilation in complex turbulent systems;
III. Model error, information barriers, multi-scale data assimilation, prediction, and control.

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NEW STRATEGIES FOR PREDICTION AND DATA ASSIMILATION FOR TURBULENT DYNAMICAL SYSTEMS IN CLIMATE SCIENCE

Award Number: W911NF-15-1-0636
Final Report

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SUMMARY.

The climate is an extremely complex multi-scale turbulent dynamical system with an extremely large phase space with a large dimension of instabilities which strongly interact and influence the large scale state. Statistical uncertainty quantification (UQ) to the response to the change in forcing or uncertain initial data in such complex turbulent systems in a grand challenge with the fundamental difficulty that by necessity imperfect models (model error) are needed both for lack of physical understanding and the overwhelming computational demands of Monte Carlo simulation with a large phase space, “the curse of ensemble size” or “the curse of dimension”. One natural way to constrain the behavior of imperfect models is to use active data assimilation and data driven methods to constrain the dynamics but these approaches are also hampered by model error and the curse of ensemble size for complex turbulent dynamical systems. Three recently developed mathematical approaches are reported here for attacking these complex dynamical systems.

PLAN.

I. Improving prediction skill of forced imperfect turbulent dynamical systems through statistical response theory and information theory;
II. Novel algorithms for multi-scale data assimilation in complex turbulent systems;
III. Model error, information barriers, multi-scale data assimilation, prediction, and control.

PUBLICATIONS.

1. Effective control of complex turbulent dynamical systems through statistical functionals, Proc. Nat. Acad. Sci., 2017.
by Andrew J. Majda and Di Qi

Turbulent dynamical systems characterized by both a high dimensional phase space and a large number of instabilities, are ubiquitous among complex systems in science and engineering including climate, material, and neural science. Control of these complex systems is a grand challenge, for example, in mitigating the effects of climate change or safe design of technology with fully developed shear turbulence. Control of flows in the transition to turbulence where there is a small dimension of instabilities about a basic mean state is an important and successful discipline. In complex turbulent dynamical systems, it is impossible to track and control the large dimension of instabilities which strongly interact and exchange energy, and new control strategies are needed. The goal of this paper is to propose an effective statistical control strategy for complex turbulent dynamical systems based on a recent statistical energy principle and statistical linear response theory. We illustrate the potential practical efficiency and verify this effective statistical control strategy on the forty dimensional Lorenz '96 model in forcing regimes with various types of fully turbulent dynamics with nearly half the phase space unstable.

2. Using statistical functionals for effective control of inhomogeneous complex turbulent dynamical systems, Physica D, 2018.
by Andrew J. Majda and Di Qi

Efficient statistical control strategies are developed for general complex turbulent systems with energy conserving nonlinearity. Instead of direct control on the high-dimensional turbulent equations concerning a large number of instabilities, a statistical functional that characterizes the total statistical structure of the complex system is adopted here as the control object. First the statistical energy equation reduces the control of the complex nonlinear system to a linear statistical control problem; then the explicit form of the forcing control is recovered through nonlocal inversion of the optimal control functional using approximate statistical linear response theory for attribution of the feedback. Through this control strategy with statistical energy conservation, the explicit form of the control forcing is determined offline only requiring the initial configuration of total statistical energy change and the autocorrelation functions in the most sensitive modes of the target statistical equilibrium, with no need of knowing the explicit forcing history and running the complex system. The general framework of the statistical control method can be applied directly on various scenarios with both homogeneous and inhomogeneous perturbations. The effectiveness of the statistical control strategy is demonstrated using the Lorenz '96 system and a turbulent barotropic system with topography and a large number of instabilities.

3. Strategies for reduced-order models for predicting the statistical responses and uncertainty quantification in complex turbulent dynamical systems, SIAM Review, 2018.
by Andrew J. Majda and Di Qi

Turbulent dynamical systems characterized by both a high-dimensional phase space and a large number of instabilities are ubiquitous among many complex systems in science and engineering including climate, material, and neural science. The existence of a strange attractor in the turbulent systems containing a large number of positive Lyapunov exponents results in a rapid growth of small uncertainties from imperfect modeling equations or perturbations in initial values, requiring naturally a probabilistic characterization for the evolution of the turbulent system. Uncertainty quantification (UQ) in turbulent dynamical systems is a grand challenge where the goal is to obtain statistical estimates such as the change in mean and variance for key physical quantities in their nonlinear responses to changes in external forcing parameters or uncertain initial data. In the development of a proper UQ scheme for systems of high or infinite dimensionality with instabilities, significant model errors compared with the true natural signal are always unavoidable due to both the imperfect understanding of the underlying physical processes and the limited computational resources available through direct Monte-Carlo integration. One central issue in contemporary research is the development of a systematic methodology that can recover the crucial features of the natural system in statistical equilibrium (model fidelity) and improve the imperfect model prediction skill in response to various external perturbations (model sensitivity).

A general mathematical framework to construct statistically accurate reduced-order models that have skill in capturing the statistical variability in the principal directions with largest energy of a general class of damped and forced complex turbulent dynamical systems is discussed here. There are generally three stages in the modeling strategy, imperfect model selection; calibration of the imperfect model in a training phase; and prediction of the responses with UQ to a wide class of forcing and perturbation scenarios. The methods are developed under a universal class of turbulent dynamical systems with quadratic nonlinearity that is representative in many applications in applied mathematics and engineering. Several mathematical ideas will be introduced to improve the prediction skill of the imperfect reduced-order models. Most importantly, empirical information theory and statistical linear response theory are applied in the training phase for calibrating model errors to achieve optimal imperfect model parameters; and total statistical energy dynamics are introduced to improve the model sensitivity in the prediction phase especially when strong external perturbations are exerted. The validity of general framework of reduced-order models is demonstrated on instructive stochastic triad models. Recent applications to two-layer baroclinic turbulence in the atmosphere and ocean with combinations of turbulent jets and vortices are also surveyed. The uncertainty quantification and statistical response for these complex models are accurately captured by the reduced-order models with only 2×10^2 modes in a highly turbulent system with 1×10^5 degrees of freedom. Less than 0.15% of the total spectral modes are needed in the reduced-order models.

4. Predicting extreme events for passive scalar turbulence in two-layer baroclinic flows through reduced-order stochastic models, Comm. Math. Sci., 2018
by Di Qi and Andrew J. Majda

The capability of using imperfect stochastic reduced-order models to capture crucial passive tracer statistics such as tracer energy spectrum, tracer intermittency, and eddy diffusivity is investigated. The passive scalar field is advected by a two-layer baroclinic turbulent flow which can generate various representative regimes in atmosphere and ocean. Much simpler and more tractable linear Gaussian stochastic models are proposed to approximate the complex and high-dimensional advection flow equations. The imperfect model prediction skill is improved through a judicious calibration of the model errors using leading order statistics of the background advection flow, while no additional prior information about the passive tracer field is required. A systematic framework of correcting model errors with empirical information theory is introduced, and optimal model parameters under this unbiased information measure can be achieved in a training phase before the prediction. It is demonstrated that crucial principal statistical quantities like the tracer spectrum and fat-tails in the tracer probability density functions in the most important large scales can be captured efficiently with accuracy using the reduced-order tracer model in various dynamical regimes of the flow field with distinct statistical structures. The skillful linear Gaussian stochastic modeling algorithm developed here should also be useful for other applications such as accurate forecast of mean responses and efficient algorithms for state estimation or data assimilation.

5. Stochastic superparameterization and multiscale filtering of turbulent tracers, SIAM-MMS, 2017.
by Yoonsang Lee, Andrew J. Majda, Di Qi

Data assimilation or filtering combines a numerical forecast model and observations to provide accurate statistical estimation of the state of interest. In this paper we are concerned with accurate data assimilation of a sparsely observed passive tracer advected in turbulent flows using a reduced-order forecast model. The turbulent flows which contain anisotropic and inhomogeneous structures such as jets are typical in geophysical turbulent flows in atmosphere and ocean science and passive tracers with a mean gradient can exhibit anisotropic transport with intermittent extreme events as shown below. Stochastic superparameterization, which is a seamless multiscale method developed for large-scale models of atmosphere and ocean models without scale-gap between the resolved and unresolved scales, generates large-scale turbulent velocity fields using a significantly smaller degree of freedoms compared to a direct fine resolution numerical simulation. In a large-scale model of the tracer transport, the tracer is advected by the large-scale velocity field generated by superparameterization with a parameterization of eddies, an additional eddy diffusion given by an anisotropic biharmonic diffusion. The turbulent tracer is sparsely observed in space in only the upper surface layer. These observations naturally mix the resolved and unresolved scales so we develop an ensemble multiscale data assimilation algorithm which provides estimates of the resolved scales using the mixed observations. The reduced-order model is 250 times cheaper than the fine resolution solution and thus enables us to increase the number of ensembles for accurate predictions of state distributions. Numerical experiments show positive results in the estimation of the resolved scales of the tracer as well as in capturing anisotropic intermittent extreme events for the unresolved portions of the tracer field.

6. Preventing catastrophic filter divergence using adaptive additive inflation for baroclinic turbulence, MWR, 2017.
By Yoonsang Lee, Andrew J. Majda, and Di Qi

Ensemble based filtering or data assimilation methods have proved to be indispensable tools in atmosphere and ocean science as they allow computationally cheap, low dimensional ensemble state approximation for extremely high dimensional turbulent dynamical systems. For sparse, accurate and infrequent observations, which are typical in data assimilation of geophysical systems, ensemble filtering methods can suffer from catastrophic filter divergence which frequently drives the filter predictions to machine infinity. A two-layer quasi-geostrophic equation which is a classical idealized model for geophysical turbulence is used to demonstrate catastrophic filter divergence. The mathematical theory of adaptive covariance inflation by Tong et al. and covariance localization are investigated to stabilize the ensemble methods and prevent catastrophic filter divergence. Two forecast models, a coarse-grained ocean code, which ignores the small-scale parameterization, and stochastic superparameterization (SP), which is a seamless multi-scale method developed for large-scale models without scale-gap between the resolved and unresolved scales, are applied to generate large-scale forecasts with a coarse spatial resolution 48×48 compared to the full resolution 256×256. The methods are tested in various dynamical regimes in ocean with jets and vorticities, and catastrophic filter divergence is documented for the standard filter without inflation. Using the two forecast models, various kinds of covariance inflation with or without localization are compared. It shows that proper adaptive additive inflation can effectively stabilize the ensemble methods without catastrophic filter divergence in all regimes. Furthermore, stochastic SP achieves accurate filtering skill with localization while the ocean code performs poorly even with localization.

7. State estimation and prediction using clustered particle filters, Proc. Nat. Acad. Sci., 2017.
by Yoonsang Lee, and Andrew J. Majda,

Particle filtering is an essential tool to improve uncertain model pre- dictions by incorporating noisy observational data from complex systems including non-Gaussian features. A new class of particle filters, clustered particle filters, is introduced for high-dimensional nonlinear systems, which uses relatively few particles compared to the standard particle filter. The clustered particle filter captures non- Gaussian features of the true signal which are typical in complex non- linear dynamical systems such as geophysical systems. The method is also robust in the difficult regime of high-quality sparse and infrequent observations. The key features of the clustered particle filtering are coarse-grained localization through the clustering of the state variables and particle adjustment to stabilize the method; each observation affects only neighbor state variables through clustering and particles are adjusted to prevent particle collapse due to the high-quality observations. The clustered particle filter is tested for the 40-dimensional Lorenz-96 model with several dynamical regimes including strongly non-Gaussian statistics. The clustered particle filter shows robust skill in both achieving accurate filter results and capturing non-Gaussian statistics of the true signal. It is further extended to the multiscale data assimilation which provides the large-scale estimation by combining a cheap reduced-order forecast model and mixed observations of the large- and small-scale variables. This approach enables use of a larger number of particles due to the computational savings in the forecast model. The multiscale clustered particle filter is tested for one-dimensional dispersive wave turbulence using a forecast model with model errors.

8. Multiscale data assimilation and prediction using clustered particle filters, Journal of Computational Physics, 2018.

by Yoonsang Lee, Andrew J. Majda

Multiscale data assimilation uses a coarse-resolution forecast model to increase the number of samples in the estimation of large-scale and long time behavior of high-dimensional complex systems along with noisy incomplete observations. A new class of multiscale particle filters, the multiscale clustered particle filter, is developed here as an effective multiscale data assimilation method for capturing non-Gaussian distributions and extreme events of high-dimensional turbulent systems using relatively few particles. The multiscale clustered particle combines the single-scale clustered particle filter with a general multiscale data assimilation framework that can handle mixed observations of both the resolved and unresolved scale components. To test the multiscale data assimilation method, we use a two-layer Lorenz system having 440 modes with important features of turbulent systems such as non-Gaussian statistics including fat-tails and intermittent extreme events. The effect of the observation model error is investigated and it is shown that the multiscale clustered particle filter captures non-Gaussian distributions using a small number of samples while an ensemble-based method fails to capture the correct distribution.

HONORS AND AWARDS.

Students/Supported Personnel Metrics for this Reporting Period.

Graduate Students: 0
Total Postdocs: 2 (Di Qi and Yoonsang Lee, full time)
Faculty: 0
Undergraduate Students: 0