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RPPR Final Report
as of 01-Nov-2017

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Final Report for Period Beginning 01-Jul-2013 and Ending 30-Jun-2017

Title: An Integrated Experimental and Modeling Study of Ductile Fracture

Begin Performance Period: 01-Jul-2013

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STEM Degrees: 2

STEM Participants: 7

Major Goals: The principal goal of the proposed research is to extend a modeling approach termed the phase-field fracture framework to include the physical behaviors relevant for modeling dynamic ductile fracture and crack evolution. This task will require the inclusion of rate and state dependent plasticity and fracture energy within the model. Additionally, it is critically important that the development of methodology is carried out in conjunction with novel experiments that allow for detailed observations and modeling of the relevant failure mechanisms. Finally, due to the coupling between the fine scales associated with the fracture process and the coarse scales related to the structural dimensions, further work on the solution of the multi-scale phase-field model is needed in order to make quantitative comparisons between the model and experiments. Additionally the last year the research program additional focus was directed towards the fundamental predictions from the phase-field approach for both static and dynamic fracture problems in brittle fracture. This studies are also complemented by experimental observations.

Accomplishments: Please refer to the uploaded document for a detailed description of the Accomplished goals.

Training Opportunities: The training during this research program involved several graduate students along with one junior faculty member that had recently left the group for a faculty position at another university. During the program there were at least two students and on some occasions three students that spent the majority of their research time directly on this project. These students were tasked with running experiments, running numerical simulations, writing code, preparing manuscripts, and delivering presentations at conferences. In addition to these students there were others that spent part of their research time on this project, primarily in a support role. These students had more advanced experience on the numerical methods and the writing of codes for performing the simulations. They acted as mentors for the more junior students and aided in training these students to modify and use the codes more efficiently.

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Results Dissemination: The results of this research have been disseminated to the communities of interest in the usual fashion for academic research including peer-reviewed publications, book chapters, dissertations, and technical progress reports.

Honors and Awards: Chad M. Landis - Elected Fellow of ASME in 2014

K. Ravi-Chandar -

-2014 Aditya Birla Visiting Chair, Indian Inst Science, Bangalore

-2014 Catedra de Excelencia (Chair of Excellence), University of Carlos III, Madrid

-2015 – present, Honorary Professor, University of Carlos III, Madrid

-2015 Daniel C. Drucker Medal, ASME

Thomas J.R. Hughes -

-2013 ACM 2013, Advances in Computational Mechanics, A Conference Celebrating the 70th Birthday of Thomas J.R. Hughes, San Diego, California, February 24-27, 2013.

-2013 World Class University Distinguished Invited Professor, School of Naval Architecture & Ocean Engineering, University of Ulsan, Ulsan, South Korea, March 7-22, May 6-26, and August 9-26, 2013.

-2013 “Isogeometric Analysis,” Raymond D. Mindlin Distinguished Lecture, Department of Civil Engineering and Engineering Mechanics, Davis Auditorium, Shapiro Center, Columbia University, April 25, 2013.

-2013 Doctorate Honoris Causa, University of A Coruña, Spain, June 21, 2013.

-2013 Elected Fellow, Engineering Mechanics Institute, American Society of Civil Engineers, August 2013.

-2013 Fellow, Engineering Mechanics Institute, September 2013.

-2013 “Isogeometric Analysis, Phase Field Modeling and Fracture,” Ted Belytschko Lecture (inaugural annual lecture honoring Ted Belytschko), Northwestern University, November 25, 2013.

-2014 Japanese Association for Computational Mechanics 2014 Computational Mechanics Award

-2014 Election to the Research Board of the Austrian Academy of Sciences

-2014 Wilhelm Exner Medal, Austrian Association für SME

-2014 International Scientific Career Award, Argentinian Association for Computational Mechanics (AMCA)

-2015 Elected to Distinguished Membership in ASCE

-2015 Billy and Claude R. Hocott Distinguished Centennial Engineering Research Award

-2014-2016 Thomas Reuters Highly Cited Researchers

-2015 Alston S. Householder Fellowship

-2015 Special Issue of the international journal Computational Mechanics dedicated to Thomas J.R. Hughes; DOI 10.1007/s00466-015-1162-1

-2015 Named Peter O'Donnell, Jr. Chair in Computational and Applied Mathematics, ICES, UT.

-2015 “Fluid-structure Interaction,” Special Issue of Computational Mechanics, published in recognition of Tom Hughes’s contributions to the broad area of fluid-structure interaction. Comput Mech (2015) 55:1057–1058, DOI 10.1007/s00466-015-1162-1.

-2016 “Isogeometric Analysis: Present, Past, and Future,” Householder Distinguished Seminars, Oak Ridge National Laboratory, Oak Ridge, Tennessee, March 8-9, 2016.

-2016 Slovak University of Technology Medal, October 3rd, -

-2016 Thomas J.R. Hughes delivered the inaugural Ernst Melan Distinguished Lecture in Engineering Science, Vienna University of Technology, Vienna, Austria

-2016 University Co-op Career Research Excellence Award

-2017 Lindbergh Lecture, University of Wisconsin, Madison, February 9th, 2017.

-2017 SIAM/ACM Prize in Computational Science and Engineering

-2017 Southeastern Universities Research Association (SURA) Distinguished Scientist Award

-2017 Identified as Clarivate Analytics Web of Science (formerly Thomson-Reuters Web of Science) Highly Cited Researcher 2017.

-2017 Piola Lecture, First Inaugural Piola Lecture, University of Pavia, Pavia, Italy, September 14, 2017.

Protocol Activity Status:

Technology Transfer: Nothing to Report

PARTICIPANTS:

Participant Type: PD/PI

Participant: Chad M. Landis

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as of 01-Nov-2017

Person Months Worked: 1.00

Funding Support:

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International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Co PD/PI

Participant: Krishnaswamy Ravi-Chandar

Person Months Worked: 1.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Co PD/PI

Participant: Thomas J.R. Hughes

Person Months Worked: 1.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: Y

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Sayedamin Anvari

Person Months Worked: 6.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Khai Pham

Person Months Worked: 6.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Isaac Lee

Person Months Worked: 6.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

RPPR Final Report
as of 01-Nov-2017

Participant: Yusheng Lo

Person Months Worked: 3.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Alexander Constable

Person Months Worked: 6.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Alexandra Woldman

Person Months Worked: 1.00

Funding Support:

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

Participant Type: Graduate Student (research assistant)

Participant: Gary Simpson

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Project Contribution:

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National Academy Member: N

Other Collaborators:

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Title: Characterization and Modeling of Mixed-mode I+III Fracture in Brittle Materials

Authors: Khai Pham

Acknowledged Federal Support: **N**

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Title: A combined experimental and modeling study of low velocity perforation of thin aluminum plates

Authors: Gary F. Simpson

Acknowledged Federal Support: **N**

Project Final Report - Grant # W911NF1310220

(Reporting Period: July 1, 2013 – June 30, 2017)

An Integrated Experimental and Modeling Study of Ductile Fracture

Chad M. Landis, K. Ravi-Chandar, Thomas J.R. Hughes
Aerospace Engineering and Engineering Mechanics, and the Institute for Computational
Engineering and Science
The University of Texas at Austin, Austin, TX 78712

Objective

The principal goal of the proposed research is to extend a modeling approach termed the phase-field fracture framework to include the physical behaviors relevant for modeling dynamic ductile fracture and crack evolution. This task will require the inclusion bulk plasticity and fracture energy within the model. Additionally, it is critically important that the development of methodology is carried out in conjunction with novel experiments that allow for detailed observations and modeling of the relevant failure mechanisms. Finally, due to the coupling between the fine scales associated with the fracture process and the coarse scales related to the structural dimensions, further work on the solution of the multi-scale phase-field model is needed in order to make quantitative comparisons between the model and experiments.

Accomplishments for Reporting Period

During the period of this project there have been three avenues of study that will be described in this report.

Phase-field Modeling of Ductile Fracture

A primary goal of this project is to extend the phase-field approach to fracture to include the phenomenological aspects of ductile fracture. We first note that allowing of plasticity within the constitutive model is relatively straightforward and was included in the first year of this project. However, this simple first step is not truly a phenomenological approach for ductile fracture since the plastic state of the material does not directly affect the local separation process. In other words, such an approach is akin to brittle fracture in the presence of plastic flow as opposed to ductile fracture where plastic deformation and the triaxiality near the crack tip play key roles in the fundamental material separation processes. In order to model the phenomenology of ductile fracture we have included the addition of accumulated plastic deformation and damage quantities within the history function that controls the distribution of the phase-field variable. The results obtained with this approach are promising [1,2]. We also extended these ideas to include accumulated degradation based upon a Johnson-Cook failure criterion, which will allow for the stress triaxiality to properly enter into the phenomenology representing ductile failure within the model. The results of this line of work are described in detail in the publications:

One significant result from this work is illustrated in Figure 1 below where we were able to simulate the fracture and large deformation of a steel plate subjected to an underwater blast loading.

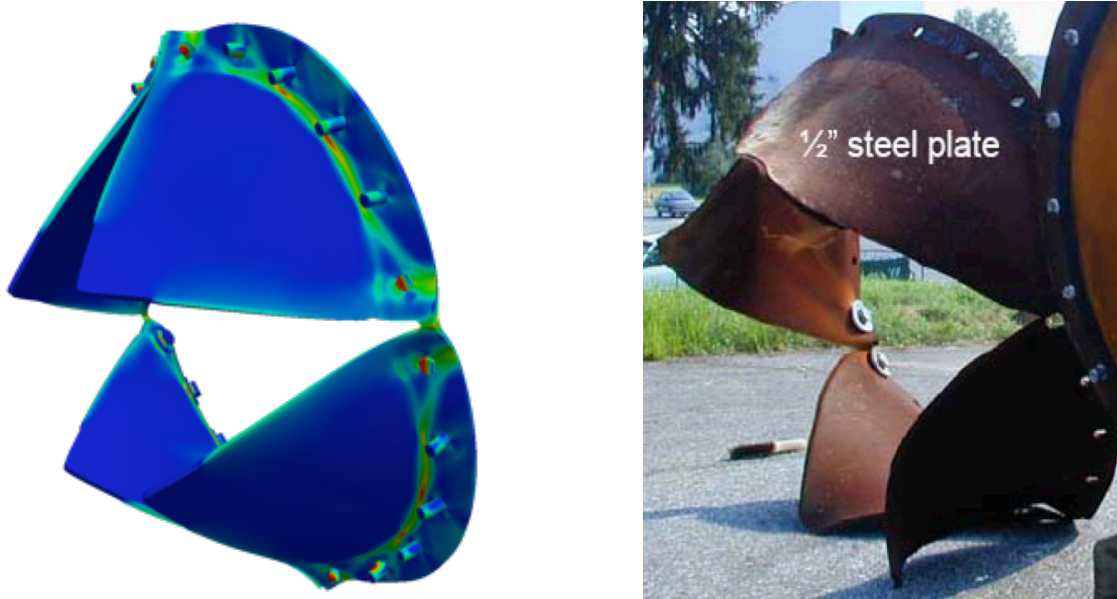


Figure 1. Our phase-field simulation of the fracture of a steel plate subjected to an underwater blast loading. Details are described in Reference (1). The experimental image is from from K.G. Webster, *Investigation of Close Proximity Underwater Explosion Effects on a Ship-Like Structure Using the Multi-Material Arbitrary Lagrangian Eulerian Finite Element Method*, Master's Thesis, Virginia Polytechnic Institute and State University, 2007.

1. M.J. Borden, T.J.R. Hughes, C.M. Landis, A. Anvari, and I.J. Lee, 2016. "A Phase-field Formulation for Fracture in Ductile Materials: Finite Deformation Balance Law Derivation, Plastic Degradation, and Stress Triaxiality Effects", *Computational Methods in Applied Mechanics and Engineering* **312** 130-166.
<https://doi.org/10.1016/j.cma.2016.09.005>

2. M.J. Borden, T.J.R. Hughes, C.M. Landis, A. Anvari, and I.J. Lee, "Phase-field Formulation for Ductile Fracture", in *Advances in Computational Plasticity: A Book in Honour of D. Roger J. Owen*, edited by Eugenio Oñate, Djordje Peric, Eduardo de Souza Neto, and Michele Chiumenti, Springer, Cham, Switzerland, pp. 45-70. <https://doi.org/10.1007/978-3-319-60885-3>

Low Velocity Perforation of Thin Aluminum Plates

This part of our work represents a combined experimental and modeling study of low velocity projectile perforation of thin AA5083-H116 aluminum plates. Experiments were performed in order to characterize the candidate material and calibrate simple and easy to implement empirical models for both the material response and ductile failure behavior. Additionally, the failure behavior and potential damage accumulation of the material were evaluated using an interrupted, multiple loading path strain-to-failure test, validating an augmented Johnson-Cook type failure model for use in numerical simulations. Low velocity ballistic impact and perforation

experiments, investigating several specific mechanisms of deformation and failure, were conducted and modeled by implementing the developed material and failure model in 3D finite element simulations. The results of this line of work are described in detail in Appendix A and are included in the Master's Thesis of Gary Simpson [3]. Figure 2 below illustrates a significant result from this work comparing the simulation results to the experimental observations.

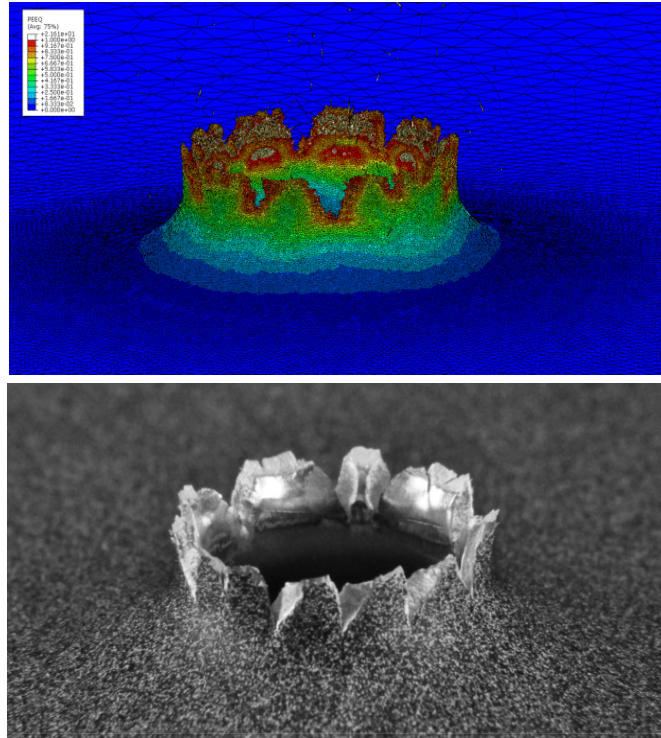


Figure 2. Comparison of petalling observed in experiments with the result of simulations; (impact speed 150 m/s).

3. Simpson, G.F., 2015. “A combined experimental and modeling study of low velocity perforation of thin aluminum plates”, MS Thesis, University of Texas at Austin.
<https://repositories.lib.utexas.edu/handle/2152/34122>

Experimental Verification of Phase-Field Modeling of Brittle Fracture

At the request of program manager, Dr. Asher Rubinstein, we studied the ability of the phase-field approach to capture the solution to the problem of a micro-crack interacting with a large semi-infinite crack under remote K-field loading conditions. We compared the phase-field results to the analytic solution for this problem and found agreement with the LEFM result to approximately 2%. Additionally, we explored a comparison between the stresses ahead of the crack tip from the phase-field approach to those predicted from the Dugdale-Barenblatt cohesive zone description. We contend that this comparison provides insights into the behavior of phase-field models, and that phase-field models have features within their solutions that are quite similar to cohesive zone models.

Verification of the phase-field method and its ability to simulate brittle fracture, has been considered through comparison to the analytical results of Rubenstein (1985). The phase-field model of fracture has been implemented in a parallel simulation framework for simulation of three-dimensional linear-elastic fracture problems. The numerical code has been verified for stationary crack problems by comparison of the crack opening displacement with the analytical linear elastic singular solution and the Dugdale-Barenblatt cohesive model solution. Based on these exercises, it was determined that the scale of the fracture process zone when utilizing the quadratic degradation function is on the order of $8l_0$, and hence the phase-field solutions will deviate from the elastic solutions for a length on the order of $\sim 8l_0$. Mixed-mode I+II crack growth was simulated using the phase-field model and the results for the crack kink angle and the stress-intensity levels at initiation were compared with models based on the maximum tangential stress, maximum energy release rate criterion, and the principle of local symmetry. All the verification exercises generated excellent agreement with the analytical solutions.

Validation of the model was also sought through comparison to experiments. Quasi-static experiments were performed under mode I and mixed-mode loading conditions on compact tension specimens in a thermoplastic polymer, polymethylmethacrylate (PMMA). The material properties were obtained through direct calibration on the same stock of material. While the elastic response was quite repeatable, noticeable scatter was observed in the case of the fracture tests. Calibration of the fracture energy (or fracture toughness) was obtained by fitting the mean trend over multiple experiments, when considering a steadily growing crack. The mode I and mixed-mode I+II experiments were then simulated using the phase-field finite element model. Very good agreement was obtained when comparing the load versus crack opening displacement response as well as the crack position versus the crack opening displacement response; crack paths were also well-predicted by the phase-field model. The results of this line of work are described in detail in Pham et al. [4]:

4. K.H. Pham, K. Ravi-Chandar, and C.M. Landis, 2017. “Experimental Validation of a Phase-field Model for Fracture”, *International Journal of Fracture*, **205** 83-101.
<https://link.springer.com/content/pdf/10.1007/s10704-017-0185-3.pdf>

The validated phase-field model was also used to examine three dimensional crack problems under mixed-mode I + III loading. The phase-field model was explored in the simulation for mixed-mode I+III problems. It was shown that the phase-field model will predict a flat crack path along the original crack plane, even for mixed-mode I+III loading, since along this path the energy release is maximum. But the failure mechanism for specific materials places an energy barrier which does not allow for such crack path. Thus, we explored the introduction of defects and discrete nucleation in these problems [5]. This allows daughter crack patterns to form, continue growing and exhibit shielding effects, and to coarsen as we observed in experiments (see Figure 3). The issue of length scale l_0 in the crack surface representation was also emphasized in these simulations. It appears that unless l_0 is chosen to be extremely small in comparison to the smallest dimension in the problem, the fragmentation of parent crack will not be captured correctly. The results are reported in Pham and Ravi-Chandar [5]

5. K.H. Pham, K. Ravi-Chandar, 2017. “The formation and growth of echelon cracks in brittle materials”, *International Journal of Fracture*, **206**: 229-244. <https://doi.org/10.1007/s10704-017-0212-4>

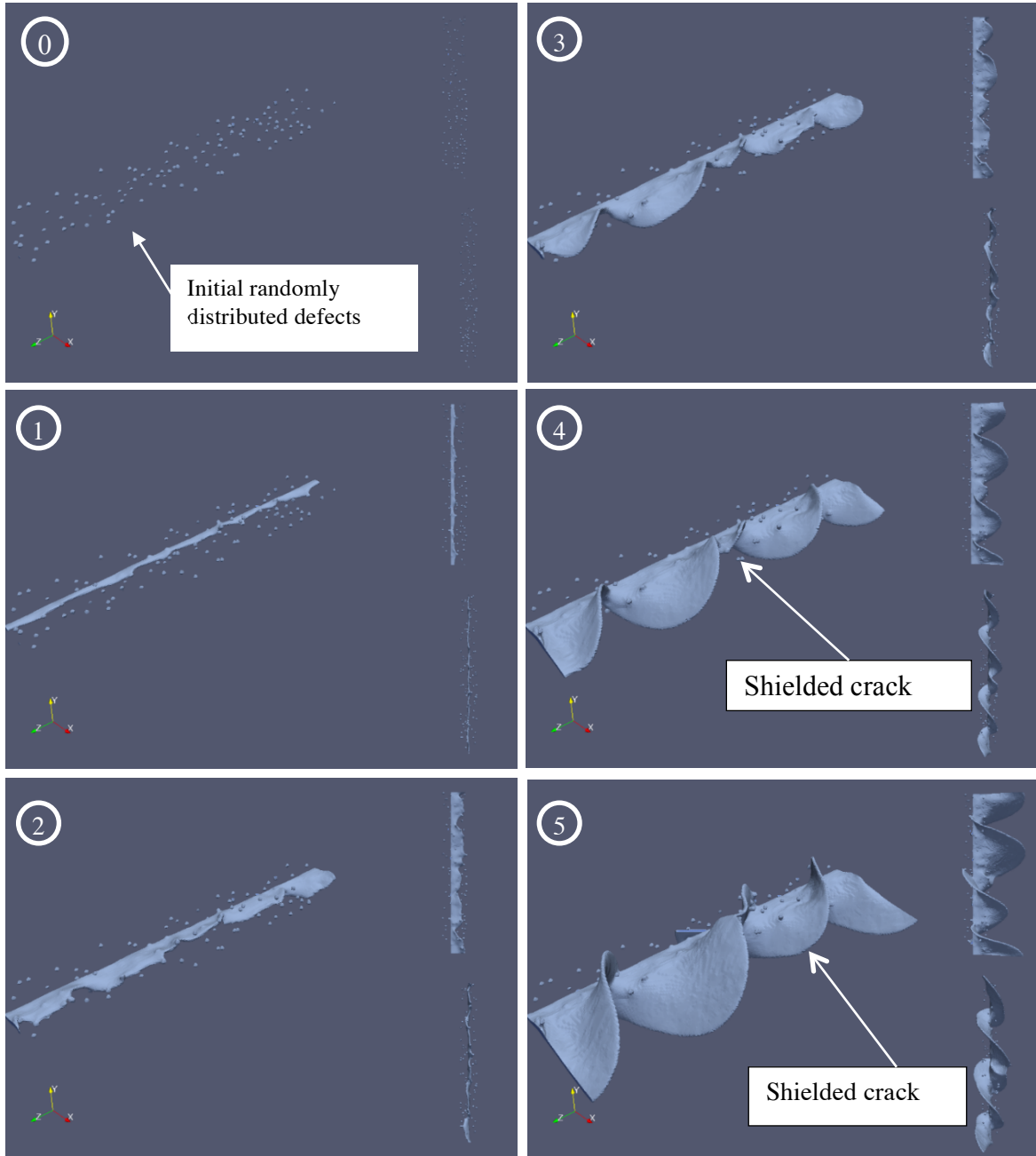


Figure 3. Pure mode III loading with defects. 0.06% of the nodes located inside a tube are assigned a damage value $c=0$ (i.e. fully damaged); the tube axis is parallel to the parent crack front line and its cross-section has the dimensions of $30l_0 \times 20l_0$ centered at the location $x = 5l_0, y = 0$.

Dynamic Fracture and Branching Studies

The results of this part of the research program have yet to be published and so we are providing a more detailed description here than for the other sections described in this report.

Spectacular branching patterns of cracks occur in nominally brittle materials under dynamic loading conditions, and multiple instances of this phenomenon results in fragmentation of the material. As a result, the problem has been investigated by a number of investigators over a very long period (Ravi-Chandar, 2004 and Bobaru and Zhang 2015 for reviews). The physical mechanisms that lead to crack branching are rather well understood: branching arises from two cooperative developments that are governed by the velocity-dependent (i) rearrangement of the stress field, known since the early work of Yoffe (1951) and (ii) the growth of the fracture process zone and consequently the increase in the fracture energy (Ravi-Chandar and Knauss, 1984a). While the underlying energetic rationale for branching was formulated quite a long time ago by Eshelby (1970), analysis of the underlying elastodynamic problem to calculate the dynamic energy release rate has been quite difficult, with only limited success (see Adda-Bedia et al 2005 for the most recent attempts). Numerical simulations of the process have been attempted through many different techniques such as numerical implementations of damage models through element erosion (Johnson 1992), cohesive zone modeling (Xu and Needleman, 1994), in XFEM and other finite element methods (see Song et al. 2008), and lattice- or particle-based methods such as molecular dynamics, peridynamics, etc (see for example, Marder and Gross, 1995, Buehler and Gao, 2006, Bobaru and Zhang 2015). Numerical implementation of the phase-field model of fracture through the finite element method has also been used in dynamic fracture simulations in our recent work (see Borden et al. 2012). While numerical simulations based on all of these methods show the emergence of crack branching, there are two limitations: first, quantitative comparisons to experimental results have not been demonstrated, with all results only implying qualitative similarity and ability for cracks to branch. Second, no systematic analyses have been performed to explore the energetic basis for branching or to incorporate the known evolution of the fracture process zone (and the commensurate increase in fracture energy).

We have investigated the dynamic crack growth branching phenomenon by using the phase-field model of Borden et al. (2012) to simulate, and compare quantitatively, the experiments carried out by one of the PIs in the late 1970's (Ravi-Chandar and Knauss, 1984ab). We have also performed additional experiments both in Homalite-100 (H-100), a brittle polyester, and polymethylmethacrylate (PMMA), a thermoplastic polymer that fails through the formation of a craze. These efforts are described below.

Dynamic Fracture Experiments with the Electromagnetic Loading Device

The electromagnetic loading scheme is based on electromagnetic interaction between two current

carrying conductors (Ravi-Chandar and Knauss, 1982). The loading is accomplished by taking advantage of the electromagnetic interaction between two conductors carrying current. The specimen is made of a large plate (500 mm x 300 mm) of thickness 4.76 mm; a crack is introduced parallel to the long side by machining a 3 mm wide slit down the middle. A natural crack tip is introduced by wedging a razor blade inside the slit and applying a small impact force. A flat copper strip, 4.76 mm by 1.2 mm thick is folded back on itself and the space between the two layers is filled with a mylar insulating strip. This assembly is then introduced into the machined slit as indicated in the schematic diagram in Figure 4a.

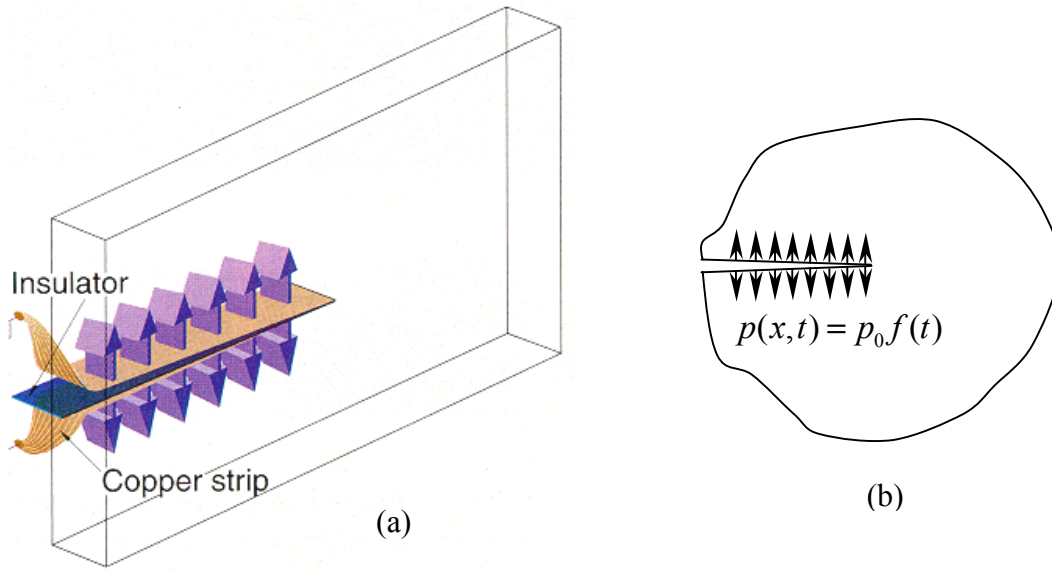


Figure 4. Principle of the electromagnetic loading scheme.

When a current flows through the copper loop, each leg generates a magnetic field surrounding it, with the magnetic field oriented normal to the current vector. The current vector in each leg interacts with the magnetic field of the other leg to produce an electromagnetic repulsion that forces the conductors apart. Since the two legs of the copper strip are confined in the slot of the machined crack, they do not move apart, but simply press upon the top and bottom surfaces of the crack with a uniform pressure. The magnitude of the pressure loading may be estimated easily from electromagnetic theory. The current in the copper-strip is generated by a discharge from a capacitor bank. The time history of the current which dictates the magnitude and duration of the pressure applied on the crack surface may be controlled by suitable choice of capacitors and inductors that form the pulse shaping circuit; Ravi-Chandar and Knauss (1982) generated a nearly trapezoidal pulse, with a rise to the peak amplitude in about 25 μ s, and a total duration of about 150 μ s. For typical values of current used in the experiments, the crack surface pressures were in the range of 1 to 20 MPa. For the large specimen, this loading configuration is equivalent to an infinite plate, with a pressurized semi-infinite crack for the duration of the current pulse. Of

course, waves from the edges of the plate arrive at the crack tip, but this may be delayed for as long as the duration of the loading pulse. The electromagnetic loading provides two major advantages. First, it provides a very repeatable, electrically synchronized loading that makes experiments easy. Second, and perhaps more important, this scheme provides crack surface loads in a configuration that can be modeled as a pressurized semi-infinite crack in an unbounded medium, a configuration that is easily analyzed; Figure 4b indicates the loading configuration.

Summary of Crack Growth and Crack Branching Results from Previous Studies:

A summary of results that were obtained by Ravi-Chandar and Knauss (1984ab) on Homalite-100 is provided in Table 1.

Table 1. Summary of results from Ravi-Chandar and Knauss (1984ab), indicating time to initiation, crack speed, and details of branching.

Test #	P_0	Time to initiate	Crack speed	Branching at	Time at branching	Number of branches	Top branch angle	Bottom branch angle	Mid branch if exists	Post branching speed	Interior branch top	Interior branch bottom
	MPa	μs	m/s	mm	μs							
1	0.63	102	240			0						
2	1.1	56	240			0						
3	5.5	20	410	50	140	2						
4	9.86	18	432	21	64.0	2	25	32	No	~440		
5	10.4	18	457	14	45.0	2	30	30	No	440		
6	15.4	14	487	5	25.0	4	30	50	2	440	15	15

An example of an observed branching pattern is shown in Figure 5. It should be noted that branching occurred in Test #3 upon arrival of reflected waves, while in Tests #4-#6, branching occurred well before the arrival of reflections. This should be taken as conclusive proof that wave reflections are not necessary for crack branching, and that dynamic cracks can spontaneously branch; this is worth reiterating, because while this result has been known for more than three decades, more recent works repeatedly cite the role of wave reflections on crack branching. Also, the number of branches that occurs increases with increased magnitude of the applied loading. We have explored this in greater detail through a new set of experiments both in H-100 and PMMA, as reported in the next Section. There are two other observations from the experiments that are important to note. The crack speed prior to branching in H-100 is in the range of $0.40C_s$ to $0.45C_s$, somewhat below the Yoffe threshold of about $0.60C_s$. Furthermore,

the crack speed of the branched cracks is almost unchanged from that of the main crack. These observations have implications on the energetic criterion for fracture (Freund, 1990; Ravi-Chandar, 2004) and for branching (Eshelby, 1970) and are being explored further through the phase-field simulations.

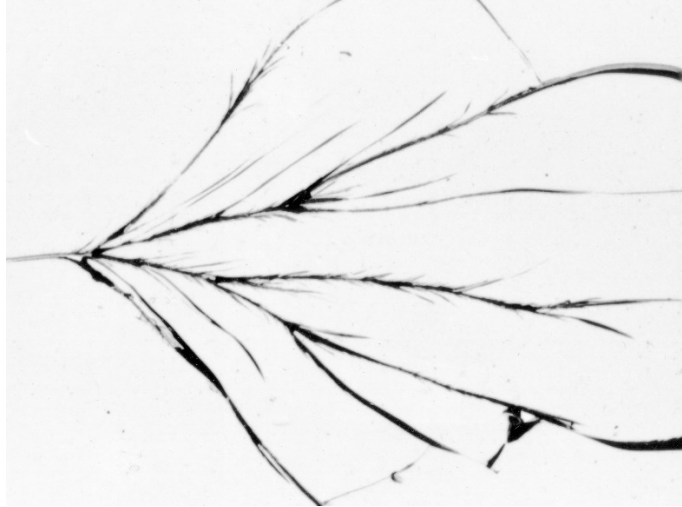


Figure 5. Pattern of Crack Branching for Test #6. Note that four cracks emerge from a single crack.

New Experiments on Crack Growth and Crack Branching in H-100 and PMMA:

New experiments were performed on both H-100 and PMMA, with the same electromagnetic loading device used by Ravi-Chandar and Knauss (1984ab). Additional experiments were also performed on the PMMA in a single-edge-notched (SEN) configuration with an unstable geometry triggering dynamic crack growth. Only the former results are reported here; the SEN experiments are still being analyzed and will be reported through archival publications. In these experiments, while the electromagnetic loading scheme was the same as in the earlier work of Ravi-Chandar and Knauss (1984ab), a modification of the copper-strip through a doubling of the coils was used to increase the loading on the crack surfaces. Table 2 indicates a summary of the tests performed to explore crack growth and branching. In addition, the method of digital gradient sensing (DGS) developed by Periasamy and Tippur (2012) was used in this work. The method relies on measuring the light ray deviations that occur in the vicinity of the crack tip in imaging a speckle pattern and inverting it to determine the dynamic stress intensity factor. A Photron FASTCAM SA1.1 was used to capture the time evolution of the crack tip and the deformation. A framing rate of 90,000 frames per second was used; this however, resulted in an image that contained only 384 x 128 pixels with a resolution of 0.228 mm per pixel. As a result, while the method of was able to resolve the crack tip deformation field under lower load levels, correlation failed at higher levels of loading approaching crack branching. These experiments would have to be performed with a higher spatio-temporal resolution that is now becoming available with newer generations of high-speed cameras.

The key results obtained are summarized in Figures 6-9. Figure 6 shows both the raw images from the high speed imaging, and the DGS processed data indicating the crack direction deviations of the light beam for Test #C from H-100 specimen. The crack surface pressure level in this experiment was 15 MPa. The contour plots indicate $\partial\sigma_{xx}/\partial x_1$ and could be used to extract the dynamic stress intensity factor. But due to the limited spatial resolution, the errors in such extraction turned out to be large. The corresponding crack position data is shown in Figure 7. For the H-100 specimens, the crack speeds were nearly constant at about 450 m/s, reproducing the earlier results from Ravi-Chandar and Knauss (1984ab). The first crack branching (into three crack branches) occurred at 100 μ s, prior to the arrival of wave reflections. The second branching of the main crack occurs at about 210 μ s, again into three branches. The post-branching crack speed is nearly indistinguishable from the speed prior to branching; this has significant implications with respect to energy-based criterion for crack branching and needs to be examined further through the results of the phase-field simulations.

Table 2. Partial list of experiments on PMMA and H-100, with DGS instrumentation.

Test Number	Test Date and Material	Charge Voltage	p_0	Time to initiate	Crack speed (fastest branch)	Branching at	Time at branching (from initial loading)	Number of branches
		kV (# Cu turns)	MPa	μ s	m/s	mm	μ s	
A	08/03 PMMA	8 kV (2 turns)	39	~10	795	2.27	<20	5
B	08/04 PMMA	5 kV (3 turns)	35	66.6	782	4.37	77.7	2
C	08/15 H100	5 kV (2 turns)	15	45	450	14.37	<100	5
D	08/21 H100	8 kV (2 turns)	39	50	448	8.9	70	12
E	08/24 H100	6 kV (2 turns)	22	50	409	2.26	80	13

Figures 8 and 9 present results for Test #A from a PMMA specimen. The crack surface pressure level in this experiment was 39 MPa. First branching, this time into five cracks, occurs at about 20 μ s (one of these was found to be a partial branch that arrested quickly). The second branching

of the main crack occurs just around $\sim 40 \mu\text{s}$, into three branches. The other branches from the first event leave the field of view and are not seen in the high-speed image; however, after recovering and reassembling the fragments from the specimens, it was found that all the branches from the first branching event exhibit further branching at about the same distance from the initial branch. An image of the reassembled specimen is shown in Figure 10. Further analysis of these results is needed to connect these measurements to the phase-field simulations and to the elastodynamic theory.

From the collection of experimental results, based on the previous tests summarized in Table 1, and the new experiments listed in Table 2, we reach the following strong conclusions:

1. Crack speed reaches a limit of about $0.45 C_s$ in H-100 and significantly larger value of $0.76 C_s$ in PMMA. This limit is lower than the Yoffe limit for H-100, but larger for PMMA.
2. While wave reflections may trigger crack branching, and influence its further propagation, branching can and does occur simply from a dynamic crack growing at speeds in the range of about $0.45 C_s$ in H-100, and $0.76 C_s$ in PMMA. Models of crack growth have to be able to predict such onset of branching.
3. The crack branches grow at speeds that are nearly unchanged from the speed of the parent crack. This brings to question crack branching criteria that are based either on energy flux or on critical crack speeds.
4. We now have a large database of experiments that can be used as benchmarks for testing with simulations based on different numerical methods such as phase-field or peridynamics.

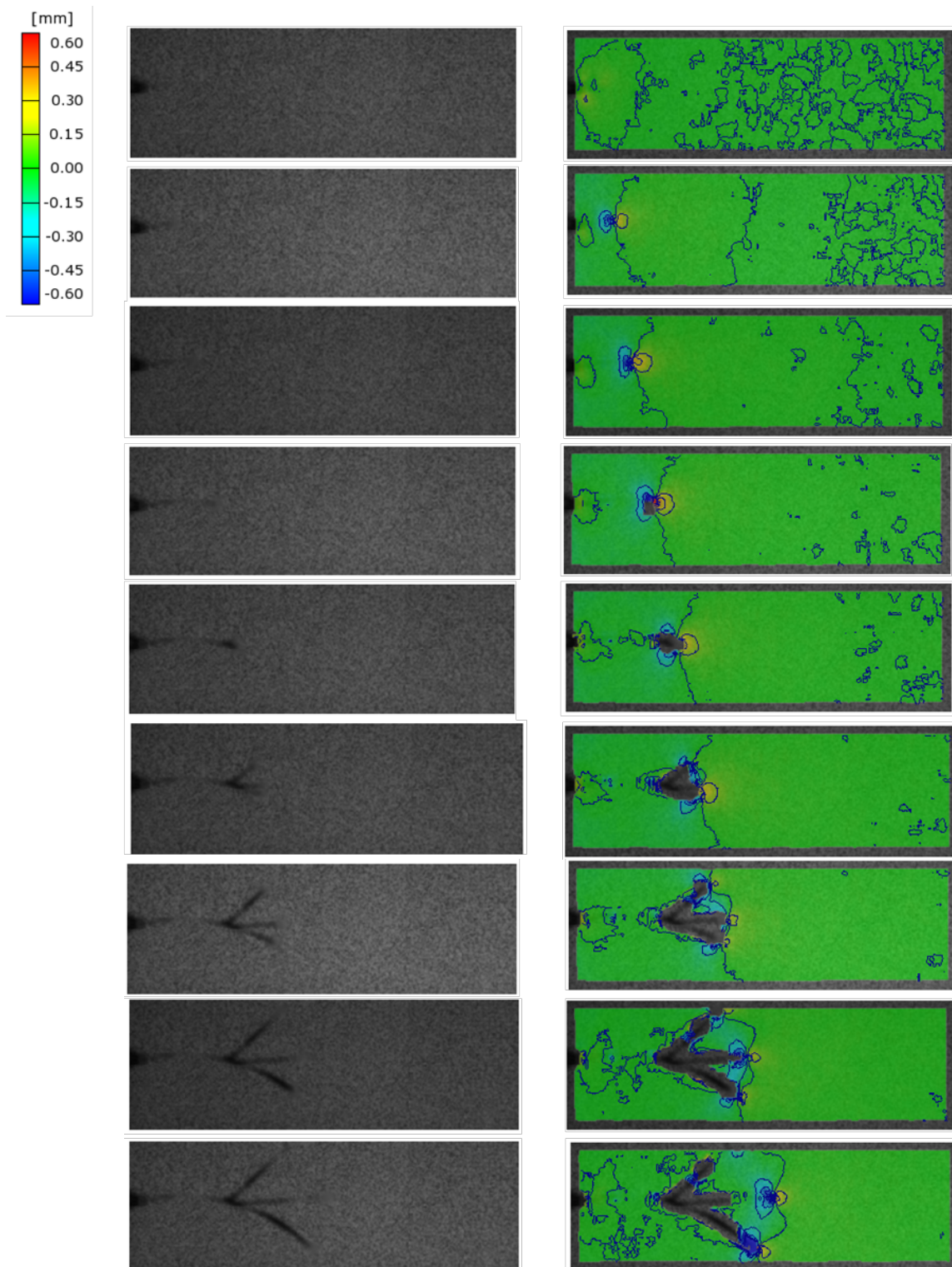


Figure 6a. (see next page for caption)

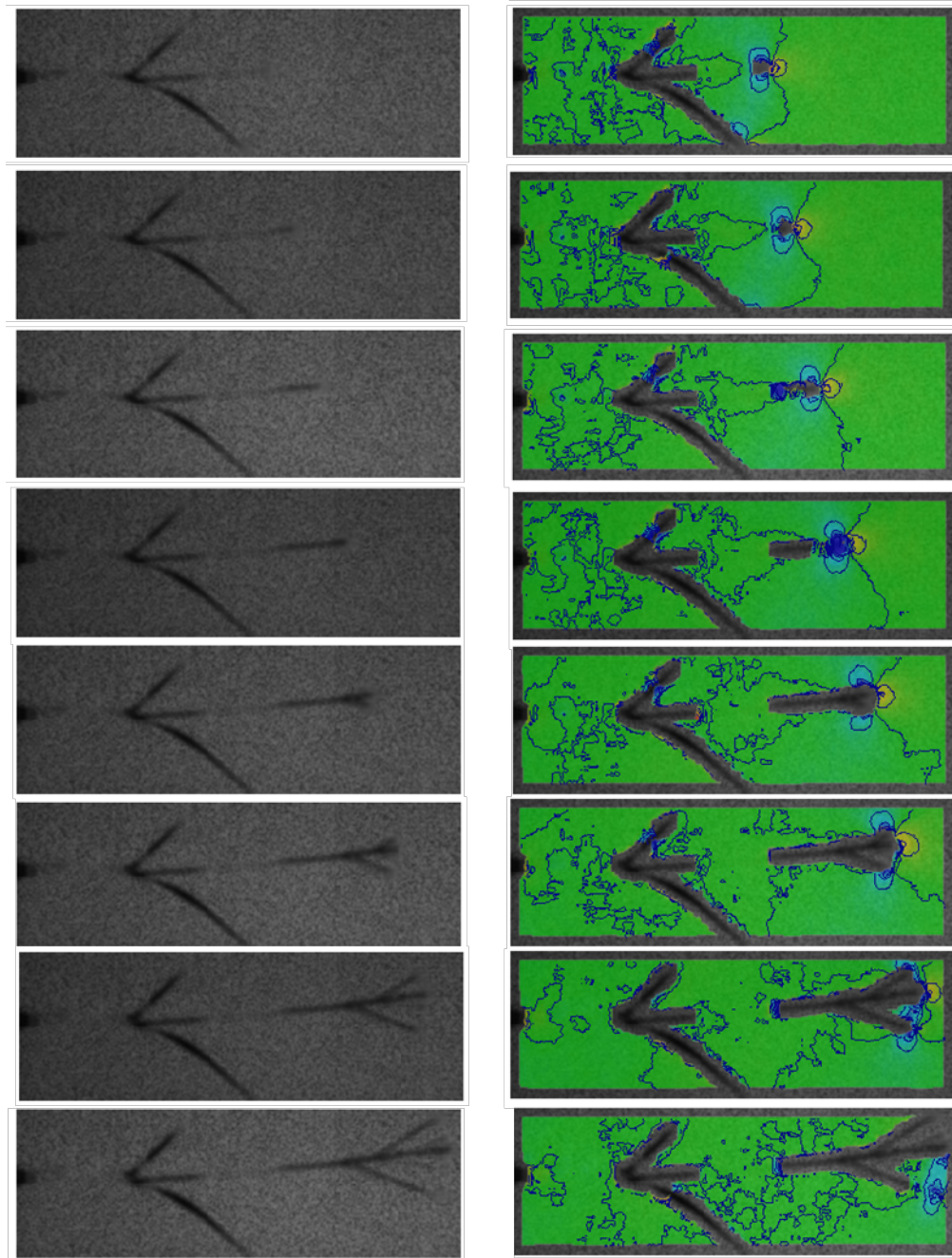


Figure 6b. Selected sequence of images (left column) every 11 μs apart from H-100 Test #C. First branching into three cracks occurs at about 100 μs , prior to the arrival of wave reflections. The second branching of the main crack occurs at about 210 μs , again into three branches. The right column images show the DGS analysis of the deformation field. Due to limited spatial resolution in the images, the fields in the vicinity of the crack tip and branches could not be resolved to obtain quantitative estimates of the dynamic stress intensity factor. The variation of the crack position with time is shown in Figure 8.

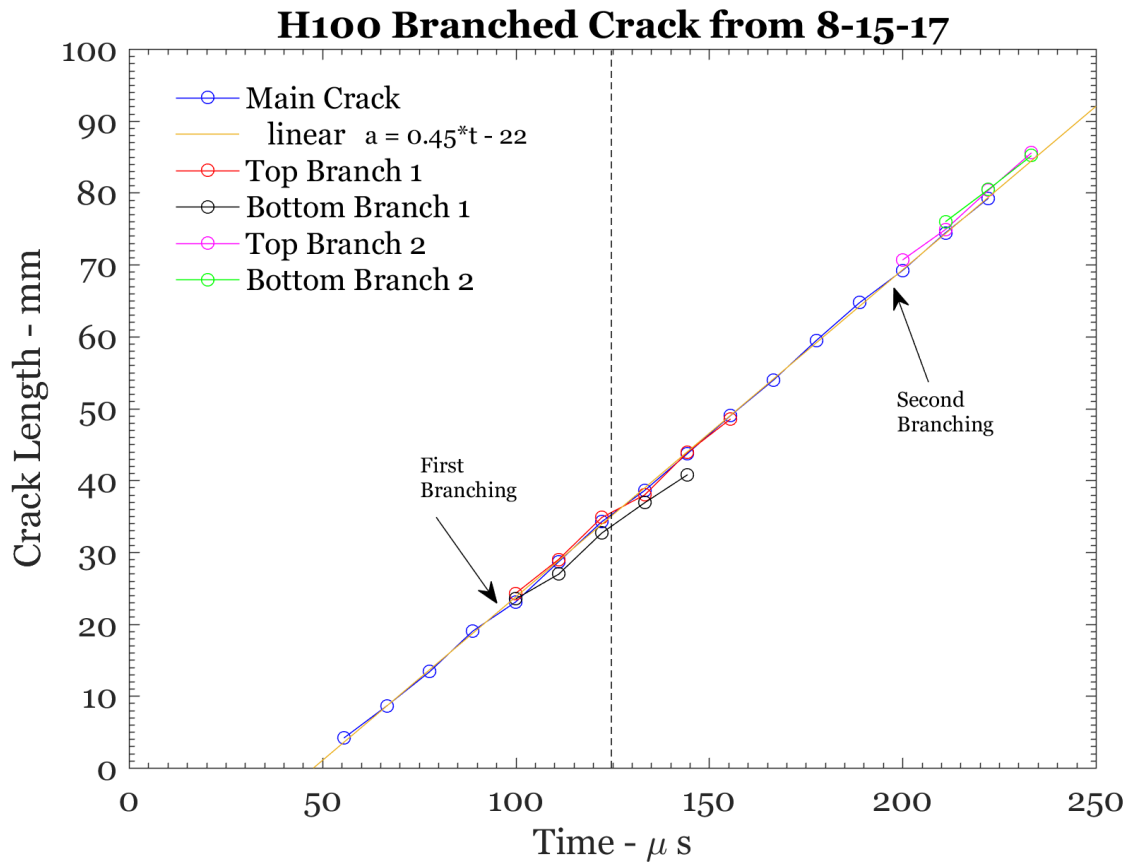


Figure 7. Crack position versus time for Test #C on H-100; raw images are shown in Figure 7. Note that the slope of these lines do not change significantly immediately after crack branching, indicating no velocity changes upon branching. Note that the trend line shown through the main crack position data indicates a nearly constant crack speed of about 450 m/s.

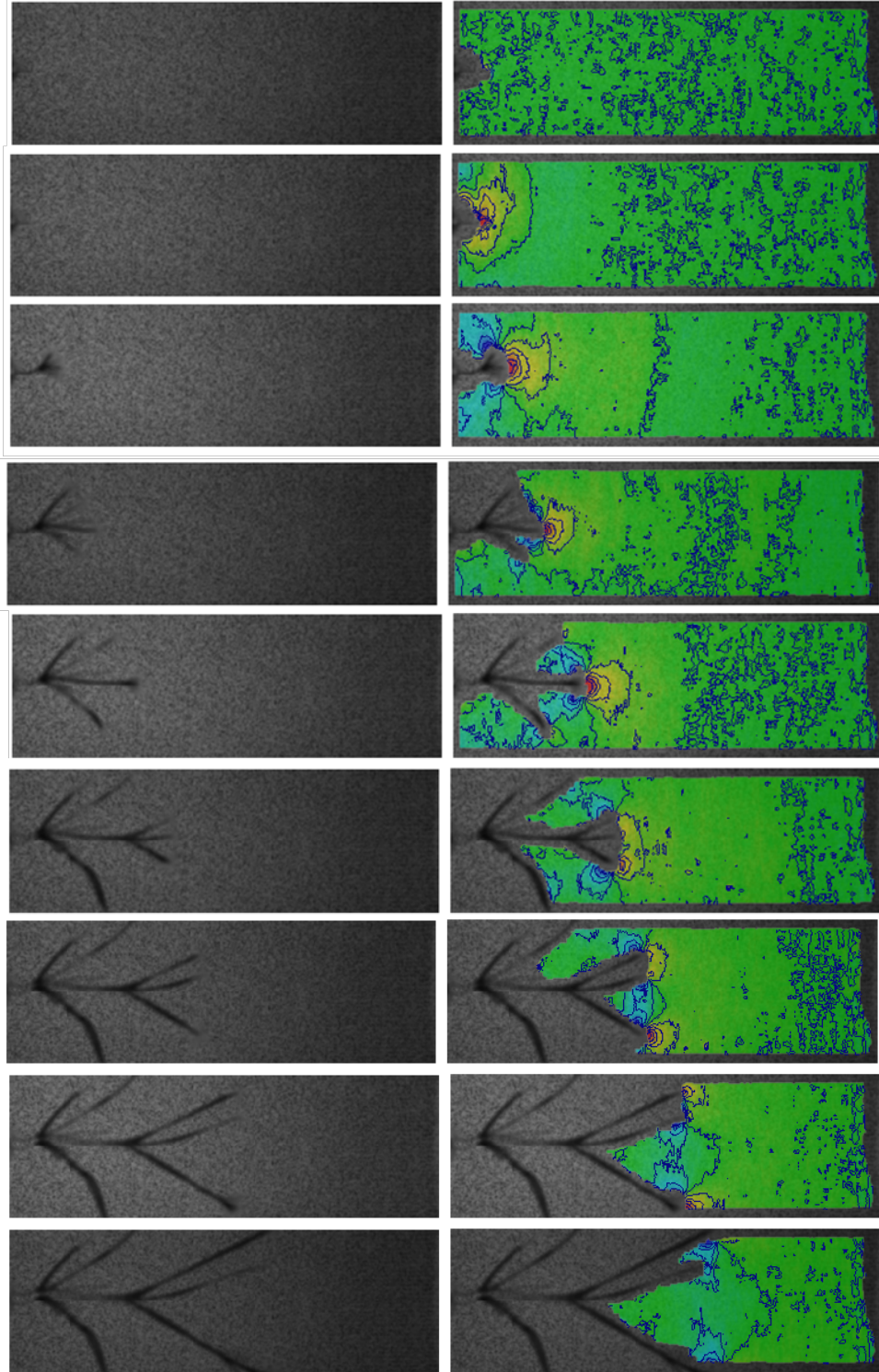


Figure 8. Selected sequence of images (left column) every $11 \mu\text{s}$ apart from PMMA **Test #B**. First branching (four cracks) occurs at about $20 \mu\text{s}$. The second branching of the main crack occurs after $\sim 40 \mu\text{s}$, into three branches. The right column images show the DGS analysis of the deformation field. The variation of the crack position with time is shown in Figure 10.

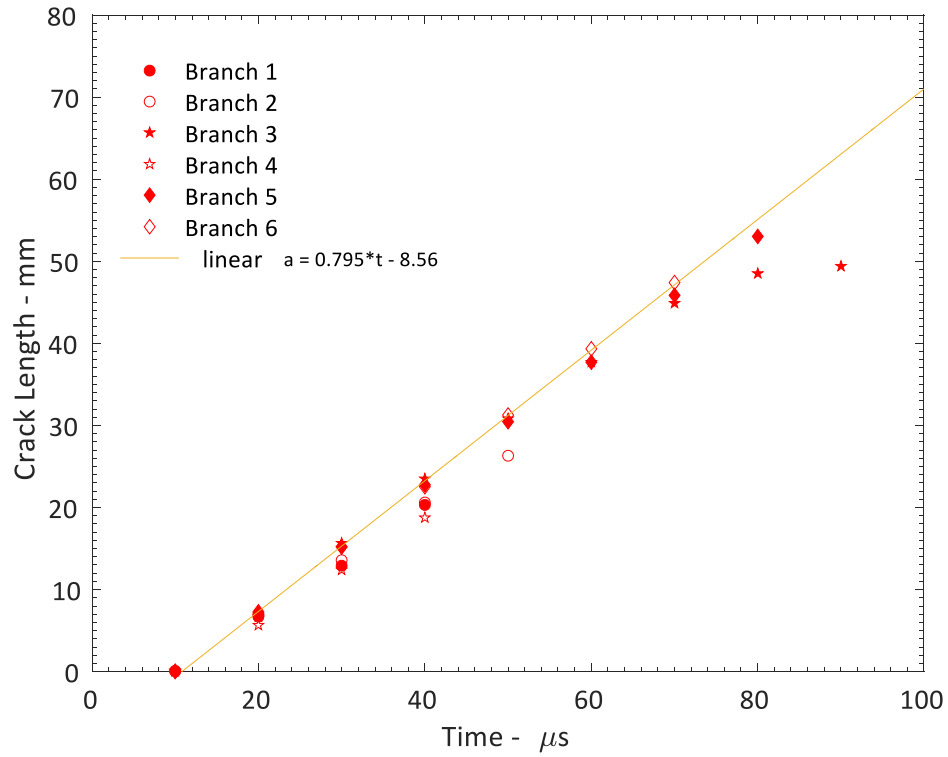


Figure 9. Crack position versus time for Test #B on PMMA; raw images are shown in Figure 9. Note that the trend line shown through the main crack (Branch 3) position data indicates a nearly constant crack speed of about 795 m/s.

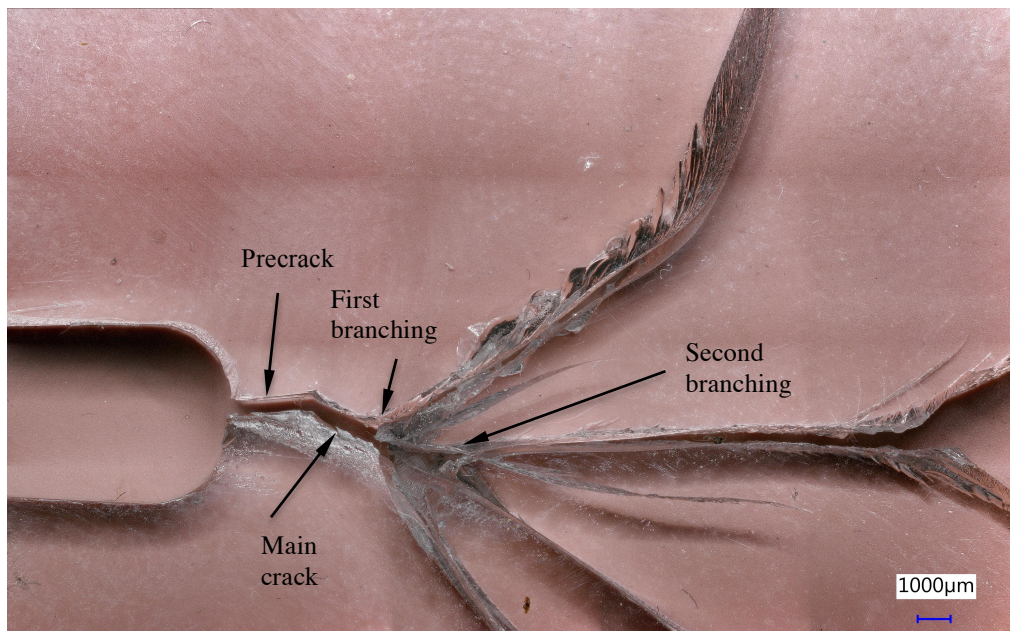


Figure 10. Crack path for Test #A on PMMA indicating the precrack, and the branching patterns.

Phase-Field Simulations of Dynamic Crack Growth and Branching in H-100:

The most basic form of the governing equations used to represent dynamic brittle crack propagation within the phase-field crack evolution modeling framework are given as,

Newton's second law: $\sigma_{ji,j} + b_i = \rho \ddot{u}_i$ in V and $\sigma_{ij} = \sigma_{ji}$ in V , and $t_i = \sigma_{ji} n_j$ on S

Micro-force balance: $\xi_{i,i} - \eta = 0$ in V , and $\lambda = \xi_i n_i$ on S

Constitutive behavior: $\sigma_{ij} = \frac{\partial \psi}{\partial \varepsilon_{ij}}$, $\xi_i = \frac{\partial \psi}{\partial \mu_{,i}}$, and $\eta = \frac{\partial \psi}{\partial \mu}$

Kinematics: $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

Free energy definition: $\psi = \frac{g(\mu)}{2} c_{ijkl} \varepsilon_{ij} \varepsilon_{kl} + G_c \left[\frac{(1-\mu)^2}{4l_0} + l_0 \mu_{,i} \mu_{,i} \right]$

Here we have used standard index notation. The symbols represent the Cartesian components of stress σ_{ij} , body force b_i , density ρ , displacement u_i , unit vector normal to the surface n_i , surface traction t_i , micro-force vector ξ_i , scalar micro-force η , strain ε_{ij} , phase-field parameter μ , and its gradient $\mu_{,i}$. The material properties include the elasticity tensor c_{ijkl} , the fracture surface energy G_c , and the characteristic fracture surface length scale l_0 . Regions of material where the phase-field parameter μ is close to zero are damaged/fractured and regions where $\mu \approx 1$ are intact. The transition from fractured to intact regions occurs over the length l_0 , which can also be interpreted as a process zone length scale. We note that these equations can also be obtained via requiring the Lagrangian to be stationary. Additionally, extensions have been made to ensure that fracture surfaces are created only in tension, and to include the effects of finite deformation and plasticity for ductile fracture. Greater detail of the modeling approach and the numerical solution of the model equations can be found in our recent paper [1].

Simulations were performed using the phase-field method implemented for dynamic problems using the formulation discussed in Borden et al. (2012). The geometry shown in Figure 5a was used with specimen dimensions of 500 mm x 300 mm. The crack surface was loaded by a pressure loading of amplitude p_0 , with a rise time of 25 μ s, and a total duration of 150 μ s, matching the loading used in the experiments. The material properties used for the H-100 are as follows: $E = 4.55$ GPa, $\nu = 0.31$, $\rho = 1230$ kg/m³. The fracture toughness was taken to be $K_C = 0.5$ MPa m^{1/2}; this corresponds to a fracture energy of $G_c = 50$ J/m². The length scale l_0 was taken to be 0.5mm. The phase-field degradation model was taken to be of the form: $g(\mu) = \mu^2$. The numerical calculations use a mesh of T-splines with 830,829 bi-cubic basis functions. The minimum discretization size for the elements around the crack is $0.3l_0$, and the time step used for the simulation was 0.55 μ s. The simulation covers 200 μ s and took

approximately 24 hours to compute on 4 computational nodes on the Stampede 2 supercomputer at UT Austin, <https://www.tacc.utexas.edu/systems/stampede2>.

Table 3. Summary of results from simulations to be compared to the experimental results indicating time to initiation, crack speed, and details of branching in Table 1.

Test #	p_0	Time to initiate	Crack speed	Branching at	Time at branching	Number of branches	Top branch angle	Bottom branch angle	Mid branch if exists	Post branching speed	Interior branch top	Interior branch bottom
	MPa	μs	m/s	mm	μs				μs			
1	0.63	-	-			0						
2	1.1	55	300*			0						
3	5.5	20	480	50	140	2	25	25	170			
4	9.86	16	447	17	55	??	25	25	??	500 - 580	-	-
5	10.4	15	441	13	48	2	25	25	150	515 - 580		
6	15.4	12	446	7	32	2	30	30	150	530 - 600		

One selected result, from a simulation corresponding to Test #5, will be described in detail. The results of other simulations are provided in Table 3. Figure 11 shows the growth of a dynamic crack in Test #5, by showing the contours of the phase-field parameter, μ . Clearly, the initiation and straight extension of the crack followed by two levels of crack branching are seen in the simulations. The time at which crack growth is initiated, the speed of the crack growth, the instant at which branching occurs, and the branching angle are all captured reasonably well by the simulations for this case. The post-branching crack speed is somewhat greater than that observed experimentally. Similar agreement was observed for all other simulations corresponding to the other tests in Table 1. It should be noted that no special criterion was imposed in generating this crack growth behavior, other than introducing the phase-field model for damage/fracture. The good quantitative agreement between the experiments in Table 1 and simulations in Table 3 suggests that the damage model contains the essential ingredients necessary to capture dynamic fracture evolution in brittle materials. Crucial parameters concerning the development of damage, flow of energy into the crack tip region, influence of the intrinsic length scale l_0 and the degradation function $g(\mu)$ in dictating the development of crack branching need to be examined further.

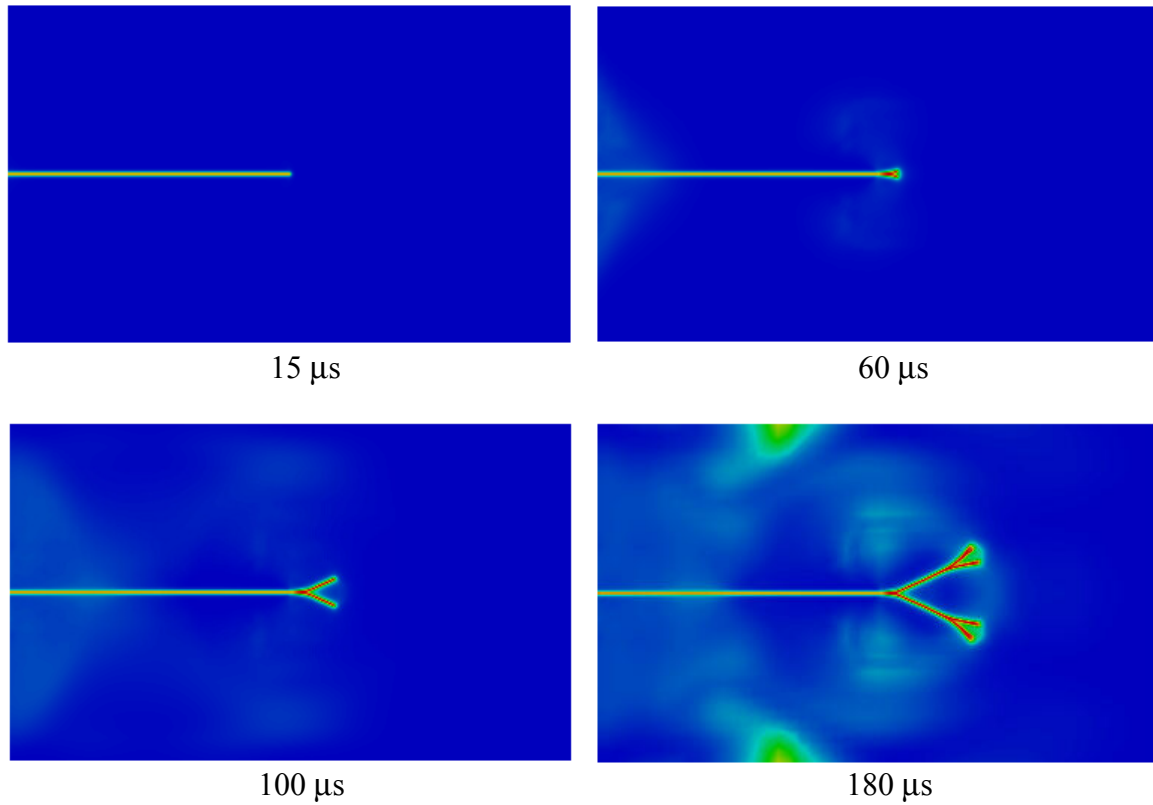


Figure 11. The growth of a dynamic crack in for a pressure of 10.1 MPa. The contours of the phase-field parameter, μ , are shown with red indicating damage/cracks and blue indicating intact material.

From the phase-field simulations we may conclude the following:

1. The crack speed reaches the experimentally observed limit of about $0.45 C_s$ in H-100.
2. Spontaneous branching of cracks can be developed without the influence of waves reflected from the boundaries, as observed in the experiments.
3. While experiments indicate branching into two or more branches emanating from a single crack, simulations have not yet produced more than two branches; this requires further examination.
4. The branches grow at speeds that are slightly greater than the parent crack; this is somewhat different from experimental observations which indicate almost no change in the crack speed.

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Appendix A

Low Velocity Perforation of Thin Aluminum Plates

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Abstract: This work represents a combined experimental and modeling study of low velocity projectile perforation of thin AA5083-H116 aluminum plates. Experiments were performed in order to characterize the candidate material and calibrate simple and easy to implement empirical models for both the material response and ductile failure behavior. Additionally, the failure behavior and potential damage accumulation of the material were evaluated using an interrupted, multiple loading path strain-to-failure test, validating an augmented Johnson-Cook type failure model for use in numerical simulations. Low velocity ballistic impact and perforation experiments, investigating several specific mechanisms of deformation and failure, were conducted and modeled by implementing the developed material and failure model in 3D finite element simulations.

1. Introduction

In order to evaluate the potential of a material to perform a protective role as armor or a component of an armor system, the response of the material and structure to impact must be evaluated. The study of projectile impact, penetration and perforation (complete penetration of a target and passage of the projectile through the target) has been a subject of intense interest for more than a century. For most of its history, studies of impact and penetration have been largely empirical in nature, observing general trends and correlations between projectile and target initial conditions and response. While certain empirical relations may still be used today, the large number of constraints necessary to maintain accuracy limits their usefulness and predictive capability, especially under conditions which see transitions between regimes of response in the target and mechanisms of deformation and failure in the material (Corbett, 1995). The quantitative engineering analysis of penetration and perforation, however, really began in earnest during WWII. From an analytical standpoint, projectile penetration and perforation was still almost entirely unexplored and unattempted due to the mechanical complexity of the problem. Therefore, the ability for predictive design and analysis of protective systems and munitions was not well developed. The challenges associated with this problem drew

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some prominent names in the scientific community. Later in the war, Clarence Zener (of the eponymous Zener diode) would work on the projectile impact for the army (Watertown Arsenal Report), but the first serious analysis of penetration and perforation was attempted in 1941 by the physicist, Hans Bethe, who performed a static analysis of the penetration of a metallic plate by a cylindrical projectile (Bethe, 1941). The potential value of a model of projectile perforation was not lost on the military, and his completed work was immediately classified, making it unavailable to Bethe who was not yet an American citizen. Taylor (1948) expanded upon this work, improving Bethe's analysis of hole expansion to account for incremental variation. This work, along with the earlier work of Bishop, Hill and Mott (1945) on quasi-static indentation in metals by cylindrical punches established the basis for the analysis of hole expansion as the fundamental process in projectile perforation problems.

During this period of time the study of ballistic impact was concerned almost entirely on munition/armor design, and so focused upon the velocity ranges experienced by military ordnance, approximately 500-1300 m/s, and the mechanisms dominating this regime. However, as the space race intensified, focus generally shifted from ordnance and sub-ordnance velocities to hypervelocity impact regimes seen during meteoroid impact. The Hypervelocity Impact Symposia of 1955-1969 spurred significant study in the area, seeing the development of cavity expansion theories (CET) and the application of those theories to penetration problems. Hopkins (1960) developed a dynamic theory of cavity expansion in metals and Goodier (1965) applied dynamic cavity expansion to the problem of projectile penetration. Cavity expansion theory would go on to form the basis of many analysis methods concerned with the penetration of tapered projectiles, and would eventually be modified by Forrestal et al. (1987) in the late 1980's for use in subsequent analyses in materials from ductile metals to soil and concrete. However, CET is inherently limited to cases where the primary mechanism of failure during perforation is the projectile induced growth of an initial impact indentation (or hole) to a size allowing passage of the projectile. Experimental studies have shown that this single mechanism only dominates impact events at velocities high above the minimum perforation velocity, or ballistic limit, with very tapered or sharp projectiles impacting intermediate thickness plates. CET was not the only analytical method developed during this period. Recht and Ipson (1963) developed an analytical model for the perforation of thin plates by blunt projectiles, in which case shear dominated plugging failure was the primary failure mechanism. Recht and Ipson utilized energy and momentum balance to develop an expression for the residual velocity of a perforating projectile assuming plugging failure. However, the expression depended upon an empirically determined ballistic limit and neglected all other mechanisms of deformation but the plugging behavior. The result of these constraints is that like CET, the model is accurate only at

velocities significantly higher than the ballistic limit (in thin plates). Of course, where armor is concerned the ballistic limit is one of the main quantities of interest. A model capable of predictive armor design must be capable of representing the full behavior of the target projectile system in this regime.

Beginning in the 1970's, focus began to shift again to capturing the multiple mechanisms of deformation and failure seen around the ballistic limit of the system – spalling, plugging, petaling, etc. Goldsmith and Finnegan (1971) and Calder and Goldsmith (1971) conducted experiments in thin plates at velocities around the ballistic limit, revealing several interesting results. They found that the maximum amount of global plastic deformation of the plate (dishing) occurred at the ballistic limit while the maximum velocity drop over the course of a successful perforation occurred at a velocity slightly higher than the limit. In addition, for a round-nose projectile they found an inverse relationship between plug size and impact velocity.

Experimental studies such as these found that around the ballistic limit, not only did significant plastic structural deformation occur, but multiple mechanisms of strain localization and failure occurred in combination such that a single mechanism analysis could not capture the perforation of a projectile near the limit. At sufficiently high velocity, only the presence of negligible deformation outside a primary perforation mechanism allowed presupposed models such as CET and Recht and Ipson (1963) to function well. Levy and Goldsmith (1984) conducted experiments in which a multiple mechanism perforation process was captured using high speed photography. They also formulated an analytical model of the penetration processes utilizing a presupposed shape of plate deformation (as measured by experiment) and projectile velocity function. While capable of accurately predicting central plate deflection for impacts below the ballistic limit, the model failed to predict the residual projectile velocity after a successful perforation, indicating the fundamental limitation of analytical methods with respect to the perforation problem. Near the ballistic limit, the additive behavior of global deformation and localized failure by multiple competing mechanisms cannot be captured by an analytical model which must place too many constraints on the problem.

In order to capture the full behavior of this complicated problem, a method capable of modeling all the above phenomena utilizing a more complicated material model must be implemented. While early suspicion of such techniques may have slowed adoption, it was eventually recognized that numerical methods capable of approximating full continuum mechanics offered the best chance of success. Wilkins (1978) conducted early numerical simulations of projectile impact utilizing a 2D finite difference method. While simple by modern computational standards, Wilkins successfully observed multiple mechanisms of failure and deformation including dishing, plugging and hole enlargement over a range of target thicknesses, material models and projectile geometries; and identified several

fundamental factors affecting the ability of a deformable projectile to perforate a target or a target to defeat a projectile. The ability of numerical methods such as finite difference or finite element methods to handle complicated material models and loadings have made them the method of choice for modeling many ballistic impact problems.

Borvik et al. (2001), building upon previous numerical work, successfully demonstrated the ability of finite element simulation to capture experimental behavior when utilizing a properly calibrated material. Utilizing both a modified Johnson-Cook plasticity and failure model calibrated from experiments on Weldox 460 E steel, they demonstrated the ability of a standard plasticity model in conjunction with a triaxiality based failure model to generate an accurate plugging and deformation response upon impact by a blunt projectile. Expanding upon this work of Borvik et al., (2001), Borvik et al. (2004) compared this finite element model to results predicted by CET for AA5083-H116, the candidate aluminum alloy. Specifically, they compared predicted results for impact by a 20 mm diameter conical projectile upon aluminum plates 15-30 mm thick at 200-400 m/s. Under such conditions, the target primarily experienced perforation by ductile hole enlargement, allowing the CET based perforation model of Forrestal to be used for analysis and comparison. Borvik et al. (2009) conducted finite element simulations of the experiment utilizing a previously calibrated Johnson-Cook plasticity model. While their 2001 work utilized Johnson-Cook failure in addition to plasticity, these simulations utilized no failure or fracture condition. Rather, a pinhole was introduced in the target mesh simulating an initial cavity or indentation; allowing full perforation to occur only by plastic enlargement and flow. The use of this configuration allowed close simulation of the ductile hole enlargement mechanism without requiring material or element failure. While successful in closely modeling the experimental result and demonstrating the capability of finite element simulation to match the accuracy of tested analytical methods, the absence of an incorporated material failure model renders this simulation scheme incomplete and once again applicable to only a very specific set of impact conditions. Gupta et al. (2006) studied the behavior of thin aluminum plates subjected to blunt and hemispherical nosed projectile impact. The Johnson-Cook plasticity and failure models were used; the numerical simulations were performed using ABAQUS finite element code with an axisymmetric numerical implementation. Gupta et al. (2006) note that the restriction to axisymmetry makes the simulations unable to capture onset of failure; in fact, one must admit that axisymmetry may be lost even upon approach towards localization of deformation.

Lee and Wierzbicki (2005) investigated the failure of impulsively loaded thin plates, utilizing a shell-element based finite element model to investigate the failure of very thin plates loaded impulsively. A Bao-Wierzbicki fracture model (2004) was implemented in order to allow failure by multiple mechanisms. Discing, and petaling were observed as a

result, rendering the perforation of the plate dependent upon multiple failure mechanisms in addition to target deformation itself. However, the treatment of the plate as a single uniform element through the thickness demands that failure occurs through the thickness of the plate simultaneously, eliminating the possibility of shear plugging failure to occur as has been documented above.

The above is clearly a non-exhaustive review of significant modeling and simulations efforts into the problem of perforation and penetration of thin plates. Even this review indicates that there are numerous possible material constitutive and failure models that have been used and all are able to replicate one or more of the penetration response observed experimentally. Considering the various assumptions regarding loading, geometry, symmetry, deformation modes, material constitutive model and failure modes, the question that arises is the following: what are the essential ingredients that must be retained in the numerical modeling and simulation in order to provide consistent, accurate predictions that can be compared quantitatively to the experimental observations? The experimental and modeling effort presented in this article leverages on the previous studies to understand and simulate the mechanisms of deformation and failure including shear plugging through the thickness of the plate and radial petalling fracture. This manuscript is organized as follows: the constitutive model for the material used is discussed in Section 2. The penetration experiments are described in Section 3. Finally, the results of the numerical modeling and simulation are presented in Section 4, with a critical discussion of the essential ingredients of the model. The main conclusions are summarized in Section 5.

2. Material characterization

The material selected for this study is AA5083 aluminum, a non-heat-treatable Al-3Mg alloy. This alloy contains 4 - 4.9% Mg as the primary alloying element (ASTM B928, ASTM B209). The specific test material was a rolled sheet of the H116 strain-hardened temper, conforming to ASTM B928 for marine grade aluminum plate. This material was chosen, in the spirit of this study, because 5083 aluminum was one of the first aluminum alloys to see common use as lightweight structural armor in vehicles on both land and sea. AA5083 is known for its exceptional corrosion resistance and performance at low temperatures, as well as its high weldability and superior strength in comparison to contemporary aluminum alloys and non-heat-treatable aluminum alloys in general.

Its notable performance in marine environments as well as its relatively low cost and high availability, has led to its acceptance as the material of choice for the construction of lightweight, high utility marine vessels. Its good weldability (and as-welded strength) has

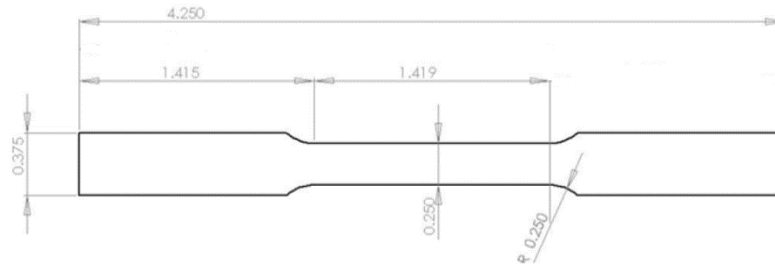


Figure 1. Quasi-static tension specimen geometry (dimensions are in inches)

also made AA5083 a desirable material for the construction of larger vessels, and the U.S. Navy has adopted 5xxx series alloys for several vessels such as variants of the Littoral Combat Ship (LCS) and Joint High Speed Vehicle (JHSV), a military adaptation of the all-aluminum high speed jet ferries in use around the world (Alcoa Defense, 2010).

2.1. *Quasi-static uniaxial material response*

The quasi-static stress strain behavior of the material was obtained by performing uniaxial tension tests. Rectangular cross-section tensile dogbone specimens, shown in Fig. 1, were cut via electrical discharge machining (EDM) from a 1.6 mm thick sheet of 5083-H116 aluminum. Note that this test was performed with the specimen pulled in the direction of rolling (x -direction). The transverse direction is labeled the y -direction and the short-transverse or thickness direction is labeled the z -direction. The first specimen was loaded under a displacement control at a constant strain rate of approximately $3 \times 10^{-4} \text{ s}^{-1}$ using an Instron electromechanical load frame, and is utilized to calibrate the uniaxial stress-strain response of the material. The second specimen was specifically used to observe the full-field strain pattern to failure and was loaded at variable displacement rate to facilitate efficient high-resolution image capture. Strain data was collected via digital image correlation (DIC). Acquired test images were analyzed using ARAMIS™ DIC software to obtain strain data as a function of time. Using the acquired load and strain data, the stress-strain behavior shown in Fig. 2 was generated prior to the onset of necking.

The uniaxial engineering stress-strain response shows significant strain hardening for a relatively strong aluminum alloy. The onset of diffuse necking begins at approximately 12% strain, and the maximum or ultimate stress, at this strain value, is approximately 325 MPa. As commonly seen in Al-3Mg alloys, the material shows a serrated stress response, associated with the Portevin-Le Chatelier (PLC) effect, for much of the hardening region.

The PLC effect begins at a critical strain of 3.6% on the stress-strain curve, and examination of ARAMIS images showed the development of PLC bands at this strain level. These PLC bands result in stationary bands of accumulating local strain that do not fully propagate longitudinally throughout the specimen, but nucleate at multiple discrete locations on the sample (Cassarotto, 2009). Nonuniform strains were found along the length of the uniaxial tensile specimen, with the fluctuations being in the range of a couple of percent. Necking of the specimen occurred at the Considère strain of approximately 12%. The necking process was dominated by the through-the-thickness sheet mode localization, resulting in a slant fracture through the thickness of the specimen as shown in Fig. 3. The axial failure strain at the surface corresponding to the last image recorded was ~35%. Based on the experiments of Ghahremaninezhad and Ravi-Chandar (2012), we anticipated that the local strain near the failure plane would be even greater; this was confirmed as discussed in Section 2.2.

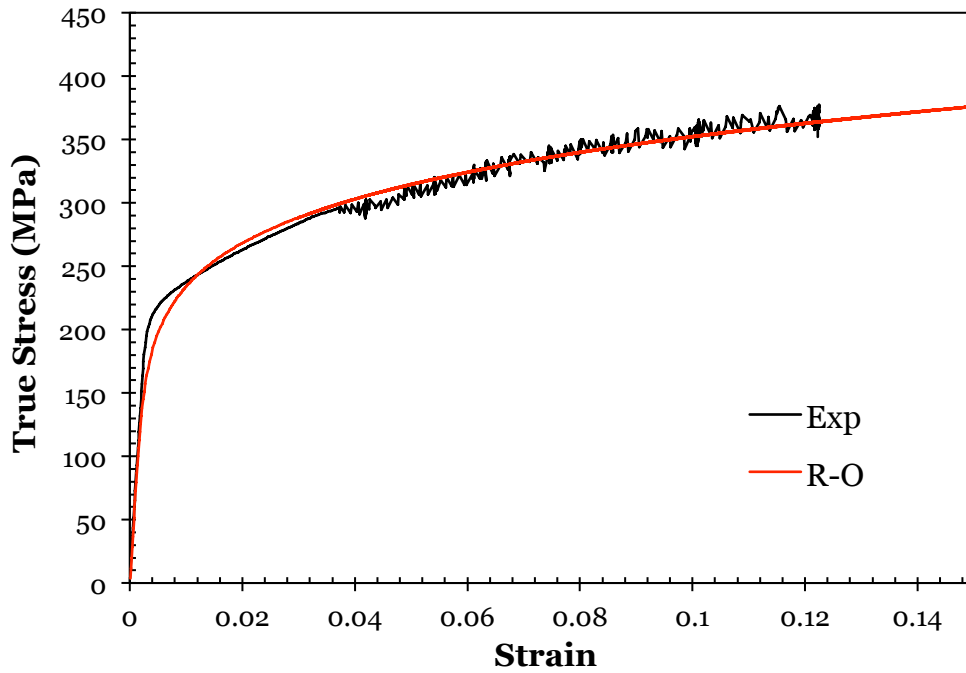


Figure 2. Quasi-static tensile stress-strain curve

The material stress-strain response shown in Fig. 2 is represented by a Ramberg-Osgood curve up to the onset of necking.

$$\varepsilon = \frac{\sigma}{E} + \alpha \frac{\sigma}{E} \left(\frac{\sigma}{\sigma_0} \right)^n \quad (1)$$

This equation requires only four constants: the elastic modulus E , the yield stress σ_0 , the hardening exponent n , and a scalar α such that $(1+\alpha)\sigma_0/E$ is the total strain at yield. This representation gives dominant elastic behavior at low strains and dominant plastic or hardening behavior at higher strains post-yielding. This behavior allows the R-O fit to closely represent many common metallic stress-strain curves using only four fitting parameters without the need for piecewise or discontinuous functions. Applied to the true stress curve vs logarithmic strain of the test material, the R-O fit closely approximates the average hardening behavior of the material (see Fig. 2). The extracted material constants are given in Table 1.

Table 1. Mechanical properties of AA 5083-H116.

Property	Value		Units
Modulus of Elasticity	E	69.9	GPa
Poisson's ratio	ν	0.33	-
Density	ρ	2,660	kg/m ³
R-O parameter	α	0.912	-
R-O parameter	n	6.49	-
Yield Strength	σ_0	203	MPa
J-C Parameter	D_1	0.3	-
J-C Parameter	D_2	1.4	-
J-C Parameter	D_3	-1.4	-

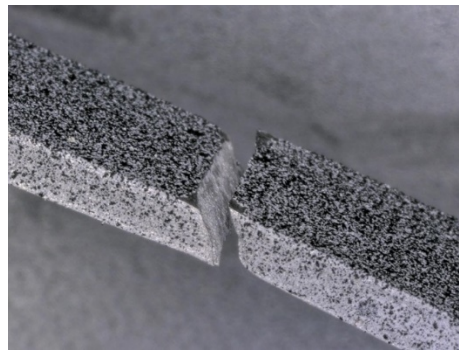


Figure 3. Tension specimen necking and fracture

While R-O clearly cannot represent the serrated region of the curve (due to PLC effect) the overall hardening behavior of the material is sufficient to capture the plastic properties of the material necessary to utilize continuum plasticity modeling methods. In addition, the R-O fit fails to capture the behavior of the material near initial yielding. The true material response shows a much sharper transition from elastic to hardening behavior which cannot be captured by the continuous linear/power-law shape of the R-O representation. However, the accuracy of Ramberg-Osgood for hardening materials at high plastic strains makes it an attractively simple material representation for the analysis of high deformation events and structural behavior assuming sufficient real hardening exists in the material in question. The R-O representation of the candidate material will be extrapolated to significantly higher levels of plastic strain (than seen under homogeneous uniaxial loading conditions) for use in numerical models.

2.2. *Failure under quasi-static loading*

The R-O model fitted to the uniaxial tension test will be extrapolated to strain levels beyond the Considère strain. In order to capture the full behavior of a structural response such as ballistic impact, not only must the plastic material response be modeled to high values of strain but the ultimate fracture or rupture of the material must be characterized. The failure of a material can be a very complex process, especially when failure occurs in a volume of material that has experienced significant global plastic strain. This condition is often seen in structures lacking clear pre-existing flaws or cracks, in which substantial permanent deformation is achieved prior to failure. This condition cannot be easily modeled using conventional fracture mechanics and must be addressed at a material level. Fracture at material scale is often modeled as the cumulative and ultimate effect of damage mechanisms present in a material volume undergoing deformation. Under ductile conditions, in which significant plastic strain occurs within the material volume, the dominant damage mechanism is that of void nucleation, growth and coalescence. As the material deforms, cavities may form at favorable sites including the interface between matrix material and inclusions or second-phase particles. Mechanically, void growth was first analyzed by McClintock (1968) and later incorporated into the plastic response by Gurson (1977) as a means of modeling ductile failure. Analysis of the micromechanics of void growth revealed a strong dependence upon stress triaxiality, defined as: $T = \sigma_m / \sigma_e$ where σ_m is the trace of the stress tensor divided by 3, and σ_e is the (von Mises) equivalent stress.

It is possible to adopt models based on micromechanical models of damage, such as the Gurson-Tvergaard-Needleman models. However, examination of the failed specimens

indicated that damage in the form of cavities occurs in a highly localized region within the neck, and the failure of the AA 5083 could be captured quite easily through a simpler strain-to-failure model. Therefore, we sought to obtain additional, more precise local strain measurements by examining the deformation at the level of the grains; the procedure developed by Ghahremaninezhad and Ravi-Chandar (2012, 2013) is used. In this procedure, we make use of measurements of grain size in the region where the specimen has failed under different loading conditions and extract a measure of the local strains at the level of the grains through a statistical comparison to the grain distribution in the undeformed specimens.

The strain-to-failure is known to be influenced significantly by the stress state, in particular by the stress triaxiality ratio T . T is a point-wise measure of the mean stress at a given stress state. Although T cannot be directly measured in specimens, computational tools enable estimation of its value, given a constitutive model. In order to vary T , in addition to the tension specimens described in Section 2.1, Arcan shear tests and compact tension fracture tests were performed on the AA 5083 specimens. The triaxiality T was extracted at potential crack initiation site for each type of experiments (geometry and loading) when first cracking occurred. Four main stress triaxiality values were identified in these specimens: approximately 1 for tensile specimens inside the necked region, 2.5 for fracture specimens, and 0 and 0.5 for the center and notches of Arcan specimens, respectively.

In order to measure the size of the deformed grains, representative specimens from tensile, Arcan shear and fracture tests were cut to reveal the x - z thickness section and x - y rolling planes. The cut pieces were then mounted in an epoxy block, and polished using standard metallographic techniques. For the Al-5083 specimens, etching was a multi-step process. Firstly, the specimens were immersed for 2-5 min into a solution composed of 50 ml of HNO_3 mixed with a 50 ml solution composed of HF (2 ml), HCl (3 ml), HNO_3 (20 ml) and water (175 ml). Then the specimens were finely polished for 20-50s using a 0.05 μm colloidal solution. Finally, the specimens were immersed for 2 min into a 10% aqueous solution of chromic acid. The micrographs shown in Figures 4 indicate the grains and grain boundaries clearly. The average grain size in the x - z plane was found to be around 10 μm in the initial rolled microstructure. Careful measurements close to the fracture surface in tensile specimens reveal 8-10 μm wide grains near the free surface. Near the center of the specimen, the average size drops to about 6-7 μm . Examination of the Arcan specimen reveals an average equiaxed grain size of about 30 μm in the x - y plane when unstrained. Near the cracks at the notches, the width of elongated and rotated grains drops to about 13 μm . However, in the shear plane near the center of the specimen, where there is no indication of crack, the average grain width is about 24 μm . The resulting equivalent plastic strains calculated based on these grain level measurements are between $\varepsilon_p \sim 0.22$ and $\varepsilon_p \sim 0.71$ in tensile specimens at the free surface and in the center, respectively. In the Arcan specimens, strain values at failure of $\varepsilon_p \sim 0.55$ and

$\varepsilon_p \sim 1.81$ were calculated at the center and near the notch, respectively. As in the case of the other aluminum alloys (see Ghahremaninezhad and Ravi-Chandar, 2012), the grain based strain levels at the onset of failure are significantly greater than that determined through DIC measurements at a macroscale. It should be noted that at these strain levels, no damage was detected using optical microscopy. This is the primary justification for the use of a strain-to-failure model without accounting for damage evolution.

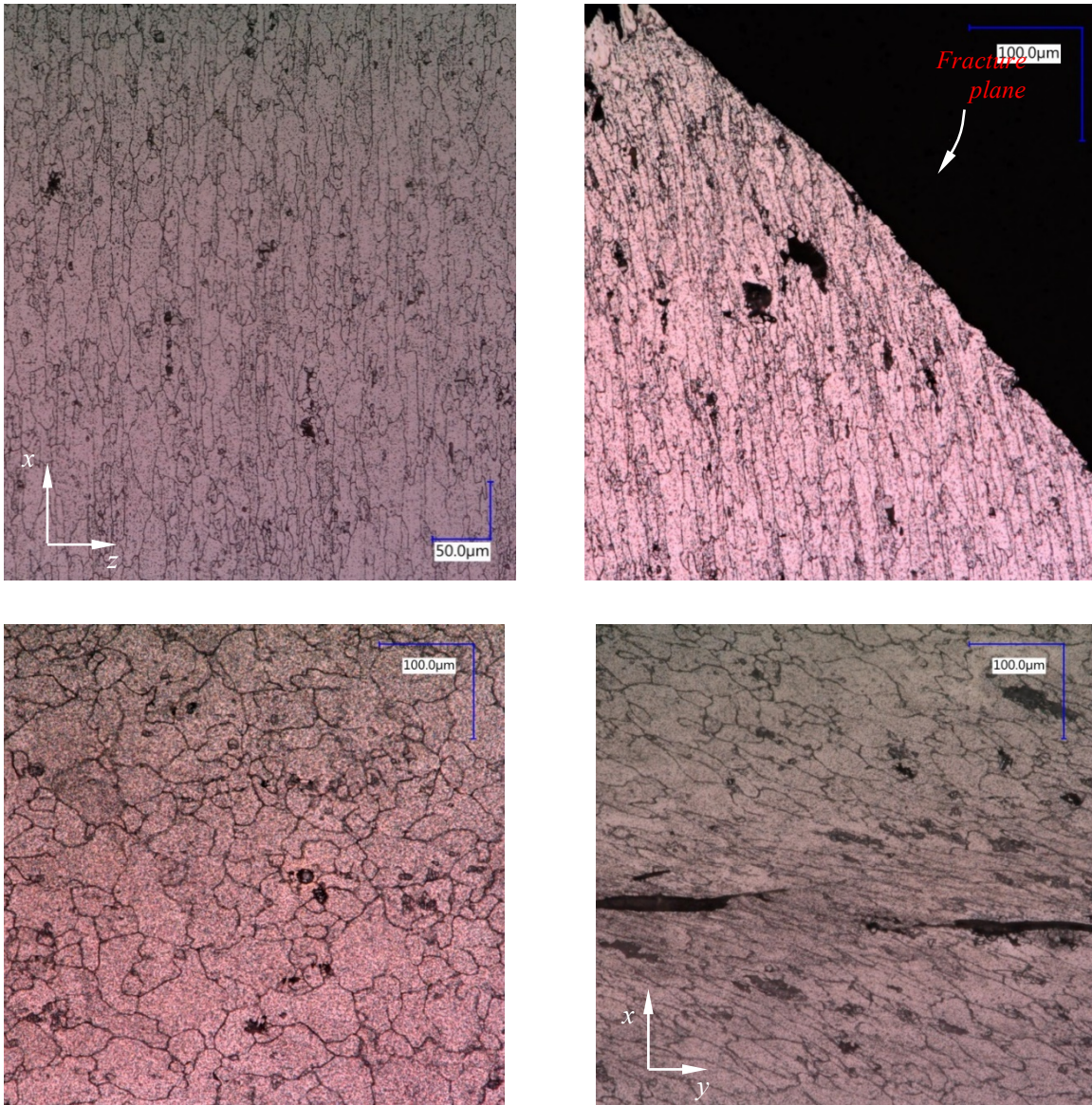


Figure 4. Micrographs of Al-5083. (a) Unstrained microstructure in the x - z plane. (b) Microstructure of the x - z plane in the vicinity of the fracture plane in a tensile test. (c) Unstrained microstructure in the x - y plane. (d) Microstructure of the x - y plane in the vicinity of the cracks developed from notch tips in the Arcan specimen

All results from tests performed on Al-5083 are gathered in Figure 5; the gage length for the measurement of strain is about two to three grain sizes. The curve of the “strain-

to-failure” ϵ_f vs. “stress triaxiality”, T is plotted in Figure 5; it should be noted that this curve represents a lower-bound; basically, we have measured these strain levels at the triaxiality levels indicated without observing failure at these points. In order for failure to occur, the strain must exceed this level. The failure curve is represented by the equation:

$$\epsilon_f = D_1 + D_2 \exp(D_3 T) \quad (2)$$

This can be considered as a simplified version of the Johnson-Cook failure model, with the strain-rate and thermal terms not considered (yet). The primary difference arises from the fact that these strain levels were obtained from grain-size measurements, and therefore are local measures, as opposed to average measures based on reduction of cross-sectional area, and other measures that are commonly used in calibrating the Johnson-Cook failure model. The DIC data lie much lower than the lower bound curve because the surface strains obtained by DIC are not representative of the actual strain at the crack initiation location.

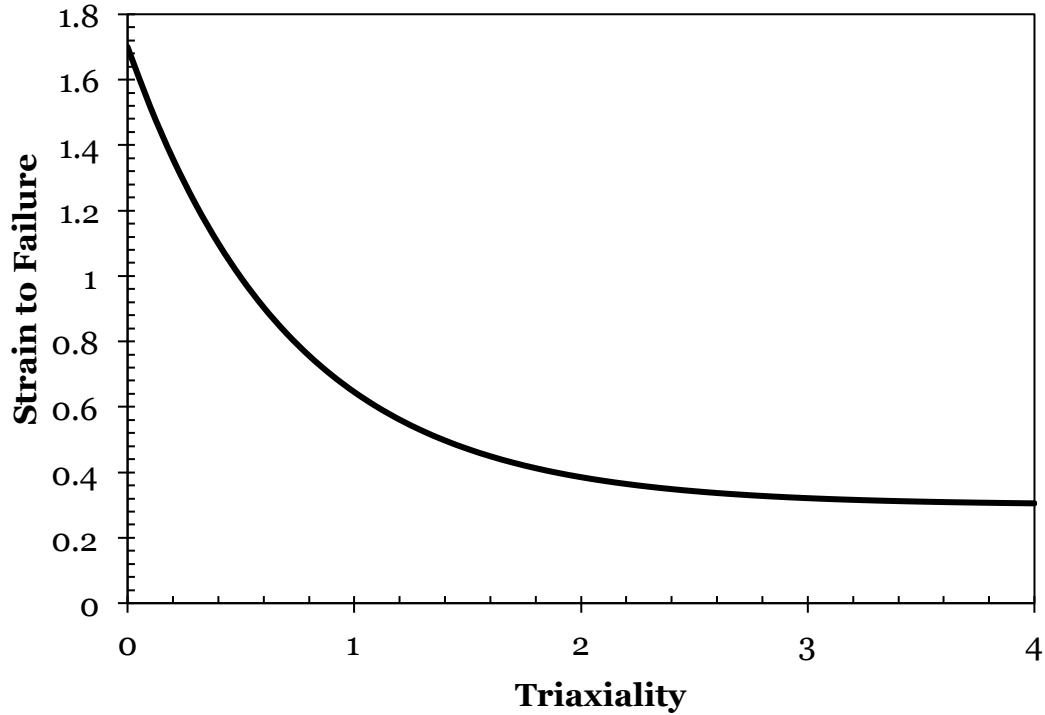


Figure 5. Johnson Cook strain-to-failure curve calibrated based on grain-scale measurements (see Eq.(2)).

2.3. Cumulative damage

In order to model the material damage present at any given state of stress achieved after a given loading history, J-C damage is usually implemented using a cumulative damage variable D , which follows a fractional or incremental summation rule similar to Miner's Rule:

$$D = \sum \frac{\Delta \epsilon}{\epsilon_f(T)} \quad (3)$$

where $\Delta \epsilon$ is the increment of plastic strain at the current triaxiality level, T and $\epsilon_f(T)$ is the failure strain level at the same triaxiality. This rule reflects a linear accumulation of damage at any given stress triaxiality. However, a damage model of this type may be implemented in several ways. A damage initiation specification is often used such that no damage is accumulated below a threshold value of effective strain at a given stress triaxiality. The evolution of the damage variable may also be coupled or uncoupled to material properties and response. A coupled analysis may be used to degrade the macroscale material properties such as elastic modulus, E , (or the effective flow stress): $E_D = (1 - D)E$. In this case, the elastic stiffness of the material is degraded linearly with increasing damage parameter. An uncoupled analysis, on the other hand, preserves the initial material response until critical damage is reached ($D = 1$), and the material fails. In the uncoupled case, no material degradation may be applied to the material response, but the damage parameter is still calculated in a linearly cumulative fashion (Lemaitre, 1996). An alternative formulation with $D = \Delta \epsilon / \epsilon_f$ as the damage parameter has no accumulation or history effect, but this is not commonly used.

A potential problem arising from this commonly utilized approach is that while the failure strain itself varies with triaxiality, the nature of damage accumulation is the same at both high triaxialities, where the dominant growth of voids have been frequently observed throughout the loading history, and at low triaxialities, where the mechanisms of material damage are less understood and void growth is more difficult to measure. At low triaxiality, matrix material flow may effectively prevent the nucleation and growth of voids during the majority of the strain history. In this case, the accumulation of damage leading to fracture must be non-linear, creating a condition where no material degradation takes place until levels of strain close to the failure strain are experienced. A uncoupled model loading history at a single value of triaxiality may not be significantly affected by this discrepancy but a loading history experiencing an evolution of triaxiality will suffer as a cumulative model will predict more damage accumulation at low triaxiality than is actually experienced.

In the case of a fundamentally non-linear mechanism for the accumulation of material damage in a real material, an uncoupled strain-to-failure model (where failure initiation occurs upon reaching the fracture locus) is a realistic empirical model of material failure. However, the presence of a Miner's style, linear cumulative rule for the determination of failure initiation retains the previously discussed issue under a variable triaxiality loading path, which is present in almost all manifestations of structural failure. Under ductile conditions, a material fracture model such as Johnson-Cook is usually implemented by means of numerical simulation. Indeed most conventional finite element programs, such as ABAQUS, implement linear accumulation of incremental damage.

In order to implement accurately a failure model of this type in a numerical finite element simulation, an appropriate fracture locus must be determined, and the method of damage initiation and damage evolution must be validated. Based on multiple loading path experiments (see Simpson, 2015), we use an uncoupled damage model in this work.

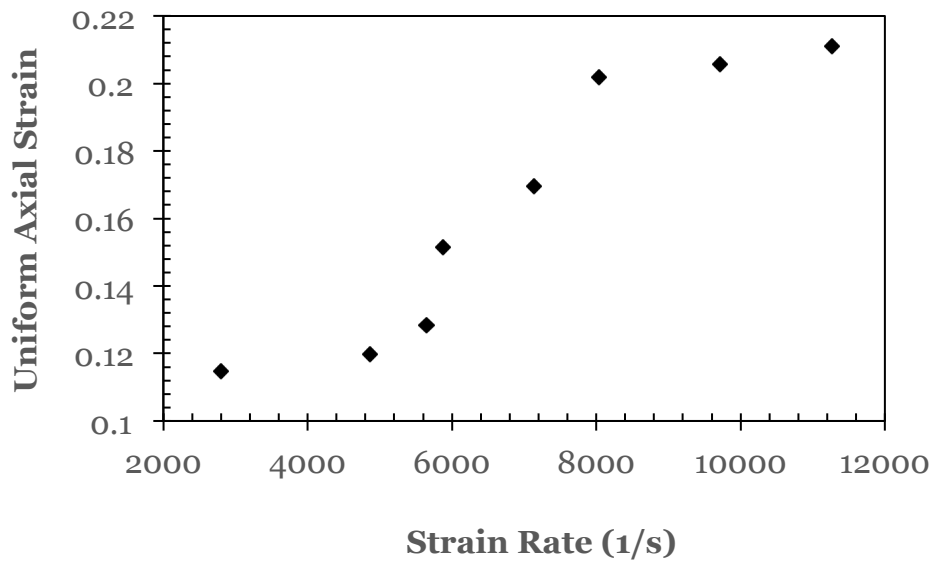
2.4. Dynamic uniaxial material response

Under quasi-static loading, the R-O material representation is capable of capturing the general plastic behavior; however, under dynamic loading at elevated strain-rates, the potential for strain-rate dependence and non-equilibrium heating of the material under deformation may require a material response characterization of significantly increased complexity in order to sufficiently capture structural behavior. A commonly implemented empirical plasticity model for use under dynamic conditions is the Johnson-Cook (1983, 1985) plasticity model. This model allows for the effect of strain rate and thermal softening, both of which may be necessary under high speed deformation events experiencing near-adiabatic heating.

The testing and characterization of materials at high strain rates has been and continues to be a challenging area of study. While the split-Hopkinson bar has emerged into a popular technique for characterization of high strain-rate data, the rates achievable, particularly under tensile loading is rather limited. Here, we used an electromagnetically driven expanding ring test, introduced by Niordson (1965) to achieve strain rates of 10^4 s^{-1} . A variant of Zhang and Ravi-Chandar's experimental apparatus (Zhang and Ravi-Chandar, 2006) was used to conduct expanding ring tests on AA 5083 aluminum ring specimens with a 1.6 mm x 1.6 mm square cross-section and a 30 mm inner diameter. Details of this investigation are reported in a Master's thesis (Simpson, 2015) and are not reproduced here; only a summary of the results is provided.

Images of the ring specimen were recorded using a Cordin 550 high speed camera at up to 200,000 images per second. The images were analyzed to obtain data from uniform

expansion, necking and fragmentation. After each test, the ring fragments were recovered and inspected post-mortem to determine the uniform strain in the unnecked region of the fragments. To evaluate the potential strain-rate dependence of the material, the homogeneous strain in the un-necked region was evaluated by making post-mortem measurements of the specimen cross-sectional dimensions and associated transverse plastic strains. Assuming conservation of volume and the dominance of incompressible plastic strains, the axial plastic strain at the measurement location was calculated from the measured transverse strains. Dimension measurements of the post-mortem fragment widths and thicknesses were made using a Keyence digital microscope and its built-in image measurement tools. Calculated values for uniform axial strains at several un-necked locations around each ring were averaged to produce an estimate of the uniaxial necking strain for each specimen. Similar measurements were made in selected arrested necks adjacent to ring fracture locations, later referred to as secondary neck locations. Figure 6a shows this average uniform axial strain as a function of strain rate, where a sharp increase in uniform strain can be seen from 5000 to 8000 1/s. Figure 6b shows the corresponding strain within the neck and should indicate a lower bound for the failure strain.



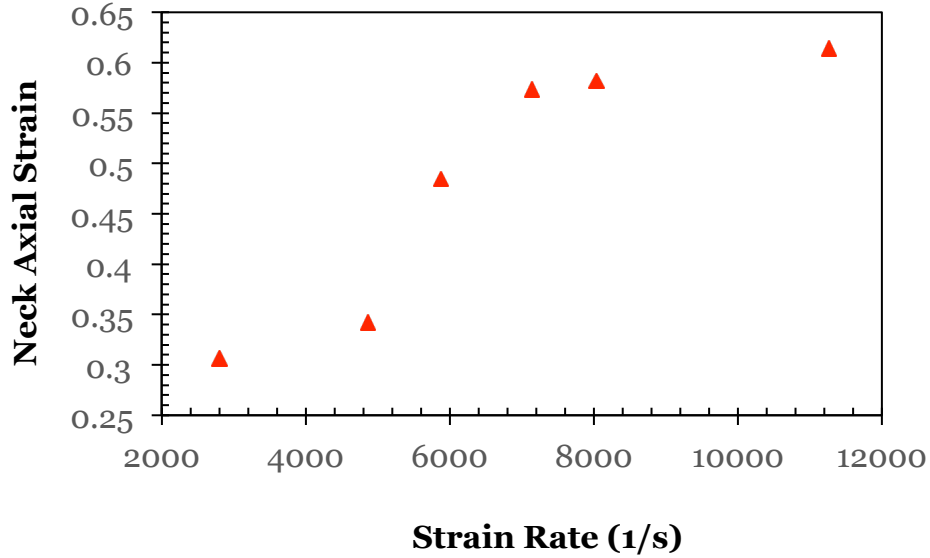


Figure 6. Uniform axial strains in un-necked regions and maximum axial strains within arrested necks

While this pattern may seem to require material strain rate dependence, it is really not due to material properties but to inertial effects associated with the physical size and shape of the specimen cross-section itself. Zhang and Ravi-Chandar (2008) observed a similar increase in specimen necking strain in rings with larger physical size (approximately 1-2 mm in largest dimension) and square cross sections. The proposed mechanism for this dependence relies upon the initiation of localization as some specific point in the cross-section of the ring, requiring the propagation of the localization across the dimension of the cross-section before the specimen scale formation of an actual neck can proceed. As the physical size of the specimen or the maximum distance across the cross-section increases, the time period between the initiation of instability and the specimen scale formation of a neck and the associated release wave increases. While this cross-sectional propagation occurs, the material outside the necking region continues to strain under the loading. This creates a timescale over which straining may continue beyond localization, prior to release. Once the neck is fully formed, the stress release wave begins to propagate outwards from the site of localization.

3. Ballistic impact experiments

The following experimental and modeling work expands the previous studies to plates of finite thickness under ballistic impact conditions that demand the multiple mechanisms of

deformation and failure including both shear plugging through the thickness of the plate and radial petalling fracture.

3.1. Experimental setup

The target specimens were machined from the same 1.6 mm thick AA5083-H116 aluminum plate utilized in the material calibration tests described in the previous section.

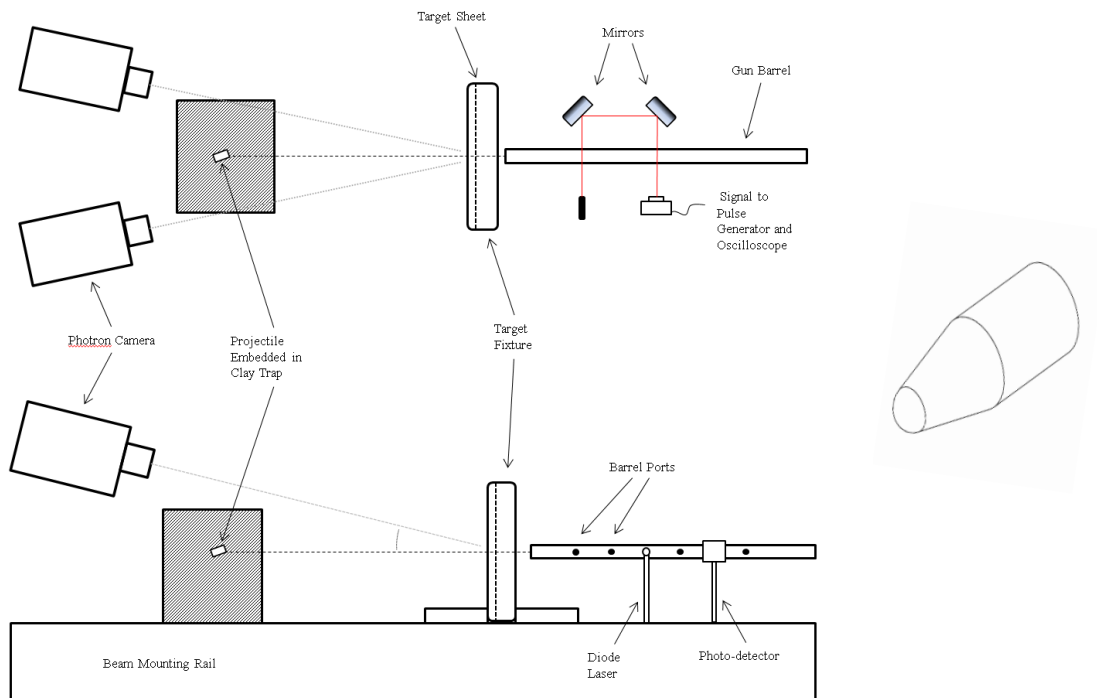


Figure 7. Experimental set up for the projectile impact experiments. The shape of the projectile is shown in the figure on the right

The target plates were clamped and bolted around the outer edge by a heavy steel fixture, leaving a 150 mm central circular region of the plate (with a clamped boundary) free to deform under impact. The fixture itself was clamped to an I-beam, serving as a mount rail for the apparatus seen in Fig. 7. A compressed air gun was used to deliver the impacting projectile to the back of the target plate. The radial center of the target fixture was aligned with the bore axis of the gun to ensure a symmetric impact location with respect to the plate boundary.

A clay trap was placed on the opposite (front) side of the target fixture to catch the perforating projectile. The last 12 inches of the 8 foot, 0.5 inch diameter gun barrel was ported, permitting the installation of a break-beam style chronograph for the measurement of the incident projectile velocity prior to impact. A diode laser beam was directed through a barrel port and reflected, via two optical mirrors, back through the adjacent port into a photo-detector. The signal from the laser photo-detector was connected to the trigger channel of a Berkeley Nucleonics pulse generator, sending a TTL pulse to a digital phosphor oscilloscope each time the beam was broken, allowing the calculation of the velocity of a projectile traveling down the barrel. The pulse generator was used to trigger (upon passage of the projectile through the beam) two synchronized Photron SA1.1 high speed cameras viewing the front surface of the target plate. These cameras were placed approximately 10° to the left and right of the bore axis and 14° above the bore axis, viewing the target plate over the front edge of the projectile trap. Nikon 24-85 mm lenses were used in conjunction with the detector settings of cameras to view a section of the target plate ranging from a central 1 inch square to the full unclamped plate surface. The stereoscopic configuration of the synchronized cameras allowed the use of 3D digital image correlation (provided by ARAMIS) on the collected surface images of the plate during the process of penetration and perforation. To utilize the DIC capability of this apparatus, the aluminum target plates were painted with an appropriate black and white speckle pattern. Illumination for the tests was provided by four high intensity halogen lamps arranged around the target fixture. The frame rate dictates the maximum resolution achievable by the camera system so that tests utilizing the higher frame rates were limited in view to the very central region of the plate around the perforation created by the projectile.

3.2. Experimental results

A series of impact tests were conducted at low velocities near the ballistic limit of the target plates, ranging from approximately 73-156 m/s. The specific details of the tests performed are listed in Table 2. For the tests analyzed here, a hardened O1 tool steel projectile was used. The 24.7 g projectile was ~0.5” in at the base, tapering to approximately 0.25” diameter round nose with a taper angle of 12° (see Fig. 7).

Synchronized images of the test were collected using by high speed camera at appropriate framing rates for each test. A selection of experimental high speed stereo-pairs of images for Plate 1 and Plate 5 are shown in Fig. 8 and Fig. 9, respectively. Full movies of the penetration is embedded in the appendix to this document. While these images could be used to obtain all three displacement components, and hence the surface

strains, it is evident that significant strain levels are developed only in the vicinity of the plug/petal formation, and the images do not have adequate spatial resolution to extract such strain variations. Hence the images were used merely to identify the sequence of events, and the final velocity of the projectile.

As noted in Table 2, Plate 1 was not successfully perforated but experienced partial plug formation and partial radial cracking necessary for the full scale development of petalling. For a better view of these phenomena see the post-mortem photos of Plate 1 given in Fig. 10. The highest velocity test (Plate 5), however, shows full perforation and petal development by the tapered projectile. To take advantage of a higher frame rate (100,000 fps), only a limited central region of the camera detector was utilized, providing a reduced spatial resolution image.

Table 2: Impact tests on AA5083-H116 aluminum target plates

Plate #	Incident Velocity (m/s)	Residual Velocity (m/s)	Kinetic Energy Retention	# of Petals	Plug Diameter (mm)	Frame Rate (kps)	Notes
1	73	NA	NA	6	4.7	30	No perforation
2	96	54	32 %	8	--	50	
3	104	65	39 %	10	--	50	
4	100	85	72 %	NA	12.5	100	Blunt-nose projectile
5	156	147	89 %	11	5.1	100	

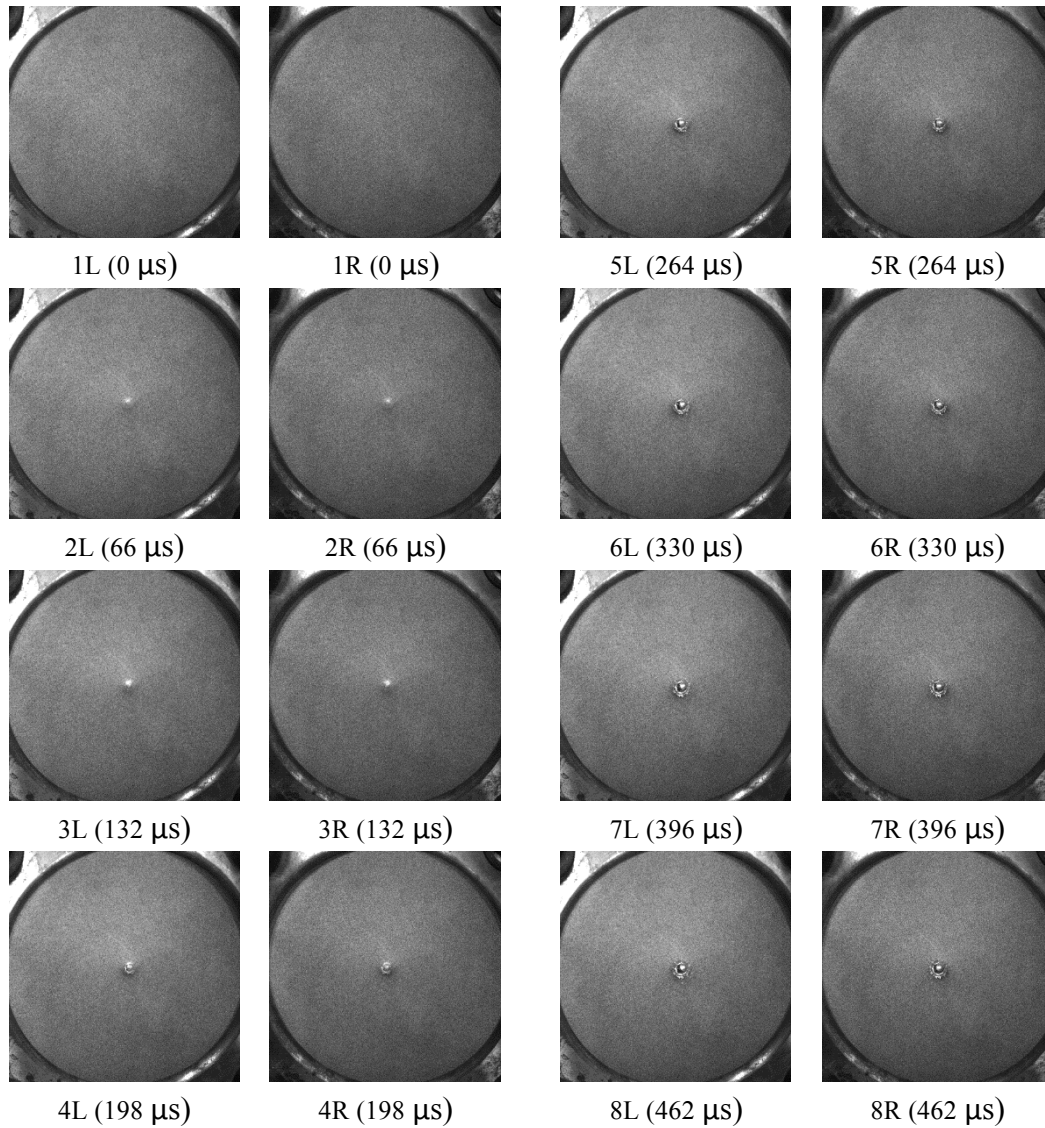
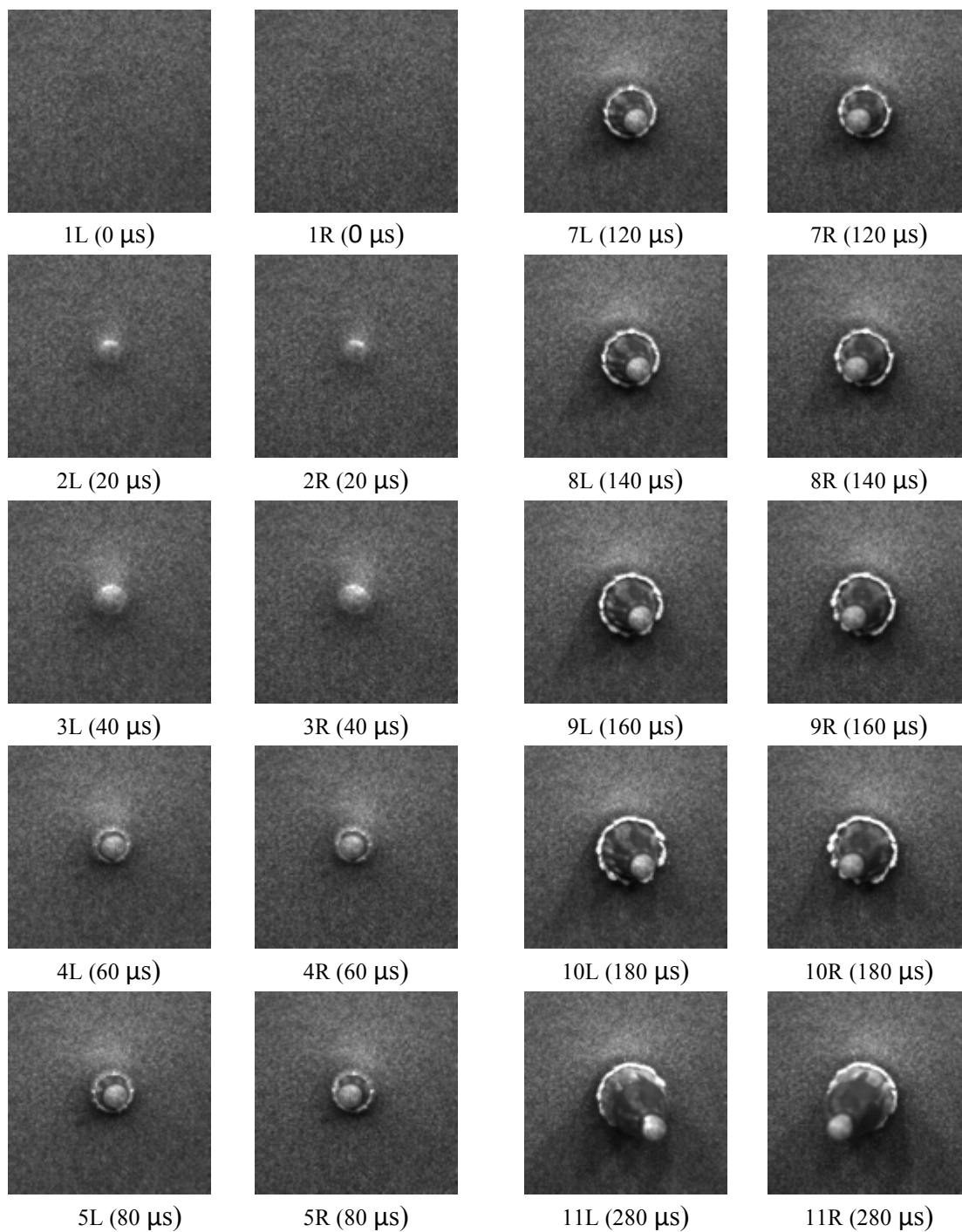


Figure 8. Plate 1 – Impact speed: 73 m/s. Selected frames are shown indicating the formation of the plug. L and R indicate left and right camera images.



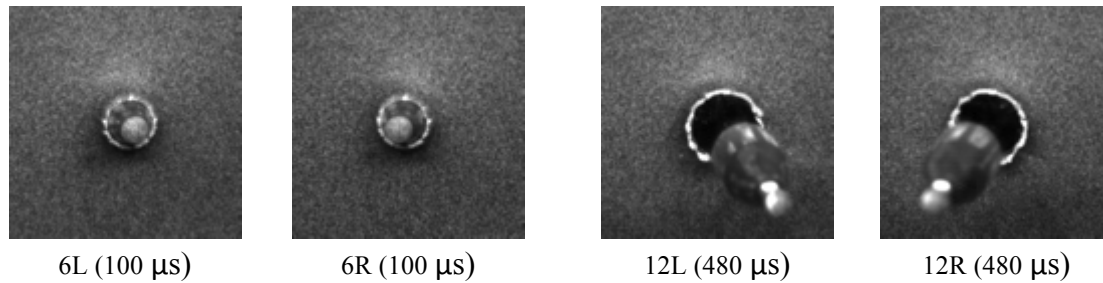


Figure 9. Plate 5 – Impact speed: 156 m/s. Selected frames are shown indicating the formation of the plug and the petalling cracks.

In Fig. 9, the basic mechanisms of plate failure considered in this work can be observed. Plug development can be observed in images 3L and 3R. By 4L, 4R full separation of the plug has occurred and the nucleation of petals by means of localization and radial fracture around the circumference of the new plug fracture surface has begun. As the projectile continues to penetrate, the passage of the tapered section drives the propagation of the radial petal cracks, growing the petals until the shoulder of the projectile is reached at approximately image 10L and 10R. Beyond this point, there is only minor frictional resistance applied to the projectile. Note that in image 12L and 12R, the detached plug has visibly separated from the nose of the projectile as the petalling process decelerates the projectile while the plug continues to move forward unhindered.



Plate
1

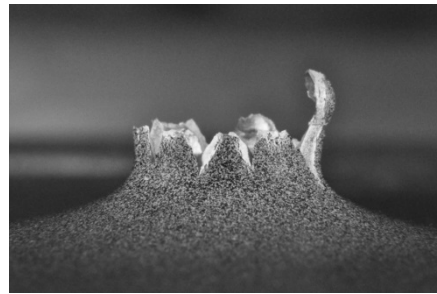
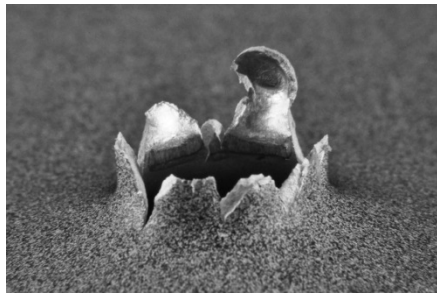


Plate
2

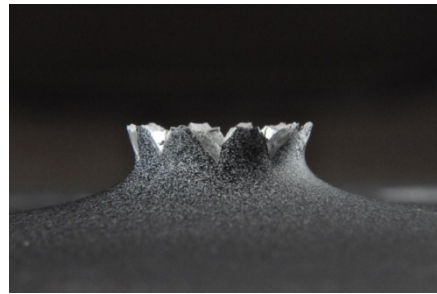


Plate
3

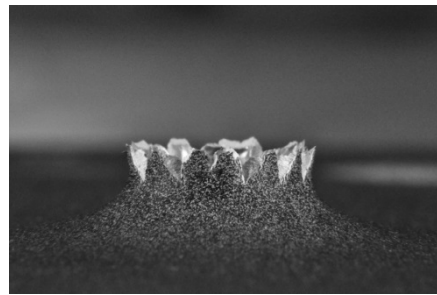
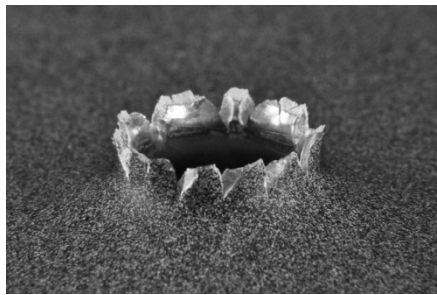


Plate
5

Figure 10. Post-mortem images of plugging and petalling features

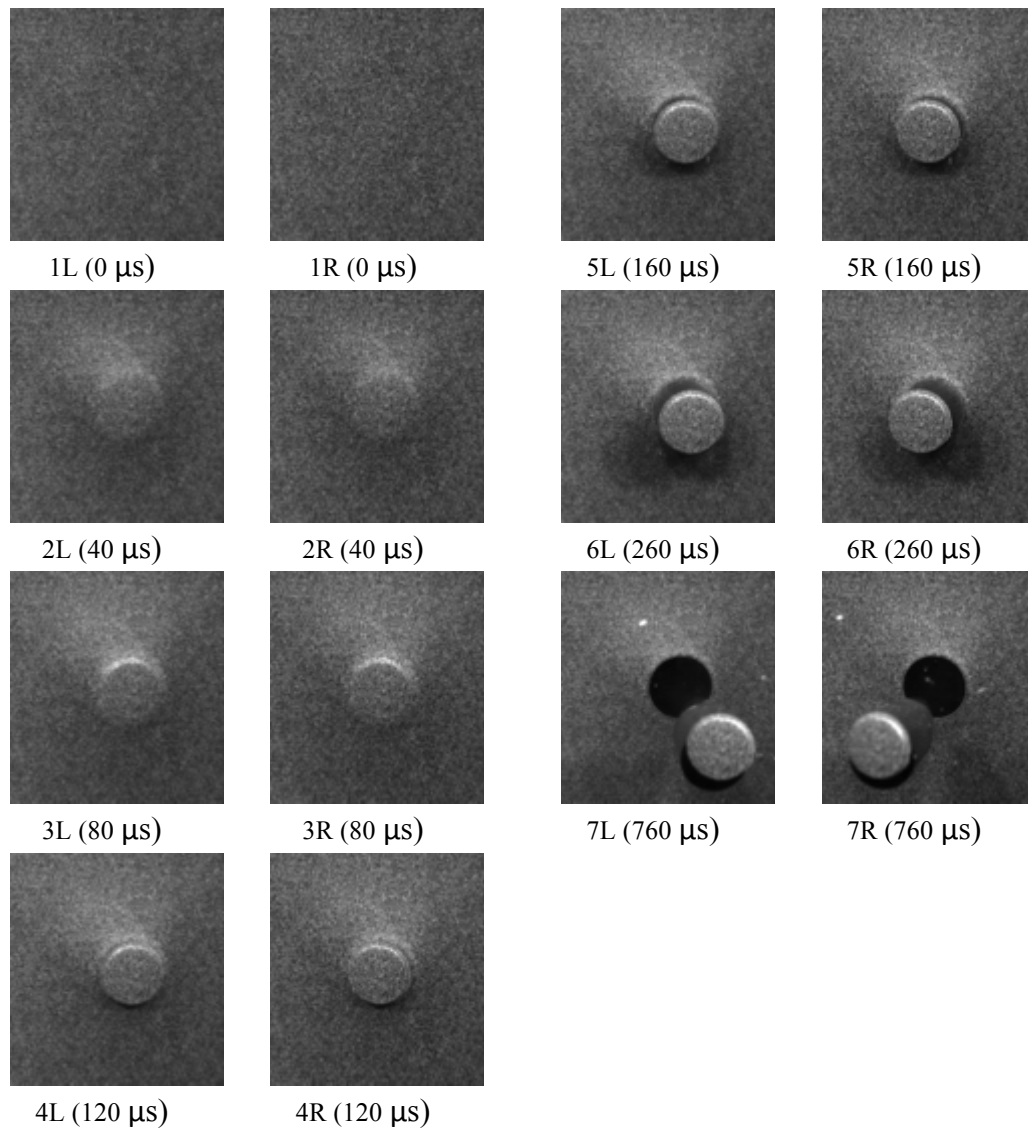


Figure 11. Plate 4 – Impact speed: 100 m/s. Blunt projectile creates a plug without any further damage to the plate.

In contrast to the combined plugging and petalling process shown above, Fig. 11 shows the results of an impact by a blunt cylindrical projectile (the impactor shown is simply the previously described projectile fired backwards). The blunt nose projectile very cleanly shears a plug from the target plate approximately the size of the projectile face itself. There is relatively little permanent plate deformation outside of the sheared region, which ultimately allows easier perforation of the target plate by the blunt projectile than the tapered projectile. As shown in Table 2, the residual velocity (at an

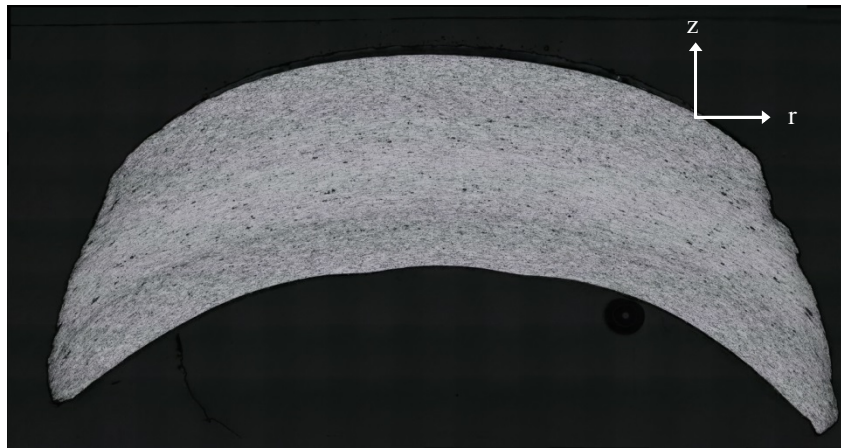
incident velocity around 100 m/s) is much higher for the blunt projectile than the tapered round nose projectile. The blunt impactor retains much more kinetic energy post-perforation, indicating that the energy consumed by the combined effect of deformation and fracture is less under the conditions created by the blunt projectile. This result matches previous experimental studies on thin plates impacted by both conical/ogive and blunt nose projectiles. Note, that the residual velocity was estimated by measuring the time required for the projectile to exit the frame and consequently travel the fixed axial distance from the target plane to the edge of the camera frame.

Post-mortem images of the plate specimens are given in Fig. 10. Petalling or incipient petal fractures can be seen in all impacts by tapered projectiles (see Table 2). In the case of the lower velocity specimens, the plug is still attached to a petal indicating the incomplete circumferential failure of the shear zone prior to radial localization and petalling fracture. The stresses at the plug are relieved as petals form, converting the connected material to what is essentially a plastic beam with a hinge point at the base of the petal. Continuing perforation by the projectile drives the propagation of radial cracks by applying bending forces to the petal itself. This mechanism increases the compliance of the petal with increasing projectile perforation until the hole is large enough to allow the unencumbered passage of the cylindrical part of the projectile.

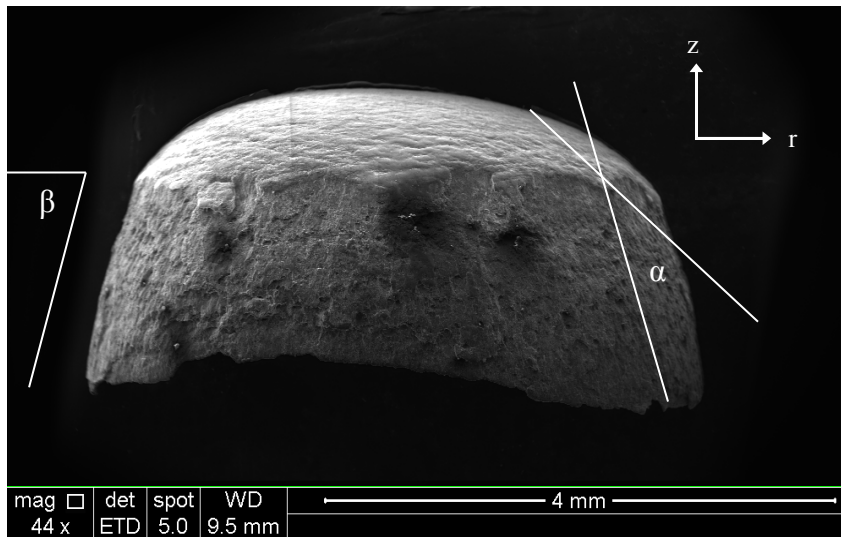
Figure 10 also shows a distinctive bend-back of the petals as the curves beyond perpendicular to the plane of the plate. This curvature is due to the initial dishing of the plate prior to petalling. As radial cracking begins and petals form, the contact point between the projectile and the petal moves toward the base of the petal and the preexisting out-of-plane curvature of the petal beyond this point of contact bends the petal slightly away from the projectile surface.

By the time this contact point reaches the shoulder of the projectile, the full petal is bent back beyond the surface of the projectile, creating the post-mortem geometry seen here. While some fractures may be difficult to see in the images above, the number of radial fractures (and therefore incipient or developed petals) can be seen to increase from Plate 1 to Plate 5 (see Table 2). The petalling fractures themselves show distinct slant character, suggesting the dominance of a sheet or through-thickness localization mode at the site of radial crack development. No clear pattern between the relative directions of adjacent slant fractures was observed—adjacent fractures may be oriented parallel or opposite to each other without any discernible pattern. Additionally, more extensive dishing at increased radial distance from the axis can be observed in the lower velocity tests. This is expected as previous experimental studies in thin plates have shown maximum center-point out of plane displacement to occur around the ballistic limit; increased perforation velocity reduces the radius of material that experiences the propagating dishing stresses generated by the impact prior to full perforation of the target.

The central region of the target initially impacted by the projectile (in the case of Plates 3, 4, 5) becomes the plug and is removed from the target completely given sufficient impact velocity. The plugs for the analyzed test were recovered for post-mortem analysis. Plug 5 was sectioned along a meridional plane along the r-axis of the target plate for analysis by optical microscope and SEM (see Fig. 12). The plug in Fig. 12 is oriented so that the labeled z-axis is in the direction of projectile motion (along the axis of the target plate).



(a) Optical Micrograph of Plug 5 Cross-section



(b) SEM Image of Plug 5 Surface

Figure 12. Optical and scanning electron microscope images of plug recovered from Plate 5

The cross-section of the plug showed center-point thickness of 1.35 mm (compared to an initial plate thickness of 1.6 mm) and a maximum width of 5.05 mm. The fracture surface of the plug occurs at an average angle $\alpha = 34^\circ$ and an angle relative to the r-axis of approximately 70° . The indentations on the bottom side of the plug show the real shape of the projectile nose, caused by the machining process. The outer fracture surface of the plug was inspected via SEM to look for signs of void growth, localized melting or recrystallization (in order to assess the possibility of thermal softening). The surface showed no signs of either, indicating normal plug development by the growth of a crack through the thickness of the target plate. The absence of any sign of excessive heating at this surface supports the conjecture by Gupta et al. (2006) that thin plates of this material—having high conductance and minimal thickness—show little temperature sensitivity and negligible thermal softening at low impact velocities.

The impact experiments performed, in summary, show several phenomena which must be adequately captured in any attempt to fully model a projectile/target interaction under the conditions given.

i. *Plugging*: Circumferential localization and fracture centered on the projectile tip leads to the separation of the target plate and the center plug, which is carried on by the projectile. The geometry of the plug depends strongly upon the shape of the projectile and the stress field induced by the interaction of the projectile with the target. A blunt projectile with sharp edges will induce very localized shear and associated plug failure, whereas a rounded projectile may produce more tearing and failure by tension at a higher triaxiality, similar to discing. Target thickness also strongly affects the development of the plug, with thicker plates favoring dominant shear plugging, especially for impact by blunt projectiles. Further discussion and analysis will be limited to thin plates, as tested experimentally, capable of significant out of plane plastic deformation.

ii. *Plug/Projectile Separation*: At the completion of the plugging process, the plug is fully separated from the target plate and in contact with the front surface of the projectile. As the projectile slows due to continued interaction with the target, the free plug separates and advances ahead of the projectile tip under its own inertia. The rate of separation varies according to the deceleration of the projectile post plug formation. At one extreme, only minimal frictional forces may be acting upon the projectile generating a minimal change in momentum between plug and projectile. Under combined mechanisms of failure, a highly tapered projectile may experience significant deceleration after plug formation, even failing to perforate the target completely. Separation of the plug from the projectile in a model response reflects the true

detachment of the plug from the plate and its continued movement under only inertia. Post-mortem analysis of a recovered plug offers several geometrical metrics that are easy to quantify with which to compare a model result, capturing elements of both the plastic behavior and failure behavior of the specimen.

iii. *Petalling*: The development of the plug leaves a hole in the target plate under hoop tension. This is similar to the hole expansion test that is typically used in forming limit estimates. Localization under this hoop tension leads to the development of radial cracks in the target plate. Propagation of these radial cracks, driven by continued penetration of the projectile, leads to petals which are bent out of plane by the projectile, allowing perforation of the target plate. Upon full perforation by a projectile, developed petals (limited to thin plates as discussed) curve beyond perpendicular to the target plane, appearing to bend back over the base of the petal. Experimental results show that the number of petals developed varies with the velocity of the projectile at impact. The number of petals, in this way, depends upon the strain-rate experienced by the target during the process of perforation. It is believed that the mechanism of this strain-rate “dependence” is much the same as that observed for the case of uniaxial tension in expanding rings discussed by Zhang and Ravi-Chandar, (2008). While the stress state is more complicated at the newly created plugging surface, the same Mott-like mechanism of localization and propagation of the associated stress-release wave (Mott, 1847) are fully capable of generating the observed dependence of petal number on impact velocity. At the plugging surface prior to radial crack formation, the dominant hoop stress in the plate gives a close qualitative analog to the expanding ring test in which the mechanism of inertia-driven, dynamic multiple localization and failure, independent of any material strain-rate dependence, was evidently supported.

iv. *Slant Fracture*: In the targets used, the fracture surfaces generated by the radial cracks forming petals display clear slant fracture character from plugging surface to petal base. In order to generate this behavior, through-the-thickness necking and localization behavior must occur. It was anticipated that this behavior may be very difficult to model under impact conditions using the methods available (see the works of El-Naaman and Nielsen, 2013; Hickey and Ravi-Chandar, 2016).

4. Numerical simulation of the ballistic impact experiments

A fully successful attempt to model ballistic impact with respect to the target-projectile system and conditions described previously must generate the full complement of

deformation and failure mechanisms observed experimentally. In addition to this, a fully accurate model must closely approximate the quantitative results of the experiment such as final target geometry and projectile velocity change. Historically, model accuracy has frequently been ensured by the tailoring of possibly several unconstrained model parameters (by means of a best fit procedure) to experiment results directly. While potential model capability can be demonstrated by means of this procedure, the predictive capability of any model is immediately compromised when formulated in this manner. The intention of this modeling effort is to assess the capability of a fundamentally simple material and failure model to capture the behavior of a projectile-target system under ballistic impact. Both the material and failure models utilized can be calibrated using reasonably simple laboratory tests, preserving an element of practical use-ability with respect to engineering analysis and design. It is recognized that the simple models utilized may not represent the material response accurately over all domains and material conditions experienced during ballistic impacts. The nature of the problem is such that extreme regimes of material response are interrogated. large plastic strains, high strain-rates, extremely localized deformation and the possibility of complicated temperature effects potentially create a large amount of uncertainty and inaccuracy in the modeling process. The complicated nature of ballistic impact and perforation in particular provides a powerful crucible with which to evaluate these material models.

In order to model the experiments detailed in the previous section, a series of dynamic finite element simulations were performed using ABAQUS Explicit. The experimental conditions were simulated initially using an axisymmetric model to evaluate the axisymmetric deformation (e.g. dishing) and plugging mechanisms observed in experimental results. Further evolution of the perforation process, with the exception of the blunt nose projectile (which is not analyzed further due to the large volume of work regarding this projectile-target configuration), becomes non-axisymmetric due to circumferential localization at the plug surface and subsequent radial cracking. Quarter-plate symmetric models were then evaluated to observe the aforementioned localization and petalling mechanisms. Both of these attempts were successful in capturing the fundamental behavior being investigated, but the implementation of symmetry in the models represented a fundamental disconnect from the real configuration and observed behavior. For these reasons only the results of complete geometry, full plate simulation are discussed from here on out, recognizing that the full plate model fundamentally contains all of the elements investigated in prior attempts.

The full target plate was modeled using a 150 mm diameter circular model plate with a rigidly clamped boundary condition along the outer circumferential surface, representing the interior deformable region of the target plate clamped on the outer boundary by the target fixture. The model plate (see Fig. 13) was meshed using 3,377,022

3D linear tetrahedral elements (C3D4) with a minimum element size of 50 μm at the center of the plate and a maximum size of 5 mm at the periphery. While the tetrahedral element allows the number of elements used through the thickness to be varied by radial location, at the center the minimum element size gives 32 elements through the plate thickness.

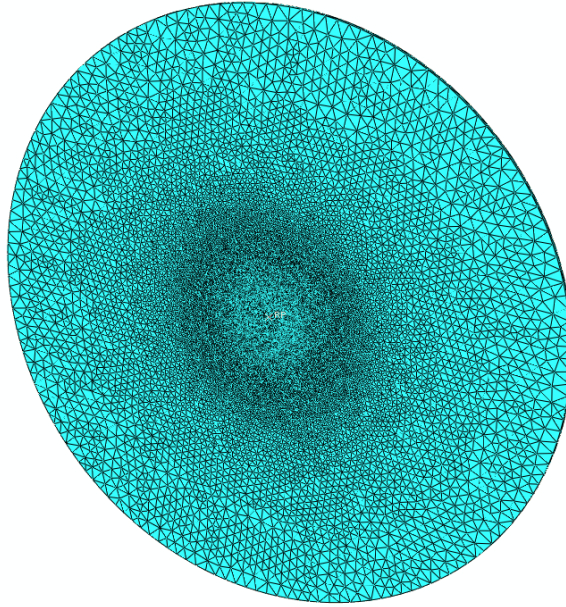


Figure 13. Target plate mesh

The partitioning and seeding of the mesh was controlled such as to generate maximally fine mesh only in the very central region in which the primary structural failure mechanisms are observed. The grain scale mesh size in the regions of failure is chosen to reflect the gage lengths over which the failure model discussed in Section 2.2 was calibrated. While material failure as implemented in dynamic finite element simulations is often based upon global measurements of specimen strain at failure, such models are appropriate to larger size elements representing the larger gage lengths over which failure strain is measured. As discussed previously, the substantial increase seen in material strain-to-failure when measured at gage lengths approaching the grain size dictates the use of more realistic gage lengths over which failure occurs in the model. While ductile material failure occurs on even smaller length scales, the use of continuum mechanics is limited by the size of some representative volume over which continuum model can be considered valid. When implementing both continuum level deformation mechanics and material failure by means of element deletion, some amount of compromise must be made. The implementation of elements on the order of grain scale

offers a potentially desirable compromise between the justifiable use of continuum mechanics and the implementation of continuum strain-to-failure models over a more realistic local gage length. While meshing at this scale pushes the justifiable limits of continuum mechanical formulations, it does afford mesh resolution sufficient to capture more intricate failure patterns than possible (while using element deletion) at coarser mesh scale. For these reasons, element size in the region of interest was chosen as to approach the grain scale. Tetrahedral elements were chosen in order to better control the number of elements through the thickness at different locations in the model plate and to ensure a more realistic, unstructured mesh in the region of interest. The projectile was modeled as a discrete rigid body using approximately 14,800 total rigid 3D quadrilateral and triangular elements (R3D4, R3D3). A summary of mesh and elastic material properties is given in Table 3.

Table 3: Model and Mesh Properties and Parameters

Target Mesh	
3,377,022 linear tetrahedrons	C3D4, free meshing
Min. element size	50 μm
Max. element size	5 mm
Max # elements through thickness	32

Standard isotropic J_2 plasticity was implemented in the model using the determined Ramberg-Osgood material fit (see Fig. 2) extrapolated to a strain level of 3. Johnson-Cook damage initiation was also incorporated in the model using the failure curve given in Fig. 5. Note that the Johnson-Cook damage as implemented in ABAQUS initiates material damage as discussed in Section 3. Specification of a means of damage evolution beyond initiation is required to generate material damage in the traditional sense, where material performance is degraded with increasing damage accumulation. In this case, instantaneous damage was implemented such that an element would reach maximum degradation and be removed from the mesh upon reaching strain-to-failure as indicated by the Johnson-Cook damage initiation criterion. As implemented in ABAQUS in this manner, material damage itself is non-cumulative but damage initiation is cumulative such that loading path influences the point of initiation of damage but material performance is not affected until the point of failure and element removal. While the results of Section 3 indicate that damage initiation should not be modeled cumulatively either, there is not a readily available way to model non-linear or non-cumulative damage initiation in ABAQUS. Table 1 gives both the R-O fit parameters utilized in the modeling

of the material plastic response and the damage initiation parameters and implemented in the ABAQUS Johnson-Cook damage model. Both the plasticity model and damage model utilized specify no strain-rate or temperature dependence, rendering the plastic response effectively identical to power-law hardening and strain-to-failure curve dependent only upon triaxiality. The objective of such simple modeling is to determine the essential (minimum) ingredients that are required to trigger the deformation and failure modes that are observed in the penetration experiments.

The material strain-rate dependence was directly evaluated in the ring expansion experiments, but material temperature dependence was not experimentally assessed in this work. While thermal softening is indeed present in materials of this type at sufficiently high temperatures, the low-velocity impacts evaluated in this work are unable to generate sufficient heating by means of plastic work to induce appreciable softening or temperature dependence. As investigated by Gupta et al. (2006), the relatively low speed, of the impact, relatively large distribution of plastic deformation (again limited to low speed impact events in thin targets), and high thermal conductivity of the material prevents significant temperature change during the perforation. The long time scale over which penetration and perforation occur (with respect to ballistic regimes) and the high thermal conductance of the aluminum alloy prevent significant localized heating as is observed in the extreme case of catastrophic adiabatic thermoplastic shear (Recht 1964) seen in some materials during high velocity blunt impact. For these reasons, temperature dependence is ignored for this analysis. However, under more violent loading, temperature effects may become significant and for the purposes of fully general analysis, would need to be evaluated and incorporated appropriately.

The rigid projectile was assigned a lumped inertia and given initial impact velocity by means of a predefined velocity field applied to the body. In order to most closely interrogate the violation of axisymmetry by means of only material localization and not geometrical asymmetry, the motion of the projectile was constrained to translation along the axis of the target/plate system (z-axis). In addition, rough contact was defined at the nose of the projectile, preventing slippage between the target and projectile nose surfaces during penetration. Due to the small minimum element size, the stable explicit time step increment as estimated by ABAQUS is very small—on the order of nanoseconds. The large overall size of the model combined with this small time increment make for a very long and intensive simulation. Simulations were performed on a Xeon processor with 8 cores and took over 3 weeks (wall-clock time) to complete. For this reason, only simulations at impact velocities of 150 m/s and 75m/s were attempted. Technical issues were encountered over the course of multiple attempts of the 75 m/s simulation, preventing its completion within the available time frame. Completed simulation results

for the 75 m/s second model are presented, but the remaining analysis will focus primarily upon the 150 m/s case.

Figure 14 shows a large field view of the 150 m/s impact simulation at the point of effective perforation. In this case, effective perforation is considered achieved when the contact patch between the target plate and projectile reaches beyond the shoulder of the tapered projectile, at which point only incidental frictional contact acts upon the projectile and no further radial expansion of the perforation hole is necessary to permit full passage of the projectile.

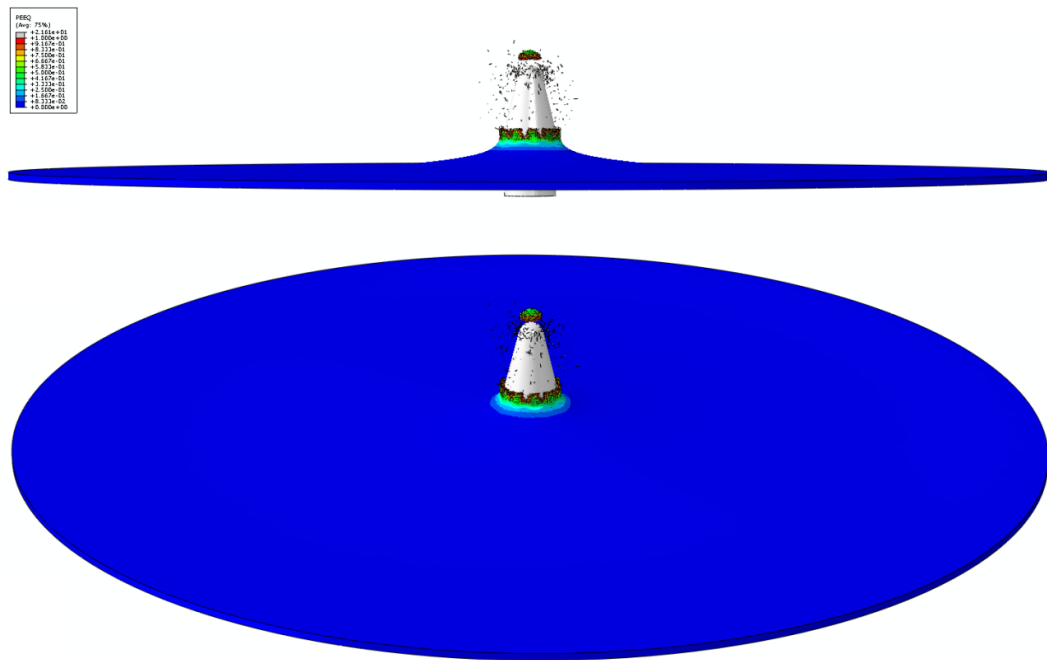


Figure 14. Full-field simulation result – 150 m/s initial velocity

Very little significant deformation occurs outside of the very center of the plate. Around the area of perforation, however, significant out of plane deformation by means of dishing can be observed. Focusing upon this central area, several mechanisms are apparent. Figure 15 shows the simulated perforation process in the center region. Note, two series of images are presented at corresponding simulation times. The A series of images gives a section view at the center plane of the target/projectile system, while the B series presents the full view of the outer surface of the plate viewed side-on.

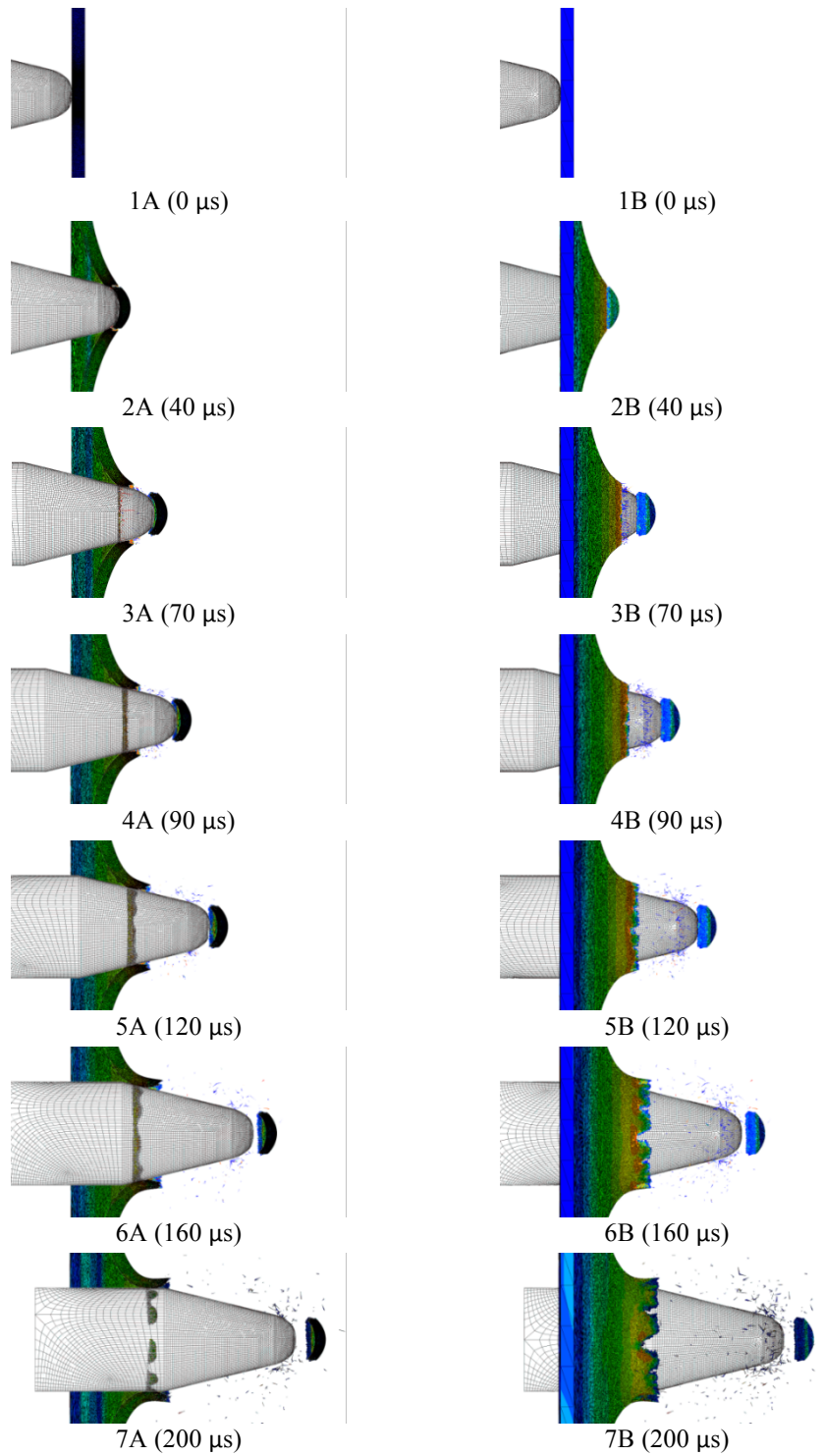


Figure 15. Simulation results – 150 m/s initial velocity

After initial plastic deformation of the target plate, plugging occurs at approximately $40\text{ }\mu\text{s}$ in 2A/2B. The continuing perforation of the plate sees the expansion of the hole axisymmetrically for a short time until the nucleation of radial cracks occurs around $70\text{ }\mu\text{s}$. The beginnings of radial cracks are seen in 3A/3B. Further progress by the tapered projectile drives the propagation of radial cracks, generating the fully formed petals observed upon the completion of effective perforation in 6A/6B. The A series of images (Fig. 15) also allows the observation of the contact patch between the projectile and target. The shape of the contact patch reveals the radial localization that breaks the axisymmetry of the event and the growth of petals as a function of perforation. Interestingly, 6A shows the remaining contact surface to be discontinuous, requiring the growth of petal cracks beyond the point of structural contact at the completion of effective perforation. Notably, plug separation is visible throughout the simulation, and while erosion at the tips of the simulated petals seems to remove some material and length from the petals themselves, back-bending of the final petal structure is visible in the model result.

A closer examination of the plugging process in the 75 m/s simulation is shown in Fig. 16. These images show a meridional slice near the region of impact. The nucleation of the plugging crack on the back surface of the target plate at an equivalent strain level of approximately 0.7 can be seen at $80\text{ }\mu\text{s}$. The circumferential plug crack propagates towards the projectile, fully separating the plug from the target plate by $118\text{ }\mu\text{s}$. The stress field generated by the projectile nose shape in conjunction with the target deformation determines the angle of the plug surface with respect to the initial target plane.

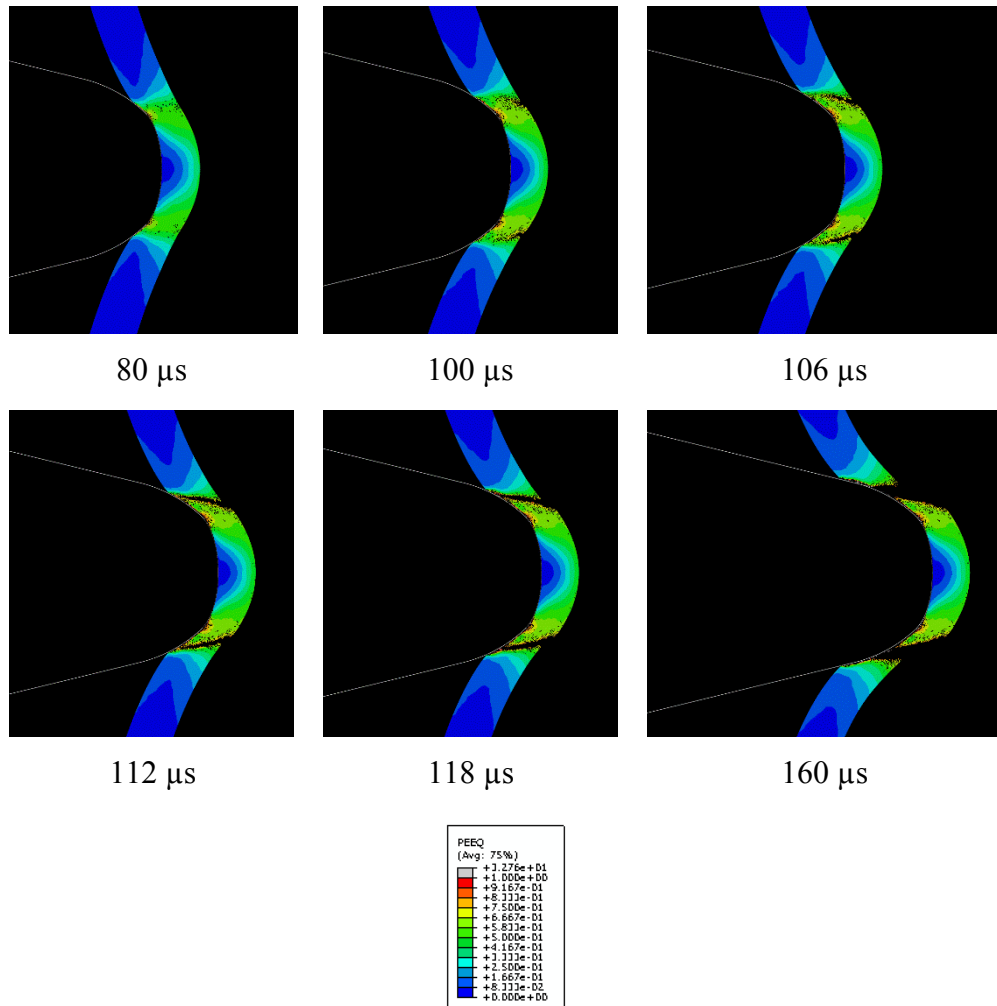


Figure 16. Plugging Process – 75 m/s initial velocity

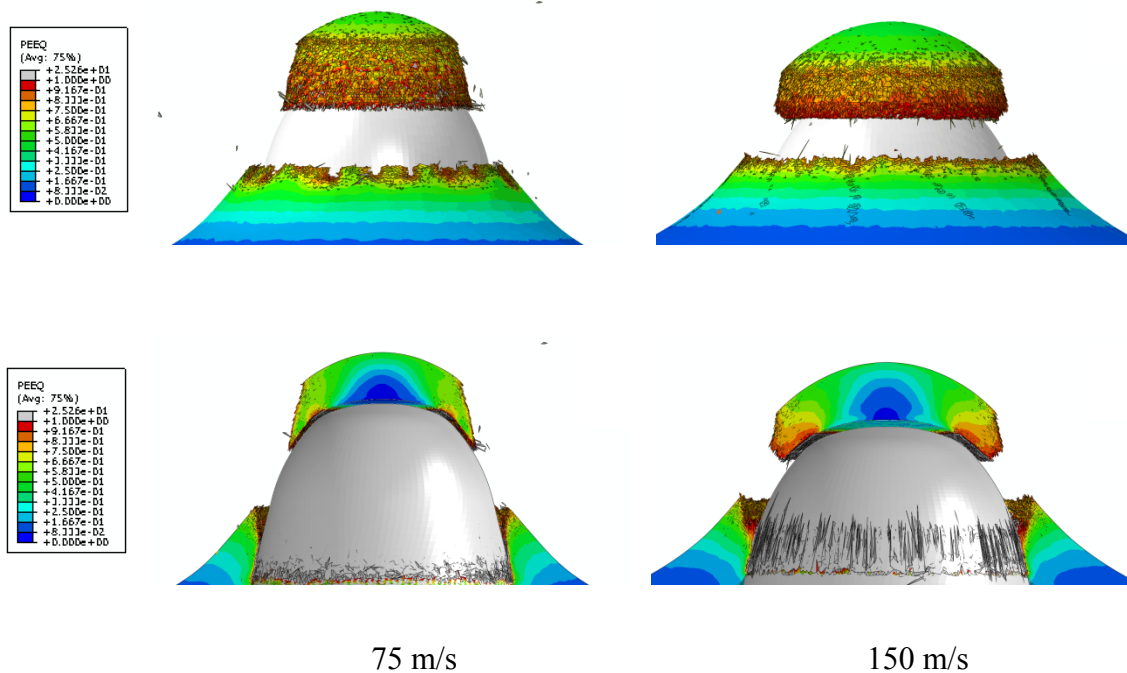


Figure 17. Plugs at 150 m/s and 75 m/s

Thin sections such as these will experience sufficient dishing, even with blunt nose projectiles, to generate a non-orthogonal plug surface angle. In contrast, a thick target may generate a truly cylindrical plug when impacted by a blunt nose projectile. Whereas the plug generated by the 75 m/s impact shows a distinct surface angle across the entire fracture surface, the 150 m/s plug shows tearing at the inner surface of the plug fracture surface (see Fig. 17). As the plugging crack extends, the effective thickness of the plug with respect to the inner surface of the target plate decreases significantly, tapering to a point as seen in the 75 m/s plug (Fig. 17). However, at 150 m/s the plugging crack “turns a corner” and completes the plug separation by means of a mode more indicative of tensile tearing. This behavior was not observed in the experimental results; however, it is possible that its presence in the model result is due to inaccuracies in the material model or local projectile geometry. Further experimentation in this study should attempt higher velocity experiments to evaluate whether this effect is real or a consequence of the finite element model. On a side note, the very beginnings of radial cracking can be seen on the surface of the plate for the 150 m/s result (Fig. 17). The maximum diameter and thickness of the plug are compared to experimental values in Table 4.

Table 4: Plug geometry measurements

Projectile speed – m/s	Central Plug Thickness (mm)	
	Experimental	Simulation
75	NA	1.28
150	1.35	1.33
	Plug Diameter (mm)	
	Experimental	Simulation
75	4.7	5.0
150	5.1	5.4

While the model predicts an absolute plug diameter slightly larger than seen experimentally, both experiment and simulation shows a very comparable change in plug diameter across the range of velocity. Likewise, simulation predicts an increase in center plug thickness with increasing velocity, a trend observed by previous works (Goldsmith and Finnegan, 1971). Note, the increase in plug thickness is expected as maximum plate deformation and dishing is expected near the ballistic limit, thinning the central region of the plate prior to plugging.

The next phase of perforation is dominated by the pettaling process. Figure 18 shows the development of petals in the 75 m/s simulation. After the separation of the plug, the effective stress is more or less axisymmetric around the newly created fracture surface (160 μ s). The highest hoop tension is seen at the edge of the hole, eventually localizing in the hoop direction and nucleating radial cracks at the outer plate surface of the hole edge. As these radial cracks grow, the petal material in between radial cracks is unloaded in the hoop direction (280 μ s). The highest stress is concentrated at the tip of the radial cracks and the associated base of the bending petal. As the contact patch between the projectile and petal begins to move backwards, the petal tip is fully unloaded (330 μ s).

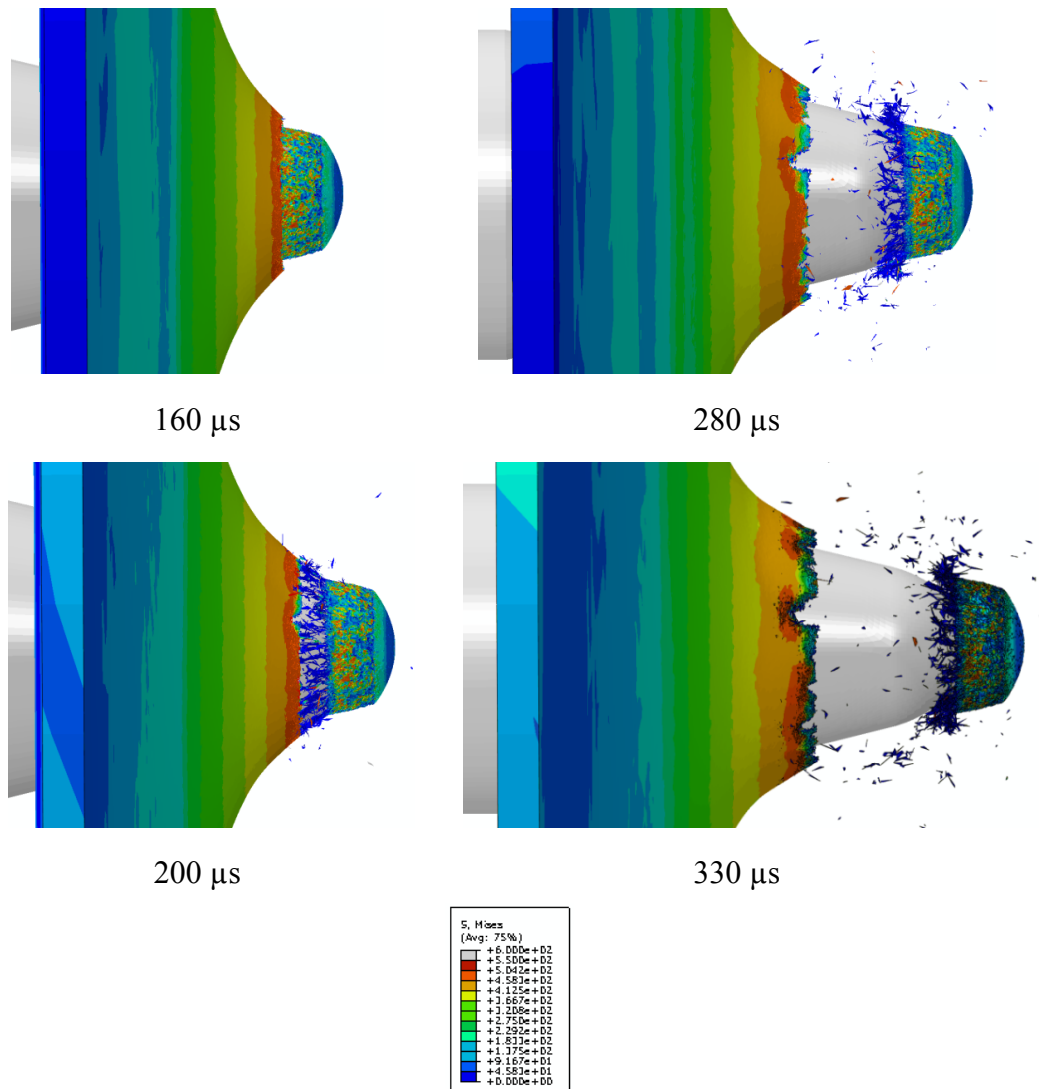
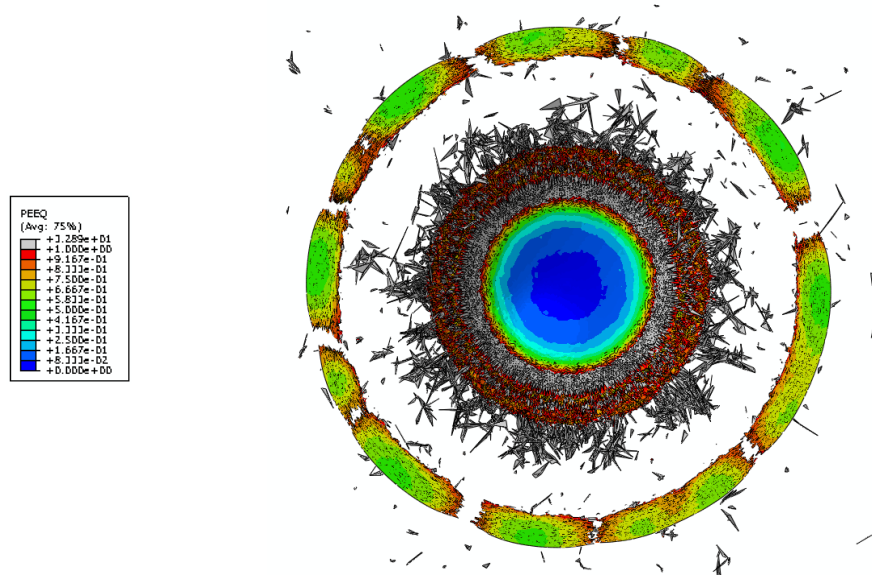


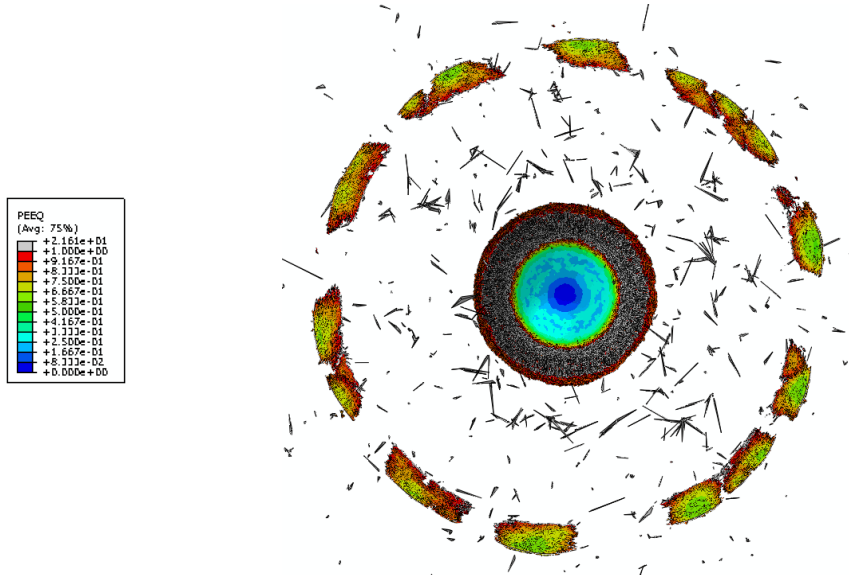
Figure 18. Development of petals in 75 m/s simulation

Both the experimental results and our expectations of dynamic petalling as an analogous process to the Mott style fragmentation predict the number of radial localizations and cracks to be dependent upon the strain rate--and ultimately the impact velocity--experienced by the material under dominant hoop tension at the edge of the plug hole. In order to observe the resulting petals clearly, Fig. 19 shows a section view parallel to the target plane, cutting through the developed petals. Note, the direction of

view is in the direction of projectile motion, but both the projectile and the majority of the target plate have been removed from view in order to clearly observe a plane where well developed petals are easily discerned. The back (concave) side of the plug is visible in the center of the images.



Impact speed: 75 m/s. 9-11 radial cracks are visible



Impact speed: 150 m/s. 11-15 radial cracks are visible

Figure 19. View cut of petal structure at 330 μ s (75 m/s) and 200 μ s (150 m/s)

While the model result generally over-predicts the number petals, a measurable increase in the number of developed petals is seen across the velocity range, as observed in experiment. Some deviation from experiment is expected in this case as the mechanism of element deletion provides a very coarse approximation of failure. However, the successfully modeled proportional variation of petals with impact velocity gives strong support to the assertion that the localization and fracture leading to the development of petals is fundamentally dynamic, and is governed by the same inertial principles present in Mott fragmentation. The combined effect of structural processes as these and material response ultimately determine the resistance of the target plate to perforation by a kinetic impactor.

The velocity of the projectiles as a function of time is given in Fig. 20. The 150 m/s and 75 m/s curves display a kink at approximately 40 μ s and 100 μ s respectively, corresponding to the plugging process seen in Fig. 16. At 190 μ s the 150 m/s projectile completes perforation. Note, the 75 m/s simulation was terminated at 330 μ s prior to full perforation.

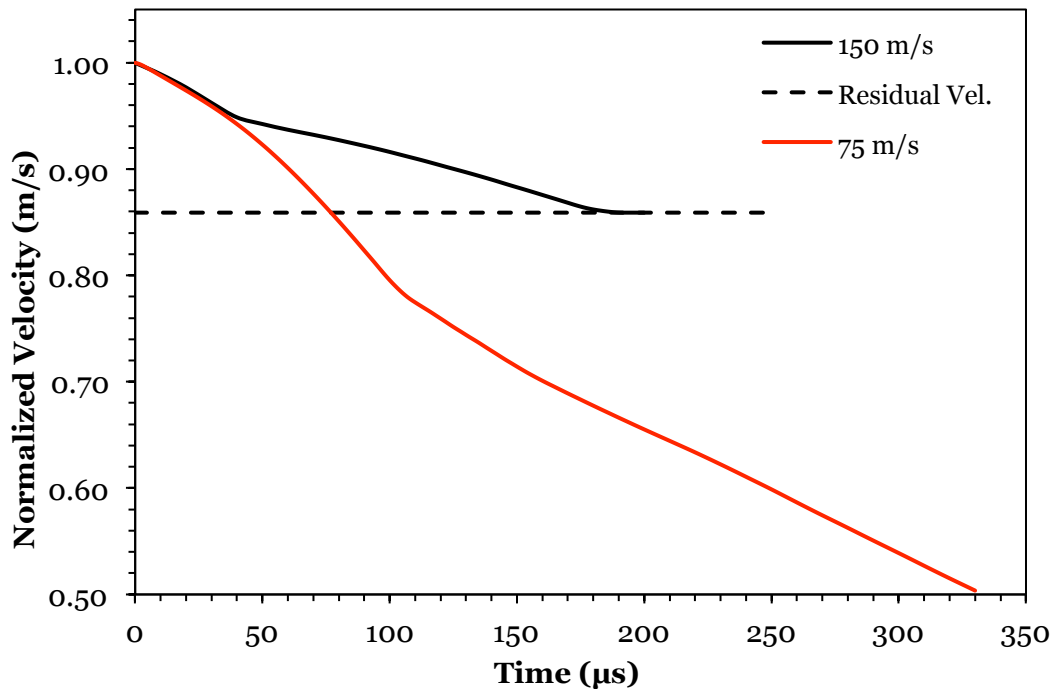


Figure 20. Normalized projectile velocity as a function of time

Prior to plugging, plastic plate deformation is responsible for the deceleration of the projectile. After plugging, failure processes dominate the resistance of the plate to perforation. Occurring over smaller volumes of material, localized failure processes such

as plugging will likely absorb less energy than global deformation mechanisms in total, leading to an overall decrease in the resistance of a target to penetration or perforation. An extreme case is the blunt nose impact experienced by Plate 4 (see Fig. 11), where the residual velocity indicates much reduced energy expenditure by the perforation process. Specifically, because plugging in the blunt case opens a hole the same size as the projectile diameter, no significant resistance is experienced subsequent to plug removal limiting target energy absorption to that achieved by relatively limited plastic deformation and extremely localized shear failure. Upon completion of effective perforation, the projectile velocity plateaus, giving the residual velocity of the projectile as predicted by the simulation. Here, the model falls significantly short, predicting a residual velocity of 129 m/s for the 150 m/s impact, well below the nearest comparable experimental value. The significantly elevated change in velocity seen in the simulation indicates excessive energy consumption by the perforation process. There are several possible sources for this discrepancy which may be effectively working in conjunction. Likely candidates are inaccuracies in the material plastic response and strain-to-failure model. Excessive hardening at large strains may contribute to the increased model stiffness. Additionally, the increased strain-to-failure measured at the length scales described may demand excessive plastic work for failure when implemented by means of element deletion across element sized volumes. Further evaluation of the total material model and its effects upon the numerical model when implemented as described must be undertaken in order to answer this discrepancy and generate a fully successful model. However, at this stage the primary goal of the simulation is to generate the entire host of deformation and failure mechanisms observed experimentally, rather than necessarily reproducing a quantitative value. Unknowns in the material model may generate the significant discrepancy seen in residual velocity, but in order to potentially solve this problem by means of more accurate material representation, the fundamental physics of the experiment must be present in the model. Successful simulation and presence of multiple mechanisms such as plugging and petalling indicate that the model is capable of achieving fundamental accuracy.

For more general comparison to experimental result, the final deformed model is given along with experimental post-mortem images of the relevant target plate in Fig. 21. The sectioned and imaged plug (150 m/s) is also compared with the simulation in Fig. 22.

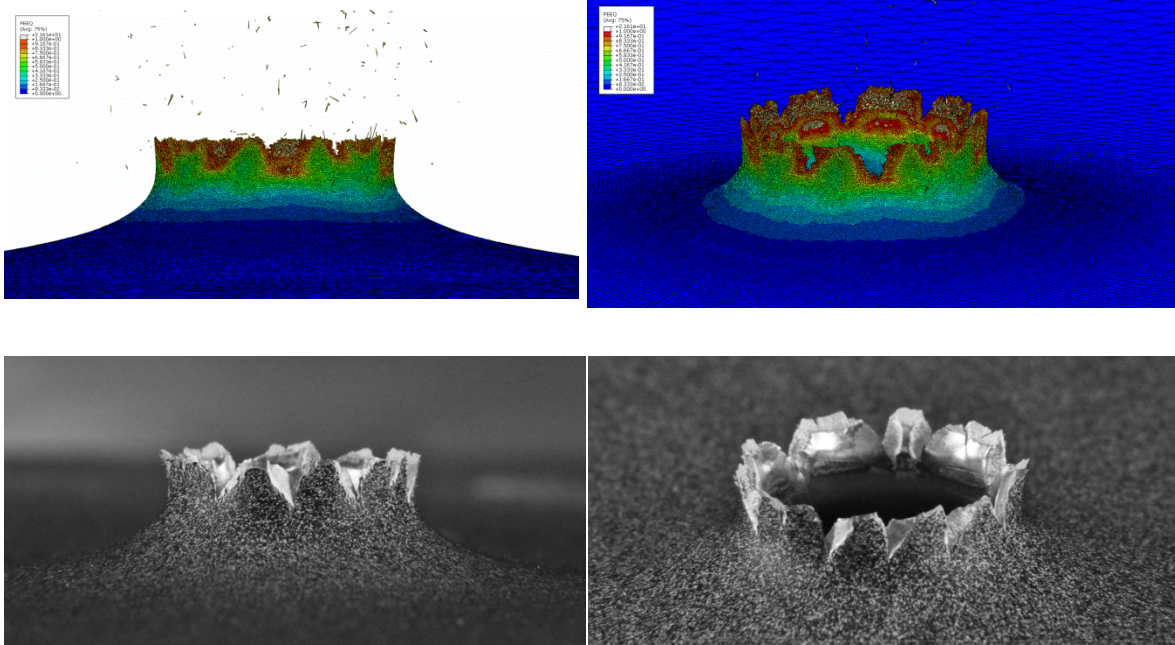


Figure 21: Post-mortem comparison of perforations (150 m/s)

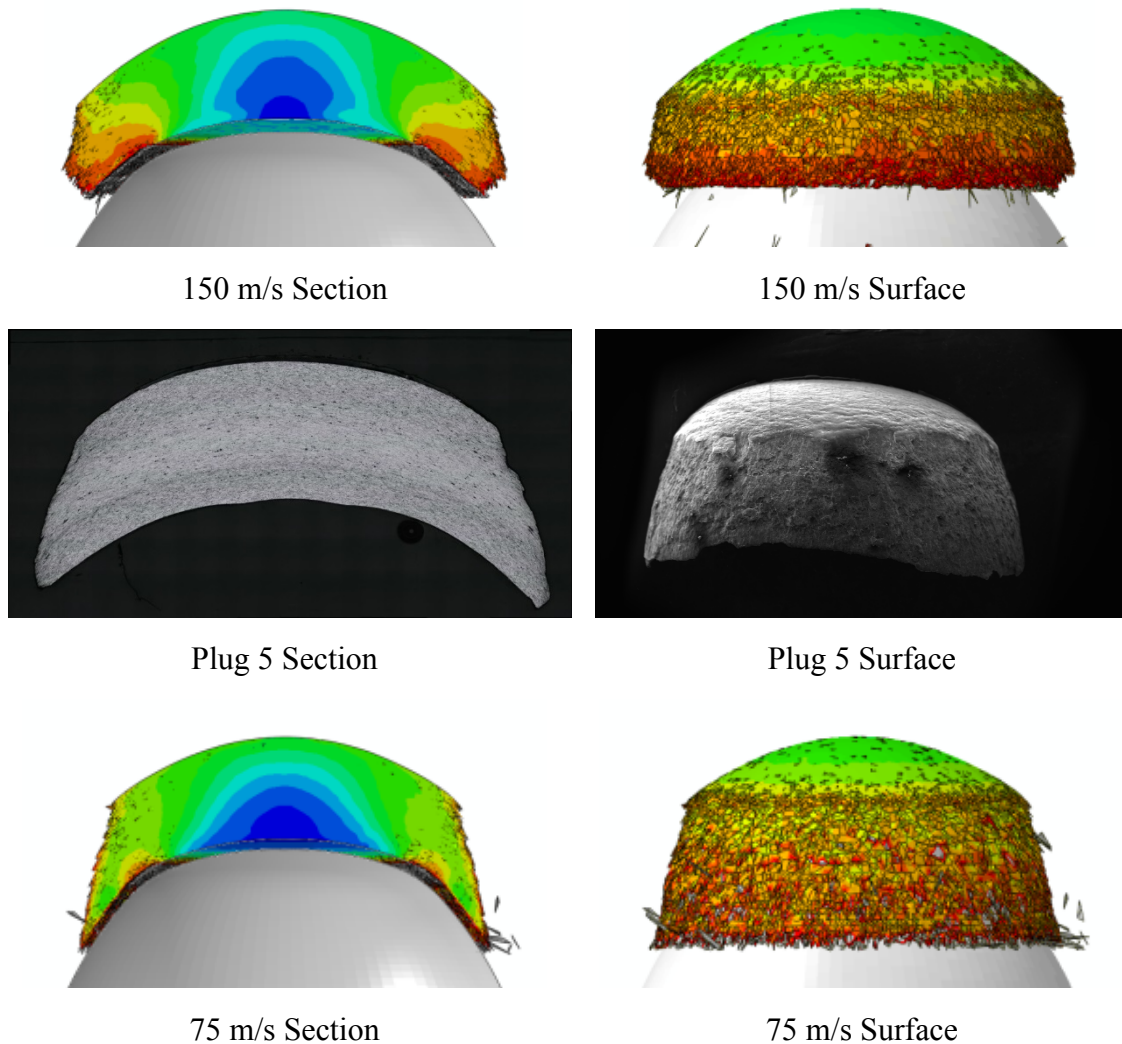


Figure 22: Comparison of the simulation plugs with Plate 5

Notably absent from the simulated result is the slant fracture observed in the petal fracture surfaces. While distinct slant fracture character can be seen in the post-mortem images from point of initiation (petal tip) to termination (petal base), definitive slant fracture was not observed in the simulation result, where the petal fracture surfaces indicated normal tensile tearing. While slant localization and fracture has been successfully modeled using similar material models previously in uniaxial tensile simulations, it is believed that in this case that the mesh and model scale utilized was

simply incapable of capturing the proper strain localization necessary to display this behavior. It is believed that even finer scale is likely necessary to generate sufficient mesh resolution through the thickness of the plate to observe slant behavior in the stock material and geometry considered here. The inability to capture this behavior is a significant failure of the given modeling attempt as slant fracture falls into the category of fundamental physical phenomena necessary to fully capture the behavior of the target/projectile system in full.

While not all the relevant behavior was captured in the attempted model, a number of fundamental phenomena were successfully generated to a level comparable to that of the experimental result. The primary failure mechanisms of plugging and petalling were captured and the important velocity dependent trends relating to these mechanisms were observed. Inaccuracy in the material model could relatively easily influence the absolute discrepancies seen in plug geometry and petal number. Physical phenomena such as plug-projectile separation were observed, ensuring proper interaction between contact surfaces and successful failure based modeling of the complete removal of the plug from the target plate. While the overall kinetic energy retention of the projectile saw significant discrepancy with experiment, a realistic and logical variation in velocity as perforation occurs was observed via simulation. Again, inaccuracy in the material or failure model may be responsible for the excessive resistance of the model target. Lastly, the model was incapable of capturing the slant fracture characteristic observed in experiment. Modification of the model to capture this behavior may very well be necessary to accurately predict residual velocity and ballistic limit of this projectile/target system without unjustifiable “a posteriori” modification of the material model in order to fit experimental results. Such tailored manipulation would of course compromise the usefulness of the modeling approach in general. Despite these failures, the successful modeling of these fundamental mechanisms of ballistic perforation utilizing only a very simple material representation and more realistically calibrated empirical failure model represents a valuable step in achieving comprehensive modeling success and potential predictive capability with respect to ballistic impacts of this type in general.

5. Conclusion

This work represents a combined experimental and modeling study of low velocity projectile perforation of thin AA5083-H116 aluminum plates. Experiments were performed in order to characterize the material and calibrate simple phenomenological models for both the constitutive response and ductile failure behavior.

- The material behavior was represented by a power-law hardening model; a Ramberg-Osgood model was calibrated to uniaxial tension experiments.
- Failure was characterized by a modified Johnson-Cook ductile failure model, with a dependence of the strain-to-failure on the triaxiality. The key point of departure was in identifying the strain-to-failure by measurements at the grain level.
- Numerical simulations were accomplished in ABAQUS, with an element deletion approach, but without degrading the material response prior to complete failure. The essential requirements for this are three-fold: first, the strain-to-failure must be calibrated at the scale of the grains. This indicates significantly larger plastic strains than from average measurements across specimen geometrical dimensions such as plate thickness. Second, as a consequence, the constitutive model must be extrapolated to significantly larger strain than achievable under homogeneous deformation conditions. Third, in order to permit large strains, and potentially strain gradients in highly localized regions, the discretization in the numerical implementation should be at a scale comparable to the grain size. This renders the simulations quite long and expensive.
- Simulations that implemented the above constraints were capable of capturing the macroscopic response in terms of the residual velocity of the projectile. In addition, the simulations were also capable of reproducing the observed localization behavior that results in the plug formation. Petalling of the plate, past the ejection of the plug was also reproduced in the simulations; however, the slant nature of the petalling fractures could not be captured correctly and remains an open problem.

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