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Formal Verification of Mathematics in HoTT-Lean

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14. ABSTRACT
This proposal supported the research group in Formalization of Mathematics in the Homotopy Type Theory system, implemented in the Lean proof assistant at Carnegie Mellon University. This research group consists of Professors Steve Awodey and Jeremy Avigad and their graduate students, in collaboration with other researchers in Computer Science. This group of expert researchers are pursuing a recent breakthrough that is reshaping the foundations of in logic, mathematics,and computer science. Using this breakthrough, new computer systems are being developed for the formal verification of mathematical theorems and critical computer software.
This proposal supported two graduate students and provided summer salary for one faculty member. This support strengthened the CMU research group and enabled a more robust effort.
Specific outcomes included two doctoral dissertations, an extensive library of formalized mathematics, and numerous conference and journal publications. The doctoral dissertation of Floris van Doorn developed a module for homotopy type theory within the new Lean proof assistant,
which is currently under development at CMU, and formalized a large amount of algebraic topology and homotopy theory therein, including the Serre spectral sequence. The doctoral dissertation of Egbert Rijke investigated synthetic homotopy theory and developed the new notion of an infinite-dimensional equivalence relation, as a step toward solving a longstanding coherence problem in homotopy type theory.
In addition to supervising the doctoral students and collaborating on their research, the PI investigated impredicative encodings of higher inductive types, including their realizability semantics.

15. SUBJECT TERMS
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Formal Verification of Mathematics in HoTT-Lean

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Summary

This project was used to support the research group in Formalization of Mathematics in the Homotopy Type Theory system, implemented in the Lean proof assistant at Carnegie Mellon University. This research group consists of Professors Steve Awodey and Jeremy Avigad and their graduate students, in collaboration with other researchers in Computer Science. This group of expert researchers are pursuing a recent breakthrough that is reshaping the foundations of in logic, mathematics, and computer science [?]. Using this breakthrough, new computer systems are being developed for the formal verification of mathematical theorems and critical computer software. The group at Carnegie Mellon University includes several faculty members, as well as postdocs and a senior researcher and several graduate students. This proposal supported one additional graduate student, as well as faculty summer salary. This support strengthened the CMU research group and enabled a more robust effort.

Specific results included research concluding with two doctoral dissertations conducted by the supported students, Floris van Doorn and Egbert Rijke under the supervision of the PI. van Doorn developed a module for homotopy type theory within the new Lean proof assistant currently under development at CMU and formalized a large amount of algebraic topology and homotopy theory, including the Serre spectral sequence. Rijke studied synthetic homotopy theory and developed the new notion of an infinite-dimensional equivalence relation, as a step toward solving a long-standing coherence problem in homotopy type theory. In addition to supervising the doctoral students and collaborating on their research, the PI investigated impredicative encodings of higher inductive types, including their realizability semantics.

*DoD grantee AFOSR Grant FA9550-15-1-0053; program manager Dr. Tristan Nguyen

Taken together, this research advances the overall project of developing new computational tools for mathematics. Several scientific papers were written and presented at international conferences and submitted for publication. In particular, the PI and each of the students each had a paper accepted to the very respected IEEE Logic in Computer Science conference, which was held in Oxford, UK in July 2018. Travel funding and summer support enabled the PI and one graduate student to participate in the Proofs, Types, and Sets research program at the Hausdorff Institute for Mathematics in Bonn, Germany, during the summer of 2018.

Technical Introduction

Homotopy type theory opens up a fundamentally new direction in the foundations of mathematics, computation, and information with powerful new principles of reasoning [**?**]; new, higher-dimensional data types not previously available in set theory and extensional type theory [**?**]; and an effective computational implementation of a kind not previously available [**?**]. Its applications range from allowing mathematicians to work with the "right" notion of equality for mathematical structures, to directly expressing higher mathematical concepts such as n-categories and homotopy types, to offering powerful and flexible new generic programming techniques that facilitate the development of certified software and formally verified proofs. At the conceptual center of these developments is the new homotopy-theoretic semantics for constructive type theories [**?**, **?**]. The new, computational proof assistants being developed on this basis will facilitate the large-scale formalization of logic and mathematics, with far-reaching practical implications for mathematics and information science.

The center of this research program is formed by the HoTT research group in the Departments of Philosophy and Mathematics at CMU, led by the PI and also consisting of Prof. Jeremy Avigad, postdocs Dr. Jonas Frey and Dr. Felix Welllen, and several doctoral students. In the past 3 years, this research group has developed into an extremely effective and productive team, advancing several different aspects of the project simultaneously, and coordinating research activities among the other researchers in CMU Computer Science and elsewhere (including collaborators at the Institute for Advanced Study (Princeton), Wesleyan University, University of San Diego, and international collaborators at MIT, Chalmers University (Sweden), University of Leeds (UK), and the University of Ljubljana (Slovenia)).

These activities are mainly in the following specific, distinct areas of research:

1. extending the original foundational system of Homotopy Type Theory by enhancements based on *cubical* methods from algebraic topoology in order to make the resulting system more algebraic, and thus more computationally tractable [**?**];

2. developing precise, mathematical semantics for the new foundational system on the basis of current homotopical methods, including various forms of cubical sets, (algebraic) Quillen model structures, and higher categories such as ∞-toposes [**?**];

3. deepening and extending applications to pure mathematics by pursuing the new, "synthetic" approach to homotopy theory made possible by the new system, specifically in the formalizaiton of the Serre spectral sequence and the elements of K-theory [**?**];

4. and collaborating with developers at Microsoft Research on the development of the new Lean proof assistant to implement the new foundations and build a library of basic results for future use [**?**].

The result of these research efforts, which is now coming into focus, will be the large-scale formalization of logic and mathematics in a powerful and flexible system of computer-verified proofs, which should ultimately enable the creation of scientific tools with impact on challenging problems of DoD interest.

Education and training

The research group in Homotopy type theory at CMU currently includes several graduate students in Pure and Applied Logic at CMU and three graduate students in Computer Science at CMU, all of whom participate actively in the research undertaken by the main investigators and postdoctoral researchers. These students thereby receive valuable training in the specific mathematical and computational skills and methods being used and developed, in addition to receiving general training in scientific practices such as planning and conducting research, writing for publication, lecturing to students and colleagues, refereeing for scientific journals, and supervising junior researchers.

Not only is much of the basic research conducted in close collaboration with these students, but their support of more senior researchers is vital in the advancement of this research as they write code for the computerized proof system, formalize existing mathematical results, verify informed conjectures, and investigate possible new directions of research.

Scope of Work

The HoTT research group around the PI currently has only limited funding for dedicated graduate students in Pure and Applied Logic, despite having the capacity for several more students. Currently, six talented and well-trained graduate students in Pure and Applied Logic are involved in the activities of the CMU research group around Awodey and Avigad, investigating the computational and semantical aspects of the foundational system, developing the code library for the new Lean proof assistant, and pursuing mathematical research in synthetic homotopy theory. Most of these students are supported directly by CMU through teaching assistant positions and must divide their time between the research group and their teaching duties.

The current proposal funded one dedicated graduate students to work full-time on the Homotopy Type Theory project. This position was divided semester-wise among the two graduate students involved in the project, Floris van Doorn and Egbert Pijke, in order to provide support for their research efforts and provide dedicated time to devote to the project. This also relieves the senior researchers of some of the more routine tasks, while involving more young scientists-in-training in the active and ongoing research program.

The proposal also included the standard travel funds for the student to attend conferences and research programs, as is required for scientific training and dissemination of results, as well as summer salary for the student researchers.

Summer salary for the PI, which is not available from other sources, was also provided for in this proposal to enabled dedicated work on the project for two summer months in Bonn, Germany. This permitted the PI to supervise and conduct joint research with the graduate student and the other researchers, all of whom are actively engaged in project research during this period.

Specific Outcomes

1. Steve Awodey, Jonas Frey, Sam Speight, Impredicative encodings of (higher) inductive types, LICS 2018, arXiv:1802.02820.

2. Simon Boulier, Egbert Rijke, Nokolas Tabareau, A coinductive approach to type valued equivalence relations, HoTT/UF Workshop 2018, Oxford.

3. Ulrik Buchholtz, Floris van Doorn, Egbert Rijke, Higher Groups in Homotopy Type Theory, LICS 2018, arXiv: 1802.04315.

4. Ulrik Buchholtz, Egbert Rijke, The real projective spaces in homotopy type theory, LICS 2018, arXiv:1704.0577.

5. Egbert Rijke, Michael Shulman, Bas Spitters, Modalities in homotopy type theory, Logical Methods in Computer Science (accepted for publication), arXiv:1706.07526.

6. Egbert Rijke, Classifying Types (CMU doctoral dissertation), July 2018.

7. Floris van Doorn, On the Formalization of Higher Inductive Types and Synthetic Homotopy Theory, (CMU doctoral dissertation), July 2018.

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Dr. Steven Awodey

Program Officer
The AFOSR Program Officer currently assigned to the award

Dr. Tristan Nguyen

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Abstract

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Taken together, this research advances the overall project of developing new computational tools for mathematics. Several scientific papers were written and presented at international conferences and submitted for publication. In particular, three separate papers were accepted to the respected IEEE Logic in Computer Science conference, which was held in Oxford, UK in July 2018. Travel funding and summer support enabled the PI and one graduate student to also participate in the Proofs, Types, Sets research program at the Hausdorff Institute for Mathematics in Bonn, Germany, during the summer of 2018.

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Archival Publications (published) during reporting period:

- Steve Awodey, Jonas Frey, Sam Speight, Impredicative encodings of (higher) inductive types, LICS 2018, arXiv:1802.02820.

- Simon Boulier, Egbert Rijke, Nokolas Tabareau, A coinductive approach to type valued equivalence relations,
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New discoveries, inventions, or patent disclosures:

Do you have any discoveries, inventions, or patent disclosures to report for this period?

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Please describe and include any notable dates

Do you plan to pursue a claim for personal or organizational intellectual property?

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Supplies			
Total			

Report Document

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Appendix Documents

2. Thank You

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