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RESEARCH INTO COORDINATING SENSING, CONTROL AND COMPUTATIONS FOR AUTONOMOUS VEHICLES

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14. ABSTRACT
The report addresses several strategies developed for coordinating sensing, estimation, control and computations in systems that must operate autonomously and satisfy constraints. The potential of the proposed algorithms is demonstrated in spacecraft simulation case studies. The developed algorithms and strategies contribute to advancing capabilities of autonomous vehicles, in particular, in the areas of handling constraints and non-standard sensing-estimation-actuation requirements. The proposed strategies also facilitate incorporating optimization-based control technology for onboard operation in autonomous vehicles.

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1. SUMMARY

The objective of the research was to advance algorithms for coordinating sensing, control and computations in systems that must operate autonomously and satisfy constraints. The focus was on situations where sensing, estimation, control and computations are inherently coupled and on developing strategies that are effective and could be straightforward to implement in real-world applications.

Specific progress has been made in the following areas:

1. Developing optimal periodic sequences of sensing and actuation actions in situations where sensing and actuation cannot be performed simultaneously,
2. Establishing a controller that guarantees the boundedness of state and state estimation errors and the eventual satisfaction of the imposed probabilistic chance constraints,
3. Demonstrating the effectiveness of this controller in simulations of an autonomous spacecraft application,
4. Developing a computational reference governor for safe online optimization. Demonstrating the potential of the computational reference governor in simulations when paired with a nonlinear model predictive controller,
5. Extending the virtual net constrained motion planning framework to the case when only output measurements are available, there is random measurement and process noise, and the states of the system are estimated. Demonstrating the potential of the proposed approach for spacecraft maneuvering while avoiding obstacle collision.

The following are publications which have appeared, been accepted or are in preparation related to the subject matter of this report:

- Sutherland, R., Kolmanovsky, I., Girard, A., Leve, F., Petersen, C., "Optimal strategies for disjunctive sensing and control" [1],
- Sutherland, R., "Advances in disjunctive and time-optimal predictive control methods," [15],
- Berning, A., Kolmanovsky, I., Girard, A., Leve, F., and Petersen, C., "Maneuver planning using chained chance constrained admissible sets," [14],
- Sutherland, R., Kolmanovsky, I., Girard, A., Leve, F., Petersen, C., "Minimum-time model predictive spacecraft attitude control for waypoint following and exclusion zone avoidance" [2],
- Sutherland, R., Kolmanovsky, I., Girard, A., Leve, F., and Petersen, C., "On closed-loop Lyapunov stability with minimum-time MPC feedback laws for discrete-time systems," [13]

2. INTRODUCTION

A key consideration in the design of control schemes for autonomous vehicles in general, and for spacecraft in particular, is the enforcement of constraints. Control actuation limits and pointwise in time and terminal state constraints must be satisfied. In this report, we summarize several research developments and outcomes in developing strategies for control of systems with constraints in situations when there is an interplay between sensing, actuation, estimation, control and computations. In particular, a non-standard setting of disjunctive sensing and control, when simultaneous actuation and sensing is not possible will be addressed. Furthermore, the notion of computational reference governor (CRG) to monitor and supervise the computational optimization within the nonlinear model predictive controller (NMPC) will be introduced and illustrated with simulation results. Finally, the previously proposed framework for motion planning/maneuvering based on chained invariant sets will be extended to the setting of output measurements and state estimation.

3. METHODS, ASSUMPTIONS, AND PROCEDURES

3.1 Optimal Strategies for Disjunctive Sensing and Control

3.1.1 Disjunctive Sensing and Actuation for Linear Systems.

Consider a system represented by a discrete-time linear model with stochastic state disturbance and measurement noise inputs, given by

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where x_k is a vector state and u_k is a vector control. The variables w_k and v_k are, respectively, the state disturbance and the measurement noise inputs, each assumed to be a sequence of zero-mean, independent (and jointly independent) identically distributed (i.i.d.) random variables with $\mathbb{E}[w_k w_k^T] = \Sigma_w = \Sigma_w^T,\ \mathbb{E}[v_k v_k^T] = \Sigma_v = \Sigma_v^T$.

We consider a fixed-gain state feedback,

$$u_k = u_T + K(\hat{x}_k - x_T), \tag{2}$$

where $\hat{x}_k$ is the state estimate generated by a fixed-gain observer of the form

$$\hat{x}_{k+1} = A\hat{x}_k + \eta_k B u_k + (1 - \eta_k)L(y_k - C\hat{x}_k). \tag{3}$$

The binary variable $\eta_k \in \{0, 1\}$ represents the system operating mode. When $\eta_k = 1$, the control is applied to the system. When $\eta_k = 0$, the control is deactivated ($u_k = 0$), and the sensed output y_k is obtained. The target equilibrium is denoted by x_T, and it is assumed that the feedforward control input, u , supports it in steady-s tate in the absence of w_k and with $\eta_k = 1$, i.e.,
$x_T = \quad _T + \quad .$

In a typical control design process, due to the Separation Principle, the gains K and L would be determined without consideration of the interference between actuation and sensing (possibly by different engineers) and then a coordination mechanism introduced by specifying $\eta_k,\ k \geq 0$. In this setting, offline-generated N-periodic switching sequences, $\{\eta_k\}$, with $\eta_k = \eta_{k+N}$ for all $k \in \mathbb{Z}_{\geq 0}$, $N \in \mathbb{Z}_{\geq 0}$, are of particular interest, so that their repeated application leads to the attainment of the control objectives, including eventual satisfaction of state constraints. This approach, based on the application of the offline generated periodic sequence, has low computational footprint and is appealing in view of limited computing power and restrictive electrical power consumption budget onboard of small spacecraft.

3.1.2 Admissible Periodic Switching Sequences.

Suppose that $\{\eta_k\}$ is fixed and define $\bar{A}_k = A + \eta_k BK,\ \tilde{A}_k = A + (1 - \eta_k)LC$. The evolution of the state, x_k, and of the state estimation error, $e_k = x_k - \hat{x}_k$, are driven by

$$\begin{aligned} x_{k+1} &= Ax_k + \eta_k BK\hat{x}_k + w_k \\ &= \bar{A}_k x_k - \eta_k Be_k + \eta_k Bu_T + w_k, \end{aligned} \tag{4}$$

and

$$\begin{aligned} e_{k+1} &= [A + (1 - \eta_k)LC]e_k + w_k + (1 - \eta_k)Lv_k \\ &= \tilde{A}_k e_k + w_k + (1 - \eta_k)Lv_k. \end{aligned} \tag{5}$$

In simulations, we will assume that $x_T = 0$ (and so $u_T = 0$). Based on the error dynamics and assumed independence and zero mean properties of the stochastic disturbance, w_k, and measurement noise, v_k, the error covariance matrix $P_k = \mathbb{E}[e_k e_k^T]$ satisfies

$$\begin{aligned} P_{k+1} &= \tilde{A}_k P_k \tilde{A}_k^{\mathrm{T}} + R_k, \\ R_k &= \begin{bmatrix} (1-\eta_k)L & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Sigma_v & \mathbf{0} \\ \mathbf{0} & \Sigma_w \end{bmatrix} \begin{bmatrix} (1-\eta_k)L^{\mathrm{T}} \\ \mathbf{I} \end{bmatrix}. \end{aligned} \tag{6}$$

Definition: Let A, B, K, L, $\bar{A}_k$, and $\tilde{A}_k$ be defined as above, with none of $\bar{A}_k$, $\tilde{A}_k$ nilpotent. An N-periodic sequence of binary integers $\{\eta_0, \eta_1, \dots, \eta_{N-1}\}$, where, for all $k \in \mathbb{Z}_{\geq 0}$, $\eta_k \in \{0, 1\}$ and $\eta_{k+N} = \eta_k$, is called admissible if the following contractivity conditions hold:

$$\rho(\bar{A}_{N-1}\bar{A}_{N-2}\cdots\bar{A}_0) = \bar{q}_A < 1, \tag{7}$$

$$\rho(\tilde{A}_{N-1}\tilde{A}_{N-2}\cdots\tilde{A}_0) = \tilde{q}_A < 1, \tag{8}$$

where $\rho(\cdot)$ denotes the spectral radius operator. A non-periodic sequence of binary integers $\{\eta_0, \eta_1, \dots\}$ is admissible if

$$\lim_{k\to\infty} \rho(\bar{A}_k\bar{A}_{k-1}\cdots\bar{A}_0) = 0, \tag{9}$$

$$\lim_{k\to\infty} \rho(\tilde{A}_k\tilde{A}_{k-1}\cdots\tilde{A}_0) = 0. \tag{10}$$

The constraint against nilpotent matrices ensures that the above limits approach zero rather than "jump" to zero. These definitions are consistent with the properties of discrete-time state transition matrices found in, for example, Chen [4]. Note that the existence of a non-periodic admissible sequence implies the existence of periodic admissible sequences.

In the existence of a periodic admissible sequence, the state and error dynamics coupled with $\mathbb{E}[v_k] = 0$, $\mathbb{E}[w_k] = 0$ imply that the state and estimation error mean, $\mu_{x,k} = \mathbb{E}[x_k]$ and $\mu_{e,k} = \mathbb{E}[e_k]$, respectively, satisfy,

$$\begin{aligned} \mu_{x,k+1} &= \bar{A}_k\mu_{x,k} - \eta_k B\mu_{e,k} + \eta_k B u_T, \\ \mu_{e,k+1} &= \tilde{A}_k\mu_{e,k}, \end{aligned} \tag{11}$$

where $\bar{A}_k$ and $\tilde{A}_k$ are periodic with the same period, N, as η_k. The following results can be proven (see [1] for details):

Proposition 1: Suppose that the contractivity conditions hold. Then, as $k \to \infty$, $\mu_{e,k} \to 0$ and $\mu_{x,k} \to \mu^s_{x,k}$ exponentially, where $\{\mu^s_{x,k}\}$ is the unique N-periodic solution of $\mu_{x,k}$ with $\mu_{e,k} \equiv 0$. Furthermore, if $u_T = 0$, then $\mu^s_{x,k} = 0$.

Proposition 2: Suppose that the second contractivity condition holds. Then, the error covariance matrix, P_k, is bounded and, as $k \to \infty$, converges to the unique N-periodic solution, $\{P^s_k\}$, with $P^s_{k+N} = P^s_k$. In addition, for any $n \in \mathbb{Z}_{\geq 0}$,

$$\left(\|P_{N(n+1)}\| - \frac{\gamma}{1-\tilde{q}_A^2}\right) \leq \tilde{q}_A^2 \left(\|P_{Nn}\| - \frac{\gamma}{1-\tilde{q}_A^2}\right), \tag{12}$$

where

$$\gamma \geq \|\tilde{A}_{N-1}\cdots\tilde{A}_1 R_0 \tilde{A}_1^{\mathrm{T}}\cdots\tilde{A}_{N-1}^{\mathrm{T}} + \cdots + \tilde{A}_{N-1}R_{N-2}\tilde{A}_{N-1}^{\mathrm{T}} + R_{N-1}\|. \tag{13}$$

Remarks:

1) The steady-state periodic solution, $\{P^s_k\}$, can be computed by solving the conventional discrete-time Lyapunov equation for the evolution of the lifted system error covariance matrix, i.e., of $P^l = \mathrm{diag}\{P^s_0, \cdots, P^s_{N-1}\}$.
2) $\limsup_{k\to\infty} \|P_{Nk}\| \leq \gamma / (1 - \tilde{q}_A^2)$.

3) The results in Propositions 1 and 2 generalize to non-constant N-periodic feedback and observer gains. However, the analysis results benefit from both $\bar{A}_k$ and $\tilde{A}_k$ having only two possible values each, which is the case when the gains are constant.

We can take advantage of the dwell time conditions for stability analysis of hybrid systems to develop simpler sufficient conditions that can inform procedures for faster determination of admissible switching sequences. The discussion of the dwell time conditions follows Theorem 4.1 in Yu [5] and its proof.

Let $\bar{\Omega}_0 = A$ and $\bar{\Omega}_1 = A + BK$, and consider the first contractivity condition. By Gelfand's theorem [6], $\lim_{k\to\infty} \|\Omega^k\|^{1\backslash k} = \rho(\Omega)$, for any matrix Ω and norm $\|\cdot\|$. This implies that there exist constants c_0, c_1 that do not depend on k such that for any $k \geq 1$,

$$\|\bar{\Omega}_0^k\|^{\frac{1}{k}} \leq c_0\rho(\bar{\Omega}_0), \quad \|\bar{\Omega}_1^k\|^{\frac{1}{k}} \leq c_1\rho(\bar{\Omega}_1). \tag{14}$$

For any consistent matrix norm, there exists a finite k^* such that $\left\|A^{k^*}\right\|^{1/k^*} \geq \|A^k\|^{1/k}$ for all $k \geq 1$. Then,

$$c_i = \frac{\|\bar{\Omega}_i^{k^*}\|^{\frac{1}{k^*}}}{\rho(\bar{\Omega}_i)}. \tag{15}$$

Let $c = \max\{c_0, c_1\}$, $n_0 < N$ be the total time spent in the mode $\eta = 0$, $n_1 < N$ be the total time spent in the mode $\eta = 1$, and n_s be the number of switches plus one. Then, the first contractivity condition holds if

$$c^{n_s}\rho(\bar{\Omega}_0)^{n_0}\rho(\bar{\Omega}_1)^{n_1} \leq \bar{q}_A < 1. \tag{16}$$

Taking the logarithm of each side, it follows that this contractivity condition holds if

$$n_s \log c + n_0 \log \bar{\rho}_0 + n_1 \log \bar{\rho}_1 < 0, \tag{17}$$

where $\bar{\rho}_0 = \rho(\bar{\Omega}_0)$ and $\bar{\rho}_1 = \rho(\bar{\Omega}_1)$. Similar analysis can be done in the case of the second contractivity condition to yield the complementary inequality

$$n_s \log c + n_1 \log \tilde{\rho}_0 + n_0 \log \tilde{\rho}_1 < 0. \tag{18}$$

When seeking admissible sequences, it is possible to restrict the search to sequences for which n_0, n_1, and n_s satisfy both logarithmic conditions, which together are sufficient (but not necessary) to ensure admissibility.

The search for admissible sequences can be made faster still by discarding sequences that replicate a known inadmissible sequence.

Definition: A sequence $\{s_N\}$ of length $N \in \mathbb{Z}$ is called reducible if there exists a subsequence $\{s_k\}$ of length $k \in \mathbb{Z}_{>0}$ such that $k < N$, N is a multiple of k, and $\{s_N\} = \{s_k\} \oplus \{s_k\} \oplus \cdots \oplus \{s_k\}$, N/k times, where $\oplus$ is used to denote sequence concatenation. A sequence which is not reducible is called irreducible.

Proposition 3: Every (non-empty) sequence $\{s_N\}$ contains a unique irreducible subsequence $\{s_n\}$.

Proposition 4: A binary sequence $\{s_N\}$ is admissible if and only if the associated irreducible subsequence $\{s_n\}$ is admissible.

Any reducible admissible sequence can be formed by propagating an irreducible admissible sequence forward in time. Thus, when investigating sequences of length N for admissibility, all sequences for which we have already evaluated the associated irreducible sub-sequence can be discarded.

3.1.3 Chance Constraints.

Consider the chance constraint,

$$\forall k \geq k^*, \ \text{Prob}(\{x_k \in X\}) \geq 1 - \delta, \tag{19}$$

where X is the set prescribed by the constraints of the problem, $0 \leq \delta < 1$, and k^* is sufficiently large. Define $z = [x^T \ e^T]^T$ and $\zeta = [v^T \ w^T]^T$. Then, $z_{k+1} = \breve{A}_k z_k + \breve{\Gamma}_k \zeta_k + \breve{G}_k u_T$, where

$$\breve{A}_k = \begin{bmatrix} \bar{A}_k & -\eta_k BK \\ \mathbf{0} & \tilde{A}_k \end{bmatrix}, \ \breve{\Gamma}_k = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ (1-\eta_k)L & \mathbf{I} \end{bmatrix}, \tag{20}$$

and $\breve{G}_k = [\mathbf{0}^T \ \eta_k B^T]^T$. Under the contractivity conditions, we repeat the analysis in Propositions 1 and 2 for z_{k+1}. Let $\breve{P}_k = \mathbb{E}\left[(z_k - \mu_{z,k})(z_k - \mu_{z,k})^T\right]$, where $\mu_{z,k} = \mathbb{E}[z_k]$. Then,

$$\breve{P}_{k+1} = \breve{A}_k \breve{P}_k \breve{A}_k^{\mathrm{T}} + \breve{\Gamma}_k \begin{bmatrix} \Sigma_v & \mathbf{0} \\ \mathbf{0} & \Sigma_w \end{bmatrix} \breve{\Gamma}_k^{\mathrm{T}}, \tag{21}$$

and $\breve{P}_k \to \breve{P}_k^s$ as $k \to \infty$, where $\breve{P}_k^s$ is the unique periodic solution to the above system. Then, the steady-state covariance matrix satisfies $\breve{P}_{x,k}^s = [\,\mathbf{I}\ \ \mathbf{0}\,]\breve{P}_k^s\,[\,\mathbf{I}\ \ \mathbf{0}\,]^{\mathrm{T}}$. Thus, x_k converges to x_k^s, which is a cyclostationary process with the N-periodic mean $\mu_{x,k}^s$ and N-periodic covariance matrix $P_{x,k}^s$.

We have made no assumptions on the probability density functions of w_k and v_k, so to treat the chance constraint we resort to the multivariate Chebyshev's inequality [7][8], which states

$$\mathrm{Prob}\left((x_k-\mu^{\mathrm{s}}_{x,k})^{\mathrm{T}}(P^{\mathrm{s}}_{x,k})^{-1}(x_k-\mu^{\mathrm{s}}_{x,k})\le\alpha_x^2\right)\ge 1-\frac{n_x}{\alpha_x^2}, \tag{22}$$

where n_x is the dimension of x and $0<\alpha_x\le 1$. Given $\delta>0$, choose $\alpha_x=\sqrt{\mathrm{n_x}/\delta}$. Suppose that for $k=0,\dots,N-1$, the following condition is verified:

$$\mu^{\mathrm{s}}_{x,k}\in X\sim\mathscr{E}\left(\mathbf{0},\frac{1}{\alpha_x^2}(P^{\mathrm{s}}_{x,k})^{-1}\right), \tag{23}$$

where $\mathcal{E}(\mathbf{0},S)=\{x\colon x^TSx\le 1\}$ is an ellipsoidal set and ~ denotes the Pontryagin's set difference. Then, in steady state, the chance constraint holds.

3.1.4 Selecting an Optimal Admissible Sequence.

Since multiple admissible sequences may exist, we can select one by minimizing a cost functional. Consider the blended cost J that penalizes the estimation, the control objective, and the control effort:

$$J=\frac{1}{N}\sum_{k=0}^{N-1}\left(\mathrm{Tr}\left(R_eP^{\mathrm{s}}_k\right)+\mathrm{Tr}\left(R_xP^{\mathrm{s}}_{x,k}\right)+r_\eta\eta_k\right), \tag{24}$$

where $R_e=R_e{}^T$ and $R_x=R_x{}^T$ are positive semi-definite weighting matrices and $r_\eta\ge 0$ is a scalar weight. The N^{-1} factor in the cost functional normalizes for sequence length, ensuring that a given reducible/irreducible sequence pair will yield the same cost. To find the optimal sequence, fix N, check sequences for admissibility, then select the admissible sequence that yields the lowest cost. If no admissible sequence exists, then extend the sequence length and search again. The equivalence of admissibility between a sequence and its associated irreducible subsequence is invoked after incrementing the sequence length; if the subsequent search produces a reducible sequence for which the irreducible subsequence has already been evaluated, then the sequence can be skipped.

3.2 Computational Reference Governors for Safe Online Optimization

As spacecraft become more autonomous they will need increasingly complex online decision-making capabilities. One of the dominant paradigms for autonomous decision making is online optimization where actions are computed by solving a suitably formulated optimization problem. Optimization based frameworks have the major advantage that constraints can be naturally incorporated into the decision making process. In the context of control, the most common type of online optimization method is Model Predictive Control (MPC) wherein a control action is computed by solving a constrained Optimal Control Problem (OCP) at each sampling instance. If the dynamics are nonlinear, the OCP is necessarily non-convex and solving it in real-time is a significant challenge. In addition, the question of optimizer reliability is paramount.

Our research into safe online optimization has led to the development of a computational reference governor (CRG). By treating the optimizer as a dynamic system, we can impose constraints on it, e.g., that its optimality error be small, using the reference governor framework [9]. Our CRG essentially extends the continuous time computational reference governor developed in [3].

to the discrete time setting common in MPC applications. We have demonstrated the efficacy of the CRG for spacecraft orientation control benchmark problem and are in the process of developing the underlying mathematical theory.

3.2.1 Assumptions.

We consider a rigid spacecraft, actuated by three orthogonal thrusters. The orientation of the spacecraft is parametrized by 3-2-1 Euler angles. It is assumed that the attitude representation does not approach singularity; this is consistent with the limitations of the target physical testbed.

The equations of motion are given by

$$I\dot{w} + w^{\times}Iw = u + d \,, \qquad \dot{\theta} = S(\theta)w \tag{25}$$

where w is the vector of angular velocities, θ are the Euler angles, $S(\theta)$ is the inverse kinematic matrix, I is the moment of inertia matrix in a body-fixed frame, u are the control torques and d are unknown, possibly state dependent, disturbances. Collected into a compact form, the model is written as

$$\dot{x} = f_c(x, u, d), \tag{26}$$

where $x = [w^T \ \theta^T]^T$.

The prediction model for NMPC is a discretized approximation of the continuous time model (using the Forward Euler integrator and zero order hold assumption) and is given by

$$x_{k+1} = x_k + t_s f_c(x_k, u_k, 0), \tag{27}$$

where t_s is the discretization time step. The discrete-time prediction model is then considered in the following form,

$$x_{k+1} = f(x_k, u_k). \tag{28}$$

Note that since the disturbances are unknown, they cannot be incorporated into the prediction model and are assumed to be zero. At time $t = kt_s$, the state estimate is denoted by $x_0 = \hat{x}(t)$.

3.2.2 Methods.

3.2.2.1 Nonlinear Model Predictive Control.

The NMPC optimal control problem (OCP) to be solved for the control to be applied, $u(t) = u_0$ has the following form,

$$\begin{aligned} \min_{z} \quad & e_i^T Q_f e_i + \sum_{i=0}^{N-1} ||e_i||_Q^2 + ||u_i||_R^2 + s_{i+1} \\ s.t. \quad & x_{i+1} = f(x_i, u_i) + x_i - f(x_{i-1}, u_{i-1}), \qquad i = 0 \dots N-1, \\ & c(x_i) \leq s_i, \qquad i = 1 \dots N, \\ & p(u_i) \leq 0, \qquad i = 0 \dots N-1, \\ & s_i \geq 0, \qquad i = 1 \dots N \end{aligned} \tag{29}$$

where $e_i = x_i - r$ is the deviation from the target, $z = [u_0^T \; x_1^T \; u_1^T \dots x_N^T]^T$ is the primal optimization variable, N is the length of the prediction horizon, $P = P^T \succcurlyeq 0$ is the terminal penalty, $Q = Q^T \succcurlyeq 0$ is the state penalty weight, $R = R^T \succ 0$ is the control penalty weight, and c, p are the stage state and control constraints respectively.

Note that the state constraints are softened using slack variables in order to ensure the problem is always feasible. The prediction model is written in rate-based form and will automatically compensate for low frequency unmodeled disturbances.

To solve the above OCP, we first write it in compact form as

$$\begin{aligned} \min_{z} \quad & F(z, \xi), \\ s.t. \quad & g(z, \xi) = 0, \\ & c(z) \leq 0, \end{aligned} \tag{30}$$

and introduce the Lagrangian,

$$L(z, \lambda, v, \xi) = F(z, \xi) + \lambda^T g(z, \xi) + v^T c(z), \tag{31}$$

where the tuple $\xi = (\hat{x}(t), r(t))$ collects the time varying parameters of the OCP and λ and v are Lagrange multipliers. The following quadratic programming problem (QPP) is then solved at the kth time-step

$$\begin{aligned} \min_{\Delta z} \quad & \frac{1}{2} \Delta z^T H \, \Delta z + q^T \Delta z, \\ s.t. \quad & G \, z = h, \\ & A \, z \leq b, \end{aligned} \tag{32}$$

where $H \approx \nabla_z^2 L(z_{k-1}, \lambda_{k-1}, v_{k-1})$, $q = \nabla_z L(z_{k-1}, \lambda_{k-1}, v_{k-1})$, $G = \nabla_z g(z_{k-1})$, $h = g(z_{k-1})$, $A = \nabla_z c(z_{k-1})$, and $b = -c(z_{k-1})$. The solution estimate is then updated as

$$z_k = \Delta z^* + z_{k-1}, \tag{33}$$

and the multipliers are updated as

$$v_k = v^*, \lambda_k = \lambda^*, \tag{34}$$

where $(\Delta z^*, \lambda^*, v^*)$ is the solution of the QP. Under certain assumptions, this strategy generates a stabilizing feedback law and will converge to the optimum solution of the original nonlinear program [9]. The convergence of the SQP algorithm can be monitored using the natural residual of the KKT conditions

$$\pi(x, \xi) = \left\| \begin{bmatrix} \nabla_x L(z, \lambda, v, \xi) \\ g(x, \xi) \\ \min(-c(x), v) \end{bmatrix} \right\|. \tag{35}$$

Written out explicitly, the feedback control law is computed by repeatedly solving QPs of the form:

$$\begin{aligned} \min_{x,u,s} \quad & \begin{bmatrix} \Delta u \\ \Delta x \end{bmatrix}^T \begin{bmatrix} H_u & 0 \\ 0 & H_x \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta x \end{bmatrix} + \begin{bmatrix} d_u \\ d_x \end{bmatrix}^T \begin{bmatrix} \Delta u \\ \Delta x \end{bmatrix} + \gamma_1 1^T s \\ s.t. \quad & A\Delta x + B\Delta u + g(\bar{x}, \bar{u}) = 0 \\ & C\Delta x + c(\bar{x}) \leq \Gamma s \\ & P\Delta u + p(\bar{u}) \leq 0 \\ & s \geq 0 \end{aligned} \tag{36}$$

where $\Gamma = I_N \otimes 1_N$, $C = \nabla_x c(\bar{x})$, $P = \nabla_u p(\bar{u})$, $A = \nabla_x g(\bar{x}, \bar{u})$, $B = \nabla_u g(\bar{x}, \bar{u})$, $H_x = diag\left(I_{N-1} \otimes Q \ , Q_f\right)$, $H_u = I_N \otimes R$.

To speed up computations we can condense the problem. Since for any sequence u the state sequence x is uniquely determined A must be invertible, allowing us to write

$$\Delta x \ = -A \ ^{-1} B \ \ \Delta u \ \ - A \ ^{-1} g \ \ (x^{-}, \quad ^{-}) = G \ \ \Delta u \ \ + w \tag{37}$$

We can then write the condensed problem as

$$\begin{aligned}\min_{\Delta u,s} & \begin{bmatrix}\Delta u\\ s\end{bmatrix}^T \begin{bmatrix}H & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta u\\ s\end{bmatrix} + \begin{bmatrix}d\\ \gamma 1\end{bmatrix}^T\begin{bmatrix}\Delta u\\ s\end{bmatrix}\\ s.t. & \begin{bmatrix}P & 0\\ CG & -\Gamma\\ 0 & -I\end{bmatrix}\begin{bmatrix}\Delta u\\ s\end{bmatrix} + \begin{bmatrix}\bar{p}\\ \bar{c}+Cw\\ 0\end{bmatrix} \leq 0,\end{aligned} \quad (38)$$

where $H = H_u + G^T H_x G$, and $d = d_u + G^T(H_x w + d_x)$. The iterates can then be updated as

$$x \leftarrow \bar{x} + G\Delta u + w, \qquad u \leftarrow \bar{u} + \Delta u. \quad (39)$$

3.2.2.2 A Computational Reference Governor for Suboptimal MPC.

Suboptimal MPC (SOMPC), where optimization algorithm iterations are distributed over time, have the potential to reduce computation times and improve the reliability of numerical algorithms for MPC controller. In SOMPC an estimate of the OCP solution is maintained within the controller and a finite number of optimization algorithm iterations are performed at each sampling instance. This practice is commonly known as *warmstarting*. The solution updating process can be seen Figure 1, the result is a pair of coupled dynamic systems representing the plant and optimizer respectively.

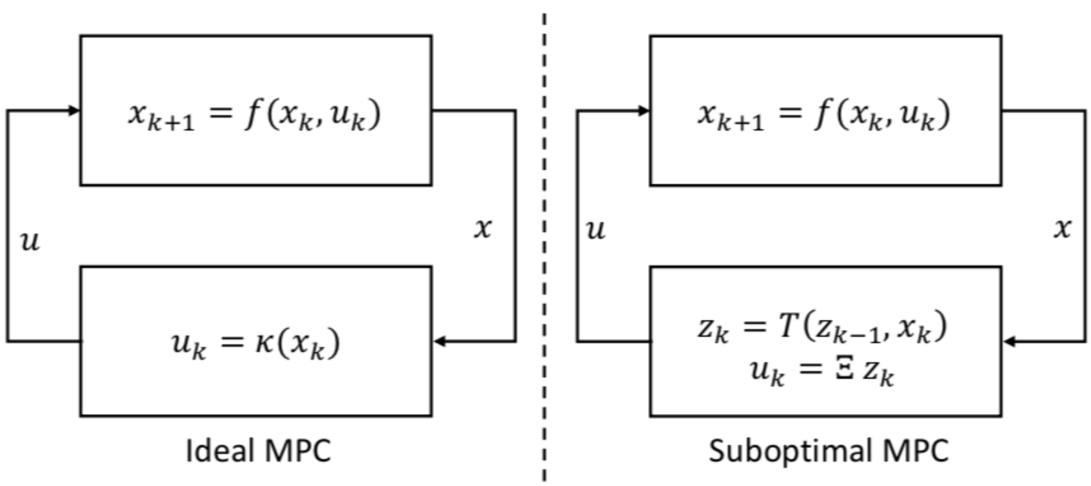

FIGURE 1 A COMPARISON OF SUBOPTIMAL MPC AND IDEAL MPC

Model predictive control imposes constraints on the plant, the objective of the CRG is to impose constraints on the optimizer. Specifically, we're interested in imposing the constraint

$$\pi(z, x) \leq \pi_{ub}, \quad (40)$$

where is the residual function for the OCP defined in (35). Enforcing this constraint ensures the quality of the numerical solution. The solver obeys the following convergence bounds

$$||z_k - z^*(\xi_k)|| \leq \eta||z_{k-1} - z^*(\xi_{k-1})|| + \beta||\Delta x_k|| + \beta||\Delta r_k||, \quad (41)$$

where z is the primal dual solution estimate, $z^*(\xi)$ is the parameter to solution mapping for the OCP and $\xi = (x, r)$ is the OCP parameter which consists of the measured state x and the reference command r.

The idea of the computational RG is to limit the residual π by controlling Δr_k. To accomplish this, we use an *add-on* unit. The CRG is attached to the NMPC controller as shown in Figure 2 below:

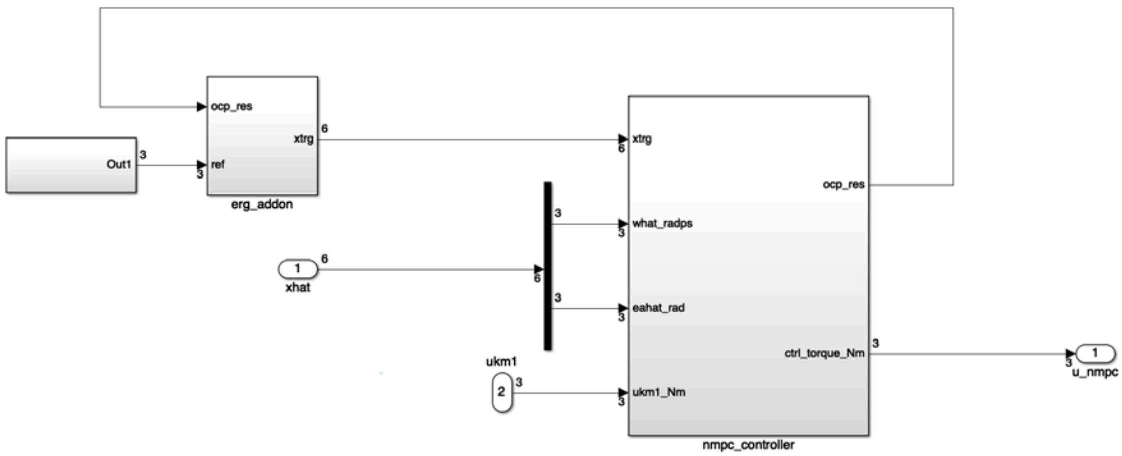

FIGURE 2 ARCHITECTURE OF THE COMPUTATIONAL REFERENCE GOVERNOR

The CRG takes in the user commanded reference r, and the current OCP residual and outputs a modified auxiliary reference v. Internally, the CRG solves the following optimization problem

$$\begin{aligned} \min_{\Delta v} \quad & ||r_k - v_{k-1} + \Delta v||_2^2 \\ s.t. \quad & |\Delta v_i| \leq \Delta v_{\text{ub}}, \qquad i = 1 \dots 3, \\ & \theta_{lb} \leq v_{k-1} + \Delta v \leq \theta_{ub}, \end{aligned} \quad (42)$$

where r_k is the reference, v_{k-1} is the virtual reference at the previous step, $\theta_{lb,ub}$ are the upper and lower bounds on the Euler angles. This is a quadratic program with only 3 decision variables; its computational cost is negligible compared to the MPC controller. The maximum step length is calculated as

$$\Delta v_{ub} = \max\left(0, \delta v \frac{(\pi_{ub} - \pi_{k-1})}{\pi_{ub}}\right) \quad (43)$$

where π_{k-1} is the residual of the OCP at the previous sampling instance and the reference is updated as

$$v_k = v_{k-1} + \Delta v. \quad (44)$$

This forms a kind of nonlinear filter; the underlying intuition is to reduce the step size proportionally to the distance from the constraint $\pi(z,x) \leq \pi_{ub}$. The maximum reference step length δv and the maximum allowable residual π_{ub} are user chosen parameters.

3.3 Constrained control framework using a virtual net of chance constraint admissible sets and output measurements only

Consider a system represented by a discrete-time linear model with stochastic state disturbance and measurement noise inputs, given by

$$x_{k+1} = Ax_k + Bu_k + \Gamma w_k, \quad y_k = Cx_k + Fv_k, \tag{45}$$

where x_k is a vector state and u_k is a vector control. The variables w_k and v_k are, respectively, the state disturbance and the measurement noise inputs, each assumed to be a sequence of zero-mean, independent (and jointly independent) identically distributed (i.i.d.) random variables with $\mathbb{E}[w_k w_k^T] = \Sigma_w = \Sigma_w^T,\ \mathbb{E}[v_k v_k^T] = \Sigma_v = \Sigma_v^T$.

We consider a fixed-gain state feedback,

$$u_k = K\hat{x}_k + Gr, \tag{46}$$

where r is the reference and $\hat{x}_k$ is the state estimate generated by a fixed-gain, L, observer. Then the error, $e = x_k - \hat{x}_k$ is normally distributed with covariance

$$P_\infty = A_o P_\infty A_o^T + B_o B_o^T \quad A_o = A + LC, B_o = [\Gamma, LF] \tag{47}$$

Consider now enforcing a state constraint of the form,

$$Hx_k \leq h, \forall k \tag{48}$$

in a probabilistic manner. The resulting chance constraint can be shown to have the form (see [14] for details):

$$Prob\left([H\ H]\begin{bmatrix}\hat{x}_k \\ e_k\end{bmatrix} \leq h - H(I - A_c)^{-1}B_c r\right) \geq 1 - \alpha, \tag{49}$$

where α is the probability threshold and e_k is the state estimation error. For this chance constraint, a chance constrained admissible set, $O_\infty(r)$, can be defined which is the set of initial state estimates such that the subsequent trajectories for the given reference, r, satisfy the chance constraint. In defining $O_\infty(r)$ certain technical assumptions need to be made such that the observed dynamics have reached the steady-state [16].

Following the same approach as in [10], now a virtual net of reference points r may be built to ensure safe transitions between reference points by declaring a connection between two nodes on the graph only exists if the a priori node lies within the chance constrained admissible set of a posteriori node. The maneuver planning problem on this graph is then solved using any standard graph search algorithm.

4. RESULTS AND DISCUSSION

4.1 Disjunctive-Based Constrained Spacecraft Relative Motion Planning

We consider a case study of spacecraft three-dimensional relative motion control. The relative motion dynamics are modeled with the linearized Clohessy-Wiltshire [11] equations,

$$\begin{aligned} \ddot{x}_1 &= 3\omega^2 x_1 + 2\omega \dot{x}_2 + \frac{1}{m}u_1, \\ \ddot{x}_2 &= -2\omega \dot{x}_1 + \frac{1}{m}u_2, \\ \ddot{x}_3 &= -\omega^2 x_3 + \frac{1}{m}u_3, \end{aligned} \tag{50}$$

where m is the chaser vehicle's mass and ω is the mean motion of the target vehicle's orbit. We form a system of first-order equations with state vector $x_k = \left[x_{1,k}, x_{2,k}, x_{3,k}, \dot{x}_{1,k}, \dot{x}_{2,k}, \dot{x}_{3,k}\right]^T$ and discretize using a Zero-Order Hold [12] with sampling period of 30 seconds, chaser vehicle mass of 140 kg, and target vehicle mean motion of 0.0010 radians per second. We choose $C = [\boldsymbol{I}_3\ \boldsymbol{0}_{3\times3}]$, which corresponds to relative position measurements. The goal in this scenario is to rendezvous the chaser vehicle with the target vehicle, i.e., to bring the chaser's state to the origin. For this controller and observer, the feedback gain matrix K is computed using LQR by solving the Discrete Time Algebraic Riccati Equation (DARE) and the observer gain matrix L computed by solving the dual DARE, with $Q = \boldsymbol{I}_6$ and $R = \boldsymbol{I}_3$ in each case.

4.1.2 Theoretical Results.

We consider a case study of spacecraft three-dimensional relative motion control. The relative motion dynamics are again modeled with the linearized CW equations,

$$\begin{aligned} \ddot{x}_1 &= 3\omega^2 x_1 + 2\omega \dot{x}_2 + \frac{1}{m}u_1, \\ \ddot{x}_2 &= -2\omega \dot{x}_1 + \frac{1}{m}u_2, \\ \ddot{x}_3 &= -\omega^2 x_3 + \frac{1}{m}u_3, \end{aligned} \tag{51}$$

where m represents the chaser vehicle's mass and ω is the mean motion of the target vehicle's orbit. We form an equivalent system of first-order equations and discretize using Zero-Order Hold [12] with sampling period 30 sec, chaser vehicle mass of 140 kg, and target vehicle mean motion of 0.0010 rad/sec. We choose $C = [\mathbf{I}_3\ \mathbf{0}_{3\times3}]$, which corresponds to relative position measurements. The goal in this scenario is to rendezvous the chaser vehicle with the target vehicle, i.e., to bring the chaser's state to the origin.

For this controller and observer, the feedback gain matrix K is computed using LQR by solving the Discrete Time Algebraic Riccati Equation (DARE) and the observer gain matrix L computed by solving the dual DARE, with $Q = \mathbf{I}_6$ and $R = \mathbf{I}_3$ in each case. The distributions of the measurement noise, v_k, and of the state disturbance, w_k, are assumed to be Gaussian with covariance matrices $\Sigma_v = 10^{-2} \cdot \mathbf{I}_3$ and $\Sigma_w = 10^{-4} \cdot \mathbf{I}_6$. The cost function is defined with $r_\eta = 0$, $R_e = \mathbf{I}$, and $R_x = \mathbf{0}$. These choices ensure accurate estimates of the relative position states.

We will now construct a sequence that satisfies the sufficient conditions, and demonstrate that it leads to stable behavior. For this example, $\bar{\rho}_0 = 1.0063$, $\bar{\rho}_1 = 0.2016$, $\tilde{\rho}_0 = 0.0332$, and $\tilde{\rho}_1 = 1.0063$. Working with the Frobenius norm, $k^* = 1$ and

$$c = \frac{\|\bar{\Omega}_1\|_F}{\rho(\bar{\Omega}_1)} = \frac{10.4716}{0.2016} = 51.950. \tag{52}$$

We begin with the (inadmissible) sequence $S_2 = \{0, 1\}$, with $n_s = 2$, $n_0 = 1$, and $n_1 = 1$. To modify the sequence such that it satisfies the sufficient conditions, increase n_1 by 1, and iterate until the first logarithmic condition is satisfied, then iterate on n_0 until the second logarithmic condition is satisfied, and repeat as necessary until both inequalities are true. For this example, the iterative process yields $n_0 = 3$ and $n_1 = 5$. The resulting sequence, $s_8 = \{0, 0, 0, 1, 1, 1, 1, 1\}$, will converge in relative position and state error estimates when propagated forward periodically, and a representative simulation is shown in Figure 3.

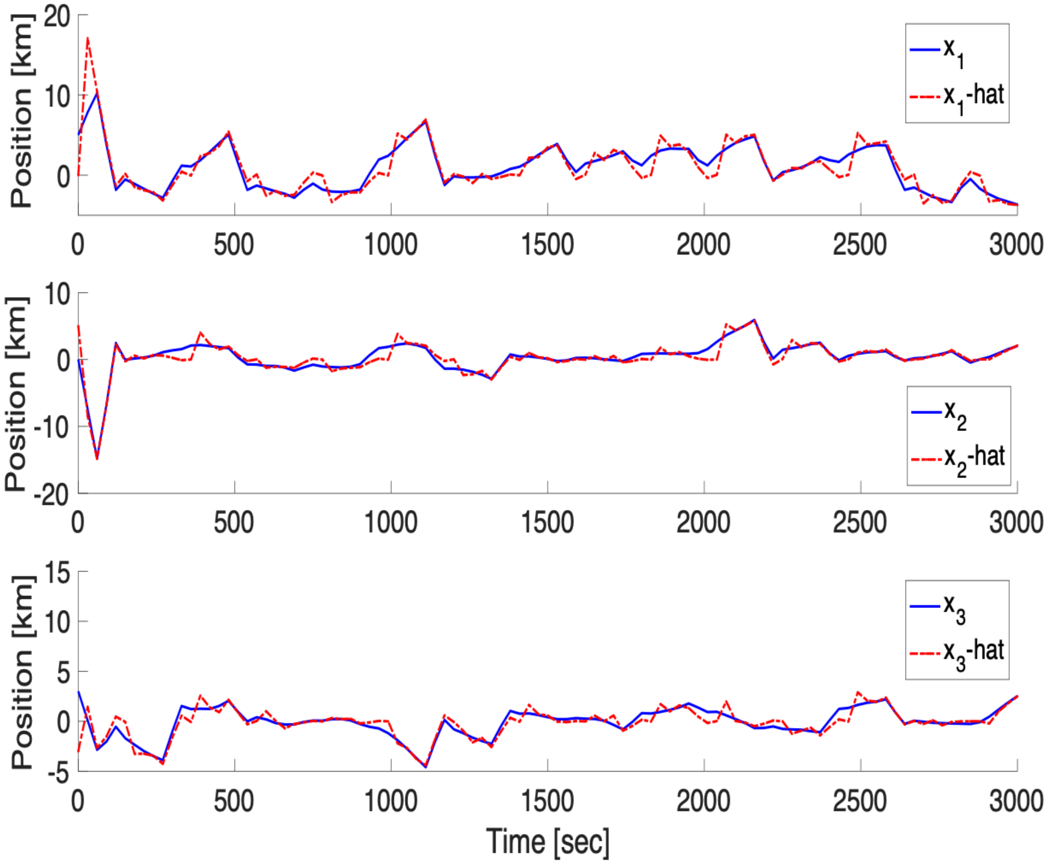

FIGURE 3 DISJUNCTIVE SENSING AND CONTROL TRAJECTORY UNDER A LENGTH 8 PERIODIC SEQUENCE

The constructed sequence satisfies the contractivity conditions but it not the optimum sequence under the chosen cost functional. To determine the optimum such sequence, we apply the algorithm discussed in Section 3.2.4. There are no admissible sequences of length 2 or length 3, thus we extend the search to sequences of length 4. The optimum such sequence is $S_4 = \{0, 0, 1, 1\}$, with $\bar{q}_A = 0.5879$ and $\tilde{q}_A = 0.0130$. This sequence, however, is optimal only over sequences of length 4; longer sequences may exist that exhibit faster convergence. When the algorithm treats sequences of length 7, the optimum such sequence is $S_7 = \{0, 0, 1, 1, 1, 0, 0\}$, with $\bar{q}_A = 0.07594$ and $\tilde{q}_A = 3.796 \times 10^{-5}$. Simulated results of propagating each optimal switching sequence forward periodically are shown in Figure 4; the sequence S_7 converges more quickly than the sequence S_4, which goes through several additional oscillations before settling to the origin.

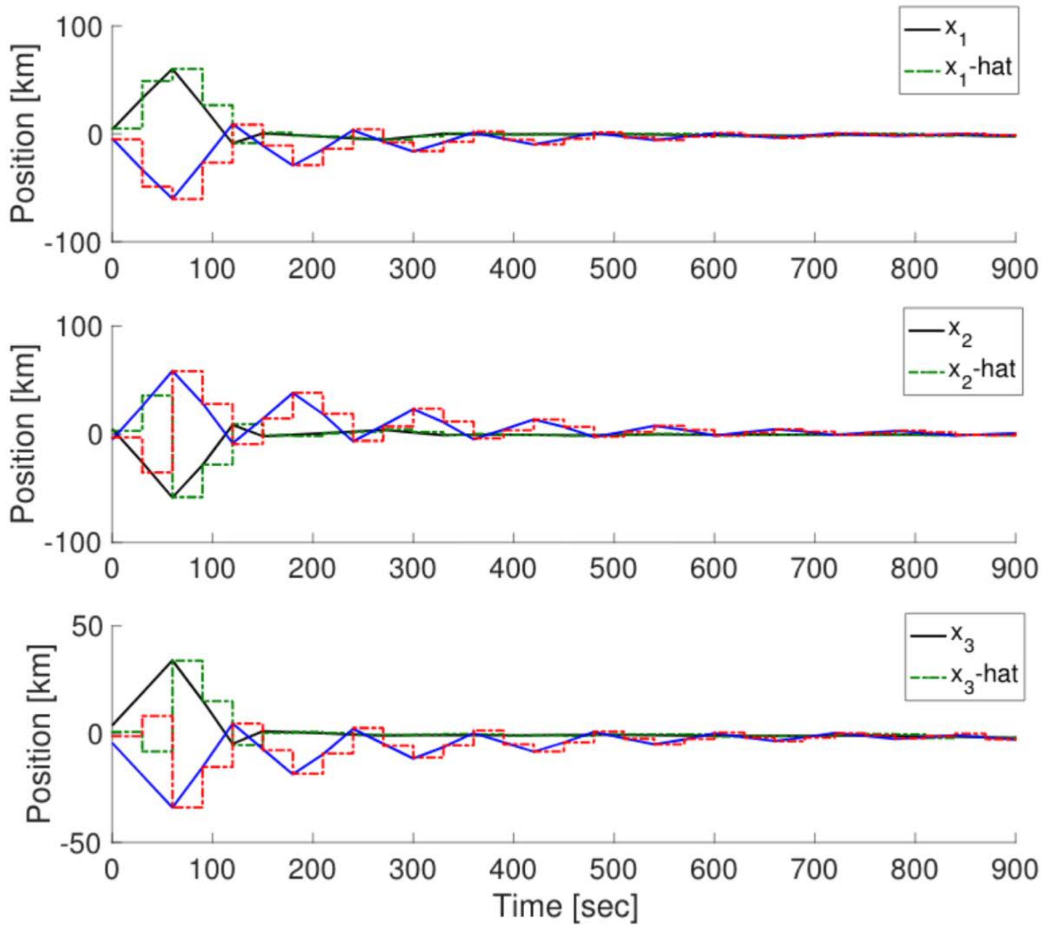

FIGURE 4 COMPARISON OF LENGTH 4 AND LENGTH 7 OPTIMAL SWITCHING SEQUENCES

Of note, when treating sequences of length 8, the optimum such sequence is $\{0, 0, 1, 1, 0, 0, 1, 1\}$, a reducible sequence that has the previous length 4 optimal sequence S_4 as the corresponding irreducible subsequence.

The remaining objective is the treatment of chance constraints. Suppose that we wish to establish a constraint on the steady-state of the form $X = \{x: \|x\|_\infty \leq b\}$, so that the chaser spacecraft remains within a box centered at the origin, of side lengths $2b$, with $\text{Prob}(X) \geq 0.95$. Consider the length 4 sequence S_4; invoking the Chebyshev inequality with $\alpha_x = \sqrt{3/0.05}$, for each of $P_{x,0}^{s}$, ..., $P_{x,3}^{s}$ yields four spheres, with minimum radius $b = 2.79$ and maximum radius $b = 9.54$. Only the maximum radius, the loosest of the four bounds, is guaranteed by the Chebyshev inequality, but even the tightest bound remains somewhat conservative. When setting $b = 2.5$, the chance constraints are violated in no more than 4% of trajectories at any given time step. Figure 5 illustrates the satisfaction of the chance constraints over the course of 200 simulation runs.

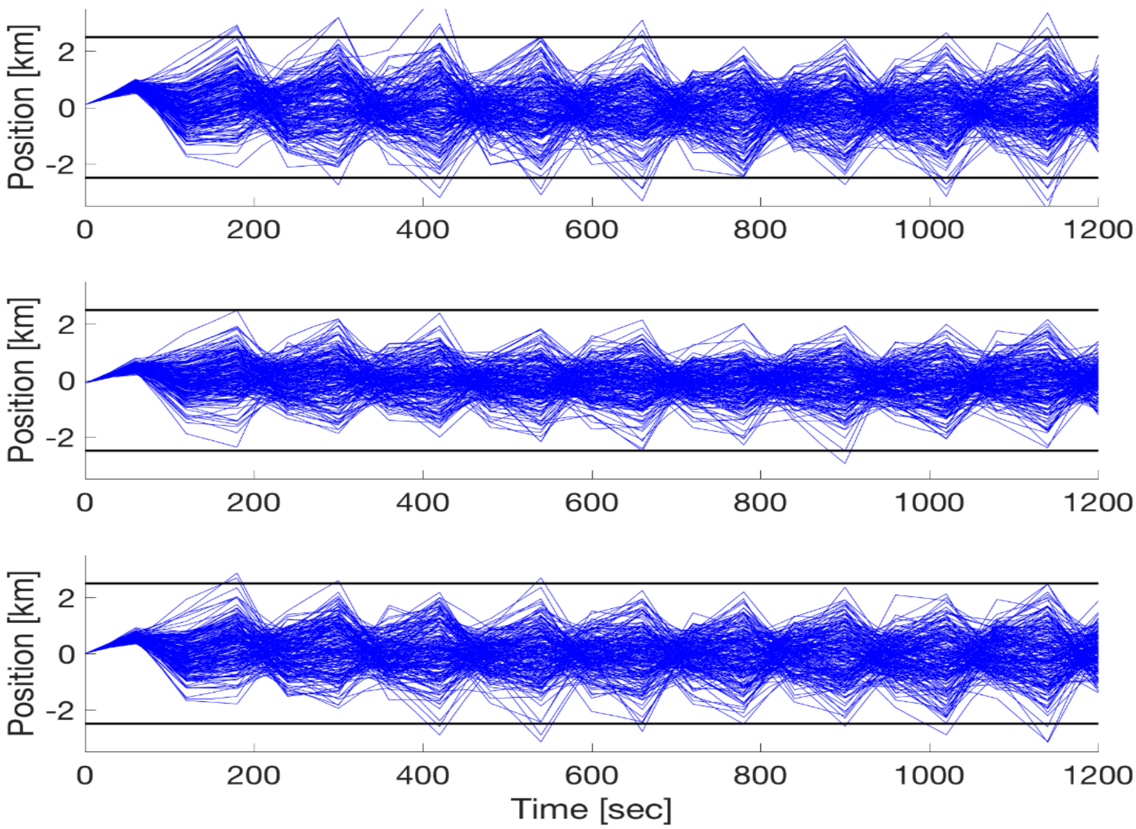

FIGURE 5 DEMONSTRATION OF SATISFACTION OF BOTH BOX AND CHANCE CONSTRAINTS

4.2 A Computational Reference Governor for Safe Online Optimization

We placed CRG and NMPC combination in closed-loop with the model described in Section 3.2.1 and commanded a pair of reorientation maneuvers. Figure 6 shows the closed-loop traces, the constraints are satisfied, and the system successfully completes the maneuver.

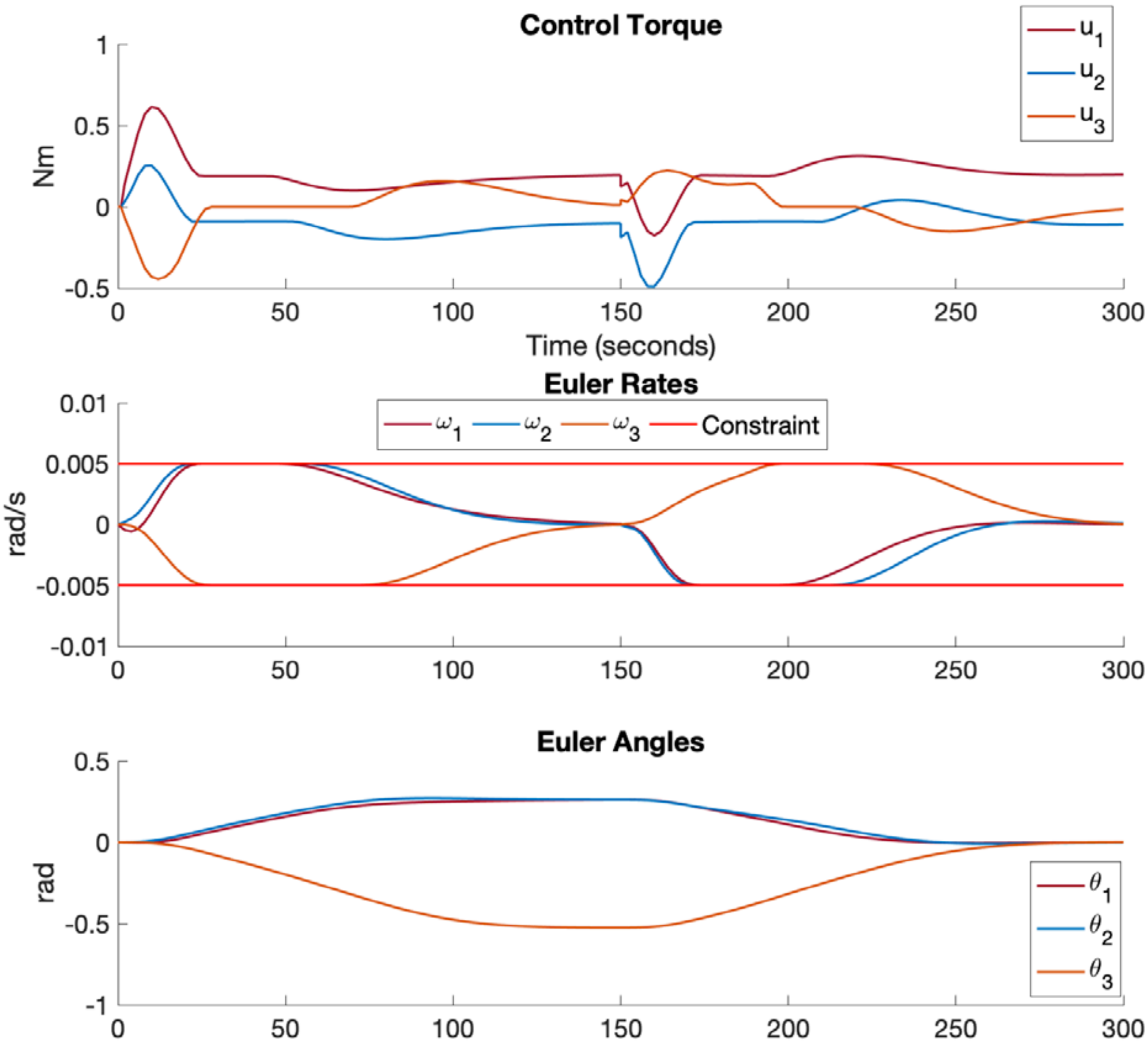

FIGURE 6 THE TIME HISTORIES OF CONTROL TORQUES, ANGULAR VELOCITIES AND EULER ANGLES IN CLOSED-LOOP SIMULATION

Figure 7 compares the angle references and OCP residual with and without the CRG. Without the CRG the OCP residual violates the maximum residual constraint ($\pi_k \leq 10^{-4}$). With the CRG active the residual constraint is satisfied; the CRG filters the reference commands to limit the change in the reference. While these simulations provide a proof of concept, further development of the theory of computational reference governors will be pursued in future research.

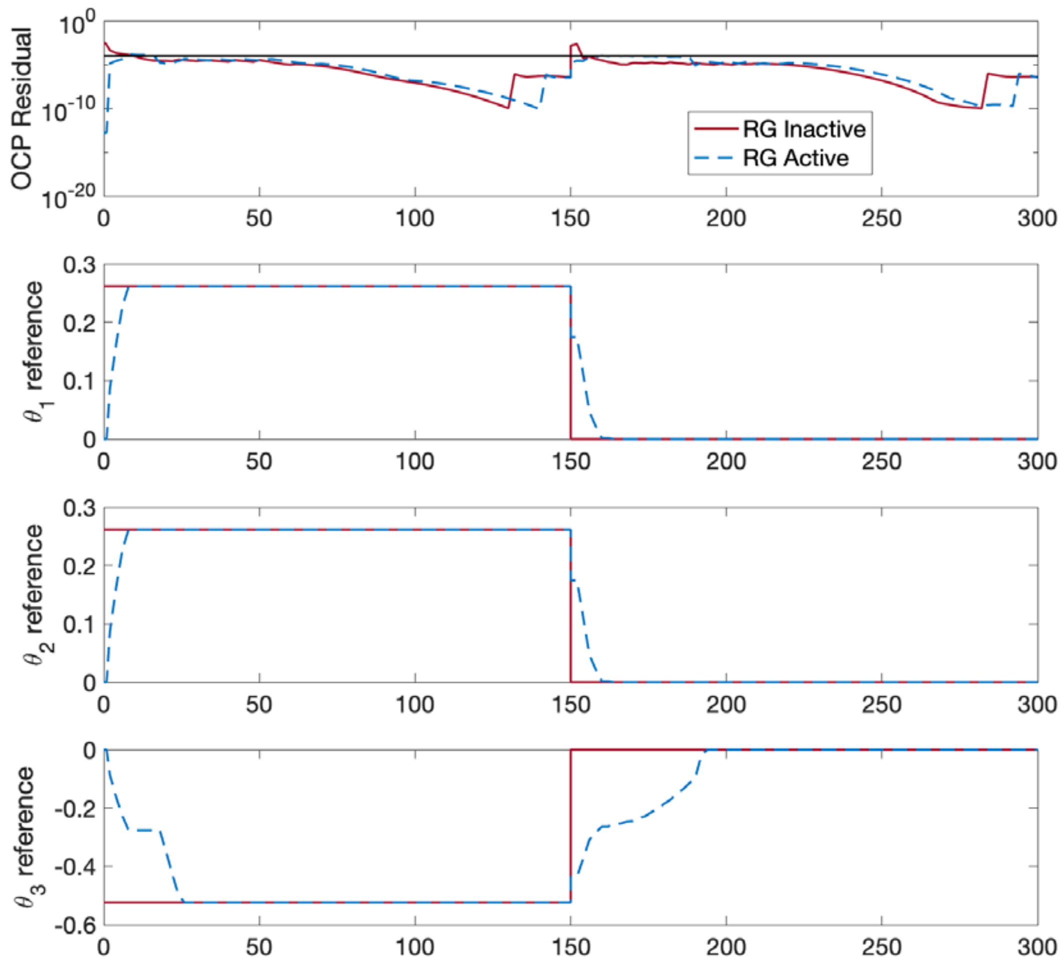

FIGURE 7 CLOSED-LOOP TRACES OF THE RESIDUALS AND REFERENCES WITH AND WITHOUT THE CRG.

4.3 Constrained Control Framework Using a Virtual Net of Chance Constraint Admissible Sets and Output Measurements Only

For illustration, a spacecraft constrained relative motion maneuver is considered. The equations of motion are based on the Clohessy-Wiltshire equations introduced in Section 4.1 modified with random disturbance terms and measurement noise. Only the spacecraft relative position coordinates are measured and an observer has been constructed to estimate the state. A virtual net of stationary reference points in the relative frame and of the corresponding chance constrained admissible sets has then been constructed. The graph search is performed with fuel usage as the cost. Figure 8 illustrates the resulting maneuver, where the constraints correspond to avoiding a cone emanating from the origin and staying with the specified box.

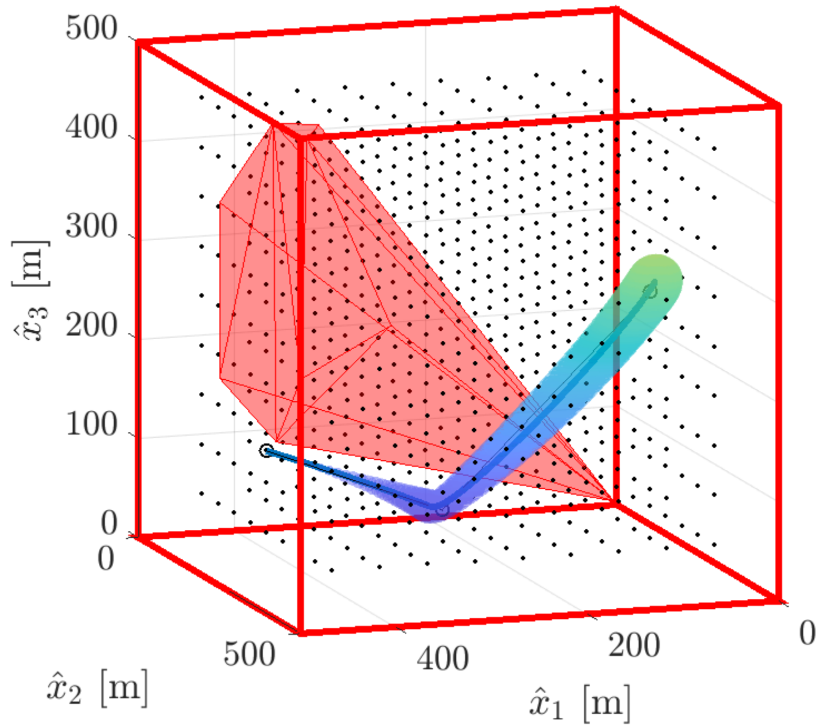

FIGURE 8 CLOSED-LOOP PATH OF SPACECRAFT IN CW FRAME. RED CONE IS THE OBSTACLE AND BLACK DOTS ARE THE REFERENCE POINT NODES IN THE GRAPH.

5. CONCLUSION

The outcomes of research into coordinating sensing, control and computations for autonomous vehicles are as follows:

- Strategies to ensure stability and eventual chance constraint enforcement have been defined for systems where simultaneous sensing and actuation cannot be performed and thus switching between sensing and actuation is necessary.
- A computational reference governor has been defined to supervise and modify if necessary reference commands to a nonlinear model predictive controller and ensure constraints on the residuals of the optimization problems to ensure acceptable accuracy of the solution within limited computational time.
- For the case of systems operating with output measurements only and a state observer used to reconstruct the full state, a methodology for constructing and chaining chance constraint admissible sets has been defined which reduces the problem of constructing constraint admissible maneuvers to a standard graph search.
- Simulations of spacecraft relative motion and attitude control have been used to illustrate the potential of the proposed strategies.

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LIST OF SYMBOLS, ABBREVIATIONS, AND ACRONYMS

CRG	Computational Reference Governor
DARE	Discrete-Time Algebraic Riccati Equation
DSC	Disjunctive Sensing and Control
IID	Independent and Identically Distributed
MILP	Mixed-Integer Linear Program
MPC	Model Predictive Control
NMPC	Nonlinear Model Predictive Control
OCP	Optimal Control Problem

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