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**Workshop on Dynamic, Control and Numerics for Fractional Partial Differential Equations**

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UNIVERSITY OF PUERTO RICO**

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Final Report**

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# Workshop on Dynamics, Control and Numerics for Fractional PDEs

**Award NO: FA9550-17-1-0524**

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## **Project Final Progress Report**

**Introduction:** The workshop was held in San Juan, Puerto Rico, at the Embassy Suites Hotel on December 5-7, 2018. There were a total of 45 participants including world class experts in the academia, researchers from the national laboratories, postdoctoral researchers and graduate students working in areas covered by the topics of the workshop. Additionally, two (2) program managers from the Air Force Office of Scientific Research were present for the entire duration of the workshop. There were five (5) keynote addresses of one hour each, covering all aspects of control theory, dynamics, numerical treatment and computational aspects of fractional PDEs. These ranged from the theoretical to the most applied aspects, and the related physical and engineering models. We had nine (9) invited talks of 45 minutes each given by junior researchers who have already obtained significant results in the field. For these talks, the speakers and the subject of their presentations covered all the above mentioned aspects. The program also included six (6) contributed talks of 30 minutes each given by postdoctoral researchers and graduate students working on the area of fractional PDEs and its applications. Finally, at the end, there was a panel discussion to evaluate the scientific activities of the workshop and to discuss perspectives for future research in the field.

**Applications of fractional differential equations:** Fractional calculus and in particular fractional partial differential equations have gained much attention lately due to the fact that they have proven more suitable for modeling a wide array of phenomena in the applied sciences and engineering. In the concerned areas, several features are captured through the use of fractional calculus, in contrast to the classical partial differential equations. Some specific topics are analytical aspects, dynamics, control, numerical experiments, and sample applications to various fields. The applications in industry are numerous and cover almost every area. From the long list of phenomena which are more appropriately modeled by fractional differential equations, we mention: viscoelasticity, anomalous transport and diffusion, hereditary phenomena with long memory, nonlocal electrostatics, the latter being relevant to drug design, and Lévy motions which appear in important models in both applied mathematics and applied probability. It is interesting to note that the fractional heat equation also emerges as the hydrodynamic limit of interacting particle systems that are super-diffusive in nature,

that is, the limit of systems in which particles may perform long jumps in the context of Lévy processes.

**Objectives of the workshop:** The present workshop deals with various aspects of fractional partial differential equations and its relevant applications to modern science and technology. The theoretical aspects involve diverse methods from analysis, operator theory, partial differential equations, stochastic differential equations and integral equations. The more applied aspects of the workshop deal with controllability, observability and stabilization as well as numerical treatment of the equations arising from models encountered in areas such as viscoelasticity, anomalous transport and diffusion, image processing and phase field models, biology and ecology. These topics are covered by the Air Force Office of Scientific Research (AFOSR) programs in "Dynamic and Control" and "Applied Mathematics and Optimization". In the area of fractional differential equations, new challenges emerge regarding the numerical treatment of the various problems involved. Due to its importance in applications, fractional calculus and anomalous diffusion have become a central point of investigation in partial differential equations, applied mathematics, computational mathematics and in many research laboratories all over the country. The workshop brought together researchers from the fields covered by the topics of the workshop: those working in pure mathematics, applied and computational mathematics and researchers working in the national laboratories. They presented some of their most relevant results so far obtained. Several international experts on the main topics of the conference gave talks about their research work, reflected about their experience and discussed perspectives for future investigations. Several very deep results and concrete applications in the industry of fractional PDEs were presented, including the emerging field of peridynamics. To summarize, we believe that the goals described in workshop proposal were accomplished.

**Impact of the workshop:** The conference provided experienced researchers, junior researchers and graduate students with the opportunity to present state of the art research on the subject of fractional differential equations and their applications. This area is currently undergoing an explosive development due to the manifold applications in technological fields mentioned above. The talks presented at the workshop covered the full spectrum of research and potential industrial and technological applications, that is, from the more theoretical aspects to the advanced applications of the field. It was also a great opportunity for researchers of various backgrounds in the above mentioned area to interact and discuss their research projects. Most importantly graduate students and post doctoral researchers found an exposition to the different research aspects and the applications in the emerging fields connected to this area. We had a very fruitful meeting where the diverse contributions came together and have served as motivation for tackling the problems already formulated, and explore many others yet to be studied. Moreover, the workshop represented an excellent opportunity to foster scientific collaboration between researchers at universities, industry and the national laboratories. To the best of our knowledge this was the first conference that brought together the above mentioned researchers to discuss the results of their research and some perspective on the above mentioned field of mathematics and its applications. We also point out that this was the only international conference in Differential Equations and its applications organized on the Island of Puerto Rico in the last 15 years. The workshop contributed to give more visibility to researchers in mathematics at various institutions in the Island of Puerto Rico. These researchers made several contacts in order to continue or initiate

collaborative work with experts in the fields of the workshop from all over the world.

**Panel discussions:** Before the closing ceremony, a panel discussion was organized in order to evaluate the quality of the talks and to explore new directions for investigation. The members of the panel were the five keynote speakers and the two program managers from the Air Force Office of Scientific Research (AFOSR). The panel appreciated the quality of the talks, the wealth of concrete models and applications presented, and rated the organization of the workshop as excellent.

The main recommendation from the panel discussion was that researchers should continue to do great research work on fractional PDEs and put more emphasis on applications that include some computations in order to further validate the discovered results. New perspectives that will significantly contribute to achieve the goal of the Mathematical Sciences program at the ARO and the AFOSR have been suggested by the program managers. Once again, it was stressed that more should be done regarding applications and related computational aspects. This will make the field extremely important and will attract all funding agencies in the country. In addition, the panel strongly recommended that the workshop be organized in a regular basis (at least every 2 years) in order to stimulate work in this area.

Finally, the panel has congratulated the PI of the grant, Mahamadi Warma, for the great initiative to reunite such world class experts working in the field to present their most important results on fractional PDEs and its applications.

**Concluding remarks:** We would like to thank the Air Force Office of Scientific Research (AFOSR), in particular, Dr. Frederick Leve, the program officer of Dynamics and Control, for his sponsorship of the workshop, and Dr. Fariba Fahroo, the program officer of Applied Mathematics and Optimization for supporting and participating to the workshop. We will need their support for our next workshop in the field of fractional PDEs and its applications that we are planning to organize in 2020. Below we include the titles of all the talks and the complete list of participants. More information on the workshop can be found at the following link: <http://math.uprrp.edu/workshop-on-dynamics-control-and-numeric-for-fractional-pdes/>

## Keynote talks

**J.A. Burns (Virginia Tech). High order approximations for modeling and control of PDE systems.**

**Z-Q. Chen (University of Washington). Fundamental solutions to time fractional Poisson equations.**

**I. Lasiecka (University of Memphis). Mathematical theory of evolution arising in flow structure interactions.**

R.C. Smith (North Carolina State University). Sensitivity analysis, uncertainty quantification, and control design for smart material systems.

E. Zuazua (Universidad Autónoma de Madrid, Spain). Control of some models in population dynamics.

## Invited talks

H. Antil (George Mason University). Fractional PDEs: Control and Applications.

U. Bicari (DeustoTech, Bilbao, Spain). Controllability of a one-dimensional fractional heat equation: theoretical and numerical aspects.

M. D'Elia (Sandia National Laboratories). Nonlocal models with nonstandard interaction domains: comparative analysis and efficient finite element methods.

C.G. Gal (Florida International University). On overview of well-posedness and regularity results for nonlocal in time PDEs.

M. Parks (Sandia National Laboratories). Subsurface applications for peridynamics.

R. Ponce (Universidad de Talca, Chile). A posteriori error estimates and maximal regularity for approximations of nonlinear fractional problems in Banach spaces.

P. Radu (University of Nebraska-Lincoln). Double nonlocality in continuum mechanics.

L. Tebou (Florida International University). Some contributions to the simultaneous and indirect stabilization of multi-component systems.

P.R. Stinga (Iowa State University). How to approximate the fractional Laplacian by the fractional discrete Laplacian.

## Contributed talks

[O. Burkovska \(Florida State University\)](#). **Model order reduction for nonlocal problems with parametrized kernels.**

[S. Charoenphon \(University of Memphis\)](#). **Vanishing relaxation time dynamics of the Jordan Moore-Gibson-Thompson (JMGT) equation arising in high frequency ultrasound (HFU).**

[J. González \(Universidad de Santiago, Chile\)](#). **Fundamental solutions for discrete dynamical systems involving the fractional Laplacian.**

[S. Guffey \(University of Memphis\)](#). **Excursions into controllability of a chemotaxis system via diffusive phenomena.**

[M. Gulian \(Brown University\)](#). **Stochastic solution of parabolic and elliptic boundary value problems for the spectral fractional Laplacian.**

[R. Khatri \(George Mason University\)](#). **Parameter estimation and fractional regularization in tomographic reconstruction.**

## List of participants

1. Harbir Antil (George Mason University)
2. Rafael Aparicio (University of Puerto Rico, Río Piedras)
3. Ernes Aragonés (University of Puerto Rico, Río Piedras)
4. Edilberto Arteaga (Inter American University, Metro)
5. Umberto Biccari (Deustotech-Bilbao, Spain)
6. Marcelo Bongarti (University of Memphis)
7. Olena Burkovska (Florida State University)

8. John A. Burns (Virginia Tech)
9. Carmen Caiseda (Inter American University, Bayamón)
10. Carlos Carvajal-Ariza (University of Puerto Rico, Mayagüez)
11. Paul Castillo (University of Puerto Rico, Mayagüez)
12. Sutthirut Charoenphon (University of Memphis)
13. Zhen-Qing Chen (University of Washington)
14. Marta D'Elia (Sandia National Laboratories)
15. Victor Díaz-Martínez (University of Puerto Rico, Mayagüez)
16. Fariba Fahroo (Air Force Office of Scientific Research)
17. Ciprian G. Gal (Florida International University)
18. Sergio Gómez (University of Puerto Rico, Mayagüez)
19. Jorge González-Camus (Universidad de Santiago, Chile)
20. Stephen Guffey (University of Memphis)
21. Mamikon Gulian (Brown University)
22. Heeralal Janwa (University of Puerto Rico, Río Piedras)
23. Valentin Keyantuo (University of Puerto Rico, Río Piedras)

24. Ratna Khatri (George Mason University)
25. Irena Lasiecka (University of Memphis)
26. Frederick Leve (Air Force Office of Scientific Research)
27. Corole Louis-Rose (Université des Antilles, Guadeloupe)
28. Mariano Marcano (University of Puerto Rico, Río Piedras)
29. Mónica Nadal-Quirós (Inter American University, Bayamón)
30. Pablo Negrón-Marrero (University of Puerto Rico, Humacao)
31. Son Luu Nguyen (University of Puerto Rico, Río Piedras)
32. Aniel Nieves (University of Puerto Rico, Río Piedras)
33. Michael Parks (Sandia National Laboratories)
34. Rodrigo Ponce (Universidad de Talca, Chile)
35. Petronela Radu (University of Nebraska-Lincoln)
36. Silvia Rueda Sánchez (Universidad de Santiago de Chile)
37. Fabián Seoanes (University of Puerto Rico, Río Piedras)
38. Ralph C. Smith (North Carolina State University)
39. Pablo Raúl Stinga (Iowa State University)

40. Lev Steinberg (University of Puerto Rico, Mayagüez)
41. Louis Tebou (Florida International University)
42. Alejandro Vélez-Santiago (University of Puerto Rico, Mayagüez)
43. Deepanshu Verma (George Mason University)
44. Mahamadi Warma (University of Puerto Rico, Río Piedras)
45. Enrique Zuazua (Deustotech-Bilbao & UAM-Madrid, Spain)