



Innovations in Statistical Image Analysis and Applications to 3D Imaging for Improved SSA

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Final Report**

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14. ABSTRACT This final report on Grant No. FA9550-15-1-0286 documents the accomplishments of the grant effort over its full period of performance, August 14, 2015 - February 14, 2019. They include (i) derivation of physical models and first-principles electromagnetic analyses of ground-based observations to estimate the 3D shape of a roughened reflecting space-object surface under solar illumination; (ii) development of novel extended frozen-flow random-walk based aero-optical turbulence modeling; (iii) derivation of quantum upper bounds on the fidelity of pair super-resolution and experimental proposal to achieve them using projective wavefront measurements; (iv) lab tests of the spiral phase plate that performs image rotation; (v) simulation of data for imaging of point sources (space debris) at varying distances from a space-based rotating PSF imager, and analysis of the numerical optimization problem for the recovery of 3D source locations and fluxes; (vi) development and validation of new computational sparse recovery techniques and algorithms; (vii) development of a three-stage method for using multispectral RPSF images to localize and classify space debris; and (viii) development of a novel hyperspectral classification method inspired by image denoising/segmentation.					
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Final Report

Innovations in Statistical Image Analysis and Applications to 3D Imaging for Improved SSA

Grant PI: Prof. S. Prasad, University of New Mexico (UNM)
Subaward PI: Prof. R. Plemmons, Wake Forest University (WFU)

May 14, 2019

Period of Performance: August 14, 2015 - February 14, 2019

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Abstract

This document represents the final report on the various scientific activities and accomplishments relating to Grant No. FA9550-15-1-0286 over its period of performance, August 14, 2015 - February 14, 2019. During this period, the project accomplished a number of technical objectives, which have been organized under three broad categories, namely theoretical, experimental, and computational accomplishments. They are listed below.

List of Technical Accomplishments of the Grant Effort

The project investigators worked to accomplish the following scientific objectives during the course of the effort:

Theoretical Accomplishments

1. Derived robust physical models for ground-based observations made by a spectral-polarimetric imaging system designed to estimate the shape, roughness, particulate contamination, and material composition of a reflecting surface of a space object under solar illumination;
2. Developed and applied first-principles electromagnetic modeling and statistical analysis tools for scattering from roughened conducting surfaces in order to validate the micro-facet based geometrical optical model for roughened-surface scattering;
3. Proposed and developed an extended frozen-flow random-walk based aero-optical turbulence modeling formalism and derived the full spatio-temporal statistics of optical waves propagated from flying aircraft, and validation with previous observational/experimental results;
4. Derived quantum mechanically rigorous upper bounds on the fidelity of pair super-resolution using rotating PSF and theoretical analysis and experimental proposal to achieve a qualitatively improved photon cost of OSR using projective, rather than image- pixel-based, measurements; and
5. Developed a mathematical framework and the requisite computational tools for modeling atmospheric propagation of optical wavefronts in order to achieve a more complete characterization of ground-based BRDF observations.

Experimental Accomplishments

1. Designed the spiral phase plate (SPP) that performs range-encoding image rotation when placed in a conjugate plane of the telescope primary;
2. Ordered and fabricated the SPP meeting its rigorous [to $O(\lambda/20)$] design specifications at a premier nano-lithographic deposition facility in Minnesota; and

3. Conducted preliminary laboratory tests of the optical images formed with it at UNM's high-resolution microscopy laboratory to validate the theoretical physical-optics model on which the design was based.

Computational Accomplishments

1. Developed a software system to provide realistic simulated data for imaging of point sources (space debris) at varying distances from a space-based rotating PSF imager. The software is implemented in Matlab and enables the specification of particular optical and detector characteristics such as pixel pitch, detector size, wavelength, and F-number as well as average distance of point sources to the imager.
2. Performed analysis of the numerical optimization problem for the recovery of transverse location, point-to-imager distance, and energy flux for multiple point sources in the field of view. As a part of this analysis, regularization strategies leading to the recovery of the global optimizer have been proposed and efficient methods are currently under development.
3. Designed and tested new, effective sparse recovery algorithms for 3D localization of point sources using the rotating PSF imager. These recovery algorithms are based on compressed sensing since recovering the 3D distribution of point sources is a large-scale sparse inverse problem. Various approximations to the ℓ_0 pseudo-norm constraint model are applied to rotating point spread function problems and comparison experiments show the advantages of the proposed methods.
4. Developed and validated new computational sparse recovery techniques and algorithms, enabled by an international team of collaborators, for extracting 3D source locations from simulated rotating-PSF-based image data for collections of closely spaced point sources. These techniques include Kullback-Leibler (KL) entropic distance minimizer with a novel non-convex regularizer for Poisson data, using an iteratively reweighted ℓ_1 (IRL1) algorithm. A new scheme of estimation of the source fluxes from the KL data-fitting term are proposed;
5. Developed and tested a Matlab code for simulating observations of unresolved point sources using a rotating-PSF element. The code adheres to physical principles and allows the user to set relevant values for optical parameters, Poisson noise SNR, and detector characteristics.
6. Developed a three-stage method for using multispectral RPSF images to localize and classify space debris. With image data from multiple bands, not only can the localization results based on the use of a single band be improved, but also classification and material composition can be estimated. By extending the single band's nonconvex optimization model, a new model was adapted for the multiple band case in a localization stage (Stage 1). Stage 2 focuses on the spectral signatures' estimation and processes each band separately. An alternative scheme was proposed which can further remove the false positives and achieve better estimation for both locations and

spectral signatures. Stage 3 was devoted to classification from the estimated spectral signatures.

7. Developed a novel two-stage hyperspectral classification method inspired by image denoising/segmentation. The method is simple yet performs effectively. In the first stage, a support vector machine method is used to estimate the pixel-wise probability map of each class. The result in the first stage has decent accuracy but is noisy. In the second stage, a convex variant of the Mumford-Shah model is applied to denoise and classify the hyperspectral image into different classes.

1 Summary Data of Grant Funded Creative Work

Over its forty-two-month performance period, the research grant supported, in part, the production of 14 publications (6 refereed and 8 conference), 1 completed PhD dissertation at UNM (Zhixian Yu, advisor Prof. S. Prasad), 1 partially completed PhD dissertation at UNM (Julian Antolin-Camarena, advisor Prof. S. Prasad, estimated completion date - August 2019), 30 professional presentations (including 13 invited). Based on research partially supported by the grant, two (2) additional manuscripts are currently in preparation for submission to peer-reviewed journals. Two US patents, while not directly supported by grant funding, were issued during the grant's period of performance.

2 Expanded Summary of Grant Accomplishments

The following subsections present an expanded summary of the grant's most important technical accomplishments. They have been organized according to their theoretical, experimental, and computational thrusts.

2.1 Theoretical Accomplishments

2.1.1 3D Space Debris Localization using Rotating PSF Imagery - Theoretical Feasibility Studies

The theoretical research led by the PI and UNM group is expanding the promise of snapshot rotating-PSF-based 3D imaging and localization, following the original publication in 2013, to high-numerical-aperture (NA) microscopy and to telescopic, the latter specifically tasked with snapshot 3D space debris localization under active illumination and being of particular interest to the SSA community. We envision our optical localization technique to work in conjunction with and cued by a radar system that will detect, identify, and track debris at larger distances than a few kms away, but then hand over these tasks to the rotating-PSF based optical system over distances out to hundreds of meters to a few kms. Such a hybrid system can be mounted on a valuable US space asset to monitor its immediate debris environment in a snapshot mode without a stringent need to refocus it continually.

Let us consider the following formula derived using a simple telescopic-imaging-based adaptation of the theory of rotating PSF microscopy [1]:

$$\frac{d\phi}{dz_O} = \frac{\pi R^2}{\lambda L z_O^2}, \quad (1)$$

in which R is the radius of the telescope primary aperture, z_O is the distance of the object from the primary, and L is the number of zones in the spiral phase plate effecting image rotation. Equation (1) represents the rate at which the image rotates with changing axial range of the unresolved (point) debris (under active illumination). Applying this formula to obtain the z resolution corresponding to a certain minimum detectable image rotation $\delta\phi$ and the Rayleigh criterion for lateral (x, y) resolution, we can construct Table 1 of the 3D localization capabilities of a rotating-PSF-based optical telescope ($R = 0.5$ m, $\lambda = 1$ μ m, $L = 7$) as a function of the range of the debris. Note that the minimum z error scales

Table 1: Debris Range vs. Transverse (x, y) and Range (z) Localization Errors

Range	(x, y) Error	z Error
100 m	0.4 mm	6 mm
500 m	2 mm	150 mm
1,000 m	4 mm	600 mm
10,000 m	40 mm	60 m

quadratically with range, while the minimum (x, y) error scales only linearly with range. This difference between the axial and transverse scaling laws accounts for the relatively rapid rise in the former with increasing range, as shown in the Table.

We have conducted computer simulations for a single unresolved debris in support of the localization precisions reported in Table 1 at various signal-to-noise ratios. Simulations involving 3D fields of randomly positioned debris have also been performed and new algorithms developed and compared with respect to a number of fidelity metrics for the problem of recovery of both the fluxes and 3D locations of the debris. Two different noise models, namely Poisson shot-noise photocount statistics and Gaussian read-noise statistics, and the different algorithms that they entail for optimum performance were thoroughly investigated. Their results have been presented in two archival publications, including a refereed and a conference publication, over the last few months. This work has been described later in this Report in the subsection on Computational Accomplishments of the project.

Since space debris will need to be actively illuminated by an on-board laser system, we must estimate the laser power needed to accomplish the localization task. This power varies with the degree of collimation of the illuminating beam, the ranges over which the debris must be localized, and the scattering cross-section of the debris. Under collimated illumination, accounting for the inverse-square-law attenuation of scattered light signal received by the on-board imager, we have calculated the following range vs. minimum laser power curve for two different illuminating-beam waists and debris of scattering cross section of order 1 mm² to provide acceptable localization errors called out in Table 1:

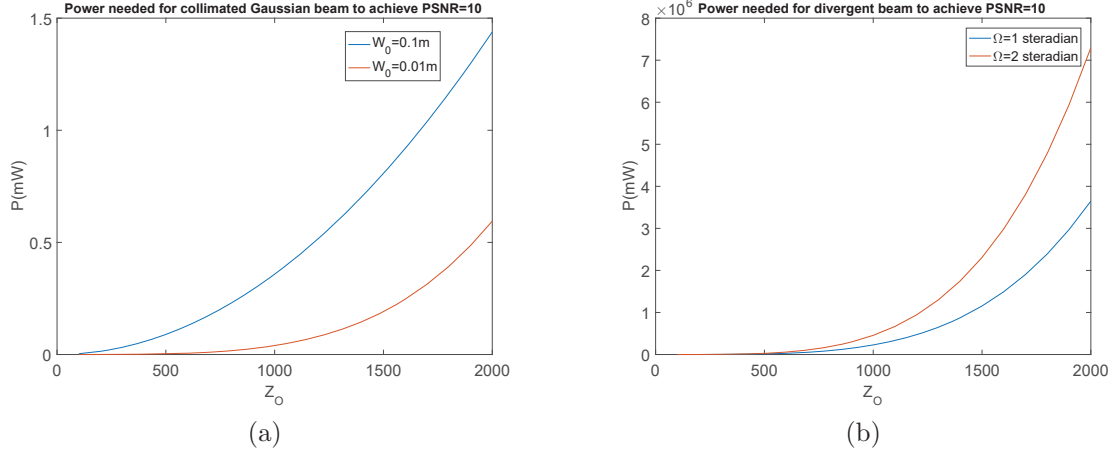


Figure 1: Laser power vs. debris range (in m) for a debris of scattering cross-section 1 mm^2 at $1 \text{ }\mu\text{m}$ wavelength for localization precisions reported in Table 1 (a) for collimated beam; (b) for divergent beam with two different divergence solid angles, 1 and 2 sr

2.1.2 Statistical Modeling of Aero-Optical Turbulence

The disturbances of the air next to a fast flying aircraft, like those of a fast flowing liquid past a stationary wall, are characterized by high Reynolds numbers and large, highly anisotropic randomness of density and pressure fluctuations in a thin aero-optically active layer no more than a few centimeters thick [2]. The air flow is typically comprised of turbulent boundary and shear layers as well as turbulent wakes, with flow-velocity-driven density fluctuations and consequent refractive index (RI) fluctuations, which can cause rapid spatio-temporal fluctuations of the phase of propagating light beams.

We have generalized the Taylor frozen-flow hypothesis to describe optical perturbations caused by an aero-optically active layer that convects at a mean velocity comparable to the ground velocity of the flying aircraft. Unlike air far from the aircraft, the air flow in this thin layer, however, also has significant velocity fluctuations. While we take the velocity fluctuations to be isotropically imposed on mean convection, we model the power spectrum of the spatial fluctuations of the RI that they transport as being highly anisotropic to account for the anisotropy of large scale coherent structures that perturb the optical phase most significantly.

We have modeled anisotropic aero-optical disturbances of the RI by a von Kármán type of extended spatial power spectral density (PSD) with three different lower cut-off frequencies, K_x, K_y, K_z , along the streamwise, spanwise, and layer-normal directions, which we label as x, y, z directions, respectively,

$$W_n(\mathbf{k}) = \frac{B_n^2}{K_x K_y K_z \left(1 + \frac{k_x^2}{K_x^2} + \frac{k_y^2}{K_y^2} + \frac{k_z^2}{K_z^2} \right)^{\gamma/2}}, \quad (2)$$

where B_n^2 is an appropriate turbulence-strength constant. We assume the inequality, $K_x \ll K_y, K_z$, to account heuristically for the much larger extension of the largest coherent struc-

tures in the streamwise direction than in the other directions, as in stream-aligned hairpin or horseshoe vortices [2]. For high Reynolds-number flow, the power index γ can be highly non-Kolmogoroff at 13/3, if the RI fluctuations arise from velocity-fluctuation-induced pressure and temperature fluctuations [4].

Our results on the spatio-temporal phase structure function (SF) are directly pertinent to observations performed by the original Airborne Aero-Optical Laboratory (AAOL) [5] and the upgraded AAOL-Transonic (AAOL-T) [6] that are designed to fly at Mach 0.5-0.7 or higher speeds and have already provided vast amounts of optical phase data. Nevertheless, our results are rather general for optical phase disturbances in transmission through any high-Reynolds-number flow near walls.

In the thin-phase-screen limit, surely valid for the thin aero-optical layer of interest, the optical phase retardation of a monochromatic laser beam over a time delay t at position $\mathbf{r}$ is given by the integral,

$$\phi(\mathbf{r}, t) - \phi(\mathbf{r}, 0) = k_0 \int_0^\sigma [n(\mathbf{r} - \zeta' \hat{\zeta}, t) - n(\mathbf{r} - \zeta' \hat{\zeta}, 0)] d\zeta', \quad (3)$$

where the beam propagates along the unit vector $\hat{\zeta}$, σ is the path length, and $k_0 = 2\pi/\lambda$ is the optical propagation constant expressed in terms of the wavelength λ . Invoking the frozen-flow hypothesis, according to which the value of the RI at position $\mathbf{r}$ and time t is the same as that at position $\mathbf{r} - \mathbf{v}_c t - \int_0^t \mathbf{v}(t') dt'$ at the initial time 0, expressing the RI in the spatial-frequency domain via its Fourier transform, squaring both sides of (3), and averaging it over the statistics of all possible RI fluctuations and Gaussian velocity fluctuations using (2) yields the temporal SF (TSF), $D_\phi^{(T)}(t)$, of phase of form

$$D_\phi^{(T)}(t) = 4k_0^2 B_n^2 \int \frac{d^3 q}{k_\zeta^2 (1 + q^2)^{\gamma/2}} [1 - \cos(k_\zeta \sigma)] [1 - \cos(q_x K_x v_c t) \exp(-\sigma_v^2 k^2 t \tau_c / 2)], \quad (4)$$

in which $k_\zeta = \mathbf{k} \cdot \hat{\zeta}$ is the projection of the spatial-frequency vector $\mathbf{k}$ given by $\mathbf{k} = (K_x q_x, K_y q_y, K_z q_z)$ along the beam propagation direction, and $q^2 = q_x^2 + q_y^2 + q_z^2$.

At long delay times, $t \rightarrow \infty$, the product of the time dependent terms contributes negligibly in (2), and the phase TSF saturates, but for short times, depending on whether the relative squared velocity fluctuation given by the ratio, $\chi = \sigma_v^2 / v_c^2$, is small or moderate, either the cosine or the exponential time-dependent term in (9) dominates the time-dependent behavior. For $\gamma = 13/3$, which exceeds 4, the corresponding dependences are either quadratic or linear at short times. If the Kolmogoroff model, $\gamma = 11/3$, holds, however, as for low-Reynolds-number flow, then the two behaviors for weak and moderate flow fluctuations are still of power-law form, but with indices that are $\gamma - 2$, *i.e.*, 5/3, and 1, respectively. The linear time dependence in the moderate-fluctuation regime, $\chi \lesssim 1$, is independent of γ , and arises from the diffusive transport of coherent spatial structures across the beam. Without the anisotropy of these structures, it would require $\chi \gg 1$, which is rarely seen in turbulent boundary and shear layers even at high Reynolds number, for the linear scaling to emerge at short time delays.

Theoretical Predictions and Comparison with AAOL Data We display in Fig. 2 a doubly logarithmic plot of the optical phase TSF versus time delay obtained by evaluating

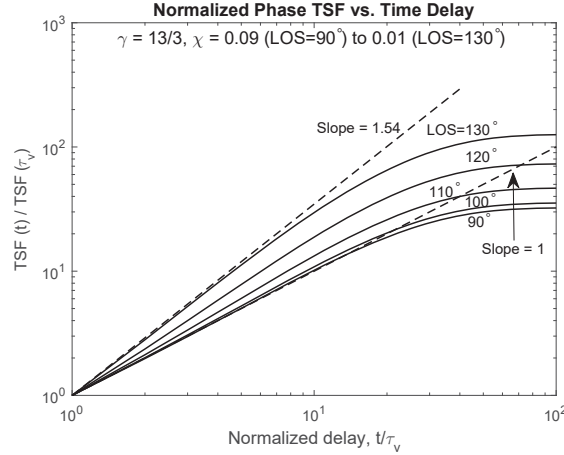


Figure 2: Phase TSF, normalized by dividing it by its starting value, that for time delay $t = \tau_c$, versus t/τ_c for five different LOS angles from 90° to 130° .

expression (9) under a sample set of parameters that apply to optical phase data acquired during actual AAOL missions and presented statistically via phase TSF. We allow for the relative flow fluctuation, $\chi^{1/2}$, to vary with the line-of-sight (LOS) angle between the optical beam and the reverse free-stream direction, in consistency with the observed AAOL data that showed higher relative fluctuation of about 30-40% at an LOS angle of 90° than the roughly 10% relative fluctuation at the higher LOS angle of 130° . The logarithmic plots, which exhibit slope variation from 0.9 to 1.4, are consistent with the behavior of phase TSF obtained from AAOL observations in Fig. 7 and Table 1 of [3]. This work was reported in Ref. [7].

Spatial Statistics of Aero-Optically Perturbed Gaussian Beam The use of frozen-flow aero-optics turbulence model was used to characterize short-time temporal statistics. The quadratic behavior of the unsaturated phase TSF when flow fluctuations are negligibly weak, as we have just discussed, corresponds to a quadratic behavior of the unsaturated spatial structure function (SSF) of the perturbed optical phase as well. The quadratic form of the phase SSF can be exploited to derive approximate, closed-form expressions for the correlation functions of the complex amplitude of an optical beam propagated through aero-optical turbulence. For a monochromatic Gaussian beam, these calculations can be performed in the Fourier domain and such correlation functions calculated to all orders, which is a rather remarkable result. We have derived all these correlation functions and used them to discuss the beam profile and intensity fluctuations, including the scintillation index, of the beam as its perturbed form propagates away from the aero-optical layer. These results are presented in great detail in a second J. Opt. Soc. Am. A publication [8]. In the next two sets of figures we present some key results of this project.

In Fig. 3, we display the beam cross-sectional shape as a function of distance for an initially focusing Gaussian beam transmitted out of a flying aircraft. The X -section of the beam, as the surface plot in Fig. 3a shows, first focuses then defocuses as it diffracts, but the

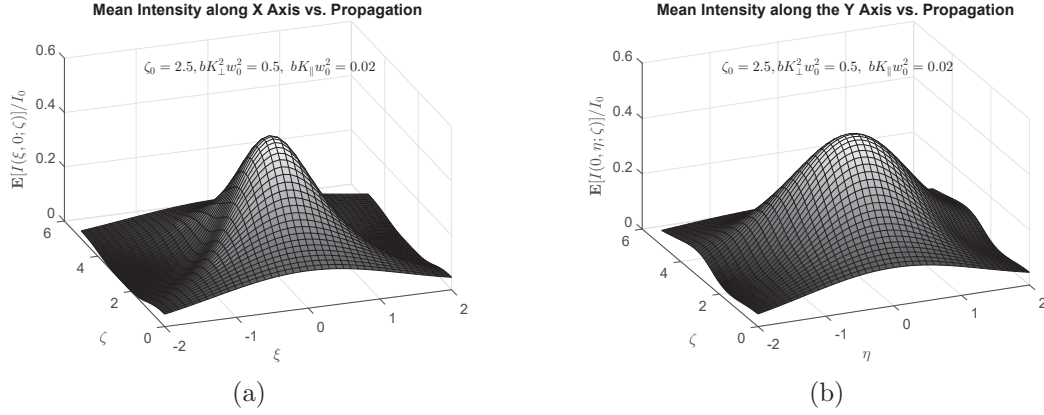


Figure 3: (a) The X -section (ξ) of the mean intensity of a focusing Gaussian beam vs. propagation distance (ζ); (b) Same as (a) except the Y -section (η).

focusing is imperfect due to the aero-optical phase perturbation of the beam. The Y -section, on the other hand, as Fig. 3b shows, does not focus at all as the perturbations along this dimension are greatly accentuated compared to the x direction by the anisotropies of the coherent structures in the aero-optical layer.

In Fig. 4a, we plot the on-axis beam scintillation index, defined as the variance of the fluctuations of the on-axis intensity to its squared mean value, as a function of the propagation distance. For both weak and strong turbulence the intensity fluctuations are most dramatic at more than twice the distance of best focus of the unperturbed beam, the latter having been taken to be 2.5 in the units plotted on the x axis. Figure 4b shows the behavior of normalized on-axis intensity moments in the first few lowest orders with propagation distance. The higher the order of the moment, the greater its peak value that is attained at a distance slightly larger than the distance of tightest focus of the unperturbed beam.

2.1.3 Analysis of the dependence of the spatial pBRDF distribution on the 3D shape

We developed a unified theoretical framework to analyze compressive sensing, sparse representation, and reconstruction of space objects in the combined spatial-spectral domain. The usual hyperspectral imaging approach gathers data over a full range of 2D spatial coordinates and 1D spectral channels in what is called a data cube. In most cases, such data typically carry relatively small amounts of information in a highly redundant manner, particularly with regard to man-made space objects like satellites that are typically composed of a much smaller number of primitives, e.g., spatially and materially homogeneous elements and geometrically simple parts, than the number of voxels in the full data cube.

Theoretical work proceeded along two different lines. The first entailed constructing simple forward models for the SOI problem. This construction begins with a low-dimensional parametric model of space-object surfaces using a superquadric parameterization. Superquadrics are generalizations of ellipsoidal, hyperboloidal, and toroidal surfaces based on changing the powers of the Cartesian-to-spherical coordinate relationships for such ordinary 3D surfaces.

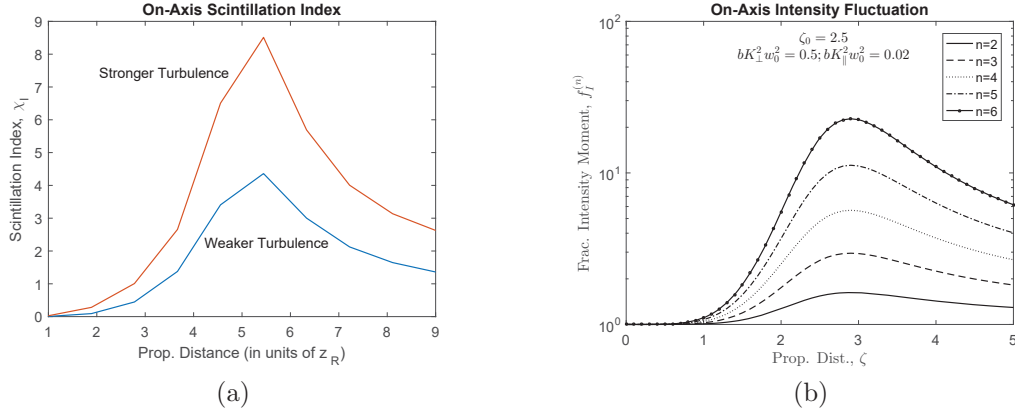


Figure 4: (a) The on-axis intensity scintillation index for a focusing Gaussian beam vs. propagation distance for weak and strong turbulence levels. (b) The on-axis intensity moments in orders 2-6 for a focusing Gaussian beam vs. propagation distance.

In particular, a super-ellipsoid may be parameterized as follows:

$$\begin{aligned} x &= a_1 \cos^{\epsilon_1} \eta |\cos \omega|^{\epsilon_2} \text{sgn}(\cos \omega); \\ y &= a_2 \cos^{\epsilon_1} \eta |\sin \omega|^{\epsilon_2} \text{sgn}(\sin \omega); \\ z &= a_3 |\sin \eta|^{\epsilon_1} \text{sgn}(\sin \eta), \end{aligned} \quad (5)$$

where the five parameters $a_1, a_2, a_3, \epsilon_1, \epsilon_2$ are chosen to be all positive. The first three determine the spatial scale of the three orthogonal dimensions in the object while the last two the geometrical shape of the solid. An example of a satellite mock-up composed of five different superellipsoids is shown in Fig. 5. By parameterizing the relatively small number of simple geometrical shapes that constitute a 3D satellite body via superquadrics, we can turn a complex surface characterization problem involving thousands of surface pixels into one involving a rather small number of parameters. These parameters can be estimated from a kinematic sequence of 2D images of such a 3D body, as we have shown previously for low-dimensional but non-superquadric parameterizations.

Once such a 3D space-object surface model is constructed, it may be endowed with material properties, specifically what material comprises what superquadric component of such a body. The surfaces are then given texture. To do so, we have modeled the surfaces of the individual superquadrics as being microscopically rough so they do not scatter sunlight simply via specular reflection. We have adapted a realistic roughness model, the Torrance-Sparrow reflectance model [9], for optical scattering from 3D surfaces that may be described statistically by means of a distribution of planar micro-facets with varying surface-normal orientations relative to the underlying idealized microscopically smooth surface. Such surfaces, since they are typically made from dielectric materials, reflect and refract, so there is both a single-scattering specular component and a multiply-scattered diffuse Lambertian component. It is the latter component, which gives an opaque surface its spectral character, *i.e.*, color, in solar reflection. Both these components contribute to the spectral power density but only the specular albedo has a polarimetric content that carries information about

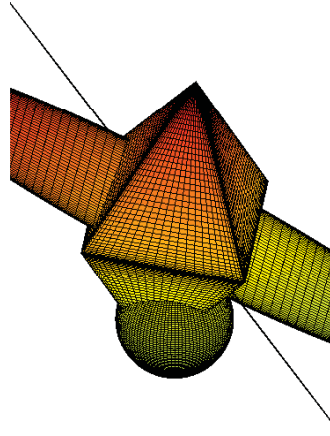
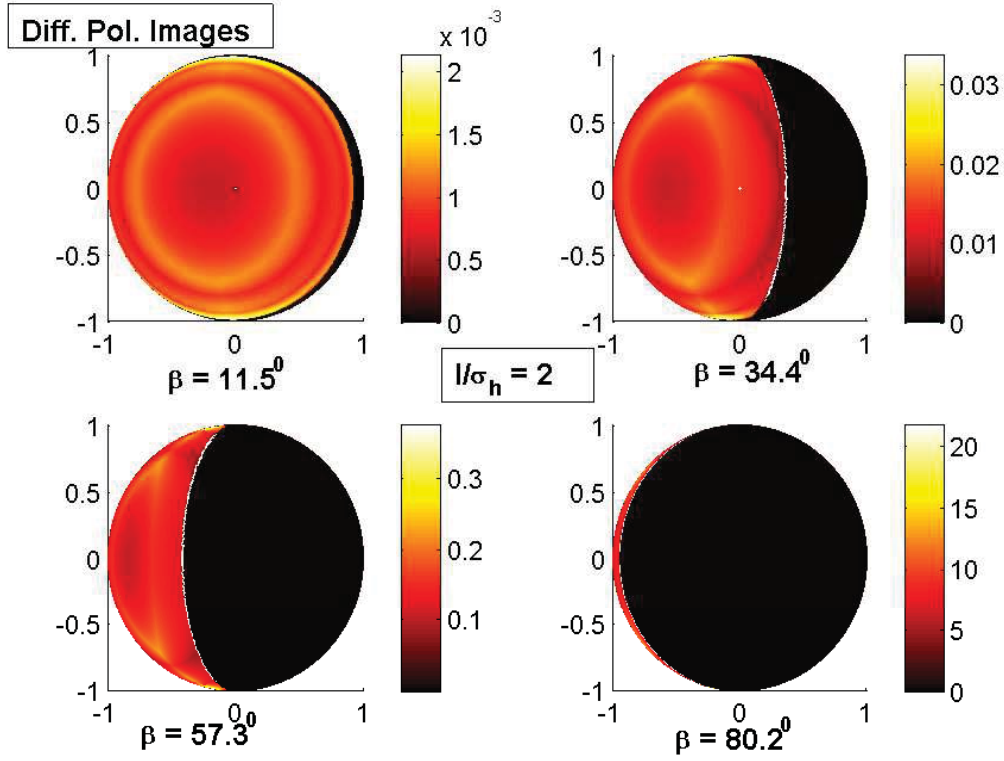


Figure 5: A satellite mock-up composed of five simple superquadrics

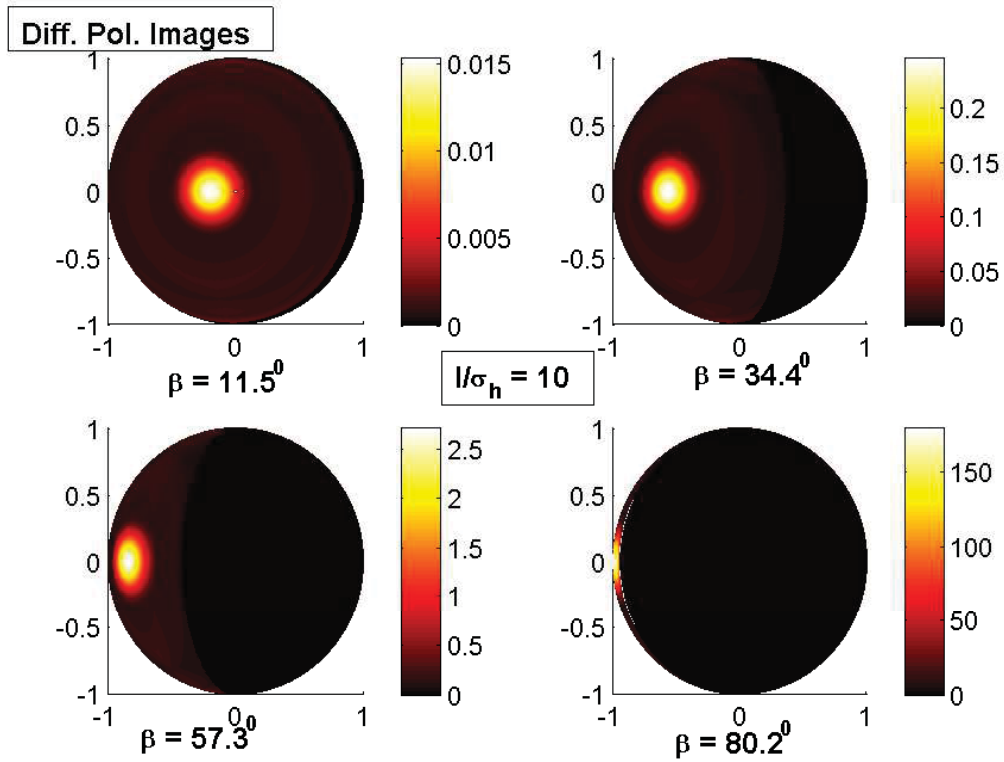
both the local shape and roughness of the space object. Thus, when supplemented with a sensor noise model, a model for atmospheric propagation and a blur model for the observing instrument, our calculations allow us to simulate the spectral and polarimetric content of any observations one might make of the solar albedo from a space object of varying local shape and roughness profile. It is this end-to-end forward model that we simulated to demonstrate the validity of our compressive spectral-polarimetric sensing and reconstruction approach.

The power spectral density (PSD) of the diffuse solar albedo depends largely on the dispersion of the materials comprising the surface, rather than its shape or roughness, and has been extensively measured for a host of materials commonly present on a satellite. We therefore focused less on the diffuse PSD in our effort, but rather more on the specular albedo which, as we have noted, depends not just on material dispersion but also more sensitively on the surface shape and roughness. We present some of the results of our simulations- in Fig. 6 where we display the difference-polarization (H-V) images of a roughened sphere of Si at a specific wavelength and four different solar angles of illumination. It is clear that surface roughness, as measured by the ratio of the standard deviation of the heights of the microfacets and its length, affects the footprint of the polarimetric signature of the surface. For highly smooth surfaces, the polarimetric difference image is limited to the glint spot, but as the surface becomes rougher, the spot containing polarimetric information spreads out in a way that is a measure of the roughness (for fixed shape), as we see in Fig. 6(b).

In Fig. 7, we plot the dependence of the angle-integrated power in the specular component as a function of the solar angle and varying roughness. As roughness increases, the specular component decreases in its fractional power when compared to the diffuse albedo, so the surface becomes more Lambertian and acquires its spectral signature that gives it a color and spectral content quite distinct from the white-light spectrum of the illuminating sunlight. By parameterizing the relatively small number of simple geometrical shapes and materials that constitute the space object and their roughness levels, represented by the ratio of the statistical scatter of microfacet heights to their lateral dimensions, we thus have an excellent



(a)



(b)

Figure 6: The 2D spatial distribution of difference-polarization intensity in solar reflection from a sphere for four different solar angles for (a) smooth surface; (b) rough surface.

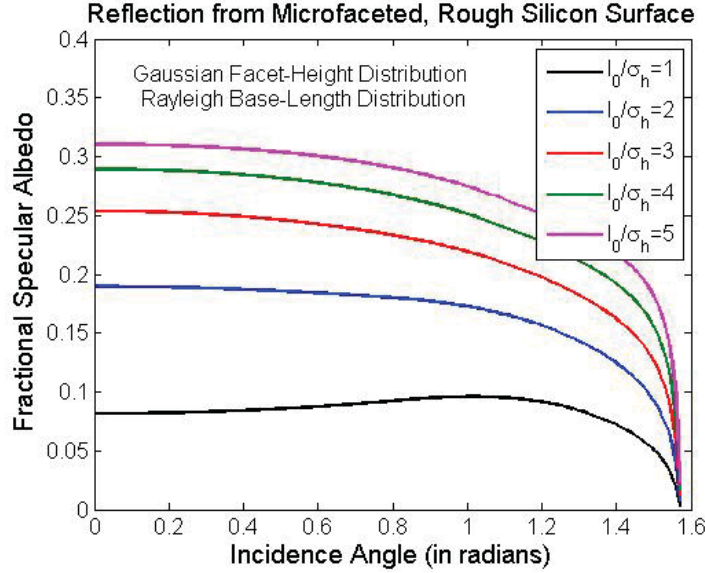


Figure 7: The total specular return fraction, sHDR, in solar reflection vs. the solar angle for different roughness levels

forward model for varying local shapes, spectral reflectances, and roughness levels of a general space object.

Results of extensive simulations of diffuse and specular scattering from a single roughened superquadric surface and recovery of its nine (shape, size, Euler orientation angles, and roughness) parameters were presented in Annual Report 2 for the grant. Two different algorithms were used, namely an alternating minimization routine and a polynomial patch parameterization based parameter recovery algorithm, which both demonstrated much promise. The reader is referred to the Annual Report for more detailed information.

2.1.4 Rigorous Electromagnetic Modeling of Optical Scattering and BRDF from Solar Illuminated Space Objects with Roughened Reflecting Surfaces

Recent theoretical work has focused on the problem of a rigorous electromagnetic-field-based description of optical scattering and reflections from a perfect electrical conductor (PEC) whose surface is in general roughened and curved, for which we have used the simplified microfacet geometrical-optics model of scattering, as we described in the previous subsection. Roughness is dealt with by allowing the surface to have statistically random deviations from a mean smooth underlying surface, which obey Gaussian statistics and have a certain standard deviation and correlation length determined by the level and scale of roughness. This work forms the main subject matter of a continuing Ph.D. dissertation at U. New Mexico, which is expected to be completed by August 2019. It will help clarify the rigorous physical basis and theoretical limits of validity of the microfacet model.

Specifically, we have analyzed a one-dimensional random surface that is translationally invariant along a Cartesian direction, which we have called y , so the surface is described by

means of an equation of form,

$$z = z(x), \quad (6)$$

with its mean being $\langle z(x) \rangle = z_0(x)$ and deviations, $\delta z(x) = z(x) - \langle z(x) \rangle$, from the mean being distributed according to Gaussian statistics with zero mean and covariance function, $C_z(x, x') = \sigma_h^2 \exp[-(1/2)(x - x')^2/l^2]$. For such PEC surfaces, the theory of electromagnetic waveguides can be applied, and complex mode solutions of form $\psi(x, z) \exp(i\beta y - i\omega t)$ for the various components of the field and surface current density may be written down. We have implemented such solution forms to write down a full vector-field electric-field integral equation (EFIE) that expresses the integral of the surface current density in terms of the tangential component of the electric field of the (known) incident plane wave that is being scattered by the PEC. We have solved this integral equation by discretizing the surface into strips of fixed x width, δx , with $\delta x \ll l, \lambda$, $\lambda = 2\pi c/\omega$ being the wavelength, and each strip being parallel to the y axis. The discretization converts the EFIE into a matrix equation that we solved numerically by matrix inversion.

Very generally, for a monochromatic incident field at frequency $\omega = ck$, the scattered electric field at position $\vec{r}$ may be expressed as

$$\vec{E}_s(\vec{r}) = \left(\mathbf{I} + \frac{1}{k^2} \vec{\nabla} \vec{\nabla} \right) \cdot \vec{A}(\vec{r}), \quad (7)$$

where $\mathbf{I}$ is the unit dyadic tensor, with elements δ_{ij} , and $\vec{A}(\vec{r})$ is the following scalar-scattering integral:

$$\begin{aligned} \vec{A}(\vec{r}) &= i\omega\mu_0 \int \vec{K}(\vec{r}'_S) G(|\vec{r} - \vec{r}'|) dS' \\ &= i\omega\mu_0 \int \int \vec{K}(\vec{r}'_S) G(|\vec{r} - \vec{r}'|) \sqrt{1 + z_{x'}'^2} dx' dy', \end{aligned} \quad (8)$$

in which $G(R) = \exp(ikR)/(4\pi R)$ is the 3D Green's function of the Helmholtz operator, $\nabla^2 + k^2$. For the 1D random surface with perfect y -translational invariance, if the exciting field is incident in the xz plane, so $\beta = 0$, and is s -polarized (*i.e.*, perpendicular to the plane of incidence), so it is along $\hat{y}$, then the current density, $\vec{K}$, is also along $\hat{y}$. In this case, the fields have no y dependence, and both $\vec{E}_s$ and $\vec{A}$ are equal to each other and also along $\hat{y}$. Equation (12) then reduces to a scalar-field relation,

$$E_s(\vec{r}) = i\omega\mu_0 \int K(\vec{r}'_S) G_0(\vec{r} - \vec{r}') \sqrt{1 + z_{x'}'^2} dx', \quad (9)$$

in which

$$\tilde{G}_0(R_\perp) = \int_{-\infty}^{\infty} dY \exp(ik_y Y) G(\sqrt{R_\perp^2 + Y^2}) = \frac{i}{4} H_0^{(1)}(k_\perp R_\perp), \quad (10)$$

where $R_\perp = [(x - x')^2 + (z - z')^2]^{1/2}$, is the Green's function of the 2D Helmholtz operator. Substituting $E_s(\vec{r}) = -E_i(\vec{r})$, where $\vec{r}$ is a general point located on the surface, into the left-hand side of Eq. (9), as the net electric field has no tangential component on the surface of a PEC, yields the requisite EFIE in this simple case. More general cases of incidence and incident polarization may be handled by means of the projections $A_y \hat{y}$ and $\vec{A}_\perp$ along the

y axis and in the xz plane of fields like $\vec{A}(\vec{r})$, for which two coupled EFIE's result, which may also be solved by discretization and matrix inversion methods, as we show in detail in a forthcoming paper.

2.1.1.1 Numerical Results Here are some graphical displays that capture the main results for the scattered fields and scattering cross-sections that are obtained from our numerical calculations for the simple case of incidence in the xz plane and s polarization.

(a) Sinusoidal Surface The surface is taken first to be non-random but with a sinusoidal height variation of amplitude z_0 on a flat background, $z(x) = z_0 \sin(2\pi\alpha x/\lambda)$, where the periodicity of the sinusoid is λ/α , with the number α varied between 0.05 and 0.5, and the total width of the sinusoidal surface was taken to be $w = 50\lambda$. We also assume an s -polarized plane wave incident in the xz plane, for which $\beta = 0$ and field variables do not have any y dependence. For such a sinusoidal surface and incidence in the xz plane, the scattering may be shown to be centered at angles, $\theta_s^{(n)}$, that are related to the angle of incidence, θ_i , as

$$\sin \theta_s^{(n)} = \sin \theta_i - n\alpha, \quad n = 0, \pm 1, \pm 2, \dots, \quad (11)$$

with only those integer values of n allowed for which $\theta_s^{(n)}$ is real (i.e., $|\sin \theta_s^{(n)}| \leq 1$). The peaks heights scale as $|J_n(2\pi(\cos \theta_s^{(n)} - \cos \theta_i)z_0/\lambda)|^2$, with J_n denoting ordinary Bessel function of order n , and peak widths scale as λ/w .

In Fig. 8, we plot, for the surface profile shown in the bottom left panel, the scattering cross-section, σ , for the case of $\theta_i = 0.64$ rad in the top right panel. We clearly see a clustering of scattering peaks around the main peak at $\theta_s = 0.64$ rad. A single large peak at $\theta_s = \pi - \theta_i \approx 2.5$ rad corresponds to forward, shadow scattering responsible for the extinction of the total field inside any PEC. The top left panel shows the total scattering cross-section, σ , given by integrating plots like that in the top right panel over all scattering angles between $-\pi$ to π , for each angle of incidence and then plotting the result in units of $w \cos \theta_i$, which is the projected width of the scattering PEC strip in a plane orthogonal to the direction of incidence. Since all of the light incident in this cross-sectional length is scattered perfectly by the PEC, with the same amount of light used to create shadow scattering in the forward direction behind the PEC to exactly cancel the incident field for zero total field inside the PEC, as we just noted, the ratio $\sigma/(w \cos \theta_i)$ should be exactly equal to 2 for all θ_i . We see this to be approximately true in this plot, which supports the validity of our numerical calculations. The surface current density, $K(x)$, obtained from our numerical evaluation is shown in the bottom right panel.

(b) Random Surface We next turned our attention to y -invariant surfaces with a random height profile along x with respect to a mean flat strip of width w . In this case, the structured scattering phenomenon that we observed for a regular, sinusoidal surface is no longer obtained. In fact, the specular scattering peaks, which are located around the specular-reflection peak expected for the flat strip at angle, $\theta_s = \theta_i$, are greatly attenuated, while the forward-scattering peak at $\theta_s = \pi - \theta_i$ is expected to remain quite pronounced and narrow, as it is responsible for the shadow scattering extinction of the incident wave inside the PEC regardless of its surface randomness or morphology. In Fig. 9, we plot, for the

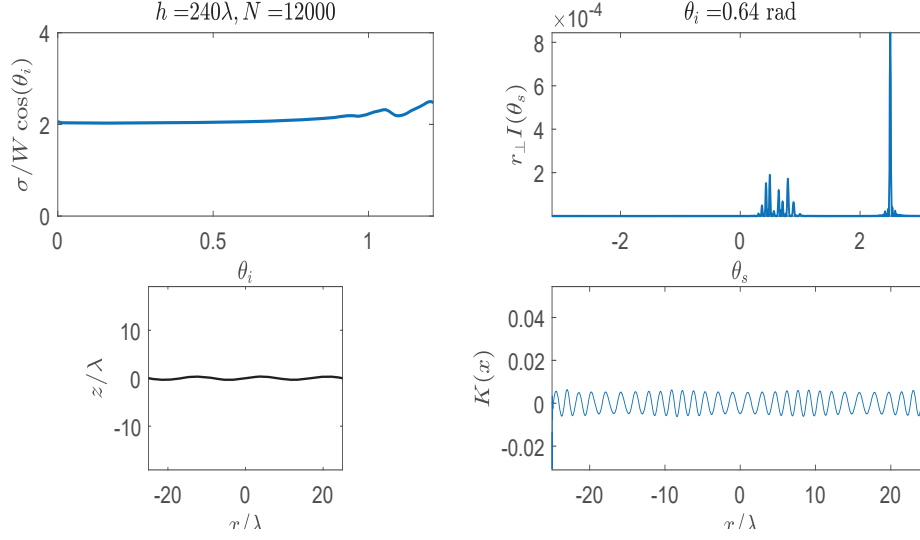


Figure 8: **Top left:** Scattering cross-section (actually a length for 2D scattering) normalized by projected strip width. **Top right:** Scattered total intensity multiplied by the distance between the PEC slit and observation point to get rid of positional dependence. **Bottom left:** The sinusoidal-surface height profile, with an amplitude, $z_0 = 1.3\lambda$. **Bottom right:** Induced surface current on the PEC strip.

random surface profile shown in the bottom left panel, the scattering cross-section, σ , for the case of $\theta_i = 0.38$ rad in the top right panel. We clearly see a clustering of scattering peaks around the main peak at $\theta_s = 0.38$ rad. A single large peak at $\theta_s = \pi - \theta_i \approx 2.76$ corresponds to forward, shadow scattering responsible for the extinction of the total field inside any PEC. Expectedly, the total scattering cross-section, as seen in the top left panel, remains close to 2 for all θ_i . The surface current density, $K(x)$, obtained from our numerical evaluation is shown in the bottom right panel, which shows a rather random structure consistent with the randomness of the surface profile.

The preceding figures present just two specific instances of our exhaustive numerical evaluations of scattering from a corrugated PEC surface that is translationally invariant in a single direction but can otherwise be either random or systematically structured along a transverse direction. These calculations are based on a rigorous electromagnetic analysis of optical scattering from a PEC surface of arbitrary surface height structure. We have also considered the geometrical optical limit of the theoretical formulation of this scattering problem, using the method of stationary phase asymptotics, and find good agreement with previous, more heuristic analyses of this problem. The latter have typically been formulated in terms of specific roughness models, like the Torrance-Sparrow micro-facet model [9], which only lend themselves to a geometric-optical analysis and cannot be extended to more general reflecting surface morphologies with fine structure on the wavelength scale for which such analyses must fail and full vector-electromagnetic-field considerations must be brought to bear on the problem, as we have shown. Our results are presently being written up both for inclusion in Julian Antolin-Camarena's Ph.D. dissertation and for submission for publication in a peer-reviewed journal.

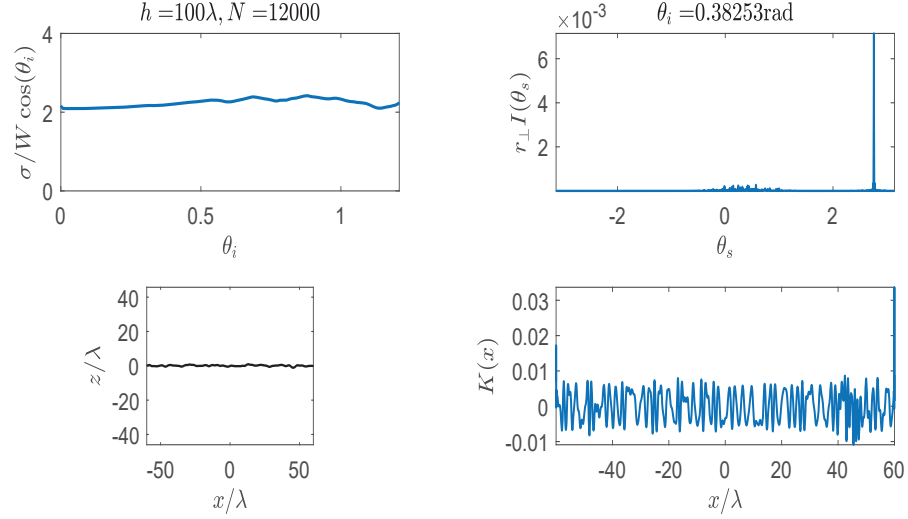


Figure 9: **Top left:** Scattering cross-section (actually a length for 2D scattering) normalized by projected strip width. **Top right:** Scattered total intensity multiplied by the distance between the PEC slit and observation point to get rid of positional dependence. **Bottom left:** The random-surface height profile, with a standard deviation, $\sigma_z = 0.25\lambda$, and correlation length, $l_z = 1.5\lambda$. **Bottom right:** Induced surface current on the PEC strip.

2.1.5 Reduced Photon Cost and Quantum-Limited 3D Pair Superresolution using the Rotating PSF and Projective Measurements

A recent paper [10] has advanced the use of wavefront projections in a method called spatial de-multiplexing (SPADE), rather than imaging, with to laterally resolve two closely spaced incoherent point sources, at a qualitatively reduced photon cost. We have applied the SPADE approach to rotating point-spread function (PSF) based 3D imaging of point sources to demonstrate its usefulness for 3D optical superresolution (OSR). Although it is now increasingly being realized that the degree of achievable OSR is simply a matter of having enough photons to overcome the Rayleigh resolution limit, the minimum photon number needed to resolve two point sources a distance d apart scales rather prohibitively as $1/d^4$ in the shot-noise limit of photon counting. The new approach qualitatively improves this scaling, making it more modest at $1/d^2$ and which furthermore is achievable even for a pair of thermal sources by means of classical projective measurements. We have adapted the calculation of the quantum mechanical limit on OSR and its achievability by the use of projective measurements and maximum likelihood estimation in the asymptotic limit of many photons for 2D OSR described in Ref. [10]. Specifically, we have derived an expression for the quantum Fisher information (QFI) matrix for the 3D OSR problem, which, when expressed in units of inverse squared transverse diffraction width (for its 11 and 22 elements)

and axial diffraction width (for its 33 element), takes the diagonal form

$$H = \begin{pmatrix} 4\pi^2 & 0 & 0 \\ 0 & 4\pi^2 & 0 \\ 0 & 0 & \frac{\pi^2}{3} \end{pmatrix}. \quad (12)$$

Rather surprisingly, QFI (12), as we recognize immediately, is independent of the pair separation vector. That suggests the possibility that projective measurements turn the pair-resolution problem into one of localization that fundamentally cannot depend on the actual source location for a space-invariant imaging aperture. Indeed, QFI for 3D localization of a single source point also has exactly the same value, as we have shown in the paper [12] published in 2018 in Physical Review Letters (PRL) and have thus confirmed this conjecture. This paper also advances the specific types of complete orthonormal bases of projection states that can attain QFI very generally.

We have also extended this calculation to a joint estimation of the pair-separation vector and pair-centroid position, showing that the two estimations are fundamentally independent and do not affect the theoretically smallest possible error of estimation, whether estimated jointly or separately. This work is presented in our second peer-reviewed paper on this subject [13], which was published recently.

In both these papers, we also introduced the choice of Zernike-polynomial basis for an efficient implementation of this approach, since the first two Zernike polynomials, Z_2 and Z_3 , which are the x and y tilts, respectively, are perfectly and exclusively matched filters with respect to the imaging wavefront corresponding to the x and y components of the pair separation vector in the limit of small separations. While there are no pure Zernikes that are exclusively matched to the quadratic defocus phase corresponding to the axial separation of the source pair, even with just the first four Zernike projections the three separation-vector components are well determined in the small-separation limit. This conclusion is dramatically verified in Fig. 10, which plots the Cramér-Rao lower bound (CRB), which is the inverse of Fisher information (FI), in the finite Zernike basis. This CRB, when compared with the inverse of QFI, which is the quantum CRB (QCRB, dashed line), shows perfect convergence to the QCRB bound in the small-separation limit.

Current Work and Future Directions This novel approach of point-source separation using wavefront projections yields a qualitative improvement of the normally quartic scaling of the photon flux needed to achieve a given degree of super-resolution (as specified, e.g., by a factor under the Rayleigh resolution limit) to a quadratic one. If the approach can be scaled up and applied to resolving extended objects, e.g., by using meter-size ground-based telescopes to achieve meter-size resolution at GEO altitudes, then the modest quadratic photon cost for achieving sub-diffractive resolution scales may mean resolvability of structures on such objects in spite of the faintness of the sunlight scattered by them and received by a ground-based telescope.

The quantum bound on the estimation of pair separations will also vary depending on the ratio of fluxes of the two sources, which we have assumed so far to be equally bright. We expect the best performance, or equivalently the highest QFI, for the case of equal

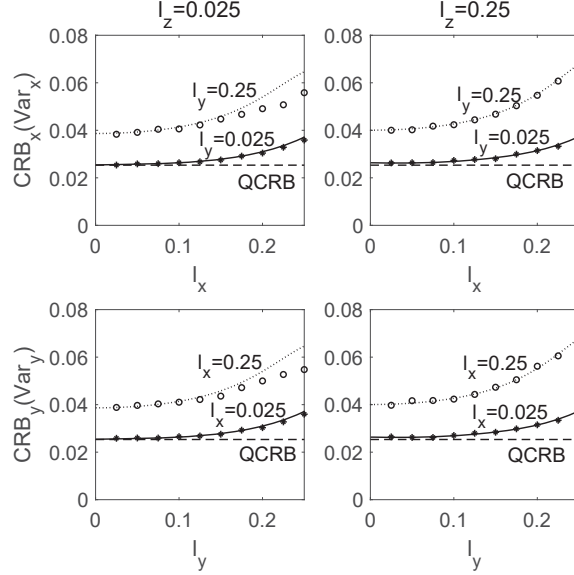


Figure 10: Plots of CRBs w.r.t. l_x for $l_y = 0.025$ (lower curve) and $l_y = 0.25$ (upper curve) and for $l_z = 0.025$ (left panels) and $l_z = 0.25$ (right panels). l_x, l_y are x, y separation coordinates in units of $\lambda f/D$, while l_z is the axial separation coordinate in units of $\lambda f^2/D^2$, which are characteristic Rayleigh scales for transverse and axial separations, respectively. The roles of l_x and l_y are interchanged in the bottom panels. Estimation variances obtained from simulation are shown by different marker symbols.

source brightness, but the QFI must degrade as the relative brightness of the two sources deviates from unity. In spite of some recent progress made by us in this problem, we have not yet achieved a complete derivation of the QFI as a function of the brightness ratio. This calculation will be critical to the calculation of QFI for superresolution imaging of extended objects with spatially varying brightness, as would be true for most extended objects of interest.

Boosting the photon count for faint objects would also require the ability to operate such an imager with significant signal bandwidths. Extending our calculations from a single spectral emission line, which has been assumed for our work to date in this problem, to multiple emission lines over a finite bandwidth is also being pursued currently.

These three generalizations of our work to date on this novel approach will critically inform our assessment of how practical the approach of using wavefront projections in the pupil plane for high-resolution imaging of extended incoherent objects under photon-starved conditions might ultimately be.

2.2 Experimental Accomplishments

2.2.1 Fabrication of a Spiral Phase Mask for Rotating PSF Imagery

One of the principal tasks for the grant proposal was to study the feasibility of a rotating PSF imager, which employs a spiral phase plate (SPP) containing multiple annular zones of

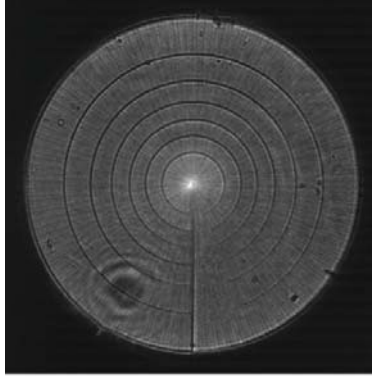


Image of Phase Plate

Figure 11: An optical image of the spiral phase plate taken by the optical set-up described in Fig. 13

spiral phase retardations. Each zone of this pattern must carry a pure vorticity index that changes in regular integer steps from one zone to the next in order to create image rotation with changing depth of a point source. Such an imager can thus fully localize unresolved sources in 3D, at least when the sources are not too far away, as we discussed earlier in this report. This device was invented by the PI in 2013 under support from a previous AFOSR grant (FA9550-09-1-0495) and has been awarded two patents during the period of performance of the present grant.

The study of the potential application of such a 3D source localizing imager as part of a more comprehensive SSA system for space debris detection and avoidance in the vicinity of a space asset has been a principal task for the grant proposal. Having established the theoretical limits of performance, including the range over which such an imager could effectively localize microscopic debris, we successfully proposed to the Program Officer, Dr. Stacie Williams, for a budget modification of the grant to allow us to test out such a device on an optical bench in the high-resolution microscopy laboratory of a colleague of the PI, Dr. Keith Lidke, at UNM. Since the basic principles of 3D localization using rotating PSF imager in the microscopy and telescopic imaging set-ups are essentially the same, the grant team felt that this kind of microscopy study in an existing laboratory would furnish an efficient, albeit indirect, validation of the space-based application without having to request additional budget to design and build an entirely new imaging set-up. A small budget for the fabrication of such a SPP was approved and awarded during the second year of the grant.

This phase plate was fabricated under two cooperative agreements between UNM, University of Arizona (Dr. Thomas Milster's lithography lab) and Minnesota Center for Nano-Technology, a state-of-the-art facility for optical lithographic etching and deposition of structures at ultrahigh precisions with printed features that are accurate down to a few tens of nanometers. The first prototype SPP that resulted from this collaboration was flawed. However, a second one, whose image appears in Fig. 11, was fabricated meeting the rigorous design specs. The spiral staircase structure consists of seven zones starting from the central disk surrounded by six different concentric annuli, with the n th annulus starting with zero

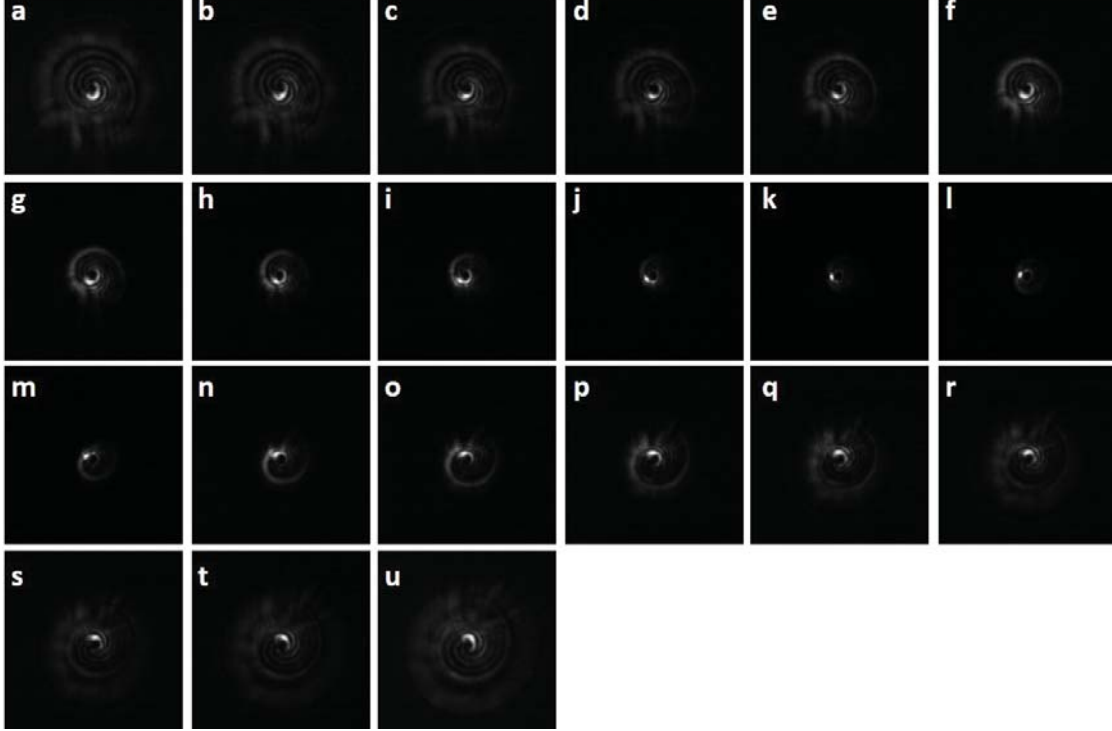


Figure 12: Surface plots of the PSFs with changing axial depth of the source for 21 different z planes over a total range of $10\ \mu\text{m}$. Note the tightest PSF form corresponding to the central, Gaussian focal plane (labeled as panel **k** in the figure).

relative thickness at a common angular position (dislocation line, shown radially downward here, independent of n) and then rising uniformly in thickness along the azimuthal angle coordinate to a final optical thickness of n optical waves after a complete 2π azimuthal rotation. Corresponding the phase retardation of an optical wave that is transmitted through it is comprised of seven zones, each zone starting with a relative phase of zero from the common dislocation line and then rising uniformly (in angle) to a progressively larger, integer number of waves before returning to that line.¹

2.2.2 Preliminary Results of Experimental Tests

The correctly fabricated SPP was not delivered to the project team until December 2018, at which time only a month remained in the (already-extended) grant duration. Only initial optical tests of the plate could be performed, which showed, however, that the plate worked according to the theoretical principles and design specifications on which it was based. The experimentally observed PSFs, particularly their rotational properties with changing axial depth of a point source, are displayed in Fig. 12.

The optical set-up that acquired both these images is presented in the next figure (Fig. 13). The final lens L_4 was moved in and out to acquire an image of the spiral phase

¹The large pit like structure seen at the bottom left of the SPP etch pattern is an artifact caused presumably by a speck of dust lying on one of the optical components near an objective-conjugate position in the beam path forming the image of SPP.

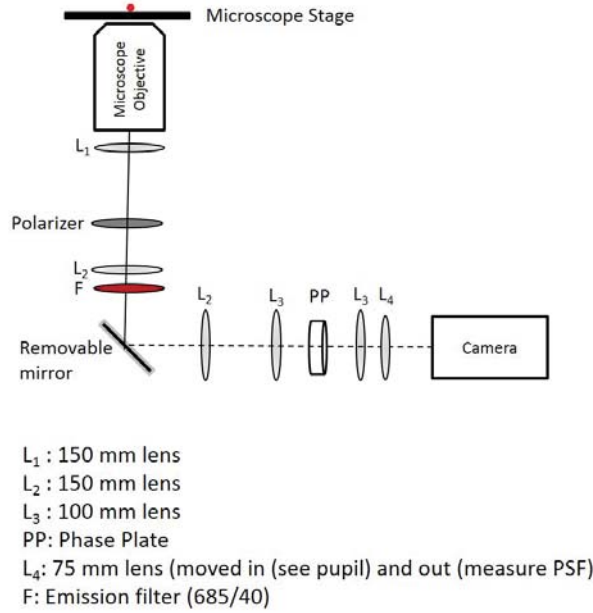


Figure 13: The optical bench set-up of the experiment to acquire both an image of the SPP and the optical PSFs, which are images of a $200\ \mu\text{m}$ bead that are placed at 21 different distances from the objective.

plate (denoted as PP), which was placed in a conjugate plane of the microscope objective, as well as the optical PSF images appearing in the previous figure.

Our future plans involve studying this device and other modified designs that can improve the overall quality of the acquired images for the tasks at hand. In particular, plans are in motion to study the modifications of the PSF arising from power radiated into near-field imaging light when a point source like a fluorophor in a biological sample on a coverslip of the microscope is very close to the glass interface. As much as 50% of light is contained in such near-field modes [14], which would be non-propagating in free space, but with the glass interface, much of this light for fluorophors very close to the glass interface can be converted into propagating photons, greatly increasing the signal power and thus enhancing the capabilities of a high-resolution imager such as the one being used in Dr. Lidke’s lab where this study is being conducted. Both theoretical and experimental studies are being planned to fully understand the capabilities of rotating PSF imagery for a high-NA microscope. As was originally envisioned by the grant proposal four years ago, we regard this new application as an excellent example of the grant serving to seed and enable important ideas and applications in a related topic, namely ultrahigh-resolution 3D single-molecule localization microscopy, which belongs in the important application domain of biophysics.

2.3 Computational Accomplishments

2.3.1 Trust-Region Methods for Nonconvex Sparse Recovery Optimization [15]

In this work we formulated a quasi-Newton trust-region method for solving non-convex penalized sparsity recovery problems arising in remote sensing. In particular, we developed

novel methods for solving the ℓ_2 - ℓ_p sparse image recovery problem. Here $0 < p < 1$, defines the regularization pseudo-norm term. We formulate the minimization problem as a smooth unconstrained optimization problem using a change of variables. We bridge the commonly-used convex ℓ_1 norm with the ℓ_0 quasi-norm using a non-convex ℓ_p norm. Numerical results show that the proposed TrustSpa- ℓ_p approach eliminates spurious solutions more effectively than using an ℓ_1 -regularization term. Furthermore, the proposed method converges faster over a 10-trial average.

Futher work includes applications of our method to sparse recovery in higher dimensional data. In particular, we plan extensions of the tests from 1D in the current paper (sparse signal recovery) to 2D images, 3D hyperspectral datacubes, and 4D spectro-polarimetric data, as well as recovery of rotating PSF images.

2.3.2 Classification Computations for Pixel-level fused Hyperspectral and LiDAR Data in Remote Sensing [16]

We investigated classification from pixel-level fusion of Hyperspectral (HSI) and Light Detection and Ranging (LiDAR) data using convolutional neural networks (CNN) for deep learning. HSI and LiDAR imaging are complementary modalities increasingly used together for geospatial data collection in remote sensing. HSI data is used to glean information about material composition and LiDAR data provides information about the geometry of objects in the scene. Two key questions relative to classification performance are addressed: the effect of merging multi-modal data and the effect of uncertainty in the CNN training data. Two recent co-registered HSI and LiDAR datasets are used here to characterize performance. One was collected over Houston, TX, by the University of Houston National Center for Airborne Laser Mapping with NSF sponsorship, and the other was collected over Gulfport, MS, by Universities of Florida and Missouri with National Geospatial-intelligence Agency (NGA) sponsorship. Our experimental results suggest that pixel-level data fusion can be an effective approach to improving accuracy of convolutional neural network classifiers, by roughly 10% compared to classification via HSI alone. For results from our processing the NGA Gulfport data see Figure 14.

2.3.3 3D Debris Localization using Rotating Point Spread Function Imagery in Gaussian noise [17]

We use a Rotating Point Spread Function (RPSF) imager to perform 3D localization of space debris from one 2D snapshot $G \in \mathbb{R}^{m \times n}$. The original 2D image is obtained by extracting the last slice of the tensor which is the 3D convolution of $\mathcal{A}$ and $\mathcal{X}$, where $\mathcal{A}$ is the RPSF and $\mathcal{X}$ represents the 3D location of the point source(s). We apply orthogonal matching pursuit (OMP) to approximately solve this ℓ_0 optimization problem

$$\begin{aligned} \min_{\mathcal{X}} \quad & \|\mathcal{X}\|_0 \\ \text{s.t.} \quad & \|T(\mathcal{A} * \mathcal{X}) - G\|_2^2 < \epsilon, \end{aligned}$$

where ϵ is the noise level and ℓ_0 denote the pseudo-norm which counts the number of nonzero entries in an array. OMP is used to localize the point sources one by one. For each iteration,



Figure 14: Gulfport dataset: CNN using HSI data only (left) and fused HSI and LiDAR data (middle). Google map view (right). Classes labeled Targets 1-4 correspond to materials placed on the ground, on elevated tables, and under trees by the NGA.

the main step of OMP is to find the most correlated atom with the remaining image and then update the flux values of the estimated point sources. In high density cases, OMP is not accurate enough. Thus we consider a more sophisticated model, an algorithm based on the continuous exact ℓ_0 (CEL0) penalty. This penalty is a nonconvex term approaching the

ℓ_0 norm and it can be written as $\Phi_{\text{CEL0}}(\mathcal{X}) = \sum_{i,j,k=1}^{m,n,d} \phi(\|\mathcal{T}(\mathcal{A} * \delta_{ijk})\|, \mu; \mathcal{X}_{ijk})$. Here

$$\phi(a, \mu; u) = \mu - \frac{a^2}{2} \left(|u| - \frac{\sqrt{2\mu}}{a} \right)^2 \mathbb{1}_{\{|u| \leq \frac{\sqrt{2\mu}}{a}\}},$$

where μ is a regularization parameter and $\mathbb{1}_{\{u \in E\}} := \begin{cases} 1, & \text{if } u \in E, \\ 0, & \text{others.} \end{cases}$. We use this penalty as the regularization term and the optimization problem becomes

$$\min_{\mathcal{X}} \frac{1}{2} \|T(\mathcal{A} * \mathcal{X}) + b\mathbf{1} - G\|_2^2 + \Phi_{\text{CEL0}}(\mathcal{X}).$$

The uniform background is denoted as $b\mathbf{1}$ with $\mathbf{1} \in \mathbb{R}^{m \times n}$ and b being a fixed constant. Our CEL0 model has many good properties. The set of all global minimizers of CEL0 contains the global minimizers of the ℓ_0 model. From each minimizer of CEL0, one can easily identify a minimizer of the ℓ_0 model. But since some local minimizers of the ℓ_0 model are not critical points of the CEL0 model, we can avoid those local minimizers. We feel that our CEL0 is a good approach to solving the 3D point source localization problem. We used the iterative reweighted ℓ_1 algorithm (IRL1) to solve this nonsmooth nonconvex optimization problem. Note that since one subproblem in IRL1 is required to solve the weighted ℓ_1 model with the rotating PSF data fitting term, we used the alternating direction method of multipliers (ADMM) to solve and calculate it more efficiently by using the 3D fast Fourier transform (FFT). Numerical results show the numerical advantages of using our CEL0 model for point source localization. See Figure 15.

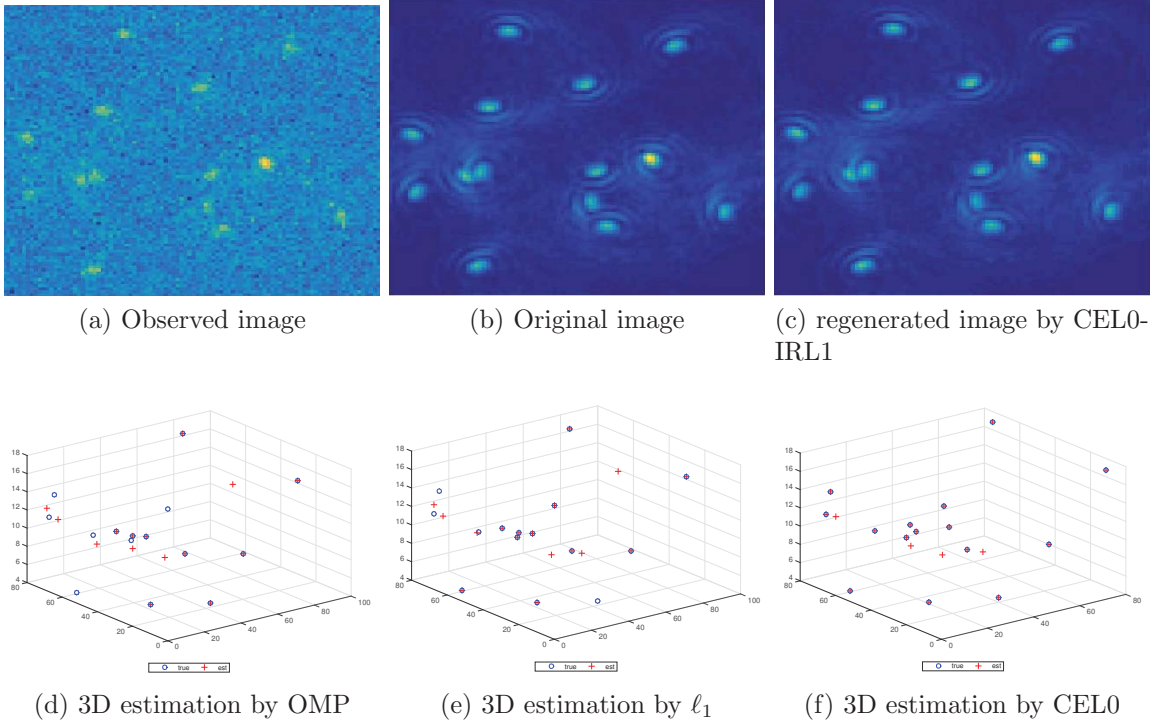


Figure 15: Localization of 15 point sources by using OMP, ℓ_1 and CEL0.

2.3.4 3D Debris Localization using Rotating Point Spread Function Imagery in Poisson noise [18, 19]

Here, we continue to consider 3D localization of space debris from one 2D snapshot $G \in \mathbb{R}^{m \times n}$. Instead of the Gaussian noise which describes conventional CCD sensors operating at low per-pixel photon fluxes and large read-out noise, we study for the Poisson noise case. The work characterizes an electron-multiplying CCD (EM-CCD) sensor operated in the photon-counting regime. For formulation, we do the same discretization but the forward model needs to be adjusted as $G = \mathcal{P}(\mathcal{T}(\mathcal{A} * \mathcal{X}) + b \mathbf{1})$, as the Poisson noise is signal dependent. Here $\mathcal{P}$ denotes the Poisson distribution, $\mathcal{A} * \mathcal{X}$ is the 3D convolution between 3D point spread function $\mathcal{A}$ and the discretized space $\mathcal{X}$ for point sources. Here we consider maximizing the likelihood distribution which is equivalent to minimizing the Kullback-Leibler (KL) divergence. Therefore, we consider the data fitting term as $D_{KL}(\mathcal{T}(\mathcal{A} * \mathcal{X}) + b \mathbf{1}, G)$, where $D_{KL}(z, g) = \langle g, \ln \frac{g}{z} \rangle + \langle \mathbf{1}, z - g \rangle$. With the KL divergence data-fitting term, the CEL0 regularization that we considered for Gaussian noise [17] fails to approximate the ℓ_0 norm and loses the good properties of global or local minimizer. Thus we need to consider another non-convex term to approximate ℓ_0 norm. We consider a new non-convex nondifferentiable function

$$\mathcal{R}(\mathcal{X}) := \sum_{i,j,k=1}^{m,n,d} \theta(\mathcal{X}_{ijk}) = \sum_{i,j,k=1}^{m,n,d} \frac{|\mathcal{X}_{ijk}|}{a + |\mathcal{X}_{ijk}|},$$

where a is a fixed constant which determines the degree of non-convexity. We can see that θ is close to the ℓ_0 norm when a is small. Thus, the minimization problem amounts to

$$\min_{\mathcal{X} \geq 0} \left\{ \langle 1, \mathcal{T}(\mathcal{A} * \mathcal{X}) - G \ln(\mathcal{T}(\mathcal{A} * \mathcal{X}) + b \mathbf{1}) \rangle + \mu \sum_{i,j,k=1}^{m,n,d} \frac{|\mathcal{X}_{ijk}|}{a + |\mathcal{X}_{ijk}|} \right\}. \quad (13)$$

We have used an iteratively reweighted ℓ_1 algorithm (IRL1) to solve the minimization problem (13). The subproblem is to minimize the KL divergence data fitting term with a weighted ℓ_1 term. We have used the popular ADMM method to solve this and deduced a closed-form solution for the subproblem involving the data-fitting term. To emphasize that our non-convex optimization model is based on the use of a KL data fitting (KL) term and a non-convex (NC) regularization term, we designate our optimization model (13) as KL-NC. Numerical results show the advantages of our KL-NC method for point source localization. See Figure 16. For the Poisson noise case we have provided an algorithm for estimating

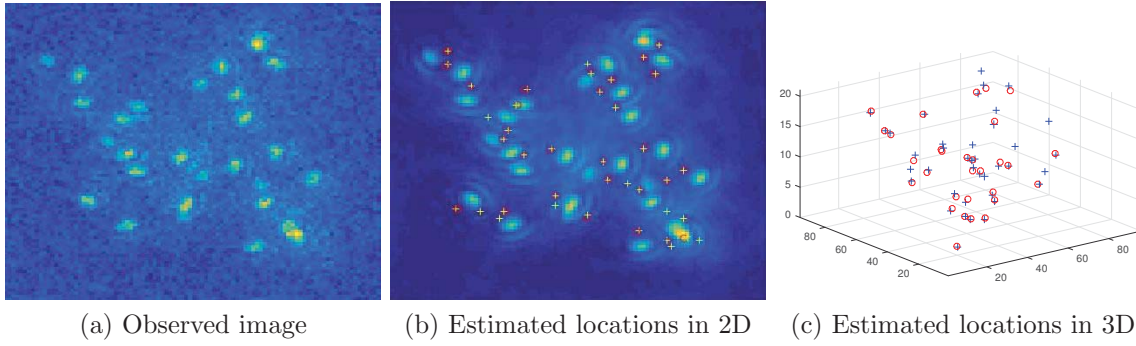


Figure 16: Localizations for the 30 point sources case. “o” is the location of the ground truth point source and “+” is the location of the estimated point source.

the locations and fluxes for the point sources. However, numerical results show that the flux values are generally underestimated. Our aim is thus to estimate the source flux values from the KL data-fitting term appropriate to the Poisson noise model when the source 3D positions have already been accurately estimated. Let the PSF corresponding to the i -th source be arranged as the column vector $\mathbf{h}_i$. The stacking of the P column vectors in the same sequence as the source labels for the P sources then defines a system PSF matrix H , with $H = [\mathbf{h}_1, \mathbf{h}_2, \dots, \mathbf{h}_P] \in \mathbb{R}^{K \times P}$, where K is the total number of pixels in the vectorized data array, so $K = mn$. The vectorized observed image is denoted by $\mathbf{g} \in \mathbb{R}^{K \times 1}$. The uniform background is denoted as the vector $b\mathbf{1}$ with $\mathbf{1} \in \mathbb{R}^{K \times 1}$. The flux vector is denoted as $\mathbf{f} \in \mathbb{R}^{P \times 1}$. Here the problem is overdetermined meaning that the number of point sources P is much smaller than the number of available data $K = mn$. Therefore we need to do some refinement of the estimates by minimizing directly data fitting term which is the KL divergence term. Its minimization with respect to the flux vector $\mathbf{f}$, performed by setting the gradient of D_{KL} with respect to $\mathbf{f}$ zero. This yields the nonlinear relation

$$\nabla D_{KL}(H\mathbf{f} + b\mathbf{1}, \mathbf{g}) = \sum_{i=1}^m \frac{\mathbf{e}_i^T (H\mathbf{f} + b\mathbf{1} - \mathbf{g})}{\mathbf{e}_i^T (H\mathbf{f} + b\mathbf{1})} H^T \mathbf{e}_i = 0, \quad (14)$$

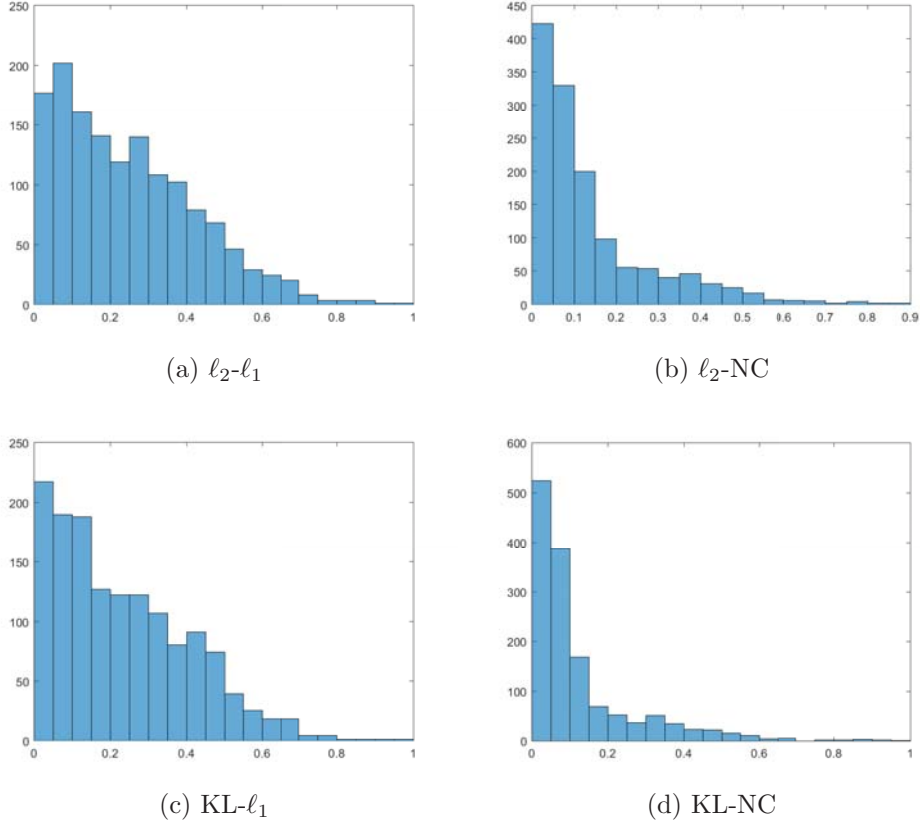


Figure 17: Histogram of relative errors on flux values in the 30 point sources case.

where $\mathbf{e}_i$ is the i -th canonical basis unit vector. Consider now an iterative solution of (14). We get an equation deduced from (14) and have a equality as

$$\mathbf{f} = \mathbf{f}_G + \mathcal{K}(\mathbf{f}), \quad (15)$$

where $\mathcal{K}(\mathbf{f}) = \sum_{i=1}^K \frac{\mathbf{e}_i^T (H\mathbf{f} + b\mathbf{1} - \mathbf{g}) \mathbf{e}_i^T H \mathbf{f}}{\mathbf{e}_i^T (H\mathbf{f} + b\mathbf{1})} H^+ \mathbf{e}_i$ and $H^+ = (H^T H)^{-1} H^T$. Here $\mathbf{f}_G = H^+ (\mathbf{g} - b\mathbf{1})$ is the solution corresponding to the Gaussian noise model. This suggests the following iterative scheme for estimating the flux values: $\mathbf{f}^{n+1} = \mathbf{f}_G + \mathcal{K}(\mathbf{f}^n)$ for estimating the flux. In Figure 17, we plot the histogram of the relative errors on these four optimization models in the 30 point sources case for 50 different observed images.

2.3.5 Joint 3D Localization and Classification of Space Debris using a Multispectral Rotating Point Spread Function [20]

Building on the success of project accomplishments described in Sections 2.3.3 and 2.3.4, we consider using multispectral data in order to not only localize the debris, but also characterize its material composition. We can exploit the fact that the behavior of the RPSF is sensitive to the wavelength in order to improve the localization and also to estimate the spectral signature of the point source.

That is, using multi-spectral images, with one RPSF per spectral band, we are able not only to localize the 3D positions of the space debris but also classify their material composition. We propose a three-stage method for achieving such joint localization and classification. In Stage 1, we adopt an optimization scheme for localization in which the spectral signature of each material is assumed to be uniform, which significantly improves efficiency and yields better localization results than possible with a single spectral band. In Stage 2, we estimate the spectral signature and refine the localization result via an alternating approach. We process classification in the final stage. Both Poisson noise and Gaussian noise models are considered, and the implementation of each is discussed. Numerical tests using data from NASA show the efficiency of our three-stage approach and illustrate the improvement of point source localization and spectral classification from using multiple bands over a single band. The simulation is illustrated by one example in Figure 18.

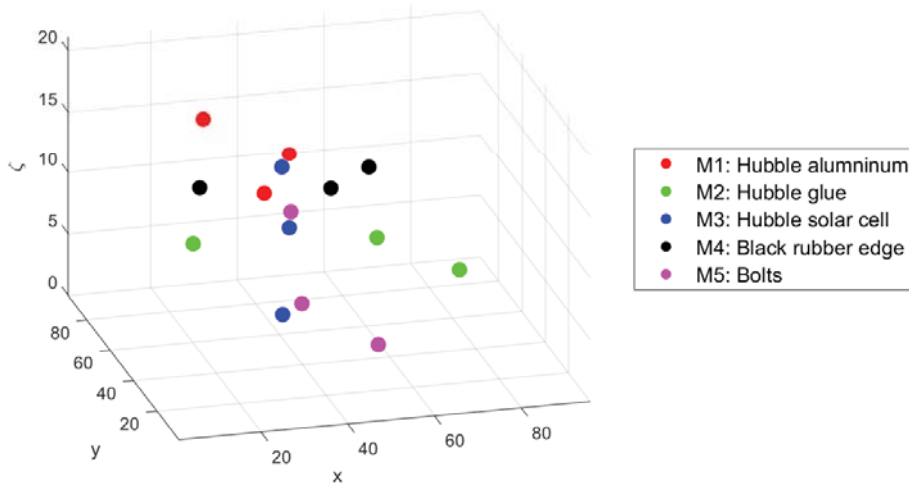


Figure 18: Simulated space debris with 5 materials

In the case of multispectral information, we have an observed image $G^{(i)}$ for each wavelength, and a different 3D RPSF $\mathcal{A}^{(i)}$ for each wavelength. Assuming at first that the spectral signatures of the point sources are uniform, the localization optimization problem becomes

$$\min_{\mathcal{X} \geq 0} \left\{ \sum_{i=1}^K \langle 1, \mathcal{T}(\mathcal{A}^{(i)} * \mathcal{X}) - G^{(i)} \log(\mathcal{T}(\mathcal{A}^{(i)} * \mathcal{X}) + b\mathbf{1}) \rangle + \mu \sum_{p,q,r=1}^{m,n,d} \frac{|\mathcal{X}_{pqr}|}{a + |\mathcal{X}_{pqr}|} \right\}, \quad (16)$$

where we use the same regularization as in Section 2.3.4. Given estimated locations of the point sources, we refine the spectral signatures of the point sources by computing the intensities $\mathbf{f}^{(i)}$ for each wavelength using the model

$$\mathbf{g}^{(i)} = \mathcal{N}(H^{(i)}\mathbf{f}^{(i)} + b\mathbf{1}) \in \mathbb{R}^{L \times 1},$$

where $\mathbf{g}^{(i)}$ is a vectorization of the observed image for the i th band and $H^{(i)}$ is a system PSF matrix that depends on the point source locations. Using the more accurate spectral signatures of the point sources, we can remove false positives more effectively than in the

single band case. Furthermore, in a third step, we can use a dictionary of known material signatures to classify the material composition of the point sources. Table 2 shows the efficiency of our proposed method.

Table 2: Comparisons of 3D localization and classification by single and multispectral images.

Localization method	No. bands		Localization		Classification	
	Stage 1	Stage 2	Recall	Precision	OA	kappa
Single	1	-	87.47%	39.22%	-	-
	1	5	86.00%	83.95%	88.09%	84.81%
Multiple	4	-	95.20%	63.78%	-	-
	4	5	94.80%	95.34%	93.28%	91.57%

2.3.6 3D Extended Object Localization and Shape Recovery using Rotating Point Spread Functions [21]

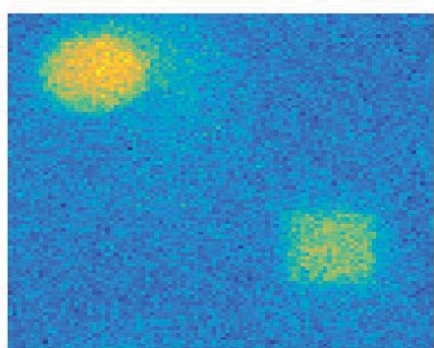
We also consider 3D imaging for extended objects. Unlike localization of point sources, this problem aims at encoding depth maps of the objects from one 2D snapshot image. However, we can formulate the problem as point sources by regarding the front surface of objects as sequences of point sources. We can see that the distribution of point sources are no longer sparse and compressed sensing tools cannot be adapted. But in the z direction, the sparsity property still exists, since we can not see any point source obscured on the back side of another object’s surface. Here, we consider the objects with the flat surface which is in front of camera. Then, the distribution of point sources is shaped like pieces of a flat plane. Therefore, in the x and y directions, the solution $\mathcal{X}$ has piecewise block characteristic. In the light of this characteristic, we consider the following optimization problem

$$\min_{\mathcal{X} \geq 0} \frac{1}{2} \|T(\mathcal{A} * \mathcal{X}) - G\|_2^2 + \mu \sum_{i=1}^d \text{TV}(X_i).$$

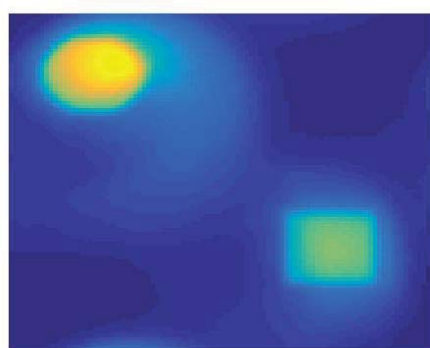
where X_i is the i -th slice in $\mathcal{X}$ with respect to certain depth. This minimization problem is solved by using ADMM and numerical results show the effectiveness of our optimization scheme, see Figure 19.

2.3.7 A Two-stage Method for Spectral-spatial Classification of Hyperspectral Images [22]

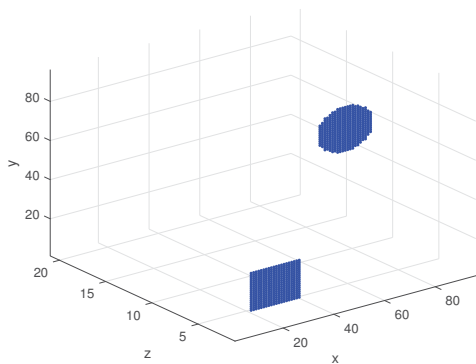
We propose a novel two-stage hyperspectral classification method inspired by image denoising/segmentation. In the first stage, we apply support vector machines to estimate the pixel-wise probability. The pixel-wise probability maps obtained are noisy since no spatial information is used. In the second stage, we denoise and segment the probability maps. Specifically, let $\mathbf{v}_k := \mathcal{V}_{:, :, k}$, $k = 1, \dots, c$, be the probability map of the k -th class obtained



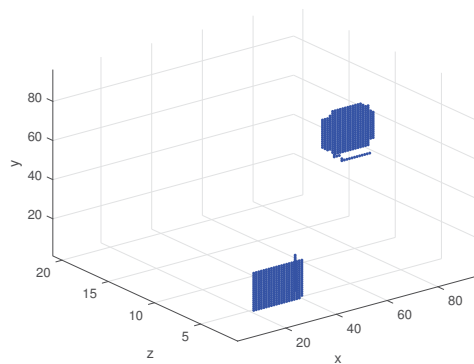
(a) Observed image



(b) Original image $\mathcal{T}(\mathcal{A} * \mathcal{X})$



(c) Ground truth of $\mathcal{X}$



(d) Solution $\mathcal{X}$

Figure 19: Extended object case.

Table 3: Number of training/testing pixels and classification accuracies for Indian Pines data set.

Class	train/test	ν -SVC	SVM-CK	EPF	SC-MK	MFASR	2-stage
Alfalfa	10/36	70.28%	81.94%	97.29%	100%	98.06%	99.17%
Corn-no till	143/1285	77.90%	89.98%	96.03%	95.44%	96.66%	97.89%
Corn-mill till	83/747	67.80%	89.68%	97.75%	97.16%	97.94%	98.73%
Corn	24/213	52.96%	86.24%	93.03%	99.25%	91.69%	99.01%
Grass/pasture	48/435	89.13%	93.31%	99.17%	96.67%	94.62%	96.92%
Grass/trees	73/657	96.15%	98.98%	96.02%	99.70%	99.56%	99.74%
Grass/pasture-mowed	10/18	93.33%	96.11%	99.47%	100%	100%	100%
Hay-windrowed	48/430	93.93%	98.42%	100%	100%	99.98%	100%
Oats	10/10	90.00%	100%	96.25%	100%	100%	100%
Soybeans-no till	97/875	72.26%	88.81%	92.21%	94.62%	96.03%	96.01%
Soybeans-mill till	246/2209	79.71%	91.57%	86.65%	98.80%	98.58%	99.54%
Soybeans-clean	59/534	67.66%	85.90%	96.26%	96.29%	97.06%	99.64%
Wheat	21/184	96.09%	98.64%	100%	99.67%	99.57%	100%
Woods	127/1138	91.89%	96.85%	95.24%	99.99%	99.89%	99.91%
Bridg-Grass-Tree-Drives	39/347	56.97%	88.01%	93.70%	98.39%	98.01%	99.14%
Stone-steel lowers	10/83	85.66%	98.43%	96.11%	97.71%	98.92%	96.39%
OA		79.78%	92.11%	93.34%	97.83%	97.88%	98.83%
AA		80.11%	92.68%	95.95%	98.35%	97.91%	98.88%
kappa		0.769	0.910	0.924	0.975	0.976	0.987

from the first stage. We improve them by minimizing:

$$\begin{aligned}
& \min_{\mathbf{u}} \frac{1}{2} \|\mathbf{u} - \mathbf{v}_k\|_2^2 + \beta_1 \|\nabla \mathbf{u}\|_1 + \frac{\beta_2}{2} \|\nabla \mathbf{u}\|_2^2, \\
& \text{s.t. } \mathbf{u}|_{\Upsilon} = \mathbf{v}_k|_{\Upsilon},
\end{aligned} \tag{17}$$

where β_1, β_2 are regularization parameters and Υ is the set of training pixels. The ground-truth labels for training pixels are known, thus we fix the training pixels. The higher-order $\|\nabla \mathbf{u}\|_2^2$ term encourages smoothness of the solution and can improve the classification accuracy. We choose this minimization functional because it has been shown to have superb performance in denoising and segmentation. For each k , once the optimization finishes, we obtain the enhanced probability map $\mathbf{u}$ for class k . We denote it as $\mathcal{U}_{:, :, k}$. After the map for each class are obtained, we get a 3D tensor $\mathcal{U}$. The final classification of the (i, j) -th pixel is given by finding the maximum value in $\mathcal{U}_{i, j, :}$, i.e. $\arg\max_k \mathcal{U}_{i, j, k}$. Essentially, the second stage first computes a smooth denoised image and then thresholding is applied to segment (i.e. classify) it.

Table 3 shows the experimental results on Indian Pines data set generated by our method (2-stage) and five other methods. We see that our method generates the best results for all three metrics (OA, AA and kappa) and outperforms the comparing methods by a significant margin. They are at least 0.95% higher than the others.

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- [22] R. Chan, K. Kan, M. Nikolova, and R. Plemmons, “A Two-stage Method for Spectral-spatial Classification of Hyperspectral Images,” arXiv preprint, arXiv:1806.00836, 2018. (Submitted to Journal of Mathematical Imaging and Vision)

3 Publications and Reports Citing Grant Support

1. Z. Yu and S. Prasad, “High-numerical-aperture Microscopy with a Rotating PSF,” J. Opt. Soc. Am. A, vol. 33, pp. B58-B69 (2016).
2. J. Antolin, Z. Yu, and S. Prasad, “Optical estimation of the 3D shape of a solar illuminated, reflecting satellite surface,” Proc. AMOS Tech. Conf. (2016). Available (8 pages) at <http://www.amostech.com/TechnicalPapers/2016.cfm>
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12. T. Torgersen, R. Plemmons, P. Pauca, D. Nikic, J. Wu and R. Rand, “Multi-Dimensional Regular Expressions for Object Detection”, Chapter 7 in *Imaging, Vision and Learning Based on Optimization and PDEs*, Springer Series on Mathematics and Visualization, 2018.
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14. C. Wang, R. Chan, R. Plemmons, and S. Prasad, “Point Spread Function Engineering for 3D Imaging of Space Debris using a Continuous Exact ℓ_0 Penalty (CEL0) Based Algorithm,” *Proceedings of International Workshop On Image Processing and Inverse Problems* (May 2019).
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4 Presentations by the Project Team

1. T. Torgersen, “Classification of Pixel-Level Fused Hyperspectral and LiDAR Data using Deep Convolutional Neural Networks,” Workshop on Hyperspectral Image and Signal Processing: Evolution in Remote Sensing, Los Angeles CA, August 2015.
2. S. Prasad, “3D Snapshot Polarimetric Imaging via Rotating PSF for Space Surveillance,” invited paper at the SIAM Imaging Science Symposium, Albuquerque, NM, May 23-26, 2016.
3. S. Prasad, “SSA Capabilities at UNM,” an invited survey presented at the Space Situational Awareness (SSA) Workshop of the NM Collaborative R&D Council, Albuquerque, NM, June 3, 2016.
4. S. Prasad, “Fast 3D Optical Localization & Tracking of Space Debris with a Rotating PSF Telescope,” an invited poster presented at the Space Situational Awareness (SSA) Workshop of the NM Collaborative R&D Council, Albuquerque, NM, June 3, 2016.
5. S. Prasad, “Innovations in Space-Object Shape Recovery and 3D Space Debris Localization,” invited talk presented at the AFOSR Space-Situational Awareness Workshop, Maui, HI, September 17, 2016.
6. S. Prasad, “Turning the Crank” on 3D Source Localization and Polarimetry in Fluorescence Microscopy,” ECE Colloquium, University of Minnesota, October 14, 2016.
7. R. Plemmons, “Computational 3D Imaging: Sparse Recovery and PSF Engineering,” Seminar, The Chinese University of Hong Kong, Hong Kong, October, 2016.
8. R. Plemmons. “Computational Imaging: Overview of Optimization in Sparse 3D Image Recovery,” International Conference on Image Processing, Computer Vision and Machine Learning based on Optimization and PDEs, Bergen Norway, Aug 29, 2016 - Sep 1, 2016.
9. R. Plemmons, “Mathematics in Hyperspectral Image Processing,” Distinguished Lecture Series, Hong Kong Baptist University, Hong Kong, February, 2017.
10. R. Plemmons, “Computational Imaging: Point Spread Function (PSF) Engineering for 3D Information from 2D Image Data,” The Twentieth Householder Symposium on Numerical Linear Algebra, Blacksburgh, VA, June 2017.
11. S. Prasad, “3D Source Localization via Pupil-Phase Engineering,” an invited seminar presented at Wake Forest University, Depts of Computer Science and Mathematics, July 20, 2017.
12. R. Plemmons, “Sparse Recovery Algorithms for 3D Imaging using Point Spread Function Engineering,” Houston Imaging Sciences Symposium, Rice University, Houston, October 3 2017.

13. R. Plemmons, "Sparse Recovery Algorithms for 3D Imaging using Point Spread Function Engineering," Isaac Newton Institute Symposium Variational methods and effective algorithms for imaging and vision Workshop 02, University of Cambridge, Cambridge, England, November 3 2017.
14. S. Prasad, "3D Source Superresolution via Wavefront Projections," an invited seminar, Department of Mathematics, Chinese University of Hong Kong, Hong Kong, January 18, 2018.
15. S. Prasad, "Optical Super-resolution via Wavefront Projections," an invited Physics and Astronomy colloquium, UNM, March 23, 2018.
16. S. Prasad, "The Modern Imaging Paradigm: There is Plenty of Room in the Middle," an invited Physics and Astronomy seminar, University of Minnesota, Twin-Cities Campus, April 16, 2018.
17. R. Plemmons, "Depth from Defocus using Point Spread Function Engineering," International Workshop On Image Processing and Inverse Problems, Beijing Computational Science Research Center, Beijing, China, April 21 2018.
18. G. Ballard, "Parallel Algorithms for CP and Tucker Decompositions," an invited talk presented at the Workshop on Low-Rank Optimization and Applications, the Max Planck Institute in Leipzig, Germany, April 2019.
19. G. Ballard, "Parallel Algorithms for the Multiple Tensor-Times-Matrix Computation," SIAM Conference on Computational Science and Engineering Spokane, WA, February 2019.
20. G. Ballard, "Communication-Optimal Algorithms for CP Decompositions of Dense Tensors," SIAM Conference on Applied Linear Algebra, Hong Kong, May 2018.
21. C. Wang, "Point Spread Function Engineering for 3D Imaging Using a Continuous Exact ℓ_0 Penalty (CEL0) based Algorithm," 15th Annual Meeting of China Society for Industrial and Applied Mathematics, Qingdao, China, October 2017.
22. C. Wang, "Nonconvex Optimization on Rotating Point Spread Methods with Applications on Microscopy and Telescope imaging," 4th AoE Symposium on Organelle Biogenesis and Function, The Chinese University of Hong Kong, Hong Kong, December 2017.
23. C. Wang, "Nonconvex optimization for 3D Point Source Localization Using a Rotating Point Spread Function," International Workshop On Image Processing and Inverse Problems, Beijing Computational Science Research Center, Beijing, China, April 2018.
24. C. Wang, "Sparse Recovery Algorithms for 3D Imaging Using Point Spread Function Engineering," SIAM Conference on Applied Linear Algebra, Hong Kong, May 2018.
25. C. Wang, "Sparse Recovery Algorithms for 3D Imaging Using Point Spread Function Engineering," SIAM Conference on Image Science, Bologna, Italy, June 2018.

26. C. Wang, "Novel Sparse Recovery Algorithms for 3D Debris Localization using Rotating Point Spread Function Imagery," AMOS Technical Conference, Maui, HI, September 2018.
27. C. Wang, "3D Debris Localization using Rotating Point Spread Function Imagery in Poisson Noise" 2019 Georgia Scientific Computing Symposium, Atlanta, GA, February 2019.
28. C. Wang, "Sparse Recovery Approaches to Imaging Processing," SIAM Conference on Computational Science and Engineering, Spokane, WA, February, 2019.
29. K. Kan, "A Two-stage Method for Spectral-spatial Classification of Hyperspectral Images," 2019 Georgia Scientific Computing Symposium, Atlanta, GA, February 2019.
30. K. Kan, "A Two-stage Method for Spectral-spatial Classification of Hyperspectral Images," SIAM Conference on Computational Science and Engineering, Spokane, WA, February, 2019.