



A Compressed Sensing Approach to Signal Fragmentation

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Final Report**

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<p>14. ABSTRACT</p> <p>This project is on a mathematical approach to signal fragmentation, as proposed by Dr. Richard Albanese, including mathematical formulation, analysis and optimization. The proposal of Albanese is for a method to send (relatively) long wavelength signals over an array of small antennas. Signal fragmentation approximates a desired signal $f(t)$ (the input current to the antenna) by a sum of wavelets, each of which has the same shape $\phi(t)$, but with the n-th term shifted in time by an amount t_n and scaled by a factor a_n. The wavelet ϕ is assumed to have compact support in time and successive wavelets are sent over different antennas, so that none of the fragments overlap in time. The objective of our project was to improve the design process, the accuracy, and the efficiency of signal fragmentation. Although we originally proposed to apply methods from compressed sensing and related fields, we found a better approach based on harmonic analysis, wavelets and approximation theory. With this approach, we succeeded in answering most of the questions that emerged from the research of Albanese, including development of a unified theory from antenna input to far field, analysis of spectral leakage for signal fragmentation, elimination of spectral leakage for sinusoidal signals, approximation of AM signals, energy efficiency of signal fragmentation, and optimal choice of wavelet. We also developed an alternative form of signal fragmentation that is more efficient than the original form. The most important of these results are the following: (1) Fourier analysis for signal fragmentation leads to a formula for the spectral leakage for signal fragmentation, showing that spectral leakage occurs only at very high wavenumbers.; (2) generalization of the Shannon-Whitaker interpolation formula can then be used to find a wavelet for which there is no spectral leakage for a sinusoid</p>		
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"A Compressed Sensing Approach to Signal Fragmentation"

Russel Caflisch, PI

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Progress and Accomplishments.

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The objective of our project was to improve the design process, the accuracy, and the efficiency of signal fragmentation. Although we originally proposed to apply methods from compressed sensing and related fields, but we found a better approach based on harmonic analysis, wavelets and approximation theory. With this approach, we succeeded in answering most of the questions that emerged from the research of Albanese, including development of a unified theory from antenna input to far field, analysis of spectral leakage for signal fragmentation, elimination of spectral leakage for sinusoidal signals, approximation of AM signals, energy efficiency of signal fragmentation, and optimal choice of wavelet. We also developed an alternative form of signal fragmentation that is more efficient than the original form. A more detailed description of these results is given below.

The unified theory that starts from input to the antenna, and includes the signal in the far field, based on a simple antenna model. This provides a direct formulation for spectral leakage and for energy efficiency, the two principal figures of merit of signal fragmentation.

Fourier analysis for signal fragmentation, including use of the Poisson summation formula, leads to a formula for the spectral leakage for signal fragmentation. For wavelet support that is small compared to the underlying wavelength, this shows that spectral leakage occurs only at very high wavenumbers.

A generalization of the Shannon-Whitaker interpolation formula can then be used to find a wavelet for which there is no spectral leakage for a sinusoidal signal. The fourier transform of

the special wavelet is a generalization of the sinc function for an AM signal. This result was a quite unexpected, and the special wavelet has not been used before, to the best of our knowledge.

These results can be generalized for bandlimited and AM signals, using the theory of frames, to find wavelets for which signal fragmentation leads to very small spectral leakage.

For the original form of signal fragmentation applied to the input current to the antennas, we found that the energy efficiency is small. As an alternative, we signal fragmentation to the time-derivative of the current, and found that the resulting energy efficiency is much higher.

For a sinusoidal signal, we also found a wavelet that optimizes the energy efficiency of signal fragmentation, among all wavelets that produce no spectral leakage and whose support is of a certain size.

Publications and Submissions

There are no publications yet for this work. We have produced one manuscript that has been submitted for publication and two research reports that are available online at www.math.ucla.edu/applied/cam:

1. Russel E. Caflisch, Hung Hsu Chou, and Jonathan W. Siegel. “Accuracy, Efficiency and Optimization of Signal Fragmentation” submitted to *Multiscale Modeling and Simulation*, 2018.
2. Russel Caflisch, Hung Hsu Chou, and Jonathan Siegel, “Efficient Frame Projection of Amplitude-Modulated Signals”, August 2018 (revised October 2018), UCLA CAM Report 18-45.
3. Jonathan W. Siegel, Shift Invariant Subspaces and Applications to Signal Fragmentation, November 2017 (revised October 2018), UCLA CAM Report 17-69.