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AEC-Air Pollution Project, Meteorological Phase

By
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NYO-1559
Waste Disposal

ANNUAL PROGRESS REPORT
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METEOROLOGICAL PHASE

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I
ABSTRACT

Studies of atmospheric turbulence were carried out in connection with a slow speed wind tunnel project.

The results reported here are based largely on observations at Upton, L. I., and concern the following subjects:

1. Methods of measuring Reynolds flux exchange coefficients.
2. Spectral properties of atmospheric turbulence.
3. Meteorological factors affecting the change of character of turbulence.
4. Wind structure in the lowest 125 meters.

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ANNUAL PROGRESS REPORT

AEC-AIR POLLUTION PROJECT

METEOROLOGICAL PHASE

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II SCOPE OF PROJECT

The A.E.C. - Air Pollution Project was set up at N. Y. University to determine whether a wind tunnel can be used to simulate full scale atmospheric conditions for the study of turbulence in the earth's boundary layer. The project has two phases. The Wind Tunnel Phase is devoted to erection and operation of a Tunnel with equipment for temperature and velocity gradient control. The Meteorology Phase is concerned with analysis of field data and scale factors for reproducing full scale observations in the Tunnel. The entire program is under the direction of Dr. Gordon H. Strom, Associate Professor of Aeronautical Engineering, Director of Wind Tunnels.

The meteorological personnel consisted of: Hans A. Panofsky, meteorologist, Irving A. Singer, Res. Ass't, Raymond K. Siffert, Res. Ass't, and Mario Mayorga, Student Ass't. Members of the meteorological group made frequent visits to the Brookhaven National Laboratories at Upton, New York, for the purpose of collecting data and discussing turbulence problems with members of the Brookhaven staff.

Dr. Panofsky visited the Round Hill Station of the Massachusetts Institute of Technology in South Dartmouth, Mass. on Jan. 25, 1951, and compared observational results with Dr. Hewson, the director of the Round Hill Station. Moreover, Dr. R. B. Montgomery, who happened to be at Round Hill at the time, expressed considerable interest in the Wind Tunnel Project and was able to give valuable advice.

Dr. Panofsky also was able to attend a number of lectures on topics in turbulence held under the auspices of the Department of Aeronautical Engineering at Princeton University, some of which had direct bearing on problems encountered in this project.

From June 4-8, Dr. Panofsky attended a conference on turbulence at Cambridge, Mass., and presented some of the results obtained during the year, especially those described in sections 1a, 2a and 2b of this report.

III

INTRODUCTION

This report describes some meteorological investigations which were carried out for the purpose of gaining a better understanding of characteristics of atmospheric turbulence, especially those characteristics which can be reproduced in a wind tunnel. Some of the results lead to recommendations concerning the instrumentation in the wind tunnel.

Four general topics were studied:

1. Eddy heat and momentum exchange coefficients were obtained by various methods and compared with each other.
2. A crude spectral analysis was made of the stress and kinetic energy, and was compared with a working hypothesis for the characteristics of the vertical velocity spectrum. Instrumentation was designed to obtain the spectrum electronically.
3. The factors governing radical changes of the characteristics of turbulence were investigated by observation and theory. The results lead to recommendations for the tunnel.
4. The variation of wind with height in the lowest 125 meters was analyzed statistically.

The observational material was very kindly made available by the Meteorology Group of the Brookhaven National Laboratories. The group, consisting of Maynard Smith, Philip H. Lowry and Daniel A. Mazzarella, continually aided the meteorologists at New York University carrying out this research by making special observations, by answering many questions concerning the equipment and observational results and by general constructive discussions. Mr. Raymond Wanta, the Weather Bureau representative at Brookhaven, has very kindly given permission to quote some of his more recent investigations relevant to this report.

In addition to the four regular members of the meteorological section of the Wind Tunnel Project, Mario Mayorga, Hans A. Panofsky, Irving A. Singer and Raymond K. Siffert, several other men participated in parts of the project: Robert McCormick, Weather Bureau Scholarship Student, was responsible for all the calculations described in sections 1a and 2a; Harold Crutcher, Weather Bureau Scholarship Student, carried out the calculations described in section 2c; Lt. Colonel Steven Pournaras, Air Force Student, analyzed the diurnal temperature wave as described in section 1d; Ernest Leonard, Graduate Student, aided in the heat flux calculations of section 1c; Prof. Greenstein and Mr. Stock of the Electrical Engineering Department designed equipment for recording wind variations on an oscillograph; Dr. Willard Pierson, Instructor of the Meteorology Department, supervised the building of equipment required for the rapid scanning of film in connection with section 2e.

A rather elaborate biannual report on the activities of the meteorological group of this project from July 15, 1950 to January 15, 1951, appeared in March, 1951. Not all the research summarized in that report is repeated here, especially if the results are not considered useful in future work. Moreover, some of the technical details in the construction of the spectrum analysis equipment has been omitted. The biannual report is referred to in the text simply as the Progress Report, and can be consulted for details if desired. The conclusions of some sections of the Progress Report have definitely been superseded by presumably more nearly correct results, especially those of sections Ic (on eddy heat flux) and IIa (on effective eddies).

IV

SECTION 1

COMPARISON OF VARIOUS METHODS USED FOR THE DETERMINATION OF EXCHANGE COEFFICIENTS

1a. Stress and Eddy Viscosity from Bivane Measurement

Published measurements of stress in the atmosphere are rare. Sheppard (1947) measured the wind stress on a plate in an oil bath and Pasquill (1950) computed the stress from observations of the drag of air on a small section of sod floating in water. Scrase (1930) computed the Reynolds stress over short periods from vertical and horizontal wind velocities, obtained with the aid of a light bivane. These studies yield the stress within a few meters of the ground.

Similar research is planned or is in progress elsewhere, for example at the University of Wisconsin and at the Round Hill Station of the Massachusetts Institute of Technology. However, results of these studies have not yet been published.

The measurements described here were obtained in a manner similar in a general way to those of Scrase; they differ from them and those of other workers in this field in the height of the instruments above the surface. This difference has a profound bearing on the instrumentation required for the measurement of the stress. Whereas at low levels the instruments must respond to eddies of periods of a second or less, most of the vertical flux of momentum at 109 m under superadiabatic conditions is produced by convective cells with periods of the order of minutes, as will appear from the observations discussed in this paper. As a result, the instruments required to measure the stress at high levels under unstable conditions do not require extremely rapid response.

The 125-meter-tower of the Brookhaven National Laboratory at Upton, New York, is equipped with aerovanes and resistance thermometers which furnish continuous records at six levels from 11 meters to 125 meters. In addition, a bivane is mounted at 109 meters. The design of this bivane is described by Mazzarella (1951).

Normally, the bivane and aerovanes record an hour of data on 3 inches of paper on Esterline-Angus recorders. However, the recorders can be speeded up by a factor 60 so that 1/2 inch represents 10 seconds of observations. Such an operation will be referred to as a "speed run".

None of the instruments is built for extremely rapid response, so that averages of recorded wind data over less than 10 seconds are probably not trustworthy, especially since the bivane has a free period of about 3 seconds at the wind speeds encountered during the intervals discussed here. It is assumed that ten-second averages of bivane and aerovane observations are reliable.

Wind tunnel observations have shown that the wind speed recorded by the aerovane equals the true three-dimensional wind speed, multiplied by the square of the cosine of the angle of attack provided this angle does not exceed 45°. Since the aerovane is mounted on a vertical axis, the angle of attack is assumed to be the vertical angle. This leads to a slight underestimate of the wind speed because the bivane does not point exactly into the direction of the horizontal wind component when the wind shifts rapidly.

The zero point of vertical angle as indicated by the bivane may be incorrect by as much as 3 degress; but the effect of this error on the vertical eddy transport of momentum is negligible. It affects only the flux produced by the mean vertical motion, which is of no particular interest here.

The ten-second averages of horizontal angle, vertical angle and indicated wind speed can be estimated by eye from the Esterline-Angus speed trace. These quantities will be referred to as α, θ, and V, respectively. The horizontal speed is then given by: $V_2 = V \sec \theta$, and the three standard orthogonal wind components u (West-East), v (South-North) and w (Vertical) by: $u = - V \sec\theta \sin\alpha$, $v = - V \sec\theta \cos\alpha$ and $w = V \sec\theta \tan\theta$. The West-East and the South-North components of the Reynolds stress are defined by:

$$\tau_x = -\overline{\overline{\overline{\rho \Delta u \Delta w}}}, \qquad \tau_y = -\overline{\overline{\overline{\rho \Delta v \Delta w}}} \qquad (1)$$

Here ρ is the density, which is assumed to have no important variations during the interval studied, and Δu, Δv and Δw are deviations from means. Three bars over a quantity designate the arithmetic mean.

For the data of October 4, 1950, the velocity deviations were computed from the simple arithmetic mean. However, during the other periods analyzed, considerable trends in some of the meteorological parameters were indicated, which must be ascribed to eddies so large that their effect on the flux of momentum cannot be taken into account. Therefore, the velocity deviations Δu, Δv, as well as Δw were computed from the line of regression of u, v or w on time. Thus, for example, $\Delta v = v - (\overline{\overline{\overline{v}}} + b_v t)$, where b_v is the slope of the line of regression of v on time t, and the origin of time is in the middle of the period studied. In the case of a linear trend of the meteorological parameters, the Reynolds stress itself is a function of time, and the quantities defined in equation (1) yield the stress at the middle of the period. In all cases the magnitude and direction of the stress were computed from the orthogonal components.

Table I shows the intervals of observation, the average lapse rate between 11 and 125 meters, the speed and direction of the mean wind, the direction and magnitude of the stress and the eddy viscosity, computed from

$$\mu = \frac{\sqrt{\tau_x^2 + \tau_y^2}}{(\partial V / \partial z)}$$

The eddy viscosities must be regarded as least accurate since the shear of the mean wind is not well determined. The stress values in table I are rather larger than those usually associated with winds below 10 m/sec. The reason can be found in the nature of the terrain which is flat grass land largely covered with clumps of trees about 10 meters high. Thus the roughness elements are large, as indicated also by the fact that the roughness length corresponding to the mean wind profile averages about 1 foot.

Table I

Stress and Eddy Viscosity at 109 Meters

Date	Time (EST)	Lapse Rate C km^{-1}	Wind Dir.	Wind Speed m. sec^{-1}	Stress Dir. from	Stress Dynes cm^{-2}	Viscosity gm cm^{-1} sec^{-1}
Oct. 4, '50	1028-1129	16	360°	7.7	360°	4.6	1300
Nov. 1, '50	1400-1500	12	221°	6.6	225°	1.7	125
Nov.15 '50	1000-1105	14	244°	6.6	247°	6.0	1300
Feb.15 '51	1043-1129	17	35°	6.5	64°	8.0	1300
Mar.27 '51	1141-1308	18	201°	5.9	230°	4.3	600

Another peculiarity of the data is the difference in the stress and wind direction of February 15 and March 27; the only explanation would be a veering of the wind with height already at 109 m, so that the wind direction is not the direction of the wind shear. In all other cases the wind presumably turns little with height so that the stress direction is that of the shear as well as of the wind.

1b. Stress and Eddy Viscosity from Pilot Balloon Observations

Rossby and Montgomery (1935) suggested a method of deriving the stress and eddy viscosities from pilot balloon observations, based on the equations of motion. This method has recently been modified by Lettau (1950). The technique is based on a knowledge of the variation of the mean wind with height. Unfortunately, pilot balloons do not stay at any one level for a long time, so that the pilot balloon winds can hardly be interpreted as mean winds. Therefore, the pilot balloon trajectories are seriously influenced by eddies. Only averages of a number of runs have so far led to acceptable results. However, Sheppard (1951) has pointed to a much more serious objection: the method is based on the assumption that important stresses are found only in the friction layer. Now Starr (1951) and others have shown that the maximum horizontal convergence of angular momentum occurs at 300 mb, so that downward flux of momentum must occur at all levels below 300 mb. Therefore, important stresses will be found at all levels up to 300 mb.

The Progress Report describes some calculations based on the method of Rossby and Montgomery, and gives details of the observations and numerical work. However, no additional studies along this line have been undertaken, partly because of the special observations required, and partly because of the objections to the theory.

The observations* summarized in the Project Report were made on June 2, 1950, under neutral stability conditions and rather weak northwesterly winds. The eddy viscosity increased to a value of 100 gm $cm^{-1}sec^{-1}$ at 200 meters, and decreased above that level. These values are somewhat smaller than the ones described in the last section. The difference is probably due partly to the smaller stability, partly due to the incorrectness of the assumptions involved in the pilot balloon method.

In conclusion, continuation of computations of stress and eddy viscosities by the pilot balloon method is not recommended.

1c. Exchange Coefficients from Heat Flux Measurements

The boom at 108 meters elevation of the Brookhaven 125 meter tower carries a sensitive thermocouple in addition to the bivane and aerovane mentioned in section 1a. From the thermocouple trace 10 second average temperatures can be read which should be accurate provided that the thermocouple circuit has been calibrated correctly. Given the temperatures, the vertical eddy heat flux then is given by:

$$F = c_p \rho \overline{\Delta w \Delta T} \qquad (1)$$

where c_p is the specific heat at constant pressure and ΔT is an eddy temperature, defined as a deviation from a mean, or, as in section 1a, from a line expressing the trend. In the particular case of temperature, the elimination of the trend throughout the period of observation is particularly desirable, since the gradual variation of temperature throughout an hour (caused by the diurnal variation) may be quite considerable, and is of a scale larger than what is to be studied here. For the situations encountered so far, the assumption of linear trends throughout the periods of observation appears sufficient, so that ΔT is the deviation of temperature from the line of regression, temperature on time.

So far, heat flux computations have been made for Nov. 1 and Nov. 15, 1950, Feb. 15 and March 27, 1951. The times coincided with those summarized in section 1a. The heat flux on the first three dates was surprisingly small, and led to estimated heat exchange coefficients about 10 times as small as the eddy viscosities. Further, a crude spectral analysis (compare section 2d) indicated that most of the flux was produced by eddies with periods longer than 8 minutes. After these reductions were completed, it was found that the thermocouple had been adjusted so that all short period temperature fluctuations were damped out. Therefore, the fluxes derived for Nov. 1, Nov. 15 and Feb. 15 were much too small;

moreover, the spectral distribution was caused by the artificial absence of short period temperature variations. As a result, only the heat flux computed from the observations of March 27 is discussed here.

Fig. 1 shows a graph of ΔT as function of Δw, indicating a large positive correlation. The heat flux comes out as

$$4.9 \times 10^{-3} \text{ cal cm}^{-2} \text{ sec}^{-1}$$

The derivation of a coefficient of heat conduction (or a "heat diffusivity") from this result presents two kinds of difficulties. In the first place, the classical equation (Brunt, 1941):

$$F = c_p \mu (\gamma - \gamma_d) \qquad (2)$$

(γ_d is the dry adiabatic lapse rate and γ the actual lapse rate) is incomplete according to Priestley and Swinbank (1947); if it is used nevertheless, it would lead to an overestimate of μ under superadiabatic conditions, and to an underestimate for stable conditions. Unfortunately, no precise estimate of the correction term in equation (2) is possible; there is some indication that the correction term should be such that no heat flows at a lapse rate of 8°km^{-1}; in that case, the dry adiabatic lapse rate in (2) could be replaced by 8°km^{-1}. However, there is no reason why the Priestley-Swinbank correction should be constant, even throughout a given day, so that the value of this change is not at all certain.

The second difficulty has its root in the nature of the temperature records at levels other than 109 meters. Accurate temperature gradients can be obtained only between 11 meters and 125 meters, and between 46 and 125 meters. Thus the lapse rate at 109 meters is not well determined. Two methods are used to estimate the temperature gradient at that level: either, the gradient is assumed constant between 46 and 125 meters, or the temperatures are plotted as function of height on semilogarithmic paper, and a smooth curve is drawn through the three points with the aid of a French curve.

Thus, there are 4 estimates of the heat exchange coefficient μ : the first two based on equation (2), with lapse rates determined from linear and non-linear vertical temperature variation; the second two based on equation (2) with the lapse rate of $8^{o}km^{-1}$ substituted for the dry adiabatic lapse rate. The result is shown in table II.

Table II

Heat Exchange Coefficients in

gm $cm^{-1}sec^{-1}$, March 27, 1951

	Linear Variation	Nonlinear Variation
classical equation	500	870
with Priestley-Swinbank correction	350	490

Apparently, the exchange coefficient for heat flux is of the same order of magnitude as the simultaneous coefficient for momentum flux found to be 600 gm $cm^{-2}sec^{-1}$ (table I). It is hoped that the relocation of the instruments to 91 meters will make better estimates of both heat and momentum coefficients possible, because the lapse rates and wind shear at this level are less uncertain than at 109 meters.

1d. Exchange Coefficients from the Diurnal Temperature Variation

Observations of the amplitude and phase of the diurnal temperature wave were made at several heights. If the exchange coefficient is assumed constant, the solution of the heat conduction equation (see Haurwitz, 1941) requires that the ratio of the amplitudes at two levels is given by:

$$A_2/A_1 = \left[e^{-\sqrt{\frac{\pi n \rho}{P\mu}}} \right](z_2 - z_1) \qquad (1)$$

and that the lag between two successive levels is given by:

$$T_2 - T_1 = \sqrt{\frac{P\rho}{4\pi n\mu}}\,(z_2 - z_1) \qquad (2)$$

Here P is the period of a day, n the number of the harmonic of the daily period (each harmonic can be treated separately), and μ is the heat exchange coefficient. The Progress Report described the calculation of μ from (1) and (2) for a clear day, and showed, in agreement with results obtained by Johnson and Heywood (1938) by the same method, that the exchange coefficient varies rapidly with height. Under these conditions the propriety of using layers with constant eddy conductivity is questionable; for then essentially a step function is assumed for the variation of eddy conductivity with height, but no attempt is made to keep the heat flux continuous across the boundaries. Thus Haurwitz (1936) showed that equation (1) certainly gives incorrect results when the exchange coefficient increases linearly with height, whereas equation (2) seems to be more reliable.

In addition to the difficulties in the integration of the heat conduction equation, the equation itself is based on the assumption that advection can be eliminated, that radiation and condensation have no terms with periods of a day. For all these reasons, the coefficients obtained by the heat conduction method should not be expected to be very reliable. Nevertheless, the diurnal daily temperature wave on Oct. 4, Nov. 1 and

Nov. 15 were investigated for the purpose of comparing the resultant heat conduction coefficients with the eddy viscosities obtained for the same days as described in section 1a.

As generally applied, the diurnal variation method is based on a heat conduction coefficient constant with time. Since the eddy viscosities computed in 1a pertain to an hour in the middle of the day, when they are much larger than the daily average, it seemed advisable to modify the diurnal wave method by the assumption of a periodic function of time for the heat exchange coefficient. Let the heat exchange coefficient be given by: $C\,(1 + \cos \frac{2\pi}{p} t)$ where the time t is counted from the time at which the heat conduction coefficient is a maximum. Actually, the coefficient is not a maximum at the same time at all levels; to compromise between the different levels, 4 p.m. was taken as the time of maximum mixing. The quantity C, the amplitude of the diurnal coefficient, is also the daily mean. The maximum exchange coefficient is, of course, 2C. If we now introduce a new time, t', defined by:

$$t' = t + \sin \frac{2\pi}{P} t$$

the heat conduction equation with a periodic heat conduction coefficient becomes:

$$\frac{\partial \theta}{\partial t'} = C \frac{\partial^2 \theta}{\partial z^2} \qquad (3)$$

where θ is the potential temperature. This equation is of the same form as the ordinary equation of heat conduction, and equations (1) and (2) still apply, with C in place of μ. They can then be solved for C, provided that the amplitude and time lag have been obtained from harmonic analysis of temperatures as a function of t'.

Advection was eliminated before the analysis was started. It was assumed that any temperature variation from 0030 one day to 0030 the next day was produced by advection. Also, the advective temperature change was assumed constant throughout the day.

For all three days, heat conduction coefficients were obtained for 8 layers from the first three harmonics, by the lag method and the amplitude method, both for μ constant in time, and for a periodic coefficient. The second and third harmonics gave extremely irregular results, mainly because they are influenced too much by the irregularities in the temperature variation. Also the assumed periodic variation of μ led to extremely scattered results, partly because equal intervals on the t' scale sometimes correspond to extremely close spacing in time, and, again, irregularities are emphasized. Table III shows the mean daily values of μ, computed by the lag method. The results obtained from amplitudes are not listed due to Haurwitz's argument (1936) discussed at the beginning of this section.

Table III

Average Exchange Coefficients Between 108 Meters and 125 Meters in gm $cm^{-1}sec^{-1}$

	Constant	Periodic
Oct. 4	454	(no result; negative lag)
Nov. 1	5.0	12.6
Nov. 15	151	(no result; negative lag)

These values show the relatively low rate of mixing on Nov. 1 which was also apparent from the eddy viscosities. Quantitatively, the agreement between eddy heat conduction and eddy viscosity is good only for Oct. 4, under the assumption of μ invariant throughout the day. A mean value of 454 is not inconsistent with a value of 1300 at 11 a.m., if one considers that the lapse rate at 11 a.m. was strongly superadiabatic. Nevertheless, method based on the diurnal variation of temperature seems to lead to so scattered exchange coefficients when applied to individual days, and is subject to so many theoretical objections that additional work along this line is not recommended.

V.

SECTION 2

PROPERTIES OF THE TURBULENCE SPECTRUM

2a. The Contribution of Eddies of Various Periods to the Stress

The results summarized in table I are incomplete in that the contribution of the eddies of duration less than 10 seconds was neglected. The relative importance of this omission can be judged from a study of the relative contributions of eddies of various periods to the total observed vertical momentum flux. The most objective manner of doing this would be to find the Fourier components of u, v and w and to find out which range of periods of these components produces most of the flux. Such an analysis of the data would be extremely time-consuming, even if a harmonic analyzer of some type was available. For not only the energy spectrum of the individual velocity components is required, but also the phase relationships between individual terms of the Fourier expansion. A somewhat crude method of determining the effect of eddies of different sizes is based on smoothing the observed vertical velocities successively. The differences between these time series of varying smoothness can be ascribed to eddy vertical velocities of several distinct groups of eddy periods. Fig. 2 indicates schematically the breakdown of the eddy vertical velocities. It shows three sets of successively smoothed series, the last being the line of regression. Let the different series in order of increasing smoothness be denoted by w, $\bar{w}$, $\bar{\bar{w}}$ and the line of regression $\bar{\bar{w}} + b_w t$ where b_w is the slope of the line of regression w on time t; the origin of time is assumed to lie at the center of the observations. Nothing has been said about the method of smoothing excepting about the last step, the line of regression. In practice, $\bar{w}$ was chosen as a 70-second running mean of w (the 10-second average observations) and $\bar{\bar{w}}$ as the 490-second running mean. Roughly, the ratios of successive time averages are equal. The eddy components of vertical velocity of various scales can now be defined as differences between the curves of differing smoothness. Thus:

$w''' = w - \bar{w}$, $w'' = \bar{w} - \bar{\bar{w}}$ and $w' = \bar{\bar{w}} - (\bar{\bar{\bar{w}}} + b_w t)$, as indicated in Fig. 2.

One of the difficulties of separating the contribution of various groups of eddies from each other by running means is that the average value of each eddy component does not vanish. Moreover, the correlation between different scale eddy components does not necessarily vanish, although it will be small. Running means also do not separate different periods in a clear-cut way; for example, a 70-second running mean completely removes any effects of 70-second periods or 35-second periods, but retains some of the energy of the 60-second period. Also this mean does not preserve all the energy of an 80-second vertical velocity period, but only a part. In other words, the cut-off characteristics of running means are gradual, not sharp, as one might wish. Nevertheless, frequency separation through use of running means is judged superior to other simple methods.

From the definition of the total eddy vertical velocity as the deviation from the line of regression, it follows that:

$$\Delta w = w' + w'' + w''' \qquad (1)$$

which can be verified from fig. 1 or the equations defining the individual eddy components. With the aid of equation (1), the stress components defined in section 1a may now be rewritten:

$$\begin{aligned} \tau_x &= -\overline{\overline{\rho w' \Delta u}} - \overline{\overline{\rho w'' \Delta u}} - \overline{\overline{\rho w''' \Delta u}} \\ \tau_y &= -\overline{\overline{\rho w' \Delta v}} - \overline{\overline{\rho w'' \Delta v}} - \overline{\overline{\rho w''' \Delta v}} \end{aligned} \qquad (2)$$

The three terms on the right hand side of these equations may be regarded as the relative contributions of certain parts of the vertical velocity eddy spectrum to the stress components. Some experimentation indicated that the largest contribution to terms like $-\overline{\overline{\rho w'' \Delta u}}$ came from $-\overline{\overline{\rho w'' u''}}$, as might be expected, where u'' expresses the contribution of the medium turbulence scale to Δu.

Table IV

Contributions to Total Stress, Dynes cm^{-2}

Date	Short-Period Eddies	Middle-Period Eddies	Long-Period Eddies
Oct. 4	0.9	2.2	1.5
Nov. 1	0.4	1.1	0.2
Nov. 15	1.0	3.4	1.6
Mar. 27	1.0	2.5	0.8

Table IV shows the contributions of the different eddy groups to the total stress. In all cases the largest contribution comes from the second group, which is produced by 70-second deviations from 490-second means. In other words, eddies with period of the order of a few minutes produce most of the vertical momentum flux under superadiabatic conditions at 109 meters. Such eddies are presumably of the nature of convective cells (Sutton, 1949). The fact that the short-period eddies are relatively unimportant makes it likely, but does not prove that eddies with periods less than 10 seconds do not produce an important contribution to the total vertical momentum flux at 100 meters elevation under superadiabatic conditions. This conclusion can be tested only with the use of more rapidly responding instruments.

It is instructive to compare the contributions of the three frequency ranges to the kinetic energy with the contributions to the stress. Table V shows the contributions to the kinetic energy.

Table V

Contributions to Twice the Kinetic Energy, $m^2 sec^{-2}$

Date	Short Period	Middle Period	Long Period
Oct. 4	.26	.24	.13
Nov. 1	.12	.04	.01
Nov. 15	.27	.33	.08
Mar. 27	.37	.34	.08

Comparison of table V with table IV shows that the maximum contribution to the energy takes place at a considerably higher frequency than the maximum contribution to the flux. Whereas most of the stress originates near the center of the medium period turbulence, that to the energy occurs near the boundary between small scale and medium scale turbulence. Physically, this difference can be understood as follows: if two eddies of different periods have the same average vertical velocities and the same average energies, the long period eddy has the larger vertical amplitude and will therefore be more efficient in mixing air of greatly different properties. It will, in particular, bring up air with much lower than average momentum, and bring down air with much above average momentum, thus producing a greater flux of momentum (stress) than the short period eddy.

2b. A Working Hypothesis for the Vertical Velocity Spectrum

If the stress or the vertical kinetic energy are determined from measurements of the velocity components at intervals δt, and averaged over a period P, the contributions of turbulence with periods less than $2\delta t$, or with periods greater than P, remain undetermined. Principally in order to estimate these remainders, a working hypothesis for the energy spectrum and the corresponding contribution to the stress is now introduced. In order to be reasonable, the working hypothesis should account for the observations of the last section. Moreover, the hypothesis makes predictions for other heights and other stability conditions which must later be checked before the working hypothesis can be promoted to the rank of a theory.

Consider the vertical velocity, w(t), expanded in a Fourier integral:

$$w(t) = \int_0^\infty A(\omega)\sin\omega t \, d\omega + \int_0^\infty B(\omega)\cos\omega t \, d\omega \quad (1)$$

Here, ω is the angular frequency and $A(\omega)$ and $B(\omega)$ are functions of ω. Now, if the vertical velocity is squared, and averaged over all time, the result is:

$$\overline{w^2} = \int_0^\infty \frac{A^2(\omega) + B^2(\omega)}{2} \, d\omega \quad (2)$$

Now, let $f(\omega)$ be defined by $f(\omega) = 1/2 \left[A^2(\omega) + B^2(\omega)\right]$. Then:

$$\overline{w^2} = \int_0^\infty f(\omega) \, d\omega \quad (3)$$

The quantity $f(\omega)$ is called the "power spectrum" and the area under the curve of $f(\omega)$ plotted as function of ω is twice the average kinetic energy. This power spectrum was first introduced by Taylor (1938) who showed how the velocity autocorrelation function (with distance lag as argument) can be derived from the power spectrum.

The spectrum can be divided into three general regions: the very high frequency region, where the energy is dissipated by friction; the middle region, in which no energy is created or dissipated, and the low frequency region in which the energy enters the turbulent motion from the organized mean motion. The spectrum in the first two domains is relatively well understood, and is independent of the geometrical surroundings. The meteorologically important domain, however, is the third, in which the geometry, especially the closeness of the ground, produces an important restriction on the turbulence spectrum. It is attempted here to take this restriction into account in a semi-empirical manner, and arrive at a working hypothesis for the spectrum in this third domain.

Let $\tilde{w}$ be a "smoothed" vertical velocity, such as might be obtained from computation of weather map information, and thus contain only the low frequencies of the complete vertical velocity spectrum. Then:

$$\tilde{w} = \int_0^{\omega} A(k)\sin kt \, dk + \int_0^{\omega} B(k)\cos kt \, dk \qquad (4)$$

Thus, $\tilde{w}$ is a function of the highest frequency observable in the smoothing process.

Squaring and averaging (4) we obtain:

$$\overline{\tilde{w}^2} = \int_0^{\omega} f(k) \, dk \qquad (5)$$

Thus, twice the kinetic energy of the smoothed motion is given by the area under the power spectrum curve between zero and ω.

The restriction imposed by the ground can be expressed by the conditions that long period (low frequency) $\tilde{w}$'s should be such that the corresponding motion does not hit the ground. Let the period T be given by $2\pi/\omega$, where ω is the upper limit in (5). Then we impose the condition, that for high periods, $\tilde{w}$ should be of order to z/T. For example, if the vertical motion represents periods a day or longer, and the observations are made at 1 km, the vertical velocities hould be of order 1 cm sec^{-1}. (which agrees with observations). For high frequencies, the integral (5) must stay finite and approach $\overline{w^2}$ as ω goes to infinity. A simple expression which satisfies these conditions is:

$$\overline{\tilde{w}^2} = \frac{1}{\left(\frac{1}{\sigma_w} + \frac{T}{z}\right)^2} = \frac{1}{\left(\frac{1}{\sigma_w} + \frac{2\pi}{\omega z}\right)^2} \qquad (6)$$

Here σ_w is the standard deviation of the vertical velocities, defined by:

$$\sigma_w = \sqrt{\overline{w^2}}$$

The power spectrum $f(\omega)$ can now be obtained by differentiation of (6) with respect to ω:

$$f(\omega) = \frac{4\pi\,\omega z^2}{\left(\frac{\omega z}{\sigma_w} + 2\pi\right)^3} \qquad (7)$$

The function $f(\omega)$ has a maximum at $\omega = \frac{\pi\sigma_w}{z}$, or, again introducing the period T, at $T = \frac{2z}{\sigma_w}$. In other words, as z increases, the spectrum is displaced to progressively higher periods. On the other hand, increasing wind and roughness will have a tendency to shift the spectrum to lower periods by increasing σ_w. At 100 meters, the period of maximum energy is near 200 seconds, at 10 meters, it should be near 20 seconds.

The use of the logarithm of frequency as dependent variable appears more natural than that of the frequency itself since the range of the frequencies may cover many powers of 10, perhaps from a 1 sec^{-1} to 10^{-6} sec^{-1}. Thus, the "short", "medium" and "long" period eddies in the computations described in section 1a were defined in terms of equal intervals of the logarithm of frequency. If the scale of ω is logarithmic, the spectral function which should be considered is $f'(\omega) = \omega f(\omega)$ since $\int f(\omega)\,d\omega = \int f'(\omega)\,d\ln\omega$. The maximum of $f'(\omega)$ occurs at a period four times as small as that of $f(\omega)$ or at about 50 seconds at 100 meters elevation.

Table VI compares the observed energy in the three frequency ranges with those predicted by the working hypothesis. σ_w was adjusted so that the combined energy in the three ranges is the same observed as predicted.

In the computation it was assumed that a running mean has a sharp cut-off at the period over which the observations are averaged.

Table VI

Comparison of Observed and Predicted Double Energies in the Three Frequency Ranges, $m^2 sec^{-2}$

Date	Short Periods Obs	Short Periods Pred	Middle Periods Obs	Middle Periods Pred	Long Periods Obs	Long Periods Pred
Oct. 4	.26	.33	.24	.28	.13	.03
Nov. 1	.12	.06	.04	.09	.01	.02
Nov. 15	.27	.35	.33	.29	.08	.03
Mar. 27	.37	.43	.34	.33	.08	.03

The agreement is fair. The poorest result is obtained for Nov. 1, where the theory predicts maximum energy in the middle eddies whereas the observations indicate maximum energy in the shortest eddies.

Next, the working hypothesis is extended to the contribution of the various frequency ranges to the stress. Let the exchange coefficient $K = \mu/\rho$ be given by:

$$K = \int_0^\infty K_\omega \, d\omega$$

where K_ω measures the relative contribution of the frequencies ω to the total coefficient, and hence also the flux of various properties. It is now assumed that K_ω is proportional to $f(\omega)/\omega$. This is dimensionally correct, and can be justified by J. E. Miller's incomplete conservation theory (1950), or by the mixing length theory, which, however, is subject to severe objections. The quantity K_ω then is of the form:

$$K = \frac{4\pi z^2 C}{\left(\frac{\omega z}{\sigma_W} + 2\pi\right)^3} \qquad (8)$$

where C is a factor of proportionality, which will be different for the flux of different properties. The quantity ωK_ω which is used with argument $\ln \omega$, has a maximum at a period four times larger than $f(\omega)$, at $T = 2\, z/\sigma_W$.

Table VII shows the predicted and observed relative contribution of the different frequency ranges to the eddy viscosity and stress:

Table VII

Comparison of Observed and Predicted Contributions to the Stress in the Three Frequency Ranges, in % of the Three Ranges Combined

Date	Short Periods Obs	Short Periods Pred	Middle Periods Obs	Middle Periods Pred	Long Periods Obs	Long Periods Pred
Oct. 4	19%	13%	48%	55%	33%	32%
Nov. 1	24%	5%	65%	45%	12%	49%
Nov. 15	17%	14%	57%	55%	27%	31%
Mar. 27	24%	16%	57%	54%	19%	30%

Again, the agreement is quite good for Oct. 4, Nov. 15 and March 27, but very poor for Nov. 1. The rather weak excuse might be made that the whole stress on Nov. 1 was so small that the contributions in the frequencies is quite uncertain. Still, even on Nov. 1, the frequency for maximum stress is much smaller than that for maximum energy, in agreement with the hypothesis.

As a consequence of the working hypothesis, less than 1% of the stress is contributed by turbulence with periods less than 10 seconds, at 109 meters, so that the instruments there are presumably sensitive enough for this type of study. On the low frequency and, an additional 10% might be added to the stress. On the other hand, the values of kinetic energy must be increased by as much as 20% to allow for the high frequency velocity variations, whereas the low frequency correction is negligible.

Integration of equation (8) over all frequencies gives:

$$K = \frac{c\,\sigma_w\,z}{2\pi} \qquad (9)$$

As a consequence of Deacon's formula (1949) and other observations in the lowest 100 meters, we know that the variation of K with height can also be described by:

$$K = c\,z^{\beta} \qquad (10)$$

where c is a constant and β is a near-constant, which is 1 for neutral lapse rates, less than 1 under stable and greater than 1 under unstable conditions. Therefore, comparing (9) with (10), we see that the standard deviation, and therefore the vertical kinetic energy, are constant with height for neutral conditions, decreases with height for stability and increases for instability. Mathematically, σ_w varies as $z^{1-\beta}$. This seems physically reasonable: for example, in a stable atmosphere, turbulence produced in the surface layers by the roughness elements is damped with height when the atmosphere is stable. Thus Barad (1950) noticed that under inversion conditions at Brookhaven, the oil smoke leaving the stack at 108 meter elevation did not noticeably spread in the vertical for several miles down stream.

When conditions are neutral, the constant c in (10) and therefore σ_w in (9) are proportional to the friction velocity u*, defined by $\sqrt{\frac{\tau_0}{\rho}}$ where τ is the surface stress. In the surface boundary layer, the friction velocity under neutral conditions is given by:

$$u^* = \frac{U}{5.75 \log z/z_0} \qquad (11)$$

We see then, that, σ_w is directly proportional to the wind at a fixed level, U, and increases with increasing roughness length z_0.

Since the stress must vanish for isotropic turbulence, we will define the "isotropic threshold frequency" as that frequency above which only 1% of the stress is produced. If we denote the isotropic threshold frequency by ω_i and the corresponding period by T_i, equation (8) gives the result that:

$$T_i = z/9\sigma_w \qquad (12)$$

Numerically, T_i comes out about 14 seconds for Oct. 4, Nov. 15 and March 27, at 109 meters. R. J. Taylor (1951) obtains almost identical results with his empirically determined formula $T_i \sim z/U$. Equation (12) differs from Taylor's result in that it indicates that increasing roughness as well as decreasing wind speed may reduce the isotropic threshold period. It should be pointed out that the smallness of the stress is only a necessary condition for isotropic turbulence. Equi-partition is also required, and may not occur even below $T = T_i$. (Lettau 1949).

The working hypothesis conspicuously fails to represent the effects of changing stability on the turbulence spectrum. As will be seen in section three, there is overwhelming evidence that, with increasing stability, relatively low period turbulence (often identified with "convection"), is suppressed. Therefore the frequency at which maximum energy is produced increases with increasing stability. This conclusion can be verified for example by comparing the observations of Nov. 1 with those of the other days (section 1a). On Nov. 1, the greatest amount of kinetic energy was produced at periods less than 70 seconds, whereas on the other days, the amounts in the small period and middle period group were about the same. Again, it should be recalled that the working hypothesis works least well for the data of Nov. 1, when the stability was relatively great. In other words, the working hypothesis seems to be most adequate when the atmosphere is most unstable. This can be understood from the conditions under which the hypothesis was derived. It was assumed that the low frequency vertical velocity, $\tilde{w}$, was equal to z/T in order that the oscillations did not hit the ground. Actually, however, there is no reason why $\tilde{w}$ cannot be less than z/T; in that case the closeness of the ground would not produce the important restriction on the spectrum, but the buoyancy of the air. In other words, the hypothesis might be expected to cover best the maximum possible energy at a given frequency, a condition which would be expected to be reached under superadiabatic conditions. In a sense, the hypothetical spectrum is analogous to the "Black Body Spectrum" in the theory of radiation in that it, too, gives the maximum energy per frequency interval.

2c. Horizontal Momentum Flux Due to Small Scale Turbulence

Several independent studies of the maintenance of the General Circulation by Widger (1949), Priestley (1951) and others are based on the poleward flux of angular momentum from its source in the tropics. The assumption is generally made that the eddies responsible for this flux are of the dimensions of cyclones, and are determined from day-by-day 6-hour by 6-hour velocity deviations from very long period means. It is assumed that velocity fluctuations on a smaller scale do not contribute significantly to the meridional momentum flux.

In order to demonstrate the properties of horizontal momentum flux, we begin by defining "horizontally isotropic turbulence" by the condition that rotation of the horizontal axes horizontally does not affect the horizontal momentum flux. This means, in particular, that no Reynolds shearing stress can exist, or that the momentum in a given direction cannot be transported at right angles to that direction, or, again, that the coefficient of correlation between any two momentum components at right angles to each other must vanish. If orthogonal wind components computed in an arbitrary coordinate system are plotted against each other, the graph will have circular symmetry if the turbulence is horizontally isotropic. If it is not, the graph will appear elongated. Computations of meridional flux of zonal momentum can stop at the threshold from horizontally isotropic to horizontally nonisotropic turbulence.

Figure 3 shows deviations of ten second average horizontal velocity components from their means plotted as function of each other. The observations were obtained from traces produced by the aerovane at the 109 meter level of the 125 meter meteorological tower of the Brookhaven National Laboratory, and ten second averages of wind direction and velocity were estimated by eye. The points plotted represent deviations from approximately one hour means. Apparently, the turbulence on this scale is not quite horizontally isotropic.

The variation of the velocity components with time indicates that considerable trends existed during both periods under discussion here. These trends must be due to eddies of periods considerably larger than one hour. Since we are primarily interested in eddies with periods less than one hour, these trends should be eliminated from the turbulent velocity components. For this reason, new turbulent components were defined by:

$$\Delta u = u - b_u t - \bar{\bar{u}}$$

$$\Delta v = v - b_v t - \bar{\bar{v}}$$

where u and v are the observed components, $\bar{\bar{u}}$ and $\bar{\bar{v}}$ the arithmetic means in the period analyzed, and b_u and b_v are the slopes of the line of regression of u and v on time t (counted from zero at the middle of the period). Figure 4 shows Δv plotted as a function of Δu for the same data of figure 3. The new eddy velocities appear to be horizontally isotropic. This is partially brought out by the fact that the meridional flux of zonal momentum produced by these eddy velocities is only 1.56 gm $cm^{-1}sec^{-2}$ on Nov. 1 and 0.48 gm $cm^{-1}sec^{-2}$ on Nov. 15, values negligible in comparison with the larger scale flux found by Widger (1949) and Starr (1951).

Similar computations were also carried out for an hour on Oct. 4, 1950 based on instantaneous wind directions and 6 second average wind speeds at 125 meters, also indicating horizontal isotropy.

The result that horizontal isotropy should be found below periods of about one hour might be compared with what might be expected from the spectrum theory of the last section (section 2b), modified to apply to meridional turbulent velocities. For vertical turbulence, the isotropic threshold period is: (equ. 12, section 2b)

$$T_i = \frac{z}{9\sigma_w}$$

The quantity z enters this equation so that the vertical oscillations do not hit the ground. In other words, z is a kind of maximum vertical amplitude.

For horizontal turbulence, the amplitude of oscillation is of order 10^6m sec^{-1}. Hence, the isotropic threshold period comes out 10^4 seconds, or about three hours. Removal of the trend from the observations removed the effect of eddies with periods longer than three hours, so that the remainder should be and was horizontally istropic. Most of the flux should be produced by a period of 18 T_i, or about 2-1/2 days, a period characteristic for weather map scale turbulence.

It should be pointed out, that, whereas the wind tunnel might imitate the characteristics of vertical turbulence, it cannot possibly simulate the characteristics of the horizontal velocities. For, whereas the horizontal amplitude in the atmosphere is 10^4 times as large as the vertical amplitude at 100 meters, the tunnel prescribes amplitudes of the same order of magnitude vertically and horizontally. Hence, in the tunnel, the isotropic threshold periods for horizontal and vertical turbulence will be about the same.

2d. The Contribution of Eddies of Various Periods to the Heat Flux

The vertical heat flux due to turbulent motion, as defined in section 1c, was

$$F = \overline{\overline{c_p \rho \Delta T \, \Delta W}} \qquad (1)$$

We can again break up the vertical turbulent velocity into contributions by fluctuations with various periods as in section 1a:

$$\Delta w = w' + w'' + w'''$$

Then the heat flux consists of three components:

$$F = \overline{\overline{c_p \rho w' \Delta T}} + \overline{\overline{c_p \rho w'' \Delta T}} + \overline{\overline{c_p \rho w''' \Delta T}} \qquad (2)$$

The three terms on the right of (2) were evaluated from the observations on March 27, 1951, described in section 1c. The result is given in Table VIII.

The table shows also the contributions expected from the working hypothesis discussed in the last section. This hypothesis should give the correct answer only if the "selective heat flux" discussed by Priestley and Swinbank (1947) is small, or has the same spectral distribution as the part of the heat flux associated with the gradient of potential temperature.

Apparently, the agreement between hypothesis and observation is quite good, with again most of the heat being transported by turbulence with periods of the order of a few minutes. In fact, the spectral distribution of heat and momentum flux is remarkably similar.

Table VIII

Spectral Analysis of Heat Flux, March 27, 1951, -

Flux in % of Total Flux

Short Eddies		Medium Eddies		Long Eddies	
Obs	Comp	Obs	Comp	Obs	Comp
23%	16%	50%	54%	27%	30%

2e. An Electronic Spectrum Analysis System

The Wave-Forecasting project at New York University, headed by Dr. Willard Pierson and sponsored by the Beach Erosion Board, has acquired a Kay Electric Company Sona-graph. This instrument is capable of analyzing the spectrum of recorded speech by electronic means. If the record of a meteorological variable is converted into sound, the Sona-graph will do the rest.

Unfortunately, the Sona-graph does not produce the analysis of the complete record, but rather analyzes short sections at a time, since it is designed to study variations of the spectrum with time. As a result, the analyses of the short sections of record is expected to be rather crude; in particular any gaps in the spectrum are likely to be accidental (Tukey, unpublished). However, the general shape of the power spectrum curve and the approximate position of the maximum should be obtainable from the analysis. It may be possible to adjust the characteristics of the Sona-graph in such a way as to analyze a larger section of spectrum at one time.

The meteorological record is converted into sound in three steps:

1. The response of the selsyn which reflects the variation of some meteorological variable, is transferred to a cathode ray oscillograph.

2. The oscillograph record is photographed with a moving film camera.

3. The film is moved rapidly past a photoelectric cell, the output is converted into sound and recorded.

The electrical system which performs item 1 for variations of wind direction was built by Prof. Greenstein and Mr. Stock of the Electrical Engineering Department at New York University and is described in some detail in the Progress Report. The equipment has been tested at Brookhaven and found to operate satisfactorily. A Fairchild Camera was borrowed for the test from the Flame and Jet Group of New York University; further tests will be made with a camera bought for the Wave Forecasting project of the N.Y.U. Meteorology Department before completion of this report.

The mechanical equipment for rapidly moving the film past a photoelectric cell has been completed; the scanning device should be finished before June 30, 1951.

Originally, the spectral analysis of wind speed was also planned; this will probably be abandoned due to shortage of man power in the Electrical Engineering Department. However, the existing equipment can easily be adjusted to record the spectrum of the vertical angle, which is essentially the spectrum of the vertical velocity component.

VI.

SECTION 3

METEOROLOGICAL CONDITIONS GOVERNING THE CHANGE OF CHARACTER OF TURBULENCE

3a. Summary of Earlier Work on the Change of Character of Turbulence

The conditions for turbulence to commence, or to persist after it has started, can be derived theoretically in two entirely different manners, with results marked by qualitative agreement only. The first method is due to Richardson (1920): it is assumed that turbulence can persist only if the energy released from large scale organized flow to the eddies is sufficient to balance the energy drained off by work done against gravity. The result is that turbulence can persist if the "Richardson Number" Ri, defined by

$$Ri = -\frac{g\frac{\partial\rho}{\partial z}}{\rho\left(\frac{\partial u}{\partial z}\right)^2}$$

(ρ density, g gravity, z height, u speed) is less than $\frac{\mu_h}{\mu_m}$ for an incompressible fluid, where μ_h is the exchange coefficient for heat and μ_m the eddy viscosity. For compressible fluids, the density must be replaced by the potential temperature, and the sign must be changed. The ratio of the exchange coefficients is often taken as one for want of a better assumption.

One drawback of this treatment is that not all the methods of energy dissipation are considered in the equation for the energy budget of eddies, as pointed out, for example, by Calder (1949) and Blackadar (1950).

The second method consists in the discussion of the stability of small perturbations superimposed on steady flow. Because of the mathematical difficulties of this technique, only incompressible fluids have been treated so far, a restriction, which should not be important in the atmosphere close to the ground. Goldstein (1931) and Taylor (1931) treated flow with linear velocity profiles; they also neglected molecular viscosity. Goldstein arrived at a limiting value for the Richardson number of 0.25, Taylor of 1.00. The form of Goldstein's equation makes it unlikely that the same criterion can be generalized to compressible flow by just expressing the Richardson number in terms of the potential temperature instead of the density, excepting close to the ground.

Schlichting (1935) generalized the perturbation treatment by including molecular friction as well as curvature of the velocity profile, and arrived at critical Richardson numbers of the order of 0.04 for high Reynolds numbers. Among his conclusions was that, no matter how high the Reynolds number, the flow could still be stable for sufficiently large positive Richardson numbers. Schlichting's results agreed with measurements in the "hot-cold tunnel"; laminar flow yielded to turbulent flow near the predicted Richardson number. Since the curvature of the velocity profile, and molecular friction, are most important near the surface, all the theoretical results lead to the general conclusion that the critical Richardson number increased from a value near .04 close to the surface to values of order unity at some distance from the surface. This conclusion goes together with the observation that in the atmosphere the Richardson number generally increases with height (in absolute magnitude).

The theories of onset of turbulence make it appear that a critical Richardson number exists which distinguishes between laminar and turbulent flow; and, indeed, in a wind tunnel this seems to be so. In the atmosphere, however, laminar flow never exists (excepting in a few millimeters above the ground). The conclusion seems reasonable, then, that, in the atmosphere, particular values of the Richardson number do not distinguish between laminar and turbulent flow, but between turbulent flow of different characteristics. Thus, for example, Giblett (1932) found that stability (high Richardson number) suppressed medium period turbulence, that is, oscillations of the velocities with periods of the order of a few minutes. Short period turbulence, on the other hand, seemed hardly affected. The same conclusions can be drawn from Paeschke's (1937) observations. Observations so far have not been conclusive as to whether this change in the characteristics of turbulence occurs at Richardson number equal to zero, or at some small positive value. Some observational results on these points are discussed in the next section.

The reason why laminar flow is observed in wind tunnels but not in the atmosphere, is probably due to the greater roughness of the ground than of the tunnel walls. As a matter of fact, practically laminar flow has been observed in the atmosphere at stack level under inversion conditions, where the effect of surface roughness has been damped out by the intervening stable air. When the walls of the tunnel are made rough to correspond to the ground roughness, the general characteristics of air and tunnel turbulence are expected to be similar.

3b. Observational Studies of the Change of Character of Turbulence Made With Not Too Sensitive Instruments

Inspection of continuous records of any variable, especially of wind direction, shows that a smooth trace may abruptly give way to a rapidly oscillating one, and vice versa, provided that the instruments are not sensitive to oscillations with periods of less than 10 seconds. These rapid changes occur especially in the morning, when the air changes from stable to unstable, and in the afternoon, when the opposite change takes place. The problem then arises whether there exists a critical Richardson number at the time at which the abrupt change occurs.

Originally, the personnel of this project had intended to investigate the effect of meteorological parameters on these changes of the character of turbulence. However, Mr. Raymond Wanta, the Weather Bureau representative at Brookhaven, had been interested in the same problem independently, and therefore no additional studies of this subject were undertaken. Mr. Wanta described some preliminary findings in the Progress Report. Since the writing of that report, Mr. Wanta had obtained some additional significant results. For a given layer of air, he compared the Richardson number just before the instability reached the lower boundary of the layer with the Richardson number after the instability had marched through the whole layer and had just become evident in the record of the upper boundary. He found that before obvious turbulence set in (small scale turbulence probably was present but not recorded), the Richardson number was generally above 1/3; after the change, it was less than 1/2 in the layer from 46 to 125 meters. Thus, a critical value of the Richardson number seemed to lie near 0.3, in fair agreement with Goldstein's theoretical work. Also, this result does not really contradict Schlincting's theory, since this applies immediately above the ground, whereas the measurements were made at heights of several tens of meters.

3c. An Attempt to Apply Perturbation Theory to Stratified, Compressible Flow

The Project Report describes an attempt to generalize Goldstein's results to compressible flow. The differential equation covering the situation has been derived by Haurwitz (1931). It is a second order equation in the amplitude of the vertical oscillation, the coefficients of which are complicated functions of the height. Even with great (and not necessarily permissible) simplifications, the solution did not lead to useful results and therefore the details are not repeated here. During the course of the work, however, it became clear that the conditions for stability in compressible flow depend on many factors besides the Richardson number. The problem is one the solution of which is highly recommended to a skilled mathematician.

VII.

SECTION 4

THE VARIATION OF THE MEAN WIND WITH HEIGHT IN THE LOWEST 125 METERS

4a. Structure of the Wind in the Lowest 400'

One of the reasons for the study of the variation of wind with height is to determine a linear scale factor between the wind tunnel and the atmosphere. Before this can be done, it is necessary to determine which mathematical expressions adequately describe the variation of wind with height in the surface layers of the atmosphere. The purpose of this section of the report is to test the use of the logarithmic law and its extension, the power law, in reproducing the wind profile of the surface layers of the atmosphere.

L. Prandtl (1932) has shown that a logarithmic law describes the variation of wind with height.

$$u = \frac{1}{k_o}\sqrt{\frac{\tau}{\rho}}\ \ln\left[\frac{z + z_o}{z_o}\right] \qquad (1)$$

The shearing stress, τ, and the density, ρ, are considered independent of the height. k_o is a non-dimensional constant which, according to Von Karman (1930), has the value of 0.4. z_o depends on the roughness of the surface and is called the "roughness parameter".

There are two main faults with the logarithmic law. First, it is restricted to adiabatic, "neutral", lapse rates, and second, the law leads to many mathematical difficulties such as transcendental equations which are too tedious to solve. Mathematically, power laws are far more convenient to use. Deacon (1949) has suggested a power law which satisfies the differential equation:

$$\frac{du}{dz} = az^{-\beta} \qquad (2)$$

which, after integration, reduces to equation (1) when $\beta \to 1$, provided that the origin of the vertical axis is shifted to a distance z_o below the level of zero wind. This law has been tested by many investigators

(Frost, 1947) for the lowest ten meters of the atmosphere. The following sections will show the validity of using the power law to a higher elevation at Upton, New York.

In addition to temperature and wind measurements at 37-1/2, 75, 150, 300, 350, and 410 feet, the wind at 37-1/2, 150, and 410 feet are recorded by means of a Friez Gradient Recorder which accurately yields the average wind for a ten minute period at normal recording speed. The present study deals with hourly averages of wind speed and temperature obtained by averaging six ten minute periods.

Deacon's power law may be integrated between two levels, z_3 and z_1, to give:

$$u_3 - u_1 = \frac{a}{1-\beta}\left[z_3^{1-\beta} - z_1^{1-\beta}\right] \quad (3)$$

Division of expressions of the form of (3) leads to:

$$\frac{u_3 - u_1}{u_3 - u_2} = \frac{z_3^{1-\beta} - z_1^{1-\beta}}{z_3^{1-\beta} - z_2^{1-\beta}} \quad (4)$$

the subscripts refer to height 1: 37-1/2', 2: 150', 3:410'.

As "a" is eliminated from equation 4 a Nomograph can be constructed to compute β with the knowledge of the wind velocity at heights 1, 2, and 3. With this value of β, "a" can be computed by means of equation 3. This was done for each hour during the four month period of April through July 1950. Using these values of "a" and β, the power law was tested as an interpolation formula for the heights of 25 and 300 feet. The standard error of estimate was 1/2 meter per second at both levels.

With the correct values of "a" and β, the power law does describe adequately the variation of wind to the height of 410 feet. The next step was to see if β and "a" depend on any meteorological parameters. Deacon found a relationship between β and the lapse rate in the lowest ten meters of the atmosphere. He observed that $\beta > 1$ for superadiabatic lapse rates, $\beta = 1$ for adiabatic lapse rates, and $\beta < 1$ for subadiabatic lapse rates.

For observations with positive lapse rates and deviations from the constant lapse rate less than .15 degrees centigrade in the height interval of 37-1/2 to 410 feet, β and "a" were plotted as function of the lapse rate. β and the lapse rate were linearly correlated and the probable error of estimate of β from the linear regression was 0.2 (Figure 5). It is interesting to note that the results obtained here concerning the variation of β for the height interval of 410 feet are comparable to the results obtained by Deacon in the lowest ten meters.

Calder (unpublished) found a relationship between β and the Richardson number better than the relation between β and the lapse rate, in the lowest ten meters of the atmosphere. An attempt to find such a relation up to 125 meters was unsuccessful. The difference of the results is probably due to the fact that $\frac{\partial u}{\partial z}$ is small and poorly determined at relatively large heights. This term occurs in the denominator of the Richardson number, raised to the second power, so that errors of observation are accentuated. Cramer (1951) had similar difficulties, even closer to the ground.

The quantity "a" also was plotted as function of the lapse rate. The relation is exponential in character, as can be seen in figure 6.

With β determined from figure 5, "a" from figure 6, and the wind at 37-1/2 feet, the wind velocities at 75 and 300 feet were computed for April through July, 1950. The standard error of estimate of the velocities at these heights was 1 m sec^{-1}.

The same technique was used on an independent sample, consisting of the wind velocities from August through November, 1950. The standard error of estimate remained of the same order of magnitude, 1.3 m sec^{-1}.

In order that Deacon's power law (equation 3) reduce to the logarithmic law (equation 1) for $\beta = 1$, the constant "a" may be taken as $\sqrt{\frac{\tau}{\beta}} / (1-\beta)$. Then, the power law can be written

$$u = \frac{\sqrt{\frac{\tau}{\rho}}}{\kappa(1-\beta)} \left[\left(\frac{z}{z_0} \right)^{1-\beta} - 1 \right] \qquad (5)$$

Given the observation at three levels, it should be possible to evaluate the three parameters, β, z_0 and τ. The computation of β from the observations at 37-1/2 feet, 150 feet and 410 feet has already been discussed. An attempt was made to derive the roughness length and surface stress from the winds at the same levels. The stress, under superadiabatic conditions, was of order 50 dynes cm^{-2}, a value unreasonably high, and incompatible with the direct bivane measurements (section 1a). The roughness length was frequently complex under stable conditions, and larger than the height of the stress when the atmosphere was stable. Probably neither stress nor roughness length computed in this manner have any physical reality.

If z_0 is assumed known for the Brookhaven sight, the observation can be fitted only if β or "a" are assumed to vary with height. But in that case, equation (5) cannot be derived by integration of equation (2), and must be taken as a rather arbitrary equation for the representation of the wind variation with height, which has the property of reducing to the logarithmic law when $\beta = 1$. But the usefulness of (5) is drastically reduced if β or τ are as yet undetermined functions of height.

In summary, it may be said that the power law, equation (5), with constant parameters β, z_0 and τ, is a convenient equation for the statistical description of the variation of the wind with height, and, as such, should be useful in diffusion problems. However, the identifaction of z_0 and τ with the physical quantities, roughness length and stress, is not permitted.

4b. Scale Considerations

The character of turbulence at a given height over a given ground is largely determined by two dimensionless numbers, the Reynolds number and the Richardson number. However, when the Reynolds number is high, the Richardson number seems to be the deciding factor. This means that if the frictional forces are small compared to the inertia forces, it is relatively unimportant just how small friction is. At high Reynold's number, a critical Richardson number decides between laminar and turbulent flow in a smooth-wall wind tunnel, and between low frequency and high frequency turbulent flow in the atmosphere. The difference between tunnel and atmosphere lies largely in the relatively much smaller roughness of the tunnel used in laboratory experiments (Schlichting, 1935).

Tunnel and atmosphere might be expected to show similar turbulence characteristics if the Richardson number was made equal at equivalent distances. Here "equivalent distance" is defined so that z/z_o in the tunnel equals z/z_o in the atmosphere. At the equivalent distance from the ground, the ratio of the wind velocity to the friction velocity, is the same in tunnel and atmosphere. The "linear scale factor" which measuresthe ratio of equivalent distances in the tunnel and atmosphere, is then equal to the ratio of the roughness lengths in the atmosphere and tunnel.

Both in the atmosphere and tunnel, the wind profile is logarithmic under neutral conditions, and the roughness parameter can be computed from the wind profile. Originally, with a smooth tunnel the linear scale between tunnel and Brookhaven came out of order 10,000. On the basis of this, it was recommended in the Progress Report to increase the surface roughness in the tunnel by introducing roughness elements with linear dimensions of about 1 inch. These elements have now been added in the shape of pieces of sponge, glued on the tunnel wall. The wind profile in the tunnel is still almost exactly logarithmic up to end of the boundary layer. The order of magnitude of the scale factor toward the end of the working section is about 30. There is still the important difference between the tunnel and the atmosphere that the thickness of the boundary layer in the tunnel increases downstream, whereas it is more or less constant in the atmosphere. Eventually, this difference might be removed by letting the roughness elements in the tunnel decrease in height downstream. Apparently, the sponges are too rough; 100 meters in the atmosphere would correspond to 3 meters in the tunnel. Hence the roughness elements should be adjusted to give a scale factor about 300.

The velocity scale factor is the ratio of the velocities in atmosphere and tunnel at equivalent heights. This is expected to be about 10, with the free air speed in the atmosphere about 10 m sec^{-1}, that in the tunnel, about 1 m sec^{-1}. The time factor is the ratio of the linear scale factor to the velocity factor, or eventually about 30. It is hoped that the periods of maximum energy in the vertical velocity spectrum in the tunnel will be equal to those in the atmosphere at equivalent heights, reduced by the time factor, provided that the Richardson number is the same. The question then remains how large temperature gradients have to be created in the tunnel in order that Richardson numbers characteristic for different types of turbulence in the atmosphere can be obtained in the tunnel.

Let subscript t refer to tunnel and subscript a to atmosphere, and $\Delta\theta$ be potential temperature differences. Then

$$\frac{\Delta \theta_t}{\Delta \theta_a} = \frac{(\Delta z)_a}{(\Delta z)_t} \quad \frac{(\Delta u)_t^2}{(\Delta u)_a^2} \tag{1}$$

In order to satisfy (1), the ratio of temperature differences in the atmosphere to that in the tunnel must be made equal to the ratio of the square of the velocity scale factor to the linear scale factor. This ratio in the rough tunnel will be adjusted to 3, so that temperature differences in the tunnel should be 3 times as large as the differences at equivalent heights in the atmosphere. This ratio seems entirely feasible with the present power supply.

VIII.

SUMMARY

1. The exchange coefficients at 100 meters under superadiabatic conditions over rough ground are between 100 and 1500 gm $cm^{-1}sec^{-1}$. No significant difference between heat and momentum coefficients was discovered. The best method for the determination of these coefficients was that based on the direct measurement of the Reynolds flux. The pilot balloon method and the diurnal temperature variation method are subject to severe theoretical objections.

2. Most of the vertical heat and momentum flux at 100 meters under superadiabatic conditions is produced by the Fourier components of the turbulence with period of order of a few minutes. Most of the kinetic energy is located at periods about four times as low. A working hypothesis is developed which accounts reasonably well for these results.

Velocity variations of periods less than an hour produce negligible meridional flux of zonal momentum.

A set of instruments has been developed which will make possible the analysis of the horizontal or vertical angle spectrum by electronic means.

3. Deacon's formula for the variation of wind with height represents a good interpolation formula. The coefficients in this formula are functions of the stability. The physical meaning of z_o and τ_o in that formula is clear only under neutral stability conditions.

4. Increasing the roughness in the wind tunnel had been found desirable previously, in order to produce the optimum scale factor between atmosphere and tunnel of 300. Some roughness elements were introduced into the tunnel which made the tunnel too rough and reduced the scale factor to 30.

BIBLIOGRAPHY AND CITED REFERENCE

Barad, M. L., 1950: Diffusion of Stack Gases in Very Stable Atmospheres, Doctorate Dissertation, New York University, 37pp.

Blackadar, A.K., 1950: The Transformation of Energy by the Large Scale Eddy Stress in the Atmosphere, Meteorological Papers, New York University, V.1, No. 4, 33 pp.

Brunt, D., 1941: Physical and Dynamical Meteorology, Cambridge University Press, London, p 225.

Calder, K. L., 1949: The criterion of Turbulence in Fluid of Variable Density with Particular Reference to Conditions in the Atmosphere. Quart J. Roy. Met. Soc., v. 75, pp. 71-88.

Cramer, H. E., 1951: Preliminary Results of A Program for Measuring the Structure Flow Near the Ground. Paper read before Cambridge. Symposium on Turbulence in Boundary Layer, June 1951.

Deacon, E. L., 1949: Vertical Diffusion in the Lowest Layers of the Atmosphere. Quart. J. Roy. Met. Soc., v. 75, p. 89.

Frost, R., 1947: The Velocity Profile in the Lowest Layers of the Atmosphere. Meteorology Magazine, London, v. 76, p. 14.

Giblett, M.A., 1932: The Structure of Wind Over Level Country. Geophysical Memoirs, Meteorological Office. No. 54.

Goldstein, A., 1931: On the Stability of S[illegible]erimposed Streams of Fluids of Differen[illegible] Densities. Proc. Roy. Soc. London, A, No. 132, p. 524.

Haurwitz, B., 1936: The Daily Temperature Period for a Linear Variation of the Austausch Coefficient. Transactions of the Royal Society of Canada, Third Series, Section III, Vol.XXX.

Haurwitz, B., 1931: Zur Theorie der Wellenbewegung in Luft und Wasser. Geophysical Institute, Univ. of Leipzig, No. 5, Heft 1.

Haurwitz, B., 1941: Dynamic Meteorology. McGraw-Hill Co., New York, 365 pp.

(Continued)

Johnson, N.K., and Heywood, G.S.P., 1938: Investigations of the Lapse Rate in the Lowest 100 meters. Geophysical Memoirs, Meteorological Office. No. 77.

Kärmän, von Th., 1930: Mechanische Ähnlichkeit und Turbulenz. Nachrichten von der Gesellschaft der Wissenschaften zu Gottingen, Math-Physical Klasse, No. 1, p. 58.

Lettau, H., 1949: Isotropic and Non-Isotropic Turbulence in the Atmospheric Surface Layer. Geophysical Research Papers, Air Force Research Laboratories, No. 1, pp. 86.

Lettau, H., 1950: A re-examination of the "Leipzig Wind Profile" considering some relations Between Wind and Turbulence in the Frictional Layer, Tellus, Stockholm, Sweden, V. 2, No.z, pp 125-129.

Mazzarella, D.A., 1951: An All-Weather, Remote-Recording Bivane, (unpublished).

Miller, J.E., 1950: Experimental Studies of Small Scale Turbulence. Progress Report 142-02, New York University.

Paeschke, W., 1937: Experimentelle Untersuchungen zum Rauhigkeits- und Stabilitätsproblem. Beitr. Phys. fr. Atm. V. 24.

Pasquill, F., 1950: The Aero-Dynamic Drag of Grassland, Proc. Roy. Soc. A., No. 202, p. 143-153.

Prandtl, L., 1932: Meteorologische Anwendung der Strömungslehre. Beitr. Phys. fr. Atm., V. 19, p.188.

Priestley, C.H.B., and Swinbank, W.C., 1947: Vertical Transport of Heat in the Atmosphere. Proc. Roy. Soc. London, A., V. 189, p. 543.

Priestley, C.H.B., 1951: Physical Interaction Between Tropical and Temperate Latitudes., Quart. Jour. Roy. Met. Soc., V. 77, No.332, pp 200-15.

Richardson, L.F., 1920: The Supply of Energy from and to Atmospheric Eddies. Proc. Roy. Soc. London, A., V. 97, p. 354.

Rossby, C.G., and Montgomery, R.B., 1935: The Layer of Frictional Influence in Wind and Ocean Currents. Papers in Physical Oceanography and Meteorology, M.I.T. V. III, No.3, pp 101.

(Continued)

Schlichting, von A., 1935: Turbulenz bei Wärmeschichtung, Zeitschrift für Angewandte Mathematik und Mechanik, V. XV, No. 6, (Reprinted in NACA Report # TM 1262).

Scrase, F.J., 1930: Some Characteristics of the Eddy Motion in the Atmosphere. Geophysical Memoirs, Meteorological Office, No.52.

Sheppard, P.A., 1947: The Aerodynamic Drag of Earth's Surface and the Value of Von Karman's Constant in Lower Atmosphere. Proc. Roy. Soc. of London, A., V. 188, pp. 208-222.

Sheppard, P.A., 1951: The Effects of Turbulent Flow. Paper read before Cambridge Symposium on Turbulence in Boundary Layer, June 1951.

Starr, V.P. and White, R.M., 1951: Hemispherical Study of the Atmospheric angular-momentum balance. Quart. Jour. Roy. Met. Soc. Vol. 77, No. 332, pp 215-225.

Sutton, O.G., 1949: Atmospheric Turbulence, Methuen and Co., Ltd., London, England, Vii, 107 pp.

Taylor, G.I., 1931: Effect of Variations in Density on the Stability of Superimposed Streams of Fluid. Proc. Roy. Soc. London, A. No. 132, p. 499.

Taylor, G.I., 1938: The Spectrum of Turbulence, Proc. Roy. Soc., A. No. 164, pp 476-490.

Taylor, R.J., 1951: Locally Isotropic Turbulence in the Lower Layers of the Atmosphere. Paper read before Cambridge Symposium on Turbulence, June 1951.

Widger, W.M., 1949: A Study of Flow of Angular Momentum in the Atmosphere. Journal of Meteorology, No.6 p. 291.

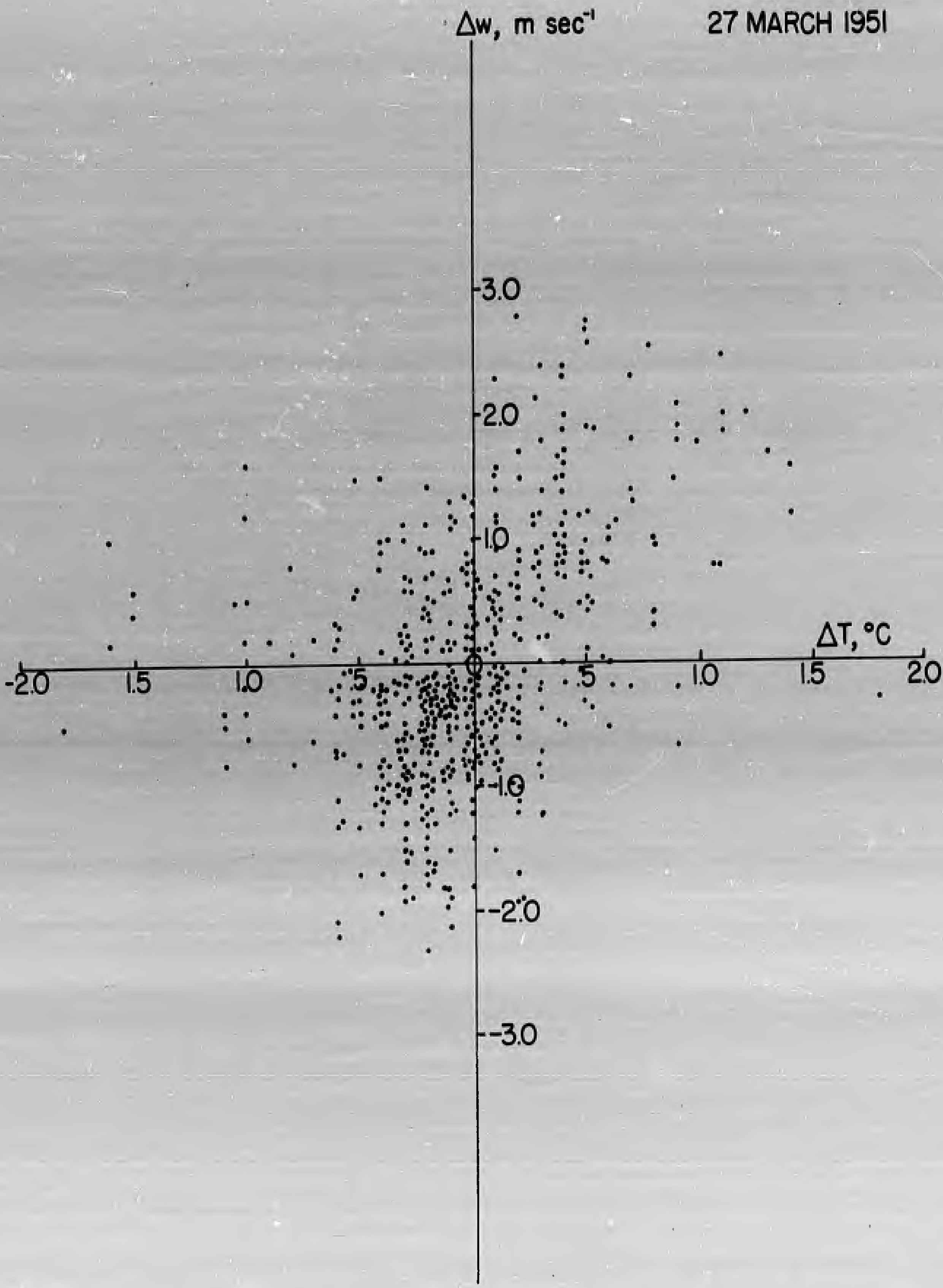

Fig. 1—Variation of Temperature Deviation with Vertical Velocity Deviation.

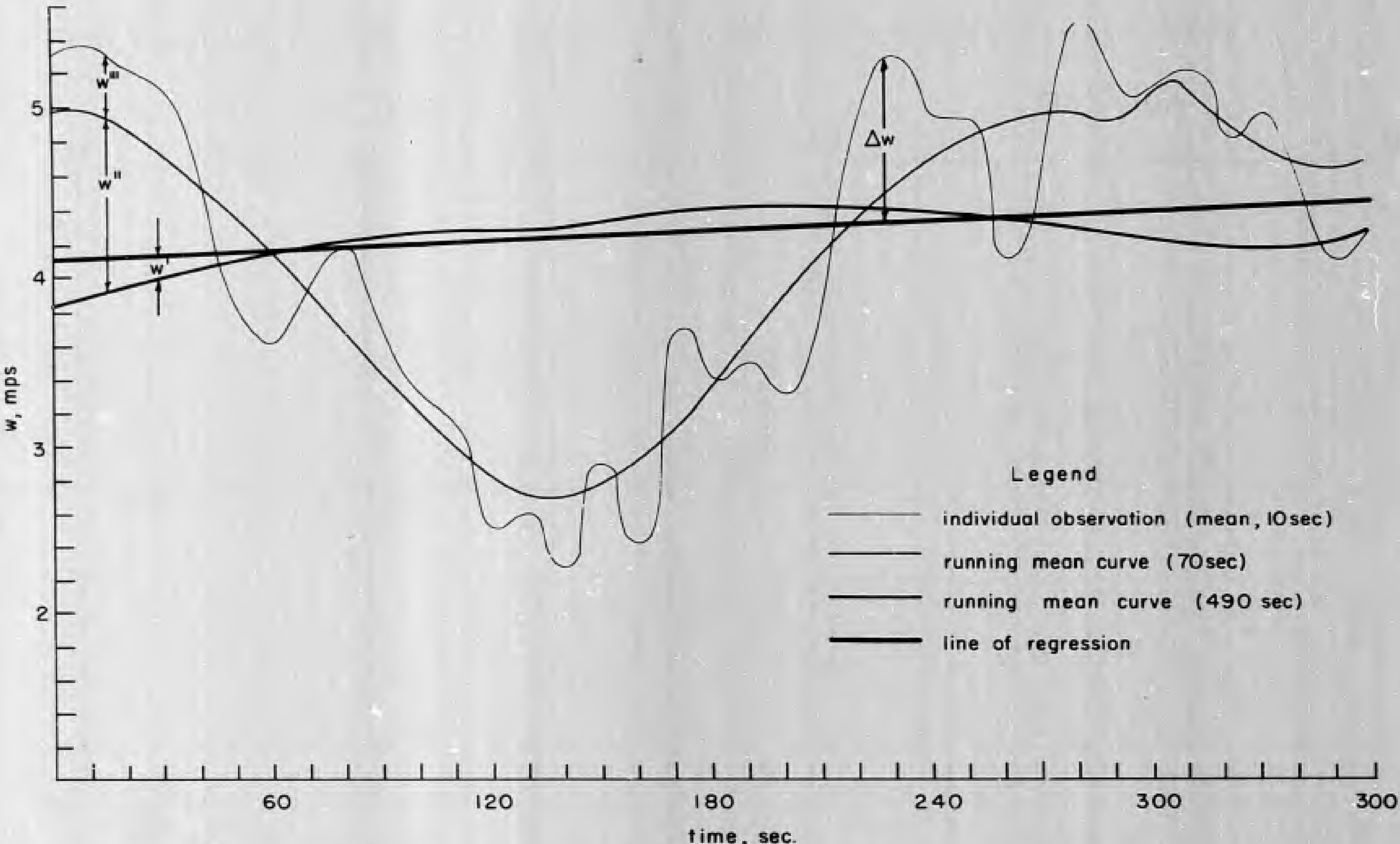

Fig. 2 — Breakdown of Eddy Vertical Velocity into Components of Different Frequency Intervals.

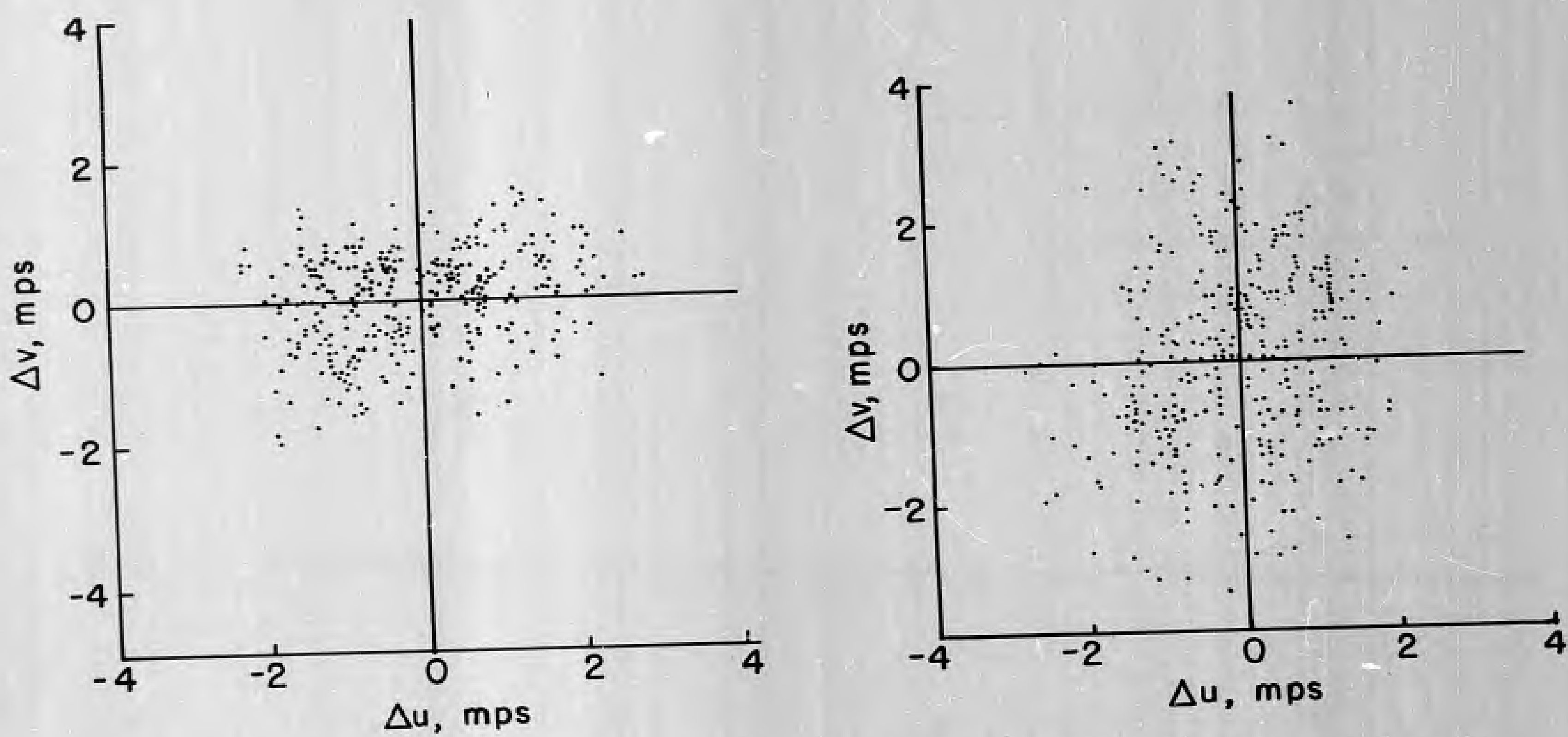

Fig. 3--Horizontal Velocity Component Deviation from Arithmetic Means Plotted as a Function of each other.

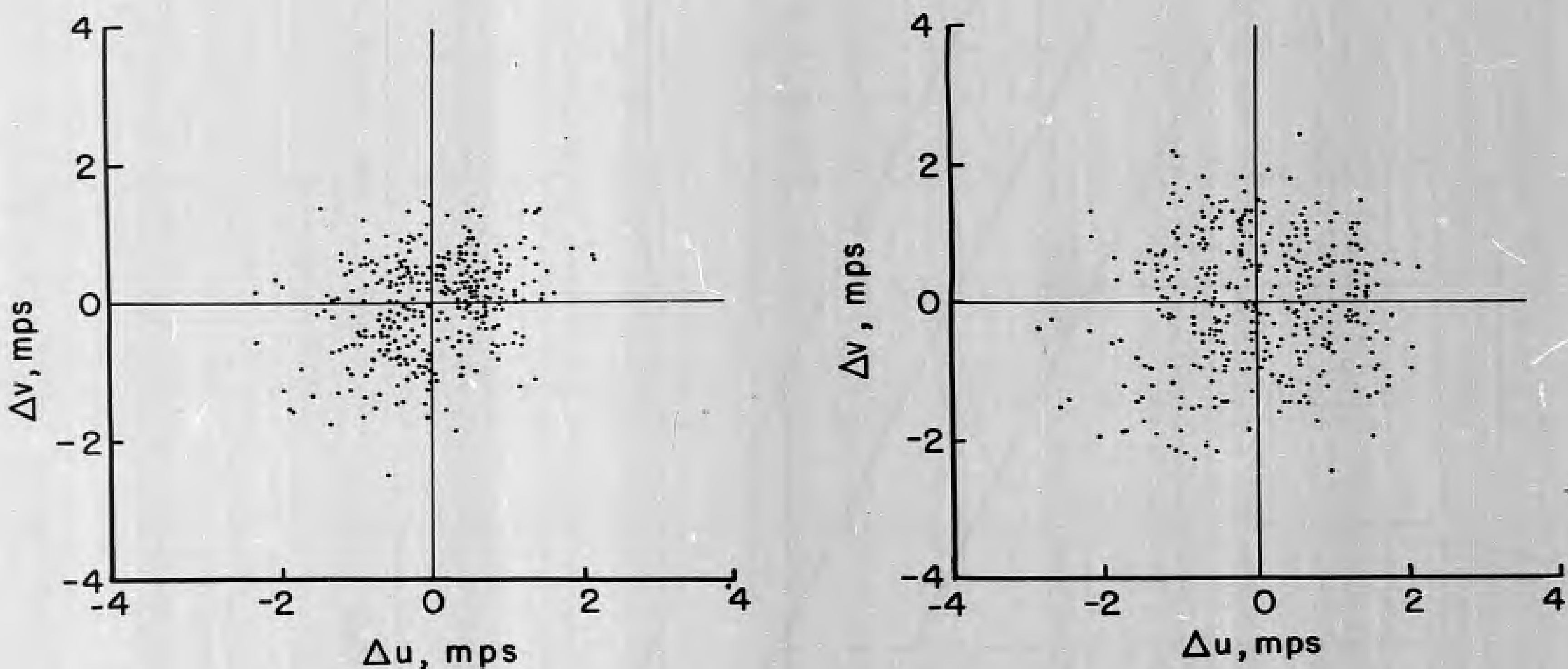

Fig. 4--Horizontal Velocity Component Deviation from Line of Regression Plotted as a Function of each other.

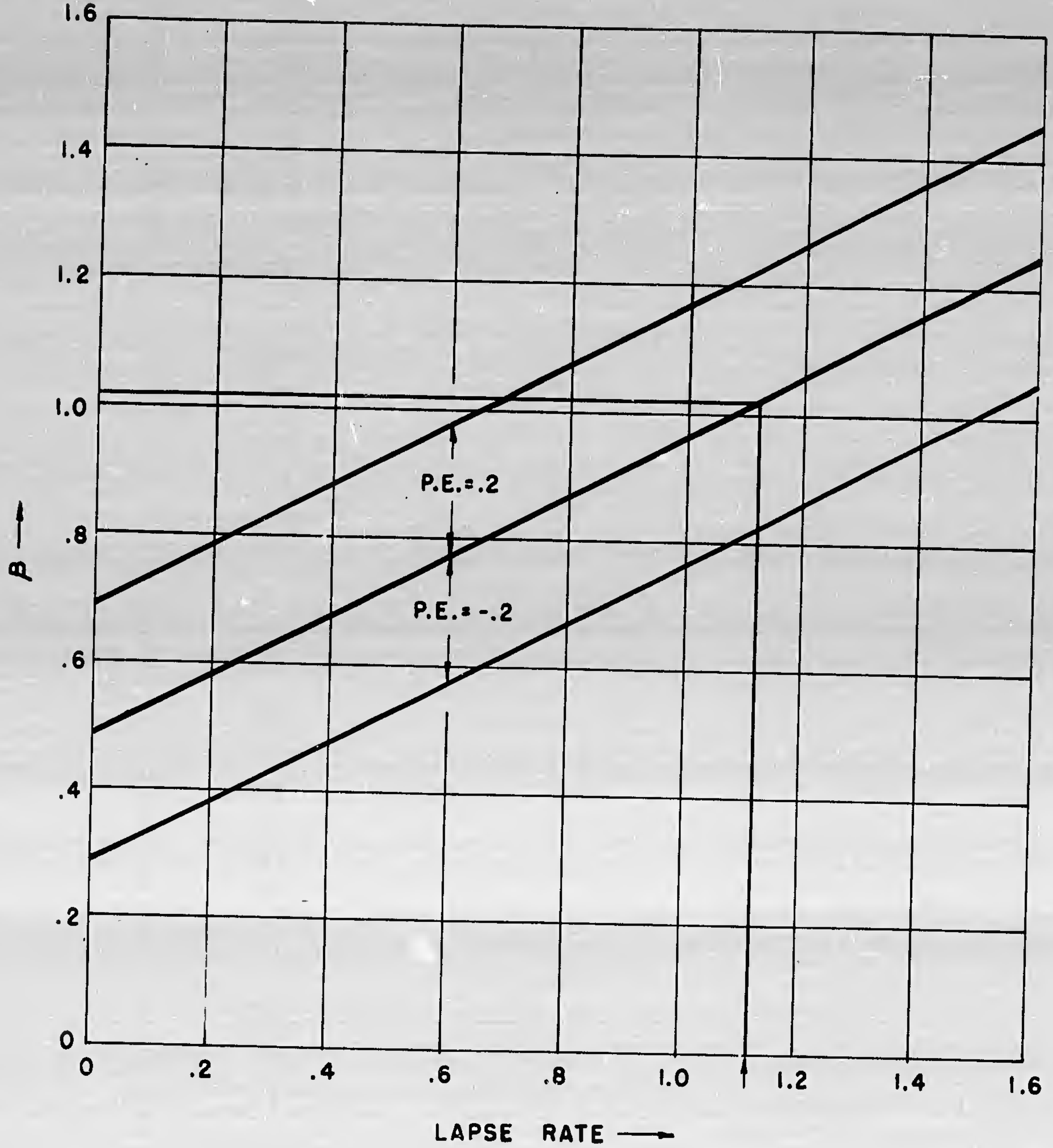

Fig. 5--Line of Regression of β on Lapse Rate Showing Probable Error of Estimate of β.

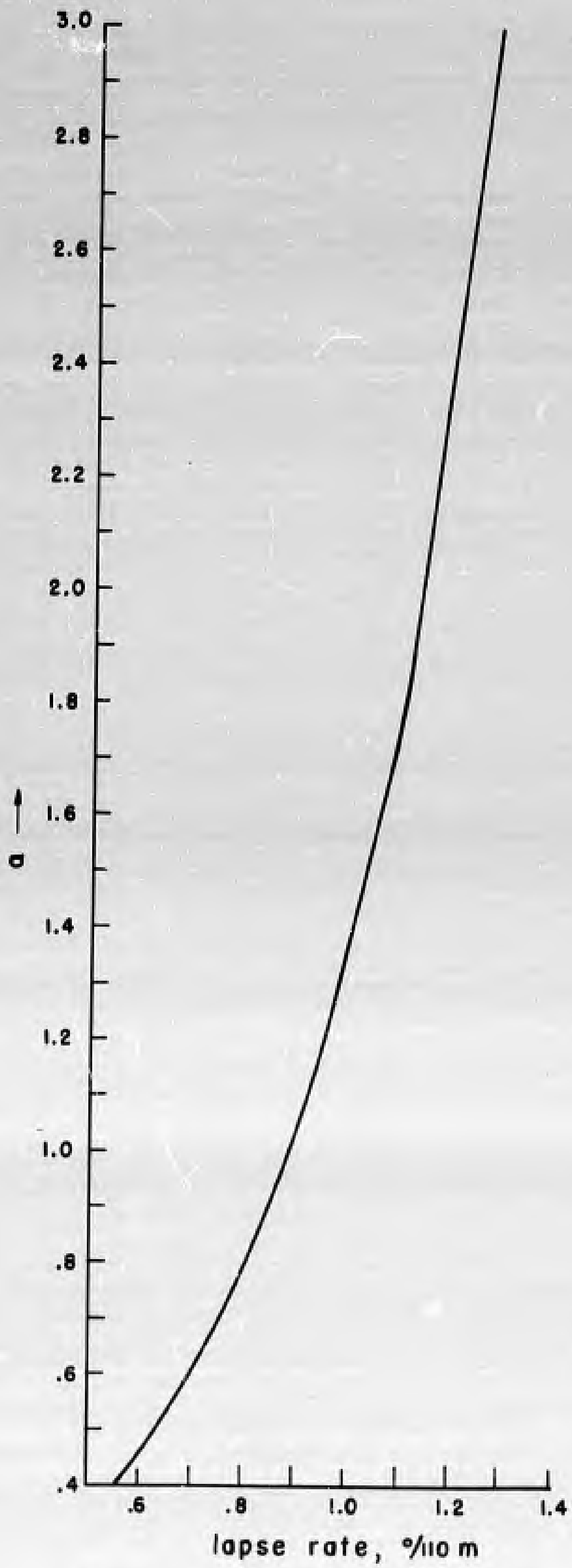

Fig. 6—Regression of "a" on Lapse Rate.

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