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Majorization-minimization procedures and multi-objective optimization

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14. ABSTRACT
This project revolved around so-called majorization-minimization methods in optimization. These types of methods are of particular importance in machine learning and statistics and applicable to a wide range of Air Force applications. In particular, Jerome established rigorous convergence results and convergence rates for a variety of non-convex problems encountered in image and phase retrieval problems. Such theoretical results guide and underpin the engineering application of these algorithms. In all, this project produced 12 journal articles, many in top journals (2 in Mathematical Programming and 2 in SIAM Optimization). Most notably, during the second year of the project, Jrme Bolte was awarded the SIAM Optimization prize with S. Sabach and M. Teboulle, for the paper Proximal Alternating Linearized Minimization for Nonconvex and Nonsmooth Problems.' Jerome will be continuing similar work with a follow-on AFOSR grant. A full list of papers and results can be found in the attached report.

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Final scientific report for grant FA9550-15-1-0500
"Majorization-Minimization methods"

September 2015 to September 2017 [1],

PI : Jérôme Bolte, TSE / Université Toulouse I Capitole.

1 Scientific activities:

The project at a glance

Budget 90% of the money in this grant was used to hire post-doctoral students. The other part helped us for research dissemination in conferences, seminars and exchanges with international researchers.

Topics The proposal revolved around Majorization-Minimization (MM) methods, problems with complex geometry and some corresponding algorithms. These questions are of first importance for today's optimization in particular for many problems arising in Machine Learning and Statistics.

Scientific outputs The project has allowed to publish 12 articles, including top research journals with in particular 3 articles in Mathematics of Operations Research, 2 Mathematical Programming, 2 SIAM Optimization.

The proposal helped participating to the following conferences (where Air Force Supported works were presented): Nesterov 60th Birthday, Les Houches, January 2016; Ben Tal 70th Birthday, Haifa, September 2016; International workshop on Mathematics and its applications, Yamaguchi University, Japan; November 2016, Workshop Optimization: Fundamentals and Algorithms for Structured Problems, Toulouse, June 2018; ISMP July 2018, Bordeaux; Variational Analysis for Ioffe's 80th birthday Erice August 2018. The participation of students to conferences and some seminars were also partially funded by our project.

Prizes During the second year of the project, Jérôme Bolte was awarded the SIAM Optimization prize with S. Sabach and M. Teboulle, for the paper "Proximal Alternating Linearized Minimization for Nonconvex and Nonsmooth Problems," see `https://bit.ly/2Tlovvm`

[1] with a one-year no cost extension 2018-19

Students

Two students were hired, 9 months each, both have started in October 1st, 2016.

1. Antoine Hochart did his PhD from École Polytechnique (Palaiseau, France).
   He is now a post-doctoral student in Santiago de Chile, Universidad Adolfo Ibanez, Chile, with Roberto Cominetti.

2. Zheng Chen did his PhD at Supélec/Paris 11 (Orsay, France).
   The year after he was a post-doctoral student in Haifa at the Technion. He is now Assistant Professor at the prestigious Zhejiang University.

An overview of our research

- The research subject of Zheng Chen concerns the complexity of MM method for problems with complex geometries in the convex world. The complexity and the convergence theory is in its infancy. We designed an algorithm called Multiprox and applied it to composite minimization. This algorithm generalizes many previous works but also offers new perspectives. For the first time we provided full complexity results for general nonlinear programming problems. The corresponding article is accepted in Mathematical Programming.
  Accelerated versions are in preparation.

- The research subject of Antoine Hochart was devoted to qualification conditions for semi-algebraic problems. We have evidenced finiteness of qualification condition failures and provided some estimates for the number of failures. Applications to MM methods and SOS techniques are provided.
  The corresponding article is published in SIAM Optimization.

- The value function approach I developed in collaboration with Pauwels brought very important results in nonlinear programming but also for imaging and phase retrieval. In particular we establish convergence and convergence rates result for a wide range of nonconvex problems.

- A new type of paradigm for tackling problems à la Bregman has given a whole new kind of MM methods (see the article with Bauschke and Teboulle). The article is published in MOR, it already has 56 citations and it was at the heart of M. Teboulle's plenary at ISMP 2018.

- Other works on SOS methods or transportation problems have been facilitated by the proposal. We identified in particular a very surprising connection between Łojasiewicz inequalities and famous functional inequalities: log-Sobolev, Gagliardo-Nirenberg, Talagrand or Poincaré inequalities. This discovery is all the more exciting that optimal transportation is now at the research front of Machine Learning.

2 Scientific productions mentioning Air Force support

Some papers[2] are not directly connected to the proposal but they share similar goals and they have benefited of the proposal synergy through discussions and scientific exchanges.

Accepted articles:

- H. Bauschke, J., Bolte, Teboulle, M., A descent Lemma beyond Lipschitz gradient continuity: first-order methods revisited and applications, to appear in Mathematics of Operations Research
- A. Beck, E. Pauwels, S. Sabach. Primal and dual predicted decrease approximation methods. Mathematical Programming (2017).
- J. Bolte, A. Blanchet, A family of functional inequalities: Łojasiewicz inequalities and displacement convex functions, to appear in Journal of Functional Analysis. *Keywords: transportation theory and Lojasiewicz inequalities*
- J. Bolte, E. Pauwels, Majorization-minimization procedures and convergence of SQP methods for semi-algebraic and tame programs, Mathematics of Operations Research 2016 (made under grant 0056 but at the origin of this new grant).
- J. Bolte, Z. Chen, E. Pauwels, The multiproximal linearization method for convex composite problems, accepted in Mathematical Programming 2019.
- J. Bolte, A. Hochart, E.Pauwels, Qualification conditions in semi-algebraic programming. SIAM J. Optim., 28(2), 1867-1891, 2018.
- J. Bolte, S. Sabach, M. Teboulle, Nonconvex Lagrangian-Based Optimization: Monitoring Schemes and Global Convergence to appear in Mathematics of Operations Research.
- J. Bolte, S. Sabach, M. Teboulle, Y. Vaisbourd, First Order Methods Beyond Convexity and Lipschitz Gradient Continuity with Applications to Quadratic Inverse Problems, SIAM J. Optim., 28(3), (2018).

  Read More: https://epubs.siam.org/doi/abs/10.1137/17M1138558
- J.B. Lasserre, E. Pauwels. Sorting out typicality with the inverse moment matrix SOS polynomial. In proceedings of the conference of Neural Information Processing Systems (2016).
- T.P. Nguyen, E.Pauwels, E. Richard, B. Suter. Extragradient Method in Optimization: Convergence and Complexity. Journal of Optimization Theory and applications (2018).
- E.Pauwels, A. Beck, Y. C. Eldar, S. Sabach. On Fienup methods for regularized phase retrieval. IEEE transactions on signal processing (2018).

[2]Mentionning the area through keywords

- E. Pauwels. The value function approach to convergence analysis in composite optimization. Operation Research Letters (2016).
  Keywords: machine learning, moments and polynomials, approximation.

- E. Pauwels, D. Henrion, J.-B. Lasserre. Positivity certificates in optimal control. In J.P. Laumond, N. Mansard, J.B. Lasserre (Editors), Geometric and Numerical Foundations of Movements, Springer Tracts in Advanced Robotics, (2017).
  Keywords: Optimal control, moments-SOS hierarchy, semidefinite programming.