Final Report
Practical implementations of parafermions and braiding of non-Abelian anyons
ONR-N00014-16-1-3158

Enrico Rossi
Physics Department, William & Mary, Williamsburg, VA 23187.

OBJECTIVES

The major goals of the project were: (i) Develop the accurate theoretical treatment of the proximity effect between superconductors, ferromagnets, and insulators in conditions that are expected to lead to the realization of non-abelian electronic states, i.e. electronic states that can be used to perform topological quantum computing; (ii) Understand how the presence of disorder, unavoidable in real systems, affects the conditions for the realization of non-abelian states in realistic set-ups, and the manipulations of such non-abelian states for quantum-computing purposes; (iii) Identify for realistic devices based on semiconductor quantum wells, graphene, and magnetic topological insulators, in proximity of superconductors and ferromagnetic insulators (or insulators with spin-orbit coupling), the conditions necessary for the realization of parafermions; (iv) Theoretically identify the conditions necessary for the realization of the braiding of non-Abelian electronic state in networks of wires taking into account the effects of disorder.

ACCOMPLISHMENTS

During the course of the project we have developed the theoretical and computational modeling of superconductor-semiconductor heterostructures [1], in particular quasi 1D structures of the type currently employed to realize non-Abelian anyons such has Majoranas, schematically shown in Fig. 1 (a), using a self-consistent Schrödinger-Poisson approach in which the semiconductor (SM) and the superconductor (SC) are treated on equal footing. This approach allowed us to obtain the following important results: (i) understand the conditions necessary to achieve a strong coupling regime between the superconductor and the semiconductor; (ii) describe the effects of external electric fields. The understanding and quantitative characterization of the effects due to the presence of electric fields is particularly significant for the progress toward the realization of SM-SC topological quantum bits. We found that an external gate voltage not only tunes the doping of the SM but it also strongly affects the hybridization of the SM and SC states, see Fig. 1 (b), and therefore the size of the superconducting gap induced, $\Delta_{\rm ind}$, into the SM, see Fig. 2 (a), and the effective g-factor of the system. The size of $\Delta_{\rm ind}$ and the effective g-factor are critical quantities for the determination of the topological phase, trivial or not, of the system. Knowing how these quantities are affected by an external gate voltage allowed us to obtain, for the first time, the topological phase diagram for a SM-SC nanowire as a function of the gate voltage, V_g, as shown in Fig. 2 (b). This is one of the most important results of this effort; previous works had obtained the topological phase diagram in terms of the magnetic field, B, and the *chemical potential* (μ), however, experimentally what is tuned is V_g, not the chemical potential. We found that the topological phase diagram in the (V_g, B) plane is quite different that the one in the (μ, B) plane.

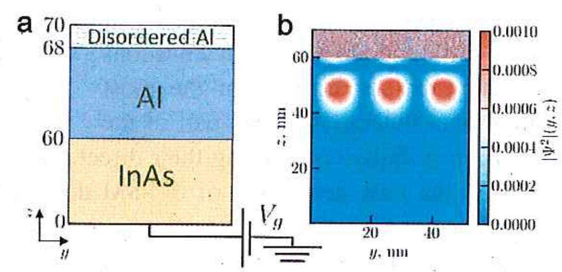

Figure 1: (a) Sketch showing the transverse cross section of a quasi 1D SM-SC heterostructure formed by superconducting Al and semiconducting InAs. V_g is the external gate voltage that can be tuned experimentally. The impurities are assumed to be concentrated in the top, 2nm thick, layer of Al. (b) Square, $|\Psi|^2$, of the wave function for one of the eigenmodes of the SC-SM nanowire taking into account the presence of disorder, for a single disorder realization.

We then proceeded to include the effects of disorder [1]. In the most recent experiments the dominant source of disorder are defects on the surface of the SC opposite to the SM-SC interface. For this reason we focused on the effect of such disorder on the phase diagram of SC-SM nanowires. We obtained the scaling of the induced gap with respect to V_g in the presence of disorder, Fig. 2 (c). The disorder, by broadening the subbands of the SC, allows for a strong coupling between the SC and the SM for a wider range of gate voltages, in very good agreement with current experiments. By changing the effective coupling between the SC and the SM, the disorder indirectly affects the system's g-factor and therefore shifts the values of the critical magnetic field necessary to drive the system into the topological phase, as shown in Fig. 2 (d). The results of Fig. 2 (d) required more than 2 millions simulations and are one of the most important outcomes of this part of the work, especially considering their direct relevance to understand current experiments on Majoranas and to design the next generation of SC-SM devices aiming to manipulate Majoranas to realize a topologically protected quantum bit.

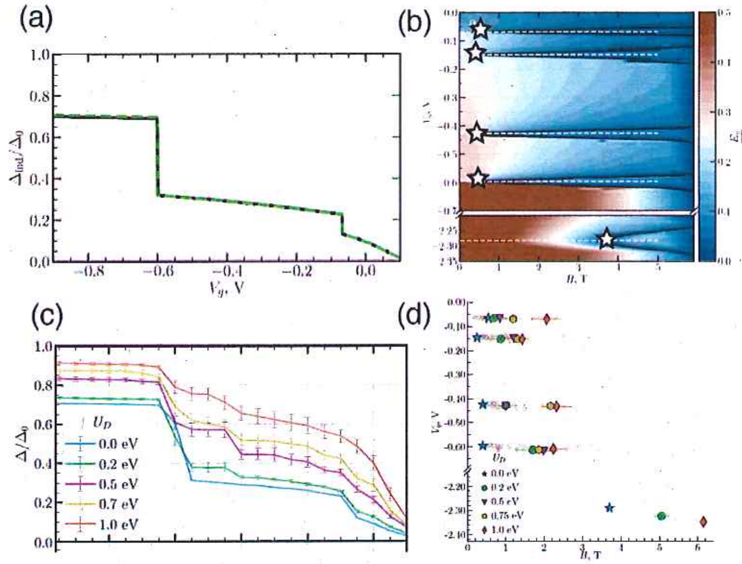

Figure 2: (a) $\Delta_{\rm ind}$ as a function of V_g for a clean SC-SM nanowire. All the results shown in this figure are obtained assuming a band offset $W = -0.25$ eV at the SC-SM interface that induces an accumulation layer in the SM. (b) Band gap at $k = 0$, E_g, as a function of B and V_g, clean limit. The points where $E_g = 0$ identify the boundaries, shown by solid black lines, between trivial and topological phase. The stars denote the lowest critical magnetic field $B_{c,\rm min}$ necessary to drive the system into the topological phase within a limited range of values of V_g. (c) $\Delta_{\rm ind}$ as a function of V_g for different values of the disorder strength U_D. (d) $B_{c,\rm min}$ for different values of the disorder strength.

In topological systems exhibiting non-abelian electronic states the presence of disorder can induce low energy states that can significantly reduce the ability to use non-abelian states to perform quantum computations. For this reason, in addition to studying how impurities can affect the topological phase diagram of quasi 1D SM-SC heterostructures, as discussed above, we investigated the effect of impurities on the low energy spectrum of such systems. We first considered the case of isolated impurities [2] focusing in particular on the situation when the impurities are located in the SC.

We found [2] that non-magnetic impurities can induce low-energy states both in the trivial and in the topological regime. Our results show that, in order to avoid the presence of low-energy impurity-induced states, it might be beneficial to be in a regime in which the SC and the SM do not hybridize too strongly [2].

We have also made significant progress in developing the theory for the effects of many impurities. As for the case of a single impurity, we found that the effect of many impurities located in the superconductor is strongly dependent on the ratio between the electrons' Fermi wave vector in the SM, $k_{F,N}$, and SC, $k_{F,SC}$, as can be seen from the qualitative differences between the results shown in the left and right panels of Fig. 3 showing the low energy spectrum of two SM-SC Majorana nanowires with different values of the ratio $k_{F,N}/k_{F,SC}$. We also find that, as in the single impurity case, the effect of the impurities becomes more important as the coupling Γ_t between SM and SC increases.

Theoretical and experimental works have shown that topological superconducting states can be realized in Josephson junctions (JJs) obtained by placing superconductors on a two-dimensional electron gas (2DEG) having spin-orbit coupling.

In this case both the phase difference between the superconductors, and a magnetic field in the plane of the 2DEG, must be present, and can be tuned to drive the system into a topological phase. Given the robustness of the topological phase in the presence of a finite phase difference between the superconductors forming the Josephson junction, this new architecture to realize Majorana modes appears to be very promising. So far Josephson junctions in which the 2DEG is created on the surface of InAs and the superconductors are made of Al, have emerged as the most promising. In the course of the project we have started to investigate these systems.

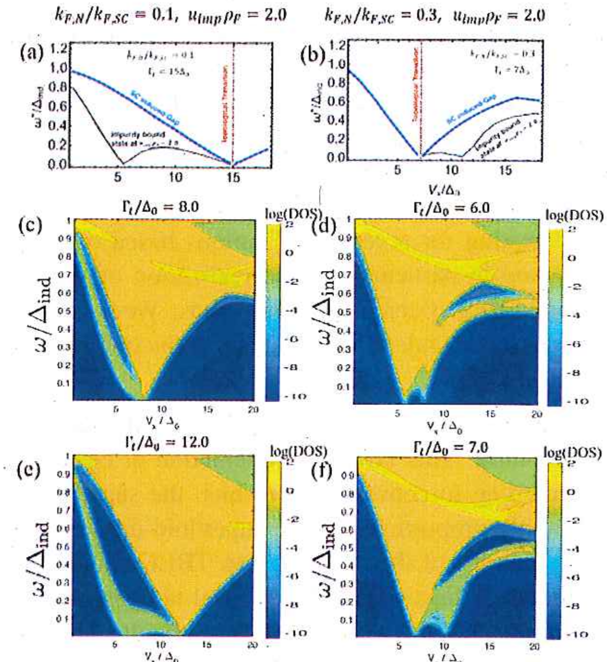

Figure 3: Spectrum of a semiconductor-superconductor Majorana nanowire in the presence of impurities in the superconductor. (a), (b) show the results for an isolated impurity. (c)-(f) Results for the case of many impurities for different strengths of the coupling, Γ_t, between the superconductor and the semiconductor.

One particular aspect of Josephson junctions in which both a magnetic field and a strong spin-orbit coupling are present in the normal region, is the possibility of the presence of an "anomalous" supercurrent, i.e. a finite supercurrent even when the phase difference between the superconductors is zero. Equivalently, such "anomalous" supercurrent can be associated to the fact that the current across the junction is zero not when the phase difference is zero but when a finite phase difference is present, i.e. for "anomalous" phase shift. The proper characterization of such anomalous phase shift and, in general, of the properties of the Al/InAs Josephson junctions is essential to elucidate the potential of these systems to realize topologically protected quantum bits. In collaboration with the experimental group of Prof. Javad Shabani (New York University), we have investigated the anomalous phase shift on Al/InAs Josephson Junctions.

One main advantage of the devices that we considered is that the strength of the spin-orbit coupling can be tuned from close to zero to as high as 150 meV·Å by simply varying the electrons' density via an external gate. We investigated how such tunability can be used to tune the properties of Josephson junctions [3].

We have shown [3] the ability to tune the anomalous phase shift, ϕ_0, via external gates that control the InAs doping. ϕ_0 is measured via a superconducting quantum interference device (SQUID), schematically shown in Fig. 4. One JJ, JJ1, is kept biased with gate voltage $V_g^1 = -4$ V, value for which the SOC is negligible, while the gate voltage V_g^2 for the other junction, JJ2, is varied. Figure 5 shows that as V_g^2 is changed from -4 V to 0 V, a phase difference, shown by the star in the figure, appears between the current phase relation of JJ1 and JJ2. This phase difference is the anomalous phase shift ϕ_0. The ability to tune ϕ_0 opens several new opportunities for superconducting spintronics, and new possibilities for the realization and characterization of topological superconductivity and Majoranas in Josephson junctions.

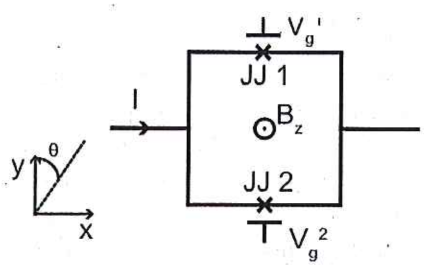

Figure 4: Schematic of the SQUID geometry used to measure the anomalous phase shift due to the interplay of spin-orbit coupling and magnetic field in InAs/Al Josephson junctions.

In the last year and a half twisted bilayer graphene (TBLG) has emerged as one of the most interesting and promising systems in condensed matter physics. For specific, "magic" values, of the twist angle between the layers, TBLG exhibits strongly correlated ground states and, in particular, a superconducting state with an extremely large ratio between the critical temperature and the Fermi temperature. This makes the system extremely interesting for several applications based on superconductivity and in particular for the realization of topological superconducting systems with Majoranas. We have investigated the mechanism underlying the superconducting behavior of magic-angle TBLG.

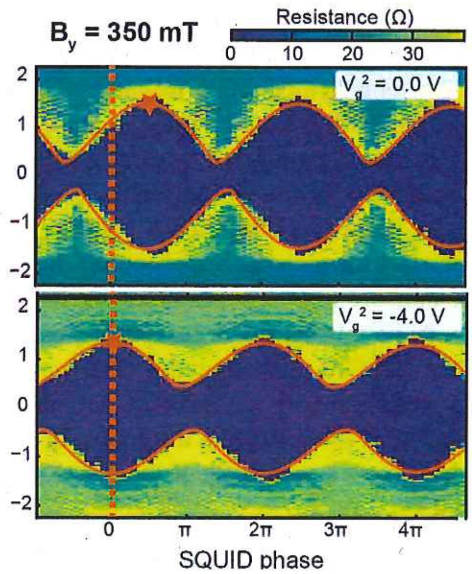

Figure 5: Results for InAs/Al Josephson junctions. Profile of the critical current as a function of the SQUID phase for two different values of the gate voltage V_g^2 for Junction JJ2 (see previous figure). We see that as V_g^2 increases an anomalous phase shift appears (top figure).

One of the main features of TBLG is the extreme flatness of its bands. This favors the interaction of electrons. However, according to conventional results, the superfluid weight, D_s, directly proportional to the superfluid density, decreases with the flatness of the bands and so TBLG should have a vanishing small superconducting critical temperature. Despite this, magic-angle TBLG superconducts at finite (and relatively high) temperatures. Motivated by this apparent contradiction we have investigated how in magic-angle TBLG the superfluid weight can be significantly different from zero despite the extreme flatness of the bands. We have found [4] that in TBLG there are two contribution to D_s: the "conventional" one that vanishes as the bands become flat, and a "geometric" one due to the quantum geometry of the occupied bands that can be zero even in perfectly flat bands. We found that at the magic angle the geometric contribution dominates. Fig. 6 (a) shows how D_s depends on the twist angle and shows that at the magic angle, $\theta = 1.05°$, the geometric contribution to D_s is larger than the conventional one.

Having obtained the temperature and twist angle dependence of D_s, we were able to calculate the Berezinskii-Kosterlitz-Thouless transition temperature, T_{KT}, as a function of different experimentally tunable parameters. Fig. 6 (b) shows T_{KT} as a function of θ, from which we can see that at the magic angle T_{KT} is visibly smaller than the critical temperature, T_c, above which the superconducting order parameter vanishes.

We have also studied twisted bilayer systems in which one layer is graphene and the other is a superconducting monolayer of $NbSe_2$ [5]. We have built a continuous low-energy effective model that takes into account the presence of a twist angle between graphene and $NbSe_2$, and of spin-orbit coupling and superconducting pairing in $NbSe_2$. We obtained the parameters entering the continuous model via *ab initio* calculations. Our results show that despite the large mismatch between the graphene's and $NbSe_2$'s lattice constants, due to the large size of the $NbSe_2$'s Fermi pockets, there is a large range of values of twist angles for which a superconducting pairing can be induced into the graphene layer. The most intriguing result is that the size of the superconducting gap induced by proximity into the graphene layer can be be significantly tuned by varying the twist angle, as shown in Fig. 7.

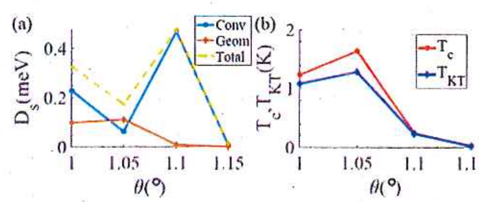

Figure 6: (a) Superfluid weight, D_s, and (b) T_c and T_{KT} as functions of twist angle for twisted bilayer graphene.

Graphene nanoribbons (GNRs) offer many potential advantages for the realization of scalable Majorana devices: (i) they are almost ideal 1D systems with very few subbands; (ii) they can now be synthesized with atomic control of their edges via bottom-up approaches; (iii) their electronic properties can be tuned via edge and width engineering.

During the course of the project, we have investigated how the electronic properties of GNRs are affected and can be tuned by the proximity of the materials that one would use to realize a GNR-based Majorana device.

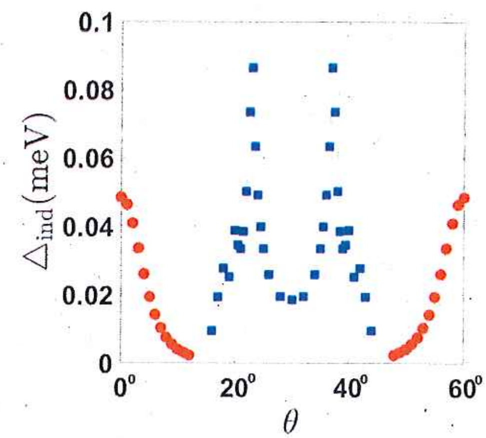

Figure 7: Results for twisted graphene/$NbSe_2$ bilayer. Dependence on the twist angle θ of the superconducting gap, $\Delta_{\rm ind}$, induced into the graphene layer by proximity.

Given that any high quality GNR-based device would most likely involve the use of high quality hexagonal boron nitride (hBN) as dielectric, our first study in this direction focused on how the presence of hBN would affect the electronic structure of GNRs [6]. There are two types of GNRs: armchair GNRs (AGNRs) in which the edges can be seen as a sequence of atomic scale armchairs, and zigzag GNRs (ZGNRs) in which the edges have a zigzag profile. ZGNRs are expected to have edge modes in which the electrons' spins align ferromagnetically (FM) along the edge and antiferromagnetically (AFM) between edges. For pristine ZGNRs the edge modes are spin degenerate: the mode with spin up on the left edge has the same energy as the one with spin down on the right side, see Fig. We found that by placing a ZGNR on hBN this degeneracy can be broken, as shown in Fig. 8 (b). This is due to the fact that the edges of ZGNRs placed on hBN can see different electrostatic potentials depending if they are close to the boron or nitrogen atoms, as shown at the bottom of Fig. 8 (a). The possibility to break the spin degeneracy in GNRs without having to introduce a magnetic field to break time reversal symmetry is very interesting because the breaking of spin degeneracy is necessary to realize Majorana states and, on the other hand, the introduction of magnetic fields is always deleterious for superconducting pairing, another necessary ingredient for the realization of Majoranas. We also find the very interesting result that the size of the spin splitting, for ZGNRs placed on hBN, can be controlled in sign and magnitude via a shift ($\delta_{||}$, $\delta_{\perp}$) of the position of the ZGNRs with respect to the hBN substrate along the ZGNR's longitudinal or transverse direction as shown in Fig. 8 (c), (d).

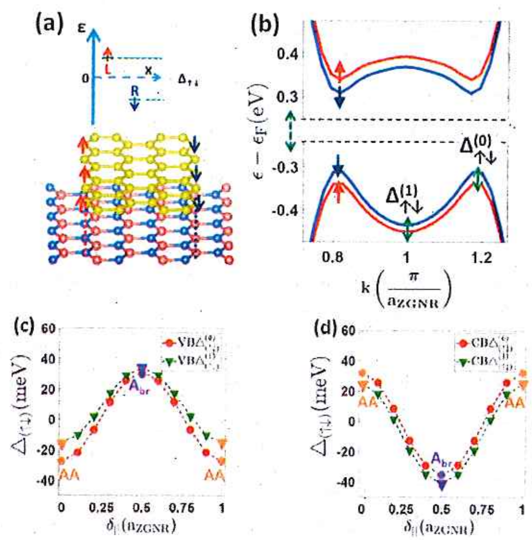

Figure 8: Results for GNR on hBN. (a) Schematic of the way in which hBN can induce the breaking of the spin degeneracy of the edge states of a ZGNR. (b) Low energy band structure of a ZGNR placed on hBN showing the spin splitting $\Delta_{\uparrow\downarrow}$ of the GNR's edge states induced by hBN. (c), (d) Spin splitting for a ZGNR on hBN as a function of parallel and perpendicular shift, respectively.

Another important ingredient for the formation of Majoranas is spin-orbit coupling (SOC). Notoriously graphene has negligible SOC. If one wants to use GNRs to realize Majoranas a way should be found to enhance their SOC. We have investigated the possibility to induce a significant SOC in GNRs by placing them in proximity of transition metal dichalcogenides (TMDs) that have a very strong SOC. In a TMD the spin splitting induced by SOC has opposite sign at the K and K' points of the Brillouin zone (BZ). By placing a GNR on a TMD the system can only preserve the periodicity along the longitudinal direction of the GNR and therefore the two dimensional BZ of the

TMD gets folded to a 1D BZ. The K and K' points of the TMD's BZ are folded on the line that denotes the direction of the GNR. Depending on the relative orientation, shown by the dashed green lines in Fig. 9 (a), (b), we have that the TMD's bands at the K and K's can either fold back to the same point, Fig. 9 (a), or to different points of the 1D BZ of the GNR-TMD system, Fig. 9 (b). In the first case, that in our notation corresponds to a 0° twist angle, we expect that the SOC effect on the GNR bands induced by the TMD will be very small due to the fact that the TMD's spin splitting is opposite at the K and K' point.

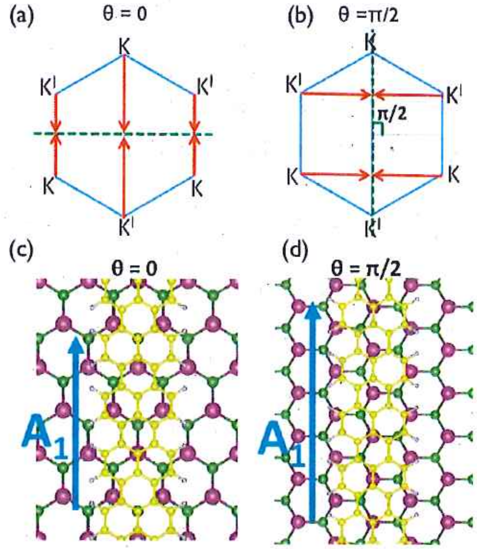

Figure 9: (a), (b) schematic showing the effect of the twist angle θ on the folding of the TMD's BZ to the 1D BZ of the GNR/TMD system. (c), (d) Primitive cells for the $\theta = 0°$, $\theta = 90°$, AGNR-$NbSe_2$ heterostructures considered, respectively

We can expect that the SOC induced into the GNRs by the TMD will be maximum when the 1D BZ of the GNR-TMD heterostructure is along the direction shown in Fig. 9 (b), i.e., $\theta = 90°$ in our notation. Using first-principles we have obtained the electronic structure of GNR-TMD heterostructures [7]. We have considered both the case when the TMD is a semiconductor, such as $MoSe_2$, or a metal, such as $NbSe_2$. Figures 9 (c), (d) show the primitive cells for the $\theta = 0°$, $\theta = 90°$, AGNR-$NbSe_2$ heterostructures considered, respectively. Figures 10 (a), (b) show the low-energy bands for a AGNR-$NbSe_2$ heterostructure for the case when $\theta = 0$ and 90°, respectively; the dots highlights the states that are localized on the ribbon. Figures 10 (c), (d) show the corresponding spin splitting. We see that the spin-splitting in Fig. 10 (c), corresponding to $\theta = 90°$, is almost an order of magnitude larger than for $\theta = 0$, shown in Fig. 10 (d). This confirms our expectations, shows that the proximity of TMDs can strongly enhance the SOC of GNRs, and that such enhancement can be tuned by changing the twist angle θ.

We have made a significant progress to characterize in realistic conditions the electronic states of heterostructures formed by a 2D systems in the quantum Hall (QH) regime and a superconductor. We have developed simplified analytical models that include the effects of spin splitting (due to the Zeeman term) and valley splitting and how the details of the interface between the QH and the SC affects the hybridization of the QH edge states with the SC's states. Figure 11 shows the profile in real space of the square of the lowest energy wave function, $\psi(x, y)$, in a QH-SC system for two slightly different values of the chemical potential, μ, in the QH system. We see that small differences in the value of μ strongly affect the profile of the wave function close to the interface between the QH system and the SC, shown by the dashed blue line.

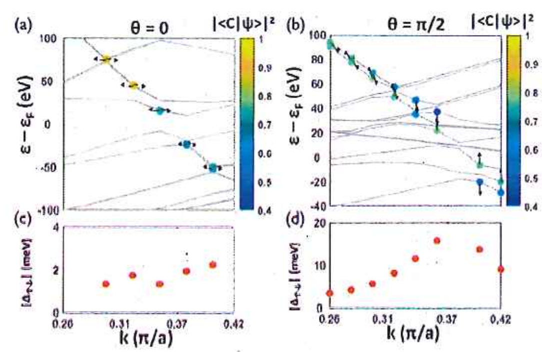

Figure 10: (a), (b) Low energy bands of AGNR-$NbSe_2$ heterostructure corresponding to $\theta = 0$, $\theta = 90°$, respectively. (c), (d) Spin splitting due to the spin-orbit coupling induced by proximity in to the armchair GNR for $\theta = 0$, $\theta = 90°$, respectively.

The details of the profile of ψ at the interface, in turn, strongly affect the transport properties and strength of the pairing when two counter-propagating QH edge modes are present. Our results show that to achieve non-negligible pairing, necessary to form Majoranas and parafermions in these systems, it is necessary to carefully tune the electrostatic potential at the interface between QH and SC.

We have obtained the transport properties of graphene, and bilayer graphene, when placed on top of a topological insulator [8]. We have found that these systems ideally combine the high mobility of graphene and the strong spin-orbit coupling of topological insulators (TIs) giving rise to strong spin-dependent transport properties. The results presented in Fig. 12 (b) and (c) show that in graphene-TI heterostructure we can have an extremely large current-driven spin accumulation, quantified by the quantity χ^{s_x,J_y}, much larger than the (already very larger) one of an isolated TI.

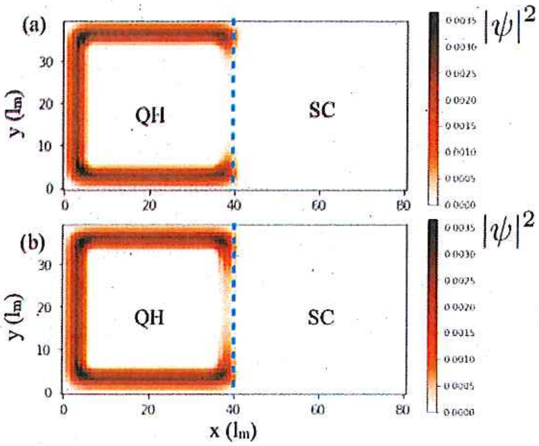

Figure 11: Results for QH-SC systems. Profile in real space of the local density of states for two slightly different values of the chemical potential in the QH system. The blue dashed line shows the boundary between QH and SC.

We have also shown that a strong coupling between spin and charge transport in systems like graphene-TI heterostructures is present even when the tunneling between the two systems is predominantly random [9], a situation that is relevant for many devices fabricated for technological applications.

Higher order topological superconductors (HOTS) are a very novel and very interesting class of systems in which non-Abelian fermionic states can be realized. In particular, 2D HOTSC might allow for novel and simpler ways to braid non-Abelian states. We have studied the superconducting instabilities of higher order topological quadrupolar semimetals [10]. We focused on superconducting states obtained through s-wave superconducting pairing, and identified higher-order topological superconductors that may manifest surface and/or hinge states, as shown schematically in Fig. 13. We found that some of the higher-order topological superconductors that we obtained are gapped in the bulk, but have 2D nodal Dirac superconductors on their surfaces, and flat bands of quasiparticle states on their hinges. We termed such superconductors *second-order Dirac superconductors*, Fig. 13 (a). We also obtained higher-order topological superconductors with gapped surfaces and non-chiral hinge states. We found that the hinge states are only present when the surfaces are gapped by a Zeeman term H_z, Fig. 13 (b), are localized at only two of the four hinges, and that the pair of corners that exhibit the 1D non-chiral states is controlled by the sign of the Zeeman term.

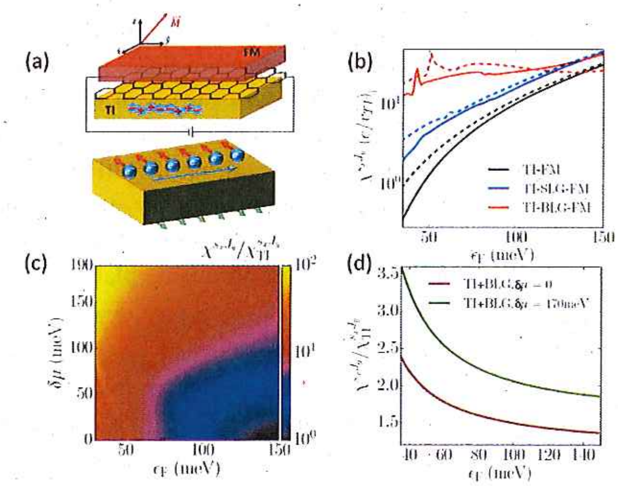

Figure 12: (a) Sketch showing a graphene-TI-FM heterostructure and the spin density accumulation on the top and bottom surface of a topological insulator (TI) induced by a current in the y direction. (b) Spin-current response $\chi^{s_x J_y}$ as a function of the Fermi energy ϵ_F when the exchange field $\Delta = 20$ meV ($\Delta = 0$), solid (dashed) lines, for different heterostructures comprising topological insulator (TI), Ferromagnet (FM), single layer graphene (SLG) and bilayer graphene (BLG). (c) Enhancement of the spin-current response $\chi^{s_x J_y}$ in a TI-BLG system compared to TI alone as a function of the Fermi energy ϵ_F and difference, $\delta\mu$, between the chemical potential of the TI and BLG. (d) $\chi^{s_x J_y}$ for TI-BLG in the limit of no tunneling between TI and BLG.

We have then considered the possibility to realize Majorana states at the end of vortex lines in superconducting higher order topological insulators [11]. We have shown that differently from the case of a "regular" superconducting TI, in a superconducting higher order TI (HOTI), the properties of the Majorana in the vortex can depend strongly on the position of the vortex with respect to the edges of the system. In addition, we have shown that vortices placed in a superconducting higher order TI, exhibit a pair of phase transitions as a function of doping, as shown in Fig. 14 (b). The first transition is a surface phase transition after which the Majoranas appear. The second transition is a topological transition of the vortex. We found that the surface transition appears because of the competition between the superconducting gap and the local time-reversal breaking gap on the surface.

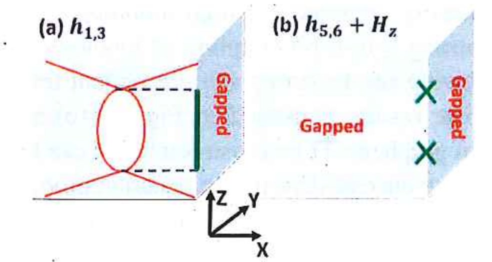

Figure 13: (a) Schematic of the band structure on the surface and at the hinges of a second-order Dirac superconductor. (b) Schematic of a higher-order topological superconductors with gapped surfaces and non-chiral hinge states induced by a Zeeman term H_z.

We have also completed an invited topical review [12] on "Van der Waals heterostructures with spin-orbit coupling" that will be published in the "Annalen der Physik" in which we discuss recent work on van der Waals systems in which at least one layer has strong spin-orbit coupling. Our work for the project has shown that these systems have extremely interesting properties and can be engineered to realize non-Abelian fermionic states.

IMPACT

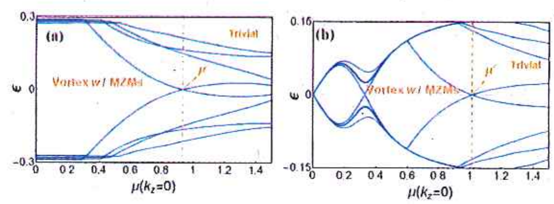

Figure 14: Spectrum of a vortex at $k = 0$ as a function of μ for (a) a superconducting TI (b) a superconducting HOTI.

Our work on the effect of electric fields on the topological phase diagram of Al/InAs Majorana nanowires is a very significant contribution toward the understanding of the properties of these systems and the improvement of their design, improvement that is needed to use them to realize topologically protected quantum bits. The impact of this work is recognized by the fact that in slightly more than a year the work has collected already more than 40 citations. Similarly, the work on the effect of impurities on the spectrum of semiconductor-superconductor heterostructure is essential to correctly detect and manipulate non-Abelian states in realistic conditions for what has so far been the most promising platform.

The work on the anomalous phase-shift of Al/InAs Josephson junctions is a significant advance in the field of Josephson junctions for two reasons: (i) it shows that in Al/InAs Josephson junctions it is possible to have a very large anomalous supercurrent; (ii) it shows that such supercurrent can be tuned simply via an external gate. Both facts are extremely significant for the prospects of "superconducting spintronics" and for the use of Josephson junctions to realize non-Abelian states and topologically protected quantum bits.

Our work on graphene nanoribbons has been the first (as far as we know) to study how the electronic structure of the ribbons can be engineered to have large spin-splittings by integrating them in heterostructures comprising transition monolayer dichalcogenides and hBN. Large, spin-orbit-coupling-induced, spin splittings are essential for the prospects of using graphene nanoribbons to realize non-Abelian states and possibly topologically protected quantum bits.

Our work on twisted bilayer graphene provides a significant advance toward the understanding of the superconducting states of this system. In the past few years the importance of geometric phases of electronic materials has become one of the central themes in condensed matter physics. For the past 15 years almost all the effects discussed of such phases were related to transport properties, in particular unusual transport properties arising from the presence of topologically protected edge states of an otherwise insulating bulk. Our work on twisted bilayer graphene is one of the few works in which the connection between the quantum geometry of the electronic states and other observables such as the Berezinskii-Kosterlitz-Thouless transition temperature is pointed out. In addition, our work does this quantitatively for a system for which several experiments are available and therefore significant progress can be made by comparing theoretical and experimental results to understand the relation between band geometry and quantities such as superfluid weight and the Berezinskii-Kosterlitz-Thouless transition temperature.

The work on graphene-$NbSe_2$ shows that this is a promising system in which the superconducting properties can be tuned and optimized. In combination with our work on graphene nanoribbons heterostructures this work offer a path toward the realization o quasi 1D systems with ideal properties to realize non-Abelian states and topologically protected quantum bits.

Our work on the transport properties of van der Waals heterostructures in which at least one of the layers has strong spin-orbit coupling is an important step to understand the potential of these systems for technological applications.

Our work on higher order topological superconductors is one of the first and necessary steps to understand if and how higher order topological materials are promising systems to realize non-Abelian states in a controlled way.

TRAINING

The members of the group working on the project had the opportunity to develop their technical skills and improve their knowledge of condensed matter physics working closely with the PI and an international group of collaborators. The PI has an open-door policy and conducted daily meetings with the members of his group. In addition, the members of the group had the opportunity to present the results of their work at APS March meetings, at the Princeton Summer school, and at the Summer school "Advances in strongly correlated electronic systems (ASCES2019)" at the University of Minnesota. During the course of the project one student earned his Ph.D. in physics, and one student earned the B.S. in physics completing a senior thesis with the PI and earned the *Thomas Jefferson Prize in Natural Philosophy*, the highest, university-wide, award for undergraduate students recognizing excellence in the sciences.

DISSEMINATION

In addition to the publications listed at the end of this report, the results of the projects were presented by the PI via several invited talks, and by the PI and the members of his group via several contributed talks at the APS March meetings. The PI posts all his group's publications to the arXiV and makes them also publicly available through his website.

PUBLICATIONS

[1] Andrey E. Antipov, Arno Bargerbos, Georg W. Winkler, Bela Bauer, Enrico Rossi, and Roman M. Lutchyn, "Effects of Gate-Induced Electric Fields on Semiconductor Majorana Nanowires," Physical Review X **8**, 31041 (2018), arXiv:1801.02616.

[2] Dong E. Liu, Enrico Rossi, and Roman M. Lutchyn, "Impurity-induced states in superconducting heterostructures," Phys. Rev. B **97**, 161408 (2018), arXiv:1711.04056.

[3] William Mayer, Matthieu C. Dartiailh, Joseph Yuan, Kaushini S. Wickramasinghe, Enrico Rossi, and Javad Shabani, "Gate Controlled Anomalous Phase Shift in Al/InAs Josephson Junctions," accepted for publication in Nature Communications, (2019), arXiv:1905.12670.

[4] Xiang Hu, Timo Hyart, Dmitry I. Pikulin, and Enrico Rossi, "Geometric and conventional contribution to superfluid weight in twisted bilayer graphene," accepted for publication in Physical Review Letters, (2019), arXiv:1906.07152.

[5] Yohanes S. Gani, Hadar Steinberg, and Enrico Rossi, "Superconductivity in twisted graphene NbSe2 heterostruc-

tures," Physical Review B **99**, 235404 (2019), arXiv:1903.00475.

[6] Yohanes S. Gani, D. S.L. Abergel, and Enrico Rossi, "Electronic structure of graphene nanoribbons on hexagonal boron nitride," Physical Review B **98**, 205415 (2018), arXiv:1808.10039.

[7] Yohanes S. Gani, Eric J. Walter, and Enrico Rossi, "Proximity induced spin-orbit splitting in graphene nanoribbons on transition metal dichalcogenides," (2019), to be sumbitted to Phys. Rev. B, arXiv:1911.08501.

[8] M. Rodriguez-Vega, G. Schwiete, J. Sinova, and E. Rossi, "Giant edelstein effect in topological-insulator–graphene heterostructures," Phys. Rev. B **96**, 235419 (2017), arXiv:1610.04229.

[9] M. Rodriguez-Vega, G Schwiete, and Enrico Rossi, "Spin-charge coupled transport in van der Waals systems with random tunneling," Physical Review Research **1**, 033085 (2019), arXiv:1904.01015.

[10] Sayed Ali Akbar Ghorashi, Xiang Hu, Taylor L. Hughes, and Enrico Rossi, "Second-order Dirac superconductors and magnetic field induced Majorana hinge modes," Physical Review B, Rapid Communitation, Editor's suggestion, **100**, 020509 (2019), arXiv:1901.07579.

[11] Sayed Ali Akbar Ghorashi, Taylor L. Hughes, and Enrico Rossi, "Vortex and Surface Phase Transitions in Superconducting Higher-order Topological Insulators," (2019), under review at Phys. Rev. Lett., arXiv:1909.10536.

[12] Enrico Rossi and Christopher Triola, "Van der Waals heterostructures with spin-orbit coupling," (2019), in print at Annalen der Physik, arXiv:1909.11674.

REPORT DOCUMENTATION PAGE				*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*

2. REPORT TYPE
Final Technical

3. DATES COVERED *(From - To)*
01-09-2016 - 30-08-2019

4. TITLE AND SUBTITLE
Practical implementations of parafermions and braiding of non-Abelian anyons

5a. CONTRACT NUMBER

5b. GRANT NUMBER
N00014-16-1-3158

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Enrico Rossi, PI

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
College of William and Mary
P.O. Box 8795
Williamsburg, VA 23187-8795

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
875 N. Randolph Street, Suite 1425
Arlington, VA 22203-1995

10. SPONSOR/MONITOR'S ACRONYM(S)
ONR

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
In this project theoretical methods and numerical codes were developed to identify realistic systems and conditions in which non-Abelian fermionic states can be realized. The effects of electric fields and imputirites on the topological phase diagram of semiconductor-superconductor nanowires were studied. Several novel material platforms for the realization of superconducting states supporting non-Abelian states were also considered, including heterostructures formed by graphene nanoribbons and transition metal dichalcogenide layers, and higher order topological superconductors.

15. SUBJECT TERMS
Superconductor, semiconductor, heterorstructure, Majorana, graphene, nanoribbon, topological insulator.

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
					Elizabeth A. Montalvo
					19b. TELEPHONE NUMBER *(Include area code)*
U	U	U	UU		757-221-3901

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39.18