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Butterfly Enumeration in Bipartite Graphs using GraphBLAS

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DM20-0037

GraphBLAS References

Mathematical Foundations of the GraphBLAS

IEEE HPEC 2016

Jeremy Kepner (MIT Lincoln Laboratory Supercomputing Center), Peter Aaltonen (Indiana University), David Bader (Georgia Institute of Technology), Aydın Buluç (Lawrence Berkeley National Laboratory), Franz Franchetti (Carnegie Mellon University), John Gilbert (University of California, Santa Barbara), Dylan Hutchison (University of Washington), Manoj Kumar (IBM), Andrew Lumsdaine (Indiana University), Henning Meyerhenke (Karlsruhe Institute of Technology), Scott McMillan (CMU Software Engineering Institute), Jose Moreira (IBM), John D. Owens (University of California, Davis), Carl Yang (University of California, Davis), Marcin Zalewski (Indiana University), Timothy Mattson (Intel)

Design of the GraphBLAS API for C

IEEE HPEC 2017

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GraphBLAS Ecosystem

GraphBLAS Forum: https://graphblas.org

gbtl
C++ Algorithms Repository
Carnegie Mellon University

Carnegie Mellon University GraphBLAS Test Framework

pggraphblas
PostgreSQL Wrapper

pygraphblas
Python Wrapper

LAGraph
C algorithms repository

redisgraph
redislabs

pyGB
Python Wrapper around gbtl
Carnegie Mellon University

GraphBLAS C API, v. 1.3.0
GraphBLAS C++ API (in progress)

Multithreaded
SuiteSparse
ATM

IBM-GraphBLAS

Galois GB
TEXAS
The University of Texas at Austin

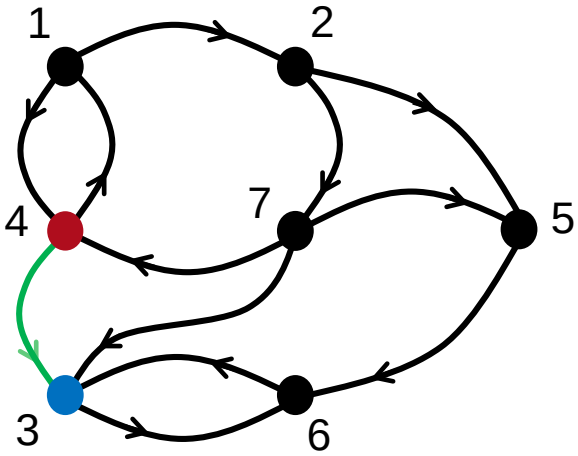
Distributed gbtl
Lawrence Livermore National Laboratory

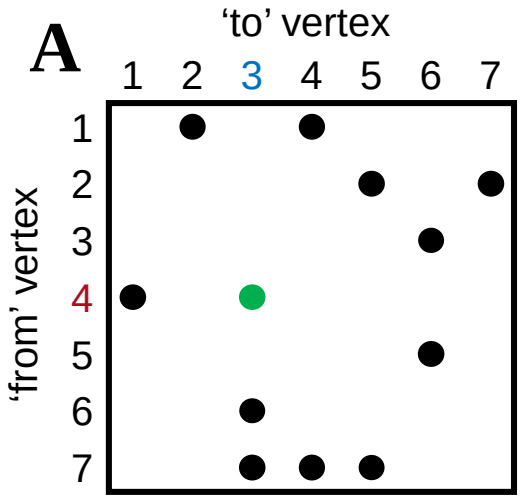
gbtl
Carnegie Mellon University
Pacific Northwest NATIONAL LABORATORY
Proudly Operated by Battelle Since 1965

GraphBLAST (GPU)
UCDAVIS
UNIVERSITY OF CALIFORNIA

GraphBLAS on EMU
Carnegie Mellon University
UMBC

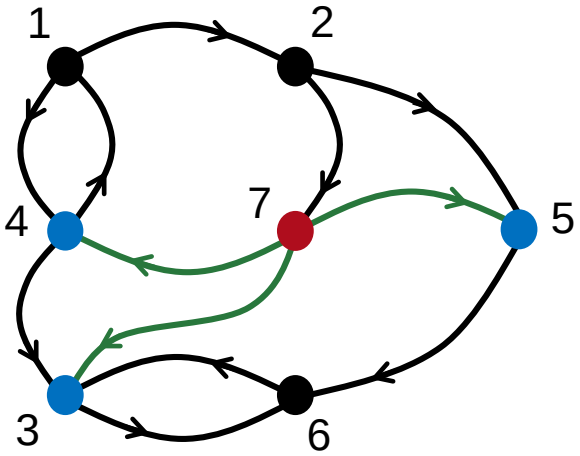
Graphs as Matrices

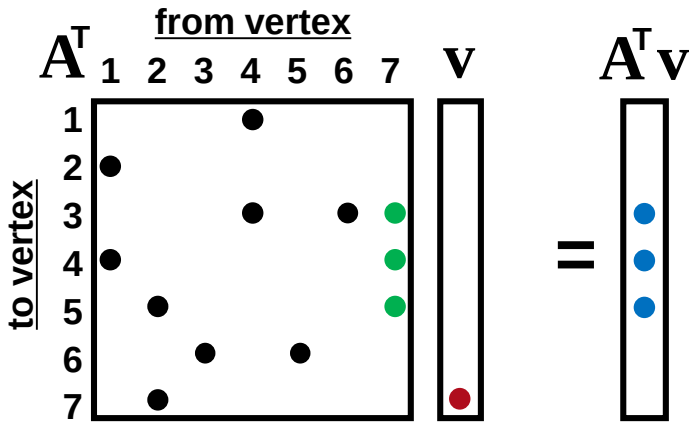




Graphs are represented as adjacency matrices that have *irregular* and *sparse* structure.

Graph Operations as Matrix Operations





- Matrix multiply → find neighbors (most important primitive)
- Used in breadth-first traversal, shortest paths, and many others
- Sparsity and irregularity of matrix structure is a barrier to high performance

K-truss Enumeration

Find all edges in graph (**A**) that are part of at least k-2 (the support) unique triangles.

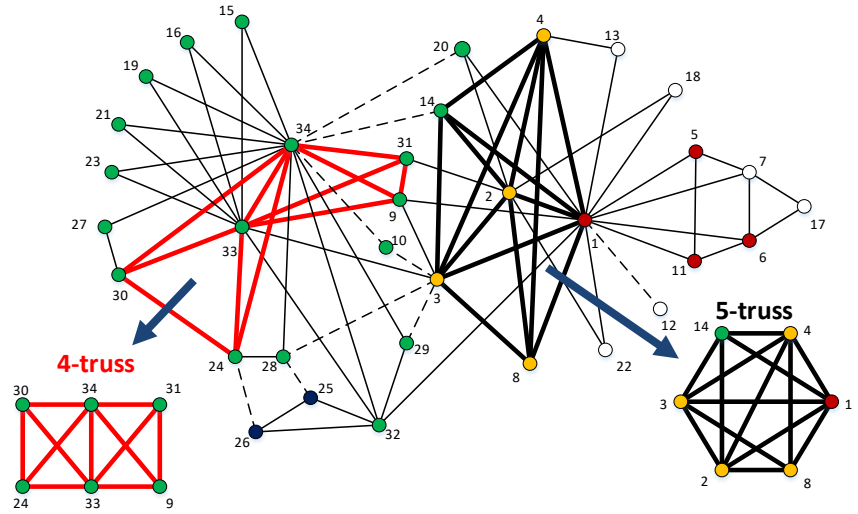
converged ← *false*
while !*converged* do**:**

S ⟨**A**⟩ = (**A**^{T} ⊕.⊗ **A**) // 1: support

M = **S** .≥ (k – 2) // 2: prune

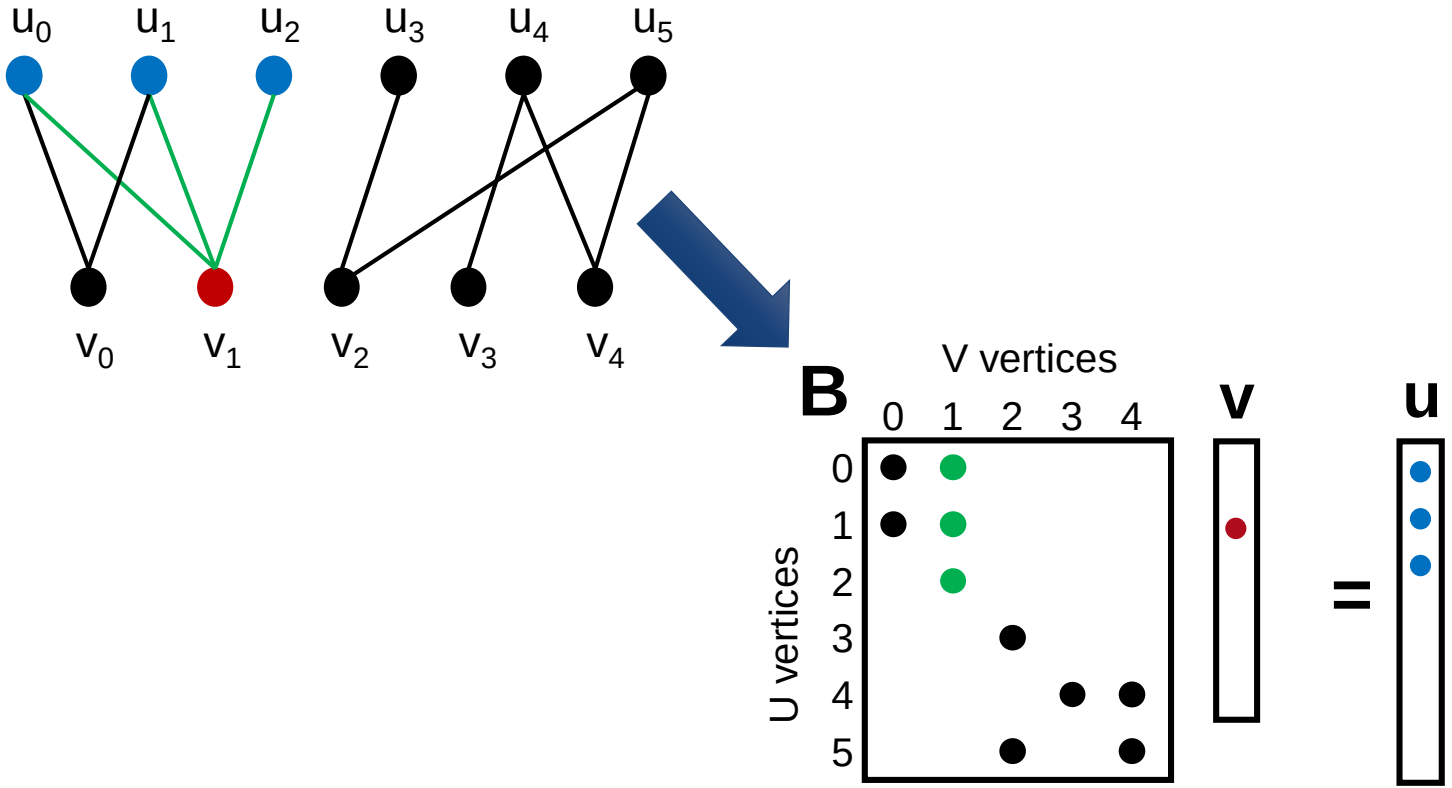
A ⟨**M**⟩ = **M** ∘ **A**

converged ← isUnchanged(**M**)




```
Matrix k_truss(Matrix &A, int k)
{
  Matrix S(A.nrows(), A.ncols());
  mxm(S, A, NoAccum(), ArithmeticSemiring<int>(),
      transpose(A), A);
  apply(M, NoMask(), NoAccum(), GreaterThan<int>(k-2), S);
  apply(A, NoMask(), NoAccum(), Identity<int>(), A);
  return A;
}
```

Bipartite Graphs as Matrices



Butterfly Enumeration

Find all edges in a bipartite graph (**B**) that are part of at least *f*(k-2) unique butterflies.

Given: **B** (|*U*| x |*V*| binary sparse matrix), and threshold k.

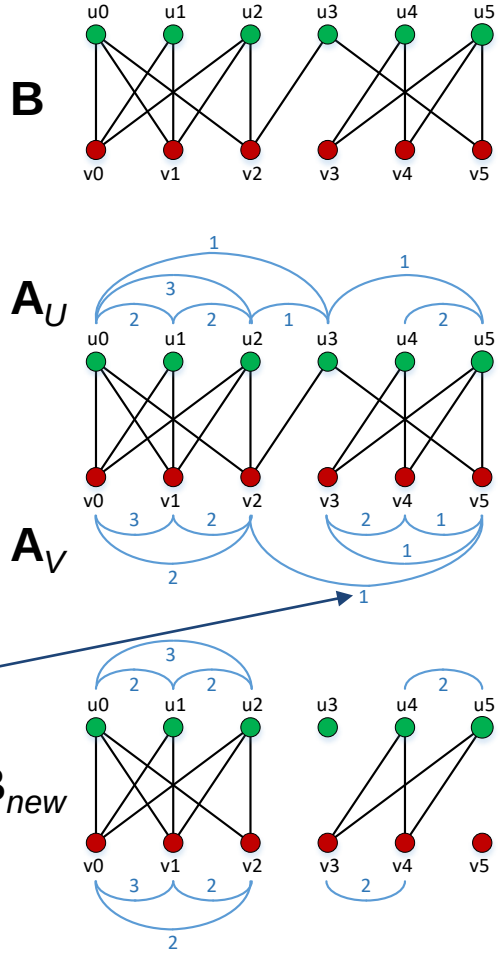
Step 1: Induce edges within U and V to create triangles
- Create edges in $\mathbf{A}_U$ b/w vertices w/in *U* that share a common neighbor in *V*.
- Create edges in $\mathbf{A}_V$ b/w vertices w/in *V* that share a common neighbor in *U*.

Step 2: Compute the triangle support for edges in $\mathbf{A}_U$ and $\mathbf{A}_V$
- Hint: support equals the number common neighbors

Step 3: Remove the edges in $\mathbf{A}_U$ and $\mathbf{A}_V$ that do not have “enough support”

Step 4: Compute the new Bipartite Graph ($\mathbf{B}_{new}$) only consisting of edges that are part of triangles

Repeat the process with $\mathbf{B}_{new}$ larger k until no edges are left.



Butterfly Enumeration

Given: **B** ($|U|$ x $|V|$ binary sparse matrix), and threshold k.

Step 1: Induce edges to create triangles
- $\mathbf{A}_U \langle \neg \mathbf{I}_{|U|} \rangle = \mathbf{B} \oplus.\otimes \mathbf{B}^T$ (boolean semiring)
- $\mathbf{A}_V \langle \neg \mathbf{I}_{|V|} \rangle = \mathbf{B}^T \oplus.\otimes \mathbf{B}$

Step 2: Compute the triangle support for edges in $\mathbf{A}_U$ and $\mathbf{A}_V$: $\mathbf{S}_U$ and $\mathbf{S}_V$
- $\mathbf{S}_U \langle \mathbf{A}_U \rangle = \mathbf{B} \oplus.\otimes \mathbf{B}^T$ (arithmetic semiring)
- $\mathbf{S}_V \langle \mathbf{A}_V \rangle = \mathbf{B}^T \oplus.\otimes \mathbf{B}$

Step 3: Remove the edges in $\mathbf{A}_U$ and $\mathbf{A}_V$ that do not have "enough support"
- $\mathbf{A}_U \langle \mathbf{M}_U \rangle = \mathbf{A}_U$, where $\mathbf{M}_U = \mathbf{S}_U .\geq (k - 2)$
- $\mathbf{A}_V \langle \mathbf{M}_V \rangle = \mathbf{A}_V$, where $\mathbf{M}_V = \mathbf{S}_V .\geq (k - 2)$

Step 4: Compute the new Bipartite Graph (**B**)
- $\mathbf{B}_{new} \langle \mathbf{B} \rangle = (\mathbf{A}_U \oplus.\otimes \mathbf{B}) \oplus (\mathbf{B} \oplus.\otimes \mathbf{A}_V)$ (boolean semiring)

Repeat with $\mathbf{B}_{new}$ and larger k, until no edges remain.