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as of 01-Nov-2019

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Major Goals: We want to revisit numerical methods for solving nonlinear hyperbolic systems of conservation laws such as:
1) Compressible Euler equations
2) Shallow water equations
3) Multi-material fluid flows (Level set method)
4) Advection-dominated multi-phase flows
5) Free-boundary problems

The Numerical methods must be invariant domain preserving on any unstructured meshes in any space dimension. The methods should respect the physical dissipation of the PDE; i.e., they must be entropy consistent. The methods must be robust: no tuning parameters, mesh-dependent coefficients, or problem dependent stabilization. They should be easy to program and parallelize. They should not require any subtle mathematical knowledge from practitioners. The methods must be at least second-order in space and open to higher-order extensions. The above objectives must be reached by stating precise mathematical statements supported either by proofs or very strong numerical evidences.

Accomplishments: 1) Robust explicit first-order method for systems.
We have constructed a general numerical method to solve any hyperbolic systems in any space dimension using forward Euler time stepping and continuous finite elements on non-uniform grids. The properties of the method are based on the introduction of an artificial dissipation that is defined so that any convex invariant sets containing the initial data is an invariant domain for the method. The invariant domain property is proved for any hyperbolic system provided a CFL condition holds. The solution is also shown to satisfy a discrete entropy inequality for every admissible entropy of the system. The method is formally first-order accurate in space and can be made high-order in time by using Strong Stability Preserving algorithms.

2) Error estimates.
We have proved error estimates for the above first-order method in the context of scalar conservation equations. The methods converges strongly to the unique entropy solution in any space dimension.

3) Maximum principle preserving second-order extension for scalar equations.
We have proposed a technique for approximating nonlinear scalar conservation equations that uses continuous finite elements and is formally (at least) second-order accurate in space and maximum principle preserving. The method is explicit in time, uses unstructured continuous finite elements in space and works in any space dimension. The stability and the accuracy of the method is achieved by locally adapting the artificial viscosity.

4) Robust explicit first-order ALE method for systems. We have extended our invariant domain preserving first-order method to the Arbitrary Lagrangian Eulerian (ALE) formulation of hyperbolic systems. The method is conservative, explicit in time, works with continuous finite elements and is first-order accurate in space. One originality of this work is that the artificial viscosity is unambiguously defined irrespective of the mesh geometry/anisotropy and does not depend on any ad hoc parameter. The proposed method is meant to be a stepping stone for the construction of higher-order methods in space by using appropriate limitation techniques.

5) Second-order approximation of the Shallow water equations. We have combined our general first-order method for hyperbolic systems and its second-order maximum principle preserving extension to scalar conservation equations to make a new second-order approximation technique for the shallow water equations with topography. The method is explicit in time, uses continuous finite elements, and is shown to be well-balanced. The technique is parameter free, works well in the presence of dry states and can be made high-order in time by using strong-stability preserving time stepping algorithms.

6) Dispersive model for shallow water flows.
We have revisits an original relaxation technique introduced in Favrie-Gavrilyuk (2017) for solving the Green-Naghdi equations. We have proposed a version of the method and a space/time approximation thereof that is scale invariant. The approximation in time is explicit and the approximation in space uses a length scale for the relaxation that is proportional to the mesh size. The method is compatible with dry states and is provably positivity preserving under the appropriate CFL condition. The method is numerically validated against manufactured solutions and is illustrated by comparison with experimental results.

7) Entropy-viscosity LES. We have investigated the idea of using the notion of entropy-viscosity as a LES technique for the incompressible Navier-Stokes equations at high Reynolds numbers.

Training Opportunities: During this project we have been approached by colleagues at the U.S. Army Engineer Research and Development Center, Vicksburg to collaborate on a shallow water project. The idea is to transition the theoretical results we have obtained during this grant period into applications that are useful for the ERDC. Our contacts at ERDC are Chris Kees and Matthew Farthing. Three graduate students have spent summer internships at ERDC (E. Tovar, M. Quezada, P. Spencer). Two post-doctoral researchers who have graduated from our group at Texas A&M have been recruited in the group of C. Kees and M. Farthing (M. Quezada, Y. Yang).

We also collaborate with the BLAST team at the Lawrence Livermore National Laboratory, Livermore. Two graduate students have spent summer internships at LLNL (M. Quezada, V. Tomov). One post-doctoral researcher who has graduated from our group at Texas A&M has been recruited in the BLAST team, (V. Tomov).

We also used the grant to support visitors who collaborate with us on this project. Pascal Azerad (Prof. Univ. Montpellier, Fr) works with us on the shallow water project. Laura Saavedra (Assist. Prof., Universidad Polit\\'ecnica de Madrid) works with us on compressible Lagrangian aerodynamics. Murtazo Nazarov (Assist. Prof., Uppsala University, Sweden) works with us on the high-order extensions of the proposed method.

Results Dissemination: The following peer-reviewed articles have been published:

J.-L. Guermond, Bojan Popov, Ignacio Tomas, Invariant domain preserving discretization-independent schemes and convex limiting for hyperbolic systems,
Computer Methods in Appl. Math. and Engin., 347 (2019) 143--175.

J.-L. Guermond, B. Popov, E. Tovar, C. Kees,
Robust explicit relaxation technique for solving the Green-Naghdi equations, J. Comput. Phys. 339 (2019), 108917.

Jean-Luc Guermond, Christian Klingenberg, Bojan Popov and Ignacio Tomas,
The Suliciu approximate Riemann solver is not invariant domain preserving,
Journal of Hyperbolic Differential Equations, 16:01 (2019) 59--72.

Jean-Luc Guermond, Bojan Popov, Laura Saavedra, Yong Yang, Arbitrary Lagrangian-Eulerian Finite Element Method Preserving Convex Invariants of Hyperbolic Systems, in Contributions to Partial Differential Equations and Applications, Chetverushkin, B. N., Fitzgibbon, W., Kuznetsov, Y.A., Neittaanmaki, P., Periaux, J., Pironneau, O., Eds., Springer International Publishing, (2019) 251--272.

C. Nore, D. Castanon Quiroz, L. Cappanera and J.-L. Guermond, Numerical simulation of the Von-Karman-Sodium dynamo experiment, J. Fluid Mech., 854 (2018) 10 November 2018, pp.~164-195.

J.-L. Guermond, M. Quezada de Luna, B. Popov, C. Kees, M. Farthing, Well-balanced second-order finite element approximation of the shallow water equations with friction.
SIAM J. Sci. Comput., 2018, 40:6 (2018) A3873--A3901.

Jean-Luc Guermond, Murtazo Nazarov, Bojan Popov, Ignacio Tomas}, Second-order invariant domain preserving approximation of the Euler equations using convex limiting, SIAM J. Sci. Comput., 2018, 40:5 (2018) A3211--A3239.

L. Cappanera, J.-L. Guermond, W. Herreman, C. Nore,
Momentum-based approximation of incompressible multiphase fluid flows,
Int. J. Numer. Fluids, (2018) 86:541--563.

J.-L. Guermond, B. Popov, Invariant domains and second-order continuous finite element approximation for scalar conservation equations, SIAM J. Numer. Anal. 55:6 (2017) 3120--3146.

P. Azerad, J.-L. Guermond, B. Popov,
Well-balanced second-order approximation of the shallow water equation with continuous finite elements, SIAM J. Numer. Anal., 55:6 (2017) 3203--3224.

J.-L.Guermond, M. Quezada de Luna, T. Thompson,
A conservative anti-diffusion technique for the level set method, J. Comput. Appl. Math., 321 (2017) 448--468.

Jean-Luc Guermond, Bojan Popov, Laura Saavedra, Yong Yang.
Invariant domains preserving Arbitrary Lagrangian Eulerian

approximation of hyperbolic systems
with continuous finite elements, SIAM J. Sci. Comput.
39 No 2 (2017) A385--A414.

Jean-Luc Guermond, Bojan, Yong Yang,
The effect of the consistent mass matrix on the maximum-principle
for scalar conservation equations, J. Sci. Comput. 70, 3 (2017) 1358--1366.

A. Ern, J.-L. Guermond, Chapter 11 - Linear Stabilization for First-Order PDEs,
Handbook of Numerical Methods for Hyperbolic Problems: Basic and Fundamental Issues,
Remi Abgrall and Chi-Wang Shu, Editors, Elsevier
17 (2016) 265--288.

J.-L. Guermond, B. Popov, Fast estimation of the maximum
wave speed in the Riemann problem for the Euler equations.
J. Comput. Phys., 321(2016) 908--926.

J.-L. Guermond, B. Popov,
Invariant domains and first-order continuous finite element
approximation for hyperbolic systems,
SIAM J. Numer. Anal., 54, 4 (2016) 2466--2489.

J.-L.Guermond, B. Popov, Error estimates of a first-order
Lagrange finite element technique for nonlinear scalar conservation
equations, SIAM J. Numer. Anal., 54:1 (2016) 57--85.

J.-L. Guermond, B. Popov, V. Tomov, Entropy–viscosity
method for the single material Euler equations in Lagrangian frame,
300 (2016) Computer Methods in Appl. Math. and
Engin., 402--426.

A. Bonito, J.-L. Guermond, S. Lee,
Simulations of bouncing jets, Int. J. Numer. Methods Fluids,
80 (2016) 53--75.

Honors and Awards: Nothing to Report

Protocol Activity Status:

Technology Transfer: Most of the work that is done in the context of this grant is transferred to
our colleagues at the U.S. Army Engineer Research and Development Center, Vicksburg. The algorithms
developed during the grant are being implemented
in the software PROTEUS at ERDC. Our current student
E. Tovar and former students M. Quezada, P. Spencer and Y. Yang
are contributing to this project under the supervision of Chris Kees and Matthew Farthing.

A software has been developed: SWFEM
Brief Project Description: The acronym SWFEM stands Shallow Water
Finite Element Method. This code solves the Saint-Venant shallow
water equations. This system of partial differential equations models
the motion of a body of water evolving under the action of gravity and
friction effects under the assumption that the deformations of the
free surface are small compared to the water height and the bottom
topography varies slowly with respect to horizontal displacements.
The model incorporates the so-called Serre-Green-Naghdi corrections
which account for essential dispersive effects.

This piece of software is written in Fortran 2003. It uses continuous
finite elements. The time stepping is done by using Strong Stability
Preserving (SSP) Runge Kutta techniques.

Branch of Service/Department/Agency: Texas A&M University
Who developed this source code?: Contractor
When was this project started?: 01/01/2016
Name: Jean-Luc Guermond
Email Address: guermond@math.tamu.edu
Phone Number: 979 7399537

Additional Comments (optional): The code is available on request. The
technology has been entirely transferred into the Proteus tool kit
developed at the U.S. Army Engineer Research and Development Center
(Contact: Christopher Kees, Christopher.E.Kees@erdc.dren.mil)
https://proteustoolkit.org/

PARTICIPANTS:

Participant Type: PD/PI
Participant: Jean-Luc Guermond
Person Months Worked: 8.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Co PD/PI
Participant: Bojan Popov
Person Months Worked: 8.00 **Funding Support:**
Project Contribution:
International Collaboration:
International Travel:
National Academy Member: N
Other Collaborators:

Participant Type: Graduate Student (research assistant)
Participant: Eric Tovar

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Other Collaborators:

Participant Type: Graduate Student (research assistant)
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Article Title: An conservative anti-diffusion technique for the level set method
Authors: Jean-Luc Guermond, Manuel Quezada de Luna, Travis Thompson
Keywords: Conservative; Level set; Two phase flow; Finite volume; Finite element; Entropy viscosity
Abstract: A novel conservative level set method is introduced for the approximation of two-phase incompressible fluid flows. The method builds on recent conservative level set approaches and utilizes an entropy production to construct a balanced artificial diffusion and artificial anti-diffusion. The method is self-tuning, maximum principle preserving, suitable for unstructured meshes, and neither re-initialization of the level set function nor reconstruction of the interface is needed for long-time simulation. Computational results in one, two and three dimensions are presented for finite element and finite volume implementations of the method.
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Authors: Andrea Bonito, Jean-Luc Guermond, Sanghyun Lee
Keywords: bouncing jet; Kaye effect; entropy viscosity; level set; projection method; shear-thinningviscosity; adaptive ?nite elements
Abstract: Bouncing jets are fascinating phenomenon occurring under certain conditions when a jet impinges on a free surface. This effect is observed when the fluid is Newtonian and the jet falls in a bath undergoing a solid motion. It occurs also for non-Newtonian fluids when the jets fall in a vessel at rest containing the same fluid. We investigate numerically the impact of the experimental setting and the rheological properties of the fluid on the onset of the bouncing phenomenon. Our investigations show that the occurrence of a thin lubricating layer of air separating the jet and the rest of the liquid is a key factor for the bouncing of the jet to happen. The numerical technique that is used consists of a projection method for the Navier–Stokes system coupled with a level set formulation for the representation of the interface. The space approximation is carried out with adaptive finite elements. Adaptive refinement is shown to be very important to capture the thin layer of air that is resp
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Article Title: Second-order, maximum-principle preserving finite element method for any scalar conservation equation
Authors: Jean-Luc Guermond
Keywords: Conservation equations, Second-order, maximum principle, entropy-viscosity, finite element method.
Abstract: We propose a technique for approximating nonlinear scalar conservation equations that uses continuous finite elements and is formally (at least) second-order accurate in space and maximum principle preserving. The method is explicit in time, uses unstructured continuous finite elements in space and works in any space dimension. The stability and the accuracy of the method is achieved by adapting the artificial viscosity.
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Authors: Pascal Azerad, Jean-Luc Guermond, Boja Popov
Keywords: Shallow water, well-balanced approximation, invariant domain, second-order method, finite element method, positivity preserving.
Abstract: This paper investigates a first-order and a second-order approximation technique for the shallow water equation with topography using continuous finite elements. Both methods are explicit in time and are shown to be well-balanced. The first-order method is invariant domain preserving and satisfy local entropy inequalities when the bottom is flat. Both methods are positivity preserving. Both techniques are parameter free, work well in the presence of dry states and can be made high-order in time by using strong-stability preserving time stepping algorithms.
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Nothing to report in the uploaded pdf (see accomplishments).