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QUANCO APPLIED TO BOLTZMANN MACHINES

MISSISSIPPI STATE UNIVERSITY

FEBRUARY 2020

FINAL TECHNICAL REPORT

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1.0 SUMMARY

The main emphasis of this one-year (plus no cost extension) ‘QuAnCo Applied to Boltzmann Machines’ project was to evaluate currently available Noisy Intermediate-Scale Quantum, NISQ, computers in order to evaluate their current and future possibilities in machine learning, ML. Emphasis was to be on quantum annealing machines, and D-Wave Systems manufactures the only Adiabatic Quantum Computer, AQC, available. The particular implementation of NISQ D-Wave machines to ML was to be via Boltzmann machines, BMs, with an emphasis on restricted Boltzmann machines, RBM graphs. The goal was to evaluate the D-Wave machine for ML, with an eye toward implementation for cybersecurity. The particular cybersecurity task was for intrusion detection in net flow traffic.

NISQ computers are an immature technology. Therefore, another emphasis for the ‘QuAnCo Applied to Boltzmann Machines’ project was to both investigate better algorithms for NISQ AQCs, and to investigate possible better algorithms for NISQ annealing machines related to ML. An additional emphasis was to develop and teach a course *Introduction to Quantum Computing* for physicists and engineering students at Mississippi State University, MSU.

The research related to Quantum annealing computers, QuAnCo also led to an unexpected breakthrough in quantum systems. In particular, it was shown a large class of atypical strongly disordered quantum models could have only local disorder correlations but exhibit phenomena not expected of typical disordered quantum systems. In particular, such quantum systems with atypical disorder can have complete electron transmission when attached to proper leads and can show order- amidst-disorder. A more full description of this breakthrough is in Sec. 4.3. The impact of this breakthrough is one has to think differently about coherent wave propagation through disordered systems. For wave functions for electrons, this means it is possible to design extremely disordered nanodevices with novel electrical properties, which could lead to improved quantum sensors. For needle-in-a-haystack types of datasets this breakthrough allows very complex datasets, even with very small data record lengths, which are extremely difficult for training ML paradigms or even humans to recognize novel properties. The novel property for this breakthrough is that with only local correlations in the Hamiltonian parameters a very disordered system is transparent to electrons ... namely all electrons with wavelengths greater than some minimum ‘discretization value’ have 100% probability of being transmitted through the disordered nanodevice so there is zero probability of reflection. Whether this quantum wave property carries over to other coherent waves is an important question to ask, and has potential applications. For example, could changing only locally parameters allow a disordered system to be invisible to coherent sound waves (with applications to sonar) or be invisible to coherent plane-wave electromagnetic waves (with applications to cloaking against radar)?

The take-away message from this project is that NISQ computers, including AQCs, are not yet ready for prime-time as of 2019, but are on-the-edge of being able to make an impact in ML. They will most likely, in the opinion of the principle investigators, PIs, be useful in the timeframe of a few years. As AQCs advance, their usefulness in the numerous subfields in defense, homeland security, engineering, and science, as well as in society, will continue to advance ... and most likely advance extremely rapidly.

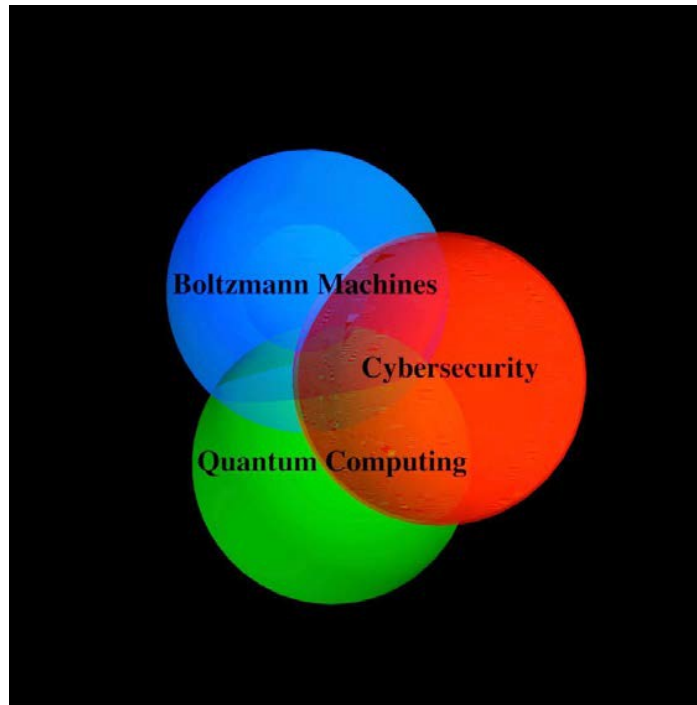


Figure 1: The overview of the proposal, namely QuAnCo (Quantum Computing, with annealing quantum machines), Boltzmann machines for ML, with an eye toward an overall future aim of applications to cybersecurity.

2.0 INTRODUCTION

NISQ computers and NISQ technology have the potential to be a disruptive technology in areas from defense to engineering to science. Therefore, studying NISQ technologies, both current and in a five-year timeframe, is critical to national security.

Under each main heading of the report, the break-up is in terms of the three sub-projects:

- 1) NISQ D-Wave machines applied to RBM for ML
- 2) Improving algorithms and hardware for AQC and ML, as well as teaching about quantum technologies
- 3) Atypical disorder in quantum systems (the unexpected breakthrough)

A schematic of the overview of the proposal is in Figure 1.

3.0 METHODS, ASSUMPTIONS, HYPOTHESIS, PROCEDURES

As research with an ultimate eye on applications gets underway with regard to the National Quantum Initiative [1], it is important to improve quantum technology, and to teach the next generation of quantum scientists and engineers.

Our main hypothesis is that quantum computing is in its infancy. In the 2012 the Nobel Prize in physics was awarded to Dr. Serge Haroche and Dr. David J. Wineland ‘for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems’. The 2012 press release by the Nobel committee included: “Perhaps the quantum computer will change our everyday lives in this century in the same radical way as the classical computer did in the last century” [2]. Reflecting on this sentence, thinking about defense, homeland security, industry, and communications should send shivers up your spine. At MSU, we partnered with the only manufacturer of a delivered quantum computing device during the timeframe of the funding period, D-Wave Systems [3]. Our mantra is: Test It! Improve It! Teach It! Use It!, with ‘It’ being quantum computers and quantum information science, with an emphasis on QuAnCo. We feel this mantra is the correct approach to any immature, but potentially game-changing, technology. Each portion of the mantra is expanded on in subsections 3.1 through 3.3.

The main method used in the research was through use of a D-Wave quantum computer and through quantum simulations on high performance classical computers. The D-Wave architecture is comprised of unit cells of 8 qubits arranged on a square grid, with the connections between qubits a Chimera graph [3]. For the D-Wave 2X, the architecture can be seen in the lower layer of Figure 2 for a 1000 qubit (actually $12^2 \times 8 = 1152$ qubits if no missing qubits were present) machine. D-Wave was kind enough to both the PIs and their group to use up to 5 hours per month on their machines, located at their headquarters in Burnaby, British Columbia, Canada. In addition, the PIs and their group were able to use, through an application to Universities Space and Research Association, USRA, and NASA Quantum Artificial Intelligence Laboratory, NASA/QuAIL, the D-Wave machine located at the NASA Ames Research Center in Moffett Field, CA. Almost all of the D-Wave time was on a D-Wave 2000Q, the 2000 qubit machine. The D-Wave 2000Q actually has 2048 qubits, but most chips have some qubits inaccessible to the user. However, there was one chip we had access to through D-Wave Systems which had all 2048 qubits available to the user.

The majority of high performance computing used for this project was on free-to-the-MSU-user MSU High Performance Computing Collaboratory, HPC², machine Shadow. Shadow is a Cray CS300-LC cluster with 4800 Intel Ivy Bridge processor cores and 28,800 Intel Xeon Phi cores. Each Shadow node has either 512 GB of RAM (45%), 128 GB of RAM (45%) or 64 GB of RAM (10%) for a system total of 70 TB of RAM. Furthermore, each node is interconnected via FDR InfiniBand (56 Gb/s) with Shadow having a peak performance of 593 teraFLOPS. The codes for real-time simulation of quantum qubit systems were written by the PIs and their group. In addition, codes run on Shadow for ML were written by the PIs and their group.

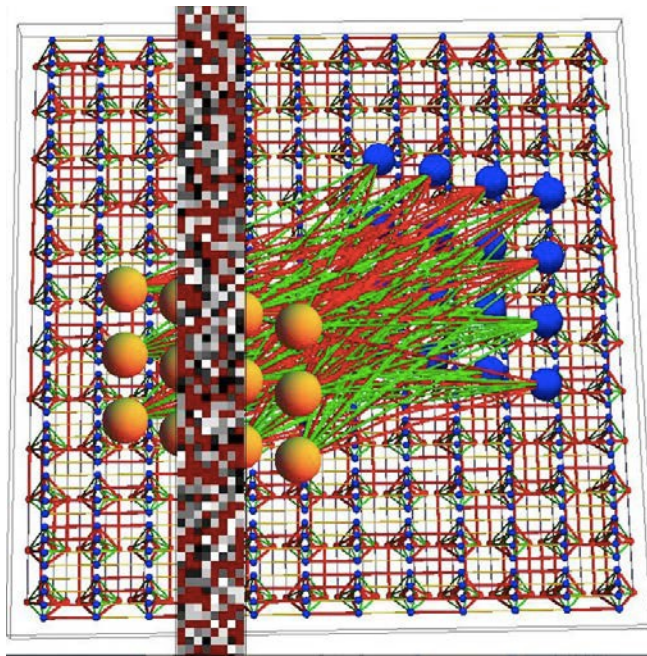


Figure 2: Schematic of the research, with net flow traffic (top level), a RBM for ML (middle level), with a D-Wave architecture on a Chimera lattice (lowest level).

3.1 NISQ D-Wave Machines Applied to RBM for ML

For the ‘Use It!’ portion of the mantra, we performed research in utilizing D-Wave machines and potential future quantum machines for ML, with possible applications to cybersecurity. The ML paradigm is a BM, with most studies on the traditional RBM. The ultimate goal of the ML is for net flow traffic applications, under the cybersecurity category of datasets. The exploratory goal is to devise procedures and try to train, test, and utilize a D- Wave AQC to enhance the ML capabilities. See Figure 2.

We implemented and studied the Boltzmann machine methodology, applied to intrusion detection in net flow traffic in cybersecurity. In particular, we utilized D-Wave quantum annealers, QuAnCo, with both 1000 qubits and for the majority of the runs 2000 qubits. This work was performed by the PI, co-PI, post-doc, and the supported graduate student which led to one conference paper [4] by the PIs. As a follow-on during the research period, we were able to study the Local Valley, LV, obtained via a D-Wave plus a Markov Chain Monte Carlo, MCMC, simulated warming algorithm on the classical machines for both the training and the testing/application parts of the ML algorithm. We are currently putting the final touches on the paper, for submission to an IEEE journal. This work was performed by the PIs and the post-doc. The take-home message is the best sampling for RBMs is to utilize both traditional MCMC methods on classical computers and also utilize available QuAnCo machines. In particular, we assumed for the spin glass Hamiltonian sent to the D-Wave the result returned was a ground state or at least a low-lying excited state. At the heart of ML is the use of the Kullback-Leibler divergence, which is a measure of how one probability distributions differs from another, and for BM implementations this means one needs to sample for the ML training portion a

Boltzmann distribution at a temperature (in appropriate units) $T=1$. Therefore the PIs implemented ‘simulated warming’ wherein a low temperature configuration from the D-Wave machine was used as the starting configuration for a MCMC procedure warming gradually from $T=0$ to the desired $T=1$. This is related to the standard MCMC technique of simulated annealing, which for training BMs starts at $T=\infty$ and slowly lowers the temperature to $T=1$.

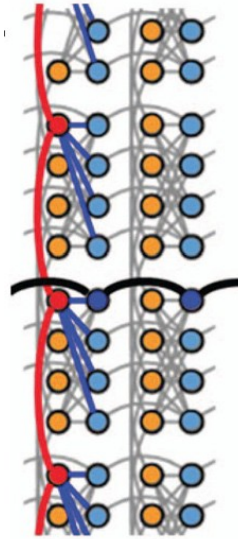


Figure 3: Chimera to RBM. A sketch of the method to turn the D-Wave Chimera graph into a graph for an RBM. This is done by chaining together physical qubits to form virtual qubits. From [4].

The method of ‘chaining’ together physical qubits on the D-Wave Chimera graph in order to obtain the graph for an RBM is shown in Figure 3. For the 2000Q machine the 2000 physical qubits yielded an RBM with 64 units in the visible layer and 64 units in the single hidden layer. Unfortunately, only this small RBM graph could fit onto the D-Wave Chimera graph with 2000 qubits.

Once the training phase is completed for any dataset, the BM has a given spin glass Hamiltonian. This Hamiltonian can be used for image reconstruction and classification. The normal classical way of doing this in RBMs is to perform on every record a long classical MCMC using simulated annealing to obtain the reconstructed record or to add a label to the record. The D-Wave allowed image reconstruction and labeling with one call to the D-Wave. In particular, the record to be reconstructed had some visible units ‘frozen in’ by adding additional bias fields to the Hamiltonian, and then with one call to the D-Wave one obtains a new reconstructed or labeled record. For current 64 bit records, both the classical and D-Wave method of reconstruction take about the same amount of wall-clock time. However, as the record length becomes larger (but still able to fit onto the next generation of QuAnCo machine) the wall-clock QuAnCo method will remain roughly the same while the classical MCMC method will require more wall-clock time. In [4] we were able to show image reconstruction of missing information and image classification on a 64 bit Bars and Stripes, BAS, dataset (the number of units allowed by the D-Wave 2000Q RBM implementation). Subsequent work during the funding period allowed us to

apply the same methodology to a 64-bit record weighted dataset which we constructed from the publicly available ISCX dataset [5] for net flow traffic.

It is important to have good datasets for testing ML, particularly when implementing and testing methodologies on immature technologies such as AQCs. We needed to test our methodology on datasets with a 64-bit record length, due to the current D-Wave 2000Q fitting a RBM graph with only 64 visible units. The traditional BAS dataset with 64 bit records was readily constructed, an 8×8 square array. We also discretized the traditional data sets for ML, the handwritten digits of the MNIST dataset, for an 8×8 square array. We were also able to use feature selection to obtain our 64-bit record ISCX dataset [5] for net flow traffic. However, we determined the BAS and MNIST datasets were too easy for such a small number of visible units. We also determined the ISCX dataset with a 64 bit record length was only marginally of the size needed for intrusion detection (basically the 64-bit record length was too short to adequately reflect the complexity of the original ISCX dataset records). Therefore, we also constructed a 64 bit dataset, which we called a quantum dragon dataset, in order to have a dataset which was well quantified but is difficult for classification from the best ML implementations (and also for humans, since no one before this work knew such a dataset was possible and we were unsuccessful in training ourselves to identify records reflecting quantum dragons). See Figure 4 for examples of records of the quantum dragon dataset, and Subsection 3.3 for a more full description. Our 64-bit quantum dragon dataset has been accepted for publication in a proceedings volume: *A 64 bit quantum dragon dataset for machine learning*, M.A. Novotny, G. Inkoom, V. Dixit, and Y. Koshka, presented at the 32nd Annual Workshop on Recent Developments in Computer Simulation Studies in Condensed Matter Physics, Athens, Georgia, Feb. 18-22, 2019. This paper is in press. The central finding is such needle-in-a-haystack datasets with records based on quantum electron transmission could be constructed, and had some counter-intuitive properties.

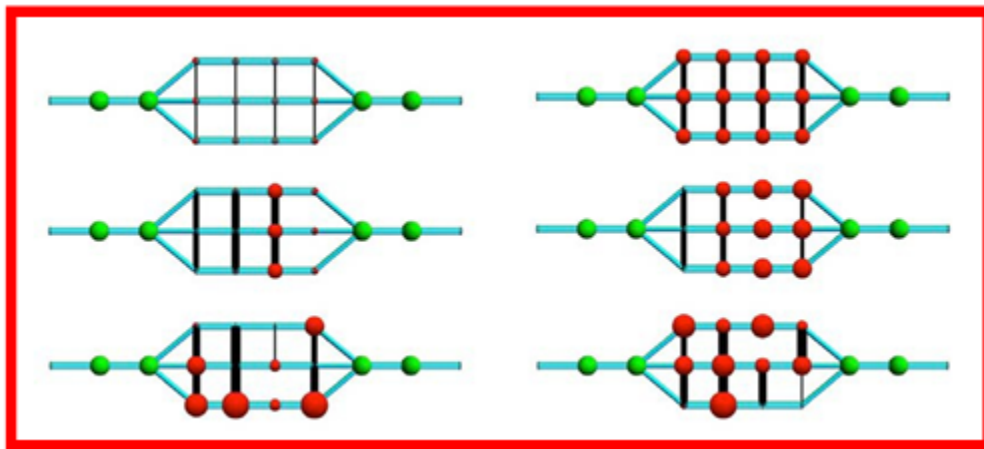


Figure 4: Quantum Dragon 64-bit dataset. Six example records. Classification is for which tight-binding models of nanodevices have complete electron transmission through the (green sphere) leads. The unexpected answer is: all of them! Even though the vast majority of disordered nanodevices do not have complete electron transmission.

3.2 Improving AQC and Teaching Quantum Computing

There are many known ways to improve future AQC machines. The best known are ones on the experimental/engineering side, namely decreasing the temperature (the current D-Wave machines operate at about 12 mK), or improving decoherence time over the entire AQC chip so the anneal times can be increased (the current D-Wave machines allow anneal times of between 20 μ s and 20 ms). Our assumption was there are other less-obvious ways to improve future AQC machines. We studied three different ways to improve future AQC machines:

- ❖ Improving the underlying graph (the AQC chip architecture)
 - All past D-Wave architectures are based on an underlying Chimera graph. This graph was chosen by D-Wave because of the ease of chip fabrication and ease of scaling to larger numbers of qubits (they have scaled from 8 qubits to 2048 qubits in various generations, with delivered QuAnCo machines at 128, 256, 512, 1024, and 2048 qubits).
 - Unfortunately Chimera graphs do not have a finite temperature Ising spin glass transition, $T_C=0$, and so D-Wave machines cannot perform using the native graph the most complicated spin glass configurations [6]. Furthermore, Chimera graphs can be embedded (coarse-grained) in $D=2$ and so are just below the lower critical dimension for an Ising spin glass.
 - Our methodology was to perform largescale classical Monte Carlo calculations of Ising spin glasses on graphs from $D=2$ plus added Small World, SW, bonds to find a buildable graph which allows the hardest Ising spin glass configurations to be studied.
- ❖ Enabling another AQC machine instruction: forward-reverse error mitigation, FREM
 - Current D-Wave machines have two types of anneals protocols
 - Forward annealing, where the initial state is an equal superposition of all 2^N states in Hilbert space, and the initial Hamiltonian is composed of only of the Pauli matrix σ^X . The anneal progresses until at the end it has a Hamiltonian composed only of the Pauli matrix σ^Z , a classical state.
 - Reverse annealing, where the initial state is a classical state, and the initial Hamiltonian is composed only of the Pauli matrix σ^Z . The anneal turns on and then turns off a Hamiltonian composed only of the Pauli matrix σ^X
 - Using large-scale classical calculations of the real-time evolution of a fully connected system of N qubits, we proposed and analyzed a third possible protocol, FREM.
 - In all three protocols the Hamiltonian at intermediate times has both matrix σ^X and σ^Z terms
- ❖ Studying the possibility an AQC machine can obtain the ground state degeneracy of a classical spin glass
 - The paper of the PI and collaborators on decoherence and thermalization at finite temperatures of a generic quantum system [7] contained the possibility of obtaining the ground state degeneracy of a large quantum system by measuring certain quantities on a small quantum system
 - There may be no Hamiltonian coupling the large and small systems
 - The wave function of the small+large system is entangled, and should be drawn from

- the canonical-thermal state at a finite but low temperature
- With a graduate student, who was unfunded by this project, the PI and co-PI supervised the building and testing of a large-scale classical computer code to enable testing of finding the ground state degeneracy of a large quantum system
 - During the funding period the codes was tested, allowing for simulations in real time of qubit systems for up to 12 qubits. The code is currently being ported to run on the MSU HPC² massively parallel machines available to the PIs and their students.

There will need to be a large number of trained scientists and engineers who are knowledgeable about quantum computing and quantum systems, as well as traditional classical topics. Therefore, the PI and co-PI proposed to teach at MSU an introductory course on quantum computing

- ❖ To be taught at the advanced undergraduate and beginning graduate student levels (at MSU the classes would be numbered 4993 and 6993, respectively) for our 3 credit hour class
- ❖ To be cross listed between the Physics department and the Electrical and Computer Engineering department

During the timeframe of the funding period, the PI and co-PI sought to obtain approval to teach this introductory class from the MSU University Courses and Curriculum Committee.

$$\mathcal{H} = \sum_j \epsilon_j c_j^\dagger c_j - \sum_{\langle ij \rangle} t_{ij} (c_i^\dagger c_j + \text{hc})$$

Figure 5: The Hamiltonian for the single-band tight-binding model, which is on a graph such as in Figure 4. The first sum is over all sites in the graph, the second overall bonds in the graph. The c_j and $c_j^\dagger$ are creation and annihilation operators at the site labeled by j . The ‘hc’ stands for Hermitian conjugate. The onsite energy value at site j is ϵ_j . The hopping term between sites labeled by i and j is t_{ij} . This Hamiltonian is for a model of electron transport through a nanodevice.

3.3 Atypical Disorder in Quantum Systems

Disorder in quantum systems is usually thought of as being disruptive, due to the scattering electrons have from impurity sites. In $D=1$ Anderson localization says any amount of disorder causes localization of electron wave functions. A useful Hamiltonian to study the effects of disorder is a tight-binding system, as in Figure 5, which is sometimes called the Anderson Hamiltonian. In higher dimensions, it was thought if the disorder become large enough the system would also have localized wave functions. In 2014 the PI showed [8] that in $D=3$ nanosystems with cylindrical symmetry one could have disorder in one direction provided there was correlation between all the tight-binding parameters within slices in the other direction.

These devices the PI called ‘quantum dragons’ [8] (see Figure 6), and showed the electron transmission could be $\mathcal{T}(E)=1$ for all electron energies E which propagate through attached semi-infinite leads. The methodology was to utilize the tight-binding Hamiltonian of Figure 5 and the knowledge of quantum dragon nanosystems in order to devise and characterize a needle-in-a-haystack dataset built on a quantum system in order to test ML for systems with small record lengths. This was to be done for 64-bit records due to the available size of D-Wave QuAnCo machines, with nanodevices such as those in Figure 4.

The unexpected outcome of this study was the counter-intuitive realization that for many $D=2$ and $D=3$ underlying graphs one could have a quantum dragon, namely have complete electron transmission of $\mathcal{T}(E)=1$, with only locally correlated disorder. This unexpected result was further studied, even though it was not anticipated at the time the proposal was submitted.

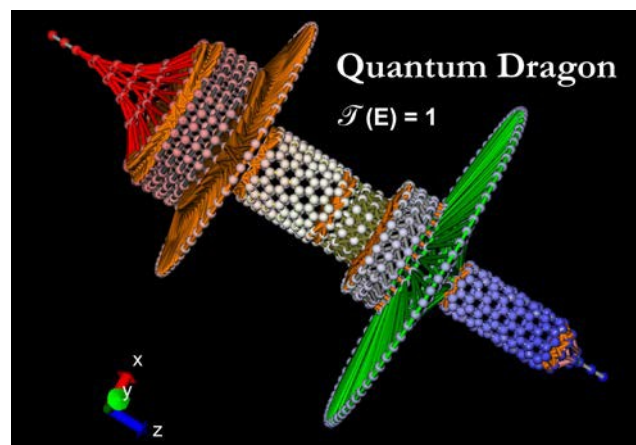


Figure 6: A quantum dragon, with axial symmetry, from the PI’s 2014 paper [8]. Only a few atoms of the semi-infinite attached thin (single atom thickness) leads are shown at each end. The nanodevice is composed of subgraphs of many different types, all with cylindrical symmetry. They include rectangular graphs, Bethe lattice graphs, and $D=2$ hexagonal graphs (in armchain and zigzag orientations).

4.0 RESULTS AND DISCUSSION

The research of this one year project was very productive. Presentations under the auspices of this funding included:

- ❖ *QuAnCo Applied to Boltzmann Machines*, M.A. Novotny and Y. Koshka, presented at program review, April 25, 2019, AFRL, Rome, New York.
- ❖ *A Small-World Search for Quantum Speed-up*, M.A. Novotny and H.G. Katzgraber, presented at AQC’18 (Adiabatic Quantum Computing 2018), June 24-29, 2018, NASA Ames Research Center, California. (Note: USRA for quantum computers is also at the NASA Ames Research Center, and since it supplied needed D-Wave computer time on a no-cost basis to this project, this was a very important meeting to attend.)
- ❖ *Quantum Oracle for #P and #BQP*, M.A. Novotny, presented at Aspen Research Center, Aspen, CO, workshop on quantum computing, May 27- June 17, 2018, Aspen Research

Center.

- ❖ *Classical and non-commutative Boltzmann Machines: An Update from the Magnolia State*, M.A. Novotny, 2018 D-Wave Users Conference, Knoxville, TN, Sept. 24-27, 2018. (Note, D-Wave supplied on a no-cost basis time on their QuAnCo machines, so making presentations at their user meeting was important.)
- ❖ *Non-commutative Boltzmann Machines*, M.A. Novotny, March Meeting of the American Physical Society, March 4-8, 2019, Boston, MA.
- ❖ *2D Short Ranged Tight Binding Models with Extensive Local Disorder but which have a Non-Localized State*, M.A. Novotny, G. Inkoom, and T. Novotný, presented at 120th Statistical Mechanics Conference, Dec. 15-18, 2018, Rutgers University, Piscataway, NJ.
- ❖ *How to Approach a Quantum Dragon*, M.A. Novotny and T. Novotný, presented at the 42nd International Conference of Theoretical Physics: Correlations & Coherence at Different Scales, Sept. 10-14, 2018, Ustron, Poland.

The work included projects which had preliminary data prior to the start of the funding period, as well as projects initiated in the funding period which are/were ongoing after the funding period. Each project will be discussed in greater detail below.

4.1 NISQ D-Wave Machines Applied to RBM for ML

4.1.1. Access to D-Wave QuAnCo Machines

The first important issue for studying ML using AQC is to have access to a NISQ machine. In the timeframe of the funding period the only vendor with an AQC product as a QuAnCo machine is D-Wave systems. Therefore, access to the latest D-Wave QuAnCo was imperative. The PI and co-PI were successful at obtaining such access on these tens-of-millions of dollar machines. Through a long-standing relation with D-Wave Systems, the PI was able to obtain access to QuAnCo machines located at the headquarters of D-Wave Systems in Burnaby, British Columbia, Canada. The amount of time, no cost to the AFRL, was up to 5 hours per month. The PI and co-PI were also able to obtain D-Wave time for this project under the auspices of USRA to the D-Wave QuAnCo machines at the NASA Ames Research Center, Moffett Blvd, Mountain View, CA, on a no-cost basis. The going rate for purchasing D-Wave time was much more than the monies spent by AFRL on this research. The PI and co-PI also taught the students and post-doc about access to and running of D-Wave machines.

4.1.2. Datasets for RBM ML on D-Wave QuAnCo Machines

It was important to have good datasets for testing ML, particularly since we were implementing and testing methodologies on immature QuAnCo technologies. We needed to test our methodology on datasets with a 64-bit record length, since a RBM with 64 hidden and 64 visible units could be embedded on the D-Wave 2000Q machine with its underlying Chimera graph architecture.

We constructed a traditional BAS dataset with 64 bit records, an 8×8 square array. We also were successful, with a graduate student funded under this project, to discretize the traditional data

set for ML of the handwritten digits of the MNIST dataset for an 8×8 square array (together with image translations and select rotations). We were also able to use feature selection to design, construct, test, and utilize a 64-bit record ISCX dataset [5] for net flow traffic. We determined the BAS and MNIST datasets were too easy for such a small number of visible units, being good for tests of QuAnCo in ML but too simple for more than cursory tests of the RBM methodology. In particular Our ISCX dataset with a 64 bit record length was only marginally of the size needed for intrusion detection, therefore not making any significant increase in productivity of ML from use of only classical computers. Again, these limitations are almost exclusively due to the size of the RBM graph (64 visible and 64 hidden units) which could fit onto a 2000 qubit Chimera graph. It is expected, for example for larger QuAnCo machines, or ones with better native graphs than the Chimera graph, the number of units (both visible and hidden) can increase substantially. This would enable better comparison between classical BM and BM enhanced by QuAnCo, and should be a reasonable ground for the MNIST and ISCX datasets with longer record lengths.

Due to these limitations of traditional ML datasets, we constructed a 64 bit dataset which we called a quantum dragon dataset, in order to have a dataset which was well quantified but is difficult for classification from the best ML implementations. In fact this dataset is also difficult for humans to classify, as tested by the PI and funded post-doc and graduate students, and indeed before this work no one knew such a dataset was even possible, as most physicists who study tight-binding models as in Figure 5 would have argued that electron scattering and Anderson localization arguments would have not allowed such a dataset to exist. Even knowing the algorithm to find quantum dragon configurations, neither the PI nor the funded post-doc or graduate students could classify by eye whether or not a record with random tight-binding parameters was or was not a quantum dragon. See Figure 4 for examples of records of the quantum dragon dataset. Our 64-bit quantum dragon dataset has been accepted for publication in a proceedings volume: *A 64 bit quantum dragon dataset for machine learning*, M.A. Novotny, G. Inkoom, V. Dixit, and Y. Koshka, presented at the 32nd Annual Workshop on Recent Developments in Computer Simulation Studies in Condensed Matter Physics, Athens, Georgia, Feb. 18-22, 2019. See Table 1 for quantification of this needle-in-a-haystack dataset, for two different boundary conditions (BC). The BC00 (free boundary conditions) dataset had only uniform devices as quantum dragons, the result expected from analysis of nanodevices with translational symmetry along the direction of electron flow. The results were unexpected for BC--, namely of the 10^{16} possible records only 10^8 are mappable, and of these about 10^5 were quantum dragons. By mappable we mean the mapping method of the PI in [8] can be used to perform a unitary transformation on the Hamiltonian which produces a $D=1$ system which is decoupled from a disordered part of the rotated-in- Hilbert-space device. By a quantum dragon we mean a nanodevice with electron transmission $\mathcal{T}(E)=1$ for all E when connected to appropriated semi- infinite leads. Thus one would have to on average look through 100 billion records to find one which is a quantum dragon. Furthermore, prior to this work it would have been expected for the graphs in Figure 4 there would be only 64 quantum dragons (all very regular arrangements) while we were able to find about 10^5 for BC--. In the PI's 2014 paper [8] we anticipated about 10^4 quantum dragons, as disorder would have to be correlated globally throughout every slice (vertical part of graph). Therefore, finding about 10^5 for BC-- was much unexpected. The critical point is one needs only local disorder correlations for a quantum

dragon, rather than the expected global disorder correlations. Our quantum dragon dataset can be expanded to other types of graphs, for example devices based on disordered $D=2$ hexagonal graphs or $D=3$ simple cubic graphs. In the future as one needs a well-quantified needle-in-a-haystack type of data, one can make use of a quantum dragon dataset to study and test ML, for any desired length of data records. See Figure 7.

Table 1: Quantum 64 bit dragon dataset quantification. Shown are two different boundary conditions (BC00 and BC--), with two bits used to label the type of boundary condition. Two bits were also used to label each record, as to whether it was mappable using the PI's method generalized method from [8], and whether or not it was a quantum dragon. The remaining 60 bits in a record allowed for many configurations based on the graph in Figure 4 and the Hamiltonian in Figure 5, the middle column in this table. It is a needle- in-a-haystack type of problem to find mappable configurations within the large number of allowed configurations, which is the column labeled # mappable in the table. The number of records that are quantum dragons were previously predicted to be only regular systems, with parameter correlations throughout the device, of which there are 64 configurations. This is the number of quantum dragons for BC00. However, for BC— a total of 91208 quantum dragons were possible ... a number which was much unexpected and went against 'tradition and folklore' within the tight-binding physics community.

Type	label	# config	# mappable	# dragons
BC00	00	$2^{60} = 1.15 \times 10^{18}$	$2^{24} = 1.68 \times 10^7$	$2^6 = 64$
BC--	01	$(8 \times 36^2)^4 = 1.16 \times 10^{16}$	$104^4 = 1.17 \times 10^8$	91208

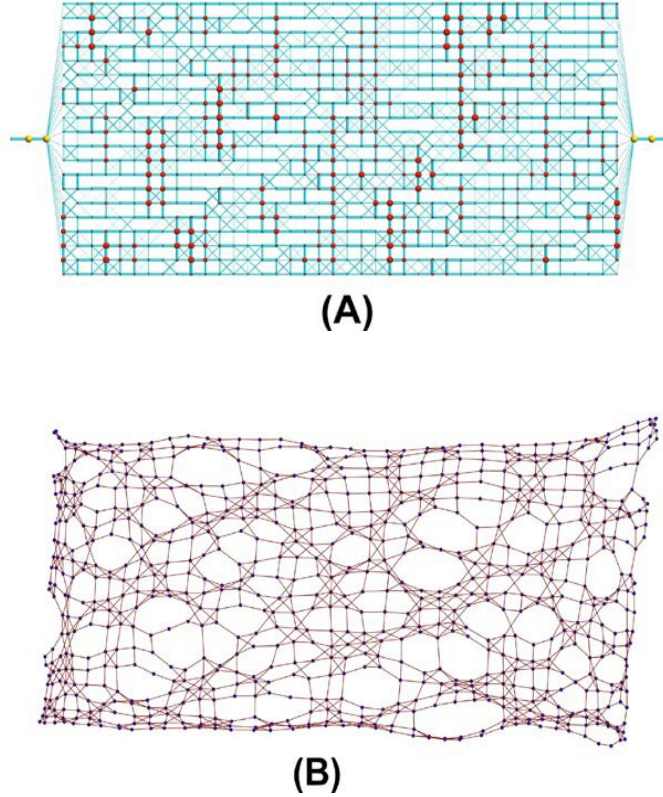


Figure 7: Example of a much larger $D=2$ square-graph quantum dragon with only local disorder. (A) shows the tight-binding model, with radii of spheres proportional to the onsite energies ε_j and of the cylinders to the hopping parameter t_{ij} . (B) shows the structure of the underlying graph, using the Mathematical built-in function GraphPlot3D.

4.1.3. Implementation and Testing of RBM on D-Wave QuAnCo Machine

The actual implementation of ML RBMs utilizing a D-Wave QuAnCo machine led to one conference paper [4] by the PIs.

The first problem addressed in the PIs' paper [4] was how to embed a RBM graph onto the Chimera graph which is native to current (and past) D-Wave architectures. The method is related to Figure 3, and a similar figure is [4] figure 2. The chaining together of qubits as in Figure 3 was addressed. However, this works only for an ideal D-Wave chip, one without any missing (actually inaccessible to the user) qubits or couplers. Therefore in [4] we introduced a modified chaining which was broken by missing qubits or couplers. This led to a slightly larger number of visible and/or hidden units, but the underlying graph now is an RBM with many missing couplers. We showed that even with the missing couplers of the RBM-from-chained-Chimera we could successfully train the size of BM we had on our datasets. Since the chaining extends the length of the chip, a larger chip has more problems with embedding a RBM onto a Chimera-with-disorder graph. Fortunately, D-Wave has announced their next generation machine will have an improved architecture, based on a Pegasus graph, which should be much better for

obtaining a trainable BM or even an RBM with more units ... even if the yield of accessible qubits and couplers remains the same from the current D-Wave 2000Q machine.

Next in the PIs' paper [4] we studied the per cent errors correct labeling of records comparing our D-Wave method (described above in Sec. 3.2) to traditional MCMC methods, using a BaS dataset with 64 bit records. As a function of the number of training iterations, we found the D-Wave labeling had always a slightly lower error rate compared with standard MCMC on the same BM graph. This was a very promising result, albeit for a small data record length on a very simple problem. Examples for the BaS dataset are shown in Figure 8 (a) and (b).

We also studied the error rate as a function of the D-Wave scale factor used to put the trained bias and coupler weights onto a D-Wave. This was necessary because for classical RBM the weights and biases can be arbitrarily large, while for the D-Wave they must be scaled to fit into the minimum and maximum allowed machine values. This is particularly true for image labeling or reconstruction, where the un-damaged (or unlabeled) portion of the record must be 'pinned' to the record values using large biases. We found there is a sweet-spot in the D-Wave scale factor where the error rate is the lowest, while the error rate grows (as could be expected) for too small or too large of a scale factor.

As a follow-on to the PIs' paper [4], during the research period we were able to study the LVs obtained via a D-Wave plus a MCMC simulated warming algorithm on the classical machines for both the training and the testing/application parts of the ML algorithm. We are currently putting the final touches on the paper, for submission to an IEEE journal. This work was performed by the PIs and the post-doc. The take-home message is the best sampling for BMs is to utilize both traditional MCMC methods on classical computers and also utilize available QuAnCo machines. For example, we used both our BAS and ISCX 64-bit datasets to study image labeling and image reconstruction. With the funded post-doctoral researcher, Dr. Vivek Dixit, we were able to submit one conference proceedings article wherein we analyzed the difference between using classical machines only or adding also a D-Wave QuAnCo machine to the ML algorithm. Sample results are shown in Figure 8. The results showed the accuracy of image reconstruction were comparable for the two cases of using only classical computers in ML or using a D-Wave QuAnCo machine. For example, for our ISCX 64-bit dataset we obtained 83.9% accuracy of reconstruction using classical computers and 82.2% using a D-Wave QuAnCo machine. These numbers, and our other numbers, illustrate that current D-Wave architectures are just on the cusp of being useful, and there is a strong possibility future QuAnCo machines will be extremely useful using RBMs (and maybe BMs) in ML.

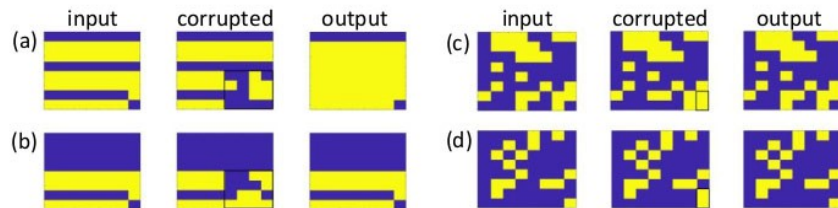


Figure 8 Examples of image reconstruction of 64-bit datasets, ones where a RBM for ML could be implemented on a D-Wave 2000Q QuAnCo machine. (a) and (b) are two examples of BAS records (the two bottom-right bits are the label) showing the original record, the corrupted record, and the output from image/label reconstruction using a D-Wave machine. Here one can visibly check the reconstruction. (c) and (d) are similar examples for our ISCX 64-bit dataset.

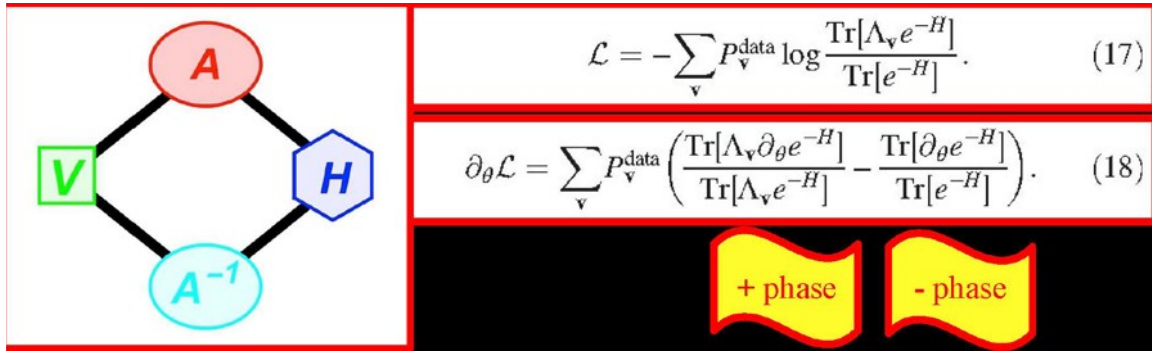


Figure 9: The left shows a graph with visible (V) and hidden (H) units, as well as the connections. In particular, using the equations shown from the quantum BM paper (Eqs. 17 and 18) [9], the geometry for a ncBM makes the ‘-’ phase easy to calculate in many cases.

4.1.4. ncBM

Building on a 2018 paper on quantum BM [9], the PI was able to generalize from having a Hamiltonian for the quantum BM to having a sequence of non-commutative operators, thereby providing a non-commutative construction for a Boltzmann machine for ML, namely a non-communicative Boltzmann machine, ncBM. From [9] the equations numbered (17) and (18) for the log-likelihood is provided by a trace over the Hamiltonian H of the quantum BM. There is also a projection operator $\Lambda_{\mathbf{v}}$ for the + phase of learning, with the projection onto a classical record.

The ncBM has almost the same equations as the quantum BM, except instead of the exponential of the Hamiltonian H entering the traces, the traces have the product of exponentials of non-commutative operators for the visible, hidden, and interconnect portions of the BM graph. As seen in Figure 9 (left), the ncBM may have one very useful concept a quantum BM does not. Namely, for certain graphs connecting visible and hidden units it is possible to make the ‘- phase’ of ML easy to calculate in the ncBM methodology. In general, the ‘- phase’ is difficult to calculate either in classical RBM, particularly in classical general-graph BM, or in quantum BM. The flexibility of ncBMs may be useful to ML in the future. Currently a graduate student, unfunded by this project, is continuing to work with the PI on ncBM studies. Another graduate student, who was partially funded by this project but who is now unfunded, is also working with the PI and co-PI to try to utilize a D-Wave QuAnCo machine to perform the sampling required for a ncBM training paradigm for ML.

4.2 Improving AQC and Teaching Quantum Computing

4.2.1 Improving Future QuAnCo Machines: Better Architectures

One way to improve a QuAnCo machine is to improve the architecture, which means improving the graph connecting qubits. The D-Wave machines to date have all had a Chimera graph architecture, as in Figure 2 and Figure 3. Unfortunately, as shown in [6], a Chimera graph does not allow for a finite T_C , and hence cannot provide ground states for the most complicated spin glasses within the native architecture. An ideal graph would be fully connected, which would allow the native architecture to even provide the ground state of an Ising spin glass with mean-field exponents as well as with a finite T_C . Unfortunately, above a few tens of qubits a fully connected graph is impossible to engineer on a chip. Therefore the PI and a collaborator looked at adding small-world connections to a $D=2$ graph, and showed such a graph had both a finite T_C and mean-field exponents. However, such a graph was still too difficult to engineer on a chip. Therefore, the PI and collaborator were able to constrain the small-world interactions, while keeping a finite T_C and mean-field exponents to design a much more engineerable chip. This work, which was initiated before the granting period, was published in 2018 [10]. Figure 10 shows a figure from that paper, together with its caption. The underlying $D=2$ graph here is a Chimera graph. However, four layers (corresponding to 4 extra fabrication layers when a chip is engineered and processed) have small-world bonds constrained to be at angles of 45° with respect to the $D=2$ lattice directions.

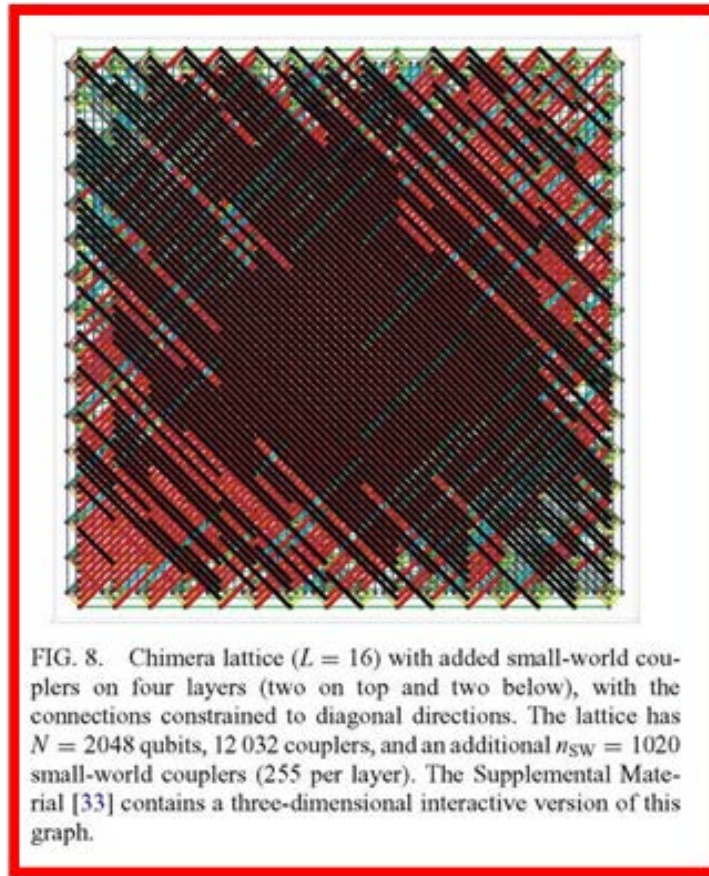


Figure 10: A small-world + Chimera QuAnCo architecture. From [10]. The two extra top layer bonds are shown in black and red. The underlying Chimera graph is in green/blue/cyan, while the bottom two added layers are similar to those of the top two.

4.2.2 Improving Future QuAnCo Machines: #BQP Oracle

Building on a previous paper of the PI and collaborators [7], we tried to design a QuAnCo machine which could provide the number of ground states (the ground state degeneracy) of an Ising spin glass. The computational complexity of this type of problem is labeled as #BQP, because it is a counting problem in BQP (bounded-error quantum polynomial time). All current QuAnCo machines are designed to return **a** ground state, a problem which is in the complexity class NP-hard. The method to devise such a QuAnCo machine for #BQP is sketched in Figure 11. By measuring a quantity δ related to thermalization of a small number of qubits (in the magenta sphere) due to a large number of qubits (in the yellow sphere), one can derive [7] an expression for the expected value of δ as a function of temperature, which in the low temperature limits approaches a value which is a function only of the ground state degeneracies for the two subsystems (in the yellow and magenta spheres).

This expression has been tested by the PI and collaborators, as seen in the upper-right figure, and the real-time quantum simulations agree well with the theory as a function of temperature (the abscissa). Thus, with a graduate student, unfunded by this project, the PI and co-PI have devised a classical computer code to simulate in real time a QuAnCo architecture and protocol to produce the ground state degeneracy of a large number of qubits by measuring an observable such as δ on a small number of qubits. This work is ongoing, with the idea that once δ is measured, from the theoretical curve (bottom-right) one can have a quantum oracle which gives the #BQP result for a large number of qubits.

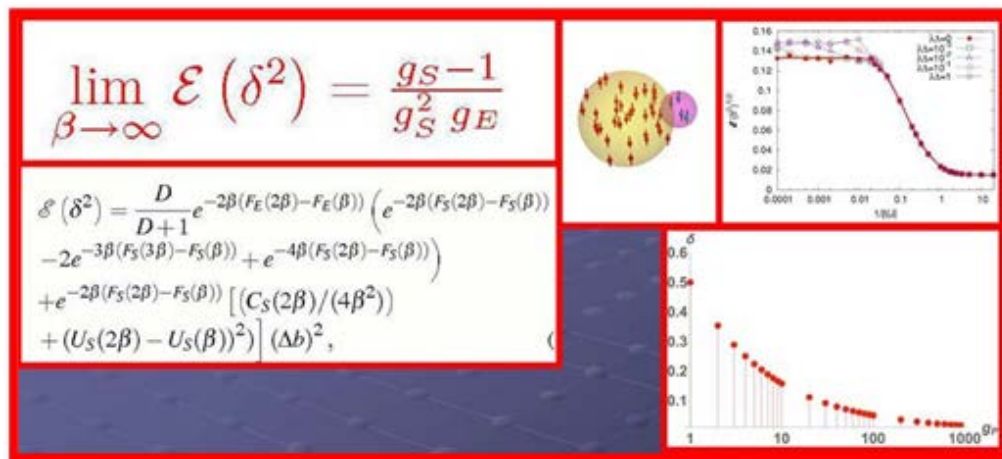


Figure 11: Designing a QuAnCo for #BQP. The theoretical predictions [7] for an observable is utilized to design a novel QuAnCo architecture and protocol. See text for further details.

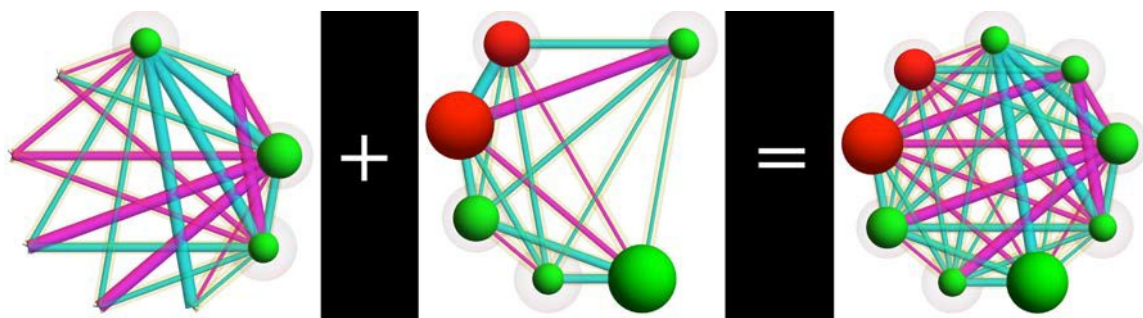


Figure 12: FREM graph composition. Showing the reverse-annealing (left) subgraph and forward-anneal (middle) which combine after to have the complete graph (right).

4.2.3 Improving Future QuAnCo Machines: FREM

The PI and a MSU undergraduate student, partially funded by this project, explored the idea of having a FREM annealing protocol for QuAnCo machines. The idea is to utilize the traditional QuAnCo forward annealing protocol on one part of a subgraph and its Hamiltonian, and the D-Wave reverse annealing protocol on the remainder of the Hamiltonian, to have a FREM procedure which at the end should give a ground state of the Ising spin glass on the complete graph. This procedure is illustrated in Figure 12. As with all QuAnCo machines, an adiabatic procedure must be used, so one must start with a ground state of the initial Hamiltonian to end up with a ground state of the final (the problem) Hamiltonian. FREM devises such a procedure. The problem Hamiltonian breakup can be performed many different ways. We labeled this as ‘error mitigation’ because it would provide a mechanism to test whether or not many different annealing protocols yield the same ground state for the problem Hamiltonian. This type of error mitigation does not require any additional qubit resources, which are the most precious resources of any NISQ quantum computer.

4.2.4 Advancing the Quantum Ready Workforce

Teaching quantum information science, including QuAnCo, to the next generation is important since future generations of engineers and scientists will be using QuAnCo and other quantum computers to solve problems, including those of importance to the warfighter. Therefore, the PI and co-PI promised as part of the funded work to develop a class ‘*Introduction to Quantum Computing*’. During the timeframe of the funding, the PI and co-PI were able to get this class approved through the MSU University Courses and Curriculum Committee. The syllabus for the developed class is shown in Figure 13. After the funding period, the PI and co-PI taught in Fall 2019 this cross-listed (between the Physics department and Electrical and Computer Engineering department) this class for advanced undergraduates and graduate students (both masters-level and PhD-level). The class is shown in Figure 14. For the class we had a textbook [11], and covered the first six chapters of the textbook. However, QuAnCo is not covered in the textbook, or in fact any textbook known to the PIs. Therefore, more than one month was spent teaching QuAnCo from notes and Mathematica programs and animations of the PI and co-PI. The students also had hands-on programming topics, and had to perform programs on both a D-Wave QuAnCo machine and on an IBM gated quantum computer. The students were all good, and all who finished earned either an A or a B in the class.

Syllabus	
Course: PH 4990 / PH 6990 <i>or</i> ECE 4990 / ECE 6990	(Fall 2019)
Section: 01	
Title: Special Topics in Physics:	Introduction to Quantum Computing
Title: Special Topics in ECE:	Introduction to Quantum Computing
Prerequisites: PH 2223 and MA 3113, or consent of Instructors	
Lectures: Tuesday – Thursday 8:00-8:50	
Location: Hilbun Hall, Room 152	
Instructor (PH):	Prof. Mark A. Novotny man40@msstate.edu 662-325-2806 Office Hilbun 131 Office Hour: Thursday 9:00-10:00
Instructor (ECE):	Prof. Yaroslav Koshka

Figure 13: Syllabus for MSU class. The class was approved during the funding period, and co-taught by the PI and co-PI after the end of the funding period.



Figure 14: The Fall 2019 ‘Introduction to Quantum Computing’ class. Shown with the PI (left) and co-PI (right). The subject on the screen is QuAnCo annealing protocols.

4.3 Atypical Disorder in Quantum Systems

One unexpected result was an advance on an open problem in mathematical physics. This advance was due to looking at quantum dragon datasets. The open problem can be seen in Figure 15, from [12], namely the problem of an extended state (extended wave function) in disordered quantum systems. We were able to show for the first time that with *extensive local* disorder there are many systems that have extended states of the discrete Schrödinger equation.



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Open Problems in Mathematical Physics

Extended States

Contributed by: M. Aizenman (Princeton Univ.), Dec. 15, 1998.

Abstract Establish the existence (in some energy range) of extended eigenstates, or continuous spectrum, for linear operators with *extensive* disorder. A prototypical example is the discrete Schroedinger operator with random potential

$$H_\omega = -\Delta + \lambda V_\omega^{(rand)}(x) + U^{(per)}(x)$$

acting in $l^2(\mathbb{Z}^d)$, with:

Delta -- the **discrete Laplacian** (n.n. difference operator),
 $V^{(rand)}$ -- a **random potential**, its values at the sites x forming a collection of *independent identically distributed* random variables, and
 $U^{(per)}$ -- a **periodic potential** (optional, its inclusion is not essential for the problem to be interesting).

Figure 15: Open problem in mathematical physics: extended states. From the url [12].

The PI had previously called such systems quantum dragons [8], but previous quantum dragons had extensive disorder but global correlations. It is this unexpected result that quantum dragons exist for Hamiltonians which have only **local** extensive disorder which allowed the quantum dragon datasets to be of interest.

The quantum dragon datasets, as shown in Figure 4 and Figure 7, were constructed from an underlying square graph. It is also possible to construct the tight-binding Hamiltonian of Figure 5 starting from a $D=2$ hexagonal graph. Placing disorder in a particular fashion then leads to quantum dragons such as shown in Figure 16 ... with AFRL ‘written’ on the hopping terms.

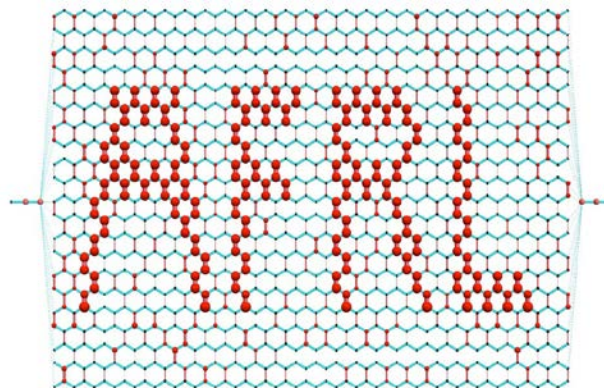


Figure 16: $D=2$ hexagonal graph AFRL quantum dragon. The underlying graph is a $D=2$ hexagonal graph, with $l=52$ vertical slices each with 21 sites, so the graph has 1092 sites. All vertical hopping terms were chosen randomly in $[0,1]$, except for some cut bonds (such as in the lower right). The AFRL was ‘printed’ with hopping terms of strength 2. The onsite energies of the tight binding Hamiltonian were chosen to make the device a quantum dragon. Only two sites of each semi-infinite lead are shown. Radii are proportional to the tight-binding Hamiltonian values.

The existence of the unexpected quantum dragons with *only locally* correlated disorder, but with arbitrarily strong disorder, may have a large impact on the study of nanodevices. For example, consider any tight-binding model based on a $D=2$ hexagonal graph. For a nanodevice composed of only regular hopping strengths and regular on site energies, but composed of carbon atoms, it was known one would have electron transmission probability $T(E)=1$ for all E , which is called ballistic electron propagation. Two examples of such all- carbon regular ballistic nanodevices are the metallic systems of a graphene ribbon with a zigzag boundary condition, and the same ribbon ‘stitched’ into a carbon nanotube (called an armchair nanotube). All researchers who have previously utilized tight-binding models to study such models of pure carbon nanodevices have used translational symmetry, which allows band structure calculations and use of Bloch wave functions. For disordered systems neither of these methods are possible. However, as in [8] these regular systems are addressable via the PI’s mapping technique. The unexpected result that only *locally correlated* disorder was needed to have a quantum dragon should allow for many other quantum nanodevices based on all carbon atoms to have complete electron transmission. Figure 17 shows two examples which would have previously been thought to only have very small electron transmission. The PI, together with collaborator Tomáš Novotný (no relation) from Charles University in the Czech Republic, is currently completing a manuscript on these carbon quantum dragons (and other $D=2$ and $D=3$ quantum dragons) for submission to Physical Review Research. With the prediction of carbon- based quantum dragons which can have many different geometries the onus is on experimentalists to synthesize and measure these interesting predicted nanodevices. For example, in a magnetic field the Möbius strip device of Figure 17 (top) is currently being studied by the PI because it may have novel and unusual electron transport properties ... the traditional model of a current circulating around the magnetic flux in a hole in a device does not apply for a Möbius strip nanodevice.

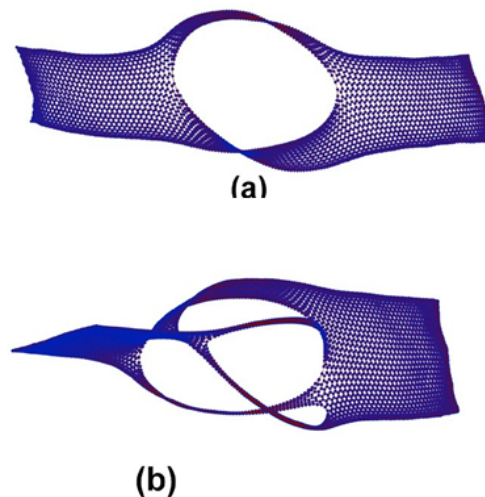


Figure 17: $D=2$ hexagonal graph quantum dragons. These are based on performing manipulations starting from a zigzag carbon nanoribbon (a graphitic sheet). (a) shows a Möbius strip nanodevice, which should have very interesting properties when a magnetic field is applied. (b) shows a more complicated quantum dragon, with ‘braiding’ of the tatty portion of the nanodevice.

5.0 CONCLUSIONS

Early introduction, testing, and education related to NISQ machines is important to national defense, as well as other human endeavors. The project research was to set the stage for future NISQ machines to actually be a useful and efficient calculation tool in solving problems that are unsolvable in a reasonable amount of time on even the largest classical computer.

For such research on NISQ machines to succeed, access to NISQ machines is imperative. For the research performed under this funded project, QuAnCo time on D-Wave machines was instrumental to the success. Simultaneously access to large-scale classical computers is also important for studying future NISQ architectures and algorithms.

The overall assessment of the research project was NISQ machines, including QuAnCo architectures, are still in their infancy. Nevertheless, it is the opinion of the PI and co-PI that these machines will be useful to the endeavors to enhance the efforts and efficiency and protection of the warfighter in the not too distant future --- with timeframes of years rather than decades. Thus considering research area overlaps, as depicted in Figure 1 and Figure 2, are predicted to become increasingly important.

5.1 NISQ D-Wave Machines Applied to RBM for ML

Future research related to ML using AQC machines is imperative. Even though these NISQ machines are only now becoming available, such as the D-Wave QuAnCo architectures. The funded research was to only utilize BM methodologies for ML, whether quantum technologies can be used for other ML paradigms was beyond the scope of the current investigation.

One take-home message is the best sampling for Boltzmann machines is to utilize both traditional methods on classical computers and newly developing methods on available QuAnCo machines. Results from the funded research, and of other researchers, illustrate that current D-Wave architectures are just on the cusp of being useful, and there is a strong possibility future QuAnCo machines will be extremely useful using BMs and RBMs in ML.

Another take-home message is that even the current D-Wave architecture allows image reconstruction and image classification to be at a similar level to that obtained with classical computers. See Figure 8. At least, that is true if the ML algorithm has a small enough number of visible units to fit onto the current QuAnCo architecture.

The algorithms for using QuAnCo architectures for ML are also in their infancy. One question is whether ML using quantum can be different compared to ML for classical computers. One advance in this regards was the introduction of quantum BMs [9] in 2018 by Amin *et al.* The PI's generalization to ncBM may be another way to advance NISQ computers, including QuAnCo, for ML. See Figure 9.

In the future whenever one needs a well-quantified needle-in-a-haystack type of dataset one can make use of a quantum dragon dataset to study ML, for any desired length of data records. See Table. I and Figures 4 and 7. In fact, one could also ask whether a dataset, like quantum dragon datasets, based on quantum mechanics can only be well characterized by ML for quantum

algorithms and quantum machines. This difficult question would be good to have addressed in future investigations.

5.2 Improving AQC and Teaching Quantum Computing

Our studies focused on three ways of improving future QuAnCo machines.

The first was to devise better QuAnCo architectures, namely graphs which can be engineering on a reasonable and scalable fashion (for example, the number of fabrication layers should not increase with the number of qubits) and can on the native graph be a quantum oracle for the most difficult Ising spin glass problems (those with a finite TC and with mean-field exponents). The PI and collaborator [10] were able to introduce constrained small-world graphs which satisfied these requirements. See Figure 10.

The PI and an undergraduate student introduced a FREM protocol for error mitigation for introduction into the instruction set for future QuAnCo releases. The FREM protocol involves using two sub-graphs and their associated Hamiltonians (as in Figure 12), and performing simultaneously forward and reverse annealing. The interesting thing is D-Wave engineers have stated they ‘almost’ have enough control wires to their current chips to be able to implement FREM for some subgraph breakups. The interesting property that this error mitigation procedure does not require any additional qubits, the most precious resource of NISQ machines, should be of interest.

As in Figure 11, the PI, co-PI, and a (unfunded by this project) graduate student began an investigation of how to design an AQC machine to be an oracle for the ground state degeneracy of a spin glass. In other words, how to design a QuAnCo machine to be an oracle for problems in the complexity class #BQP rather than today’s QuAnCo machines which are oracles for problems in the complexity class NP-hard.

The PI and co-PI also devised and worked through the MSU bureaucracy a new split-level dual-department class on ‘Introduction to Quantum Computing’. One goal was to develop notes related to QuAnCo machines, as well as Mathematica programs and simulations to help students understand concepts related to QuAnCo architectures. Current quantum computing textbooks do not go into any depth, and most do not mention, QuAnCo systems. The PI and co-PI co-taught this class after the funding period in the Fall of 2020, see Figures 13 and 14. The conclusion is there is a great deal of interest in such educational opportunities from both the engineering and the physics students. We believe such quantum-ready workforce development will be critical to national security, homeland security, and other areas in the future.

5.3 Atypical Disorder in Quantum Systems

The breakthrough not envisioned in the proposal to AFRL was the emergence of an advance to a solution of an unsolved problem in mathematical physics (see Figure 15), namely the relationship between disorder (which must be extensive, and which can have at most locally correlations) and extended quantum states. The discrete Schrödinger equation was found for ***extensive, local*** disorder to allow extended wave functions (rather than localized wave functions). The tight-binding Hamiltonian of Figure 5, which is a traditional Hamiltonian for electron transport, was studied. The goal of the study was to develop a quantum dragon needle-in-a-haystack dataset (see Table I) to test ML for both classical and QuAnCo implementations. This built on the PI's former studies of quantum dragon nanodevices [8], which allowed extensive disorder which was globally correlated to have extended wave functions (as in Figure 6). The new, unexpected development allowed for building quantum dragon datasets with the 64 bit lengths required for an RBM architecture on current D-Wave QuAnCo machines (as in Figure 4), but could have much larger record lengths (as would be the case in Figure 7), which are built on $D=2$ square graphs. These newly discovered quantum dragons with *locally* correlated, *extensive* disorder can also be devised on a $D=2$ hexagonal graph (Figure 16), and may be of interest in quantum nanodevices and quantum sensing as perhaps in the novel configuration of all- carbon quantum dragon nanodevices predicted as in Figure 17. Thus the unexpected result of quantum dragons with only locally correlated disorder may lead to novel nanodevices with novel quantum properties, a completely unanticipated result.

Whether this quantum wave property carries over to other coherent waves is an important question to ask, and has far-reaching potential applications. For example, could changing only locally parameters allow a disordered system to be invisible to coherent sound waves in a wavelength range extending to $+\infty$ (with applications to sonar)? Similarly, could changing only locally parameters allow a disordered system to be invisible to coherent plane-wave electromagnetic waves in a wavelength range extending to $+\infty$ (with applications to cloaking against radar)? These questions would not have been asked previously, because it was believed invisibility to an extended wavelength range required systems to be regular (uniform or crystalline) or at least have globally correlated disorder.

6.0 REFERENCES

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7.0 LIST OF SYMBOLS, APPREVIATIONS, AND ACRONYMS

LIST OF SYMBOLS, ABBREVIATIONS AND ACRONYMS

AQC	Adiabatic Quantum Computer
BAS	Bars and Stripes patterns
BM	Boltzmann Machine
D	Dimension of space
E	Electron Energies
FREM	Forward-Reverse Error Mitigation (annealing protocol)
HPC ²	High Performance Computing Collaboratory
LV	Local Valley
MCMC	Markov Chain Monte Carlo (Gibbs sampling)
ML	Machine Learning
MNIST	MNIST dataset of handwritten digits
MSU	Mississippi State University
N	Number of qubits
NASA/QuAIL	NASA Quantum Artificial Intelligence Laboratory
ncBM	Non-Commutative Boltzmann Machine
NISQ	Noisy Intermediate-Scale Quantum
PI	Principle Investigator
QuAnCo	Quantum Annealing Computer
RBM	Restricted Boltzmann Machine
SW	Small World (connections of graphs)
$\mathcal{T}(E)$	Electron transmission probability at energy E
T_C	Critical temperature (for Ising spin glass)
USRA	Universities Space Research Association

Note: Adding a small s to any abbreviation/acronym makes it plural.