

REPORT DOCUMENTATION PAGE			Form Approved OMB NO. 0704-0188		
<p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</p>					
1. REPORT DATE (DD-MM-YYYY) 18-12-2019		2. REPORT TYPE Final Report		3. DATES COVERED (From - To) 1-Sep-2014 - 31-Aug-2019	
4. TITLE AND SUBTITLE Final Report: PECASE (topic 1.3.2): The Case for Morphologically Modulated Dynamics			5a. CONTRACT NUMBER W911NF-14-1-0573		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER 611102		
6. AUTHORS			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES University of Michigan - Ann Arbor 3003 South State Street  Ann Arbor, MI 48109 -1274			8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS (ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSOR/MONITOR'S ACRONYM(S) ARO		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S) 65587-EG.15		
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:		17. LIMITATION OF ABSTRACT		15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU	UU		Shai Revzen
				19b. TELEPHONE NUMBER 734-763-3561	

**RPPR Final Report**  
as of 07-Jan-2020

Agency Code:

Proposal Number: 65587EG

**Agreement Number: W911NF-14-1-0573**

**INVESTIGATOR(S):**

**Name:** Shai Revzen  
**Email:** shrevzen@umich.edu  
**Phone Number:** 7347633561  
**Principal:** Y

Organization: **University of Michigan - Ann Arbor**

Address: 3003 South State Street, Ann Arbor, MI 481091274

Country: USA

DUNS Number: 073133571

EIN: 386006309

**Report Date:** 30-Nov-2019

Date Received: 18-Dec-2019

**Final Report** for Period Beginning 01-Sep-2014 and Ending 31-Aug-2019

**Title:** PECASE (topic 1.3.2): The Case for Morphologically Modulated Dynamics

**Begin Performance Period:** 01-Sep-2014

**End Performance Period:** 31-Aug-2019

**Report Term:** 0-Other

Submitted By: Michele Feldkamp

Email: careymrz@umich.edu

Phone: (734) 647-1813

**Distribution Statement:** 1-Approved for public release; distribution is unlimited.

**STEM Degrees:** 1

**STEM Participants:** 2

**Major Goals:** The objective of the project is to provide predictive models of animal and robot locomotion. We are focused on capturing the robustness and survivability of the most reliable terrestrial locomotion systems known—land-based arthropods—and transferring this essence into new mathematical representations and instantiation techniques, resulting in robotic technologies that move in fundamentally more robust ways than existing legged systems (be they for transport of soldiers and materiel in the field, or for improved prosthetic devices back home).

This work builds on fundamental advances in the theory of hybrid dynamical systems and coupled oscillators (informed by insights gained through a previous ARO YIP grant). The initial phases of this work consisted of publishing results from previous work and applying these findings to establish a new robot platform for further experimentation. We have continued to advance our methods for identifying the body dynamics of robots and animals while developing more advanced forms of instantiating and modulating dynamics. We also continued work on “connection-based” models. These methods form the foundation of a coherent methodology for producing robots whose body morphology contributes to simplifying their control, and whose software can identify and re-instantiate desired behaviors if they fail.

Our approach is grounded in oscillator theory, and accounts for both the crucial role of body morphology and mechanics in the dynamics of motion, and the interactions of the body with the environment. This fundamental progress in anchoring templates is proving to be a novel approach to control, applicable anywhere the control of high-dimensional nonlinear systems is of interest to the Army. Moreover, theoretical and computational advances in the theory of oscillators and their application may impact a range of engineering fields outside biomechanics and robotics, including vibration mitigation in mechanical and civil engineering and power systems engineering. It also shows promise in other fields, such as econometrics, where these advances could aid in the analysis of seasonality in markets.

**Accomplishments:** Used our enhanced motion tracking arena to collect new data sets on BigANT 6, Enepod (highly flexible legs, for investigating dynamic motion), and Multipods (especially pertaining to gaits and backbone morphology options in 12-legged Multipod).

Built “Enepod,” a robot that implements the measurement of dominant energy flows within the body of the robot, significantly enhancing the robot’s ability to recover from damage. The deformations of the Enepod’s highly flexible legs can be measured directly using motion tracking systems improved through this grant. The results were used to develop a novel method for allowing robots to recover locomotion ability after being damaged (featured in chapter 2 of Council PhD thesis).

## RPPR Final Report as of 07-Jan-2020

Extended previous work on event-selected vector fields, showing that a broad class of hybrid oscillators exhibit the same topological dynamics as smooth oscillators through a piecewise smooth conjugacy. Demonstrated that the interaction of legs with the ground can, in and of itself, serve to synchronize legs, without need for any additional control and implemented this in a version of the RHex robot. Work on implementation of these findings relied strongly on robot components acquired through this grant and related DURIP ARO grant #W911NF-17-1-0243. (See Council PhD thesis section 3.9.)

Finished work on implementing tools for numerical computation of Bouligand derivatives of the flows of event selected vector fields (see Council PhD these section 3.5–3.7).

Documented how event-selected controllers can be used to produce reference trajectory tracking with reduced proprioceptive requirements and lower bandwidth than conventional controllers. (See section 3.8 of Council PhD thesis.)

Developed and expanded the “Multipod” family of robots (see Figure 1, left side; and Figure 5). Their modular design extends from four to 12 legs, enabling us to examine the oscillator-template covering relationship and how gaits map up and down the complexity hierarchy. Used enhanced motion tracking arena in conjunction with these robots to prepare datasets for work under this grant.

Furthered advanced RHex and BigANT robot development. This included applying trajectory-based control and gait recovery to a simplified model of the RHex robot, and developing thermal safety circuits for individual RHex leg motors (as these can draw 6kW), preparatory to pursuing more ambitious gait development. Continued work on the extended water-resistant corrugated-plastic plate and reinforced flexure (PARF) fabrication methodology developed under this grant.

Integrated force- and torque-transducers (purchased under DURIP Grant #W911NF-17-1-0243) into a hexapedal robot, as proposed under the DURIP grant (see Figure 1, right side). These are intended for collecting data to investigate the surprising success of the multi-legged geometric mechanics motion model developed over the last two years.

**Training Opportunities:** Continued support of Graduate student, George Council, who finished his PhD in December, 2019.

Continued support of Graduate student, Dan Zhao.

## RPPR Final Report as of 07-Jan-2020

**Results Dissemination:** Bittner, B., Hatton, R., & Revzen, S., (2018). Geometrically optimal gaits: a data-driven approach. *Nonlinear Dynamics*. <https://dx.doi.org/10.1007/s11071-018-4466-9>

Bittner, B., & Revzen, S., Optimizing gaits for coverage on lie groups, *Dynamic Walking 2019*, Calgary, AB, Canada, June 2019.

Council, G., Data-driven methods to build robust legged robots, PhD thesis, University of Michigan, 2019.

Council, G., & Revzen, S. Energy and phased-based gait recovery, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.

Council, G., & Revzen, S. (2019, June). Energy and phased-based gait recovery. Retrieved from <https://sinews.siam.org/Details-Page/energy-and-phased-based-gait-recovery-2>

Eldering, J., Kvalheim, M., & Revzen, S. (2018). Global linearization and fiber bundle structure of invariant manifolds. *Nonlinearity*, 31(9), 4202. <https://dx.doi.org/10.1088/1361-6544/aaca8d>

Fitzner, I., Sun, Y., Sachdeva, V., & Revzen, S. (2017). Rapidly prototyping robots: Using plates and reinforced flexures. *IEEE Robotics & Automation Magazine*, 24(1), 41-47.

Kvalheim, M., Bittner, B., & Revzen, S., (2019). Gait modeling and optimization for the perturbed Stokes regime. *Nonlinear Dynamics*. <https://dx.doi.org/10.1007/s11071-019-05121-3>

Kvalheim, M., Bittner, B., Revzen, S., Reduced-order models for locomotion in the perturbed Stokes regime, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.

Kvalheim, M., & Revzen, S. (2019). Existence and uniqueness of global Koopman eigenfunctions for stable fixed points and periodic orbits. arXiv preprint arXiv:1911.11996

Revzen, S. (2019). Moving with more legs is different: a geometric mechanics perspective. *Integrative and Comparative Biology*, 59, pp. E191–E191

Revzen, S., How many legs become a snake? CMU-RI 2019, Carnegie Mellon University, Pittsburgh, PA, July 25, 2019.

Revzen, S., Geometric mechanics and robots with multiple contacts, LCSR Seminar, John Hopkins University, Baltimore, MD, April 24, 2019.

Revzen, S., Council, G., & Kvalheim, M., Is legged locomotion almost smooth? *Dynamic Walking 2019*, Calgary, AB, Canada, June 2019.

Revzen, S. & Kvalheim, M., Hybrid oscillators: Phase and amplitude in a class of non-smooth systems, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.

Revzen, S. & Wu, Z., Viscous friction-like relationship arises from a simple Coulomb friction locomotion model, *APS Meeting Abstracts*, 2019.

Wu, Z., Zhao, D., & Revzen, S. (2019). Coulomb friction crawling model yields linear force-velocity profile. *Journal of Applied Mechanics*, 85(5), pp. 054501-1–054501-6

Sun, Y. and Revzen., S. A hexapedal robot designed for ground contact force analysis, *Dynamic Walking 2019*, Calgary, AB, Canada, June 2019.

Zhao., D., Li., H., & Revzen., S., Design and undulatory gait tests of a meter-size modular centipede robot, *Dynamic Walking 2019*, Calgary, AB, Canada, June 2019.

**Honors and Awards:** Nothing to Report

**Protocol Activity Status:**

**RPPR Final Report**  
as of 07-Jan-2020

**Technology Transfer:** Nothing to Report

**PARTICIPANTS:**

**Participant Type:** PD/PI

**Participant:** Shai Revzen

**Person Months Worked:** 2.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**Participant Type:** Graduate Student (research assistant)

**Participant:** George Council

**Person Months Worked:** 1.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**Participant Type:** Graduate Student (research assistant)

**Participant:** Dan Zhao

**Person Months Worked:** 6.00

**Funding Support:**

Project Contribution:

International Collaboration:

International Travel:

National Academy Member: N

Other Collaborators:

**CONFERENCE PAPERS:**

**Publication Type:** Conference Paper or Presentation

**Publication Status:** 1-Published

**Conference Name:** 2019 SIAM Conference on Applications of Dynamical Systems

Date Received: 17-Dec-2019      Conference Date: 19-May-2019      Date Published: 21-Jun-2019

Conference Location: Snowbird, Utah

**Paper Title:** Energy and Phased-Based Gait Recovery

**Authors:** George Council, Shai Revzen

Acknowledged Federal Support: **Y**

**DISSERTATIONS:**

**Publication Type:** Thesis or Dissertation

**Institution:** University of Michigan

Date Received: 17-Dec-2019      Completion Date: 11/13/19 9:19PM

**Title:** Data-Driven Methods to Build Robust Legged Robots

**Authors:** George Council

Acknowledged Federal Support: **N**

**RPPR Final Report**  
as of 07-Jan-2020

# PROJECT SUMMARY - GRANT #W911NF-14-1-0573

*(Reporting Period: September 2018 – August 2019)*

## *Morphologically Modulated Dynamics*

Shai Revzen

EECS

U. Michigan, Ann Arbor, MI, 48109

## Major goals

The objective of the project is to provide predictive models of animal and robot locomotion. We are focused on capturing the robustness and survivability of the most reliable terrestrial locomotion systems known—land-based arthropods—and transferring this essence into new mathematical representations and instantiation techniques, resulting in robotic technologies that move in fundamentally more robust ways than existing legged systems (be they for transport of soldiers and materiel in the field, or for improved prosthetic devices back home).

This work builds on fundamental advances in the theory of hybrid dynamical systems and coupled oscillators (informed by insights gained through a previous ARO YIP grant). The initial phases of this work consisted of publishing results from previous work and applying these findings to establish a new robot platform for further experimentation. We have continued to advance our methods for identifying the body dynamics of robots and animals while developing more advanced forms of instantiating and modulating dynamics. We also continued work on “connection-based” models. These methods form the foundation of a coherent methodology for producing robots whose body morphology contributes to simplifying their control, and whose software can identify and re-instantiate desired behaviors if they fail.

Our approach is grounded in oscillator theory, and accounts for both the crucial role of body morphology and mechanics in the dynamics of motion, and the interactions of the body with the environment. This fundamental progress in anchoring templates is proving to be a novel approach to control, applicable anywhere the control of high-dimensional nonlinear systems is of interest to the Army. Moreover, theoretical and computational advances in the theory of oscillators and their application may impact a range of engineering fields outside biomechanics and robotics, including vibration mitigation in

mechanical and civil engineering and power systems engineering. It also shows promise in other fields, such as econometrics, where these advances could aid in the analysis of seasonality in markets.

## Accomplished

- Used our enhanced motion tracking arena to collect new data sets on BigANT 6, Enepod (highly flexible legs, for investigating dynamic motion), and Multipods (especially pertaining to gaits and backbone morphology options in 12-legged Multipod).
- Built “Enepod,” a robot that implements the measurement of dominant energy flows within the body of the robot, significantly enhancing the robot’s ability to recover from damage. The deformations of the Enepod’s highly flexible legs can be measured directly using motion tracking systems improved through this grant. The results were used to develop a novel method for allowing robots to recover locomotion ability after being damaged (featured in chapter 2 of Council PhD thesis).
- Extended previous work on event-selected vector fields, showing that a broad class of hybrid oscillators exhibit the same topological dynamics as smooth oscillators through a piecewise smooth conjugacy. Demonstrated that the interaction of legs with the ground can, in and of itself, serve to synchronize legs, without need for any additional control and implemented this in a version of the RHex robot. Work on implementation of these findings relied strongly on robot components acquired through this grant and related DURIP ARO grant #W911NF-17-1-0243. (See Council PhD thesis section 3.9.)
- Finished work on implementing tools for numerical computation of Bouligand derivatives of the flows of event selected vector fields (see Council PhD these section 3.5–3.7).
- Documented how event-selected controllers can be used to produce reference trajectory tracking with reduced proprioceptive requirements and lower bandwidth than conventional controllers. (See section 3.8 of Council PhD thesis.)
- Developed and expanded the “Multipod” family of robots (see Figure 1, left side; and Figure 5). Their modular design extends from four to 12 legs, enabling us to examine the oscillator-template covering relationship and how gaits map up and down the complexity hierarchy. Used enhanced motion tracking arena in conjunction with these robots to prepare datasets for work under this grant.
- Furthered advanced RHex and BigANT robot development. This included applying trajectory-based control and gait recovery to a simplified model of the RHex robot, and developing thermal safety circuits for individual RHex leg motors

(as these can draw 6kW), preparatory to pursuing more ambitious gait development. Continued work on the extended water-resistant corrugated-plastic plate and reinforced flexure (PARF) fabrication methodology developed under this grant.

- Integrated force- and torque-transducers (purchased under DURIP Grant #W911NF-17-1-0243) into a hexapedal robot, as proposed under the DURIP grant (see Figure 1, right side). These are intended for collecting data to investigate the surprising success of the multi-legged geometric mechanics motion model developed over the last two years.

## Training

- Continued support of Graduate student, George Council, who finished his PhD in December, 2019.
- Continued support of Graduate student, Dan Zhao.

## Dissemination

- Bittner, B., Hatton, R., & Revzen, S., (2018). Geometrically optimal gaits: a data-driven approach. *Nonlinear Dynamics*.  
<https://dx.doi.org/10.1007/s11071-018-4466-9>
- Bittner, B., & Revzen, S., *Optimizing gaits for coverage on lie groups*, Dynamic Walking 2019, Calgary, AB, Canada, June 2019.
- Council, G., *Data-driven methods to build robust legged robots*, PhD thesis, University of Michigan, 2019.
- Council, G., & Revzen, S. *Energy and phased-based gait recovery*, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.
- Council, G., & Revzen, S. (2019, June). *Energy and phased-based gait recovery*. Retrieved from  
<https://sinews.siam.org/Details-Page/energy-and-phased-based-gait-recovery-2>
- Eldering, J., Kvalheim, M., & Revzen, S. (2018). Global linearization and fiber bundle structure of invariant manifolds. *Nonlinearity*, 31(9), 4202.  
<https://dx.doi.org/10.1088/1361-6544/aaca8d>
- Fitzner, I., Sun, Y., Sachdeva, V., & Revzen, S. (2017). Rapidly prototyping robots: Using plates and reinforced flexures. *IEEE Robotics & Automation Magazine*, 24(1), 41-47.
- Kvalheim, M., Bittner, B., & Revzen, S., (2019). Gait modeling and optimization for the perturbed Stokes regime. *Nonlinear Dynamics*.  
<https://dx.doi.org/10.1007/s11071-019-05121-3>
- Kvalheim, M., Bittner, B., Revzen, S., *Reduced-order models for locomotion in the perturbed Stokes regime*, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.

- Kvalheim, M., & Revzen, S. (2019). *Existence and uniqueness of global Koopman eigenfunctions for stable fixed points and periodic orbits*. arXiv preprint arXiv:1911.11996
- Revzen, S. (2019). Moving with more legs is different: a geometric mechanics perspective. *Integrative and Comparative Biology*, 59, pp. E191–E191
- Revzen, S., *How many legs become a snake?* CMU-RI 2019, Carnegie Mellon University, Pittsburgh, PA, July 25, 2019.
- Revzen, S., *Geometric mechanics and robots with multiple contacts*, LCSR Seminar, John Hopkins University, Baltimore, MD, April 24, 2019.
- Revzen, S., Council, G., & Kvalheim, M., *Is legged locomotion almost smooth?* Dynamic Walking 2019, Calgary, AB, Canada, June 2019.
- Revzen, S. & Kvalheim, M., *Hybrid oscillators: Phase and amplitude in a class of non-smooth systems*, 2019 SIAM Conference on Applications of Dynamical Systems, May 2019.
- Revzen, S. & Wu, Z., *Viscous friction-like relationship arises from a simple Coulomb friction locomotion model*, APS Meeting Abstracts, 2019.
- Wu, Z., Zhao, D., & Revzen, S. (2019). Coulomb friction crawling model yields linear force-velocity profile. *Journal of Applied Mechanics*, 85(5), pp. 054501-1–054501-6
- Sun, Y. and Revzen., S. *A hexapedal robot designed for ground contact force analysis*, Dynamic Walking 2019, Calgary, AB, Canada, June 2019.
- Zhao., D., Li., H., & Revzen., S., *Design and undulatory gait tests of a meter-size modular centipede robot*, Dynamic Walking 2019, Calgary, AB, Canada, June 2019.

## Honors

—na—

## Tech Transfer

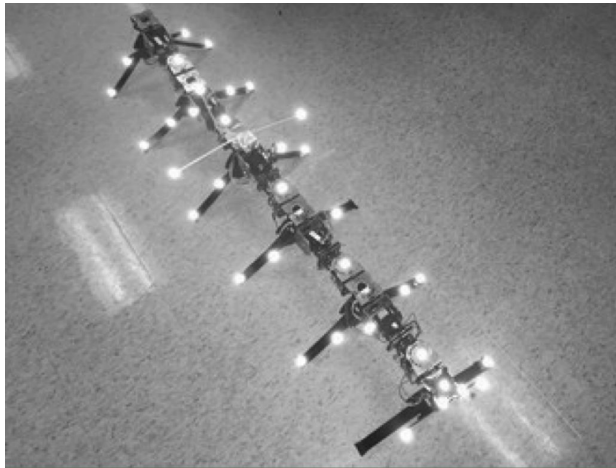
—na—

## Products

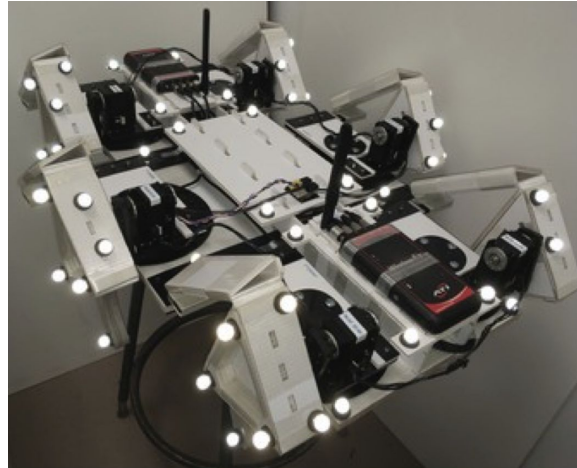
- “Multipod” family of robots: Modular design extends from four to 12 legs, enabling investigation of how gaits map up and down the complexity hierarchy
- Advanced RHex robot: A simplified model of the RHex robot enhanced with trajectory-based control and gait recovery, thermal safety circuits for individual RHex leg motors, new 3D printed leg mounts with specialized electronics that optically couple leg contact sensors to the main body of the robot

- “Enepod”: Implements the measurement of dominant energy flows within the body of the robot, significantly enhancing the robot’s ability (and developing novel methods) to recover from damage
- BigANT 6: Water-resistant corrugated-plastic plate and reinforced flexure (PARF) construction, with integrated force- and torque-transducers
- Used our enhanced motion tracking arena to collect new data sets on BigANT 6, Enepod (highly flexible legs, for investigating dynamic motion), and Multipods (especially pertaining to gaits and backbone morphology options in 12-legged Multipod)

## FIGURES



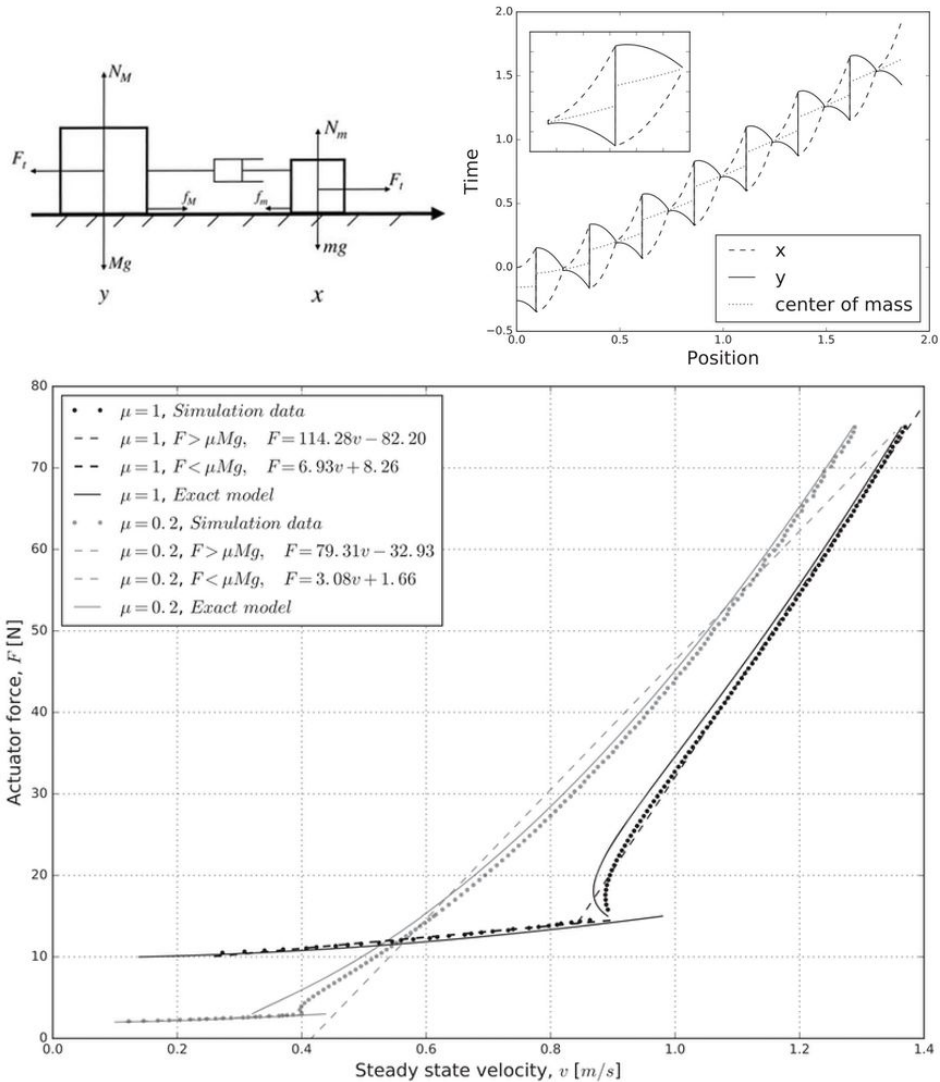
2019 new data; complete



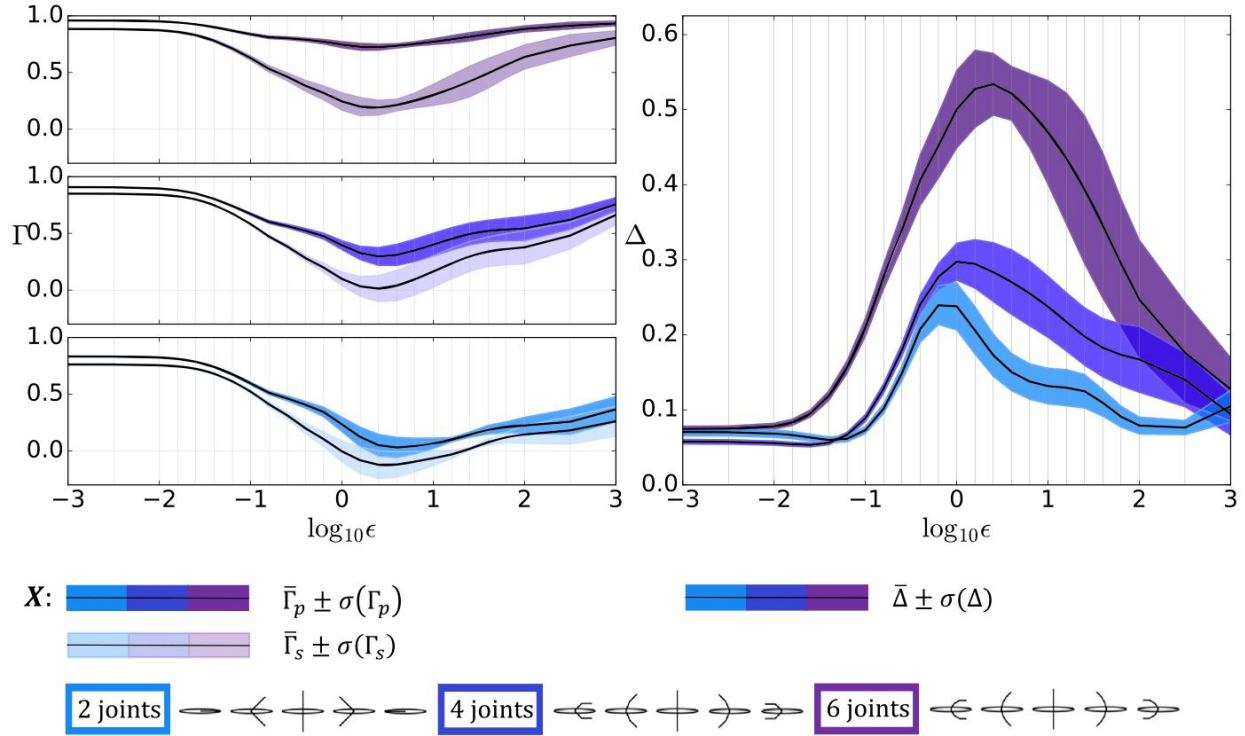
2019 new data; in progress

*Figure 1. Robotic platforms producing model data. Multipod with 12 legs used to study gaits at various levels of morphological complexity (left); Mechapod instrumented with force and torque measurement capabilities (right).*

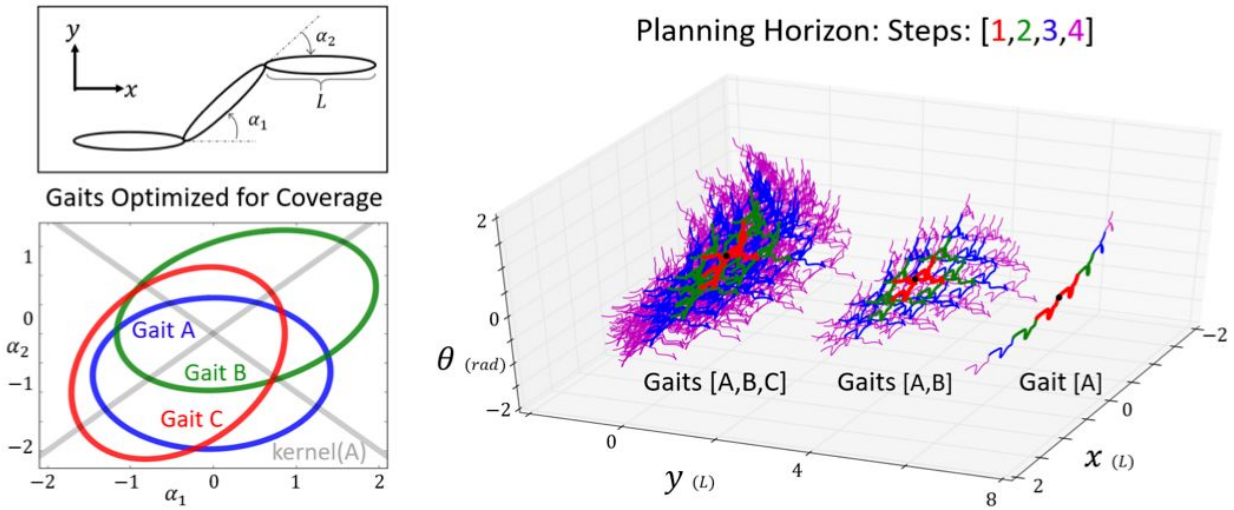
**Figure 2. upper left:** Schematic representation of the mechanical model.  $M$  and  $m$  are the larger and smaller point masses, respectively, with  $y$  and  $x$  their positions. The friction forces acting on the point masses are  $f_m$  and  $f_M$ .  $F_t$  is the actuator force, which is positive when pushing the masses apart and negative when pulling them together.  $g$  is the gravitational



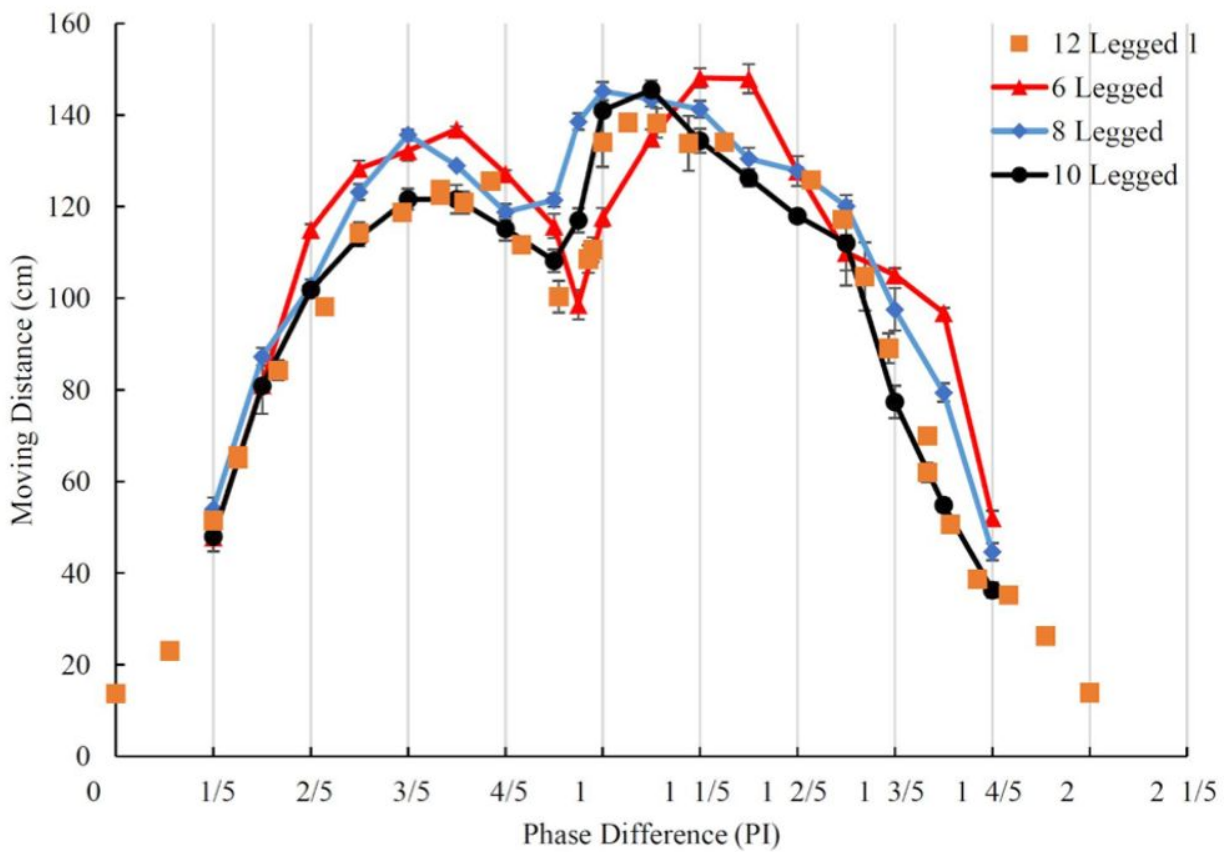
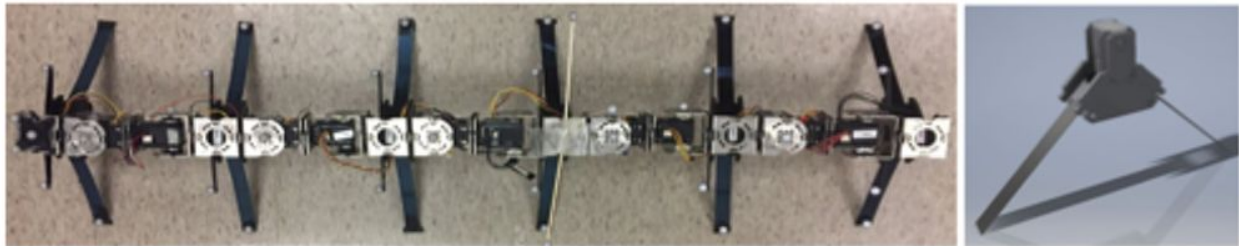
acceleration. **upper right:** Position of  $m$ ,  $M$ , and the center of mass versus time in an example simulation. The positions of  $m$  (dashed line),  $M$  (solid line), and center of mass  $(mx+My)/(m+M)$  (dotted line) over time are shown. Inset graph shows an enlarged cycle starting at  $y - x = L_{min}$ . **bottom:** Constant force versus steady-state velocity under Coulomb friction model,  $\mu=1$ , and  $\mu=0.2$ . Our notable result is that in each regime, the  $F$  to  $v_{ss}$  relationship can be described fairly accurately as linear, as demonstrated by the high values of regression  $R^2$ . Note that intermediate values of  $\mu$  interpolate those plotted and were omitted for clarity.



**Figure 3.** Comparison of model prediction quality when using the perturbed Stokes regressors versus the Stokes regressors on paddles with different dimensions of the shape space, shown in terms of the  $\Gamma$  and  $\Delta$  quality metrics. We plotted  $\Gamma$  and  $\Delta$  of three swimmers with different numbers of paddle segments: one segment per paddle (light blue), two segments (blue), and three segments (purple). We used a symmetric flapping gait. The paddles moved symmetrically with total angles of all joints summing up to a sinusoid of amplitude  $\pi$ . We plot the  $X$  components of  $\Gamma_p$  (left column; one plot per model; saturated colors  $\Gamma_p$ ; pale colors  $\Gamma_s$ ) and  $\Delta$  (right column). Results show that over the two orders of magnitude range of  $10^{-0.5} < \epsilon < 10^{1.5}$ , the perturbed Stokes regressors consistently provide improvements. The relative improvement  $\Delta$  increased markedly with shape space dimension, by as much as 0.5 in  $\Delta$ .

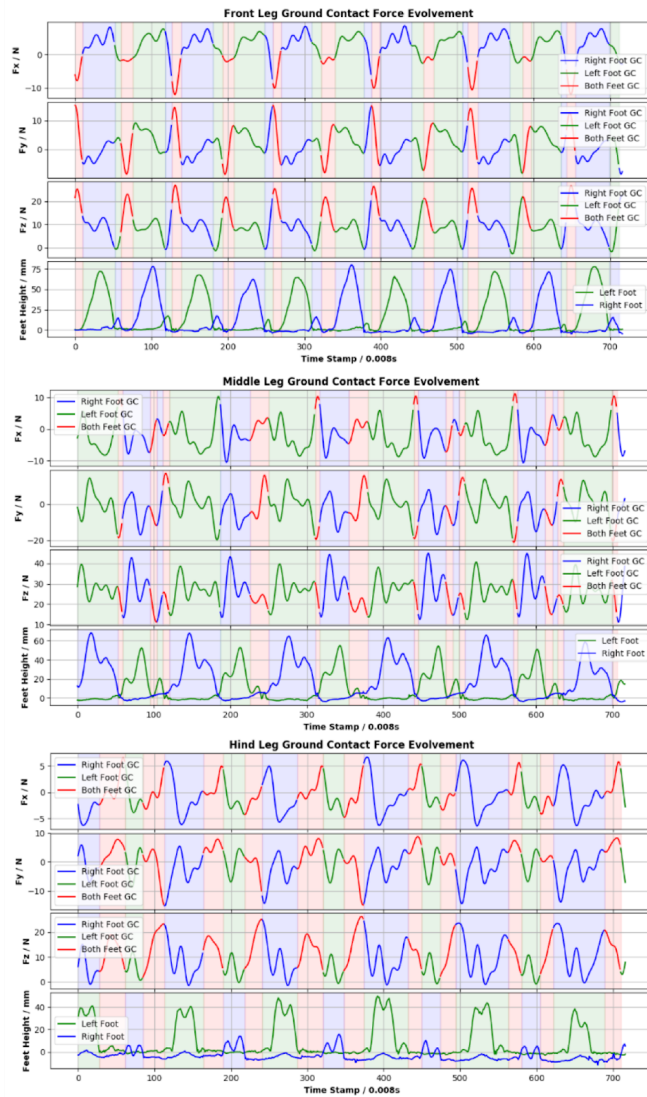
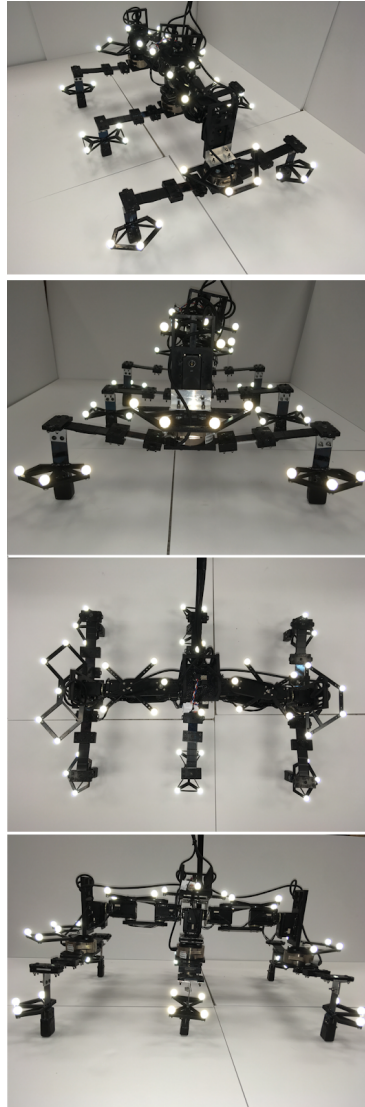


**Figure 4.** We constructed  $G$  from randomly sampling 100 points of unit ballon  $SE(2)$ . Ten iterations of gradient descent on the loss function (seeded with a unit circle gait), resulted in the gaits (bottom left) for the 3-link Purcell swimmer (top left). Gaits can be connected along the kernel of  $A(\cdot)$  (grey), preserving the exactness of the four step planner (right). The two and one gait planners are shifted in  $y$  by  $5L$  and  $7L$  respectively.



**Figure 5.** *top:* Multipod robot and leg segment CAD. **bottom:** Moving distance at 5 strides ( $N=3$  trials) vs. phase difference between segments. Different numbers of legs produce similar motion.

**Figure 6.** *left:* The constitutive equations governing multi-legged contact with the ground, although known in theory, are rarely used in practice. This hexapodal robot is equipped with integrated



force- and torque-transducers, allowing us to track the actual contact forces and validate and extend this new class of models. **right:** Ground contact force evolution. The colors represent the type of ground contact (left foot: green; right foot: blue; both feet: red). Subfigures illustrate the ground contact forces decomposition along x, y, z-axis of the world frame. The bottom chart in each plot shows the height of the feet.