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FINAL REPORT: TRANSPORT THEORY FOR PROPAGATION AND REVERBERATION

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INTRODUCTION AND LONG-TERM GOALS

Propagation or reverberation modeling is important for many naval applications, and this project is dedicated to improving the accuracy of such modeling under realistic conditions. Propagation and reverberation modeling are typically accomplished using ray tracing, normal mode, or energy flux methods, with PE a common option for propagation. However, these methods usually rely on approximating the effects of forward scattering from roughness at the sea surface or sea floor by using a boundary reflection loss, or by simply ignoring these effects. In order to more accurately treat the full complexity of the environment, approximate wave method (such as rough surface PE) or full wave methods (such as the Boundary Element Method (BEM)) could be employed, and a “Monte Carlo” approach can in principle be used with realizations of rough boundaries. The average field or average intensity, as well as higher statistics of the field, can then be obtained through averaging results over an ensemble of realizations. The computational burden, however, makes such approaches only suitable for obtaining “benchmark” solutions for comparison with other methods.

Transport theory, a fast computational method, has been under continuing development, and as implemented it can account for the effects of forward scattering from the sea surface in both propagation and reverberation for frequencies up to the mid-frequency range (e.g., at least up to 3 kHz). The term transport theory applies to any method that attempts to develop evolution equations for the moments (or averages) of the field, and it has been applied to diverse topics quite separate from acoustics. Our approach is based on expanding the acoustic field in modes, and therefore would most readily apply at mid-frequencies and below, and in relatively shallow water environments such as on the continental shelf. With transport theory, the need for a Monte Carlo approach with computations using many rough surface realizations is avoided, and the average intensity associated with each mode amplitude is propagated directly. The description of the rough sea surface is embodied in the spectrum of the surface roughness, and the effect of scattering from the rough surface is to lead to mode coupling that can affect both transmission loss in propagation and the reverberation level.

An example illustrating these points is shown in Figure 1. For this case the frequency is 3 kHz, the rough sea surface is modeled with an isotropic Pierson-Moskowitz roughness spectrum for a wind speed of 7.7 m/s (15 knots) giving an rms wave height of 0.31 m (or a significant wave height of 1.2 m), the sound speed is taken as isovelocity at 1500 m/s, and the bottom roughness is described by a typical roughness model for a medium sand bottom with a sound speed of 1600 m/s. The water depth is 50 m.

Figure 1 shows two sets of reverberation curves out to a time of 60 s, the lower set of three curves is for surface reverberation only, and the upper set is for surface and bottom reverberation. For the lower set the bottom is taken as flat with no roughness. It is evident for this isovelocity case with typical surface and bottom roughness that bottom reverberation is dominant and the surface contribution can be neglected. The upper set of curves can be considered due to bottom reverberation alone. Three ways of modeling the reverberation are shown: ignoring effects of surface forward scattering (red curve), using the coherent reflection loss for the surface interaction (green curve), and fully accounting for the effects of forward scattering with transport theory (blue curve). The differences between the three ways of modeling the reverberation are significant and can exceed 10 dB for this example. It should be noted that transport theory results show that the reverberation level is much more sensitive to surface forward scattering than one-way transmission loss.

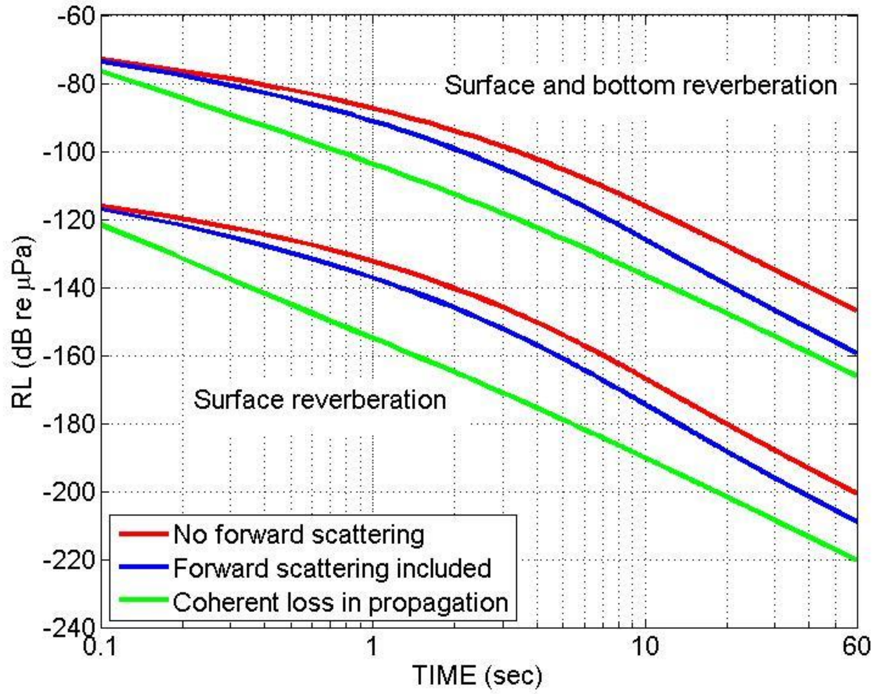

Figure 1. Reverberation predictions at 3 kHz obtained with transport theory. The red curves ignore all effects of boundary roughness during propagation. The blue curves account for surface forward scattering. The green curves approximate the effect of surface forward scattering in terms of a coherent loss.

The results shown in Figure 1 imply that significant modifications may be needed to existing reverberation modeling methods for naval applications, since uncertainties of 10 dB or even greater should not be ignored. However, it should also be recognized that there are approximations that enter into our transport theory formulation, and it is important to understand if these approximations significantly impact the reverberation results shown in Figure 1. These approximations fall into two classes. First, transport theory propagation has been formulated in two space dimensions (range and depth), which means that the rough surface is treated as one-dimensional. For propagation modeling, all forward scattering will be in a vertical range-depth plane, and forward scattering out of that plane will not be included. For reverberation modeling with transport theory, the full three-dimensional geometry is employed approximately. If the roughness spectrum is not isotropic, propagation paths out to and back from the reverberant scattering patch are run on multiple radials (the commonly used "N by 2-D" method), and for an isotropic roughness spectrum a single radial is sufficient. Second, there are approximations required to develop a transport method in two space dimension that will model effects of rough surface forward scattering. Prominent among the approximations are (1) that the propagation equations of the mode amplitude intensities are fully linearized in the surface height and (2) the neglect of cross-mode coherences (the "Dozier-Tappert" approximation) in the propagation equations.

The long range goals for this project include confirmation of transport theory accuracy along two main tracks: 1) comparison with experiments, and (2) comparison with more accurate numerical methods. If these approaches indicate that modifications of our transport theory method are needed to improve accuracy, that would be undertaken. There are several additional long range goals for this project. Our present version of transport theory has been formulated for a range-independent environment, aside from the surface roughness. This is the most natural environment for a method based on normal modes, but is not sufficient for modeling under realistic conditions. Therefore, another long range goal is to modify the mode-based approach to allow for range dependence in the sound speed profile and water depth. This will lead to mode coupling due to the range dependence of the environment in addition to the sea surface forward scattering. Another long range goal is to develop transport theory for another second moment of the propagated field: the vertical coherence. An important reason for modeling the vertical coherence is that a transform of the vertical coherence will yield the vertical angular distribution of the propagated intensity. Applying this to sources of ambient noise can lead to better understanding the effects of surface forward scattering on the vertical angular distribution of ambient noise.

A final long range goal is to support development of a method to allow the effects of surface forward scattering as modeled with transport theory to be accurately modeled with codes used in naval applications, such as CASS-GRAB and ASPM. These codes treat the surface interaction as a simple reflection, and with the standard surface coherent reflection loss the full effects of surface forward scattering cannot be treated as shown in Figure 1. An approach for developing an effective surface reflection loss (referred to as TOTLOS) that allows effects of surface forward scattering to be modeled with ray codes such as CASS-GRAB or energy flux codes such as ASPM is being developed under a separate 6.4 project (supported by ONR 322). (The term "TOTLOS" stands for an effective surface reflection loss for the total field, made up of the coherent field and the scattered field.) Since the approach involves determining the effective surface reflection loss that matches in detail the results obtained with transport theory, the two projects are closely coupled, and considerable effort from this project has been used to support TOTLOS development.

SPECIFIC OBJECTIVES

The specific objectives for the reporting period ending on 8/31/19 will be summarized here. A key objective has been confirmation of transport theory accuracy by comparison with experimental data or by comparison with more accurate simulation methods. The reverberation results obtained during TREX13 have been the focus of data-model comparisons. Because this data set was obtained as part of a basic research effort, the extensive environmental characterization that was associated with the acoustic measurements would suggest that this is an ideal source of measurements for data-model comparisons. The focus for comparison with a more accurate simulation method has been with a mode-based approach based on the Differential Algebraic Equation (DAE) method. This method works with individual surface realizations followed by averaging results over an ensemble of these surface realizations. Therefore, transport theory approximations used to obtain equations that evolve moments of the field are not made with the DAE method. Also, the equations of motion for the mode amplitudes are obtained at a higher level of accuracy than those used with the transport theory method.

Another specific objective was to develop a method based on normal modes that can allow for range dependence in sound speed profile or water depth. This supports the long-range goal of developing transport theory for range-dependent environments, since our transport theory method is based on normal modes. For the case of a range-dependent depth, for example, the depth is usually represented as a set of discrete small steps in depth. It is possible in principle to obtain a new set of normal modes at each step, and then transform the field onto the new mode set at each step. However, this is a numerically intensive approach, best used for benchmark solutions. The objective here is to obtain a more numerically efficient method that requires far fewer instances when new sets of modes are required.

PE methods are also commonly used for propagation modeling in range-dependent environments, though the PE approach does not readily lend itself to reverberation modeling. Even so, there are additional reasons for pursuing a mode-based method for range dependent environments. The focus here is on the effects of sea surface forward scattering on propagation and reverberation, and it is possible to generalize the PE method to treat realizations of rough surfaces using a rough surface PE. An issue comes up in this case, however, that surface forward scattering leads to scattering into all grazing angles, including relatively high grazing angles where the PE method can be suspect. This is not important is most uses of the PE method that ignores rough boundary scattering, but does matter here. Another issue is that the rough surface PE method is a Monte Carlo method, where averages are made over realizations of rough surfaces. For the development of the TOTLOS model (mentioned below), the average of the mode amplitudes as a function of range are used to infer an effective reflection loss at the surface that includes the effects of surface forward scattering. The mode amplitude averages obtained from the rough surface PE method exhibit a noisy structure about the average. Because it is the slope of the average mode amplitudes that are actually used for developing the TOTLOS model, the smooth mode amplitudes as a function of range obtained with transport theory are much preferred.

For TOTLOS model development, the specific objective during the reporting period has been to move beyond the initial limitations of the assumed environment of a range-independent, isovelocity waveguide. The main objective was to generalize the TOTLOS model to include arbitrary sound speed profiles, including surface ducts. An additional objective was to examine the application of TOTLOS to environments with a range-dependent depth.

The long-range goal of developing transport theory for the vertical coherence was not addressed directly during the period of performance ending on 8/31/19. However, this topic is the prime objective in the continuing work on transport theory under the Task Force Ocean (TFO) program with performance period from 7/15/2019 to 7/14/2022.

RESULTS

Confirmation of transport theory accuracy: Comparison with experiments

The goal in this effort has been to compare transport theory mid-frequency predictions for reverberation level with experiments in circumstances when surface forward scattering occurs during the propagation phase out to and back from the scattering patch. TREX13 provided excellent reverberation data sets in a well-characterized environment, and transport theory was used to model the measured reverberation in this shallow water environment at mid frequencies. In doing this, it was necessary to confront the issue of the directional nature of the wave field relative to the “reverberation track,” a region of about 2 degrees in azimuth from which the reverberation was measured with a horizontal array. It must be emphasized that our transport theory method has been formulated in 2-D (range and depth), so that out-of-plane forward scattering (a 3-D effect) is not treated directly. This fact could be important when attempts are made to verify the accuracy of transport theory reverberation predictions by comparisons with measurements.

It is also true that the reverberation levels can be quite different when the surface waves are propagating in a direction closely aligned with the reverberation track compared to when the waves are propagating in a direction approximately perpendicular to the reverberation track. In the first (parallel) case, surface scattering will lead to enhanced energy loss into the bottom reducing the expected reverberation, but the higher grazing angles incident on the bottom because of the surface scattering will mitigate that reduction to some extent. In the second (perpendicular) case, surface scattering will reduce the energy loss into the bottom, even compared to a calm surface, which should increase the energy that reaches longer ranges with a tendency to increase the longer range reverberation. But at the same time, the surface scattering will have the effect of decreasing the grazing angles at the bottom, which will mitigate the expected increase to some extent. The picture just described is rather simplified, however, because for the TREX13 data sets examined in detail, the measured directional wave spectra extended azimuthally about 90°. Therefore, it is necessary to model the surface forward scattering over an extended range of wave propagation directions. The details of this treatment are described in the previous final report (for FY13–15). The result of those data-model comparisons can be summarized as follows: The importance of properly accounting for surface forward scattering and the directional nature of the wave field in reverberation modeling is evident in the measured reverberation itself. This shows the importance of accounting for the full 3-D nature of the forward scattering process when modeling the reverberation. The attempt to account for these effects by converting from 3-D to 2-D with an approximate method that preserves the vertical distribution of scattering angles shows some promise, but is not sufficiently accurate at this time. It should be noted that the TREX13 reverberation data were taken with relatively modest sea states, and at higher sea states these effects are predicted to be substantially greater.

During TREX13 there was extensive environmental characterization, appearing to make this data set ideal for such a comparison. However, even in this case complexities and uncertainties associated with working with measured data turned out to be important. The sea state levels accessed in the experiment were only modest, and when the sea state increased significantly during the one major weather event, the measurements were suspended and the supporting ship transited to port. In order to carry out the detailed bottom characterization required for a basic research effort, the site was chosen to be in very shallow water (about 19 m water depth) near shore to allow extensive diver activity on the bottom. But then there were changes in the bottom roughness and composition immediately after the one major weather event, complicating comparisons between data sets for different surface conditions. Finally, there has been a lingering mystery about consistently modeling propagation and reverberation measurements using the same assumptions about the bottom properties. Work by TREX13 investigators (DJ Tang and Todd Hefner) has shown discrepancies on the order of 3–5 dB between modeled and measured propagation loss or reverberation level when the same bottom properties were used in the modeling for both. These differences are of the same order as the reverberation changes observed due to the modest sea state conditions present when the reverberation data were obtained, adding ambiguity into the entire effort. One hypothesis for this discrepancy is related to volume scattering in the regions of mud in the swales of the ridge and swale structure of the shallow water region where TREX13 was held. The volume scattering is likely due to inclusions within the mud, such as shells or clumps of sand. The volume scattering could increase the transmission loss compared to modeling predictions, and could also increase the backscattering compared to modeling predictions. Therefore, even when propagation and reverberation measurements are accompanied with environmental characterization at a basic research level, it can be difficult to end up with the kind of data-model comparisons that are desired to give a rigorous test of transport theory. This suggests that the best method for testing the accuracy of transport theory is with comparisons with more rigorous numerical methods, since in such comparisons the environment is perfectly defined.

One additional complication encountered in modeling TREX13 reverberation data was the presence of fish schools, and scattering from these schools, when they were present, led to enhancements of reverberation that would not be included in transport theory reverberation modeling. The presence of fish school scattering was seen in the data during the night, and it was somewhat of a mystery why the scattering from the fish could apparently be entirely absent during the day. This led to a joint effort with Wu-Jung Lee (a postdoc at APL-UW at that time), DJ Tang, and Tim Stanton using available TREX13 data to investigate the behavior of the fish schools to better understand the fish school contribution to reverberation. The main finding is that the fish at the TREX13 site almost entirely hide out during the day in several nearby ship wrecks, emerge at dusk, and return at dawn. This work led to an article in JASA (Vol. 144 (3), Sept. 2018, 1424–1434).

Confirmation of transport theory accuracy: Comparison with a more accurate simulation method

The effects of sea surface roughness on forward scattering are treated in our formulation of transport theory by coupling modes propagating in a shallow water waveguide. The mode coupling is modeled using first-order perturbation theory, and our main scenario of interest is in mid-frequency propagation and reverberation, with emphasis in the 1–3 kHz range. As work progressed by reviewing the issues that would be addressed in a publication, it became apparent that more work was needed to fully understand the limitations of the present version of transport theory, since it is based on first-order perturbation theory. It also became apparent that it was important to determine if extensions are needed, and if so, if they can be made that will improve accuracy in the mid-frequency region. This has led to a fundamental re-examination of the formulation through comparisons with Monte Carlo methods that have greater accuracy, and through consideration of extensions to the transport method itself.

To fully understand the issues at stake and the recent accomplishments, it is necessary to give some details of the method. The acoustic field is expanded in "unperturbed modes," that is, in terms of normal modes for a flat sea surface. Thus, we write

$$p(x,z) = \sum_{n=1}^{N} a_n(x)\phi_n(z), \quad (1)$$

where the $a_n(x)$ are the mode amplitudes and the $\phi_n(z)$ are the mode functions. The goal is to determine the "equations of motion" that determine the evolution of the mode amplitudes with range in the presence of a rough sea surface that leads to mode coupling. The boundary condition at a rough sea surface is that the total pressure is zero on that surface. Thus, $p(x,z) = 0$ for $z = h(x)$ where $h(x)$ is the rough surface. With first-order perturbation theory, the boundary condition is re-expressed as an effective boundary condition on the mean plane of the rough surface (i.e., at $z = 0$). This effective boundary condition is obtained by the lowest-order Taylor series expansion about the mean plane and leads to the result

$$p(x,0) + h(x)\frac{\partial p(x,z)}{\partial z}|_{z=0} = 0\,. \quad (2)$$

This effective boundary condition for $p(x,0)$ is linear in $h(x)$, and in our present version of transport theory only terms up to linear in $h(x)$ are retained in the remaining development for the equations that determine the evolution of the first and second moments of the mode amplitudes with range. Thus, a consistent treatment linear in $h(x)$ has been employed. One complication that can be seen right at the beginning is that the unperturbed modes in (1) satisfy the boundary condition $\phi_n(0) = 0$, and from (1) then it would follow that $p(x,0) = 0$, which would violate the boundary condition given by (2). To avoid this problem, one extra mode, referred to as a "pseudo mode," is introduce that is non-zero at *z* = 0, which allows the boundary condition to be satisfied, and is orthogonal to the other *N* modes.

The equations of motions for the $N + 1$ mode amplitudes are obtained with a variational principle, starting from the appropriate Lagrangian (the difference between the kinetic energy density and the potential energy density), and the boundary condition is enforced with an added term consisting of the product of the left-hand side of (2) and a Lagrange multiplier $\lambda(x)$. Thus, the Lagrangian is given by

(kinetic energy density) – (potential energy density) +λ(x)$\left[p(x,0) + h(x)\frac{\partial p(x,z)}{\partial z}|_{z=0}\right]$. (3)

Variation of the Lagrangian leads to the coupled equations of motion for the $N + 1$ mode amplitudes, but they also contain the Lagrange multiplier $\lambda(x)$ that also needs to be determined. When solving for $\lambda(x)$, terms arise that are non-linear in $h(x)$, and the non-linear terms are not included in our present transport theory development. The reason the non-linear terms have not been kept in the past is they complicate the process that leads to equations of motion for the first and second moments of the field, and propagating these moments is the key element of the transport theory method. When only the first-order terms in $h(x)$ are retained (linear in $h(x)$ or the LIN model), the following equation of motion for the mode amplitudes is obtained:

$$\frac{da_n(x)}{dx} = ik_{xn}a_n(x) + i\frac{h(x)}{2k_{xn}}\frac{d\phi_n(z)}{dz}|_{z=0}\sum_{m=1}^{N} a_m(x)\frac{d\phi_m(z)}{dz}|_{z=0}. \quad (4)$$

It is then possible to use this evolution equation for the mode amplitudes with realizations of the rough surface $h(x)$. This gives a Monte Carlo LIN method with mode coupling due to the rough surface given by the last term in (4). Averaging over many realizations, the first and second moments of the field can be obtained. The sum in (4) does not extend to $N + 1$ because $a_{N+1}(x)$ is found to be of order $h(x)$, and when combined with the $h(x)$ in front of the sum, that leads to a term of order $h^2(x)$, which is not included in the LIN model.

For transport theory, we need evolution equations for the moments of the mode amplitudes. For the first moment $< a_n(x) >$, for example, we have by taking the first moment of (4) that

$$\frac{d < a_n(x) >}{dx} = ik_{xn} < a_n(x) > + \frac{i}{2k_{xn}}\frac{d\phi_n(z)}{dz}|_{z=0}\sum_{m=1}^{N} < h(x)a_m(x) > \frac{d\phi_m(z)}{dz}|_{z=0} \, (5).$$

The key step in obtaining the equations for transport theory is to approximate the moment $< h(x)a_m(x) >$. A good reference on this step is the book by N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry*, Chapter XVI. The result is that

$< h(x)a_n(x) > = < a_n(x) > \times$ (expression involving the spectrum of $h(x)$). (6)

For the second moment (the intensity) an analogous evolution equation is obtained for $< a_n^*(x)a_m(x) >$. And again, transport theory approximations give

$< h(x)a_n^*(x)a_m(x) > = < a_n^*(x)a_m(x) > \times$ (expression involving the spectrum of $h(x)$). (7)

Comparisons between the range dependence for mode amplitudes from transport theory (LIN) and rough surface PE agree well. In spite of this, analysis shows that the LIN model may become inaccurate as *N* becomes large. This is based on an analysis of the field structure near the rough surface as *N* increases. Also, as *N* increases, modes are included that correspond to higher grazing angles, where PE itself can become suspect. These considerations have led to an effort to go beyond the LIN model to obtain a formulation that has greater accuracy.

Propagation methods that couple modes due to the rough sea surface and that work directly with individual rough surface realizations can be developed that include the non-linear terms that arise when solving for $\lambda(x)$. Frank Henyey, a co-investigator on this project, has developed a rough surface mode code based on a Differential Algebraic Equation (DAE) method. The DAE method uses a modification to the linear boundary condition given by (2), but beyond that is not based on perturbation theory, and conserves energy exactly. It is used with realizations of rough surfaces in a Monte Carlo format, and averaging the mode amplitudes from the realizations gives results that can be compared with the mode amplitude moments from transport theory. In addition, averaged mode amplitudes from the DAE method can be used to model reverberation and compared with transport theory reverberation results.

The important steps in developing the DAE method can be outlined as follows. The Lagrangian given by (3) is converted to a 1-way Lagrangian $L(x)$. Variation of $L(x)$ leads to the equations of motion for the mode amplitudes, known as the Euler Equations. There are now $N+2$ unknown functions: $a_n(x)$ for $n = 1 \ldots N, N+1,$ and $\lambda(x)$. The equations of motion are of the form

$$\frac{da_n(x)}{dx} = \ \ldots \ \text{terms} \ldots + \lambda(x)(\ldots \text{terms} \ldots), \quad (8)$$

plus the boundary condition given by (2). It is possible to solve for $\lambda(x)$ and plug back into the equations of motion. The resulting equations of motion have terms in $h(x)$, $h^2(x)$, and $dh(x)/dx$ in the numerator and terms in $h(x)$ and $h^2(x)$ in the denominator. Even with this level of complexity, can still attempt to solve the evolution equations for the mode amplitudes in a Monte Carlo method. However, in doing this, it turns out that at certain values of $h(x)$ singularities of the form (0/0) occur. In a purely analytical treatment, such singularities could be dealt with, but they stop cold the numerical integration of the equations of motion.

It is at this point that the Differential Algebraic Equation (DAE) method makes its appearance. In this method, one replaces the boundary condition term in (3)

$$\lambda(x)\left[p(x,0) + h(x)\frac{\partial p(x,z)}{\partial z}\Big|_{z=0}\right] \qquad (9)$$

with the Lagrange multiplier by the derivative of the boundary condition:

$$\lambda(x)\frac{d}{dx}\left[p(x,0) + h(x)\frac{\partial p(x,z)}{\partial z}\Big|_{z=0}\right]. \qquad (10)$$

Amazingly, the equations of motion are no longer singular, and can be integrated. The replacement of (9) by (10) is legitimate, because if the expression in brackets in (9) is a constraint that should be zero for all *x*, then its *x*-derivative should also be zero for all *x*, and can also be a legitimate constraint. The DAE method can thus be used in a Monte Carlo approach to compare with transport theory predictions.

A mode amplitude comparison between transport and the LIN Monte Carlo method for an example at 3 kHz is shown in Figure 2. The conditions are the same as for Figure 1, and results for 50 surface realizations have been averaged to obtain the LIN result given by the fluctuating red line. Both transport theory and the LIN Monte Carlo method keep only terms up to linear in $h(x)$. Even so, good agreement is not a foregone conclusion, because certain transport theory approximations are made in going from the LIN Monte Carlo method to transport theory, e.g., Eq. (7) Therefore, it is reassuring that reasonable agreement is indeed obtained. If even more surface realizations were used, it is likely that the LIN result would converge near the top of its range of fluctuations, which is near the transport theory result.

The corresponding mode amplitude comparison between transport and the DAE Monte Carlo method is shown in Figure 3. In this case the agreement is not as good, with the transport theory curve several dB higher than the DAE Monte Carlo result. Assuming the DAE result is more accurate, there would be a clear benefit to incorporating the improved accuracy into transport theory itself. Note that differences in the mode amplitude decays between Figures 2 and 3 are for one-way propagation, and these differences would be doubled for two-way propagation that enters into reverberation modeling. It would be even better if fully exact results could be obtained with a method such as the Boundary Element Method in order to confirmed the level of accuracy of the DAE method, but such comparisons are not available at present.

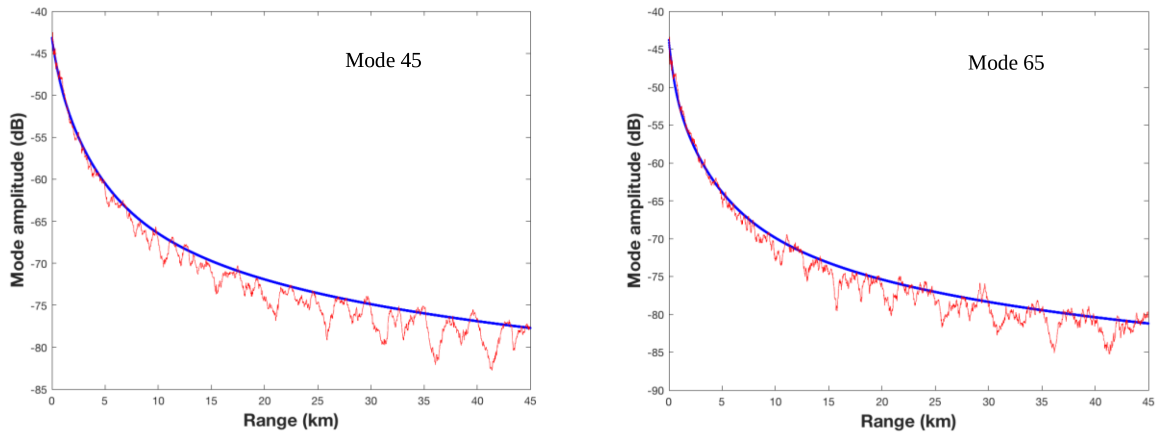

Figure 2. A comparison between transport theory (blue) and LIN Monte Carlo results (red) for two mode amplitudes versus range. The frequency is 3 kHz and the rough sea surface corresponds to a fully developed sea at a wind speed of 7.7 m/s (15 knots). Other conditions are the same as for Figure 1.

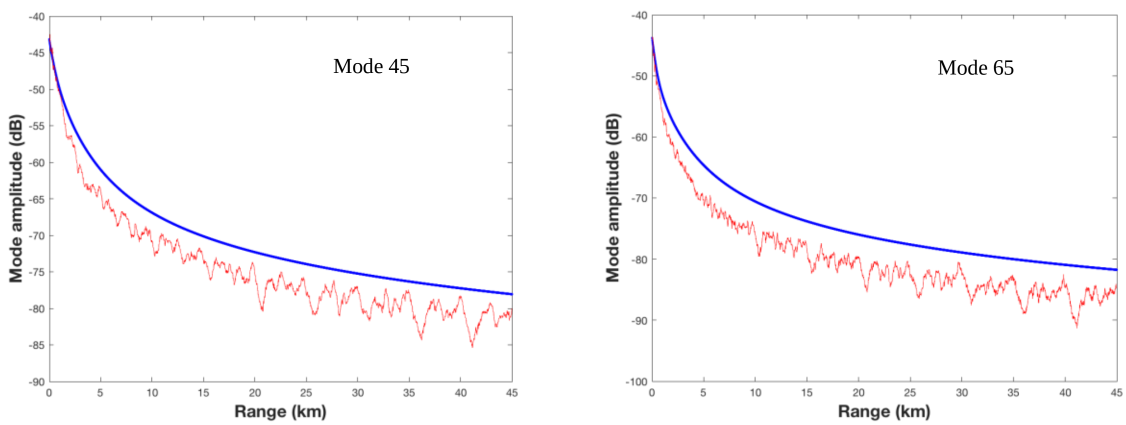

Figure 3. A comparison between transport theory (blue) and DAE Monte Carlo results (red) for two mode amplitudes versus range. The conditions are the same as for Figure 2.

There is some prospect of using the DAE method in transport theory, in spite of the complexity of the mode amplitude equations given by the method. For example, with the LIN method, the first moment equation involves the expression given in (6). The corresponding moment for the DAE method would be given by

$$< F(x) a_n(x) > = < a_n(x) > \times \text{(expression involving the spectrum of } F(x)\text{)}, \quad (11)$$

where $F(x)$ has terms in $h(x)$, $h^2(x)$, and $dh(x)/dx$ in the numerator and terms in $h(x)$ and $h^2(x)$ in the denominator. It should be possible to obtain the spectrum of $F(x)$ numerically with a Monte Carlo method to enable a DAE transport version for some test cases. This would help in gaining full confidence in the DAE numerics. If it appears warranted in the future, it may be possible to obtain the spectrum of $F(x)$ analytically using results from the mathematical literature [e.g., A. M. Mathai and S. B. Provost, *Quadratic Forms in Random Variables*, 1992].

An extensive investigation has been made and is continuing on the results from the DAE method, and how they compare with a fully linear Monte Carlo method (the LIN method) and the transport theory method. Just one aspect will be mentioned here. One expected result is that as the number of modes is increased in these simulations the results will converge, meaning that the contribution of high modes (corresponding to high grazing angles) is minimal. As the dependence on the number of modes was investigated, indications were obtained suggesting a lack of convergence. While most simulations utilize a point source, it has been useful to also examine cases with a single mode starting field as a way of isolating certain aspects of the physics. Also, instead of modeling the rough sea surface as a fully developed sea, it has been useful to examine cases with a much narrower wave number spectrum, such as would occur in a swell spectrum. Figure 4 shows such an example at 2 kHz, with a starting field composed of mode 10 and with the rough sea surface modeled as a swell spectrum. The intensity distribution as a function of mode number at a range of 1 km is shown, with mode 10 still essentially at its initial intensity value of 1.0. The grazing angle corresponding to mode 10 is at 4.3°. Scattering from the peak of the spectrum leads to the “first Bragg” peak at about modes 20–25, and smaller higher-order peaks can be observed in the higher modes. What is unexpected is the leveling off of the mode intensity at very high mode numbers, which correspond to high grazing angles. For this particular simulation, the bottom has been modeled as a perfectly reflecting boundary with a zero field boundary condition, so energy is not lost into the bottom at grazing angles above a critical angle. In this very idealized case, scattering is occurring to high mode numbers even though the surface roughness is concentrated at low wave numbers.

This has led to a continuing investigation and the development of integral equation methods to give reference solutions. Two integral equation methods have been developed, and in both cases instead of using the boundary condition that the total field is zero on the rough surface, the total field is subject to the boundary condition given by (2) on the mean plane at $z = 0$. One method assumes an infinite periodic geometry, best suited to sinusoidal surfaces. The second method assumes finite length surfaces, and thus can be conveniently applied to more general cases. Both integral equation approaches yield a solution with assumptions made that are very similar to those used in the DAE method. That is, the boundary condition given by (2) is imposed on the mean plane, which is an approximation, but otherwise no other approximations are made. What this effort has found so far is that the solutions obtained by both integral equation methods appear to contain excessive contributions of evanescent waves when the surface roughness increases. Similar effects of these evanescent waves are not found with fully exact integral equation methods using the boundary condition that the total field is zero on the rough surface. Because of the usefulness of building approximate methods based on starting with the boundary condition given by (2) on the mean plane, effort is continuing to see if modifications can be made that will avoid the appearance of the excessive contributions of evanescent waves.

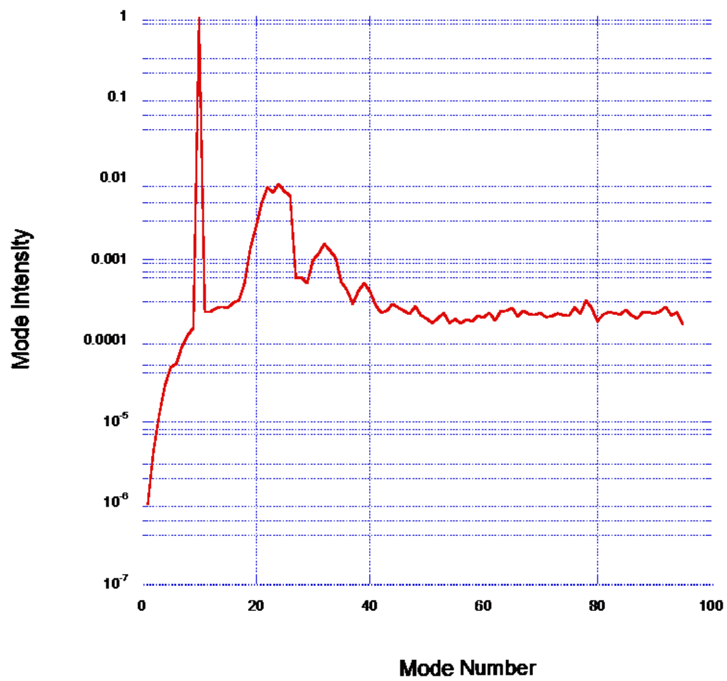

Figure 4. DAE method result for the mode intensity as a function of mode number at a range of 1 km. The frequency is 2 kHz, the starting field is mode 10, and the rough sea surface is modeled with a swell spectrum.

Development of method based on normal modes that accommodates range dependence

Because our transport theory formulation is based on modes, it most naturally applies to range-independent environments. A graduate student, Feilong Zhu, from the Institute of Acoustics, Beijing, China worked with the PI from the fall of 2015 to the fall of 2016 on generalizing the mode method to apply to range-dependent environments where the water depth changes with range. The idea was to generalize previous work in the literature by Higham and Tindle (JASA, 2003) that only applies at low frequency (< 250 Hz) so it would be accurate up into the mid-frequency range. Higham and Tindle used a perturbation method to obtain new mode sets following steps in the bottom profile, limiting the number of times new mode sets need be computed. In the new extended perturbation method developed in our work, an iterative scheme was employed to obtain more accurate results and allow the method to be used at higher frequencies, e.g., at 1 kHz. Reference solutions were obtained with COUPLE, and the modes functions used with the method were obtained with KRAKENC.

Figure 5 shows an example range-dependent environment, where the water depth decreases linearly from 50 m to 26 m over a range interval of 2 km, and then increases linearly back to 50 m over the next 2 km. In the computation the linear slope regions are modeled with a set of stairsteps. Figure 6 shows that the use of adiabatic modes for this environment gives relatively poor results in comparison to the reference solution obtained with the code COUPLE. Figure 7 shows that excellent agreement with the reference solution is obtained when modes are found with the extended perturbation method.

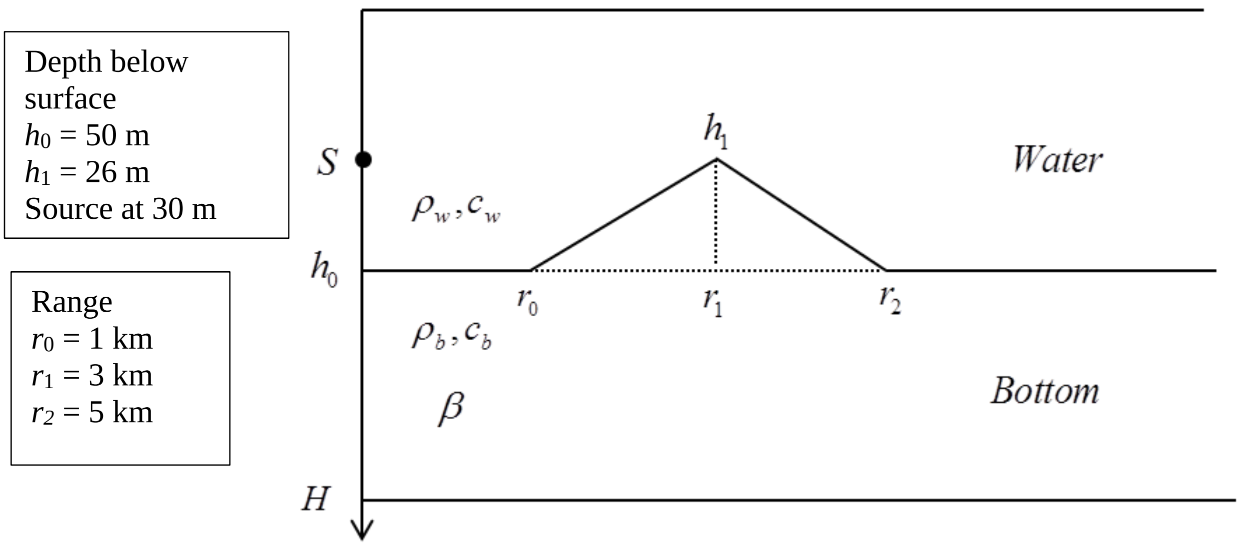

Figure 5. Example range-dependent environment.

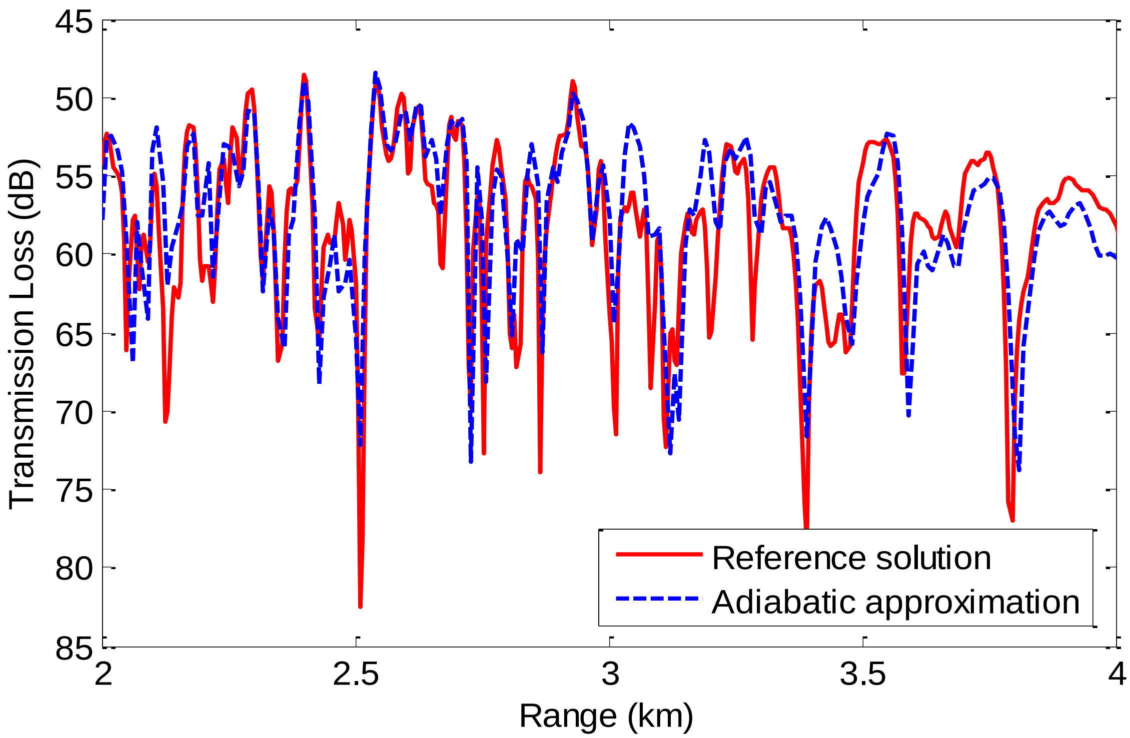

Figure 6. Transmission loss comparison at a depth of 20 m for a frequency of 1 kHz. Adiabatic modes were used in this comparison.

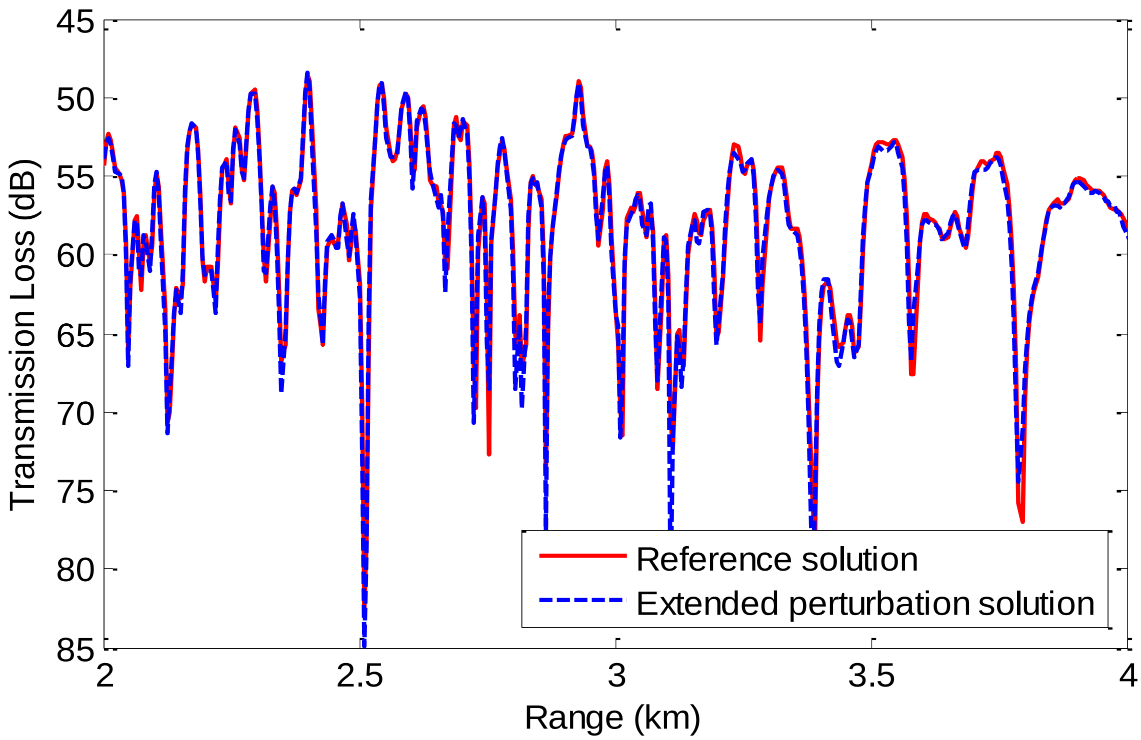

Figure 7. Transmission loss comparison at a depth of 20 m for a frequency of 1 kHz. Modes obtained with the extended perturbation method were used in this comparison.

The time to obtain a new mode set by the extended perturbation method is about 300 time shorter than the time to obtain the new set directly with KRACKENC. On the upward slope, a new mode set was obtained after every 10 steps, on the downward slope, a new mode set was obtained after every 5 steps. This development leads the way to incorporating this type of range dependence into our transport method in the future. Range dependence of the sound speed profile is even easier to handle with the extended perturbation method.

Feilong presented results at the ASA meeting in the fall of 2016 in Honolulu. The details of the method are described in a recent publication: F. Zhu, E. I. Thorsos, F. Li, "Coupled Perturbed Modes over Sloping Penetrable Bottom," Chinese Physics Letters, Vol 34, No. 7 (2017) 074302

TOTLOS model development

The main objective was to generalize the TOTLOS model to include arbitrary sound speed profiles, including surface ducts. An additional objective was to examine the application of TOTLOS to environments with a range-dependent depth.

Considering the additional objective first, we have investigated the extent to which the TOTLOS model can be used in environments with range-dependent depths by using the local depth in the existing TOTLOS model as the depth varies with range. Comparisons have been made for one-way propagation using rough surface PE as ground truth. Starting with relatively gentle bottom slopes (e.g., depth decreasing from 80 m to 50 m over a range interval of 10 km, the agreement between CASS-GRAB with TOTLOS and PE has been excellent using the metric of the vertically averaged absolute square of the pressure. As the region with slope has been reduced so the slope increases, no obvious issues with the TOTLOS model has been observed, but instead CASS-GRAB itself has been giving what appear to be incorrect results in comparison to PE and to expectations generally when there is no surface scattering. Contact has been made with the CASS-GRAB designer (Weinberg) in an attempt to resolve this issue.

For the main objective of generalizing the TOTLOS model to include arbitrary sound speed profiles, we have found it necessary to make important modifications to the approach used with isovelocity profiles. The method used in developing the TOTLOS model is to first use the transport theory method to obtain mode amplitude decays versus range for the environment being considered in the presence of a rough sea surface. Each mode amplitude has a particular grazing angle at the surface, and an effective surface reflection coefficient (which is the output of the TOTLOS model) is determined as a function of grazing angle (and range) that will reproduce the transport theory results for transmission loss and reverberation. One potential complication, even for the case of an isovelocity sound speed profile, is that the initial values of the mode amplitudes depend on the assumed source depth. This would suggest that the TOTLOS model would need to depend on the source depth in addition to other aspects of the environment. For the isovelocity case, however, it has been found that this complication can be avoided, and reasonable accuracy can be obtained by using transport theory results in developing the TOTLOS model that are obtained with all modes having equal starting amplitudes. This procedure leads to reverberation results when using TOTLOS in CASS-GRAB that are all within 3 dB of the transport theory results where the source depth is varied throughout the water column in both CASS-GRAB and transport theory modeling. For general sound speed profiles, the goal has been to obtain the same level of accuracy, but it has been found that the approach of using equal starting mode amplitudes in transport theory runs when developing the TOTLOS model is no longer sufficient. This means information on the source depth needs to be part of the algorithm that determines the TOTLOS model.

The issue can be understood in the case of a constant gradient upward refracting SSP. In a ray picture of propagation, rays that are launched in the upper part of the water column with shallow grazing angles will have upward refracting turning points before reaching the bottom. In the corresponding mode picture, the lowest few mode functions are concentrated in the upper part of the water column with some enhancement in amplitude near the water depth of the ray turning points. It was found that when developing the TOTLOS model, the starting amplitudes of the lowest few modes need to be modified depending on the source depth. The following algorithm was found to give acceptable error for constant gradient environments. For source depths below ray turning points, the starting mode amplitudes are modified to be proportional to magnitude of mode function at the source depth. The same modification is also made in ray turning point enhancement region. Equal starting mode amplitudes are used for higher modes when the ray turning point enhancement region is below source depth, or for modes that correspond to no turning points.

While the procedure of modifying a limited number of starting mode amplitudes in transport runs to obtain the TOTLOS model leads to acceptable error, it does not provide a final model because it would require a transport run for each case being considered. It does, however, clarify what level of modification is needed for acceptable results. To obtain a final model, the idea is to use the fact that the field structure near and below ray turning points can be modeled with Airy functions. Thus, the goal going forward is to use Airy functions to determine the modified starting mode amplitudes without the need for using properties of the mode functions.

An environment with a surface duct has also been considered for TOTLOS development. This can be an important case to consider, since sound can be trapped in a surface duct, but also scattered out of the duct by surface forward scattering when the sea surface has roughness. A problem was soon discovered during this investigation. When surface forward scattering is not present (so TOTLOS is not needed), the standard version of CASS-GRAB gives > 20 dB transmission loss error at long times when the source is in surface duct. This was traced to excessive leakage out of the duct for the case when the surface is calm. In reality, the error might not be as serious as it seems, because there is usually some roughness at the surface which will lead to scattering out the duct (and which is not modeled in the standard version of CASS-GRAB). But detailed data-model agreement would then be somewhat of a coincidence, since the scattering out of the duct should depend on the sea state. Also, in order to develop a TOTLOS model for the scattering out of the duct, a version of CASS-GRAB is needed that has a realistic amount of duct leakage when the surface is calm.

After some discussion with the developer of CASS-GRAB (Weinberg), a version of CASS-GRAB was made available to us with all duct leakage turned off. It would appear that this version would not model the expected but normally small amount of duct leakage that occurs because of wave penetration out of the duct. At the same time, however, an effort is underway to modify CASS-GRAB so that is will correctly model the duct leakage when the surface is calm. Ruth Keenan at ARL-UT is involved in the effort, and the approach is to incorporate Cathy Clark's normal mode code into CASS-GRAB to handle propagation within surface ducts. Ruth has indicated that the developer version of this code will be made available to us as it is being developed. This will allow us to move forward with working toward a TOTLOS model that applies to surface duct environments.

RELATED PROJECTS

TOTLOS model development is being supported by a 6.4 program of ONR 332, POC Bill Schulz.

Work on developing a transport theory model for the effect of surface forward scattering on vertical coherence is ongoing in a project support by Task Force Ocean.

PUBLICATIONS

F. Zhu, E. I. Thorsos, and F. Li, "Coupled Perturbed Modes over Sloping Penetrable Bottom," Chinese Physics Letters **34** (7), 074302-1 to 074302-4 (2017). [published, refereed]

W-J Lee, D. Tang, T. K. Stanton, and E. I. Thorsos, "Macroscopic observations of diel fish aggregation movements around a shallow water artificial reef using a mid-frequency horizontal-looking sonar," J. Acoust. Soc. Am **144** (3), 1424-1434 (2018). [published, refereed]

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14. ABSTRACT
There were four specific objectives for the work described in this final report for the reporting period ending on 8/31/19. The first objective has been confirmation of transport theory accuracy by comparison with experimental data, and in particular by the reverberation results obtained during TREX13. The second objective has been confirmation of transport theory accuracy by comparison with more accurate simulation methods. The third objective was to develop a method based on normal modes that can allow for range dependence in the sound speed profile or water depth. This supports the long-range goal of developing transport theory for range-dependent environments, since our transport theory method is based on normal modes. The fourth objective is to use transport theory methods to support TOTLOS model development, where the specific objective was to generalize the TOTLOS model to include arbitrary sound speed profiles, including surface ducts. The progress made and issues that arose in the course of this work are summarized in this final report.

15. SUBJECT TERMS
Underwater acoustics, surface forward scattering, transport theory, reverberation modeling

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