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Cooperative Evasion and Pursuit for Autonomous Vehicles

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Cooperative Evasion and Pursuit for Autonomous Vehicles

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Final Technical Report **(Covering the period: 1 September 2015 - 31 August 2018)**

Principal Investigator: Prof. Tal Shima

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Summary

The research focused on the development of novel cooperative pursuit and evasion strategies, while taking into account task assignment and estimation considerations. In this three years research we explored the possible synergy available through cooperation between multiple autonomous vehicles, specifically in scenarios where multiple missiles are homing onto multiple targets, protected possibly by multiple defender missiles.

The research in the first year, summarized in the first year's annual report, was conducted in several paths:

- Optimal evasion in a switched system - First we have investigated the interception engagement where a single target tries to evade from a single missile. We have analyzed the case where the missile may be switching between different, known, guidance laws during the engagement. Switches are either known *a priori* or not. For these cases optimal evasive strategies were developed for the target aircraft.
- Cooperative nonlinear guidance - Nonlinear robust guidance strategies, for the target and defender, were derived using the sliding mode approach. These laws utilize the extra degree of freedom of having two controllers (target and defender) in the design of several different cooperative guidance strategies. Protection envelopes for the aircraft, ensuring successful interception of the incoming missile, were also obtained.
- Intertwined guidance and estimation - Multiple model adaptive evasion strategies for a single target evading from a single missile were derived. The missile's guidance strategy is assumed to belong in a finite set of linear guidance laws and to be fixed throughout the engagement. The target simultaneously identifies the guidance law used by the missile and evades from the homing missile.
- Cooperative guidance for estimation - A reduced-order estimation scheme based on information sharing between multiple defenders and a target cooperatively estimating the relative state and the unknown parameters of the attacking missile was developed. The observability was considerably enhanced by cooperatively imposing relative intercept angles between consecutive defenders and the attacking missile.

The research in the second year, summarized in the second year's annual report, was conducted in the following several paths:

- Game based guidance in a switched system - We have developed robust, differential games based, pursuit and evasion guidance strategies in a one-on-one engagement where the system overcomes multiple abrupt changes. These changes, e.g. in the dynamics of the systems, are either known *a priori* or not (faults).
- Cooperative guidance and task assignment - Using the adjoint system representation we have developed computationally inexpensive defender-missile allocation algorithms coupled with cooperative evasion strategies for the protected target. A comparison to the optimal allocation demonstrates the fact that the proposed allocation algorithms can provide close-to-optimal performance with a significant computational runtime improvement.
- Intertwined guidance and estimation - We have developed a unique "blinding" guidance strategy for a target trying to evade two homing missiles that are sharing line-of-sight measurements. The developed strategy minimizes the information gained by the group of missiles, thus reducing their homing performance. Subsequently, an appropriately timed evasion maneuver helps the target to increase its survivability likelihood from the blinded homing missiles.

This final report summarizes the results obtained during the third year of the research. The research in this year was conducted with my research team (mainly researcher Dr. Martin Weiss, Dr. Vladimir Turetsky, postdoc Dr. Robert Fonod, and graduate student Zheng Wen Tan) in several parallel paths:

- Cooperative guidance and estimation - we developed a novel cooperative defensive guidance law for a two-on-two engagement. What is unique is that the target pair lures the pursuing missiles into collision with one another. In addition to the proposed cooperative defensive strategy, a decentralized multiple-model estimation scheme was also presented. Guidance law and estimation performance were demonstrated using nonlinear simulations showing the viability of the proposed approaches. This work was performed with my M.Sc. student Zheng Wen Tan and postdoc Dr. Robert Fonod.
- Intertwined guidance and estimation - we developed a novel wingman-based guidance and estimation concept for a sensorless homing missile. The wingman (e.g. the defended aircraft) is assumed to accurately track the relative position between the homing missile and itself, and to acquire bearings-only measurements. Analytical observability analysis of the proposed concept suggests an optimal wingman trajectory, in terms of relative wingman-missile geometry. The suggested wingman trajectory ensures maximum observability of the missile-target line-of-sight angle, crucial for a proportional navigation guidance law implementation. This work was performed with my postdoc Dr. Robert Fonod.
- Task assignment dependent guidance - we have developed a minimum effort intercept guidance algorithm for a missile that faces two potential targets and takes the decision to intercept one of the targets at a specified moment of time during the flight. The probability of the decision to engage either of the targets is assumed to be known. The algorithm is a time varying linear combination of Augmented Proportional Navigation APN guidance laws for each of the two targets. This work was performed with my research team including Dr. Martin Weiss and Dr. Vladimir Turetsky.

Over the course of this research we have also performed experimental demonstration and verification of some of the developed algorithms in our indoor, Optitrack motion capture equipped, 6m x 6m x 3m testbed. The collaborating vehicles were emulated by Kobuki ground robots and Parrot Bebop 2 quads and controlled via a Simulink-ROS operating environment that we have developed.

The results of this three years research effort were summarized in the following published / accepted / submitted journal publications:

- Fonod, R. and Shima, T., "Multiple Model Adaptive Evasion Against a Homing Missile", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 7, 2016, pp. 1578-1592, doi: 10.2514/1.G000404 .
- Turetsky, V. and Shima, T., "Target Evasion from a Missile Performing Multiple Switches in Guidance Law", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 10, 2016, pp. 2364-2373, doi: 10.2514/1.G000461 .
- Kumar, S. and Shima, T., "Cooperative Nonlinear Guidance Strategies for Aircraft Defense", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 1, 2017, pp. 124-138, doi: 10.2514/1.G000659
- Fonod, R. and Shima, T., "Estimation Enhancement by Cooperatively Imposing Relative Intercept Angles", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 7, 2017, pp. 1711-1725, doi: 10.2514/1.G002379 .
- Shalumov, V. and Shima, T., "Weapon-Target-Allocation Strategies in Multi-Agent Target-Missile-Defender Engagement", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 10, 2017, pp. 2452-2464, doi: 10.2514/1.G002598 .
- Turetsky, V. and Shima, T., "Pursuit-Evasion Guidance in a Switched System", *SIAM Journal on Control and Optimization*, Vol. 50, No. 4, 2018, pp. 2613-2633, doi: 10.1137/17M1142764 .
- Fonod, R. and Shima, T., "Blinding Guidance Against Missiles Sharing Bearings-only Measurements", *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 54, No. 1, 2018, pp. 205-216, doi: 10.1109/TAES.2017.2747098 .

- Tan, Z. W., Fonod, R., and Shima, T., "Cooperative Guidance Law for Target Pair to Lure Two Pursuers into Collision", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 41, No. 8, 2018, pp. 1687-1699, doi: 10.2514/1.G003357 .
- Fonod, R. and Shima, T., "Wingman-based Estimation and Guidance for a Sensorless PN-Guided Pursuer", submitted to the *IEEE Transactions on Aerospace and Electronic Systems*.

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I. Introduction

This research aimed at developing algorithms and concepts for cooperative pursuit and evasion applicable to groups of autonomous vehicles. To fix the ideas, we adopted a scenario that we used to motivate our problem formulation and to apply our research results. The scenario that we have addressed is an extension of the Target-Missile-Defender engagement in which in order to increase the probability of its survival, an aircraft (denoted as target) may also deploy a protecting missile (denoted as defender) to intercept the incoming attacking interceptor (denoted as missile). The extended scenario deals with multiple targets, missiles, and defenders. It can be described briefly as follows. A group of aircraft is attacked by a salvo of guided missiles and applies a set of hardkill and softkill measures to defend the group members against the attack. The hardkill measures consist of launching multiple defender missiles. The softkill measures consist of performing evasive maneuvers. These measures could be employed by the attacked aircraft and/or by their wingman.

In our three years long research, we have developed pursuit and evasion guidance algorithms that are naturally suited to a multi-agent missile engagement, taking into account such issues as homing and robustness performance. For the allocation of the multiple defenders launched from multiple aircraft to multiple attacking missiles, we have derived task assignment algorithms that make efficient use of the group resources and are robust with respect to failure of agents within the group. As the problem of high-level task assignment of the defenders to attacking missiles is intertwined with that of end-game guidance of each engagement, we also derived coupled guidance/task assignment algorithms. We also examined the cooperation in guidance taking into account estimation considerations. We have developed a "target-tracking-oriented cooperative guidance law" as a guidance law that combines the traditional guidance algorithms requirement (e.g., small miss distance and reduced maneuvering effort) with the requirements of the cooperative target-tracking algorithms (e.g., minimum variance of the error).

In the third year of the research, summarized in this report, together with my research team (mainly researcher Dr. Martin Weiss, Dr. Vladimir Turetsky, postdoc Dr. Robert Fonod, and graduate student Zheng Wen Tan) we have conducted research in the following paths:

- Cooperative guidance and estimation: two targets, two missiles; imperfect information
- Intertwined guidance and estimation: single target, single missile, single defender; imperfect information
- Task assignment dependent guidance: two missiles, two defenders; perfect information

The remaining part of this report has three sections, presenting in detail the research performed in the third year in the above three directions.

In the next section a planar engagement between two maneuvering targets and two homing missiles is considered, assuming imperfect information. A novel cooperative defensive guidance law is presented for the target team in this 2-on-2 engagement. Instead of classical strategies in which evasive maneuvers are performed or additional agents (e.g., defender missiles) are deployed, the target pair lures the pursuing missiles into collision. The optimal cooperative strategy is solved using State Dependent Riccati Equations method. Linearized kinematics and arbitrary-order linear adversaries' dynamics are assumed in the guidance law derivation. Imperfect information is assumed on the relative states and the guidance laws employed by the missile pair. Guidance strategies that the pursuing missiles may employ are assumed to belong to a finite set of linear guidance laws. In addition to the proposed cooperative defensive strategy, a decentralized estimation scheme based on the multiple model adaptive estimation approach is also presented. Guidance law and estimation performance are demonstrated using nonlinear simulations. Simulation results show the viability of the proposed guidance law and highlights the sensitivity of the guidance law to range measurement accuracy. The results of this effort were recently published in the AIAA Journal of Guidance, Control, and Dynamics.

The following section continues with the assumption of imperfect information and examines an engagement where a sensor-less missile is aided by a wingman, possibly the defended aircraft, to intercept the incoming threat. The contribution of this part of the work is twofold. First, a wingman-based target tracking estimator is developed to estimate the missile-target kinematic variables, needed for the sensorless missile proportional navigation (PN) guidance law implementation. Second, a novel observability metrics for the missile-target range and LOS angle are derived analytically by transforming the wingman's bearings-only

target measurements into the missile frame and then constructing the variance of the resulting pseudo-measurements using analytical linearization. Insights gained from these metrics suggest that the “best” wingman trajectory for a PN-guided missile is to be on the same line-of-sight (LOS) as the missile-target. These results are further validated through numerical simulations. The results of this effort were submitted for publication in the IEEE Transactions on Aerospace and Electronic Systems.

In the last section a perfect information engagement is considered where a missile needs to intercept one of two targets. We assume that the targets are well separated and tracked from the beginning of the flight. However, they do not have the same priority and there is a given probability that at the time that the interceptor will have to decide on the specific target to engage, one of the targets may or may not be destroyed already. Therefore, at the decision time, the interceptor will have to choose the highest priority target that is still alive. As a consequence, we assume that there is a certain probability that each target will be engaged at the decision moment. We propose a guidance algorithm that minimizes the expected guidance effort, where the expectation is computed over all possible outcomes of the engagement decision. The solution turns out to be a weighted sum of Augmented Proportional Navigation (APN) guidance commands against each of the targets, with weighting coefficients that are dependent on time-to-go values to each target and on the outcome probabilities for the engagement decision. The results of this effort were accepted for presentation in the upcoming ICSEE International Conference on the Science of Electrical Engineering, Eilat, Israel. We are currently working on expanding this work and submitting it to the AIAA Journal of Guidance, Control, and Dynamics.

It should be noted that as this report is based on three different papers (mentioned above), there may be differences between the sections in some of the definitions and notations.

II. Cooperative Guidance & Estimation

A. Engagement

Consider a scenario with 2 missiles homing on 2 targets. The objective of the targets in this engagement is to conduct cooperative guidance so that the pursuers are lured into interception with each other without any of the targets being intercepted by the missiles. In addition, we assume the problem can be simplified to a point-mass, planar engagement, and effects of gravity can be neglected. All vehicles use skid-to-turn control with roll-stabilization. All vehicles are assumed to have constant speeds and perform lateral maneuvers only. Imperfect information on the relative states between each missile and target is assumed while perfect information on those between the targets is considered. Lag-free communication of all states between the targets are also assumed. Each target in the engagement is assumed to be pursued by one missile, and each missile pursues only one target. The missile pair is also assumed to have no knowledge that they are on a collision course with each other.

Schematic of the engagement can be seen in Figure 1, where $X_I - O_I - Y_I$ denotes the Cartesian, inertial reference frame.

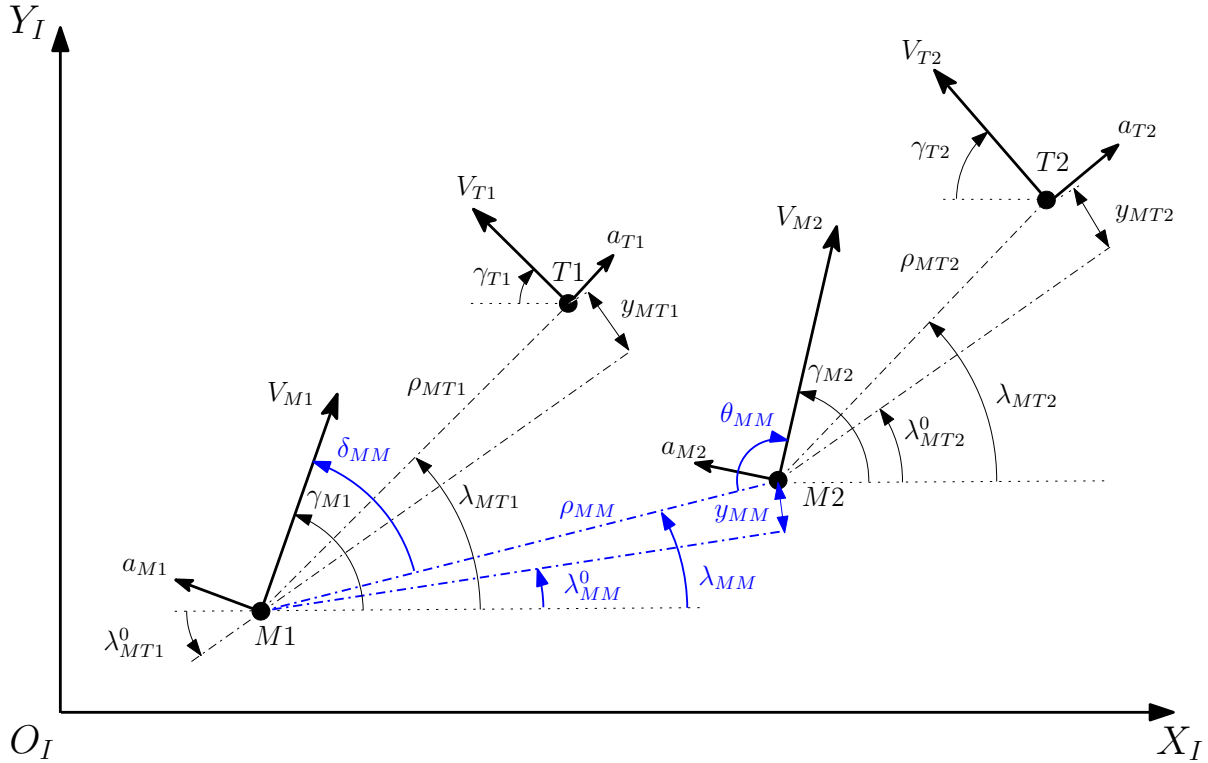


Figure 1: Schematic of a 2-on-2 Engagement.

1. Nonlinear Kinematics

Consider first the kinematics of the M_i-T_i engagement, $i \in \{1, 2\}$, in the 2-on-2 scenario (Figure 1). Defining the latter in polar coordinates $(\rho_{MTi}, \lambda_{MTi})$ referenced to M_i , T_i 's state vector of M_i is

$$\mathbf{x}_{MTi} = [\rho_{MTi} \quad \lambda_{MTi} \quad \mathbf{x}_{Mi} \quad \gamma_{Mi} \quad V_{Mi}]^T \quad (1)$$

where $\mathbf{x}_{Mi}$ is the internal state vector of M_i associated with its dynamics. Assuming the velocity of each vehicle $v \in \{M_i, T_i\}$ is constant near the endgame and their dynamics can be represented by an arbitrary-order linear system, i.e.,

$$\left. \begin{aligned} \dot{\mathbf{x}}_v &= \mathbf{A}_v \mathbf{x}_v + \mathbf{B}_v u_v \\ a_v &= \mathbf{C}_v \mathbf{x}_v + d_v u_v \end{aligned} \right\} \quad (2)$$

the equations of motion (EOM) associated with $\mathbf{x}_{MTi}$ are:

$$\left. \begin{aligned} \dot{\rho}_{MTi} &= -V_{C,MTi} \\ \dot{\lambda}_{MTi} &= V_{\lambda,MTi}/\rho_{MTi} \\ \dot{\mathbf{x}}_{Mi} &= \mathbf{A}_{Mi}\mathbf{x}_{Mi} + \mathbf{B}_{Mi}u_{Mi} \\ \dot{\gamma}_{Mi} &= a_{Mi}/V_{Mi} \\ \dot{V}_{Mi} &= 0 \end{aligned} \right\} \quad (3)$$

where the closing speed of the Mi - Ti engagement, $V_{C,MTi}$ is

$$V_{C,MTi} = V_{Ti} \cos(\gamma_{Ti} + \lambda_{MTi}) + V_{Mi} \cos(\gamma_{Mi} - \lambda_{MTi}) \quad (4)$$

the speed orthogonal to the Mi - Ti LOS, $V_{\lambda,MTi}$, is

$$V_{\lambda,MTi} = V_{Ti} \sin(\gamma_{Ti} + \lambda_{MTi}) - V_{Mi} \sin(\gamma_{Mi} - \lambda_{MTi}) \quad (5)$$

and γ_{Ti} is defined by the following:

$$\dot{\gamma}_{Ti} = a_{Ti}/V_{Ti} \quad (6)$$

where a_{Ti} is defined in Eq. (2).

In addition to kinematics between the missiles and targets, the relative motion between $M1$ and $M2$ is also described in a similar manner as Eq. (3), i.e.,

$$\left. \begin{aligned} \dot{\rho}_{MM} &= -V_{C,MM} \\ \dot{\lambda}_{MM} &= V_{\lambda,MM}/\rho_{MM} \end{aligned} \right\} \quad (7)$$

where

$$V_{C,MM} = -V_{M2} \cos(\gamma_{M2} - \lambda_{MM}) + V_{M1} \cos(\gamma_{M1} - \lambda_{MM}) \quad (8)$$

$$V_{\lambda,MM} = V_{M2} \sin(\gamma_{M2} - \lambda_{MM}) - V_{M1} \sin(\gamma_{M1} - \lambda_{MM}) \quad (9)$$

2. Measurement Model

It is assumed that the targets are using electro-optic seekers and/ or radars to acquire measurements. Therefore, each target may either measure ρ_{MTi} and λ_{MTi} or only λ_{MTi} , where $i \in \{1, 2\}$. The discrete time measurements, $\mathbf{z}_{MTi}(k) \in \mathbb{R}^{n_z}$, are corrupted by a zero-mean, mutually independent, white Gaussian measurement noise, $\mathbf{v}_{MTi}(k) \in \mathbb{R}^{n_z}$. Therefore, the measurement model of the i th estimator, when both ρ_{MTi} and λ_{MTi} are available, is

$$\mathbf{z}_{MTi}(k) = \mathbf{H}\mathbf{x}_{MTi}(k) + \mathbf{v}_{MTi}(k) = \begin{bmatrix} \rho_{MTi}(k) \\ \lambda_{MTi}(k) \end{bmatrix} + \mathbf{v}_{MTi}(k) \quad (10)$$

where

$$\mathbf{v}_{MTi}(k) \sim \mathcal{N}([0]_{n_z \times 1}, \mathbf{R}_{MTi}), \quad \mathbf{R}_{MTi} = \text{diag}(\sigma_{i,\rho}^2, \sigma_{i,\lambda}^2) \quad (11)$$

such that $\mathbf{H}$ is the corresponding measurement matrix and $\mathbf{R}_{MTi}$ is the covariance matrix with $\sigma_{i,\rho}^2$ and $\sigma_{i,\lambda}^2$ being the variances for the range and LOS angle measurements acquired by i th target respectively. In the case when only λ_{MTi} is acquired, the first row of $\mathbf{H}$ is removed and $\mathbf{R}_{MTi} = \sigma_{i,\lambda}^2$.

B. Optimal Cooperative Guidance Law for the Target Pair

In this section, the optimal cooperative guidance law for the target pair is derived. The optimal strategy is solved using linearized kinematics. It is assumed that the missiles are using a linear guidance law that is known to the target pair through an identification scheme such as the one that will be presented in Section 1.

1. Linearized Kinematics for Guidance Law Derivation

Classical linearization [1] about the collision triangles in the engagement is performed with reference to each triangle's initial LOS. The state y_{MTi} , $i \in \{1, 2\}$ is defined as the separation between Mi and Ti orthogonal to the initial LOS of Mi and Ti while y_{MM} is the separation between $M1$ and $M2$ orthogonal to the initial LOS of $M1$ and $M2$ (Figure 1).

Since the objective for the target pair defined in this work is to survive the engagement by cooperatively luring the missiles into collision, the guidance law should drive the final time of the $M1 - M2$ engagement, $t_{f,MM}$, to occur earlier than both $M1 - T1$ and $M2 - T2$ engagements, i.e.,

$$t_{f,MM} < \min(t_{f,MT1}, t_{f,MT2}) \quad (12)$$

However, depending on initial positions and flight path angles of the missiles, condition Eq. (12) need not necessarily hold. Referring to Figure 1, it is evident that in order to reduce $t_{f,MM}$, the target pair should minimize the magnitude of δ_{MM} and θ_{MM} so as to increase the closing velocity of the $M1 - M2$ engagement. To this end, the linearized model is formulated incorporating the latter two states.

Thus, to solve for the optimal guidance law, the following linearized state vector $\mathbf{y}$ is defined:

$$\mathbf{y} = [\mathbf{y}_{MT1}^T \quad \mathbf{y}_{MT2}^T \quad \mathbf{y}_{MM}^T \quad \delta_{MM} \quad \theta_{MM}]^T \quad (13)$$

where

$$\mathbf{y}_{MM} = [y_{MM} \quad \dot{y}_{MM}]^T, \quad \mathbf{y}_{MTi} = [y_{MTi} \quad \dot{y}_{MTi} \quad \mathbf{y}_{Mi}^T \quad \mathbf{y}_{Ti}^T]^T, \quad i \in \{1, 2\} \quad (14)$$

The internal state vectors $\mathbf{y}_{Mi}$ and $\mathbf{y}_{Ti}$ are the components of the $\mathbf{x}_{Mi}$ and $\mathbf{x}_{Ti}$ orthogonal to the initial line-of-sight of the $Mi - Ti$ engagement and

$$\left. \begin{aligned} \delta_{MM} &\triangleq \gamma_{M1} - \lambda_{MM} \\ \theta_{MM} &\triangleq \pi - (\gamma_{M2} - \lambda_{MM}) \end{aligned} \right\} \quad (15)$$

The linearized EOM are:

$$\dot{\mathbf{y}} = \begin{cases} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= a_{T1}^\perp - a_{M1}^\perp \\ \dot{\mathbf{y}}_{M1} &= \mathbf{A}_{M1}\mathbf{y}_{M1} + \mathbf{B}_{M1}u_{M1}^\perp \\ \dot{\mathbf{y}}_{T1} &= \mathbf{A}_{T1}\mathbf{y}_{T1} + \mathbf{B}_{T1}u_{T1}^\perp \\ \dot{y}_{n_{MT1}+3} &= y_{n_{MT1}+4} \\ \dot{y}_{n_{MT1}+4} &= a_{T2}^\perp - a_{M2}^\perp \\ \dot{\mathbf{y}}_{M2} &= \mathbf{A}_{M2}\mathbf{y}_{M2} + \mathbf{B}_{M2}u_{M2}^\perp \\ \dot{\mathbf{y}}_{T2} &= \mathbf{A}_{T2}\mathbf{y}_{T2} + \mathbf{B}_{T2}u_{T2}^\perp \\ \dot{y}_{n_{MT1}+n_{MT2}+5} &= y_{n_{MT1}+n_{MT2}+6} \\ \dot{y}_{n_{MT1}+n_{MT2}+6} &= -a_{M2}^\perp C_{MM}^\theta / C_{MT2}^\delta - a_{M1}^\perp C_{MM}^\delta / C_{MT1}^\delta \\ \dot{y}_{n_{MT1}+n_{MT2}+7} &= \dot{\gamma}_{M1} - \dot{\lambda}_{MM} \\ \dot{y}_{n_{MT1}+n_{MT2}+8} &= \dot{\lambda}_{MM} - \dot{\gamma}_{M2} \end{cases} \quad (16)$$

where for $i \in \{1, 2\}$, $n_{Mi} = \dim(\mathbf{A}_{Mi})$, $n_{Ti} = \dim(\mathbf{A}_{Ti})$ and

$$n_{MTi} = n_{Mi} + n_{Ti} \quad (17)$$

and for $E \in \{MT1, MT2, MM\}$, $\eta \in \{\delta, \theta\}$

$$C_E^\eta = \cos \eta_E \quad (18)$$

with δ_{MTi} and θ_{MTi} defined as

$$\left. \begin{aligned} \delta_{MTi} &= \gamma_{Mi} - \lambda_{MTi} \\ \theta_{MTi} &= \gamma_{Ti} + \lambda_{MTi} \end{aligned} \right\} \quad (19)$$

and δ_{MM} and θ_{MM} as presented in Eq. (15).

In Eq. (16), the superscript $\perp$ represents the component of the acceleration orthogonal to the associated LOS, i.e., for $i \in \{1, 2\}$

$$a_{Ti}^{\perp} = a_{Ti} C_{MTi}^{\theta} \quad (20)$$

$$a_{Mi}^{\perp} = a_{Mi} C_{MTi}^{\delta} \quad (21)$$

Assuming linearization holds, $\dot{\lambda}_{MM}$ defined in Eq. (7) is incorporated into Eq. (16) in its linearized form as:

$$\dot{\lambda}_{MM} = \frac{y_{MM} + \dot{y}_{MM} t_{go,MM}}{V_{C,MM} t_{go,MM}^2} \quad (22)$$

where $V_{C,MM}$ is defined in Eq. (8) and $t_{go,MM}$ is defined as

$$t_{go,MM} = t_{f,MM} - t \quad (23)$$

Next, the missiles are assumed to be using a linear guidance law in the following form:

$$u_{Mi}^{\perp} = \mathbf{K}_{MTi} \mathbf{y}_{MTi} + K_{u_{Ti}} u_{Ti}^{\perp} \quad (24)$$

where

$$\mathbf{K}_{MTi} = \begin{bmatrix} K_1^{Mi} & K_2^{Mi} & \mathbf{K}_{Mi} & \mathbf{K}_{Ti} \end{bmatrix}, \quad i \in \{1, 2\} \quad (25)$$

Remark 1. In this work, the cooperative guidance law is derived against missiles using PN [2], APN [3] and OGL [4]. For the sake of brevity, gains of these well-established linear guidance laws, $\mathbf{K}_{MTi}$ and $K_{u_{Ti}}$ are not explicitly defined in this work. Readers are referred to Section V.B in [5] for the detailed formulation of these guidance laws.

Assuming that information about $\mathbf{K}_{MTi}$ and $K_{u_{Ti}}$ is known to the target group, Eq. (16) can be formulated as a one-sided optimal control problem, i.e:

$$\dot{\mathbf{y}} = \mathbf{A}(t) \mathbf{y}(t) + \mathbf{B}(t) \mathbf{u}_T(t) \quad (26)$$

such that

$$\begin{aligned} \mathbf{A}(t) &= \begin{bmatrix} \mathbf{A}_{MT1} & [0] & [0] \\ [0] & \mathbf{A}_{MT2} & [0] \\ \mathbf{A}_{MM}^{M1} & \mathbf{A}_{MM}^{M2} & \mathbf{A}_{MM}^0 \end{bmatrix}, \quad \mathbf{B}(t) = \begin{bmatrix} \mathbf{B}_{MT1} & [0] \\ [0] & \mathbf{B}_{MT2} \\ \mathbf{B}_{MM}^{T1} & \mathbf{B}_{MM}^{T2} \end{bmatrix}, \\ \mathbf{u}_T(t) &= \begin{bmatrix} u_{T1}^{\perp} & u_{T2}^{\perp} \end{bmatrix}^T \end{aligned} \quad (27)$$

where $[0]$ is a matrix of zeros with appropriate dimensions.

2. Optimal Cooperative Guidance Law Derivation

As discussed above, Eq. (12) may not hold for all initial conditions. Thus, in order to derive a guidance law that can drive the system towards achieving Eq. (12), states δ_{MM} and θ_{MM} need to be incorporated into the cost function of the optimal control problem.

The cost is defined using the finite horizon, linear-quadratic formulation:

$$J = \mathbf{y}^T(t_{f,MM})\mathbf{Q}_f\mathbf{y}(t_{f,MM}) + \int_{t_0}^{t_{f,MM}} [\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t) + \mathbf{u}_T^T(t)\mathbf{R}\mathbf{u}_T(t)] dt \quad (28)$$

where

$$\begin{aligned} \mathbf{Q}_f &= \begin{bmatrix} [0] & [0] \\ [0] & \mathbf{w}_f \end{bmatrix}, & \mathbf{w}_f &= \begin{bmatrix} w_{ms} & [0] \\ [0] & [0] \end{bmatrix} \\ \mathbf{Q} &= \begin{bmatrix} [0] & [0] \\ [0] & \mathbf{w}_r \end{bmatrix}, & \mathbf{w}_r &= \begin{bmatrix} w_\delta & 0 \\ 0 & w_\theta \end{bmatrix} \\ \mathbf{R} &= \begin{bmatrix} w_{u1} & 0 \\ 0 & w_{u2} \end{bmatrix} \end{aligned} \quad (29)$$

The weights in Eq. (29) are defined as follows: w_{ms} is the weight on $y_{MM}(t_{f,MM})$; w_δ and w_θ are weights on the angles δ_{MM} and θ_{MM} that were defined in Eq. (15) and shown in Figure 1; w_{u1} and w_{u2} are weights on $u_{T1}^\perp$ and $u_{T2}^\perp$ respectively.

The objective of the optimal controller, $\mathbf{u}_T^*(t)$, is to minimize J in Eq. (28), i.e.,

$$\mathbf{u}_T^*(t) = \arg \min_{\mathbf{u}_T \in \Omega} J \quad (30)$$

where Ω is the set of admissible controls.

Solution to the problem defined in (30) is given by:

$$\mathbf{u}_T^*(t) = -\mathbf{R}^{-1}\mathbf{B}^T(t)\mathbf{P}(t)\mathbf{y}(t) \quad (31)$$

where $\mathbf{P}(t)$ is the solution to the differential Riccati equation [6]:

$$-\dot{\mathbf{P}}(t) = \mathbf{P}(t)\mathbf{A}(t) + \mathbf{A}^T(t)\mathbf{P}(t) - \mathbf{P}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}(t) + \mathbf{Q}(t), \quad \mathbf{P}(t_f) = \mathbf{Q}_f \quad (32)$$

Physically, $\mathbf{u}_T^*(t)$ not only minimizes the miss distance between the missiles at $t_{f,MM}$ with minimal control effort, it also minimizes the weighted sum of δ_{MM}^2 and θ_{MM}^2 over $[t_0, t_{f,MM}]$ to maximize $V_{C,MM}$ and drive the missiles to collision before they could hit the targets. Note that this is unlike classical linearization assumptions for missile-target engagements [1] where the geometry of the collision triangle is assumed to be near constant since $\mathbf{u}_T^*$ is altering the collision geometry at each time step by minimizing δ_{MM}^2 and θ_{MM}^2 .

Implementing the Linear Optimal Strategy in the Nonlinear Engagement

The optimal controller, $\mathbf{u}_T^*$, in Eq. (31) is solved in the linearized frame and represents the required control that is orthogonal to corresponding line-of-sight. This subsection describes how $\mathbf{u}_T^*$ is implemented in the proposed guidance strategy to obtain the nonlinear controller $\mathbf{u}_T^{NL}$.

As noted in Section 2, in such 2-on-2 engagements there exist initial conditions in which the missiles are not on the required collision triangle such that condition Eq. (12) is met. In such conditions, the targets are required to maneuver in a way such that the initial collision geometry is altered to meet condition Eq. (12). Thus, the classical linearization assumption of the collision geometry remaining constant is no longer valid and in order to implement the proposed linear controller, this work employs the State Dependent Riccati Equations (SDRE) method [7]. Under the SDRE framework, the nonlinear engagement is re-linearized at every time step k and Eq. (32) is solved online to obtain $\mathbf{u}_T^*(k)$. Finally, in order to obtain the required nonlinear control at each time step, $\mathbf{u}_T^{NL}(k)$, $\mathbf{u}_T^*(k)$ is resolved to the direction orthogonal to the target's velocity vector, i.e.,

$$u_{Ti}^{NL}(k) = \frac{u_{Ti}^*(k)}{\cos[\gamma_{Ti}(k) + \lambda_{MTi}(k)]}, \quad i \in \{1, 2\} \quad (33)$$

In order to solve for $\mathbf{u}_T^*$, integration of Eq. (32) from $t_{go,MM} \in [0, t_{f,MM}]$ is required. Since it is assumed that the missile is using classical missile guidance laws, there exist elements that are inversely proportional to $t_{go,MT1}$ in $\mathbf{A}(t)$ and $\mathbf{A}(t)$ becomes singular when $t_{go,MM} = \Delta$.

In this work, we use the classical linearized method to approximate $t_{go,E}$ of the collision geometry E , $E \in \{MT1, MT2, MM\}$, at each time step. Let $t_{go,E}^{lin}$ be the linear approximation for the $t_{go,E}$, then

$$t_{go,E}^{lin} = \rho_E / V_{C,E}, \quad E \in \{MT1, MT2, MM\} \quad (34)$$

This method of estimating $t_{go,E}^{lin}$ can deviate far from the true value especially when there are large changes to the initial collision geometry. Depending on initial conditions, such errors in $t_{go,E}^{lin}$ could violate the condition in Eq. (12) although actual values of $t_{go,E}$ do not. Therefore, for the implementation of the SDRE-based controller in this work, to avoid the aforementioned singularity issue, $\bar{t}_{go,MM}$ is used to approximate $t_{go,MM}$, where $\bar{t}_{go,MM}$ is defined by the following conditional statement:

Condition 1. If $t_{go,MM}^{lin} \neq \min(t_{go,MT1}^{lin}, t_{go,MT2}^{lin}, t_{go,MM}^{lin})$, then

$$\bar{t}_{go,MM} = \min(t_{go,MT1}^{lin}, t_{go,MT2}^{lin}) \quad (35)$$

else

$$\bar{t}_{go,MM} = t_{go,MM}^{lin} \quad (36)$$

While Eq. (35) clearly does not accurately depict the true kinematics of the problem, it is a heuristic method that allows us to avoid the singularity problem. It also makes physical sense to assume that the missile-missile collision, if it succeeds, should occur no later than earliest missile-target collision. Note while more accurate methods of approximating time-to-go exists in open literature (e.g., in [8]) the method proposed using Condition 1 is much simpler to implement and, as shown in the simulation results in Section D, proved to be sufficient for the implementation of the cooperative guidance law.

C. MMAC-based Cooperative Guidance Law

In order for the target pair to maneuver optimally against the pursuing missiles and lure them into collision, the targets are assumed to know exactly what guidance laws the missile team is using. In this section, an MMAE approach is proposed to identify the guidance laws employed by the missile team, the relative states of the engagement, as well as the various time-to-go values necessary for the computation of the cooperative guidance strategy. The section begins with a brief review of the fundamental principles of MMAE before presenting the implementation of the estimator together with guidance law described in Section B under a MMAC framework.

1. Multiple-Model Adaptive Estimator

The MMAE approach [9, 5, 10, 11, 12] is a well-established method of estimating unknown system parameters. The system is assumed to have a known finite set of possible regimes and the true regime is fixed. Filters are run in parallel and each filter matches one of the possible regimes (see for instance [9]).

In this work, it is assumed that the pursuing missiles are employing one of the p possible missile guidance strategies in the set $\mathcal{U}_i$, $i \in \{1, 2\}$, i.e.,

$$u_{Mi} \in \mathcal{U}_i \triangleq \{u_{Mi}^1, \dots, u_{Mi}^j, \dots, u_{Mi}^p\} \quad (37)$$

In addition, we assume in this work that the target pair has perfect information on its own parameters associated to the estimation process ($\mathbf{x}_{Ti}$, γ_{Ti} , and V_{Ti} , $i \in \{1, 2\}$). It is also assumed that the target pair has exact information on the missile dynamics (for methods to identify the latter, readers are directed to [9, 5]). The estimation process is assumed to be decoupled between the two targets and the estimator of each target need only the measurements acquired by itself. Therefore, the j th regime dynamics is defined by the EOM presented in Eq. (3) with $u_{Mi} = u_{Mi}^j$ and can be compactly written in discrete time as:

$$\mathbf{x}_{MTi}^j(k) = \mathbf{f}_{k-1}^j(\mathbf{x}_{MTi}^j(k-1), \mathbf{u}_T(k-1)), \quad i \in \{1, 2\} \quad (38)$$

where $\mathbf{x}_{MTi}^j(k)$ is the discretized version of the state vector $\mathbf{x}_{MTi}(t_k)$ in Eq. (1) associated with the j th regime and $t_k = kT_s$, T_s is the sampling period used for estimation. The vector function $\mathbf{f}_{k-1}^j$ is obtained by

integrating the EOMs of the j th regime from t_{k-1} to t_k . In this work, EKF matching the regime dynamics is used to calculate the time update state estimate since the engagement kinematics is nonlinear.

Next, using Bayes' rule, the j th regime probability at the k th time step, $\mu_i^j(k)$, can be determined by the following recursive formula [11], based on a given initial probability, $\mu_i^j(0)$:

$$\mu_i^j(k) = \frac{\Lambda_i^j(k)\mu_i^j(k-1)}{\sum_{l=1}^p \Lambda_i^l(k)\mu_i^l(k-1)}, \quad j \in \{1, \dots, p\} \quad (39)$$

where $\Lambda_i^j(k) \triangleq f_p(\mathbf{z}_i(k)|\mathbf{z}_i(1:k-1), u_{Mi}^j)$ is the j th regime-conditioned likelihood function computed based on the innovations process statistics of the j th filter of the i th missile, $\mathbf{z}_i(1:k)$ is the measurement acquired by the i th missile from 1st time step to the k th, and $f_p(\mathcal{A}|\mathcal{B})$ is the conditional probability density function of $\mathcal{A}$ given $\mathcal{B}$.

Under linear-Gaussian assumptions, $\Lambda_i^j(k)$ is also Gaussian and is thus

$$\Lambda_i^j(k) = \mathcal{N}(\boldsymbol{\nu}_i^j(k); [0]_{n_z \times 1}, \mathbf{S}_i^j(k)) \quad (40)$$

where $\boldsymbol{\nu}_i^j(k)$ and $\mathbf{S}_i^j(k)$ are the innovation and its covariance from the j th regime-matched filter.

2. MMAE for Identification of Classical Guidance Laws

In this work, the cooperative targets guidance law is derived against a missile team using the well-established classical guidance laws PN, APN and OGL. Application of these guidance laws is usually in its nonlinear form rather than the linearized formulation presented in Eq. (24). For the sake of brevity, the index for the i th missile or target, $i \in \{1, 2\}$ is dropped in the formulation of this subsection:

$$u_M^j = N'_{GL} \frac{Z_{GL}}{t_{go,MT}^2 \cos(\gamma_M - \lambda_{MT})} \quad GL \in \{PN, APN, OGL\} \quad (41)$$

where N'_{GL} is the effective navigation gain, Z_{GL} is the missile's ZEM distance, which is unique for each guidance law:

$$Z_{PN} = V_{C,MT} \dot{\lambda}_{MT} t_{go,MT}^2 \quad (42a)$$

$$Z_{APN} = Z_{PN} + \frac{t_{go,MT}^2}{2} a_T^\perp \quad (42b)$$

$$Z_{OGL} = Z_{APN} - a_M^\perp \tau_M^2 \psi(\xi_{MT}) \quad (42c)$$

where $\dot{\lambda}_{MT}$ and $V_{C,MT}$ are defined in Eq. (3) and Eq. (4) respectively. Variables $a_T^\perp$ and $a_M^\perp$ are the components of the target and missile accelerations orthogonal to the missile-target LOS as presented in Eq. (20) and Eq. (21). And $\psi(\xi)$ is defined as

$$\psi(\xi_{MT}) = \exp(-\xi_{MT}) + \xi_{MT} - 1 \quad (43)$$

where $\xi_{MT} \triangleq t_{go,MT}/\tau_M$ is the non-dimensionalized time-to-go.

While N'_{GL} are constants for PN and APN, N'_{OGL} is a function of ξ_{MT} and is defined as:

$$N'_{OGL} = \frac{6\xi_{MT}^2 \psi(\xi_{MT})}{3 + 6\xi_{MT} - 6\xi_{MT}^2 + 2\xi_{MT}^3 - 3\exp(-2\xi_{MT}) - 12\xi_{MT}\exp(-\xi_{MT})} \quad (44)$$

Thus to determine the guidance law of the missiles, the MMAE is required to identify the ZEM values, Eq. (42), and in the case of PN and APN, N'_{GL} .

3. On Estimating the Range and LOS Angle Between M1 and M2

Besides the states associated with the missile-target collision geometries, the cooperative guidance law defined in Section B requires also the states ρ_{MM} and λ_{MM} for computation of the linearized states. In this work, these states are computed separately out of the estimation loop using the estimates of ρ_{MTi} and λ_{MTi} ,

$i \in \{1, 2\}$ and the relative position of the targets. This way estimation can be decoupled between the targets and they need only compute the number of filters that corresponds to the p number of possible guidance laws.

Let ρ_{TT} and λ_{TT} (see Figure 2) define the relative position between the targets in polar coordinates and assuming we have perfect information on these parameters. Then by taking reference from one of the targets, say $T1$, ρ_{MM} and λ_{MM} can be computed:

$$\left. \begin{aligned} \rho_{MM} &= \sqrt{\Delta X_{MM}^2 + \Delta Y_{MM}^2} \\ \lambda_{MM} &= \arctan(\Delta Y_{MM} / \Delta X_{MM}) \end{aligned} \right\} \quad (45)$$

where

$$\Delta X_{MM} = \rho_{TT} \cos \lambda_{TT} - \rho_{MT2} \cos \lambda_{MT2} + \rho_{MT1} \cos \lambda_{MT1} \quad (46)$$

$$\Delta Y_{MM} = \rho_{TT} \sin \lambda_{TT} - \rho_{MT2} \sin \lambda_{MT2} + \rho_{MT1} \sin \lambda_{MT1} \quad (47)$$

Estimates $\hat{\rho}_{MM}$ and $\hat{\lambda}_{MM}$ can then be computed using equations Eq. (45) to Eq. (47) based on the estimates $\hat{\rho}_{MT1}$, $\hat{\rho}_{MT2}$, $\hat{\lambda}_{MT1}$, $\hat{\rho}_{MT2}$, and variables ρ_{TT} and λ_{TT} in which we assumed to have perfect information of.

Remark 2. If the estimation process is coupled between $T1$ and $T2$, i.e., ρ_{MM} and λ_{MM} are estimated by incorporating their dynamics Eq. (7) into the EKF equations, then the number of filters increases drastically with p since in addition to the number of possible guidance laws employed by each missile, the MMAE scheme has to compute p^2 number of filters in parallel to account for the possible combinations of guidance laws employed by the 2 missiles. Also, the size of the covariance matrix to be solved by each filter increases from 5×5 (5 states for each $Mi - Ti$ engagement, $i \in \{1, 2\}$) to 12×12 (10 states for both $Mi - Ti$ engagements plus ρ_{MM} and λ_{MM}). This can become very computationally intensive with increasing p .

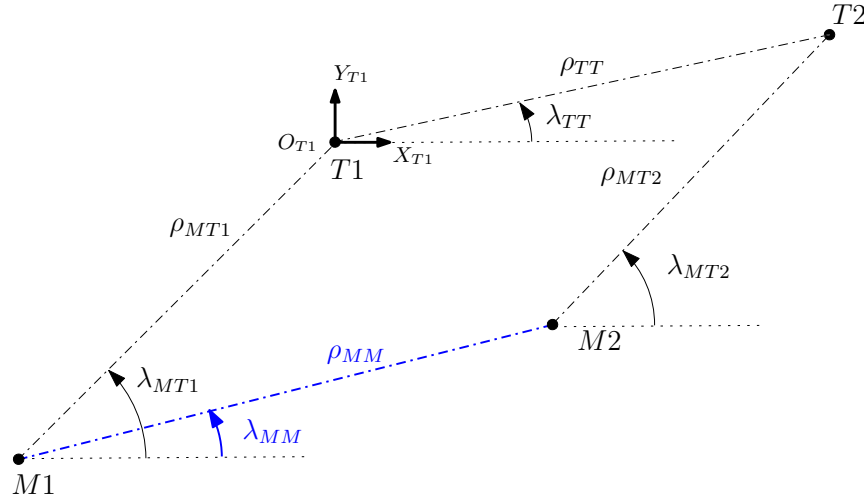


Figure 2: Relative positions of $M1$ & $M2$ referenced to $T1$.

4. MMAC-based Cooperative Guidance Law

In order to compute the maneuver commands for the target pair when information on the missiles is imperfect, this work employs the MMAC approach [13]. In this approach, the state estimates from each filter are fed into a regime matched controller. The final target commands can then be computed by either: 1) minimum mean square error (MMSE), or 2) maximum a posteriori probability (MAP) approach.

MMSE Approach

In the MMSE approach, the final target command u_T is determined by a weighted average of the regime-matched target commands. However, each regime-matched controller varies with different combinations of guidance laws used by the missile team, and in order to compute the final target commands using the MMSE

approach would require multiple solutions of the Riccati equation Eq. (32) each accounting for every possible combination.

Consider each target, Ti , computes p number of regimes that corresponds to the list of possible guidance laws employed by its pursuer, Mi . Let $\mathcal{U}_{comb}$ denote the set that consists of all possible combinations of guidance laws between $M1$ and $M2$ and $\mathbf{u}_M^l$ the l th combination in this set, i.e.,

$$\mathbf{u}_M^l = \begin{bmatrix} u_{M1}^{j1}, & u_{M2}^{j2} \end{bmatrix}, \quad l \in \{1, \dots, p^2\}, \quad j1, j2 \in \{1, \dots, p\}, \quad \text{and} \quad \mathbf{u}_M^l \in \mathcal{U}_{comb} \quad (48)$$

If $\mathcal{U}_{comb}$ is arranged in the following order:

$$\mathcal{U}_{comb} \triangleq \left\{ \begin{array}{ll} \mathbf{u}_M^1 & \triangleq \begin{bmatrix} u_{M1}^1, & u_{M2}^1 \end{bmatrix}^T, & \dots, & \mathbf{u}_M^p & \triangleq \begin{bmatrix} u_{M1}^1, & u_{M2}^p \end{bmatrix}^T \\ \mathbf{u}_M^{p+1} & \triangleq \begin{bmatrix} u_{M1}^2, & u_{M2}^1 \end{bmatrix}^T, & \dots, & \mathbf{u}_M^{2p} & \triangleq \begin{bmatrix} u_{M1}^2, & u_{M2}^p \end{bmatrix}^T \\ & \vdots & & & \\ \mathbf{u}_M^{(p-1)p+1} & \triangleq \begin{bmatrix} u_{M1}^p, & u_{M2}^1 \end{bmatrix}^T & \dots, & \mathbf{u}_M^{p^2} & \triangleq \begin{bmatrix} u_{M1}^p, & u_{M2}^p \end{bmatrix}^T \end{array} \right\} \quad (49)$$

then the index l can be written as a function of $j1$ and $j2$,

$$l = (j1 - 1)p + j2 \quad (50)$$

The combined regime probability of $\mathbf{u}_M^l$ at time step k , $\mu^l(k)$, is calculated based on the regime probabilities associated with the $j1$ -th and $j2$ -th regimes in the MMAE of $T1$ and $T2$, i.e.,

$$\mu^l(k) = \mu_1^{j1}(k) \mu_2^{j2}(k), \quad l \in \{1, \dots, p^2\} \quad (51)$$

The final command at time step k in a MMSE sense is,

$$\mathbf{u}_T^{(mmse)} = \sum_{l=1}^{p^2} \mu^l(k) \mathbf{u}_T^{*l}, \quad (52)$$

where $\mathbf{u}_T^{*l} = \begin{bmatrix} u_{T1}^{*l}, & u_{T2}^{*l} \end{bmatrix}^T$, $l \in \{1, \dots, p^2\}$ is the optimal maneuver based on the cooperative guidance strategy as formulated in Section B against the l th combination of missile guidance laws.

Remark 3. If the estimation is coupled, then both MMAE scheme and Riccati equation Eq. (32) will be computed p^2 number of times in parallel. Instead of Eq. (51), $\mu^l(k)$ will simply correspond to regime probability of the l th regime in the p^2 number of filters in the coupled MMAE scheme.

MAP Approach

In the MAP approach, $\mathbf{u}_T$ is determined as the command against the combination of missile guidance laws that has the maximum a posteriori probability. In this approach, only one solution to Eq. (32) is required since we only need to compute the SDRE controller for the combination of guidance laws that corresponds to the regimes with the highest probability from each target's MMAE. Consider the following:

$$\left. \begin{array}{l} j1 = \arg \max_{j \in \{1, \dots, p\}} \mu_1^j(k) \\ j2 = \arg \max_{j \in \{1, \dots, p\}} \mu_2^j(k) \end{array} \right\} \quad (53)$$

then

$$\mathbf{u}_T^{(map)} = \mathbf{u}_T^{*l} \quad (54)$$

where l relates to indices $j1$ and $j2$ through Eq. (50), and it is the index in $\mathcal{U}_{comb}$ as defined by Eq. (49).

Remark 4. If the estimation process is coupled, then a total of p^2 number of filters need to be run in parallel. However, unlike in MMSE approach, $\mathbf{u}_T^{(map)}$ only needs Eq. (32) to be solved once at each time step for $\mathbf{u}_T^{*l}$ instead of solving all p^2 number of regime-matched optimal maneuvers as shown in Eq. (52).

5. Sensitivity of Guidance Law to Estimation Performance

The cooperative guidance law derived in Section B follows a similar geometric rule to the classical PN in which collision is achieved if the associated vehicles are able to remain on the collision triangle. The targets must therefore ensure that the LOS rate between the missiles are nullified towards the end of the engagement. Hence, accuracy of the estimate for the LOS angle between the missiles, λ_{MM} , becomes paramount to achieving good miss-distance performance. Figure 3 depicts the effects of missile-target estimation errors on the estimation error of λ_{MM} . Here, the accent “ $\sim$ ” denotes the estimation error of the associated parameter. For the purpose of this discussion, it is assumed that states of $M1$ are known perfectly and $\tilde{\lambda}_{MM}$ is attributed to errors in state estimates of the $M2 - T2$ engagement. We introduce another parameter in this discussion, $\tilde{y}_{MM}$, which represents the difference in the true and estimated position of $M2$. The parameter $\tilde{y}_{MM}$ can also be viewed as the miss-distance associated with estimation error, $\tilde{\lambda}_{MM}$. Assuming estimation errors are small near the end of $M1 - M2$ engagement, using small angle approximations, $\tilde{y}_{MM}$, can be written as:

$$\tilde{y}_{MM} \simeq \rho_{MM} \tilde{\lambda}_{MM} = \sqrt{\tilde{y}_{MT2}^2 + \tilde{\rho}_{MT2}^2} \quad (55)$$

such that

$$\tilde{y}_{MT2} \simeq \rho_{MT2} \tilde{\lambda}_{MT2} \quad (56)$$

From Eq. (55) and Eq. (56), it is clear that contribution of $\tilde{\lambda}_{MT2}$ to $\tilde{y}_{MM}$ diminishes as the range between the $M2$ and $T2$ decreases. Assuming $\tilde{\lambda}_{MT2}$ and $\tilde{\rho}_{MT2}$ converges to its asymptotic value towards the end of the engagement and $\dot{\rho}_{MT2} < 0$, the influence of $\tilde{\lambda}_{MT2}$ on $\tilde{y}_{MM}$ reduces as compared to $\tilde{\rho}_{MT2}$ as the missile approaches the target. Thus, the quality of the range estimate between the missile and the target becomes the dominant factor influencing the cooperative guidance law’s miss distance performance.

In order to have some quantitative assessment on the influence of errors in range and LOS angle estimates between the missiles and targets towards $\tilde{y}_{MM}$, we compute $\tilde{y}_{MM}$ using Eq. (55) for various error levels in the missile-target state estimates for the case when $\hat{\rho}_{MT2} = 2000$ m at $t_{f,MM}$. Here it is assumed that the missiles have a non-zero miss (i.e., $\rho_{MM} \neq 0$) at $t_{f,MM}$.

	$\tilde{\lambda}_{MT2}$ [mrad]	$\tilde{\rho}_{MT2}$ [m]	$\tilde{y}_{MT2}$ [m]	$\tilde{y}_{MM}$ [m]
Nominal	0.25	1.5	0.5	1.581
$0.1 \times \tilde{\lambda}_{MT2}$	0.025	1.5	0.05	1.501
$0.1 \times \tilde{\rho}_{MT2}$	0.25	0.15	0.5	0.522

Table 1: Influence of the estimation errors in the missile-target engagement on missile-missile miss distance estimation using (55).

As seen in Table 1, a significant reduction in $\tilde{y}_{MM}$ is achieved for a given ρ_{MM} only in the case when $\tilde{\rho}_{MT2}$ is reduced. This influence of the range estimation accuracy between the missile-target on $\tilde{y}_{MM}$ will be further validated in the Monte Carlo studies in Section 2.

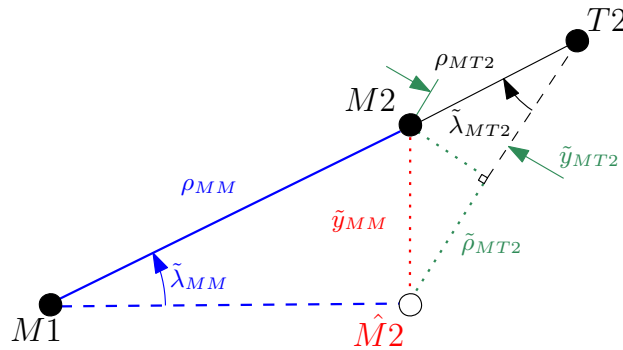


Figure 3: Effects of missile-target estimation errors on missile-missile miss-distance estimation, $\tilde{y}_{MM}$.

D. Simulation Analysis

In this section, the performance of the cooperative target guidance strategy derived in Section A is presented. The section begins by evaluating the proposed guidance law using perfect information, where we demonstrate the importance of the weights on δ_{MM} and θ_{MM} in the cost function Eq. (28). A “collision map” is presented thereafter in which the ability of the target team to lure the missile into collision based on different initial conditions is shown.

In the subsequent subsection, performance of the guidance law using the estimated states (i.e., “estimation-in-the-loop”) is analyzed in Monte Carlo (MC) simulations. Results from a sample run are also presented in this subsection to demonstrate the performance of the estimator. Sensitivity of the cumulative distribution function (CDF) of the missile-missile miss distance to measurement noise is also evaluated.

1. Guidance Law Performance with Perfect Information

Simulation Setup

The simulation setup is as follows. All vehicles have constant speeds: $V_{M1} = V_{M2} = 600 \text{ m/s}$, $V_{T1} = V_{T2} = 400 \text{ m/s}$. Flight path angles were set such that $\gamma_{T1} = \gamma_{T2} = 0^\circ$ and missiles have zero initial heading error. The guidance laws for $M1$ and $M2$ are APN, with $N'_{APN} = 5$, and OGL respectively. All vehicles have 1st-order dynamics with time lag constants, $\tau_M = 0.25 \text{ s}$ for both missiles and $\tau_T = 0.5 \text{ s}$ for both targets. Accelerations were bounded at $a_T^{max} = 10 \text{ g}$ for both targets and at $a_M^{max} = 20 \text{ g}$ for both missiles where $g = 9.80665 \text{ m/s}^2$ is the gravitational constant. Control update frequency is set at 50 Hz. Unless otherwise stated, weights on the cost function Eq. (28) are set as such: $w_{ms} = 1e4$, $w_{u1} = w_{u2} = 0.003$ and $w_\delta = w_\theta = 30$.

Influence of Weights on Guidance Performance

In this subsection, the weights on δ_{MM} and θ_{MM} from the cost function (28), w_δ and w_θ respectively, are varied to demonstrate its influence on the guidance performance. Referring to Figure 4, when $w_\delta = w_\theta = 0$, the targets simply maneuver to minimize the change in LOS between the missiles (through minimizing $y_{MM}(t_{f,MM})$ in the linear system). While the targets were able to guide the missiles to be on a collision course, they were not able to do so before $M2$ collided with $T2$. When $w_\delta = w_\theta = 30$, the targets maneuver to reduce δ_{MM} and θ_{MM} , thus driving the missiles to collide at an earlier time. It is important to note that the weights were tuned to ensure zero miss between the missiles: values too high compared to the weight on $y_{MM}(t_{f,MM})$ will result in non-zero miss while values too low (similar to $w_\delta = w_\theta = 0$) lead to collision between missile(s) and target(s).

Remark 5. *In the proposed guidance law, collision avoidance between the targets is not explicitly accounted for. Thus, in the singular cases, e.g., when the engagement has perfect symmetry (i.e., in position, target/missile velocities, missile guidance laws), the targets may collide before the missiles do. In order to avoid target-target collision, the target pair can 1) fly in an asymmetric formation as shown in Figure 4, 2) fly at different velocities and/ or 3) implement an anti-collision guidance strategy.*

Collision Map

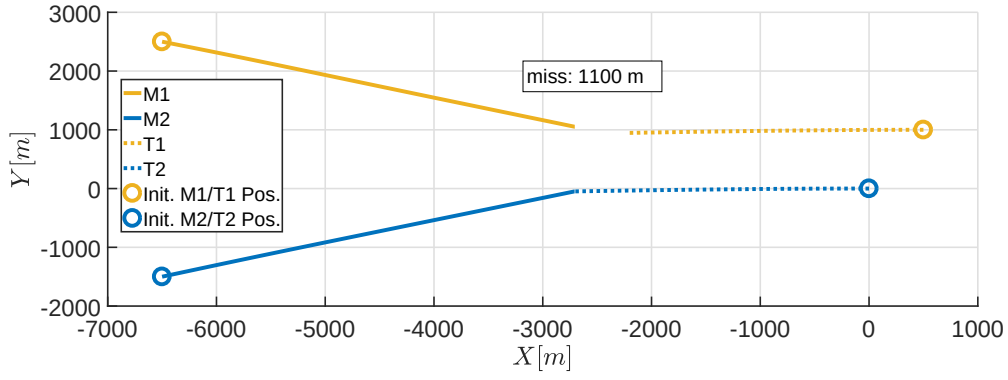
The performance of the guidance law is evaluated over a range of initial conditions and presented in a “collision map”. Missile and target parameters are the same as described in Section 1. Target positions are fixed at the same location as seen in Figure 4 while positions of the missiles are varied.

Positions of the missiles are varied as follows. Referring to Figure 5, the missiles’ initial LOS angle is fixed at $\lambda_{MM}^0 = -\pi/2$ (i.e., parallel to the Y_I axis). The center of missile-missile LOS is denoted as c and the initial inertial position of c is (X_c^0, Y_c^0) . The initial range between the missiles ρ_{MM}^0 , together with X_c^0 and Y_c^0 are varied over the intervals $\rho_{MM}^0 \in [100, 6000] \text{ m}$, $X_c^0 \in [-10000, -1000] \text{ m}$, and $Y_c^0 \in [-3000, 3000] \text{ m}$. These parameters are normalized by the initial range between the targets, ρ_{TT}^0 and are denoted as $\bar{\rho}_{MM}^0$, $\bar{X}_c^0$ and $\bar{Y}_c^0$, i.e.,

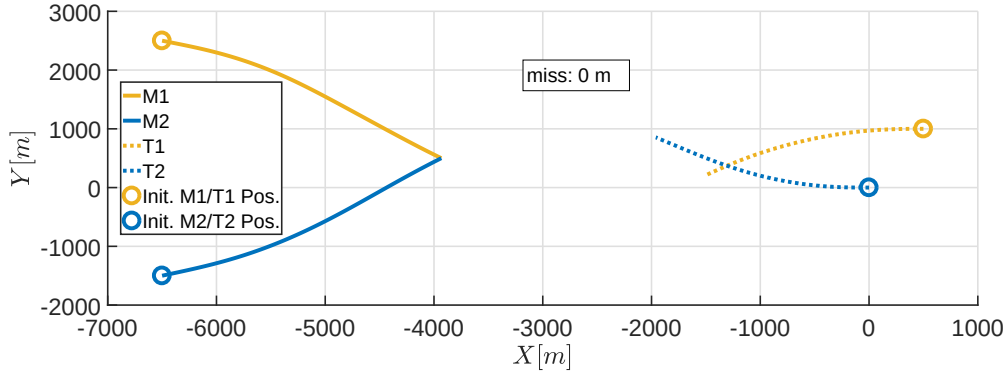
$$\bar{\rho}_{MM}^0 \triangleq \rho_{MM}^0 / \rho_{TT}^0, \quad \bar{X}_c^0 \triangleq X_c^0 / \rho_{TT}^0, \quad \bar{Y}_c^0 \triangleq Y_c^0 / \rho_{TT}^0 \quad (57)$$

The collision maps obtained from the simulations are presented in Figure 6 in which initial conditions that resulted in missile-missile collision are marked with blue circles while conditions which led to a miss are marked by yellow dots. Missiles are considered to have “collided” if $\rho_{MM}(t_{f,MM}) \leq 0.1 \text{ m}$.

As shown in Figure 6, the guidance law is most effective when $\bar{\rho}_{MM}^0 > 1$, $\bar{X}_c^0$ is large and when $\bar{Y}_c^0$ is small. This is intuitive as observed in the sample trajectory in Figure 4, the targets require sufficient initial



(a) $w_\delta = w_\theta = 0$



(b) $w_\delta = w_\theta = 30$

Figure 4: Influence of the weights on the relative angles between missiles' heading and the LOS between them.

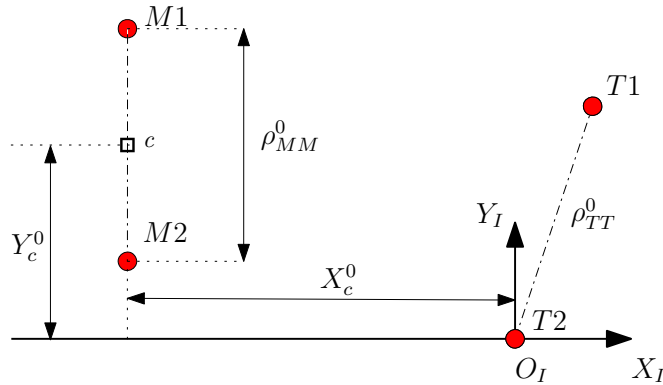


Figure 5: Relative positions of $M1$ & $M2$ referenced to $T1$.

separation between themselves and the missiles so that the target pair has sufficient time to lure the pursuers into collision. The guidance is less effective when $\bar{\rho}_{MM}^0 < 1$, i.e. $\rho_{MM}^0 < \rho_{TT}^0$. In such scenarios, the missile velocities are initially pointing away from each other (since we assumed that the missiles have zero initial heading error) and with decreasing $\bar{X}_c^0$, the initial difference between the missile headings becomes too large for the target pair to turn the missiles into head-on collision before one of the targets is intercepted. Also, as seen in Figures 6c and 6d, the targets were able to enforce collision if $\bar{Y}_c^0 \sim [-1, 2]$. This is also intuitive since a large offset in the center point c meant that one of the missile is closer to the target than the other

and target interception would occur earlier than the missile-missile collision.

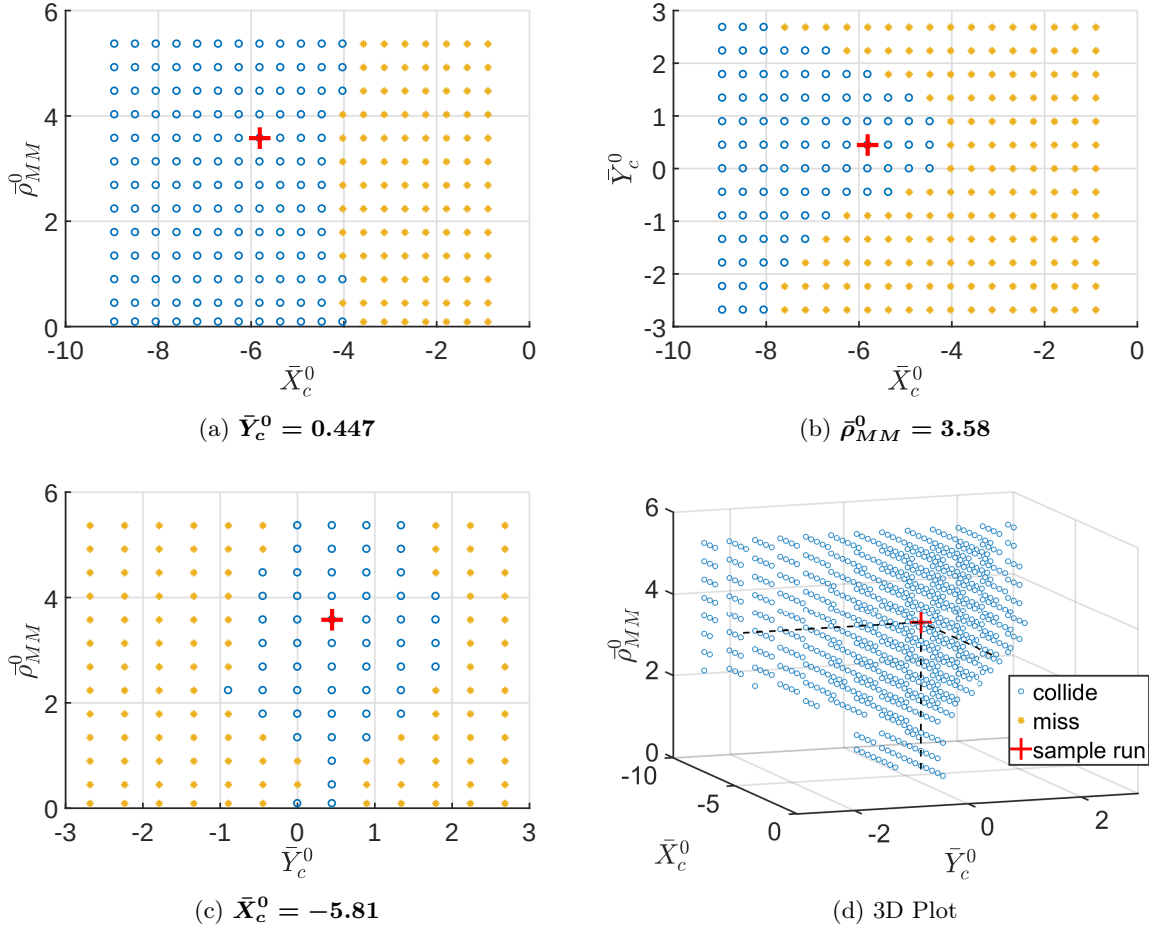


Figure 6: Collision map. Data points where a missile hits its target were excluded in (d) for clarity.

2. Monte Carlo Study with “Estimator-in-the-loop”

Simulation Setup

A Monte Carlo study was conducted to evaluate the “estimator-in-the-loop” performance of the cooperative guidance law. Inertial positions are the same as that shown in Figure 4 and are fixed in this study. Initial flight path angles of the targets are uniformly selected from the interval $[-10, 10]^\circ$ while the initial heading errors of the missiles are drawn from a zero-mean normal distribution with 3° standard deviation. The guidance law that is employed by each missile is assumed to be one of the 7 regimes considered by each target in this study. This includes: PN, APN, and OGL with $N'_{GL} \in \{3, 4, 5\}$ for $GL \in \{\text{PN}, \text{APN}\}$. Selection of the missiles’ guidance laws at the beginning of each run is based on the initial regime probability, $\mu_i^j(0)$, $i \in \{1, 2\}$, $j \in \{1, \dots, 7\}$. $\mu_i^j(0)$ is set such that each type of guidance law has $1/3$ probability and each N'_{GL} has a $1/9$ chance of occurring within the guidance law. Each MC simulation includes 500 runs.

In this study, the MAP approach is employed for MMAC (see Section 4) as it is computationally more efficient and is more practical to implement on the on-board processor of an aircraft/missile. Each filter of the i th estimator is initiated with the same state estimate, $\hat{\mathbf{x}}_{MTi}^j(0|0) \sim \mathcal{N}(\mathbf{x}_{MTi}(0), \mathbf{P}(0|0))$, $i \in \{1, 2\}$, $j \in \{1, \dots, 7\}$, where $\mathbf{x}_{MTi}(0)$ is the true initial state and the initial covariance $\mathbf{P}(0|0) = \text{diag}\{650^2, (3\pi/180)^2, (5g)^2, (3\pi/180)^2, 60^2\}$.

The targets are assumed to be acquiring both range and bearing measurements that are corrupted by noises as defined in Eq. (11). Variances of the measurement noises are varied in this study to assess the sensitivity of the cooperative guidance law to noise. Measurements are taken at a frequency of 50 Hz.

Sample Run

A sample run is presented here to demonstrate the performance of the estimator and the guidance law when using imperfect information. In this sample run, $\sigma_\rho = 10 \text{ m}$ and $\sigma_\lambda = 1 \text{ mrad}$. Missile and target parameters follow exactly as described in Section 1.

Convergence of the regime probabilities μ_1^j and μ_2^j are shown in Figure 8. As seen in the figure, regime probabilities for both missiles converged to 1 by $t = 4 \text{ s}$. Convergence of μ_2^j is slower as the guidance behaviors for OGL and APN, $N'_{APN} = 3$ (i.e., the optimal guidance gain for APN) are very similar. They differ only in the term associated with the $M2$'s first order lag, τ_{M2} (see Eq. (42)), and since $\tau_{M2} = 0.25 \text{ s}$, contribution from this term is hardly discernible. While this is true, $T2$'s estimator correctly identified *OGL* as the most probable guidance law being employed by $M2$ throughout the engagement, and since it is using the MAP approach to derive its optimal maneuver, it is still behaving optimally against $M2$.

For brevity, only the estimation performance of the states relating to the $M2 - T2$ and $M1 - M2$ engagements are presented as similar results are seen in the $M1 - T1$ case. Despite the slower convergence in the regime probabilities in $T2$'s estimator, the state estimation errors converges to their asymptotic values at around 2 s , as seen in Figure 8. Note that since $\hat{\rho}_{MM}$ and $\hat{\lambda}_{MM}$ are computed using Eq. (45) and not within the estimator, hence only the sample errors without the variances are shown in Figure 9. For other estimates presented in Figure 8, the error performance is obtained based on the blended estimate $\hat{\mathbf{x}}_{MTi}$ and blended covariance $\mathbf{P}_{MTi}$ and are calculated using:

$$\hat{\mathbf{x}}_{MTi}(k|k) = \sum_{j=1}^p \mu_i^j(k) \hat{\mathbf{x}}_{MTi}^j(k|k) \quad (58a)$$

$$\mathbf{P}_{MTi}(k|k) = \sum_{j=1}^p \mu_i^j(k) \left[\mathbf{P}_{MTi}^j(k|k) + \bar{\mathbf{P}}_{MTi}^j \right] \quad (58b)$$

where $\hat{\mathbf{x}}_{MTi}^j(k|k)$, $j \in \{1, \dots, p\}$ and $\mathbf{P}_{MTi}^j(k|k)$ are the j th regime estimate and covariance of the i th estimator respectively and

$$\bar{\mathbf{P}}_{MTi}^j \triangleq \left[\hat{\mathbf{x}}_{MTi}^j(k|k) - \hat{\mathbf{x}}_{MTi}(k|k) \right] \left[\hat{\mathbf{x}}_{MTi}^j(k|k) - \hat{\mathbf{x}}_{MTi}(k|k) \right]^T \quad (59)$$

Engagement trajectories of the sample run are shown in Figure 7 and are very similar to what was depicted in Figure 4b. Missile-missile miss distance in this run was 0.599 m . Acceleration profiles of the target pair in this sample run when using imperfect and perfect information (labeled a_{Ti}^{map} and a_{Ti}^* respectively, $i \in \{1, 2\}$) are plotted in Figure 10. Initial disparities between a_{Ti}^{map} and a_{Ti}^* are observed due to large estimation errors. As estimation errors converge at $t \sim 2 \text{ s}$, a_{Ti}^{map} behaves close to that of a_{Ti}^* . The kink observed in a_{Ti}^{map} near $t = 5.4 \text{ s}$ was due to divergence of $\tilde{\lambda}_{MM}$ as observed in Figure 9. The divergence is consistent with what was derived in Eq. (55), as $\tilde{\lambda}_{MM}$ increases exponentially as ρ_{MM} approaches zero.

Monte Carlo Study

Results of the MC simulations are presented in this subsection. The performance metric in this study is the CDF of the miss distance between the missiles. The sensitivity of the miss CDF to measurement noise is evaluated here. Measurement noise variances are the same for both targets, i.e., $\sigma_{i,\rho} = \sigma_\rho$ and $\sigma_{i,\lambda} = \sigma_\lambda$, for $i \in \{1, 2\}$. Figure 11 shows the CDF of the missile-missile miss distance for different levels of noise in the range and bearing measurements. As shown in Figure 11, the miss CDF improves drastically with the reduction in σ_ρ . The results shown in Figure 11 substantiates the discussion in Section 5 as the accuracy of the range estimates between the missile-target significantly affects the target pair's ability to accurately nullify the missile-missile LOS and keep the pursuers on their collision triangle.

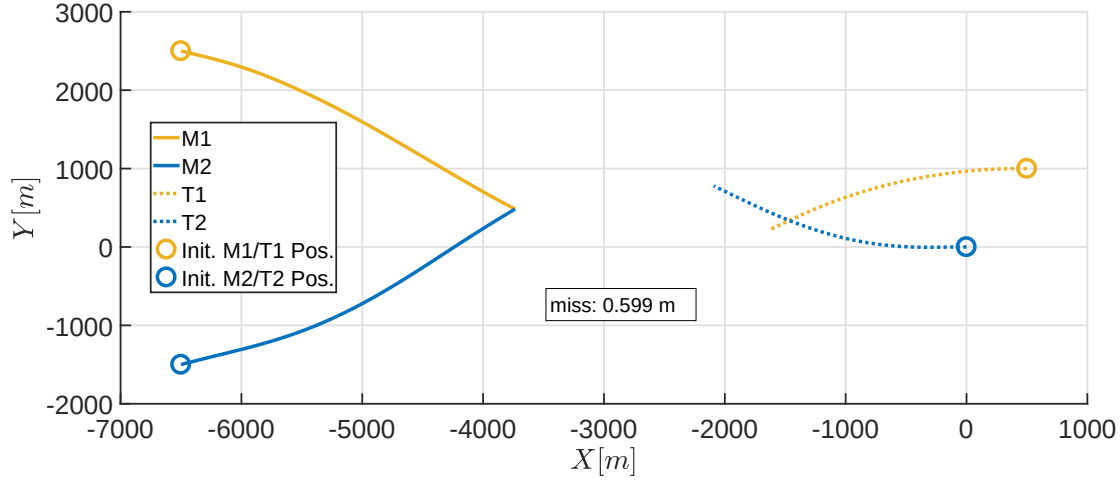


Figure 7: Engagement trajectories when targets are using MMAC with **M1** using APN, $N'_{APN} = 5$, and **M2** using OGL.

E. Results, Discussion, & Conclusions

A novel optimal cooperative defensive strategy in a 2-on-2 engagement was derived. The proposed strategy capitalizes on the presence of multiple adversaries to lure them into collision. With the proposed MMAE scheme, the targets are able to identify the missiles' guidance laws and predict their future trajectories. This information allows the target pair to maneuver cooperatively and set their pursuers on a collision triangle such that the missiles will hit each other before they could reach any of their targets.

When using perfect information, the proposed guidance strategy was shown to be effective over a wide range of initial conditions, as demonstrated by the collision map. This is despite the fact that the targets had half the maximum allowable acceleration and double the time lag compared to the missiles. This result highlights the advantage of the proposed cooperative strategy over conventional defensive strategies such as evasion or deploying defenders as the target pair was able to survive the engagement despite being less agile and not carrying any defending missiles.

For the case when the targets use imperfect information, a decentralized MMAC scheme was presented. Unlike the conventional MMAE approach in which the number of required filters for guidance law identification grows quadratically with the number of possible missile guidance strategies, number of filters in the proposed decentralized scheme grows only linearly. Sensitivity of the proposed guidance law to the estimation error of the missile-target range was also presented analytically and the findings were verified by Monte Carlo simulations.

Finally, it is important to note that the effectiveness of the guidance law also depends on the appropriate tuning of the weights in the cost function. Therefore, it is critical for the designer to make adjustments to the weights, if necessary, based on the required engagement conditions when applying the proposed guidance law.

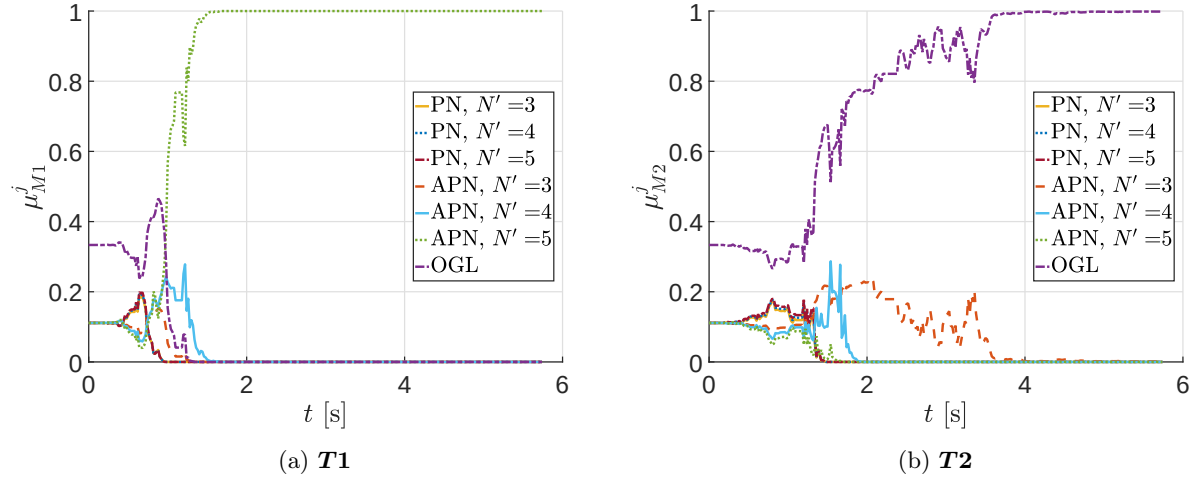


Figure 8: Regime probabilities of $T1$'s and $T2$'s estimator. $M1$ is using APN with $N'_{APN} = 5$ while $M2$ is using OGL.

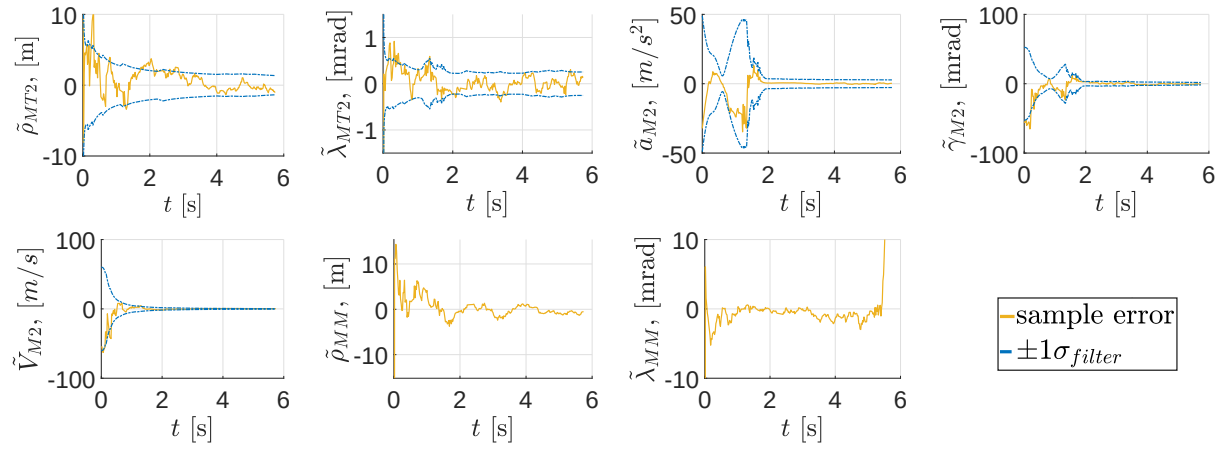


Figure 9: Sample estimation error performance.

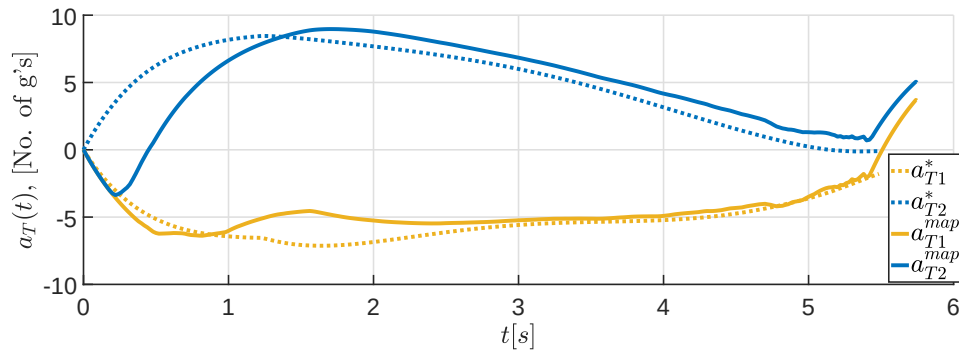


Figure 10: Target acceleration profiles.

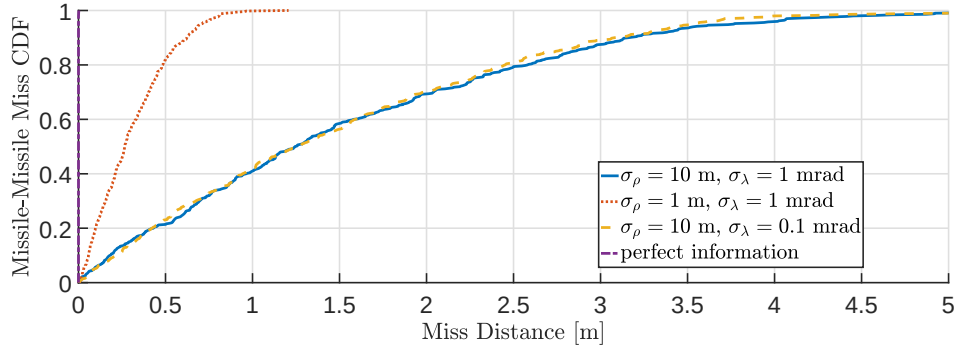


Figure 11: Sensitivity of missile-missile miss distance to measurement noise.

III. Intertwined Guidance & Estimation

A. Engagement

This section presents the kinematics, dynamics, and timeline of the considered engagement. The wingman's measurement model is introduced alongside with the missile's guidance law. The underlying assumptions are also discussed.

1. Kinematics and Dynamics

Consider a planar engagement scenario shown in Fig. 12, where a sensorless homing missile M pursues a maneuvering aerial target T with a help of a wingman vehicle W . For brevity, in the following text, the homing missile is referred to as missile or pursuer, the wingman vehicle as wingman, and the aerial target as target.

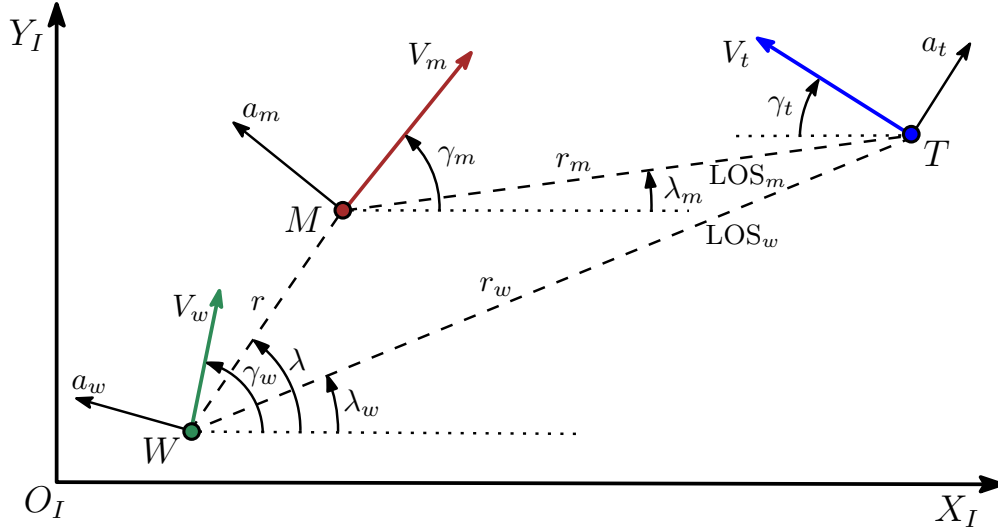


Figure 12: Planar engagement geometry.

In Fig. 12, the Cartesian inertial reference frame is denoted by $X_I-O_I-Y_I$. The speed, normal acceleration, and flight-path angle are given by V , a , and γ ; subscript m , w , and t denotes the missile, wingman, and target, respectively. The range between the wingman-missile ($W-M$), missile-target ($M-T$), and wingman-target ($W-T$) are denoted as r , r_m , and r_w , respectively. The angle between the missile-to-target LOS (LOS_m) and the X_I axis is denoted as λ_m . Similarly, the angle between LOS_w and the X_I axis is denoted as λ_w , whereas the angle between the missile-to-wingman LOS and the X_I axis is denoted as λ . All angles are measured in a counter-clockwise direction from the positive X_I axis.

All three vehicles are assumed to be skid-to-turn roll-stabilized. Additionally, assuming M , W , and T being point-masses and neglecting the effects of gravity, the M - T and the W - T engagement kinematics can be expressed in polar coordinates (r_i, λ_i) , $i \in \{m, w\}$, as follows¹

$$\dot{r}_i = -V_i \cos(\gamma_i - \lambda_i) - V_t \cos(\gamma_t + \lambda_i) \triangleq V_{r,i}, \quad (60a)$$

$$\dot{\lambda}_i = \frac{-V_i \sin(\gamma_i - \lambda_i) + V_t \sin(\gamma_t + \lambda_i)}{r_i} \triangleq \frac{V_{\lambda,i}}{r_i}, \quad (60b)$$

where $V_{r,i}$ is the relative velocity along, and $V_{\lambda,i}$ normal, to LOS _{i} . It is assumed that $r_i(0) > 0$ and $|\lambda_i(0)| \leq \pi/2$.

During the endgame, all vehicles are assumed to fly at constant speeds and to perform lateral maneuvers only, therefore

$$\dot{\gamma}_v = a_v/V_v, \quad (61a)$$

$$\dot{V}_v = 0, \quad (61b)$$

where $v \in \{m, w, t\}$. In addition, first-order maneuver dynamics is considered for all vehicles, i.e.,

$$\dot{a}_v = (u_v - a_v)/\tau_v. \quad (62)$$

In Eq. (62), u_v is the vehicle's piecewise continuous acceleration command and $\tau_v > 0$ is the time constant of the vehicle's dynamics. We also assume that all vehicles have maneuverability limitations defined as

$$|u_v| \leq a_v^{max}, \quad (63)$$

where $a_v^{max} > 0$ is the vehicle's maximal lateral acceleration.

2. Timeline

We denote the running time as t . The engagement starts at $t = t_0 \triangleq 0$ with $\dot{r}_m(t_0) < 0$. The endgame terminates at $t = t_f$, where t_f is the M - T interception time defined as

$$t_f \triangleq \arg \inf_{t \geq 0} (r_m(t) \dot{r}_m(t) = 0). \quad (64)$$

The interception time t_f allows to define the nonnegative missile-to-target time-to-go t_{go} as

$$t_{go} \triangleq \begin{cases} t_f - t, & t \leq t_f \\ 0, & t > t_f \end{cases} \quad (65)$$

At $t = t_f$, the M - T range, $r_m(t_f)$, is minimal and will be referred to as “miss distance” or compactly as “miss”.

Since the exact value of t_f is hard to compute, a common approximation of t_{go} is

$$t_{go} \approx -r_m/V_{r,m}, \quad V_{r,m} < 0, \quad (66)$$

where $V_{r,m}$ is given in Eq. (60a).

3. Wingman's Measurement Model

For the wingman-missile team, we assume that only the wingman is equipped with sensors that are able to track the target motion. Particularly, we assume that the wingman is equipped with an IR sensor, measuring the wingman-target LOS angle λ_w . These measurements, z_k , $k \in \mathbb{N}$, are assumed to be acquired at discrete-time instances $t = t_k \triangleq kT$, where $T_s > 0$ is a fixed measurement sampling period. Additionally, the measurements are assumed to be contaminated by a zero-mean white Gaussian noise sequence, $v_{\lambda_w;k}$, $k \in \mathbb{N}$, having a time-invariant standard deviation $\sigma_{\lambda_w} > 0$. Based on the above assumptions, the physical measurement model of the wingman's sensor measurements is

$$z_k = \lambda_{w;k} + v_{\lambda_w;k}, \quad v_{\lambda_w;k} \sim \mathcal{N}(0, \sigma_{\lambda_w}^2), \quad (67)$$

where $\lambda_{w;k}$ is the LOS angle λ_w at $t = t_k$, i.e., $\lambda_{w;k} \triangleq \lambda_w(t_k)$. Henceforth, the subscript k , separated by a semicolon if necessary, will denote the discrete-time t_k .

¹For notational simplicity, the reference to the independent time variable “ t ” will be omitted whenever the context is clear.

4. Missile's Guidance Law

Most of the guidance laws are implemented using the kinematics and dynamics variables. For the M - T engagement, these variables can be lumped into the following state vector

$$\mathbf{x}_m \triangleq \begin{bmatrix} r_m & \lambda_m & \gamma_t & a_t & V_t \end{bmatrix}^T. \quad (68)$$

The acceleration command u_m normal to the velocity vector of a PN-guided missile can be expressed using the state vector $\mathbf{x}_m$ and the known flight-related parameters of the missile (γ_m and V_m) as follows

$$u_m(\mathbf{x}_m) = N' \frac{V_{c,m} \dot{\lambda}_m}{\cos(\gamma_m - \lambda_m)}, \quad (69)$$

where N' is the effective navigation gain, normally having an integer value of 3, 4, or 5, and $V_{c,m}$ is the M - T closing velocity defined as $V_{c,m} \triangleq -V_{r,m}$. Variables $\dot{\lambda}_m$ and $V_{r,m}$ are given in Eq. (60). The term $\cos(\gamma_m - \lambda_m)$ in the denominator of Eq. (69) accounts for the inertial-to-body frame transformation.

5. Assumptions

Figure 13 summarizes the missile-target engagement dynamics in a block diagram form. It also includes the role of the wingman and the information flow between the vehicles.

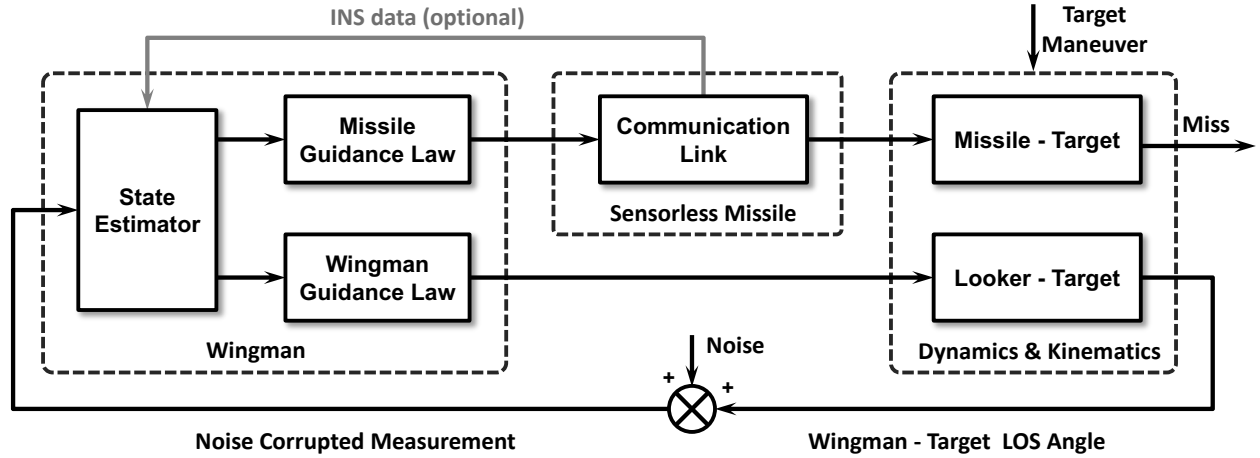


Figure 13: Block diagram of the engagement dynamics.

In this work, we adopt the following assumptions on the W - M kinematics, inertial sensors, and communications:

Assumption 1 The wingman has perfect information on the missile's and its own flight-related parameters, such as flight-path angle, lateral acceleration, and speed. This is a common assumption and can be accomplished by installing an inertial navigation system (INS) on both the wingman and the missile, and transmitting the missile's parameters (γ_m , a_m , and V_m) to the wingman.

Assumption 2 The relative geometry between the wingman and the missile, see r and λ in Fig. 12, is known accurately to the wingman (via some navigation system installed on the wingman).

Assumption 3 The missile's acceleration command u_m is computed in the wingman's on-board computer and is transmitted to the missile with zero lag. This minimizes the W - M communication overhead as, instead of all kinematic variables needed for the missile guidance law implementation, only u_m is transmitted to the missile.

Assumption 4 The missile and the wingman are launched simultaneously from the same location, i.e., $r(t_0) \approx 0$. To avoid wingman-target clash ($r_w = 0$), the wingman shall fly at a lower speed than the missile ($V_w/V_m < 1$) and shall stay behind the missile at all times ($r_w > r_m$).

Assumption 5 The target acceleration $a_t(t)$ is viewed as a random process with unknown statistics.

Remark 6. Given u_m and accurate W - M relative position information (r, λ) , the missile's flight-related parameters (γ_m , a_m , and V_m) can be directly estimated by the wingman. Consequently, the assumption on the availability of the INS for the missile, as discussed in Assumption 1, can be dropped.

B. Estimator Design

The missile's state vector $\mathbf{x}_m$ needs to be estimated from the available data to the wingman. In this section, we will design a relatively simple target tracking estimator which can be run on the wingman.

1. Measurement Model in the Missile Frame

Consider the engagement geometry depicted in Fig. 12 and the assumption that the relative position between W and M is known accurately to W . Let ϕ denote the W - M relative position (r, λ) . Then, using the trigonometric law of cosines, the LOS angle λ_w can be expressed as a function of $\mathbf{x}_m$ and ϕ , i.e.,

$$\lambda_w = h_w(\mathbf{x}_m, \phi) \triangleq \lambda + s \left[\pi - \cos^{-1} \left(\frac{r^2 + r_w^2 - r_m^2}{2rr_w} \right) \right]. \quad (70)$$

Here, the W - T range, r_w , can be also expressed as a function of $\mathbf{x}_m$ and ϕ , that is

$$r_w = g_w(\mathbf{x}_m, \phi) \triangleq \sqrt{r^2 + r_m^2 - 2rr_m \cos(\lambda_m - \lambda)}. \quad (71)$$

Note that in Eq. (70) and Eq. (71), the values of r_m , λ_m , r , and λ are substituted with corresponding entries from $\mathbf{x}_m$ and ϕ .

In Eq. (70), s stands for a sign function, which depends on the relative geometry between M and W , defined as

$$s \triangleq \begin{cases} 1, & \lambda_w > \lambda, \\ -1, & \lambda_w < \lambda, \\ 0, & \text{otherwise.} \end{cases} \quad (72)$$

Finally, the physical measurement model of the wingman Eq. (67) can be related to $\mathbf{x}_{m;k} \triangleq \mathbf{x}_m(t_k)$ and $\phi_k \triangleq (r(t_k), \lambda(t_k))$ as follows

$$z_k = h_w(\mathbf{x}_{m;k}, \phi_k) + v_{w;k}. \quad (73)$$

As will be shown next, the indirect measurement model of Eq. (73) allows the wingman to run an estimator on its own and to compute an estimate of $\mathbf{x}_{m;k}$ using ϕ_k and $\mathbf{z}_{1:k} \triangleq \{z_1, z_2, \dots, z_k\}$.

2. Target Maneuver Model Considerations

Since the target dynamics and target maneuver command u_t are rarely known in reality, an assumption must be established on the target maneuver model, i.e., on $a_t(t)$, in order to design a target tracking estimator.

In this work, we adopt the Singer's exponentially correlated acceleration (ECA) model [14]. The ECA model is widely used for miscellaneous maneuvering target tracking problems. It suggests that maneuvers (turns, evasive maneuvers, and accelerations due to atmospheric disturbances), acting on constant-velocity moving targets, can be viewed as perturbations of the constant velocity trajectories and represented as random accelerations correlated in time.

To proceed, the ECA model assumes that the correlation function $C(\tau)$ of the target acceleration $a_t(t)$ is a scalar stochastic process with exponentially decaying autocorrelation

$$C(\tau) = \sigma_t^2 e^{-\alpha|\tau|} \quad (74)$$

where σ_t^2 is the instantaneous variance of the target acceleration and $\alpha > 0$ is the reciprocal of the time constant of the target acceleration autocorrelation [15]. For example, $\alpha \simeq 1/60$ for a lazy turn, $\alpha \simeq 1/20$ for an evasive maneuver, and $\alpha \simeq 1$ for atmospheric turbulence [14]. According to Singer [14], to provide satisfactory representation of the target's instantaneous maneuver characteristics, σ_t shall be chosen as follows:

$$\sigma_t = \Psi a_t^{max}, \quad (75)$$

where a_t^{max} is the target's maximal lateral acceleration and $\Psi = \sqrt{(1 - P_0 + 4P_{max})/3}$. Here, P_{max} stands for the probability of the target accelerating at $\pm a_t^{max}$ and P_0 for the probability of the target not maneuvering. Since a_t^{max} , P_0 , and P_{max} are rarely available in practice, Ψ is often used as the tuning parameter of the filter.

The process $a_t(t)$ corresponding to Eq. (74) can be represented by a linear time-invariant system as

$$\dot{a}_t(t) = -\alpha a_t(t) + w_t(t), \quad (76)$$

where $w_t(t)$ is a zero-mean white Gaussian process with the following autocorrelation function

$$E[w_t(t + \tau)w_t(t)] = 2\alpha\sigma_t^2\delta(\tau), \quad (77)$$

with $\delta(\tau)$ being the Dirac delta function.

Using Eq. (60), Eq. (61), and Eq. (76), the equations of motion (EOM) for the estimator design become

$$\dot{\mathbf{x}}_m = \mathbf{f}(\mathbf{x}_m) + \mathbf{G}w_t, \quad (78)$$

where

$$\mathbf{f}(\mathbf{x}_m) \triangleq \begin{bmatrix} V_{r,m} & V_{\lambda,m}/r_m & a_t/V_t & -\alpha a_t & 0 \end{bmatrix}^T \quad (79)$$

and $\mathbf{G} \triangleq \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T$. The expressions for $V_{r,m}$ and $V_{\lambda,m}$ are given in Eq. (60).

Remark 7. Other target maneuver models might be more appropriate in specific situations, see [16] for a good survey on target maneuvering models. For instance, if the target is assumed to perform optimal evasive maneuvers, which are known to have a “bang-bang” structure against a PN-guided missile [17], such maneuvers may be better described by a shaping filter [18, 19]. However, due to Assumption 5, the ECA model presented in this section is considered to be a good approximation to many random processes with unknown statistics.

3. Discretization

The discrete-time version of Eq. (78) can be written as

$$\mathbf{x}_{m;k} = \mathbf{f}_{k-1}^d(\mathbf{x}_{m;k-1}) + \mathbf{w}_{t;k}, \quad (80)$$

where $\mathbf{f}_{k-1}^d(\cdot)$ is a vector function obtained by integrating Eq. (78) from t_{k-1} to t_k with initial condition $\mathbf{x}_m(t_{k-1}) = \mathbf{x}_{m;k-1}$ and setting $w_t(\tau)$ to zero for $\tau \in (t_{k-1}, t_k)$. In Eq. (80), $\mathbf{w}_{t;k}$ represents a vector valued zero-mean white noise sequence which relates to the scalar continuous-time process $w_t(t)$ as follows

$$\mathbf{w}_{t;k} = \int_{t_{k-1}}^{t_k} e^{(t_k-\tau)\mathbf{F}_{k-1}} \mathbf{G} w_t(\tau) d\tau, \quad (81)$$

where $\mathbf{F}_{k-1}$ is the Jacobian matrix associated with Eq. (79) and evaluated at $\mathbf{x}_{m;k-1}$, i.e.,

$$\begin{aligned} \mathbf{F}_{k-1} &\triangleq \left. \frac{\partial \mathbf{f}(\mathbf{x}_m)}{\partial \mathbf{x}_m} \right|_{\mathbf{x}_m = \mathbf{x}_{m;k-1}} \\ &= \begin{bmatrix} 0 & F_{12} & F_{13} & 0 & F_{15} \\ F_{21} & F_{22} & F_{23} & 0 & F_{25} \\ 0 & 0 & 0 & 1/V_t & -a_t/V_t^2 \\ 0 & 0 & 0 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \bigg|_{\mathbf{x}_m = \mathbf{x}_{m;k-1}} \end{aligned} \quad (82)$$

where

$$\begin{aligned}
F_{12} &= V_t \sin(\gamma_t + \lambda_m) - V_m \sin(\gamma_m - \lambda_m) \\
F_{13} &= V_t \sin(\gamma_t + \lambda_m) \\
F_{15} &= -\cos(\gamma_t + \lambda_m) \\
F_{21} &= \frac{1}{r_m^2} [V_m \sin(\gamma_m - \lambda_m) - V_t \sin(\gamma_t + \lambda_m)] \\
F_{22} &= \frac{1}{r_m} [V_m \cos(\gamma_m - \lambda_m) + V_t \cos(\gamma_t + \lambda_m)] \\
F_{23} &= \frac{1}{r_m} V_t \cos(\gamma_t + \lambda_m) \\
F_{25} &= \frac{1}{r_m} \sin(\gamma_t + \lambda_m)
\end{aligned}$$

It is assumed that $\mathbf{F}_{k-1}$ is fixed during the time interval (t_{k-1}, t_k) . With the zero-mean and white assumption on $w_t(t)$, it follows that

$$E[\mathbf{w}_{t;k}] = \mathbf{0}, \quad E[\mathbf{w}_{t;k} \mathbf{w}_{t;j}^T] = \mathbf{Q}_k \delta_{k,j}, \quad (83)$$

where $\delta_{k,j}$ is the Kronecker delta ($\delta_{k,j} = 1$ if $k = j$, else $\delta_{k,j} = 0$) and $\mathbf{Q}_k$ is the covariance matrix of $\mathbf{w}_{t;k}$, satisfying

$$\begin{aligned}
\mathbf{Q}_k &\triangleq \text{cov}(\mathbf{w}_{t;k}) \\
&= 2\alpha\sigma_t^2 \int_{t_{k-1}}^{t_k} e^{(t_k-\tau)\mathbf{F}_{k-1}} \mathbf{\Theta} e^{(t_k-\tau)\mathbf{F}_{k-1}^T} d\tau,
\end{aligned} \quad (84)$$

where $\mathbf{\Theta} \triangleq \mathbf{G}\mathbf{G}^T$. If the exponential terms in Eq. (84) are replaced by their 1st-order Taylor series approximations, i.e.,

$$\begin{aligned}
e^{(t_k-\tau)\mathbf{F}_{k-1}} &\approx \mathbf{I} + (t_k - \tau)\mathbf{F}_{k-1}, \\
e^{(t_k-\tau)\mathbf{F}_{k-1}^T} &\approx \mathbf{I} + (t_k - \tau)\mathbf{F}_{k-1}^T,
\end{aligned}$$

then the integral in Eq. (84) can be easily solved and $\mathbf{Q}_k$ approximated by

$$\begin{aligned}
\mathbf{Q}_k &\approx 2\alpha\sigma_t^2 T_d \\
&\times \left[\mathbf{\Theta} + \frac{T_d}{2} \left(\mathbf{F}_{k-1} \mathbf{\Theta} + \mathbf{\Theta} \mathbf{F}_{k-1}^T \right) + \frac{T_d^2}{3} \mathbf{F}_{k-1} \mathbf{\Theta} \mathbf{F}_{k-1}^T \right],
\end{aligned} \quad (85)$$

where $T_d \triangleq t_k - t_{k-1}$ is the discretization sampling time. In this work, we assume that $T_d = T_s$, where T_s is the measurement sampling time, see Section 3.

4. Filtering Equations

Both the system Eq. (80) and the measurement Eq. (73) equations are nonlinear. Thus, a suitable estimation technique must be considered. In this work, we will employ an extended Kalman filter (EKF) based approach to obtain an approximate solution to the optimal filtering problem of finding $E[\mathbf{x}_{m;k} | \mathbf{z}_{1:k}]$. Obviously, other sub-optimal estimation techniques, such as unscented Kalman filter (UKF), particle filter, etc., might be considered, although we expect similar trends in the estimation performance.

Assuming that at time $t_0 = 0$, an initial estimate, $\hat{\mathbf{x}}_{m;0|0}$, of the missile state $\mathbf{x}_{m;0}$ is available, satisfying

$$\hat{\mathbf{x}}_{m;0|0} \sim \mathcal{N}(\mathbf{x}_{m;0}, \mathbf{P}_{0|0}), \quad (86)$$

where $\mathbf{P}_{0|0}$ is the covariance matrix of the initial estimation error ($\mathbf{x}_{m;0} - \hat{\mathbf{x}}_{m;0|0}$), then the filtering process can be divided into two steps of time propagation (TP) and measurement update (MU).

TP: The a posteriori state estimate $\hat{\mathbf{x}}_{m;k-1|k-1}$ is time-propagated from t_{k-1} to t_k using

$$\hat{\mathbf{x}}_{m;k|k-1} = \mathbf{f}_{k-1}^d(\hat{\mathbf{x}}_{m;k-1|k-1}), \quad (87)$$

where $\mathbf{f}_{k-1}^d(\cdot)$ was defined shortly after Eq. (80). The a posteriori covariance matrix $\mathbf{P}_{k-1|k-1}$ is propagated using

$$\mathbf{P}_{k|k-1} = e^{\mathbf{F}_{k-1} T_d} \mathbf{P}_{k-1|k-1} e^{\mathbf{F}_{k-1}^T T_d} + \mathbf{Q}_k, \quad (88)$$

where $\mathbf{Q}_k$ is given in Eq. (85) and $\mathbf{F}_{k-1}$ is the Jacobian matrix Eq. (82) evaluated at $\mathbf{x}_m = \hat{\mathbf{x}}_{m;k-1|k-1}$.

MU: If the measurement z_k becomes available at time t_k , the a priori state estimate $\hat{\mathbf{x}}_{m;k|k-1}$ is updated using

$$\hat{\mathbf{x}}_{m;k|k} = \hat{\mathbf{x}}_{m;k|k-1} + \mathbf{K}_k [z_k - h_w(\hat{\mathbf{x}}_{m;k|k-1}, \phi_k)], \quad (89)$$

where $h_w(\cdot, \cdot)$ was defined in Eq. (70) and $\mathbf{K}_k$ is the Kalman gain computed as

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + R)^{-1}. \quad (90)$$

In Eq. (90), R stands for the variance of the measurement noise v_k , i.e., $R = \sigma_{\lambda,w}^2$, and $\mathbf{H}_k$ is the Jacobian of the measurement model Eq. (70), derived as

$$\begin{aligned} \mathbf{H}_k &\triangleq \left. \frac{\partial h_w(\mathbf{x}_m, \phi)}{\partial \mathbf{x}_m} \right|_{\mathbf{x}_m = \hat{\mathbf{x}}_{m;k|k-1}, \phi = \phi_k} \\ &= \begin{bmatrix} H_1 & H_2 & 0 & 0 & 0 \end{bmatrix} \Big|_{\mathbf{x}_m = \hat{\mathbf{x}}_{m;k|k-1}, \phi = \phi_k}, \end{aligned} \quad (91)$$

with H_1 and H_2 being defined as

$$\begin{aligned} H_1 &= -s \frac{r \sqrt{|\cos^2(\lambda - \lambda_m) - 1|}}{g_w^2(\hat{\mathbf{x}}_{m;k|k-1}, \phi_k)}, \\ H_2 &= s \frac{r r_m \sin(2\lambda - 2\lambda_m) - 2r_m^2 \sin(\lambda - \lambda_m)}{2g_w^2(\hat{\mathbf{x}}_{m;k|k-1}, \phi_k) \sqrt{|\cos^2(\lambda - \lambda_m) - 1|}}, \end{aligned}$$

where $g_w(\cdot, \cdot)$ and s were defined in Eq. (71) and Eq. (72), respectively. The values of r_m , r , λ_m , and λ_m in H_1 and H_2 are substituted with corresponding entries from $\hat{\mathbf{x}}_{m;k|k-1}$ and ϕ_k . The a priori covariance matrix $\mathbf{P}_{k|k-1}$ is updated as

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1}. \quad (92)$$

Note that to compute Eq. (89) and Eq. (91), the sign function s , defined in Eq. (72), needs to be evaluated. It requires the knowledge of $\lambda_{w;k}$. This angle is not precisely known, but is directly measured, see Eq. (67). Thus, for practical implementation purposes, $\lambda_{w;k}$ in Eq. (72) can be replaced by z_k or, better yet, by its a priori estimate $\hat{\lambda}_{w;k|k-1} = h_w(\hat{\mathbf{x}}_{m;k|k-1}, \phi_k)$.

Remark 8. To compute an estimate of $\mathbf{x}_{m;k}$ using the missile own-ship LOS angle measurements, only slight modifications to the above equations are necessary. The indirect measurement model $h_w(\cdot, \cdot)$ in Eq. (89) is replaced by $h(\mathbf{x}_m) \triangleq \lambda_m$, the measurement Jacobian $\mathbf{H}_k$ simplifies to $\mathbf{H}_k = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$, and R in Eq. (90) is set to $R = \sigma_{\lambda,m}^2$, where $\sigma_{\lambda,m}^2$ is the variance of the missile's sensor noise.

Remark 9. The proposed wingman-based estimation scheme can be extended with a little extra effort to a multiple wingman-based measuring scheme. This can be accomplished by transforming all the measurements from the wingmans' to the missile's moving frame and fusing them similarly as done in [20].

C. Observability Analysis

In this section, we will analytically study the accuracy of the wingman-based estimation concept developed in the previous section.

1. Observability Metrics Derivation

The estimation accuracy of a_t , γ_t , and V_t is mainly driven by the availability of an accurate target maneuver model, and the actual target maneuvers. Moreover, as will be discussed in the next section, the accuracy of these parameters is not very crucial for the implementation of the PN guidance law of Eq. (69). Therefore, the focus of this section will be only on the observability of λ_m and r_m .

We will investigate the observability of λ_m and r_m from a geometric perspective. Consider the engagement geometry depicted in Fig. 12. From the law of cosines, the M - T range can be expressed as

$$r_m = g_m(r_w, \lambda_w, \phi) \triangleq \sqrt{r^2 + r_w^2 - 2r r_w \cos \Delta_\lambda}, \quad (93)$$

where Δ_λ is defined as the difference between the W - M and W - T LOS angles, i.e.,

$$\Delta_\lambda \triangleq \lambda - \lambda_w. \quad (94)$$

Inserting r_m of Eq. (93) in the expression of r_w given in Eq. (71), we can easily isolate λ_m as follows

$$\lambda_m = h_m(r_w, \lambda_w, \phi) \triangleq \lambda + \cos^{-1} \left(\frac{r - r_w \cos \Delta_\lambda}{g_m(r_w, \lambda_w, \phi)} \right). \quad (95)$$

The above expressions for r_m and λ_m will form the baseline for observability analysis.

To proceed, we need to take into account the stochastic nature of the measurements and the state estimate. Consider that, at time step k , r_w in Eq. (93) and Eq. (95) is replaced by its a priori state estimate, i.e.,

$$\hat{r}_{w;k} = g_w(\hat{\mathbf{x}}_{m;k|k-1}, \phi_k) = r_{w;k} + v_{r_w;k}. \quad (96)$$

In this context, $\hat{r}_{w;k}$ is viewed as a random variable, where $v_{r_w;k}$ is assumed to be a zero-mean random process with a time-varying variance $\sigma_{r,w;k}^2$. Note that expression Eq. (96) is the same as the one used at the MU stage of the EKF, see $h_w(\cdot, \cdot)$ in Eq. (89) and Eq. (91). In both cases, $r_{w;k}$ is evaluated using the a priori state estimate $\hat{\mathbf{x}}_{m;k|k-1}$ and the definition of $g_w(\cdot, \cdot)$ given in Eq. (71). Consequently, the accuracy of $\hat{r}_{w;k}$ will be driven by the filter's performance, i.e., $\sigma_{r,w;k}^2$ is essentially a function of $\mathbf{P}_{k|k-1}$.

For observability analysis purposes, λ_w in Eq. (93) and Eq. (95) is replaced by the wingman's noise-corrupted measurement $z_k = \lambda_{w;k} + v_{\lambda_w;k}$. Consider that the accuracy of $\hat{r}_{w;k}$ and z_k is quantified by their standard deviations $\sigma_{r,w;k}$ and $\sigma_{\lambda,w}$, respectively, then

$$\lambda_{m;k}^\dagger = h_m(\hat{r}_{w;k}, z_k, \phi_k), \quad (97a)$$

$$r_{m;k}^\dagger = g_m(\hat{r}_{w;k}, z_k, \phi_k), \quad (97b)$$

can be viewed as pseudomeasurements of $\lambda_{m;k}$ and $r_{m;k}$, respectively.

Next, the accuracy of the above pseudomeasurements will be analyzed. Because $\hat{r}_{w;k}$ in Eq. (97) is expressed using the a priori state estimate $\hat{\mathbf{x}}_{m;k|k-1}$, the two random processes of $v_{r_w;k}$ and $v_{\lambda_w;k}$ are mutually independent, i.e., $E[v_{r_w;k}, v_{\lambda_w;k}] = 0$. Therefore, the associated variance of the LOS angle pseudomeasurement, $\lambda_{m;k}^\dagger$, can be exactly computed, as

$$\begin{aligned} \sigma_{\lambda,m;k}^2 &= \int_{\mathbb{R}^2} \left[(h_m(\hat{r}_{w;k}, z_k, \phi_k) - \mu_{\lambda,m;k})^2 \right. \\ &\quad \left. \times f_r(v_{r_w}) f_\lambda(v_{\lambda_w}) \right] dv_{r_w} dv_{\lambda_w}, \end{aligned} \quad (98)$$

where $\mu_{\lambda,m;k}$ is the mean value of $\lambda_{m;k}^\dagger$,

$$\mu_{\lambda,m;k} = \int_{\mathbb{R}^2} h_m(\hat{r}_{w;k}, z_k, \phi_k) f_r(v_{r_w}) f_\lambda(v_{\lambda_w}) dv_{r_w} dv_{\lambda_w}, \quad (99)$$

and $f_r(\cdot)$ and $f_\lambda(\cdot)$ are two scalar Gaussian density functions with zero means and variances $\sigma_{r,w;k}^2$ and $\sigma_{\lambda,w}^2$. In a similar manner, the variance of $r_{m;k}^\dagger$, denoted as $\sigma_{r,m;k}^2$, can be computed using the relations above with $h_m(\cdot, \cdot, \cdot) \rightarrow g_m(\cdot, \cdot, \cdot)$.

Unfortunately, the integrals in Eq. (98) and Eq. (99) are not trivial to compute. Hence, in this work, we will attempt to solve these integrals by the *analytical linearization* (AL) technique [21]. The AL, also used in the EKF algorithm derivation, aims at obtaining the variance of the transformed random variable(s) using linearization of the underlying nonlinear function and evaluating at expected values. Thus, the variance of the M - T LOS angle pseudomeasurement, $\lambda_{m;k}^\dagger$, obtained using the AL method, becomes

$$\begin{aligned} \sigma_{\lambda,m;k}^2 &\approx \sigma_{r,w;k}^2 \left(\frac{\partial h_m(\hat{r}_w, z, \phi_k)}{\partial \hat{r}_w} \right)^2 \bigg|_{\hat{r}_w=E[\hat{r}_{w;k}], z=E[z_k]} \\ &\quad + \sigma_{\lambda,w}^2 \left(\frac{\partial h_m(\hat{r}_w, z, \phi_k)}{\partial z} \right)^2 \bigg|_{\hat{r}_w=E[\hat{r}_{w;k}], z=E[z_k]} \\ &= \sigma_{r,w;k}^2 \left(\frac{r \sin \Delta_\lambda}{g_m^2(r_w, \lambda_w, \phi)} \right)^2 + \sigma_{\lambda,w}^2 \left(\frac{r r_w \cos \Delta_\lambda - r_w^2}{g_m^2(r_w, \lambda_w, \phi)} \right)^2. \end{aligned} \quad (100)$$

Similarly, using AL, the variance of the M - T range pseudomeasurement, $r_{m;k}^\dagger$, becomes

$$\begin{aligned}\sigma_{r,m;k}^2 &\approx \sigma_{r,w;k}^2 \left(\frac{\partial g_m(\hat{r}_w, z, \phi_k)}{\partial \hat{r}_w} \right)^2 \bigg|_{\hat{r}_w = E[\hat{r}_{w;k}], z = E[z_k]} \\ &\quad + \sigma_{\lambda,w}^2 \left(\frac{\partial g_m(\hat{r}_w, z, \phi_k)}{\partial z} \right)^2 \bigg|_{\hat{r}_w = E[\hat{r}_{w;k}], z = E[z_k]} \\ &= \sigma_{r,w;k}^2 \left(\frac{r \cos \Delta_\lambda - r_w}{g_m(r_w, \lambda_w, \phi)} \right)^2 + \sigma_{\lambda,w}^2 \left(\frac{r r_w \sin \Delta_\lambda}{g_m(r_w, \lambda_w, \phi)} \right)^2.\end{aligned}\tag{101}$$

The analytical expressions of Eq. (100) and Eq. (101) provide powerful means to analyze the observability of λ_m and r_m as a function of the W - M relative geometry.

2. Observability Metrics Evaluation

The AL method is often criticized to give inaccurate results. Therefore, we will verify the analytical results of Eq. (100) and Eq. (101) using a *Monte Carlo* (MC) statistical method. Unlike AL, the MC method does not yield an explicit solution for the variance of the transformed random variable(s), but instead uses sufficiently large number of realizations of the random variable(s) to numerically approximate the underlying distribution. This method reassembles the unscented transformation used in the UKF. Here, instead of using only few ‘‘sigma points’’, a large number, N , of realizations is generated from the prior distribution of $\hat{r}_{w;k} \sim \mathcal{N}(r_{w;k}, \sigma_{r,w;k}^2)$ and $z_k \sim \mathcal{N}(\lambda_{w;k}, \sigma_{\lambda,w}^2)$, which are then propagated through the nonlinear functions $h_m(\cdot, \cdot, \cdot)$ and $g_m(\cdot, \cdot, \cdot)$, and the sample variances are computed:

$$\sigma_{\lambda,m;k}^2 \approx \frac{1}{N-1} \sum_{i=1}^N \left(\lambda_{m;k}^{\dagger(i)} - \frac{1}{N} \sum_{j=1}^N \lambda_{m;k}^{\dagger(j)} \right)^2, \tag{102a}$$

$$\sigma_{r,m;k}^2 \approx \frac{1}{N-1} \sum_{i=1}^N \left(r_{m;k}^{\dagger(i)} - \frac{1}{N} \sum_{j=1}^N r_{m;k}^{\dagger(j)} \right)^2, \tag{102b}$$

where the superscript ‘‘ i ’’ denotes the i th realization of the pseudomeasurement $\lambda_{m;k}^\dagger$ or $r_{m;k}^\dagger$.

Without loss of generality, for the subsequent analysis, we will assume that $\sigma_{r,w;k}$ is constant, i.e., $\sigma_{r,w;k} = \sigma_{r,w}$ for all $k \in \mathbb{N}$ and that $r_w > 0$ (this is aligned with Assumption 4). The normalized values of $\sigma_{\lambda,m}$ and $\sigma_{r,m}$ are depicted in Fig. 14 for different W - M angle separations Δ_λ and ranges r . The obtained results are normalized by $\sigma_{\lambda,w} = 0.001$ and $\sigma_{r,w} = r_w/100$ to better appreciate the relative uncertainty change with respect to these reference uncertainties. The W - M range, r , is made dimensionless by r_w . For the MC method, a total number of $N = 1,000,000$ points were generated for each Δ_λ and r/r_w combination, see the circles in Fig. 14. It can be observed that the AL method very closely matches with the MC method. To reflect on Assumption 4, the infeasible W - M relative geometries are depicted as filled circles. These regions represent unfavourable W - M geometries, when the wingman is closer to the target than the missile ($r_w < r_m$). Note that the results are not dependent on the choice of $r_w > 0$.

It can be observed from Fig. 14, that if the position of the missile and the wingman coincide, i.e., $r = 0 \Rightarrow r/r_w = 0$, the estimation accuracy of λ_m will be purely driven by the accuracy of the wingman’s measurements $\sigma_{\lambda,w}$ and $\sigma_{r,m} = \sigma_{r,w}$ for any Δ_λ . This is obvious as this case is identical to a missile having own-ship measurements with accuracy of $\sigma_{\lambda,w}$. However, if the wingman and the missile are apart, i.e., $r/r_w > 0$, a contradictory behavior in the estimation accuracy of the M - T kinematic variables (range r_m and LOS angle λ_m) can be observed for $|\Delta_\lambda| \rightarrow 0$. While for $\Delta_\lambda = 0$, the observability of the M - T line-of-sight angle λ_m is maximized, the observability of the M - T range r_m is minimized. Similarly, increasing $|\Delta_\lambda|$, opposing trends can be observed for $\sigma_{\lambda,m}$ vs. $\sigma_{r,m}$. Increasing r/r_w leads to improving accuracy of r_m w.r.t. the accuracy of r_w . This trend can be explained by the fact that increasing r/r_w means that the missile is approaching the target and the accurately known W - M relative position (r, λ) has an improving effect on the M - T range estimate as the accuracy of the approximation $r_m \approx r \sin(\lambda - \lambda_w)$ becomes more and more valid. Note that both r and λ are assumed to be known and λ_w is directly measured.

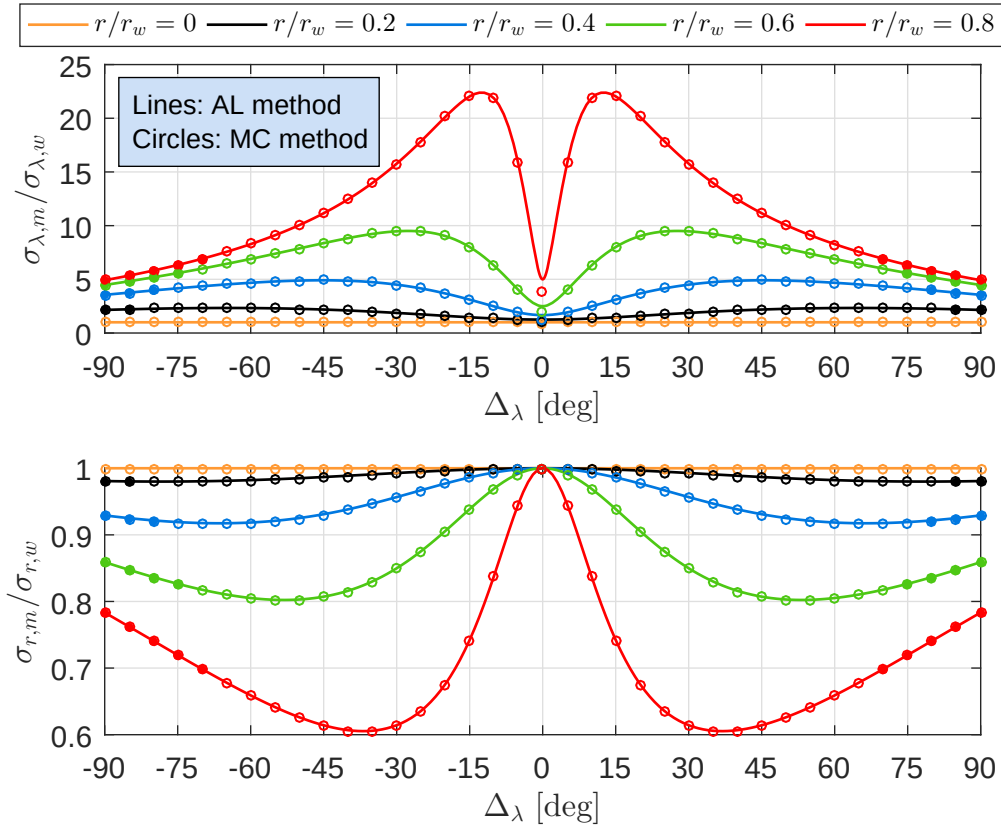


Figure 14: Effect of W - M relative geometry on the M - T range and LOS angle accuracy

Figure 14 presents a rather static grasp of the underlying observability issue. Next, we will attempt to address some dynamical aspects of the considered engagement scenario. Based on Assumption 4, we have $r_m(t_0) = r_w(t_0)$ at the beginning of the engagement, thus $r(t_0) = 0$. In a perfect interception scenario, $r_m(t_f) = 0$. This implies $r(t_f) = r_w(t_f) > 0$. Considering Assumption 4 again, it becomes evident that $r(t)/r_w(t)$ is continuous and monotonously increasing from 0 to 1 on the interval $t \in \langle t_0, t_f \rangle$, hence takes values exclusively in the closed interval $\langle 0, 1 \rangle$. This insight enables us to parameterize Eq. (100) and Eq. (101) for all possible engagement trajectories by considering ($r_w > 0$)

$$-\pi/2 \leq \Delta_\lambda \leq \pi/2, \quad 0 \leq r/r_w < 1. \quad (103)$$

We are interested in the feasible LOS separation angles Δ_λ which minimize $\sigma_{\lambda,m}^2$ or $\sigma_{r,m}^2$ for a given ratio r/r_w . This can be mathematically formulated as:

$$\Delta_\lambda^{\min}(r/r_w) = \underset{-\pi/2 \leq \Delta_\lambda \leq \pi/2}{\operatorname{argmin}} \{ \sigma_{j,m}^2 \}, \quad j \in \{\lambda, r\}. \quad (104)$$

The resulting angles $\Delta_\lambda^{\min}$ for $0 \leq r/r_w < 1$ are depicted in Fig. 15. Absolute value is used for the y-axis as $\sigma_{\lambda,m}^2$ and $\sigma_{r,m}^2$ are symmetric functions around $\Delta_\lambda = 0$ [deg], see Fig. 14. Note that the limit case $r/r_w = 1$ yields to singularity issues in Eq. (100) and Eq. (101) and is also not relevant from observability perspective as $r/r_w = 1$ occurs only at the end of the engagement when $r_m(t_f) = 0$, therefore $r/r_w = 1$ is considered only as a limit in Fig. 15. Again, the results obtained by the AL method are verified by the MC method using Eq. (102).

It becomes evident from Fig. 15 that simultaneous minimization of uncertainties associated with both λ_m and r_m is not possible. Moreover, keeping Δ_λ close to zero will on one hand minimize $\sigma_{\lambda,m}^2$ for all $0 \leq r/r_w < 1$, but on the other hand will maximize $\sigma_{r,m}^2$, see the lower subfigure in Fig. 14 for $\Delta_\lambda = 0$ [deg].

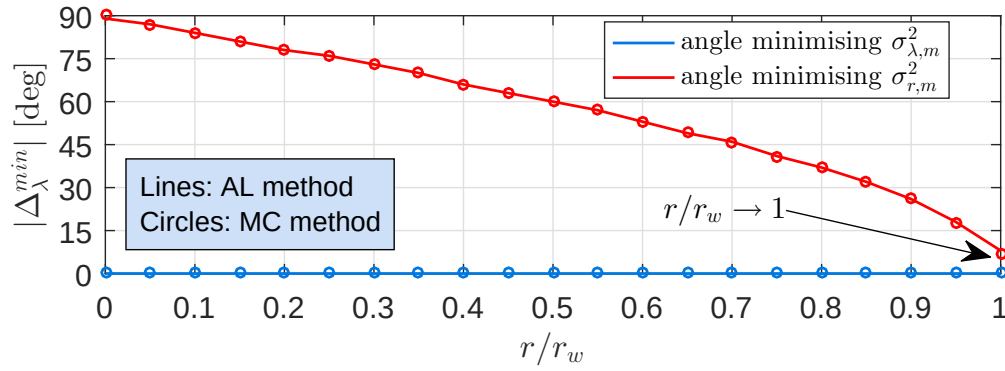


Figure 15: LOS separation angles Δ_λ minimizing uncertainties in the M – T range and LOS angle.

D. Wingman Trajectory Implications

Based on the preceding observability analysis, possible trajectory implications for the wingman are discussed in this section.

1. Implications for a PN-Guided Missile

The trajectory of the missile is predetermined by the employed PN guidance law Eq. (69), therefore the focus of this section will be on the remaining degree of freedom, which is the choice of the wingman's trajectory. The resulting wingman trajectory should maximize the missile's homing performance.

To implement the PN guidance law Eq. (69), only the missile-target LOS angle rate $\dot{\lambda}_m$ and the closing velocity $V_{c,m}$ shall be provided. The closing velocity is typically constant throughout the endgame and can be easily computed using the estimated state $\hat{\mathbf{x}}_m$. However, absence of an accurate estimate on $V_{c,m}$, in general, does not significantly affect the homing performance of a PN-guided missile. Often $V_{c,m}$ is approximated as $V_{c,m} \approx V_m$, where V_m is the missile speed.

On the other hand, an effective implementation of the PN guidance law requires accurate information on the LOS rate $\dot{\lambda}_m$ [22]. In a typical one-on-one M – T engagement, this is accomplished by directly measuring the LOS angle λ_m or the LOS angle rate $\dot{\lambda}_m$. Therefore, in such a scenario, the estimation accuracy of the LOS angle (rate) is directly governed by the accuracy of the sensor's measurements. However, in the proposed wingman-based estimation scheme, only the LOS angle λ_w of the W – T engagement is measured, see Sec. 3. The accuracy of the λ_w measurement is only indirectly related to the M – T LOS angle (rate), see λ_w in Eq. (95).

Direct knowledge on the target acceleration a_t is not required for the PN guidance law implementation, but a_t must be included in $\mathbf{x}_m$ to enable estimation of γ_t , which is, together with V_t , essential for estimating r_m and λ_m , see Eq. (60).

Based on the above discussion, it might seem natural for the wingman to try to maximize the observability of the LOS angle λ_m , as this is the most critical variable for the PN guidance law implementation. Figure 15 clearly suggests that in order to maximize the observability of λ_m , the relative LOS separation Δ_λ must be kept zero throughout the entire engagement. This leads to a straightforward formulation of the wingman's trajectory, which is to maintain Δ_λ zero at all times, i.e.,

$$\Delta_\lambda(t) = 0, \quad \forall t \in \langle t_0, t_f \rangle. \quad (105)$$

Consider the desired wingman trajectory described in terms of Eq. (105) and the simplified engagement geometry depicted in Fig. 16. It becomes evident that the wingman's guidance problem can be directly related to the well known LOS guidance concept. This concept is a three-point guidance problem and its basic principle is to keep the pursuer on a LOS connecting the target and the (stationary) launch platform. In our case, the wingman acts as moving launcher which tries to maintain its position on the extended M – T LOS line. A similar LOS guidance problem with a moving platform was studied in [23], where a defending missile followed LOS guidance concept to maintain its position on the LOS connecting the targeted aircraft with the homing missile.

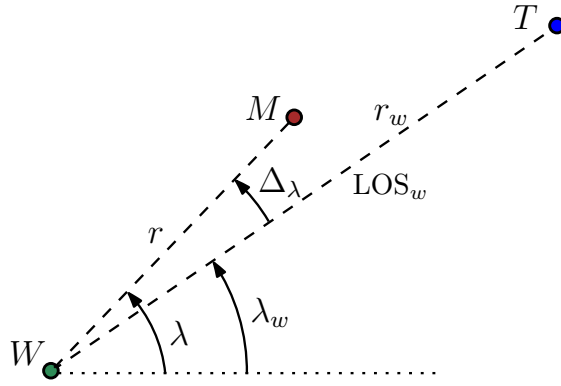


Figure 16: CLOS guidance problem visualization.

2. LOS Guidance Law Mechanization

Mechanization of the “classical” three-point LOS guidance concept is commonly achieved by implementing either command to LOS (CLOS) or beam rider guidance law [22]. Neither of these guidance laws is readily suitable for the wingman as they were devised for the missile instead.

Perhaps the simplest mechanization of the proposed wingman trajectory is by employing a proportional-integral-derivative (PID) controller to minimize the LOS separation angle Δ_λ . The discrete-time implementation of such a PID-based LOS guidance strategy takes the following structure:

$$u_{w;k} = K_p \hat{\Delta}_{\lambda;k} + K_i T \sum_{j=0}^k \hat{\Delta}_{\lambda;j} + K_d \frac{\hat{\Delta}_{\lambda;k} - \hat{\Delta}_{\lambda;k-1}}{T}, \quad (106)$$

where $u_{w;k}$ is the wingman’s commanded lateral acceleration at time t_k , $T = t_k - t_{k-1}$ is the discretization step, K_p , K_i , K_d are the PID controller’s tuning parameters [24], and $\hat{\Delta}_{\lambda;k}$ is the estimate of $\Delta_{\lambda;k} = \lambda_k - \lambda_{w;k}$. If $T = T_s$, then the state estimate $\hat{\mathbf{x}}_{m;k|k}$ can be used to compute $\hat{\Delta}_{\lambda;k}$ as follows

$$\hat{\Delta}_{\lambda;k} = \lambda_k - h_w(\hat{\mathbf{x}}_{m;k|k}, \phi_k), \quad (107)$$

where $h_w(\cdot, \cdot)$ was defined in Eq. (70). The use of $h_w(\cdot, \cdot)$ (instead of z_k) in Eq. (107) shall reduce the sensitivity of $\hat{\Delta}_{\lambda;k}$ to noise.

The PID algorithm of Eq. (106) can be written in a recursive form

$$u_{w;k} = u_{w;k-1} + K_1 \hat{\Delta}_{\lambda;k} - K_2 \hat{\Delta}_{\lambda;k-1} + K_3 \hat{\Delta}_{\lambda;k-2}, \quad (108)$$

with $K_1 = K_p + K_i T + K_d/T$, $K_2 = K_p + 2K_d/T$, and $K_3 = K_d/T$. This form of the wingman’s guidance law facilitates its on-board implementation. A proper tuning of the PID parameters is important, because a relatively small deviation of Δ_λ from zero leads to a significant deterioration in the estimation accuracy of λ_m , especially at the end of the engagement, see the upper subplot of Fig. 14. Note that Eq. (106) or Eq. (108) requires only the availability of λ_k and not r_k , hence Assumption 2 can be relaxed for the wingman guidance strategy implementation.

3. Other Missile Guidance Laws and Their Implications

The preceding developments assumed PN guidance law for the missile. Obviously, other missile guidance strategies might lead to different (optimal) trajectory implications for the wingman. For instance, the optimal guidance law [25] or the impact time/angle guidance law [26, 27] require an accurate estimation of the time-to-go (t_{go}) variable. The estimation accuracy of t_{go} is highly dependent on the M – T range (r_m) estimate accuracy, see the typical t_{go} approximation in Eq. (66).

The W – M engagement parametrization Eq. (103) enables to pose the following weighted optimization problem

$$\Delta_\lambda^*(r/r_w, \varepsilon) = \underset{-\frac{\pi}{2} \leq \Delta_\lambda \leq \frac{\pi}{2}}{\operatorname{argmin}} \left\{ \varepsilon \sigma_{\lambda,m}^2 + (1 - \varepsilon) \sigma_{r,m}^2 \right\}, \quad (109)$$

where $\sigma_{\lambda,m}^2$ and $\sigma_{r,m}^2$ are given in Eq. (100) and Eq. (101), $0 \leq \varepsilon \leq 1$ is a weight factor trading the uncertainty between λ_m and r_m , and Δ_λ^* is the optimal LOS separation angle between W and M which minimizes the weighted uncertainty for a given ε and r/r_w . The physical interpretation of ε can be difficult due to the fact that $\sigma_{\lambda,m}^2$ and $\sigma_{r,m}^2$ might operate at different magnitudes. Therefore, proper scaling shall be introduced in Eq. (109).

By an adequate choice of ε in Eq. (109), an optimal wingman trajectory (parameterized in terms of r/r_w and Δ_λ^*) can be obtained, which reflects the estimation accuracy needs for r_m and/or λ_m . These accuracy needs shall be driven by the employed missile guidance strategy and its implementation requirements. The resulting trajectory should then serve as a baseline for the wingman guidance law derivation.

Remark 10. *More sophisticated wingman guidance algorithms might require the availability of the W - T state vector $\mathbf{x}_w$, defined in a similar manner as $\mathbf{x}_m$, i.e.,*

$$\mathbf{x}_w \triangleq \begin{bmatrix} r_w & \lambda_w & \gamma_t & a_t & V_t \end{bmatrix}^T. \quad (110)$$

An estimate of $\mathbf{x}_w$ can be easily obtained either by running a separate estimator for the W - T engagement, or by using $\hat{\mathbf{x}}_{m;k|k}$ and the following relations

$$\hat{r}_{w;k} = g_w(\hat{\mathbf{x}}_{m;k|k}, \phi_k), \quad \hat{\lambda}_{w;k} = h_w(\hat{\mathbf{x}}_{m;k|k}, \phi_k). \quad (111)$$

Estimates on γ_t , a_t , and V_t are contained in $\hat{\mathbf{x}}_{m;k|k}$, see Eq. (68).

E. Simulation Results

In this section, numerical simulations are introduced to demonstrate the closed-loop (estimator in the guidance loop) performance of the proposed wingman-based sensorless missile guidance concept. The effect of different wingman guidance strategies and measurement accuracies on the missile homing performance are also presented.

1. Engagement Scenario and Parameters

For all simulations, the missile and the wingman are launched simultaneously from the same initial location. The initial horizontal separation of the target from the wingman-missile team is 5 [km] in the positive X_I direction. The missile and the target are flying with the same constant speed $V_m = V_t = 500$ [m/s] and have first-order lateral dynamics with identical time constants $\tau_m = \tau_t = 0.2$ [s]. The wingman is assumed to fly at a lower speed of $V_w = 400$ [m/s] and to have time constant $\tau_w = 0.05$ [s]. The maximal maneuverability of the target, missile, and the wingman is $a_t^{max} = 5$ [g₀], $a_m^{max} = 15$ [g₀], and $a_w^{max} = 30$ [g₀], respectively. Here, $g_0 = 9.80665$ [m/s²] represents the standard acceleration due to gravity.

The parameters of the PID-based LOS guidance law were fine tuned to $K_p = 10^5$, $K_d = 5 \cdot 10^6$, and $K_i = 10$. Both, the estimation and guidance loops run at identical sampling rates of 100 [Hz]. The ECA model parameters are set to $\alpha = 1/5$ and $\Psi = \sqrt{5/3}$. The latter corresponds to $P_{\max} = 1$ and $P_0 = 0$, see Eq. (75). At each run, the filter's initial state is randomly sampled as

$$\hat{\mathbf{x}}_{m;0|0} \sim \mathcal{N}(\mathbf{x}_{m;0}, \mathbf{P}_{0|0})$$

where $\mathbf{x}_{m;0}$ is the true initial state vector defined in Eq. (68) and

$$\mathbf{P}_{0|0} = \text{diag} \left\{ 50^2 \quad (\pi/180)^2 \quad (\pi/180)^2 \quad (5g_0)^2 \quad 25^2 \right\}$$

is the initial estimation error covariance matrix of the filter.

2. Sample Run Simulations

Before turning to a statistical MC evaluation, first four sample run examples are illustrated. In all cases, the wingman (if engaged) acquires bearings-only measurements of the target with $\sigma_{\lambda,w} = 1$ [mrad] accuracy. The missile's navigation constant is $N' = 4$ and the initial flight path angle of the target is $\gamma_{t;0} = 15$ [deg]. Both the missile and the wingman are initially on a perfect collision course with the target, i.e.,

$$\gamma_{j;0} = \sin^{-1}(V_t \sin(\gamma_{t;0} + \lambda_{j;0})/V_j) + \lambda_{j;0}, \quad j \in \{m, w\}.$$

At first, the target applies a constant maneuver turn at $u_t = 5 \text{ [g}_0\text{]}$ and then, one second before the estimated end of the engagement, i.e., when $t_{go} = 1$, a maneuver direction switch occurs to the opposite side, i.e., $u_t = -5 \text{ [g}_0\text{]}$. Different missile-wingman guidance strategies combinations are demonstrated in Figs. 17–20.

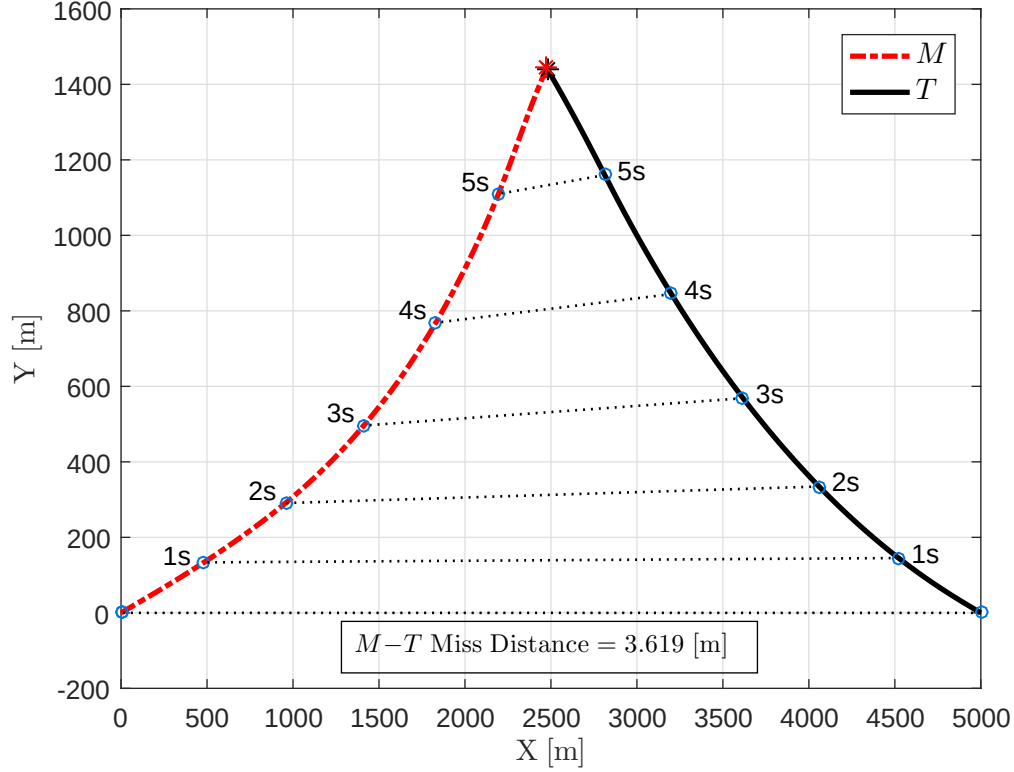


Figure 17: Missile guided using its own-ship LOS angle measurements.

In Fig. 17, a classical one-on-one scenario is considered, i.e., without the wingman being engaged, where the missile acquires own-ship LOS angle measurements and runs an estimator on its own, designed as described in Remark 8. As expected, a relatively small miss distances is achieved in this case. This is because the accuracy of the M – T LOS angle estimate is directly governed by the accuracy of the missile’s own sensory measurements, which in this example were set to $\sigma_{\lambda,m} = 1 \text{ [mrad]}$.

The second example, depicted in Fig. 18, considers a sensorless missile being guided by the wingman. The wingman employs the suggested PID-based LOS guidance law of Section D. Thanks to the wingman’s agility and its measurements accuracy ($\sigma_{\lambda,w} = 1 \text{ [mrad]}$), the LOS separation angle Δ_λ is kept close to zero throughout the entire engagement (see the overlayed dotted lines in Fig. 18). The resulting miss is only slightly larger than in the case of the missile having own-ship measurements with the same accuracy.

Fig. 19 demonstrates a case when the W – M relative geometry is similar to that of Fig. 18, but the separation angle Δ_λ is not kept zero throughout the engagement. Here, the wingman employs a PN guidance law Eq. (69) with $N' = 3$. The state needed for the wingman’s guidance law implementation, $\mathbf{x}_w$, was obtained using $\hat{\mathbf{x}}_m$ and Eq. (111), see the discussion in Remark 10. The obtained M – T miss is inferior to the preceding two cases. This result emphasizes the importance of Δ_λ being zeroed by the wingman, see the upper plot in Fig. 14 for non-zero Δ_λ angles.

The last example considers a scenario where the wingman performs an acceleration maneuver of $5 \text{ [g}_0\text{]}$ to the opposite side of the target’s flight direction. The obtained trajectories are shown in Fig. 20. The results indicate very poor homing performance of the sensorless missile. This might come as a consequence of poor M – T relative state estimate. Notice that the relative W – M geometry does not follow any of the two optimal relative geometries suggested by Fig. 15.

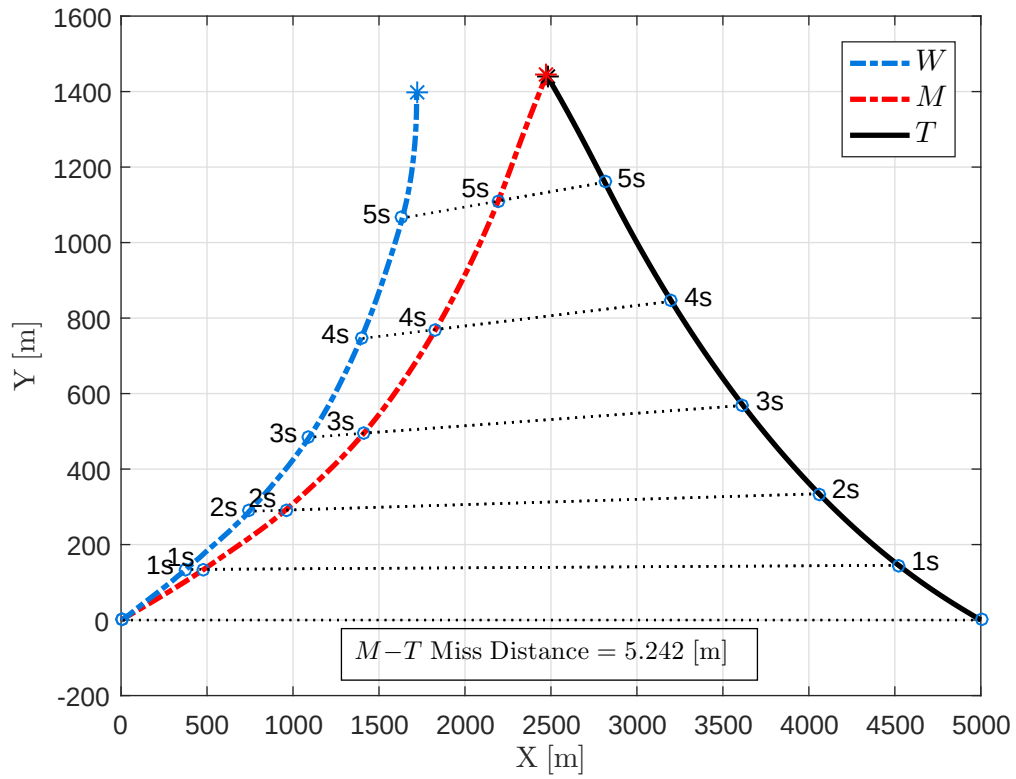


Figure 18: Sensorless missile guided by the Wingman which employs CLOS guidance law.

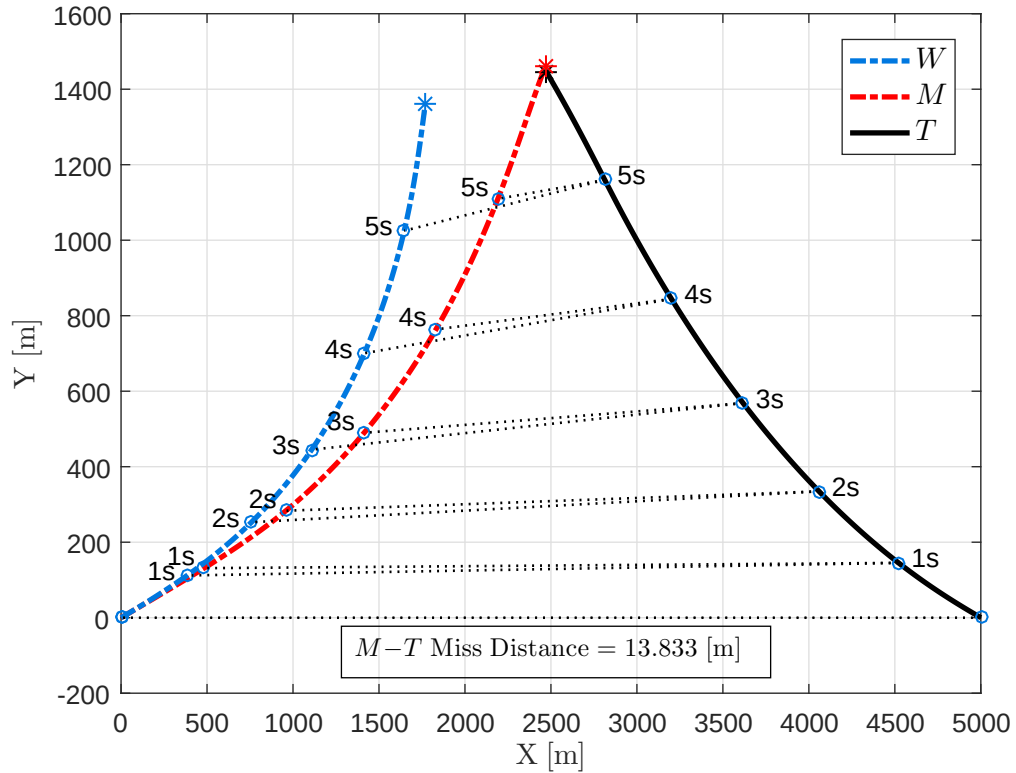


Figure 19: Sensorless missile guided by the Wingman which employs PN guidance law with $N' = 3$.

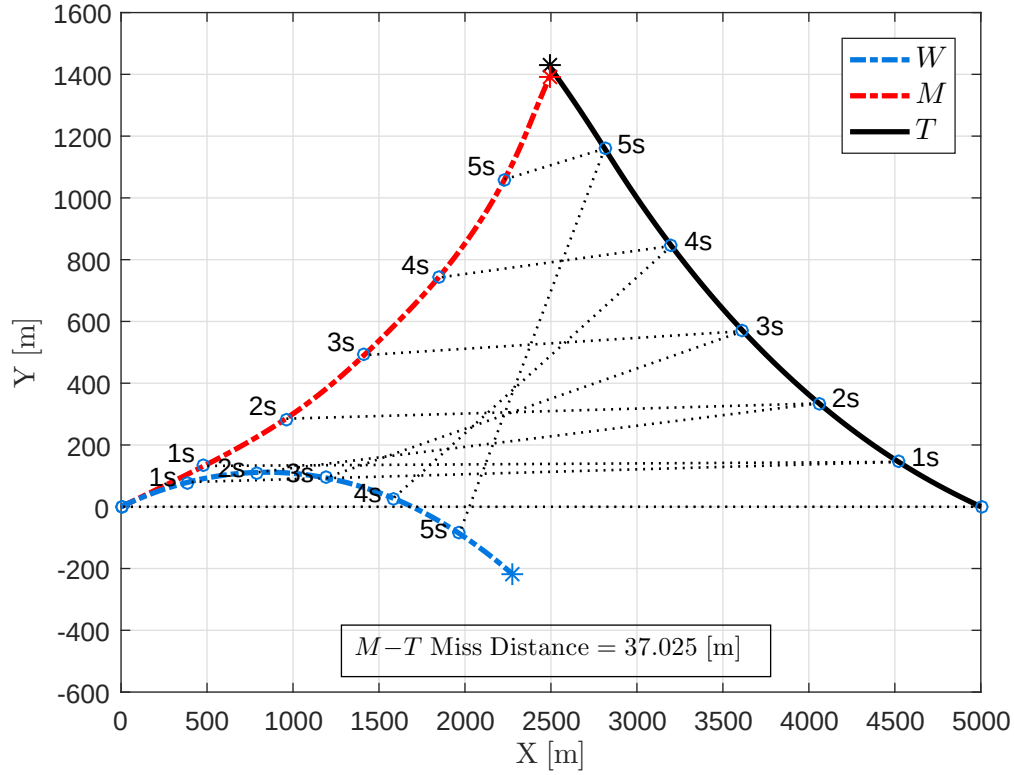


Figure 20: Sensorless missile guided by the Wingman which performs a single direction maneuver at 5 [g₀].

3. Monte Carlo Simulation Results

The four missile–wingman guidance strategy combinations from the previous subsection are further evaluated here using an extensive Monte Carlo campaign. Furthermore, two different levels of noise intensities are investigated, namely $\sigma_\lambda \in \{0.1, 1\}$ [mrad]. A set of 1,000 MC simulations is run for each case. For each run, the target’s initial flight path angle is drawn uniformly from the closed interval $\langle 0, 20 \rangle$ [deg]. The initial flight path angles of the missile and the target have a 2 [deg] heading error from the perfect collision course. These heading errors are uniformly distributed. The target’s 5 [g₀] maneuver direction switch occurs uniformly between zero and two seconds before the end of the engagement. The missile’s navigation gain is selected uniformly from $\{3, 4, 5\}$.

Figure 21 presents the empirical cumulative distribution function (CDF) of the M – T miss distance for each considered case. Table 2 compiles the obtained results in terms of the warhead lethality range ensuring a 95% kill probability (CDFs’ cross point values with the dotted horizontal line in Fig. 21). The obtained MC results reaffirm the sample run results of Figs. 17–20. The results are also in line with the wingman guidance law implications discussed in Sec. D, i.e., the “best” wingman guidance strategy to maximize the sensorless missile homing performance is to employ the suggested PID-based LOS guidance. It is interesting to observe that, in order for the wingman-missile team to achieve similar homing performance as in the traditional one-on-one engagement, the wingman shall have an order of magnitude better LOS measurements.

Sensor accuracy	M only	W – CLOS	W – PN	W – SIDE
$\sigma_\lambda = 10^{-4}$ [rad]	4.71	6.19	20.15	71.78
$\sigma_\lambda = 10^{-3}$ [rad]	6.06	11.33	39.97	68.91

Table 2: Missile’s homing performance in 95% of runs (in meters)

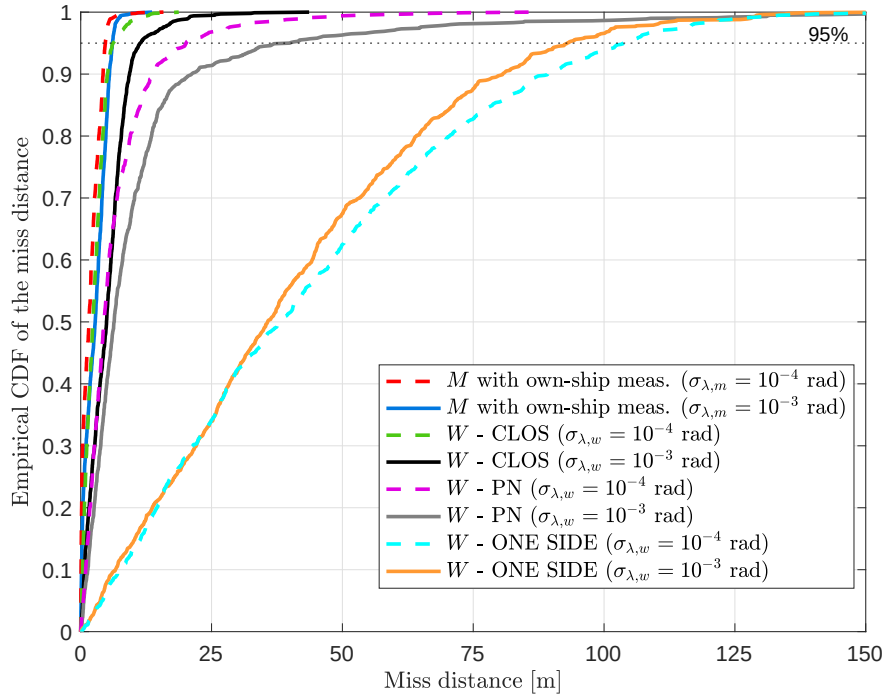


Figure 21: CDF of the missile-target miss distance for each considered case.

F. Results, Discussion, & Conclusions

A novel wingman-based guidance and estimation concept for a sensorless PN-guided homing missile was proposed. This concept considers a wingman vehicle that tracks the target's motion using bearings-only measurements and guides the missile into collision with the maneuvering target. Only the wingman is assumed to be equipped with sensors that allow to track the target motion and the relative position between the missile and the wingman. The missile lateral acceleration command is computed by the wingman. The proposed concept allows to reduce the weight, on-board computational requirements, and the cost of a missile guided towards an aerial target with the help of a wingman vehicle.

Analytical observability analysis of the wingman-based navigation concept suggested that, in order to achieve maximum observability of the missile-target LOS angle, known to be crucial for a PN-guided missile, the wingman shall fly at a predefined trajectory with respect to the missile. This resulting wingman trajectory can be related to the well-known LOS guidance principle with a moving launchpad. Mechanization aspects of such guidance law in the presented missile-wingman-target scenario were also discussed.

Monte Carlo simulation results verified the analytical findings. Additionally, the simulation results revealed that in order to achieve similar homing performance as the conventional own-ship measurement approach, the wingman, when employing the suggested PID-based LOS guidance law, shall have an order of magnitude better bearings-only measurements. Other guidance strategies for the wingman might lead to better range observability due to rotating wingman-target LOS, however, the simulation results revealed that they significantly deteriorate the homing accuracy of the PN-guided missile, which is less sensitive to range errors.

In this work, only the PN missile guidance law was analyzed. Other missile guidance laws may be more sensitive to estimation errors in other kinematic variables, and hence different guidance strategies for the wingman shall be considered. The homing accuracy of the sensorless missile could be further improved by introducing additional sensors for the wingman vehicle, such as radar measuring the wingman-target range. Furthermore, the proposed approach could be extended to a shoot-look-shoot strategy where, based on the kill assessment of the missile, the wingman could be actively engaged in the pursuit.

IV. Task assignment dependent guidance

A. Engagement

Assume a scenario in which one missile M is tracking two potential targets T1 and T2 (see illustration in Fig. 22).

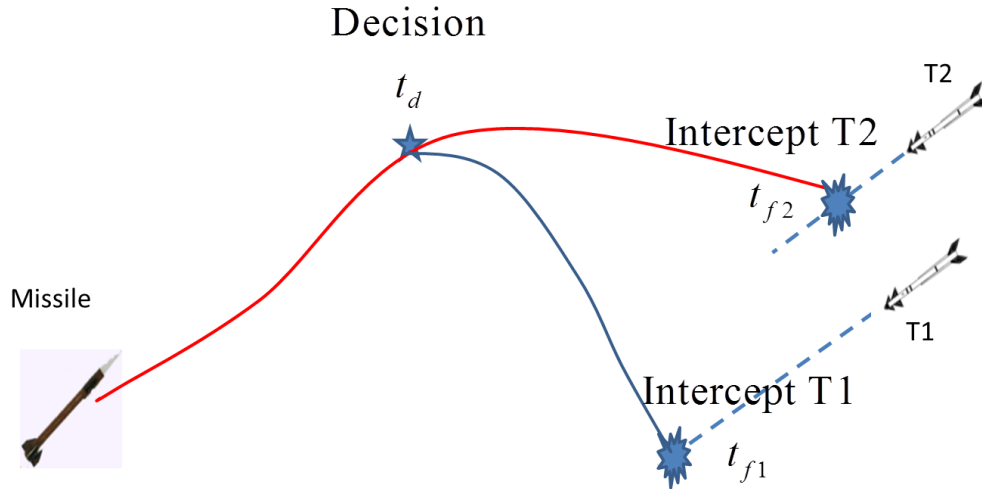


Figure 22: Intercept scenario

The missile will have to intercept one of the targets, but at the current time t_0 which of the targets will have to be intercepted is not known. The decision will be taken at time $t_d > t_0$. If the decision will be to intercept target T1, then the intercept time will be $t_{f1} > t_d$, whereas if the decision will be to intercept target T2, the intercept time will be $t_{f2} > t_d$. We make no assumption here on the relative size of the two times of flight. Although it is not known which decision will be taken at time t_d , it is known that there is a probability p that the missile will have to intercept T1. Correspondingly, there is a probability $1 - p$ that the missile will go after T2.

There are two possible scenarios that would warrant this assumption. The first possibility is that one of the targets, say T2, is already under intercept from a missile M_p launched before and that the intercept of T2 is planned to be completed at time t_d . In the case that T2 is not destroyed by missile M_p , missile M will have to intercept T2 because it is a higher priority target (e.g. $t_{f1} > t_{f2}$) so that the intercept of T1 is left to a trailing missile. The chance that missile M will have to go after T2 is equal to the probability that M_p will not be successful. If p denotes the probability of success of M_p , then there is indeed a probability p that M will go after T1 and a probability $1 - p$ that M will go after T2. Another possible scenario that justifies our assumptions is that one of the targets is actually a decoy and that only at time t_d it will be clear which of them is the actual target. In this case, it is harder to assign a specific value to the probability p , but it represents the probability that T1 is the real target and T2 is the decoy. This probability may be based on target tracking information and, in fact, it may vary as more information is accumulated during the flight of the missile before the decision time t_d .

B. Problem formulation

Let

$$y_j = y_{T_j} - y_M, \quad j = 1, 2, \quad (112)$$

be the relative normal separation between T_j and M. Then, the linearized intercept model is

$$\ddot{y}_j = a_{T_j} - a_M, \quad j = 1, 2, \quad t_0 \leq t \leq t_{f_j}, \quad (113)$$

where $a_M(t)$ is the missile lateral acceleration, a_{T_j} , $j = 1, 2$, are the targets' lateral accelerations which are assumed to be constant and known.

The intercept effort of the missile is measured by the quadratic integral of its lateral acceleration

$$\text{IE} = \int_{t_0}^{t_f} a_M^2(t) dt, \quad (114)$$

where $t_f = t_{f1}$ if the decision at time t_d is to intercept T1 and $t_f = t_{f2}$ is the decision is to intercept T2. If we denote by $a_{M_j}(\cdot)$ the acceleration if the missile is directed to intercept Tj ($j = 1, 2$), then the average of the intercept effort is computed as

$$J = p \int_{t_0}^{t_{f1}} a_{M_1}^2(t) dt + (1 - p) \int_{t_0}^{t_{f2}} a_{M_2}^2(t) dt. \quad (115)$$

This cost function has to be minimized by choosing the accelerations a_{M_j} under the following constraints:

- The miss with respect to the target Tj when using a_{M_j} is zero for $j = 1, 2$:

$$y_j(t_{f_j}) = 0. \quad (116)$$

- The two accelerations coincide before the decision time t_d :

$$a_{M_1}(t) = a_{M_2}(t) = a_M(t), \quad t \in [t_0, t_d]. \quad (117)$$

C. Solution

Define the zero-effort miss distance (ZEM) to each of targets

$$z_j(t) = y_j(t) + \dot{y}_j(t)(t_{f_j} - t) + \frac{1}{2} a_{T_j}(t_{f_j} - t)^2, \quad j = 1, 2. \quad (118)$$

By virtue of (113) and (118), the ZEMs satisfy the differential equations

$$\dot{z}_j = -(t_{f_j} - t) a_{M_j}, \quad j = 1, 2. \quad (119)$$

Moreover,

$$z_j(t_{f_j}) = y_j(t_{f_j}), \quad j = 1, 2. \quad (120)$$

Thus, for $t \in [t_d, t_{f_j}]$ (after the decision moment), the problem is to minimize the intercept efforts

$$J_j = \int_{t_d}^{t_{f_j}} a_{M_j}^2(t) dt \rightarrow \min_{a_{M_j}(\cdot)}, \quad j = 1, 2. \quad (121)$$

subject to the scalar differential equations (119) with boundary conditions

$$z_j(t_{f_j}) = 0, \quad j = 1, 2. \quad (122)$$

Let us denote

$$z_j(t_d) = z_{j_d}, \quad j = 1, 2. \quad (123)$$

Then, by the Pontryagin's Maximum Principle, the optimal accelerations are

$$a_{M_j}^* = c_j z_{j_d}(t_{f_j} - t), \quad j = 1, 2, \quad (124)$$

where

$$c_j \triangleq \frac{3}{(t_{f_j} - t_d)^3}. \quad (125)$$

This yields the optimal costs:

$$J_j^* = \frac{3}{(t_{f_j} - t_d)^3} z_{j_d}^2. \quad (126)$$

The cost function (127) is

$$J_p = \int_{t_0}^{t_d} a_M^2(t) dt + pJ_1^* + (1-p)J_2^*. \quad (127)$$

Due to (126),

$$J_p = pc_1 z_{1d}^2 + (1-p)c_2 z_{2d}^2 + \int_{t_0}^{t_d} a_M^2(t) dt, \quad (128)$$

where

$$z_{jd} = z_{j0} - \int_{t_0}^{t_d} (t_{f_j} - t) a_M(t) dt, \quad j = 1, 2. \quad (129)$$

It is verified directly that

$$z_{jd}^2 = \max_{l_j \in \mathbb{R}} \left[l_j z_{jd} - \frac{1}{4} l_j^2 \right] = \max_{l_j \in \mathbb{R}} \left[l_j \left(z_{j0} - \int_{t_0}^{t_d} (t_{f_j} - t) a_M(t) dt \right) - \frac{1}{4} l_j^2 \right], \quad j = 1, 2. \quad (130)$$

Thus,

$$J_p = \max_{l_1, l_2 \in \mathbb{R}} \varphi(a(\cdot), l_1, l_2), \quad (131)$$

where

$$\begin{aligned} \varphi(a(\cdot), l_1, l_2) &\triangleq pc_1 z_{10} l_1 + (1-p)c_2 z_{20} l_2 - \frac{pc_1}{4} l_1^2 - \frac{(1-p)c_2}{4} l_2^2 + \\ &\int_{t_0}^{t_d} \left[a_M^2(t) - \left(pc_1(t_{f_1} - t)l_1 + (1-p)c_2(t_{f_2} - t)l_2 \right) a_M(t) \right] dt \end{aligned} \quad (132)$$

Lemma

$$\min_{a_M(\cdot)} \max_{l_1, l_2 \in \mathbb{R}} \varphi(a_M(\cdot), l_1, l_2) = \max_{l_1, l_2 \in \mathbb{R}} \min_{a_M(\cdot)} \varphi(a(\cdot), l_1, l_2). \quad (133)$$

The lemma is proved straightforwardly.

Due to this Lemma

$$\min_{a_M(\cdot)} J_p = \max_{l_1, l_2 \in \mathbb{R}} \min_{a_M(\cdot)} \varphi(a(\cdot), l_1, l_2). \quad (134)$$

For any fixed $l_1, l_2 \in \mathbb{R}$, the minimizing function is

$$a_M^*(t) = a_M^*(t; l_1, l_2) = \frac{1}{2} \left(pc_1(t_{f_1} - t)l_1 + (1-p)c_2(t_{f_2} - t)l_2 \right). \quad (135)$$

By substituting (135) into (132),

$$\begin{aligned} \psi(l_1, l_2) &\triangleq \varphi(a^*(\cdot; l_1, l_2), l_1, l_2) = pc_1 z_{10} l_1 + (1-p)c_2 z_{20} l_2 - \frac{pc_1(pc_1 I_1(t_0) + 1)}{4} l_1^2 - \\ &\frac{p(1-p)c_1 c_2 I_2(t_0)}{2} l_1 l_2 - \frac{(1-p)c_2((1-p)I_3(t_0) + 1)}{4} l_2^2, \end{aligned} \quad (136)$$

where

$$I_1(t) \triangleq \int_t^{t_d} (t_{f_1} - t)^2 dt, \quad (137)$$

$$I_2(t) \triangleq \int_t^{t_d} (t_{f_1} - t)(t_{f_2} - t) dt, \quad (138)$$

$$I_3(t) \triangleq \int_t^{t_d} (t_{f_2} - t)^2 dt. \quad (139)$$

The maximizing elements satisfy the equations

$$\frac{\partial \psi}{\partial l_1} = 0, \quad \frac{\partial \psi}{\partial l_2} = 0, \quad (140)$$

which reduces to the linear system w.r.t. to $l = (l_1, l_2)^T$:

$$D(t_0)l = d(z_{10}, z_{20}), \quad (141)$$

where

$$D(t) = \begin{bmatrix} pc_1 I_1(t) + 1 & (1-p)c_2 I_2(t) \\ pc_1 I_2(t) & (1-p)c_2 I_3(t) + 1 \end{bmatrix}, \quad d(z_1, z_2) = \begin{bmatrix} 2z_1 \\ 2z_2 \end{bmatrix}. \quad (142)$$

Note that by the Cauchy-Schwarz inequality,

$$I_2(t)^2 = \left(\int_t^{t_d} (t_{f_1} - t)(t_{f_2} - t) dt \right)^2 \leq \int_t^{t_d} (t_{f_1} - t)^2 dt \cdot \int_t^{t_d} (t_{f_2} - t)^2 dt = I_1(t)I_3(t). \quad (143)$$

Therefore,

$$\det D(t) = pc_1 I_1(t) + (1-p)c_2 I_3(t) + 1 + p(1-p)c_1 c_2 (I_1(t)I_3(t) - I_2(t)^2) > 0, \quad (144)$$

yielding the unique solution

$$l^* = \begin{bmatrix} l_1^* \\ l_2^* \end{bmatrix} = D^{-1}d = \frac{2}{\det D} \begin{bmatrix} ((1-p)c_2 I_3 + 1) z_{10} - (1-p)c_2 I_2 z_{20} \\ -pc_1 I_2 z_{10} + (pc_1 I_1 + 1) z_{20} \end{bmatrix}. \quad (145)$$

By substituting (145) into (135) and by replacing (t_0, z_{10}, z_{20}) with (t, z_1, z_2) , the optimal feedback is obtained as

$$a^*(t, z_1, z_2) = N_1(t)z_1 + N_2(t)z_2, \quad (146)$$

where

$$N_1(t) = N_1(t; t_d, t_{f_1}, t_{f_2}) = \frac{\kappa}{\det D(t)} \left[\left(I_3 + \frac{1}{(1-p)c_2} \right) (t_{f_1} - t) - I_2(t_{f_2} - t) \right], \quad (147)$$

$$N_2(t) = N_2(t; t_d, t_{f_1}, t_{f_2}) = \frac{\kappa}{\det D(t)} \left[-I_2(t_{f_1} - t) + \left(I_1 + \frac{1}{pc_1} \right) (t_{f_2} - t) \right], \quad (148)$$

$$\kappa \triangleq p(1-p)c_1 c_2. \quad (149)$$

Let us define the time-to-go

$$t_{gj} = t_{f_j} - t, \quad j = 1, 2, \quad (150)$$

and the quantities

$$t_{gd_j} = t_{f_j} - t_d, \quad j = 1, 2; \quad t_{gd} = t_d - t_0. \quad (151)$$

Then, by explicit calculating the integrals (137) – (139), the optimal feedback (146) is rewritten as

$$a^*(t_{g_1}, z_1, t_{g_2}, z_2) = N_1 z_1 + N_2 z_2, \quad (152)$$

where

$$N_1 = N_1(t_{g_1}, t_{g_2}, t_{gd_1}, t_{gd_2}, t_{gd}) = \frac{1}{A} (A_3 t_{g_1} - A_2 t_{g_2}) t_{g_1}^2, \quad (153)$$

$$N_2 = N_2(t_{g_1}, t_{g_2}, t_{gd_1}, t_{gd_2}, t_{gd}) = \frac{1}{A} (-A_2 t_{g_1} + A_1 t_{g_2}) t_{g_2}^2, \quad (154)$$

$$A_1 = \frac{t_{g_1}^3 - t_{gd_1}^3}{3} + \frac{1}{3p} t_{gd_1}^3, \quad (155)$$

$$A_2 = \frac{1}{3} t_{gd}^3 + \frac{1}{2} t_{gd} (t_{g_1} t_{gd_2} + t_{g_2} t_{gd_1}), \quad (156)$$

$$A_3 = \frac{t_{g_2}^3 - t_{gd_2}^3}{3} + \frac{1}{3(1-p)} t_{gd_2}^3, \quad (157)$$

$$A = A_1 A_3 - A_2^2. \quad (158)$$

D. Numerical simulation

Numerical simulations were carried out in the case of two diverging targets for the parameters presented in Table I. For these parameters, $t_{f_1} = 4.5$ s, $t_{f_2} = 6.25$ s.

Table 3: Simulation parameters

Parameter	Symbol	Value	Unit
Velocity T1	V_{T_1}	400	m/s
Initial Position T1	$(x_{0_{T_1}}, y_{0_{T_1}})$	(3600, 0)	m
Lateral acceleration T1	a_{T_1}	30	m/s ²
Velocity T2	V_{T_2}	400	m/s
Initial Position T2	$(x_{0_{T_2}}, y_{0_{T_2}})$	(5000, 0)	m
Lateral acceleration T2	a_{T_2}	-30	m/s ²
Velocity M	V_M	400	m/s
Initial Position M	(x_{0_M}, y_{0_M})	(0, 0)	m
Decision time	t_d	3	s

In Fig. 23, the trajectories of the missile and the targets are shown in the cases of the homing to the target T1 or to the target T2 for different values of the probability p . It is seen that for $t \in [t_0, t_d]$, the missile trajectory lies "between" the targets and "closer" to T1 for larger p . For $t \in [t_d, t_f]$ (after the decision), the missile trajectories are shaped differently making the homing on T1 easier for a larger probability p .

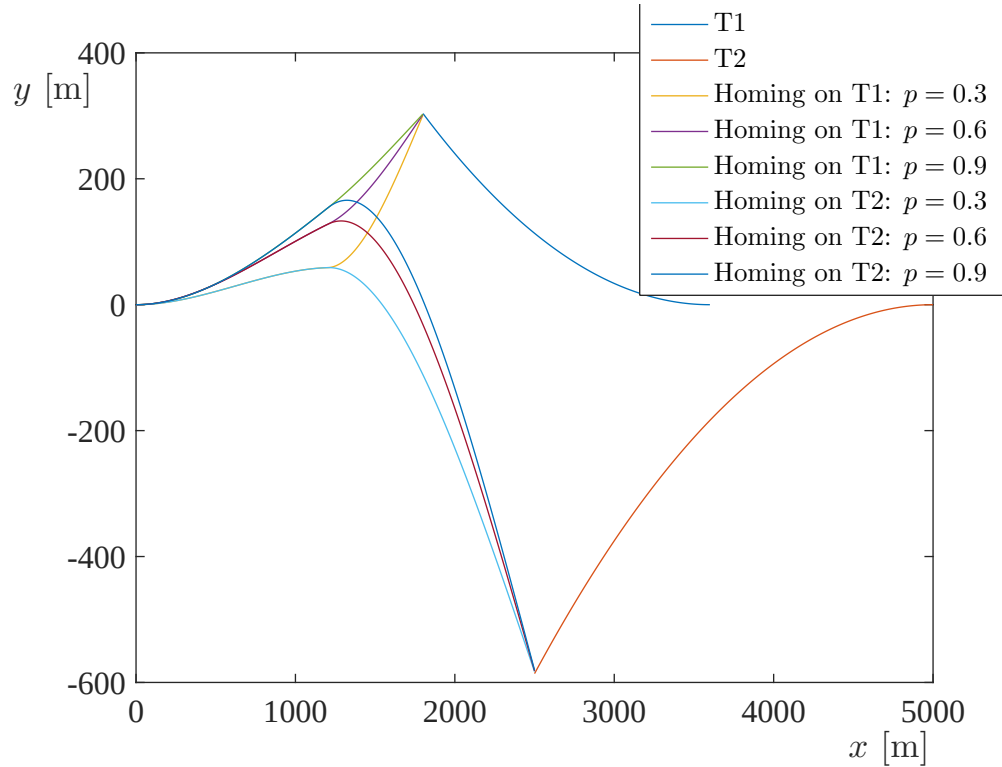


Figure 23: Missile and targets' trajectories

The respective missile acceleration profiles are shown in Fig. 24. It is seen that if the decision was to intercept T1, then for larger values of p the acceleration effort is lower. For the opposite decision, the dependence is opposite.

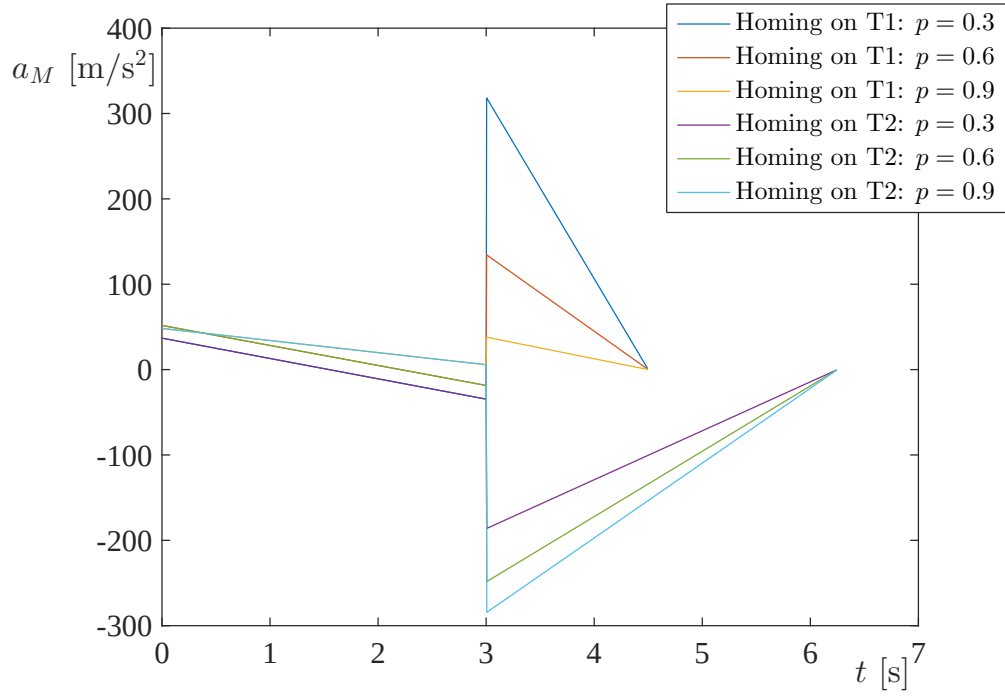


Figure 24: Missile acceleration profiles

E. Results, Discussion, & Conclusions

A minimum effort guidance algorithm was proposed for the case that an interceptor is confronted with two targets and the decision on which of the targets should be engaged is taken only later in the flight, assuming that the probability of each outcome is known. The algorithm is a weighted sum of Augmented Proportional-Navigation type guidance commands, each corresponding to the engagement of one of the targets, so the structure of the algorithm is very simple. However, expression of the weighting coefficients is complex as it depends on the time-to-go to each of the targets and the outcome probabilities of the engagement decision. In this work, we assumed that these probabilities are known prior to the interceptor launch and remain constant until the decision time. In practice, the probabilities may change during the flight as more information becomes available. The consequences of this fact will be explored as part of future work.

References

- [1] Shinar, J. and Steinberg, D., “Analysis of Optimal Evasive Maneuvers Based on a Linearized Two-Dimensional Kinematic Model,” *Journal of Aircraft*, Vol. 14, No. 8, 1977, pp. 795–802.
- [2] Zarchan, P., *Tactical and strategic missile guidance*, American Institute of Aeronautics and Astronautics, Reston, VA, 2012, Chap. 2.
- [3] Garber, V., “Optimum intercept laws for accelerating targets,” *AIAA Journal*, Vol. 6, No. 11, 1968, pp. 2196–2198.
- [4] Cottrell, R. G., “Optimal intercept guidance for short-range tactical missiles,” *AIAA Journal*, Vol. 9, No. 7, 1971, pp. 1414–1415.
- [5] Fonod, R. and Shima, T., “Multiple Model Adaptive Evasion Against a Homing Missile,” *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 7, 2016, pp. 1578–1592.
- [6] Bryson, A. E., “Applied optimal control: optimization, estimation and control,” CRC Press, 1975, pp. 148–176.
- [7] Friedland, B., *Advanced Control System Design*, Prentice-Hall, Saddle River, NJ, 1996, pp. 110–112.
- [8] Tahk, M.-J., Ryoo, C.-K., and Cho, H., “Recursive time-to-go estimation for homing guidance missiles,” *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 38, No. 1, Jan 2002, pp. 13–24.
- [9] Shaferman, V. and Shima, T., “Cooperative Multiple-Model Adaptive Guidance for an Aircraft Defending Missile,” *Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 6, 2010, pp. 1801–1813.
- [10] Magill, D., “Optimal adaptive estimation of sampled stochastic processes,” *IEEE Transactions on Automatic Control*, Vol. 10, No. 4, 1965, pp. 434–439.
- [11] Bar-Shalom, Y. and Li, X.-R., *Estimation with Applications to Tracking and Navigation*, John Wiley & Sons, Inc., New York, 2001, Chap. 11.
- [12] Oshman, Y., Shinar, J., and Weizman, S. A., “Using a Multiple-Model Adaptive Estimator in a Random Evasion Missile/Aircraft Encounter,” *Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 6, 2001, pp. 1176–1186.
- [13] Lainiotis, D. G., “Partitioning: A unifying framework for adaptive systems, II: Control,” *Proceedings of the IEEE*, Vol. 64, No. 8, 1976, pp. 1182–1198.
- [14] Singer, R. A., “Estimating optimal tracking filter performance for manned maneuvering targets,” *IEEE Transactions on Aerospace and Electronic Systems*, , No. 4, 1970, pp. 473–483.
- [15] Bar-Shalom, Y., Li, X. R., and Kirubarajan, T., *Estimation with applications to tracking and navigation: theory algorithms and software*, John Wiley & Sons, Inc., New York, NY, 2001.
- [16] Li, X. R. and Jilkov, V., “Survey of maneuvering target tracking. Part I: Dynamic models,” *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 39, No. 4, 2003, pp. 1333–1364.
- [17] Shinar, J. and Steinberg, D., “Analysis of optimal evasive maneuvers based on a linearized two-dimensional kinematic model,” *AIAA Journal of Aircraft*, Vol. 14, No. 8, 1977, pp. 795–802.
- [18] Zarchan, P., “Representation of Realistic Evasive Maneuvers by the Use of Shaping Filters,” *Journal of Guidance, Control, and Dynamics*, Vol. 2, No. 4, 1979, pp. 290–295.
- [19] Fitzgerald, R. J., “Shaping filters for disturbances with random starting times,” *Journal of Guidance, Control, and Dynamics*, Vol. 2, No. 2, 1979, pp. 152–154.
- [20] Fonod, R. and Shima, T., “Estimation Enhancement by Cooperatively Imposing Relative Intercept Angles,” *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 7, 2017, pp. 1711–1725.

- [21] Rhudy, M., Gu, Y., and Napolitano, M. R., “An Analytical Approach for Comparing Linearization Methods in EKF and UKF,” *International Journal of Advanced Robotic Systems*, Vol. 10, No. 4, 2013, pp. 208.
- [22] Shneydor, N., *Missile Guidance and Pursuit: Kinematics, Dynamics and Control*, Horwood Publishing, Chichester, England, 1998.
- [23] Ratnoo, A. and Shima, T., “Line-of-sight interceptor guidance for defending an aircraft,” *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 2, 2011, pp. 522–532.
- [24] Skogestad, S., “Simple analytic rules for model reduction and PID controller tuning,” *Journal of Process Control*, Vol. 13, No. 4, 2003, pp. 291–309.
- [25] Cottrell, R. G., “Optimal Intercept Guidance for Short-Range Tactical Missiles,” *AIAA journal*, Vol. 9, No. 7, 1971, pp. 1414–1415.
- [26] Jeon, I.-S., Lee, J.-I., and Tahk, M.-J., “Impact-time-control guidance law for anti-ship missiles,” *IEEE Trans. Control Syst. Technol.*, Vol. 14, No. 2, 2006, pp. 260–266.
- [27] Shima, T., “Intercept-Angle Guidance,” *Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 2, 2011, pp. 484–492.